Non-thermal Thick Target Recombination

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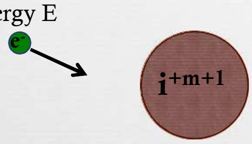
Based on Reep & Brown 2016, ApJ, 824, 90

(Two-body)

Radiative Recombination

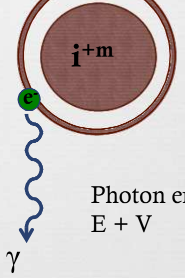
$$X_{i'}^{+m+1} + e^- \rightarrow X_i^{+m} + \Delta E.$$

Kinetic Energy E



Incoming electron recombines with ion, releasing a photon with energy equal to the kinetic energy less the bound energy

Bound level energy (-V)



Photon energy E + V

(Apologies for the Bohr atom)

Non-thermal Recombination

Non-thermal electrons accelerated in a solar flare recombine with ambient ions, resulting in an X-ray emission spectrum. The general form for emission from a bound state with effective charge state Z_{eff} is given by:

$$J_{RZ_{\text{eff}}}(\epsilon) = \int_{E_0}^{\infty} \mathfrak{F}_0(E_0)\ \eta(\epsilon, E_0)\ dE_0$$

where $\mathfrak{F}_0(E_0)$ is the initial electron distribution and $\eta(\epsilon, E_0)$ the photon yield per unit photon energy for an electron.

Equations

Substituting for the photon yield, and reversing the integral, we rewrite this as

$$\begin{aligned} J_{RZ_{\text{eff}}}(\epsilon) &= \int dE_0\ \mathfrak{F}_0(E_0) \int dE\ \frac{E}{2\pi e^4 \Lambda} \frac{dQ_R}{d\epsilon} \\ &= \int dE\ \frac{E}{2\pi e^4 \Lambda} \frac{dQ_R}{d\epsilon} \int dE_0\ \mathfrak{F}_0(E_0) \end{aligned}$$

The integral of many electron distributions is analytic, while the photon yield requires numerical evaluation for all but the simplest cross-section.

Equations

In the simple case of a sharp cut-off and Kramers cross-section, we find

$$J_{RZ_{\rm eff}}(\epsilon) = \frac{16 r_e^2 \chi^2 Z_{\rm eff}^4}{3\sqrt{3}\alpha n^3 e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\rm eff}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\rm eff}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\rm eff}}}{n^2} \end{cases}$$

where the limits account for the collisional loss in energy in the thick target and the fact that photons must have energy greater than the ionization state of the bound state.

(Full derivation in Reep & Brown 2016)

Equations

We then sum over all bound states and all ions:

$$J_R(\epsilon) = \frac{16 r_e^2 \chi^2}{3\sqrt{3}\alpha e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \sum_{Z_{\rm eff}} \sum_{n \geq n_{min}} p_n \zeta_{RZ_{\rm eff}} \frac{1}{n^3} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\rm eff}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\rm eff}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\rm eff}}}{n^2} \end{cases}$$

Although the cross-section is not suitable for general use, we can use this form to estimate the relative importance of NTR to the total non-thermal spectrum in flares.

(Note this form corrects an algebraic mistake in Brown & Mallik 2008.)

Remarks

$$J_R(\epsilon) = \frac{16 r_e^2 \chi^2}{3\sqrt{3}\alpha e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \sum_{Z_{\text{eff}}} \sum_{n \geq n_{min}} p_n \zeta_{RZ_{\text{eff}}} \frac{1}{n^3} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\text{eff}}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\text{eff}}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\text{eff}}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\text{eff}}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\text{eff}}}}{n^2} \end{cases}$$

1. Linearly proportional to electron rate, as with bremsstrahlung. More electrons, more recombination.

Remarks

$$J_R(\epsilon) = \frac{16 r_e^2 \chi^2}{3\sqrt{3}\alpha e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \sum_{Z_{\text{eff}}} \sum_{n \geq n_{min}} p_n \zeta_{RZ_{\text{eff}}} \frac{1}{n^3} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\text{eff}}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\text{eff}}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\text{eff}}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\text{eff}}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\text{eff}}}}{n^2} \end{cases}$$

2. Strongly weighted by states with smaller principal quantum number n. Higher states mostly negligible.

Remarks

$$J_R(\epsilon) = \frac{16 r_e^2 \chi^2}{3\sqrt{3}\alpha e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \sum_{Z_{\rm eff}} \sum_{n \geq n_{min}} p_n \zeta_{R Z_{\rm eff}} \frac{1}{n^3} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\rm eff}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\rm eff}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\rm eff}}}{n^2} \end{cases}$$

3. There exist 'plateaus' in the emission with width equal to the initial cut-off energy. Diagnostic tool?

Remarks

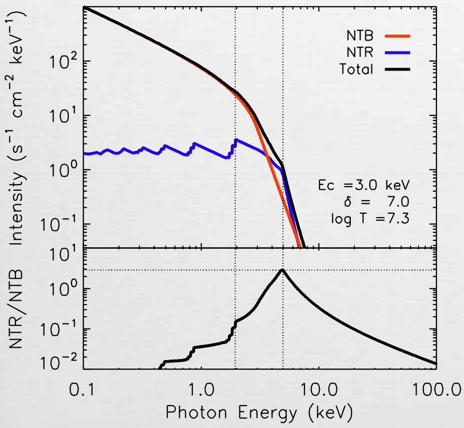
$$J_R(\epsilon) = \frac{16 r_e^2 \chi^2}{3\sqrt{3}\alpha e^4 \Lambda} \frac{\mathfrak{F}_{0c}}{\epsilon} \sum_{Z_{\rm eff}} \sum_{n \geq n_{min}} p_n \zeta_{R Z_{\rm eff}} \frac{1}{n^3} \times \begin{cases} \left[\frac{\epsilon - V_{Z_{\rm eff}}/n^2}{E_{0c}}\right]^{1-\delta} & \text{if } \epsilon \geq E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 1 & \text{if } \frac{V_{Z_{\rm eff}}}{n^2} \leq \epsilon < E_{0c} + \frac{V_{Z_{\rm eff}}}{n^2} \\ 0 & \text{if } \epsilon < \frac{V_{Z_{\rm eff}}}{n^2} \end{cases}$$

4. The derivatives are discontinuous! This implies substantial errors in inversion of photon spectra if NTR is not accounted for (cf. Brown & Mallik 2008)

Caveats

1. **The abundances matter!** NTR depends linearly on an ion's abundance relative to H, so photospheric vs. coronal abundance strongly alters the resultant spectrum. (The plots in this talk all use coronal abundance)
2. **Non-equilibrium ionization important!** NTR depends on ionization fractions as well, so a proper treatment needs to account for the possibility that the ionization states are not described by the temperature. (This talk assumes equilibrium ionization)
3. **Many ions contribute to the emission!** NTR must be calculated with contributions from many ions and ionization states. (This talk assumes contributions from Fe XXI – Fe XXV only)

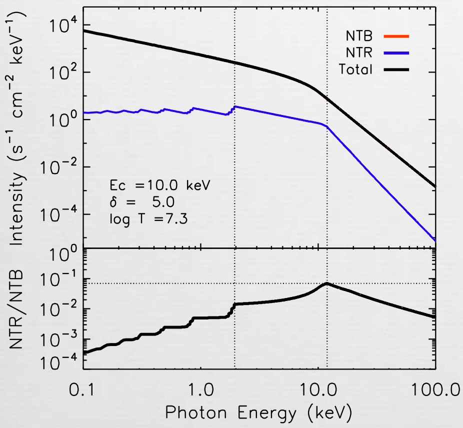
Example spectrum



At small cut-offs and/or large spectral indices, NTR can exceed NTB

→ Important consideration for microflares

Example spectrum 2



For larger cut-offs and smaller spectral indices, NTR is small compared to NTB at all photon energies

→ In very large flares, it is negligible compared to NTB

Inversion depends on the derivatives of the spectrum (Brown 1971) so NTR still should be considered!

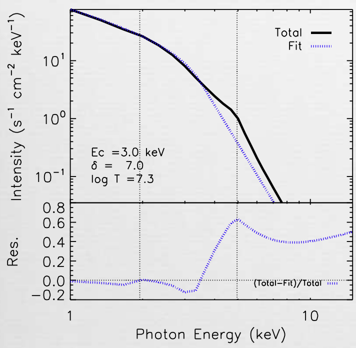
Simple scaling law

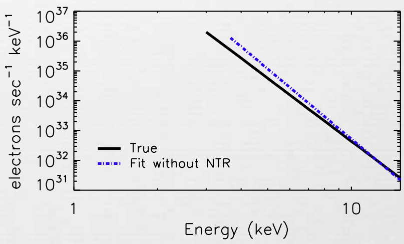
Relative importance of NTR compared to NTB empirically found to approximately follow:

$$\max(\mathrm{NTR}/\mathrm{NTB}) > 0.5 \text{ if } \delta \gtrsim 6\left(\frac{E_{0c}}{4\,\mathrm{keV}}\right)^{0.4}$$
$$\max(\mathrm{NTR}/\mathrm{NTB}) > 0.1 \text{ if } \delta \gtrsim 4\left(\frac{E_{0c}}{4\,\mathrm{keV}}\right)^{0.4}$$

This can be used to quickly estimate whether to include NTR in spectral fitting

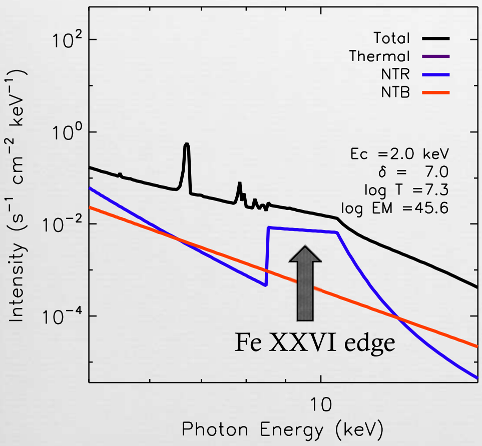
Fitted spectrum





Fitted electron rate, cut-off, and spectral index over-estimated!

Observable?



Two major constraints:

1. **Thermal emissions.** Need a small EM for NTR to not be totally masked.
2. **Spectral resolution.** Need a detector that can resolve recombination edges in the data. RHESSI likely insufficient!

Conclusions

I. NTR contributes to the non-thermal spectrum. In microflares, we expect it to be comparable to or stronger than NTB.

II. Spectral inversion depends on the derivatives of the photon spectrum. NTR causes discontinuities, which causes errors in derived parameters. (Brown & Mallik 2008)

III. Many factors affect the relative strength of NTR: abundance, ionization fractions, electron rate, cut-off, and spectral index

IV. The edge widths may provide diagnostic information about the low energy cut-off, but are difficult to observe

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