Solar Flare Energetics and collisional Relaxation of Electrons in a Warm Plasma

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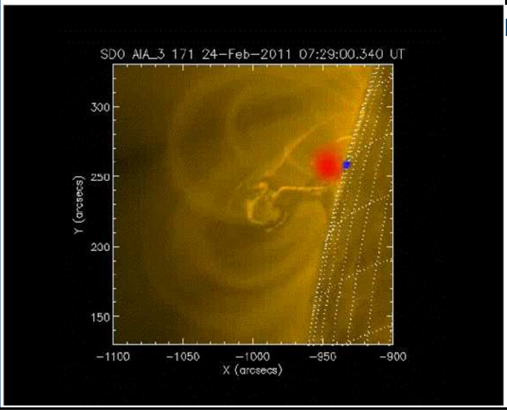
X-rays and flare accelerated electrons

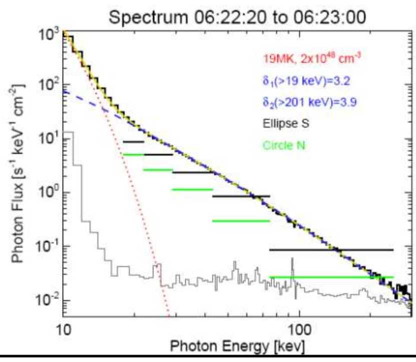
Observed X-rays — Unknown electron distribution — Emission cross-sections

$$I(\epsilon,\mathbf{\Omega},t)=\int_{\ell}\int_{\Omega'}\int_{\epsilon}^{\infty} n(\mathbf{r})\bar{F}(E,\mathbf{\Omega}',\mathbf{r},t)Q(\mathbf{\Omega},\mathbf{\Omega}',\epsilon,E)\,\mathrm{d}E\,\mathrm{d}\mathbf{\Omega}'\,\mathrm{d}\ell,$$

Thin-target case: For the electron spectrum **$F(E) \sim E^{-\delta}$** ,

bremsstrahlung (free-free, free-bound)





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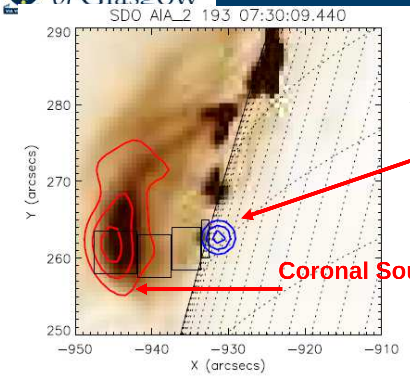
Motivation1:X-ray emission from typical flares

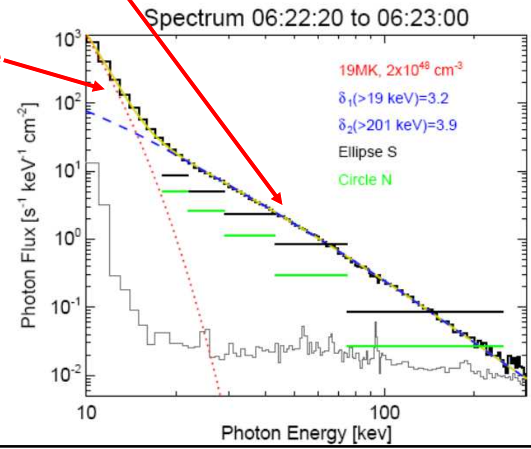


Battaglia & Kontar 2012

Soft X-ray coronal source
HXR chromospheric footpoints

Footpoints

Coronal Source



Flaring region
T ~ 1-3x10^7 K
=> 1-3 keV per particle

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How do we determine electron energetics?

can we determine the acceleration rate and hence the power of non-thermal electrons in solar flares using standard thick-target model, e.g. f_thick in ospex?

only the lower limit

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How do we estimate electron energetics?

Assuming isotropic electron distribution:

$$I(\epsilon) = \frac{1}{4\pi R^2} \int_{\epsilon}^{\infty} \sigma(\epsilon, E) \langle nVF \rangle (E) dE$$

Photon flux spectrum

Mean electron flux spectrum

Normally collisional thick-target is used to estimate the mean electron flux spectrum:

$$\langle nVF \rangle (E) = \frac{E}{2K} \int_{E}^{\infty} A F_0(E_0) dE_0 \ .$$

Brown, 1971,
Brown et al 2003

Injected or accelerated electron spectrum

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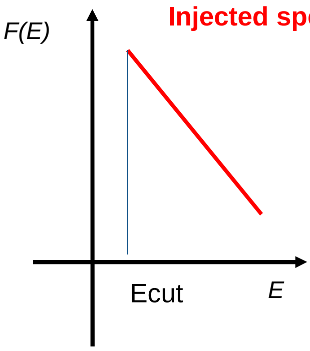
Motivation 2:

Injected spectrum:



$F(E) \sim E^{-\delta}$

Using spectroscopy (or imaging spectroscopy) we normally infer electron power or/and total rate above some energy or lower limit.
We do not know the upper limit.

Can we better determine the lower energy cut-off and upper limits on power and injection rate?

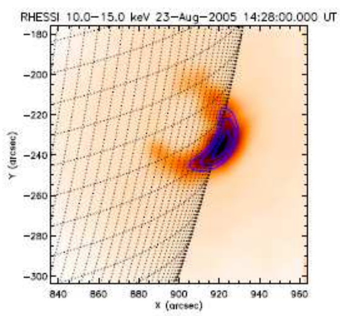
Now we can determine the upper limit for electron energetics

The model involves hot-corona and cold chromosphere

f_thick_warm in ospex

Warm-corona and cold chromosphere model

The model in
1. pictures
2. simulations
3. equations



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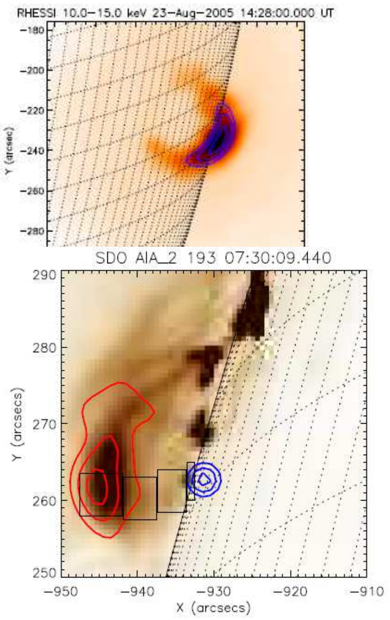
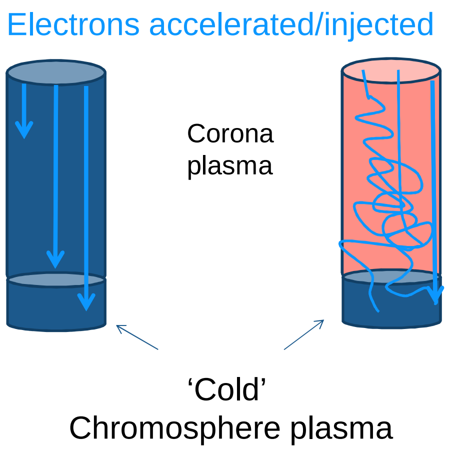
The model in pictures



'Cold'
Plasma Model

Our 'Warm-cold'
Plasma Model



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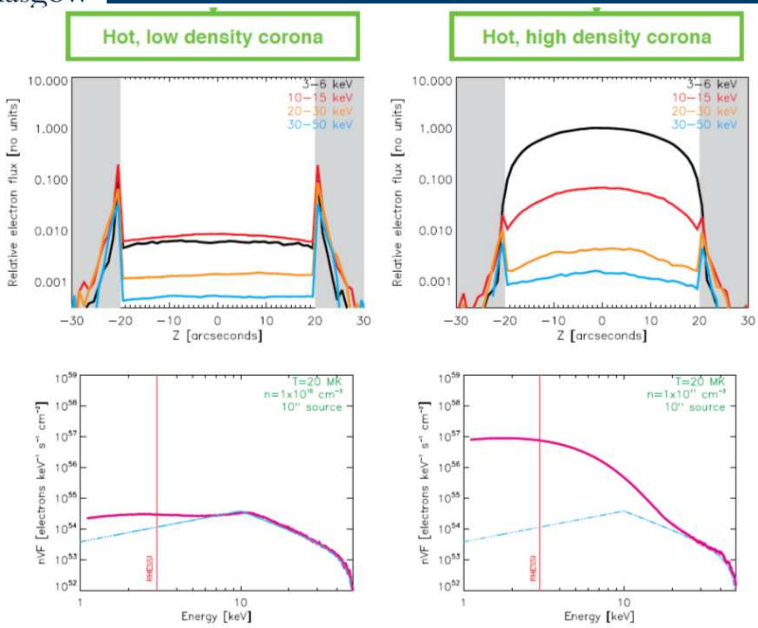
The model from simulations



See Jeffrey et al 2015

Warm plasma collisional transport - Equations

To describe warm plasma environment we can use Fokker-Planck equation:

Collisional drag · Collisional diffusion

$$\mu\frac{\partial F}{\partial z} = 2Kn\left\{\frac{\partial}{\partial E}\left[G\left(\sqrt{\frac{E}{k_BT}}\right)\frac{\partial F}{\partial E}+\frac{1}{E}\left(\frac{E}{k_BT}-1\right)G\left(\sqrt{\frac{E}{k_BT}}\right)F\right]+\right.$$
$$\left.+\frac{1}{8E^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\left(\mathrm{erf}\left(\sqrt{\frac{E}{k_BT}}\right)-G\left(\sqrt{\frac{E}{k_BT}}\right)\right)\frac{\partial F}{\partial\mu}\right]\right\}+F_0(E)\delta(z)\ .$$

Collisional scattering of electrons · Source of particles (injected spectrum)

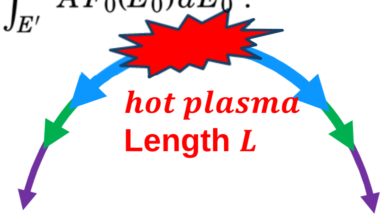
Finite temperature effects: e.g. Emslie, 2003,Galloway et al 2005,Jeffrey et al, 2014

The Model in equations

Integrating (twice) the kinetic equation one finds:

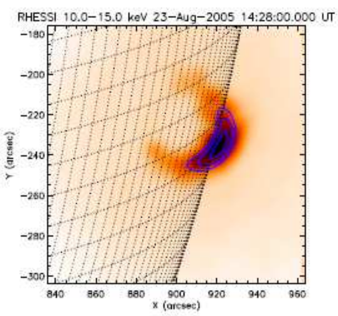
$$\langle nVF\rangle(E)=\frac{1}{2K}E\,e^{-E/kT}\int_{E_{\min}}^{E}\frac{e^{E'/kT}\,dE'}{E'\,G\left(\sqrt{\frac{E'}{kT}}\right)}\int_{E'}^{\infty}A\,F_0(E_0)\,dE_0\ .$$

To find Emin we consider warm plasma loop and cold chromosphere.



In a stationary state the number of electrons in the target is balanced between injection and diffusive escape of thermalized electrons:

$$\frac{3\sqrt{\pi}}{2K}\sqrt{\frac{kT}{E_{min}}}\dot{N}=\sqrt{\frac{8}{\pi m_e}}\frac{nN}{(kT)^{3/2}}$$



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Mean electron flux

Integrating, one obtains the mean electron flux

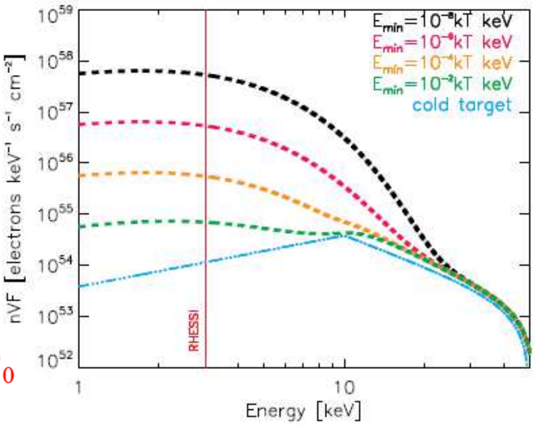
$$\langle nVF\rangle(E) = \frac{1}{2K} E\, e^{-E/kT} \int_{E_{\min}}^{E} \frac{e^{E'/kT}\, dE'}{E' G\left(\sqrt{\frac{E'}{kT}}\right)} \int_{E'}^{\infty} A F_0(E_0) dE_0 \,,$$

$$\frac{E_{\min}}{kT} \simeq 3 \left(\frac{5\lambda}{L}\right)^4 ,$$

$$\lambda = (kT)^2/2Kn$$

c.f. cold target result:

$$\langle nVF\rangle(E) = \frac{1}{2K} E \int_{E}^{\infty} A F_0(E_0) dE_0$$



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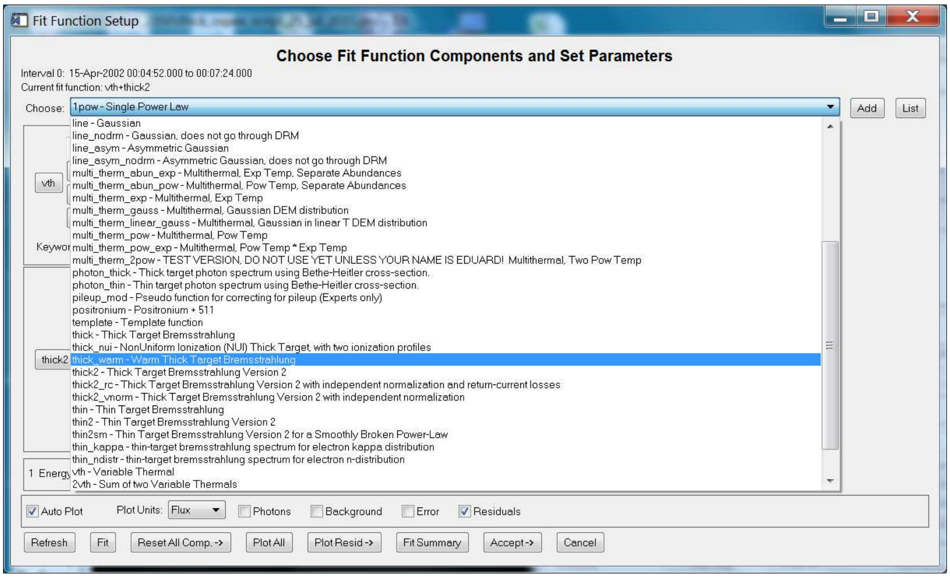
Model in ospex



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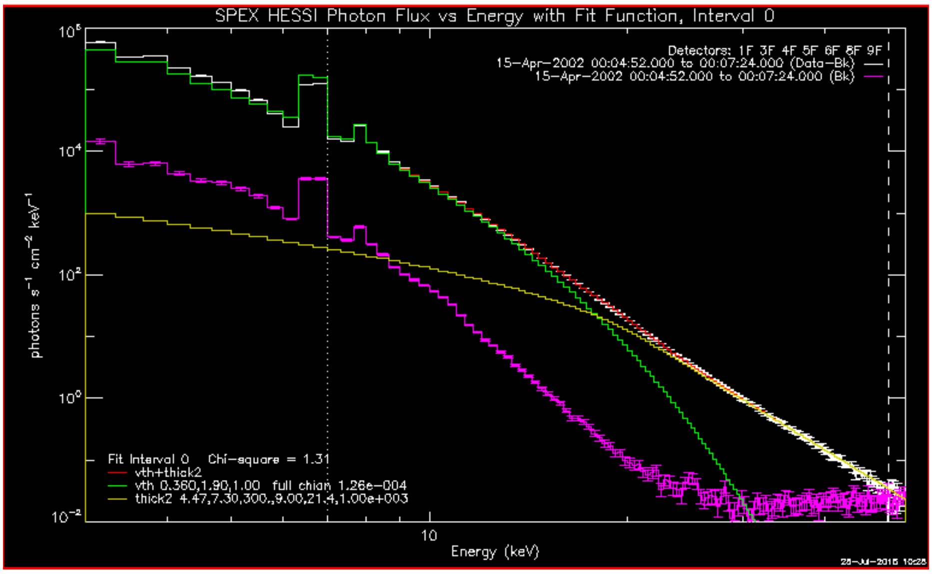
Cold thick target 1



E_cut =21.4 keV

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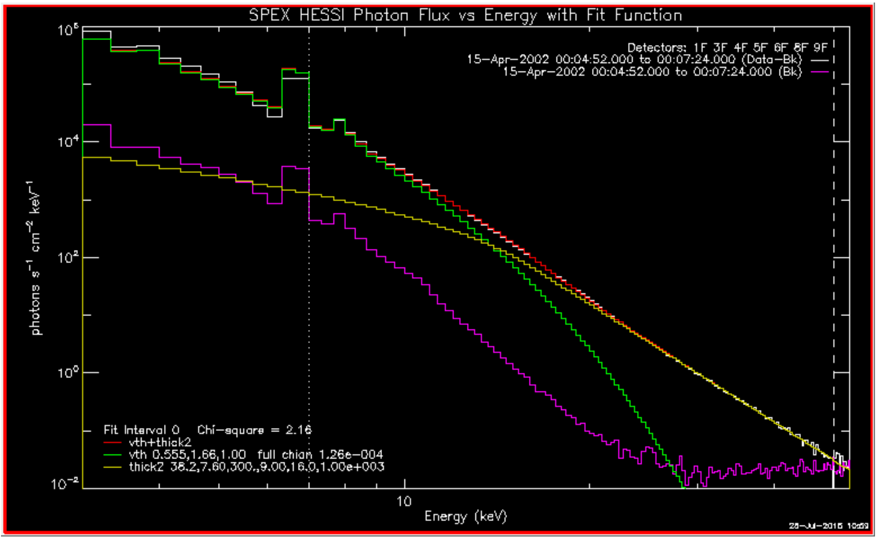
Cold thick target 2



E_cut =16.4 keV

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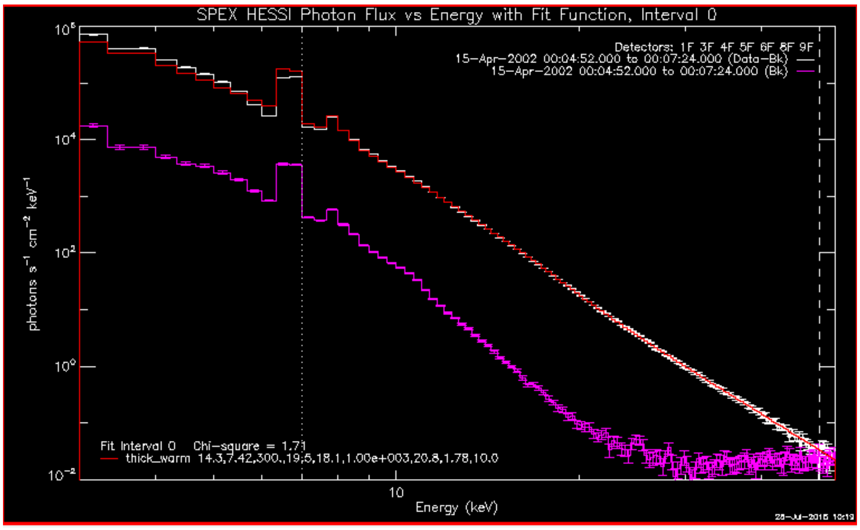
Warm thick target



E_cut =19.5 keV

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Recent additions

- to test how element abundance affects the fit, (Brian’s wish)
- relative abundance of elements default = 1 relative to CHIANTI coronal values.
- the same implementation as in f_vth.pro (thanks Kim)

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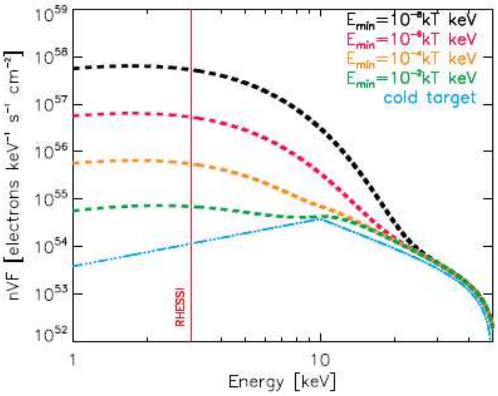
Conclusions



Warm target effects play important role for

1) Dense warm plasma
2) Steep injected spectra (e.g. microflares, or loops)
3) Low energies ~kT

$$L_{Loop} \lesssim E_0^2/(2Kn)$$

Warm target model gives (at least) minimum low energy cut-off and provides the upper limit on the total number or injected electrons or the total injected power.

In the example, we determined the low-energy cut-off +/-1 keV!

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Extra slides….

Target averaged spectrum

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Averaging over pitch-angle and integrating over the emitting volume (Kontar et al, 2014) gives the relationship between the source-integrated electron spectrum and the (pitch-angle averaged) injected electron spectrum:

Warm plasma environment (this model):

$$F_0(E_0) = -\frac{2K}{A}\frac{d}{dE}\left[G\left(\sqrt{\frac{E}{kT}}\right)\left\{\frac{d\langle nVF\rangle(E)}{dE} + \frac{1}{E}\left(\frac{E}{kT}-1\right)\langle nVF\rangle(E)\right\}\right]_{E=E_0} .$$

Warm target drag model (Emslie 2003):

$$F_0(E_0) = -\frac{2K}{A}\frac{d}{dE}\left[\frac{1}{E}\left(\frac{E}{kT}-1\right)G\left(\sqrt{\frac{E}{kT}}\right)\langle nVF\rangle(E)\right]_{E=E_0} .$$

Cold target model (Brown 1971, Brown et al 2003):

$$F_0(E_0) = -\frac{K}{A}\frac{d}{dE}\left[\frac{\langle nVF\rangle(E)}{E}\right]_{E=E_0}$$

How important is the diffusion term?

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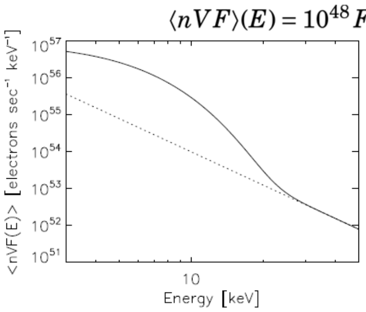
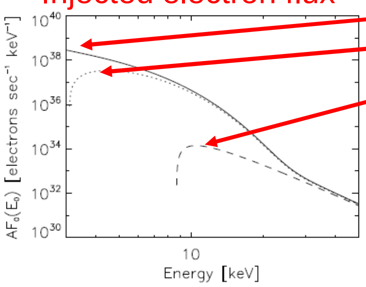
$\langle nVF\rangle(E) = 10^{48}F_M(E) + 10^{54}(E/10\ \mathrm{keV})^{-3}$ (electrons s^{-1} keV^{-1})



<=Mean electron flux

Injected electron flux



Cold target

Warm drag

Warm target model

Note appearance of cut-off:

$$E_{\mathrm{c,eff}} \simeq (\zeta + 2)kT$$

Warm plasma and collisional relaxation

To make a model useful for data analysis, we want to integrate the kinetic equation to find the mean electron flux *<nVF(E)>:*

$$F_0(E_0) = -\frac{2K}{A}\frac{d}{dE}\left[G\left(\sqrt{\frac{E}{kT}}\right)\left\{\frac{d\langle nVF\rangle(E)}{dE} + \frac{1}{E}\left(\frac{E}{kT}-1\right)\langle nVF\rangle(E)\right\}\right]_{E=E_0} .$$

Similar to thick –target model *:*

$$\langle nVF\rangle(E) = \frac{E}{2K}\int_E^\infty A\,F_0(E_0)\,dE_0 \ .$$

We can formally integrate the kinetic equation*:*

$$\langle nVF\rangle(E) = \frac{1}{2K}E\,e^{-E/kT}\int_0^E \frac{e^{E'/kT}\,dE'}{E'G\left(\sqrt{\frac{E'}{kT}}\right)}\int_{E'}^\infty A\,F_0(E_0)\,dE_0 \ ,$$

Warm plasma and collisional relaxation

However, the mean electron flux *<nVF(E)>* is divergent:

$$\langle nVF\rangle(E) = \frac{1}{2K}E\,e^{-E/kT}\int_0^E \frac{e^{E'/kT}\,dE'}{E'G\left(\sqrt{\frac{E'}{kT}}\right)}\int_{E'}^\infty A\,F_0(E_0)\,dE_0 \ ,$$

Unlike standard thick-target model we have the collisional operator conserving the number of particles and injection of electrons, hence infinite number of electrons or infinite mean electron flux.

$$F_0(E_0) = -\frac{2K}{A}\frac{d}{dE}\left[G\left(\sqrt{\frac{E}{kT}}\right)\left\{\frac{d\langle nVF\rangle(E)}{dE} + \frac{1}{E}\left(\frac{E}{kT}-1\right)\langle nVF\rangle(E)\right\}\right]_{E=E_0}$$

Source **Collisional operator**

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Warm plasma and collisional relaxation

However, the mean electron flux <nVF(E)> is infinite:

$$\langle nVF\rangle(E) = \frac{1}{2K} E\, e^{-E/kT} \int_0^E \frac{e^{E'/kT}\, dE'}{E' G\left(\sqrt{\frac{E'}{kT}}\right)} \int_{E'}^{\infty} A F_0(E_0)\, dE_0 \ ,$$

Unlike standard thick-target model we have the collisional operator conserving the number of particles and injection of electrons, hence infinite number of electrons or infinite mean electron flux.

$$F_0(E_0) = -\frac{2K}{A} \frac{d}{dE} \left[G\left(\sqrt{\frac{E}{kT}}\right) \left\{ \frac{d\langle nVF\rangle(E)}{dE} + \frac{1}{E}\left(\frac{E}{kT} - 1\right) \langle nVF\rangle(E) \right\} \right]_{E=E_0}$$

Source **Collisional operator**

=>Warm (finite temperature with diffusion) thick target model does not exist

See Jeffrey et al 2015