The determination of high energy particle spectra via gamma ray regularized inversion

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15 RHESSI meeting Graz

Joint work with:

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A γ-ray spectrum

Share, Murphy, APS Conference Series

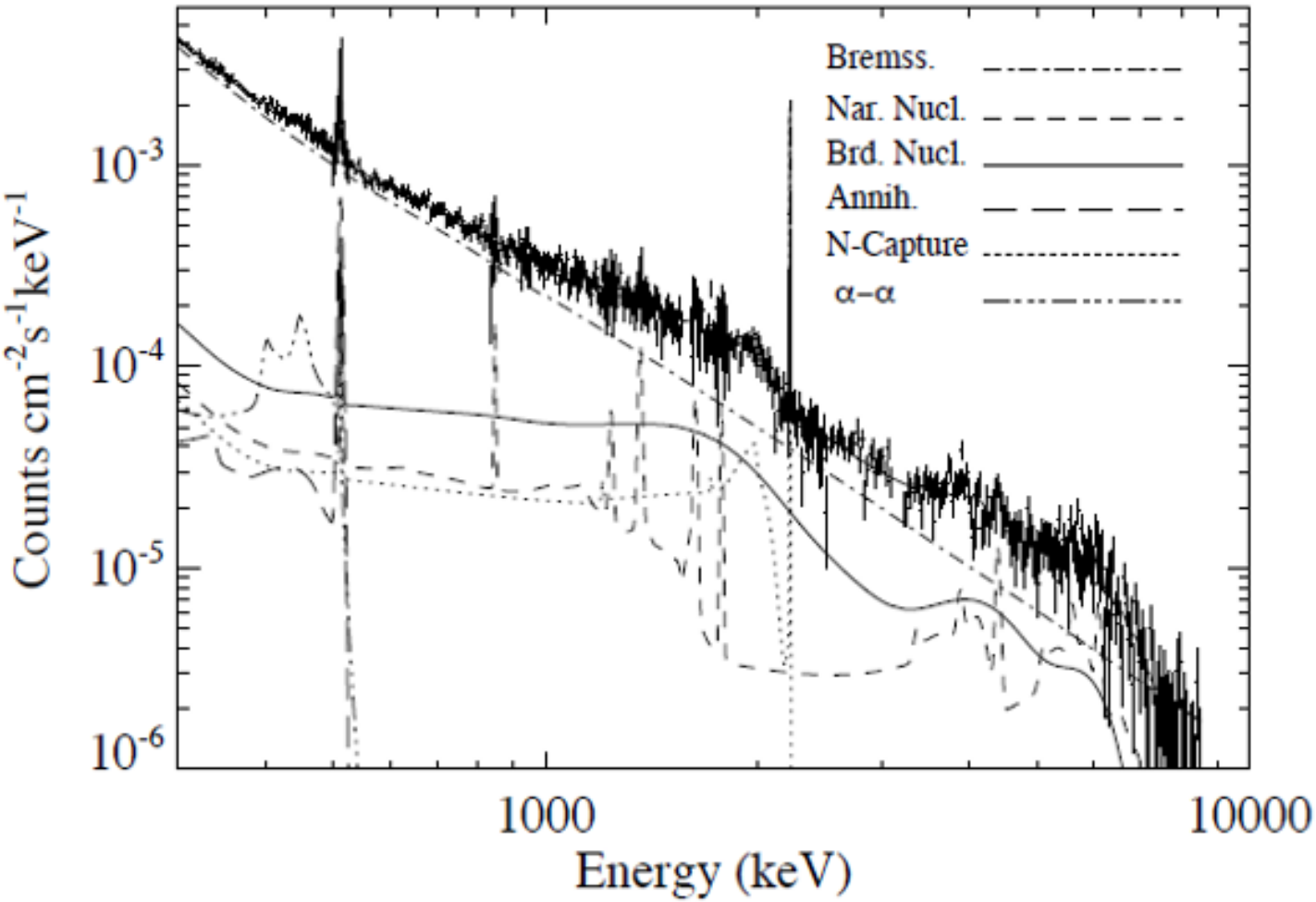


Figure: Count spectrum of the 2002 July 23 solar flare with fits revealing different components.

Ingredients for γ-ray solar flare spectra

Energy range : $100 KeV - 8 MeV$

Many different interactions and physical processes contributing to X-ray and γ-ray solar flare spectra.

- **electrons**

  X-ray and γ-ray Bremsstrahlung

- **ions**

  γ-ray line radiation (1-8 MeV) (narrow and broad lines)
  $\alpha - \alpha$-particle interaction
  neutron capture line (2.223 MeV)s

Ions

Line radiation model

- L: number of ambient species
- K: number of accelerated species

$$I_{line}(\epsilon)\,\Delta\epsilon = \sum_{k=1}^{K}\sum_{\ell=1}^{L} a_\ell \int_0^\infty j_{k,\ell}(\epsilon, E)\, F_k(E)\, dE. \tag{1}$$

- $\{F_k(E)\}_{k=1}^{K}$ Accelerated ion spectra
- $j_{k,l}(\epsilon, E)$ Interaction integral kernel
- $I(\epsilon)\Delta\epsilon$ emergent γ-line spectrum
- a_ℓ Ambient abundances

Nuclear deexcitation gamma-ray lines from accelerated particle interactions,

ApJS, Kozlovsky Murphy Ramaty 2002

Interaction kernel

What does the interaction integral kernel contain ?

- 24 spectra for the narrow lines
- - accelerated H, 4He and 3He interacting with the eight ambient elements 4He, 24 12C, 14N, 16O, 20Ne, 24Mg, 28Si, 56Fe
- 14 spectra for the broad lines
- - accelerated 12C, 14N, 16O, 20Ne, 24Mg, 28Si, 56Fe interacting with ambient H and 4He

Ions

Discretized framework

- $x_p \sim (F_1(E), \ldots, F_K(E))$
- $H_{i,\ell,p} \sim j_{k,\ell}(\epsilon, E) \qquad (E,k) \to p \qquad \epsilon \to i$

Let us consider the problem to find a, x and g once y is given and described by the equation:

Inverse Problem

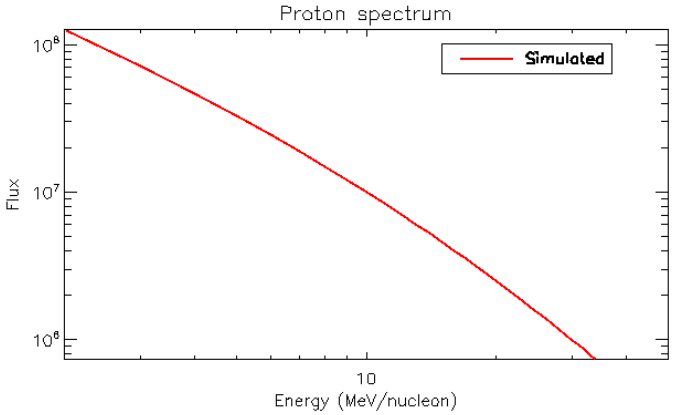
$$y_i = f(a,x) = \sum_{\ell=1}^{L} a_\ell \sum_{p=1}^{KM} H_{i,\ell,p}\, x_p + b_i$$

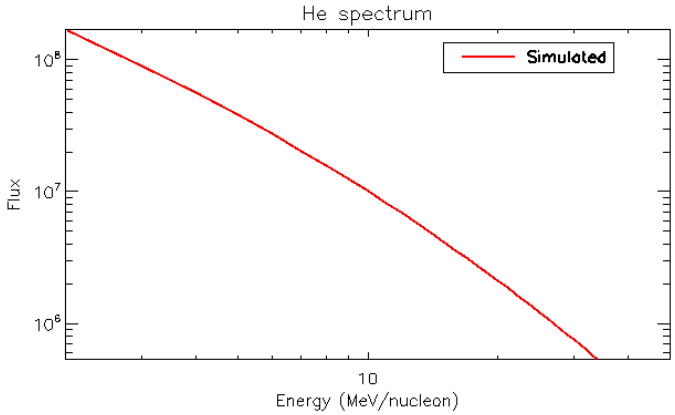
Take a look at the forward model

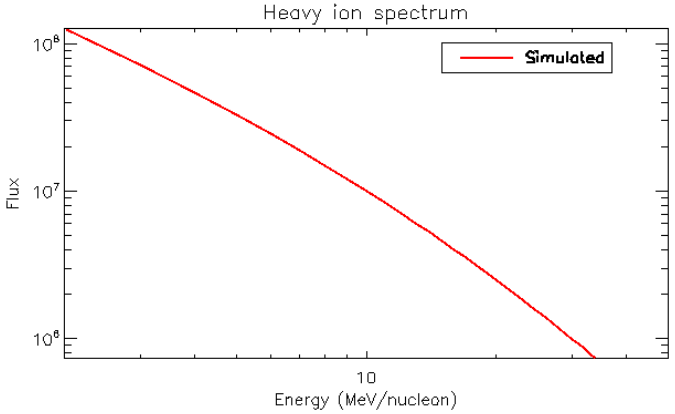
Ambient abundances

H : 1.00E-00
HH : 1.00E-01
C : 2.96E-04
N : 7.90E-05
O : 6.37E-04
Ne : 9.55E-05
Mg : 1.25E-04
Al : 1.00E-05
Si : 9.68E-05
S : 2.03E-05
Ca : 6.75E-06
Fe : 8.54E-05

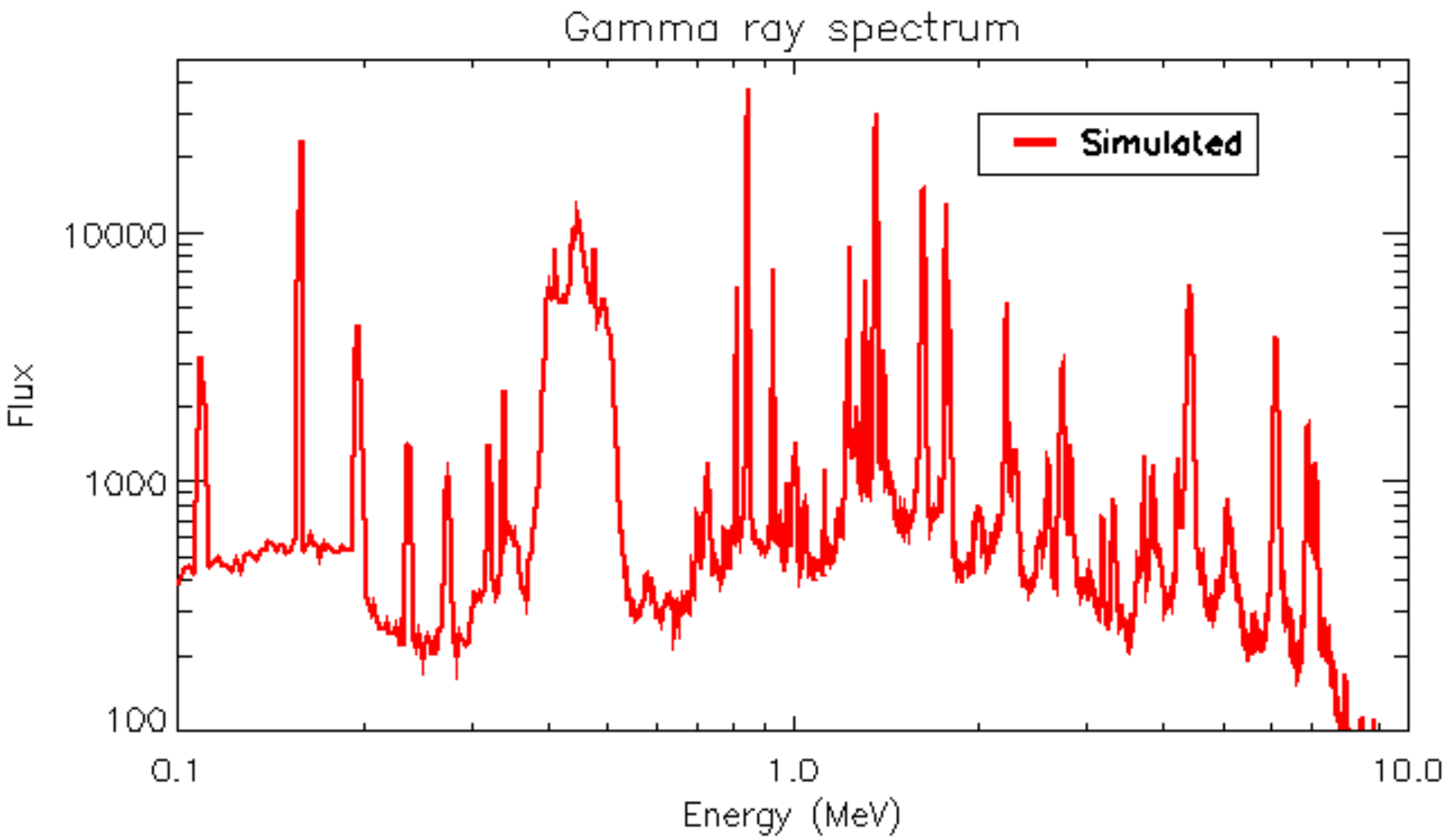
Input spectra: Bessel functions







Take a look at the forward model



Generated using the 'Murphy code'

Inverse Problem as optimization

Assumptions:

- the data y has to be realizations of a Poisson random variable.
- x, a and b have non-negative components;

The standard **Maximum Likelihood** approach boils down to find the solution x, a which minimizes the

C-statistic

$$C(x, y) = \frac{2}{N} \sum_{i=0}^{N} y_i \log \left(\frac{y}{aHx + b} \right)_i + (aHx + b)_i - y_i \tag{2}$$

Inverse methods

If you consider the abundances to be known $= a$, it is a linear problem: a popular **non parametric** inversion method is the

Richardson Lucy

$$x_{k+1} = x_k \; P \star \frac{y}{P * x_k}$$

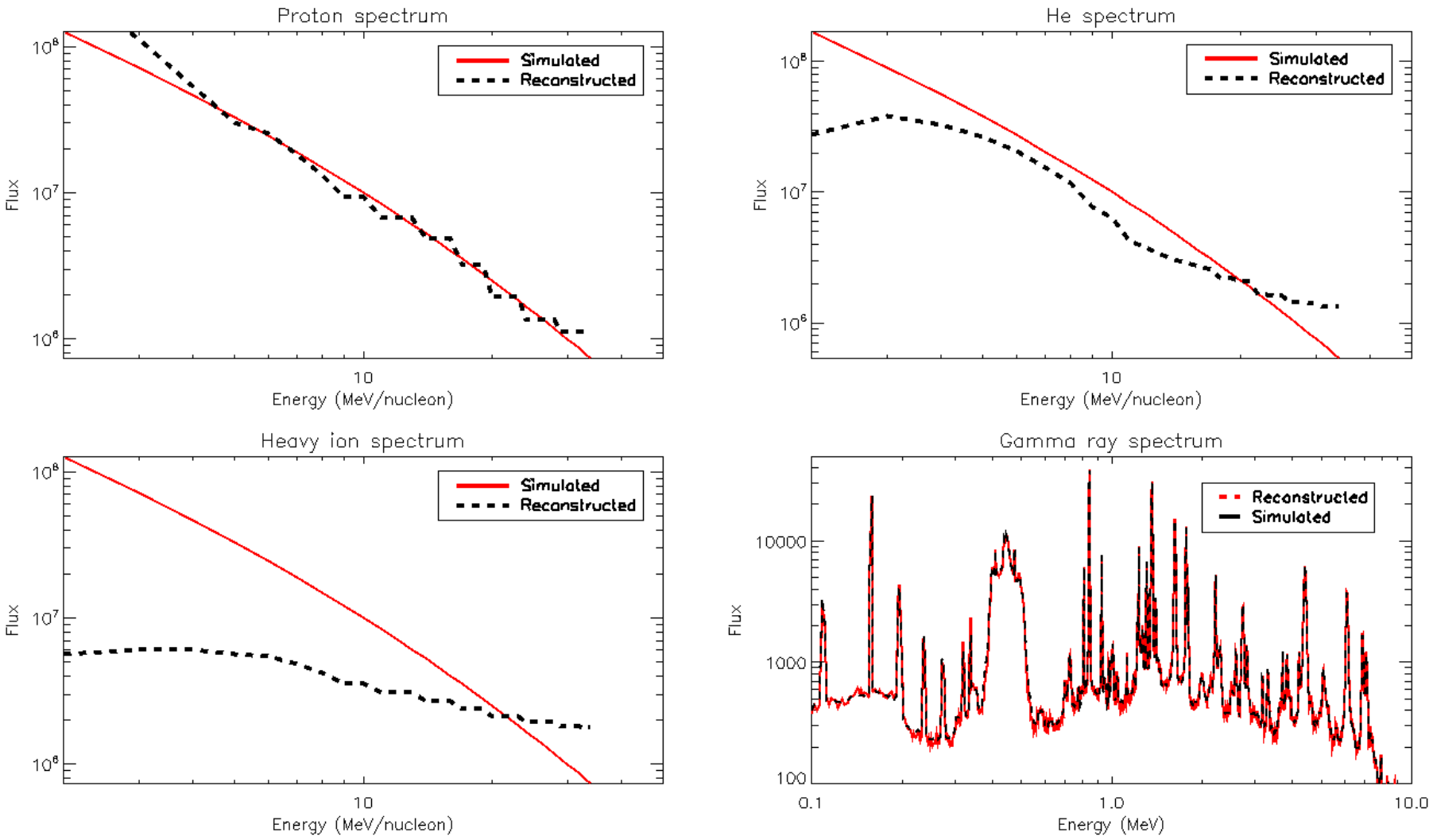
where P is a point spread function, $P*$ and $P\star$ indicate convolution and correlation.
Here we use its generalization:

Expectation Maximization

$$x_{k+1} = \frac{x_k}{(aH)^T 1}(aH)^T \frac{y}{aHx_k}$$

where we have aH (k), row-column multiplication (convolution) and column-row multiplication (correlation).

Inversion with known abundances



Blind expectation-maximization

Take another step towards reality

$$y = f(a, x, z) = aHx + Qz + b$$

- background b unknown (+ Bremsstrahlung effect)
- ambient abundances a unknown

Inspired by the blind deconvolution scheme

INIT : set x_0, z_0 and a_0

$$STEP\ 1: \quad x_{k+1} = \frac{x_k}{(a_k H)^T 1}(a_k H)^T \frac{y}{a_k H x_k + Q z_k + b}$$

$$STEP\ 2: \quad a_{k+1} = \frac{a_k}{(H x_{k+1})^T 1}(H x_{k+1})^T \frac{y}{a_k H x_{k+1} + Q z_k + b}$$

$$STEP\ 3: \quad z_{k+1} = \frac{z_k}{Q^T 1} Q^T \frac{y}{a_{k+1} H x_{k+1} + Q z_k + b}$$

Repeat STEP 1,2 and 3 until a given condition is met (stop criterion)

Inversion method

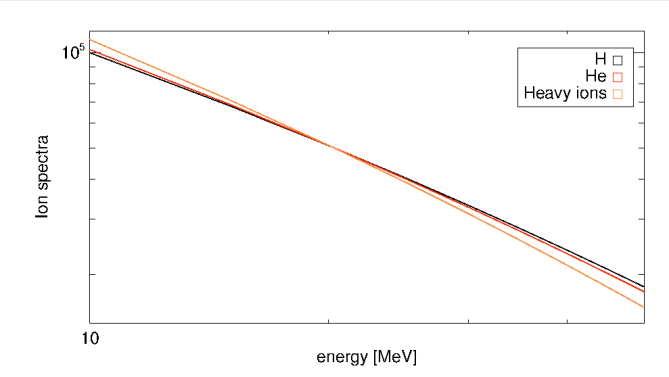
Some comments:

- The used method is convergent as shown in Lee & Seung, Neural Information Processing Systems (2000) to an optimized solution.
- From a numerical point of view the solution is unique, but the key point is the $\alpha - \alpha$ interaction.
- Perform several inversions (several initializations x_0, a_0, z_0) with fixed carbon ambient abundance. Select the optimal one.

Simulations

Input spectra for energetic ions:

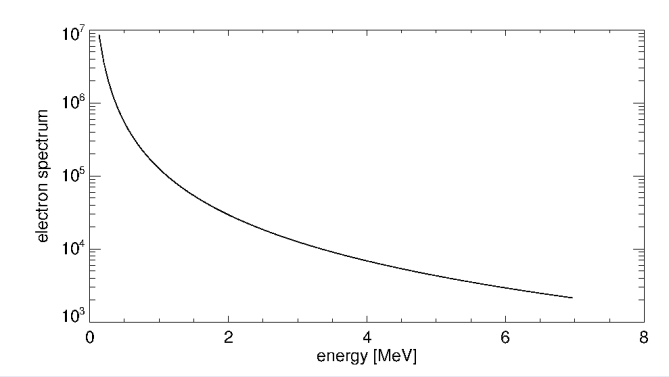
$$N(E) \simeq AK_2 \left[\left(\frac{E}{mc^2} \right)^{\frac{1}{4}} \right]$$



Ramaty R., Murphy R. J., *Nuclear processes and accelerated particles in solar flares*, 1987

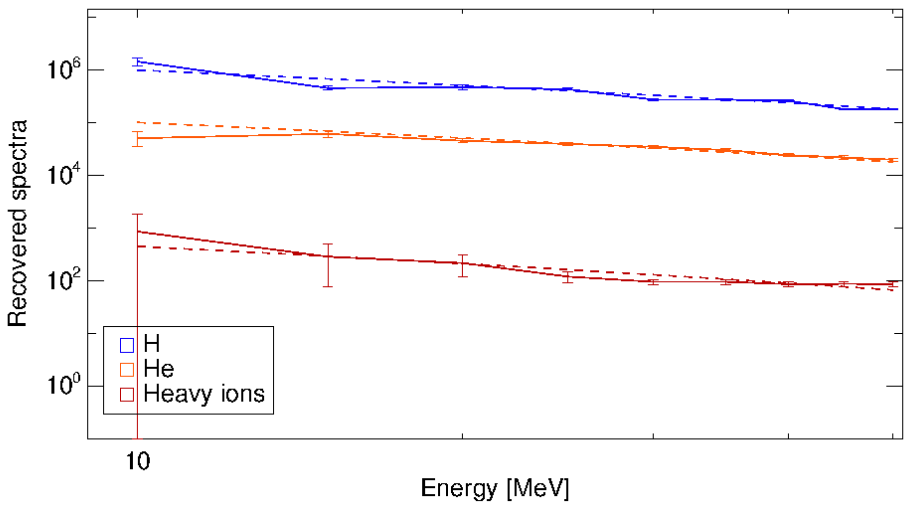
Input spectra for energetic electrons:

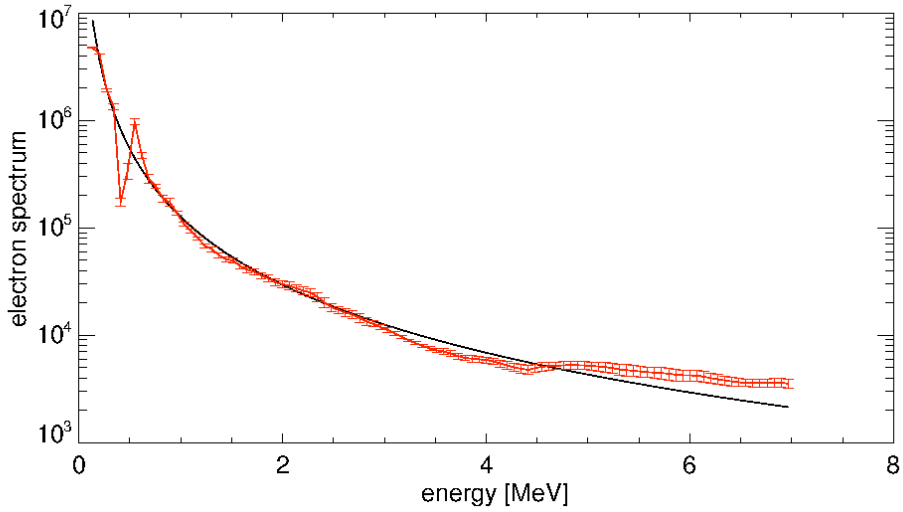
$$F(E) = B \left(\frac{E_0}{E} \right)^{2.9}$$

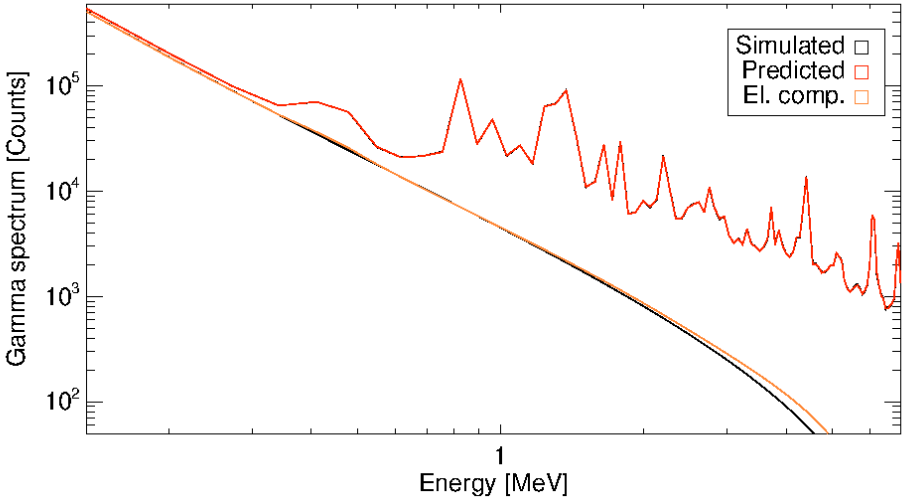


Share G., H. Murphy R. J., *Solar gamma-ray line spectroscopy - physics of a flaring star*, 2004

Simulation: high statistic







Black line under the red one represents simulated data (Fermi bins)

Simulation: high statistic

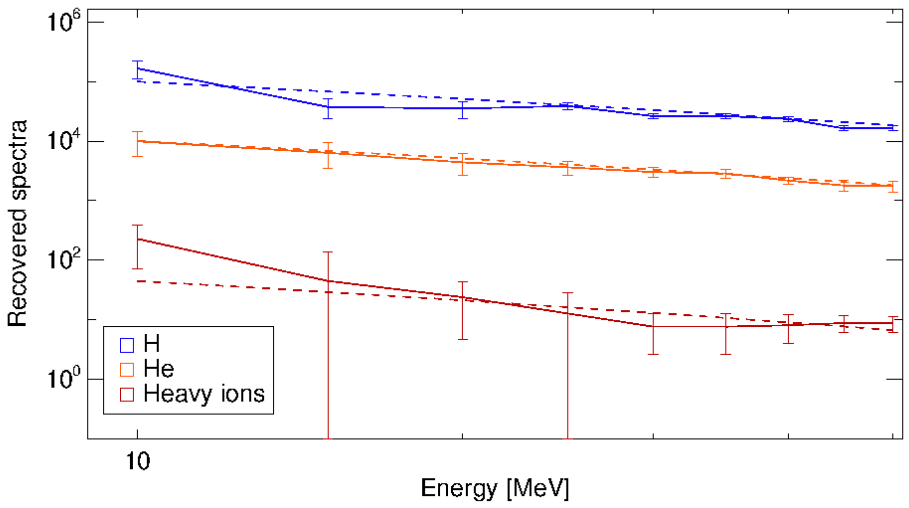
Ambient Abundances

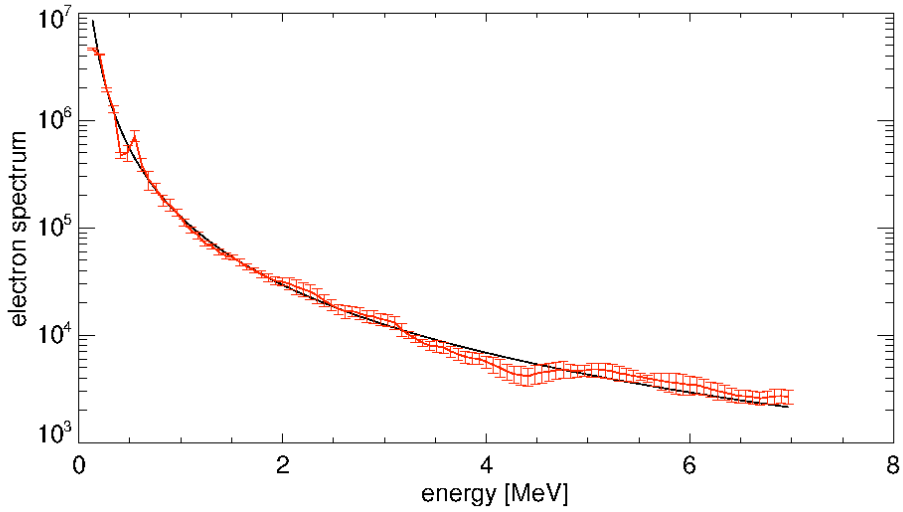
H	1.0 ± 0.0	1.0
He	0.084 ± 0.005	0.1
C	$(4.34 \pm 0.00)10^{-4}$	$4.34\ 10^{-4}$
N	$(1.67 \pm 0.11)10^{-4}$	$1.57\ 10^{-4}$
O	$(1.058 \pm 0.024)10^{-3}$	10^{-3}
Ne	$(4.11 \pm 0.12)10^{-4}$	$4\ 10^{-4}$
Mg	$(4.20 \pm 0.08)10^{-4}$	$4.08\ 10^{-4}$
Al	$(6.8 \pm 0.6)10^{-5}$	$6.8\ 10^{-5}$
Si	$(3.68 \pm 0.07)10^{-4}$	$3.52\ 10^{-4}$
S	$(1.205 \pm 0.022)10^{-4}$	$1.17\ 10^{-4}$
Ca	$(9.21 \pm 0.25)10^{-5}$	$8.8\ 10^{-5}$
Fe	$(1.116 \pm 0.022)10^{-3}$	$1.08\ 10^{-3}$

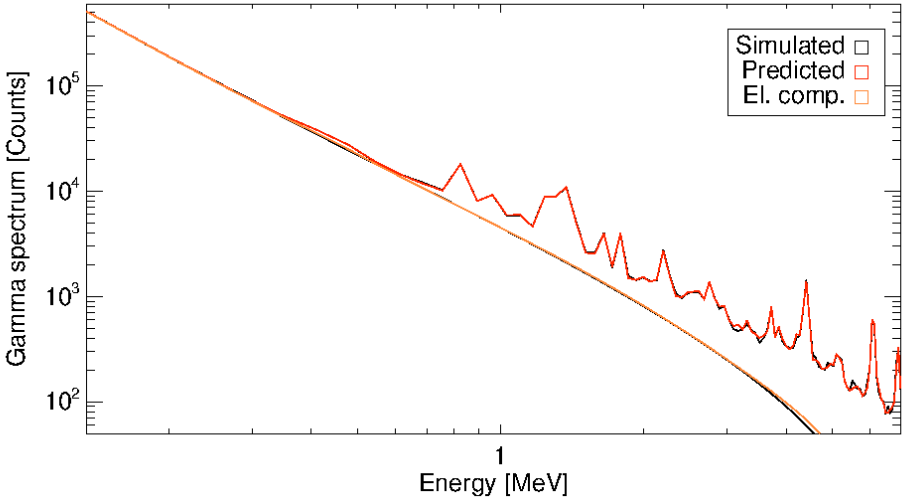
Energetic Abundances

H	1.00 ± 0.19	1.0
He	0.082 ± 0.018	0.1
C	$(5.8 \pm 3.6)10^{-4}$	$4.34\ 10^{-4}$
N	$(1.52 \pm 1.01)10^{-4}$	$1.57\ 10^{-4}$
O	$(1.7 \pm 1.4)10^{-3}$	10^{-3}
Ne	$(2.8 \pm 0.9)10^{-4}$	$4\ 10^{-4}$
Mg	$(4.9 \pm 2.7)10^{-4}$	$4.08\ 10^{-4}$
Al	$(0.55 \pm 1.04)10^{-4}$	$6.8\ 10^{-5}$
Si	$(3.59 \pm 2.09)10^{-4}$	$3.52\ 10^{-4}$
S	$(1.31 \pm 1.09)10^{-4}$	$1.17\ 10^{-4}$
Ca	$(2.81 \pm 3.07)10^{-4}$	$8.8\ 10^{-5}$
Fe	$(8.6 \pm 2.4)10^{-4}$	$1.08\ 10^{-3}$

Simulation: medium statistic







Black line under the red one represents simulated data (Fermi bins)

Simulation: medium statistic

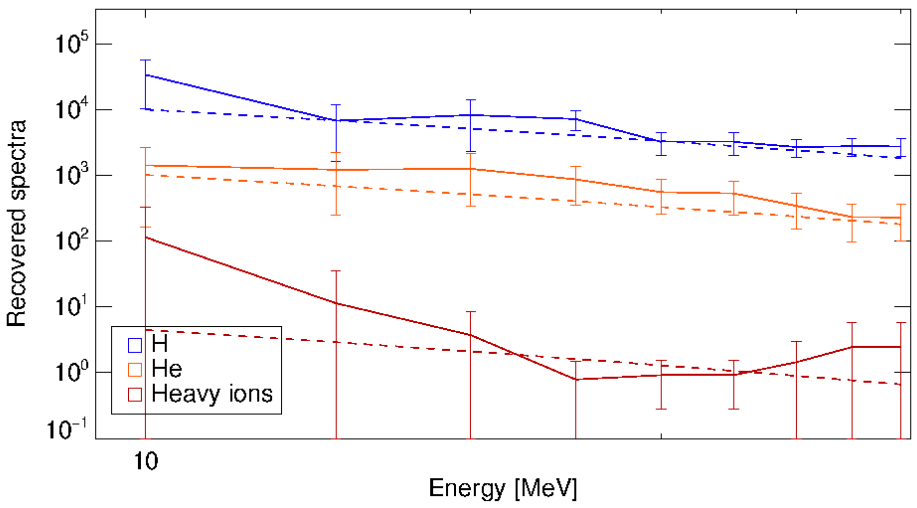
Ambient Abundances

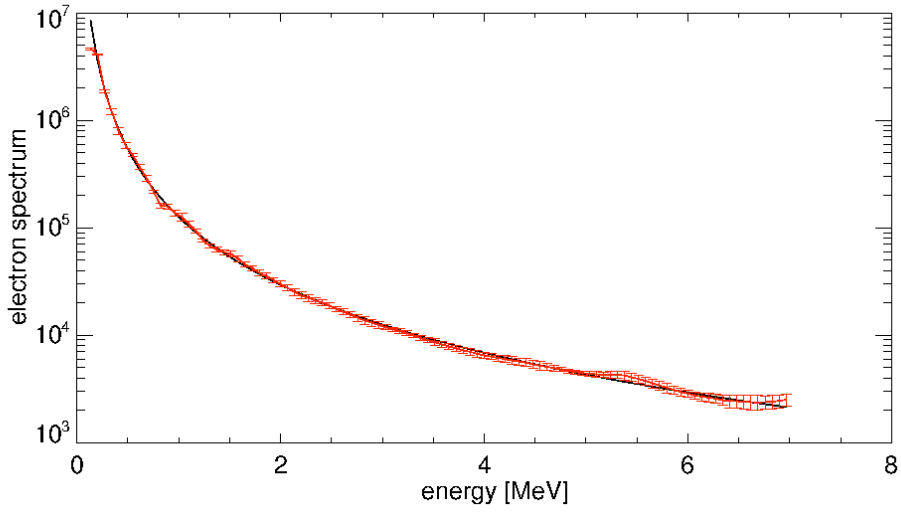
H	1.0 ± 0.0	1.0
He	0.02 ± 0.014	0.1
C	$(4.34 \pm 0.0)10^{-4}$	$4,34\ 10^{-4}$
N	$(1.9 \pm 0.5)10^{-4}$	$1.57\ 10^{-4}$
O	$(1.18 \pm 0.09)10^{-3}$	10^{-3}
Ne	$(4.4 \pm 0.5)10^{-4}$	$4\ 10^{-4}$
Mg	$(4.6 \pm 0.3)10^{-4}$	$4.08\ 10^{-4}$
Al	$(4.2 \pm 5.2)10^{-5}$	$6.8\ 10^{-5}$
Si	$(4.2 \pm 0.3)10^{-4}$	$3.52\ 10^{-4}$
S	$(1.29 \pm 0.11)10^{-4}$	$1.17\ 10^{-4}$
Ca	$(9.89 \pm 1.05)10^{-5}$	$8.8\ 10^{-5}$
Fe	$(1.22 \pm 0.09)10^{-3}$	$1.08\ 10^{-3}$

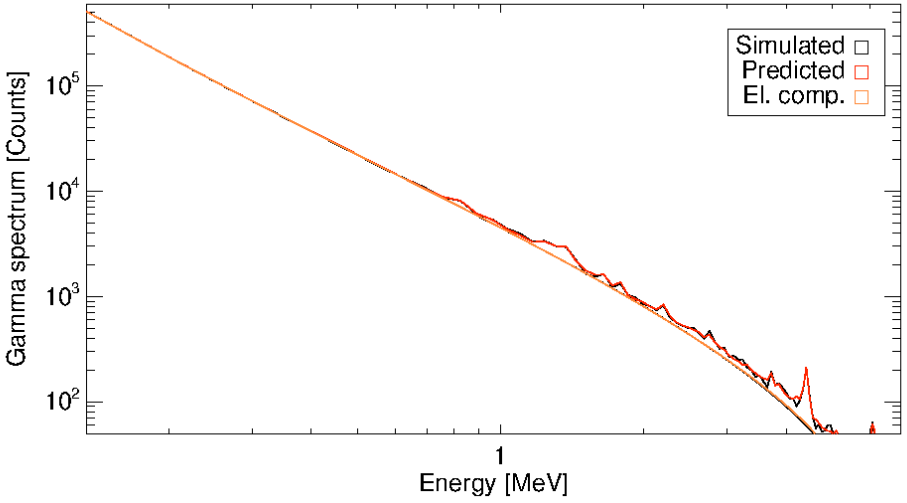
Energetic Abundances

H	1.0 ± 0.5	1.0
He	0.09 ± 0.06	0.1
C	$(1.4 \pm 2.1)10^{-3}$	$4.34\ 10^{-4}$
N	$(6.6 \pm 8.9)10^{-4}$	$1.57\ 10^{-4}$
O	$(3.4 \pm 3.6)10^{-3}$	10^{-3}
Ne	$(2.9 \pm 2.9)10^{-4}$	$4\ 10^{-4}$
Mg	$(0.79 \pm 1.02)10^{-3}$	$4.08\ 10^{-4}$
Al	$(0.86 \pm 2.8)10^{-4}$	$6.8\ 10^{-5}$
Si	$(4.9 \pm 9.9)10^{-4}$	$3.52\ 10^{-4}$
S	$(0.61 \pm 1.12)10^{-4}$	$1.17\ 10^{-4}$
Ca	$(5.3 \pm 7.4)10^{-4}$	$8.8\ 10^{-5}$
Fe	$(1.32 \pm 1.16)10^{-3}$	$1.08\ 10^{-3}$

Simulation: low statistic







Black line under the red one represents simulated data (Fermi bins)

Simulation: low statistic

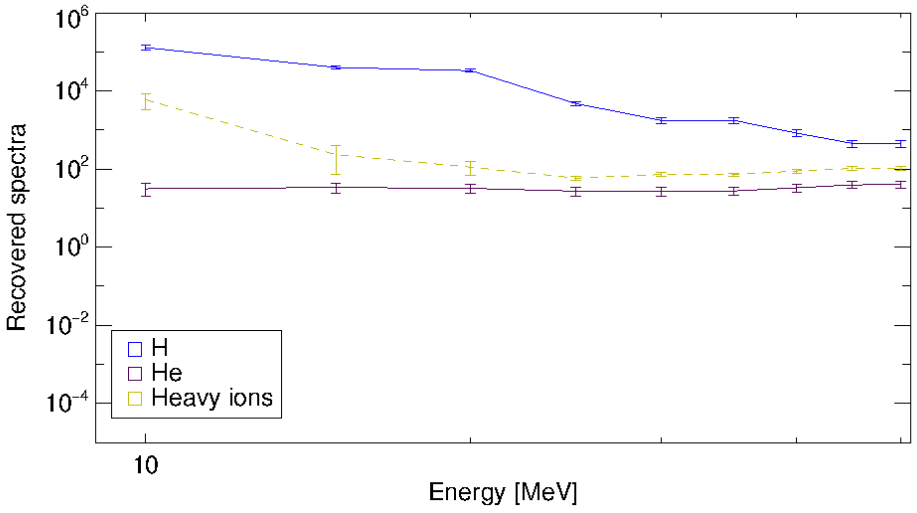
Ambient Abundances

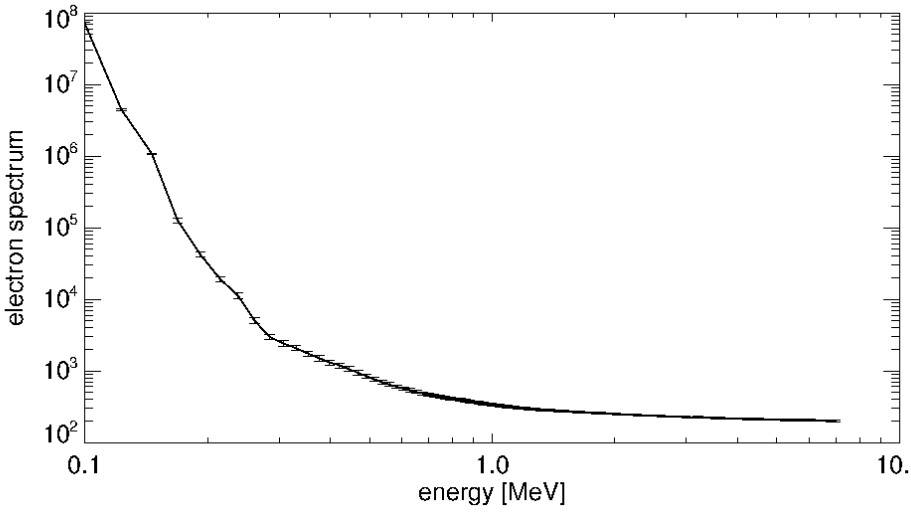
H	1.0 ± 0.0	1.0
He	$(3.0 \pm 5.2)10^{-7}$	0.1
C	$(4.34 \pm 0.0)10^{-4}$	$4.34\ 10^{-4}$
N	$(0.8 \pm 1.1)10^{-4}$	$1.57\ 10^{-4}$
O	$(6.4 \pm 1.3)10^{-4}$	10^{-3}
Ne	$(3.2 \pm 0.8)10^{-4}$	$4\ 10^{-4}$
Mg	$(3.0 \pm 0.5)10^{-4}$	$4.08\ 10^{-4}$
Al	$(0.5 \pm 1.7)10^{-4}$	$6.8\ 10^{-5}$
Si	$(2.6 \pm 0.9)10^{-4}$	$3.52\ 10^{-4}$
S	$(8.9 \pm 2.6)10^{-5}$	$1.17\ 10^{-4}$
Ca	$(5.6 \pm 2.2)10^{-5}$	$8.8\ 10^{-5}$
Fe	$(6.5 \pm 1.2)10^{-5}$	$1.08\ 10^{-8}$

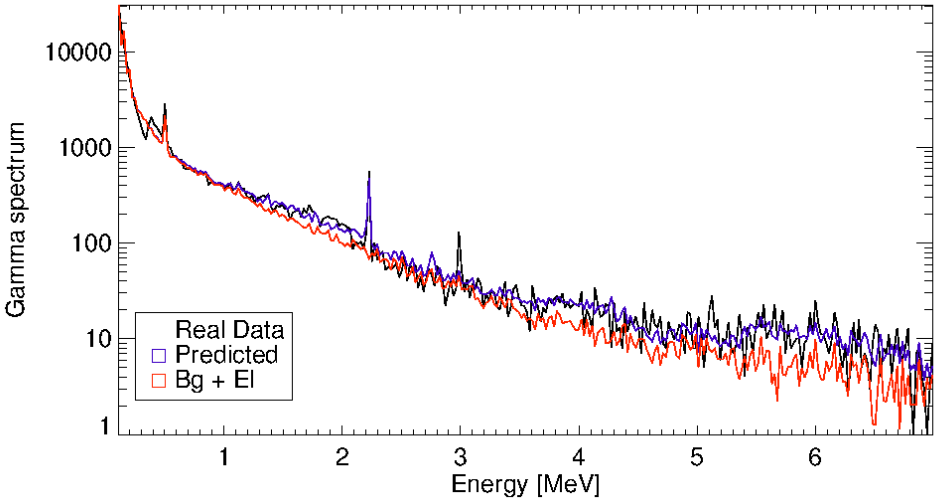
Energetic Abundances

H	1.0 ± 1.2	1.0
He	0.09 ± 0.12	0.1
C	$(0.7 \pm 1.7)10^{-3}$	$4.34\ 10^{-4}$
N	$(1.5 \pm 2.5)10^{-4}$	$1.57\ 10^{-4}$
O	$(2.2 \pm 5.3)10^{-3}$	10^{-3}
Ne	$(5.4 \pm 7.1)10^{-4}$	$4\ 10^{-4}$
Mg	$(3.3 \pm 4.8)10^{-3}$	$4.08\ 10^{-4}$
Al	$(0.6 \pm 1.5)10^{-2}$	$6.8\ 10^{-5}$
Si	$(1.2 \pm 2.6)10^{-5}$	$3.52\ 10^{-4}$
S	$(2.6 \pm 9.9)10^{-4}$	$1.17\ 10^{-4}$
Ca	$(0.4 \pm 1.3)10^{-3}$	$8.8\ 10^{-5}$
Fe	$(6.4 \pm 9.2)10^{-3}$	$1.08\ 10^{-3}$

28 November 2003 event







28 November 2003 event

Ambient Abundances

H	1.0 ± 0.0
He	$(3.7 \pm 0.4)10^{-5}$
C	$(2.2 \pm 2.5)10^{-4}$
N	$(3.9 \pm 3.2)10^{-5}$
O	$(1.8 \pm 2.4)10^{-4}$
Ne	$(9.1 \pm 5.6)10^{-5}$
Mg	$(3.28 \pm 1.19)10^{-4}$
Al	$(3.1 \pm 4.9)10^{-5}$
Si	$(4.9 \pm 1.7)10^{-4}$
S	$(2.45 \pm 0.11)10^{-3}$
Ca	$(5.5 \pm 6.4)10^{-5}$
Fe	$(3.3 \pm 1.6)10^{-6}$

Energetic Abundances

H	1.00 ± 0.24
He	$(1.4 \pm 0.5)10^{-3}$
C	$(2.52 \pm 1.06)10^{-3}$
N	$(3.90 \pm 1.15)10^{-3}$
O	$(3.0 \pm 0.8)10^{-1}$
Ne	$(4.04 \pm 1.09)10^{-3}$
Mg	$(1.6 \pm 0.8)10^{-4}$
Al	$(2.6 \pm 2.4)10^{-4}$
Si	$(1.6 \pm 0.7)10^{-2}$
S	$(3.30 \pm 1.06)10^{-3}$
Ca	$(1.5 \pm 1.5)10^{-3}$
Fe	$(9.2 \pm 3.3)10^{-4}$

To be continued

- A fundamental issue is the background subtraction
- RHESSI spectra have a high energy resolution
- Shift effect can make the fit procedure totally ineffective.

Possible solutions:

- Produce a family of models parametrized by the acceleration angle.
  - choose which model gives the best fit (if any)
- Introduce a parameter for each species for modeling the shift.
  - Is there an efficient optimization strategy for solving the inverse problem?
- Extract the main peaks of a RHESSI gamma ray spectrum and shift them at the 'right' energy (those of the Murphy code).
  - what to do with broader lines ?
- Convolve both the gamma ray data and the gamma ray model output by a psf in order to dump the doppler shift.
  - how large the psf should be ?