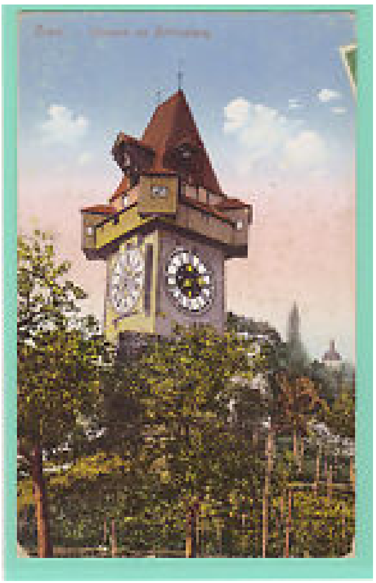
Combinatorial and Additive Number Theory 2016

January 4 - 8, 2016

Graz, Austria



CONFERENCE BOOKLET

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CONFERENCE INFORMATION

Date

January 4 - 8, 2016

Place of Event

University of Graz
Institute for Mathematics and Scientific Computing
Heinrichstraße 36
8010 Graz
Austria

Organizers: University of Graz

Alfred Geroldinger, Andreas Reinhart,
Daniel Smertnig, Qinghai Zhong

Conference Website

http://additive2016.uni-graz.at/en

Plenary Speakers

- Matt DeVos (Simon Fraser University):
  On the structure of very small product sets
- David J. Grynkiewicz (The University of Memphis):
  The Freiman $3k-4$ Theorem
- Vsevolod Lev (The University of Haifa at Oranim):
  Some Problems in Combinatorial Number Theory
- Alain Plagne (École Polytechnique):
  The Davenport constant of a box
- Imre Z. Ruzsa (Alfréd Rényi Institute of Mathematics):
  More differences than multiple sums
- Wolfgang A. Schmid (LAGA, University Paris 8 and 13):
  Characteristic Sets of Lengths
- Pingzhi Yuan (South China Normal University):
  Unsplittable minimal zero-sum sequences over C_n

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in the staterooms of Graz Burg Hofgasse 15, 8010 Graz

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Galliano, Harrachgasse 22 (closed on January 6)
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January 4 - 8, 2016

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Menu available from 12.00 p.m. on

4.1.2016 Lunch Menu | UniCafe Campus

Oriental-style lentil soup with bread	3,90 €
Spinach and sheep's cheese strudel with salad (vegetarian)	6,90 €

5.1.2016 Lunch Menu | UniCafe Campus

Carrot and ginger soup with bread	3,90 €
Thai chicken soup with rice	6,90 €
Gnocchi in gorgonzola sauce with salad (vegetarian)	6,90 €

6.1.2016 Lunch Menu | UniCafe Campus

Apple and celery soup with bread	3,90 €
Lasagne bolognese with salad	6,90 €
Pasta alla verdura (pasta with vegetables), with salad (vegetarian)	6,90 €

7.1.2016 Lunch Menu | UniCafe Campus

Potato and leek soup with bread	3,90 €
Pasta bolognese with salad	6,90 €
Polenta ratatouille bake with salad (vegetarian)	6,90 €

8.1.2016 Lunch Menu | UniCafe Campus

Minestrone soup with bread	3,90 €
Chili con carne with potato bread	6,90 €
Gnocchi in tomato sauce with salad (vegetarian)	6,90 €

ABSTRACTS

Some classical Ramsey-type theorems: Early and recent applications

SUKUMAR DAS ADHIKARI

Early Ramsey-type theorems include the theorem of Ramsey, and the other results include the celebrated theorem of van der Waerden and a result of Schur. Origins of some of the famous recent developments in mathematics can be traced back to these results.
Here we dwell on the Schur and the van der Waerden theme - giving some interrelations, looking mostly at the classical results, and some early and recent applications of these results.

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Pair correlations and additive energy

Christoph Aistleitner

Pair correlations play a role in mathematical physics, where they are used to characterize the properties of the energy levels of certain integrable systems. For an infinite sequence of numbers in $[0, 1]$, the asymptotic distribution of the pair correlations can be either "Poissonian" (that is, the same as in the random case) or "non-Poissonian". For example, the asymptotic distribution of the pair correlations of the fractional parts of $n^2\alpha$ is asymptotically Poissonian for almost all α, while it is non-Poissonian for the fractional parts of $n\alpha$ for all α. In the more general setting of fractional parts of sequences of the form $a(n)\alpha$, from work of Rudnick, Sarnak and Zaharescu it is known that the metric behavior of the pair correlations is related to the number of solutions of certain Diophantine equations. In this talk, which reports on current work in progress with Gerhard Larcher (JKU Linz) and Mark Lewko (UCLA), we show that the metric behavior of the pair correlations can actually be linked to the additive energy of the sequence $a(n)$. As a rule of thumb, low additive energy means Poissonian pair correlations for almost all α, while high additive energy means non-Poissonian pair correlations for almost all α.

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A contribution to zero-sum problem with some applications

Armen Bagdasaryan

In this talk, we present a simple and general technique, which (at least to our knowledge) is new, for investigating problems of zero-sum type in finite sets with certain structure. The main theorem is quite general in the sense that we do not require for operations to obey concrete algebraic rules (e.g. commutativity, associativity, etc.). The next result, stated as another theorem and obtained by applying the technique of the main theorem, is related to a zero-sum problem in general finite groups. With this latter theorem, applied to multiplicative group of residues mod p, where p is prime, we obtain some results that complement the Wilson's theorem and Fermat's little theorem. Finally, we try to outline further research by mentioning several new possible developments of the quoted theorems.

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Elasticity in Arithmetic Congruence Monoids

Paul Baginski

For integers $0 < a \leq b$, the arithmetic progression $M_{a,b} := a + b\mathbb{N}$ is closed under multiplication if and only if $a^2 \equiv a \mod b$. Any such multiplicatively closed arithmetic progression is called an arithmetic congruence monoid (ACM). Though these $M_{a,b}$ are multiplicative submonoids of $\mathbb{N}$, their factorization properties differ greatly from the unique factorization one enjoys in $\mathbb{N}$. In this talk we will explore the known factorization properties of these monoids, with a particular emphasis on recent results about the elasticity. When $a > 1$, these monoids are not Krull and thus do not have a class group which fully captures the factorization behavior. Nonetheless, an ACM can be associated to a finite abelian group associated which aids our understanding of factorization properties.
For both Krull monoids and ACMs, factorization properties can be understood using the additive combinatorics of the monoid's associated finite group. For Krull monoids, the key combinatorial idea is that of a zero-sum sequence. In contrast, for ACMs, the key combinatorial idea will be sequences which attain certain sums while avoiding others. We will demonstrate how these sequences specifically relate to problems regarding the elasticity of ACMs.

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Open Problems About Sumsets in Finite Abelian Groups

Béla Bajnok

For a positive integer h and a subset A of a given finite abelian group, we let hA, $h\hat{}A$, and $h_{\pm}A$ denote the h-fold sumset, restricted sumset, and signed sumset of A, respectively. Here we review some of what is known and not yet known about the minimum sizes of these three types of sumsets, as well as their corresponding critical numbers. In particular, we discuss several new open direct and inverse problems.

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Additive combinatorics methods in associative algebras

Vincent Beck

Kneser's theorem has been generalized to separable field extensions by Hou, Leung and Xiang[1]: they give a lower bound for the dimension of the Minkowski product of two subspaces in this context. We generalize their result to associative algebras. Our method also apply to generalize Hamidoune results [2] on atom and Tao results [3] on space of small doubling. Moreover applying these results to a group algebra enable us to recover the group results.

References

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Upper Bounds for the Davenport's Constant

Gautami Bhowmik

The Davenport's constant of an abelian group is the smallest positive integer such that each sequence of that length k contains a zero-sum subsequence. In general an exact estimation for it is not known. We will present some reasonable bounds.

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Rokhlin's lemma, a generalization, and combinatorial applications

Pablo Candela

Rokhlin's lemma is a fundamental tool in ergodic theory which allows one, roughly speaking, to approximate an arbitrary aperiodic invertible measure preserving map by a periodic map. I will describe a recent generalization of this result to several non-invertible transformations, and describe several applications in additive combinatorics. Joint work with Artur Avila.

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On a conjecture of Sárközy and Szemerédi

YONG-GAO CHEN

Two infinite sequences A and B of non-negative integers are called *infinite additive complements*, if their sum contains all sufficiently large integers. In 1994, Sárközy and Szemerédi conjectured that there exist infinite additive complements A, B with $\limsup A(x)B(x)/x \leq 1$ and $A(x)B(x) - x = O(\min\{A(x), B(x)\})$, where $A(x)$ and $B(x)$ are the counting functions of A and B, respectively. In this paper, we prove that, for infinite additive complements A and B, if $\limsup A(x)B(x)/x \leq 1$, then, for any given $M > 1$, we have

$$A(x)B(x) - x \geq (\min\{A(x), B(x)\})^M$$

for all sufficiently large integers x. It follows that the answer to the above Sárközy-Szemerédi conjecture is negative. We also pose several problems for further research.

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On some properties of the generalised multinomial measure

Ligia L. Cristea

Okada, Sekiguchi and Shiota [1] introduce the multinomial measure on the unit interval, that is defined with the help of the digital expansions of the numbers in the unit interval in a certain integer base.

In recent work [2] we introduced the *generalised multinomial measure*. Here a generalisation consists, roughly speaking, in the fact that instead of dividing the unit interval into a finite number of subintervals of equal length, we divide it into infinitely (and denumerably) many intervals, such that the j-th subinterval has length pq^{j-1}, where $p, q > 0$, and $p + q = 1$. One way to define the generalised multinomial measure is the following. We consider the set $\mathcal{W}$ of all (finite and infinite) words over the finite alphabet $\{0, 1, \dots\}$ and a probability measure $\mathbb{P}_{\mathbf{r}}$ defined on the set of all words. A function value associates to every word $\omega \in \mathcal{W}$ a real number $\mathsf{value}(\omega) \in [0, 1)$, such that the closure of the set of all such values $\mathsf{value}(\omega)$, is the interval $[0, 1]$. Then the measure of an interval $\mu_{\mathbf{r},q}([0, a))$, with $0 \leq a \leq 1$ can be defined in a natural way with the help of the probability $\mathbb{P}_{\mathbf{r}}$, where $\mathbf{r} = (r_0, r_1, \dots)$ is a sequence of real numbers (parameters) with $0 \leq r_i \leq 1$ and $\sum_{k=0}^{q-1} r_k = 1$.

We study the behaviour of the average minimum value a_n among n words of $\mathcal{W}$ chosen intependently at random with respect to the generalised multinomial measure $\mu_{\mathbf{r},q}$ for certain values of r_j, $j = 0, 1, \dots$. Futhermore, we study the analogous problem for the average maximum value among n words. We note that the final formulae obtained for the asymtotics show a certain duality.

This is a joint work with Helmut Prodinger, funded by the Austrian Science Fund FWF (project number P27050-N26).

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[1] T. Okada, T. Sekiguchi, and Y. Shiota. A Generalization of Hata-Yamaguti's Results on the Takagi Function II: Multinomial Case. *Japan J. Indust. Appl. Math.*, 13:435–463, 1996.

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Connections between zero-sum theory and invariant theory

Kálmán Cziszter

It is well known that there is a one-to-one correspondence between the zero-sum sequences over an abelian group A and the monomials spannig the ring of polynomial invariants of a certain representation of A. This correspondence has been used in [2] and [3] to obtain some informations about the ring of invariants of some non-abelian groups G which contain A as a normal subgroup or as a homomorphic image. I will review in this context some interesting applications of zero-sum theoretical results to invariant theory and I will also formulate some new problems in zero-sum theory which are motivated by this kind of invariant theoretical applications.

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On the structure of very small product sets

Matt DeVos

The Cauchy-Davenport Theorem gives a natural lower bound on the size of a sumset in the group $\mathbb{Z}/p\mathbb{Z}$ when p is prime. This was later sharpened by Vosper who classified the pairs (A, B) of subsets of such a group for which $|A+B| < |A|+|B|$. Kneser gave a lower bound on the size of a sumset in an abelian group, and this result was later refined by Kemperman who classified the pairs (A, B) of finite subsets of an abelian group for which $|A+B| < |A|+|B|$. We extend these classification theorems to the realm of nonabelian groups, which we write multiplicatively. That is we will give a classification of all pairs (A, B) of finite subsets of an arbitrary group for which $|AB| < |A|+|B|$.

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Second order differences between primes, for thin (but not too thin) sequences of primes

Christian Elsholtz

This is joint work with Jörg Brüdern:
Rényi (1950) studied the curvature of the prime number graph. To explain this, consider at least three distinct points $z_1, \dots, z_N$ in the complex plane, and define the total curvature of the polygonal line connecting z_{n-1} with z_n for $2 \le n \le N$ by

$$K = \sum_{n=1}^{N-2} \left| \arg \frac{z_{n+2} - z_{n+1}}{z_{n+1} - z_n} \right|.$$

Let (p_n) denote the sequence of all primes arranged in increasing order, and put $z_n = n + \mathrm{i} \log p_n$. Let K_N be the curvature with this special choice of z_n. In this notation, Rényi obtained the bound $K_N \gg \log\log\log N$. Erdős and Rényi (1950) improved this to

$$\log N \ll K_N \ll \log N.$$

As a corollary it follows that for any $n_0 \in \mathbb{N}$, the sequence $(\log p_n)_{n \ge n_0}$ is neither concave nor convex, and this implies that the sequence $p_{n+1}^2 - p_n p_{n+2}$ changes sign infinitely often. We take a fresh look at this problem from a modern view point. Are there infinitely many sign changes of $p_{n+1}^2 - p_n p_{n+2}$, for a thin sequence of primes? We prove:

Theorem: Let $\delta : [1, \infty) \to (0, 1)$ be monotonically decreasing with $\delta(x) \ge (\log x)^{-1}$, and let $\mathcal{P}$ be a set of primes satisfying:

$$\#\{p \in \mathcal{P} : p \le x\} \ge \frac{\delta(x) x}{\log x}$$

for all $x \ge x_0$. Then, for $N \ge 3$, one has

$$\delta_{p_N}^3 \log N \ll K_N(\mathcal{P}) \ll \delta_{p_N}^{-1} \log N.$$

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An integral formula for a finite sum of inverse powers of cosines

Carlos M. da Fonseca

We present a new and elegant integral approach to computing the Gardner-Fisher trigonometric power sum, which is given by

$$S_{m,v} = \left(\frac{\pi}{2m}\right)^{2v} \sum_{k=1}^{m-1} \cos^{-2v}\left(\frac{k\pi}{2m}\right),$$

where m and v are positive integers. This method not only confirms the results obtained earlier by an empirical method, but it is also much more expedient from a computational point of view. By comparing the formulas from both methods, we derive several new interesting number theoretic results involving symmetric polynomials over the set of quadratic powers up to $(v-1)^2$ and the generalized cosecant numbers. The method is then extended to other related trigonometric power sums including the untwisted Dowker sum. This is a joint work with M. Lawrence Glasser, Clarkson University, Potsdam, USA, and Victor Kowalenko, The University of Melbourne, Victoria, Australia, funded by the Kuwait University Research Grant No. SM03/13.

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Fractional Hypergeometric Zeta Functions

Hunduma Legesse Geleta

In this paper we investigate a continuous version of the hypergeometric zeta functions for any positive rational number “a” and demonstrate the analytic continuation. The fractional hypergeometric zeta functions are shown to exhibit many properties analogous to its hypergeometric counter part, including its intimate connection to Bernoulli numbers.

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The Freiman $3k - 4$ Theorem

David J. Grynkiewicz

The Freiman $3k - 4$ Theorem, in its original form, gives a fairly precise structural sumset result for a k-element subset $A \subseteq \mathbb{Z}$ with small sumset

$$|A + A| \leq 3|A| - 4 = 3k - 4.$$

Namely, letting $r = |A+A| - (2|A| - 1) \geq 0$ denote the amount $|A+A|$ is above the trivial lower bound $|A + A| \geq 2|A| - 1$, then there must be an arithmetic progression P that contains A having few "holes" $|P \setminus A| \leq r$. The bound on $|P \setminus A|$ is known to be tight, giving a rare instance of the more general Freiman's Theorem in which the constants are known exactly. Since its original proof, there have been many generalizations and partial extensions of the $3k - 4$ Theorem, including allowing distinct summands, partial versions valid in $\mathbb{Z}/p\mathbb{Z}$, and bounds on the length of the longest arithmetic progression inside the sumset itself. We will give an overview of these newer versions and end with a very recent extension to two-dimensional sumsets $A + B$ in a torsion-free abelian group below the threshold $|A + B| < |A| + \frac{7}{3}|B| - 5$, where $|B| \leq |A|$. Indeed, imposing some additional geometric constraints on A and B (that they be covered by two parallel lines), the latter result extends to the threshold $|A+B| \leq |A| + \frac{19}{7}|B| - 5$ and gives a single two-dimensional progression P that simultaneously contains A and B with few holes.

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Character sum estimations for various problems in combinatorial number theory

Norbert Hegyvári

Let $f : \mathbb{Z}_p^2 \to \mathbb{Z}_p$. f is said to be *expander* if for any $A, B \subseteq \mathbb{Z}_p$, $|A| \asymp |B|$ then $f(A, B) := \{f(a, b) : a \in A; b \in B\}$ is ampler (in some uniform meaning) than $|A|$.
We say that a map $f : \mathbb{F}_p^k \mapsto \mathbb{F}_p$ is a covering polynomial if there is a threshold $\gamma(f, p)$ such that for every $A_i \subseteq \mathbb{F}_p; i = 1, \dots k$ with $|A_1| \cdots |A_k| > \gamma(f, p)$ we get $f(A_1, \dots, A_k) = \mathbb{F}_p$.
We discuss some problems in prime fields: distribution of two-variable polynomials, expander polynomials, covering polynomials, character sums on Hilbert cubes, e.t.c.
The main tool at the proofs are character sum estimations on different structures.
The results are partially joint work with François Hennecart.

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Expanders and good distribution in $\mathbf{F}_p$

François Hennecart

An expander is a binary function $f(x, y)$, $x, y \in \mathbf{F}_p$ taking its value in $\mathbf{F}_p$ such that $\mathrm{Card}(f(A, A)) \geq (\mathrm{Card}(A))^{1+\delta(a)}$ with $\delta(a) > 0$ whenever A verifies $\mathrm{Card}(A) \asymp p^a$. For instance, since Bourgain (2005), we know that the function $xy + x^2$ defines an expander. Our aim is to consider some questions related to the distribution of the values taken by certain classes of binary functions $f(x, y)$. We shall use the notion of good distribution in $\mathbf{F}_p$ for binary functions.

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An inverse theorem in $\mathbb{F}_p$ and rainbow free colorings

Mario Huicochea

Let $\mathbb{F}_p$ be the field with p elements with p prime, $X_1, \ldots, X_n$ pairwise disjoint subsets of $\mathbb{F}_p$ with at least 3 elements such that $\sum_{i=1}^{n} |X_i| \leq p-5$, and $\mathbb{S}_n$ the set of permutations of $\{1, 2, \ldots, n\}$. If $a_1, \ldots, a_n \in \mathbb{F}_p^*$ are not all equal, we characterize the subsets $X_1, \ldots, X_n$ which satisfy

$$\left| \bigcup_{\sigma \in \mathbb{S}_n} \sum_{i=1}^{n} a_{\sigma(i)} X_i \right| \leq \sum_{i=1}^{n} |X_i|.$$

This result has the following application: for $n \geq 2$, $b \in \mathbb{F}_p$ and $a_1, \ldots, a_n \in \mathbb{F}_p^*$ with $a_i \neq a_j$ for some $i, j \in \{1, \ldots, n\}$, we characterize the colorings $\mathbb{F}_p = \bigcup_{i=1}^{n} C_i$ where each color class has at least 3 elements such that $\sum_{i=1}^{n} a_i x_i = b$ has not rainbow solutions.

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Long arithmetic progressions in subset sums and a conjecture of Alon

Gyula Károlyi

Let $f(\ell, m)$ denote the maximum cardinality of a set $A \subset \{1, \ldots, \ell\}$ such that there is no $B \subset A$ the sum of whose elements is m. We determine the exact value of $f(\ell, m)$ in the range $\ell \ln \ell \ll m \ll \ell^2 / \ln^2 \ell$, thus verifying a conjecture of Alon. Our proof depends on the existence of long homogeneous arithmetic progressions with small common difference in the set of subset sums of dense enough sets of integers. We give a more precise result in the case when the density exceeds $1/2$, which confirms a conjecture of Lev. The proofs, combinatorial in nature, rely on earlier works of Lev and the Dias da Silva–Hamidoune theorem.

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Some Problems in Combinatorial Number Theory

Vsevolod Lev

I discuss a number of not particularly famous, but challenging open problems I came across at various stages of my research.

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Long zero-sum free sequences and n-zero-sum free sequences over finite cyclic groups

Yuanlin Li

In an additively written abelian group, a sequence is called zero-sum free if each of its nonempty subsequences has sum different from the zero element of the group. In this paper, we consider the structure of long zero-sum free sequences and n-zero-sum free sequences over finite cyclic groups $\mathbb{Z}_n$. Among which, we determine the structure of the long zero-sum free sequences of length between $n/3+1$ and $n/2$, where $n \geq 50$ is an odd integer, and we provide a general description on the structure of n-zero-sum free sequences of length $n+l$, where $\ell \geq n/p+p-2$ and p is the smallest prime dividing n.

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Bijective Proof of Extensions of the Sury's Identity

Ivica Martinjak

We present two families of Fibonacci-Lucas identities, with the *Sury's identity* $\sum_{k=0}^{n} 2^k L_k = 2^{n+1} F_{n+1}$ being the best known representative of one of the family. While these results can be proved by means of the basic identity relating Fibonacci and Lucas sequences we also provide a combinatorial proof. In particular, we demonstrate that for the Fibonacci sequence $(F_n)_{n\geq 0}$ and the Lucas sequence $(L_n)_{n\geq 0}$ there is a family of identities

$$\sum_{k=0}^{n} m^k [L_k + (m-2)F_{k+1}] = m^{n+1} F_{n+1},$$

where $m \geq 2$. Our bijective proof is based on the fact that the product $m^n f_n$, where $f_n = F_{n+1}$, represents the number of *colored n-board tilings* with *squares* in m colors and *dominoes* in m^2 colors.

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The use of additive tools in solving arithmetic anti-Ramsey problems

Amanda Montejano

The study of the existence of rainbow structures in colored universes falls into the anti-Ramsey theory. Canonical versions of this theory prove the existence of either a monochromatic structure or a rainbow structure. Instead, in the most recent so called *Rainbow Ramsey Theory* [2], the existence of rainbow structures is guaranteed regardless of the existence of monochromatic structures. Arithmetic versions of this theory were initiated by Jungic, Fox, Mahdian, Nesetril and Radoicic [1] studying the existence of rainbow arithmetic progressions in colorings of cyclic groups and of intervals of integers. In this setting it happens, most of the times, that to ensure the existence of a rainbow structure the color classes have to satisfy some density conditions. Our particular interest is in describing colorings containing no rainbow structures, called *rainbow-free colorings*. Beyond of studying density conditions to force a rainbow set our aim is to characterize the structure of rainbow-free colorings. In this talk we present how to use classical inverse theorems in additive number theory in order to obtain such results. We present a characterization of rainbow-free 3-colorings in the following cases: (1) Abelian groups with respect to three term arithmetic progressions [3,4]; (2) Cyclic groups of prime order with respect to solutions of any linear equation on three variables [5].

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Sums of sets of lattice points

Melvyn B. Nathanson

This talk will review some recent problems and results concerning polytopes and sums of sets of lattice points. Of particular interest is the following question: Let P be a lattice polytope. When is every lattice point in the sumset hP a sum of h lattice points in P?

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On some Multiplicative Problems of Erdős

Péter Pál Pach

How large can a set of integers be, if the equation $a_1 a_2 \dots a_h = b_1 b_2 \dots b_h$ has no solution consisting of distinct elements of this set? How large can a set of integers be, if none of them divides the product of h others? The first question is about a generalization of the multiplicative Sidon-sets and the second one is of the primitive sets. In answering the above mentioned questions some lemmas on product representation of integers and extremal combinatorial tools can help. In the results not only the asymptotics are found, but very tight bounds are obtained for the error terms, as well. For example, if the numbers are from the set $\{1, 2, \dots, n\}$, the precious answer to the second question has both lower- and upper bounds in the form $\pi(n) + cn^{2/(h+1)}/(\log n)^2$ with $c > 0$. Here, n has to be large enough compared to h, but the constants do not depend on h.

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Translated Dot Products in Finite Fields

Giorgis Petridis

A theorem of Iosevich and Hart states that if $E \subseteq \mathbb{F}^2$ is a set in the 2-dimensional vector space over a finite field with q elements that satisfies $|E| > q^{3/2}$, then the set of dot products determined by E is the whole of $\mathbb{F}^*$. The exponent $3/2$ of q essentially being best possible.

If we give ourselves a little more leeway and ask for the existence of $x \in E$ so that the set of dot products

$$E \cdot (E - x) = \{u \cdot (v - x) : u, v \in E\}$$

has cardinality, say, at least $q/2$, then one only requires the (up to a multiplicative constant) best possible lower bound $|E| > 3q$.

The talk will include a general in nature discussion about the original question and some recent progress in the continuous setting by Orponen (related to his breakthrough on the Falconer distance conjecture).

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The Davenport constant of a box

Alain Plagne

The Davenport constant of a group is a central invariant in combinatorial group theory. In this talk we generalize this concept to a more general setting. Given an additively written abelian group G and a set $X \subseteq G$, we let $\mathscr{B}(X)$ denote the monoid of zero-sum sequences over X and $\mathsf{D}(X)$ the Davenport constant of $\mathscr{B}(X)$, namely the supremum of the positive integers n for which there exists a sequence $x_1 \cdots x_n$ of $\mathscr{B}(X)$ such that $\sum_{i \in I} x_i \neq 0$ for each non-empty proper subset I of $\{1, \ldots, n\}$. In this talk, we mainly study the case when G is a power of $\mathbb{Z}$ and X is a box (i.e., a product of intervals of G). Some mixed sets (e.g., the product of a group by a box) are studied too, and some inverse results are obtained.
Joint work with Salvatore Tringali.

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Structural sum-product problems

Oliver Roche-Newton

A common variation of the sum-product problem is the question of determining whether certain sets defined by a combination of additive and multiplicative operations are guaranteed to be large. Recent years have seen progress in this area, with a number of essentially optimal results being established. A natural follow-up problem concerns the structural question of when these lower bounds can be attained. This talk will give an introduction to problems of this type, discussing some of the many open problems in the area and giving some details of the few results that are known.

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More differences than multiple sums

Imre Z. Ruzsa

I will tell, based on an 1973 paper of Haight, how to construct a set of integers such that

$$|kA| < |A - A|^{1-\delta_k}.$$

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An effective van der Corput inequality

Sumaia Saad Eddin

Let $d(n)$ be a divisor function. In this talk, we will prove inequalities of the shape

$$d^s(n) \leq K(s, \delta, \beta) \sum_{\ell | n, \ell \leq n^\delta} d^\beta(\ell)$$

with explicit values of $K(s, \delta, \beta)$ and optimal values for β. When $s = 1$ and $\delta \in (0, 1/2]$, any $\beta \geq 2$ of the form $\beta = \epsilon + \delta^{-1} + (\delta \log \delta + (1 - \delta) \log(1 - \delta))/(\delta \log 2)$ for some $\epsilon > 0$ is admissible, with a constant $K(s, \delta, \beta)$ equal to $(\delta^{-5} \min(1, \epsilon))^{-3/\delta^3}$. This elaborates on work of Munshi in that the constants are effective (and even explicit) and that the inequality is also valid for non-especially square-free integers. We also give more specific inequalities for square-free integers.

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Mertens' theorems for Galois extensions

Seiken Saito

Mertens' theorem is the asymptotic formula for the product $\prod_{p\leq x}(1-1/p)$ with primes p less than or equal to a positive number x (see [1]). In 1974, K.S. Williams gave a generalization of Mertens' theorem for arithmetic progressions (see [2]). More precisely, Williams' theorem is the asymptotic formula for the product $\prod_{p\leq x,\ p\equiv a \pmod q}(1-1/p)$ for given natural numbers a and q with $(a,q)=1$. By the class field theory, the congruence condition $p\equiv a \pmod q$ is equivalent to the condition $\varphi_p=\{g\}$ for the Frobenius map φ_p. Here g is an element of the Galois group G of an abelian extension $L/\mathbf{Q}$, and $\{g\}$ denotes the conjugacy class of g in G. In this point of view, we give a generalization of Mertens-Williams theorem for Galois extensions of number fields. This talk is based on my joint work with Takehiro Hasegawa (Shiga University, Japan).

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Characteristic Sets of Lengths

Wolfgang A. Schmid

For a (finite) abelian group $(G, +)$, one calls a sequence $g_1 \dots g_n$ a zero-sum sequence when $g_1 + \cdots + g_n = 0$. A zero-sum sequence can be decomposed (or factored) into minimal zero-sum sequences, that is zero-sum sequences that do not have a proper subsequence that is also a zero-sum sequence. In general, there are various ways to do this. One calls the number of minimal zero-sum sequences occurring in such a factorization the length of the factorization. The set of lengths of a zero-sum sequence is the set formed by the lengths of all the factorizations of the zero-sum sequence.
The investigation of these sets of lengths is central in Factorization Theory as it models the phenomena one encounters when studying factorizations in a variety of different algebraic structures of interest.
The guiding theme of the talk is the following pair of questions.

- Which types of sets are typical sets of lengths, in the sense of being sets of lengths for almost all groups?
- Which types of sets of lengths are distinctive, in the sense of being sets of lengths for a few special groups only?

An overview over some classical and recent results is given.

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On Zeros of a Polynomial in a Finite Grid: the Alon-Füredi Bound

John R. Schmitt

A 1993 result of Alon and Füredi gives a sharp upper bound on the number of zeros of a multivariate polynomial over an integral domain in a finite grid in terms of the degree of the polynomial. This result was recently generalized to polynomials over an arbitrary commutative ring, assuming a certain "Condition (D)" on the grid which holds vacuously when the ring is a domain. We give a further Generalized Alon-Füredi Theorem, which provides a sharp upper bound when the degrees of the polynomial in each variable are also taken into account. This yields in particular a new proof of Alon-Füredi. We then discuss the relationship between Alon-Füredi and results of DeMillo-Lipton, Schwartz and Zippel.

This is joint work with Pete L. Clark (Georgia), Anurag Bishnoi (Ghent) and Aditya Potukuchi (Rutgers).

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On the structure of sets with a small doubling property in torsion free groups

Yonutz V. Stanchescu

In this talk we will present some new results about sets with a small doubling property in torsion free groups.

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Some new problems and results in combinatorial and additive number theory

Zhi-Wei Sun

In this talk we introduce various new conjectures of the speaker in combinatorial and additive number theory as well as related progress. We mainly focus on additive problems related to permutations and combinatorial properties of the prime-counting function $\pi(x)$. For example, we conjecture that for any finite subset A of an abelian group G with $|A| = n > 3$. there is a numbering $a_1, \ldots, a_n$ of all the n elements of A such that

$$a_1 + a_2 + a_3,\ a_2 + a_3 + a_4,\ \ldots,\ a_{n-2} + a_{n-1} + a_n,\ a_{n-1} + a_n + a_1,\ a_n + a_1 + a_2$$

are pairwise distinct, and confirm this when G is torsion-free. Our problems on the prime-counting function depend on some exact values of $\pi(x)$, in this direction we show that for any integer $m > 4$ this is a positive integer n such that $\pi(mn) = m + n$.

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Maximum Size of a Set of Integers with no Two adding up to a Square

Endre Szemerédi

Erdös and Sárközi ask the maximum size of a subset of the first N integers with no two elements adding up a perfect square. We will obtain a tight answer namely will prove that the size of the largest set is $\frac{11}{32} \cdot N$ for sufficient large N. We will describe the structure of the extremal sets and we are going to prove some stability results.
This is a joint work with Ayman Khalfalah and Simao Herdade.

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Upper densities and Darboux properties

Salvatore Tringali

Let μ^* be a real-valued function on $\mathcal{P}(\mathbf{N})$, the power set of $\mathbf{N}$. We say that μ^* is an upper quasi-density if $\mu^*(\mathbf{N}) = 1$ and, for all $X, Y \subseteq \mathbf{N}$ and $h, k \in \mathbf{N}^+$, it holds $\mu^*(X \cup Y) \leq \mu^*(X) + \mu^*(Y)$ and $\mu^*(k \cdot X + h) = \frac{1}{k}\mu^*(X)$, where $k \cdot X + h := \{kx + h : x \in X\}$. If μ^* is an upper quasi-density and $\mu^*(X) \leq \mu^*(Y)$ whenever $X \subseteq Y \subseteq \mathbf{N}$, then μ^* is called an upper density. Upper densities include some, though not all, of the most remarkable "densities" that have been considered in number theory and additive combinatorics to study, loosely speaking, the relation between the "structure" of a set of integers and information about its "largeness": In particular, the upper asymptotic, upper logarithmic, upper Banach, upper Buck, upper Pólya, and upper analytic densities, together with the upper α-densities, are all examples of upper densities in the sense of the above definition, see [4, Sections 3 and 4 and Examples 4-7]. On the other hand, it is known that non-monotone upper quasi-densities do actually exist, see [4, Theorem 1], which makes it interesting, at least from a fundamental point of view, to understand if certain properties of upper densities depend or not on the assumption of monotonicity.

With this in mind, we say that μ^* has the strong Darboux property if for all $X, Y \subseteq \mathbf{N}$ and $a \in [\mu^*(X), \mu^*(Y)]$ there exists a set A such that $X \subseteq A \subseteq Y$ and $\mu^*(A) = a$.

We will show that if μ^* is an upper quasi-density then it has the strong Darboux property, and so does the associated lower density $\mu_* : \mathcal{P}(\mathbf{N}) \to \mathbf{R} : X \mapsto 1 - \mu^*(\mathbf{N} \setminus X)$, see [5].

In addition, if time permits, we will hint how the proof of this theorem can be revisited, see [6], to make it work for a much larger class of "densities" and, more in general, of real-valued set functions (including weighted densities, Dirichlet densities and measures with suitable properties), which unifies, extends and, in some case, strengthens previous, and seemingly unrelated, results by various authors, see, e.g., [9], [2], [8], [1], [7], and [3].

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Solutions of polynomial equation over $\mathbb{F}_p$ and new bounds of energy of multiplicative subgroups

Ilya Vyugin

We study an algebraic equation $P(x,y) = 0$ over a field $\mathbb{F}_p$, where p is a prime. Let $P \in \mathbb{F}_p[x,y]$ be a polynomial of two variables x and y, G be a subgroup of $\mathbb{F}_p^*$. We study the upper bound of the number solutions of the polynomial equation, such that $x \in g_1G$, $y \in g_2G$. The estimate

$$\#\{(x,y) \mid P(x,y) = 0,\ x \in g_1G,\ y \in g_2G\} \leqslant 16mn^2(m+n)|G|^{2/3}.$$

is obtained using Stepanov method. This estimate was obtained by a different method in the paper [1]. We improve this estimate in average.

Let us consider a homogeneous polynomial $P(x,y)$ of degree n such that $\deg P(x,0) \geqslant 1$, $P(0,0) \neq 0$ and $l_1,\ldots,l_h$ belong to different cosets g_iG. We estimate the sum N_h of numbers of solutions of the set of equations:

$$P(x,y) = l_i, \qquad i = 1,\ldots,h, \quad x \in g_1G_1,\ y \in g_2G.$$

Then the sum N_h does not exceed $32h^{3/4}n^5|G|^{2/3}$.

Now let us consider some generalization of the additive energy which we call *polynomial energy*. Polynomial energy is the following

$$E_P^q(A) = \#\{(x_1,y_2,x_2,y_2) \mid P(x_1,y_1) = P(x_2,y_2),\ \ x_1,y_1,x_2,y_2 \in A\},$$

where $P(x,y) \in \mathbb{F}_p[x,y]$ is a polynomial.

Theorem *Let us suppose that* $100n^3 < |G| < \left(\frac{p}{3}\right)^{\frac{12}{17}}$, $P \in \mathbb{F}_p[x,y]$ *is a homogeneous polynomial. Then the following holds: if* $q \leqslant 3$ *then* $E_P^q(G) \leqslant C(n,q)|G|^{\frac{7q+16}{12}}$;
if $q = 4$ *then* $E_P^4 \leqslant C(n,q)|G|^{1+\frac{2q}{3}} \ln |G|$;
if $q \geqslant 5$ *then* $E_P^q(G) \leqslant C(n,q)|G|^{1+\frac{2q}{3}}$, *where* $C(n,q)$ *depends only on* n *and* q.

The results of the talk can be found in the paper [2].

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Additive properties of sequences on semigroups

Guoqing Wang

Let $\mathcal{S}$ be a commutative semigroup, and let T be a sequence of terms from the semigroup $\mathcal{S}$. We call T an (additively) *irreducible* sequence provided that no sum of its some terms vanishes. The Davenport constant of $\mathcal{S}$, denoted $\mathrm{D}(\mathcal{S})$, is defined to be the least $\ell \in \mathbb{N} \cup \{\infty\}$ such that every sequence T of terms from the semigroup $\mathcal{S}$ of length at least ℓ is reducible. In this talk, we shall present some recent results on additive properties of sequences of terms from semigroups, which mainly focus on the Davenport constant and irreducible sequences in commutative semigroups.

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Unsplittable minimal zero-sum sequences over C_n

Pingzhi Yuan

In this talk, we discuss the latest results on the structure and the index of unsplittable minimal zero-sum sequences over finite cyclic groups.

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The characterization of minimal zero-sum sequences over finite cyclic groups

Xiangneng Zeng

In Zero-sum Theory, one of the main objects is to study the minimal zero-sum sequence. In this talk, we mainly consider the minimal zero-sum sequences over finite cyclic groups and characterize the structures in some cases. This talk contains two parts. Let G be a cyclic group of order n and S be a minimal zero-sum sequence. First we characterize the structure of S provided that n is odd and $|S| \geq \lfloor n/3 \rfloor + 3$. Next we show that $\mathrm{Ind}(S) = 1$ if $\gcd(n, 30) = 1$ and $|S| = 4$.

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Discrete spheres and arithmetic progressions in product sets

Dmitrii Zhelezov

We prove that if B is a set of N positive integers such that $B \cdot B$ contains an arithmetic progression of length M then $N \geq \pi(M) + M^{2/3-o(1)}$. On the other hand, there are examples for which $N < \pi(M) + M^{2/3}$. This improves previously known bounds of the form $N = \Omega(\pi(M))$ and $N = O(\pi(M))$, respectively.
The main new tool is a lower bound for the size of an additive basis for the 3-sphere in $\mathbb{F}_3^n$ which is the set of 0-1 vectors with exactly three non-zero coordinates. Namely, we prove that such a set cannot be contained in a sumset $A + A$ unless $|A| \gg n^2$.

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