How to choose between fixed and variable rate loans

Edwin O. Fischer, Lisa-Maria Kampl



Working Paper 2018-04

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JEL: G21, G31, G32

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September 2018

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Abstract

The current low interest rate landscape influences the decision whether to secure these low long-term interest rates with a fixed rate agreement or to benefit from the current negative short-term rates with a variable rate loan. The decision is based on expected future interest rates. We propose two criteria, namely the effective interest rate and the total repayments, to decide what type of loan is more advantageous. In this context, we also present the effects of varying future interest rates on the three selected repayment agreements; i.e. lump sum, constant principal and annuity repayments.

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Introduction

Since March 2016, the European Central Bank's (ECB) main refinancing rate, which allows banks to borrow from the ECB, is at its lower bound of 0%. Before the crisis in 2008, this rate was still at 4%. This low and enduring interest rate environment has led to a gradual reduction in lending rates on loans. While the average lending rate on loans for housing purpose before the crisis was 5%, the average interest rate currently is 2%. Another important development, due to the low interest rates, is the allocation of long-term fixed interest loans with a maturity of 20 and up to 30 years. Such long-term fixed interest loans were not existent before the crisis. Demand for these long-term fixed rate loans is on the rise, as the par yield curve, which serves as a reference for these loan rates, is flattening after 15 years. This fact leads to relatively low fixed interest rates for loans with a term over 20 years. By taking out a fixed rate loan, the debtor can secure the currently prevailing low interest rates for a long period of time. However, due to the currently negative Euribor rates that serve as reference rates for variable loans, the current lending rates for the variable variant are around 1% for debtors with a high creditworthiness. Yet there is a risk of rising interest rates that is expected soon. Nevertheless, in the future, possible higher variable interest rates will only affect the outstanding face value of the loan when calculating interest payments. When deciding whether to take a fixed rate or variable rate loan consumers and companies are faced with a decision where they have to make a prediction of the path of future interest rates. The optimal choice between a fixed rate and variable rate loan is mostly made in practice based on the effective interest rate as it would be if only fixed rate loans are compared. A decision based on the advertised effective interest rates in loan offers is problematic since banks make non-representative assumptions regarding future interest rates and therefore future interest payments of variable rate loans are not realistic. In loan offers it quite often appears that a variable rate loan is always advantageous since the most used reference rates (the 3M Euribor as well as the 6M Euribor) are both negative and the risk premium which is additionally charged is around 1.25% for debtors with a high creditworthiness. However, there is a risk of rising interest rates that is expected in the near future after the ECB terminates its asset purchase programs and raises the main refinancing rate again.

Although the decision has a great impact on future payments only little literature deals with this specific decision-making process. Most literature focuses on determinants of loan choice. One of these determinants is the spread between long-term yields that are linked to long-term fixed interest rates and the average of recent short-term interest rates, which serve as reference rates for variable rate loans (Koijen, Hemert and van Nieuwerburgh, 2009). However, no explicit key figure or method is presented in order to make the optimal decision between two or more loan offers when variable rate loans are involved.

As a result, we propose two different criteria, namely the effective interest rate (dynamic criterion) and the total repayments/average monthly repayments (static criterion) arising from a loan, which serve as a basis for a loan decision. In addition to that, we offer an instruction how financial advisors can find the optimal loan by some modifications of their calculations. In order to understand why this topic is relevant Section 1 gives a short overview of the current interest landscape in the Eurozone as well as its development since the financial crisis in 2008. In that context, we also present the current lending rates for different types of loans in Austria and Germany in Section 2. In Section 3 we then take a closer look at the EU and national regulations and laws regarding the calculation of the effective interest rate, which have to be stated in a loan offer if the debtor is a consumer. In Section 4, we then give instructions how the optimal loan choice for different payback agreements is made. Moreover, in Section 5 we show the effects of data modifications and the payback to payout ratio. Section 6 gives a short conclusion.

1 Current interest rates in the euro zone

Since the financial crisis in 2008 interest rates are declining in the euro zone. As a reaction to the global financial crisis and the following European sovereign debt crises, the ECB lowered the key interest rate from 4% to 1% and finally to the zero lower bound in March 2016 in order to stimulate the European economy and to prevent financial instability. The main refinancing rate serves as an indicator for other interest rates, e.g., the 3M, 6M or 12M Euribor that are most commonly used as reference rates for variable rate loans in the euro zone. In addition to lowering the base rate to its effective lower bound the ECB implemented various asset purchase programs after undertaking some longer-term refinancing operations since 2008 to fulfil the task of price stability in the euro zone. This price stability is defined as an annual inflation of below but close to 2% measured by the Harmonised Index of Consumer Prices (HICP). These asset purchase programs lowered long-term yields as can be obtained from the following figures. Through various transmission channels these asset purchase programs were designed to stimulate the GDP growth of the euro zone and consequently boost inflation.

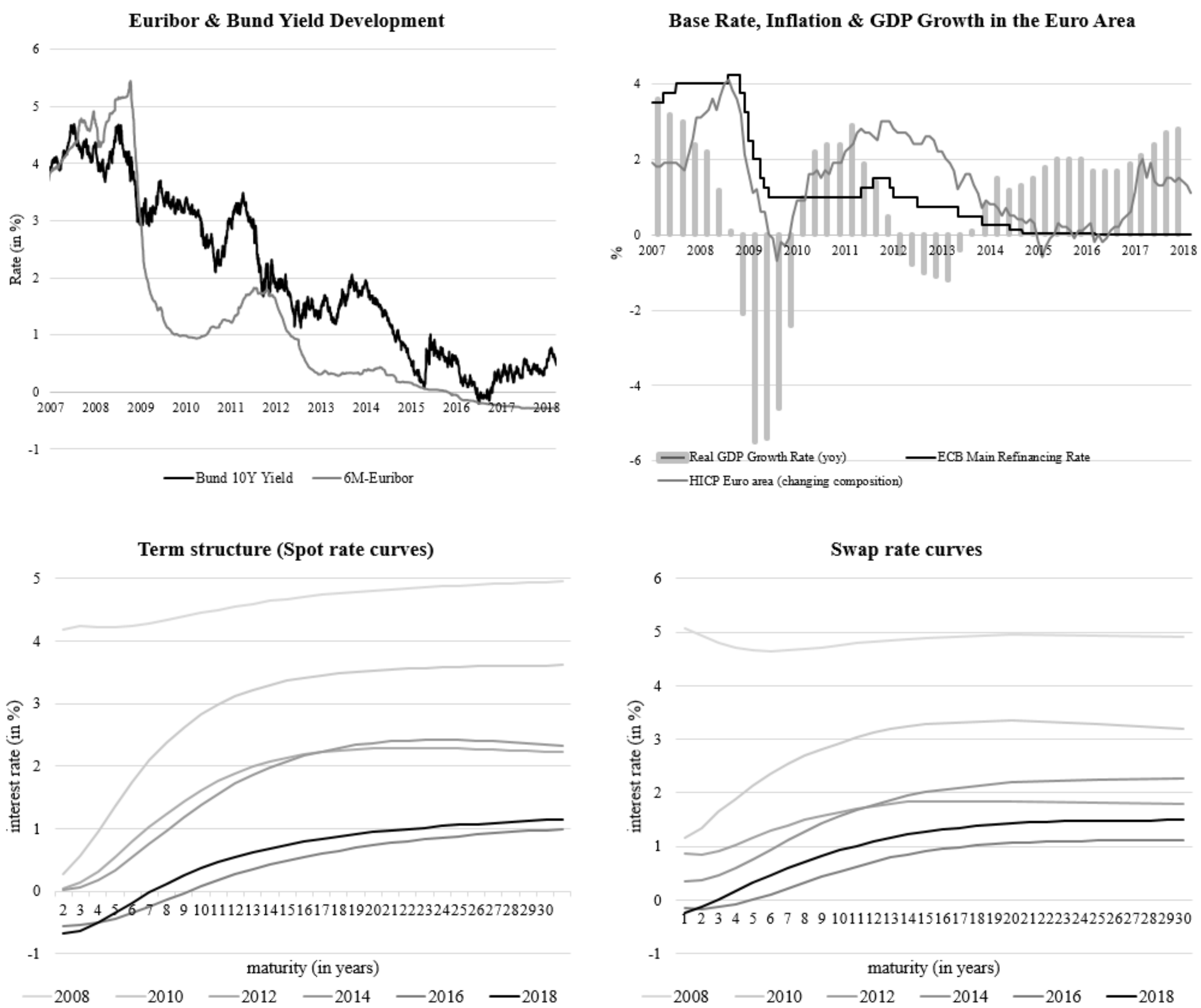


Figure 1: Interest landscape in the Eurozone

The term structure that is derived of AAA-rated, EUR denominated government bonds in the euro zone serves as an indicator for the current state of economy and gives an overview of current interest rates. Figure 1 shows the term structure (spot rate curves) from 2007 to 2018 for a maturity range up to 30 years. The development (from light to dark) shows that spot rates for all maturities decreased over the last ten years after the financial crisis. In the last years, the spot rates for maturities up to nine years were even below zero. The latest curve (black line) shows that the spot rates until a maturity of six years are negative. The right side of figure 1 shows the development of swap rate curves (interest rate swap) for the same period of time and maturity range. The trend of swap rates is the same as for spot rates.

2 Loan business in Austria and Germany

To understand why the decision between fixed and variable rate loans is currently a topic of great importance, we look at lending rates and volume of new loans in Austria and Germany. Before the financial crisis lending rates for both fixed and variable rate loans increased due to the tense economic environment until they reached their peak in 2008. With the burst of the real estate bubble and the following global financial crisis, lending rates fell swiftly. Since then a constant downward trend for lending rates for fixed rate loans to households for housing purpose as well as to non-financial institutions can be observed. The development of interest rates in Austria for fixed rate loans to households for consumption slightly differs from the above-mentioned ones. Not only is the level of these rates higher, also the trend is not downward. Instead, a sideways trend is observed. That can be explained because these loans do not require any collateral as it would be mandatory for residential purpose loans. Therefore, a higher risk premium is charged.

When comparing lending rates for variable and fixed rate loans in Austria it can be seen that the rate for fixed loans is higher for housing purpose loans as well as for loans to companies. However, the reverse is true if loans for consumption purpose are examined. The current lending rates for housing purpose loans to households are 1.52% (2.22%) for variable rate loans (fixed rate loans). As mentioned above, lending rates are higher for consumption loans and amount to 5.30% (5.07%) for variable rate loans (fixed rate loans). Lending rates for loans to non-financial institutions with a face value of more than one million euro amount to 1.29% (2.15%) for variable rate loans (fixed rate loans).

Due to the persistently low interest rate environment especially in the longer-term, the share of fixed interest loans increased consistently and amounts to 58.34% (2017: 43,71%) in May 2018 for housing purpose loans and to 25.13% (2017: 19.66%) for consumption loans. The higher share of variable rate consumption loans can be explained because these loans generally have a shorter maturity than loans for housing purpose and thus there is less risk of rising interest rates. Nevertheless, due to low current interest rates there is also a trend towards fixed rate loans. The share of fixed rate loans to non-financial institutions is much lower but also shows an increasing trend to 19.19% (2017: 17.30%) in May 2018. That development shows that both private and institutional debtors want to secure the current low interest rates over a longer time period as rising interest rates are only a matter of (short) time. In addition, fixed income loans offer the advantage of being able to plan the future amount of repayments.

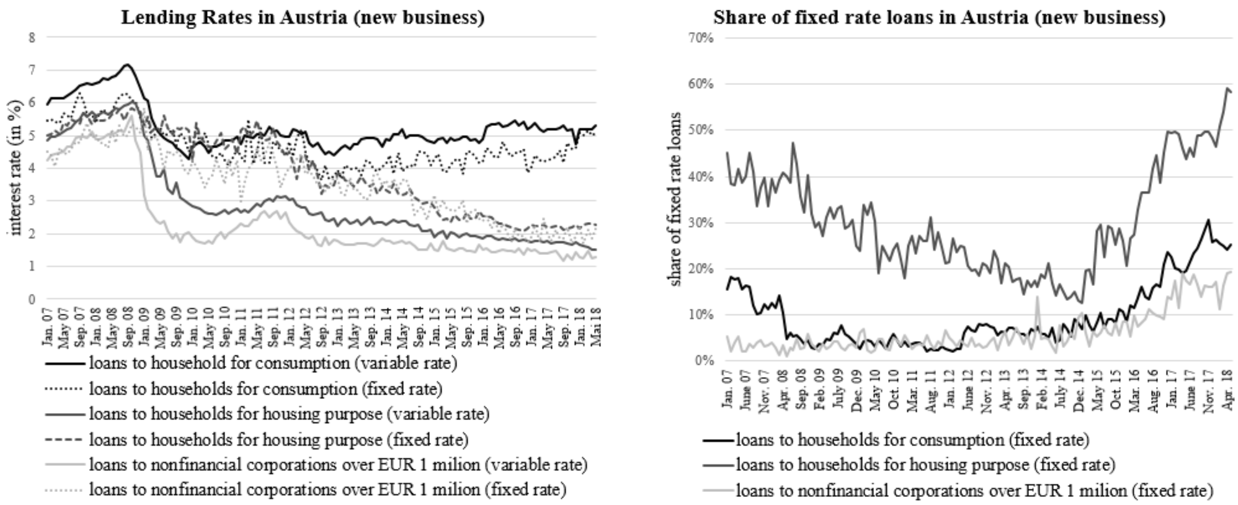


Figure 2: Loan business in Austria (new business)

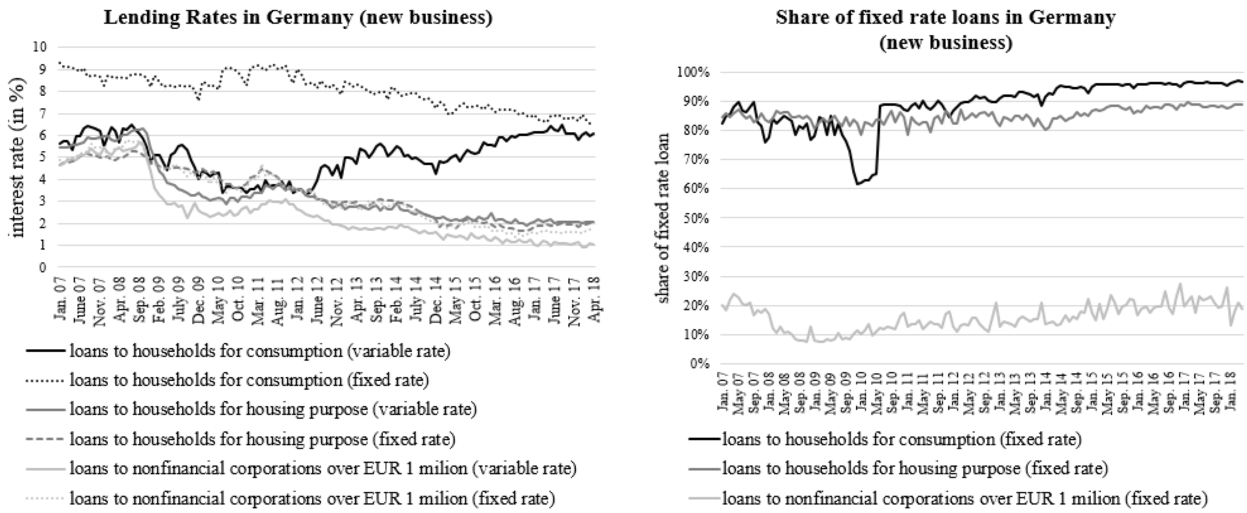


Figure 3: Loan business in Germany (new business)

In comparison to Austria, lending rates in Germany are slightly higher for consumption loans to private households. In addition to the difference in the level, also the development differs as an upward trend can be observed for fixed lending rates for consumption loans to private households. Another difference between these two countries is the share of fixed rate loans. While the share of fixed rate loans in Germany varies between 70% and nearly 100% (96.76% in May 2018), the share of fixed rate loans to private households in Austria is far below that level (25.12% in May 2018). The share of fixed rate loans to non-financial institutions with a face value over one million euro is almost the same and amounts to 19.93% (19.19%) in May 2018 in Germany (Austria). While the share of fixed rate loans to companies has been almost constant over the last decade in Germany an upward trend can be observed in Austria since 2016. Before the share of fixed rate loans in Austria was below 10%. The two figures show that the lending rates as well as the share of fixed rate loans differ between the two countries nowadays and in the past.

3 Determinants of loan choice

The main question is now based on which determinant or key figure a debtor should choose a variable rate loan or a fixed rate loan. Therefore, we provide a short overview of the different components of fixed and variable loan rates. Whereas the interest rate of both forms depends on the face value, term and payback agreement other factors vary. The interest rate of a fixed rate loan is composed of a riskless interest rate which is linked, e.g., to the riskless par yield and is fixed at the beginning of the loan. A margin that represents the creditor's operating costs as well as a risk premium that reflects the creditworthiness of the debtor are charged in addition to get the fixed interest rate i^{fix} of a fixed rate loan. When deriving the interest rate i_t^{var} for a variable rate loan the basis is a reference rate, which is a short interest rate. The development of the variable interest rate is tied to this short interest rate. In comparison to the fixed interest rate the variable interest rate will be updated in specific time intervals to the determined reference rate at the beginning of each adjustment period. These interest adjustments are made mainly quarterly, semi-annually or annually, as in the case of savers' building loans. To this varying short-term interest rate, a margin and a risk premium RP^{var} are added to get the variable interest rate.

The selection of possible determinants that can be used to make a decision between fixed and variable rate loans is diverse and ranges from the term structure of interest rates to the term of the loan. Other possible determinants are the expected future reference rates, disbursement discount as well as the risk premia for fixed and variable rate loans. In this matter, we propose two different criteria that serve as a decision variable in this article. The first key figure we propose is the effective interest rate, which serves as the exact or dynamic criterion. The effective interest rate is a dynamic criterion as it takes into account the chronological sequence of payments. From a financial perspective, the decision for an investment should be based on maximizing the shareholder value or in other words the net present value. However, a decision based on the effective interest rate can differ from the decision that would be made based on the net present value. In our specific case, it is not problematic because of the assumptions we made regarding the occurring payments. The second criterion, namely the total repayments (or average monthly repayments) is a static criterion and serves as an approximation for the dynamic one. Payments include principal repayments as well as interest payments. We use this static criterion in order to offer a more simple decision variable in practice.

3.1 Effective interest rate

As we propose the effective interest rate as a key figure to choose between fixed and variable rate loans we first want to define this criterion. The effective interest rate or rather the annual percentage rate of charge (APRC) represents the total costs of the credit to the debtor with the exception of any charges payable by the debtor that are not linked to any of his commitments laid down in the underlying credit agreement. In other words, the effective interest rate is the discount rate for which the present value of the payments from the creditor equals the present value of the repayments to the creditor. The effective interest rate is calculated to ensure comparability between different loan offers for debtors. In order to protect consumers special rules are applied concerning the effective interest rates and its calculation. In EU Directive 2008/48/EC on credit agreements for consumers and in Directive 2014/17/EU on credit agreements for consumers relating to residential immovable property the rules used to calculate the effective interest rate of fixed and variable rate loans are found. These two EU Directives were set up to enable consumers to appraise a bank's loan offer by the effective interest rate and to choose the optimal loan from different offers by means of this key figure. Directive 2014/17/EU lists all costs that are taken into account when calculating the effective interest rate. These costs include interest payments, costs of maintaining an account recording both payment transactions and drawdowns, commissions for credit intermediaries, administration fees at the beginning of the loan (except for notarial costs) and other costs, e.g., fee for a creditworthiness check of the debtor. Total costs exclude costs that the debtor pays to or receives from third parties, e.g., immovable property or land-associated taxes, notarial costs or costs for land registration or mortgage registration and possible commissions to loan brokers. These costs are excluded although they can be rather high compared to the face value of the loan. The effective interest rate solely reflects all payments between the creditor and the debtor.

In these two Directives, also the underlying assumptions used by banks to calculate the effective interest rate are specified. One of these assumptions is that the credit agreement is to remain valid for the period agreed and creditors fulfil their obligations under the terms and by the dates specified in the credit agreement. Due to this assumption, no prematurity compensation costs have to be taken into account. The most important assumption regarding variable rate loans is that if the credit agreement allows variation in the borrowing rate, the effective interest rate is calculated on the assumption that the borrowing rate will remain fixed in relation to the initial level based on the value of the agreed reference rate at the time. In addition, charges that are unquantifiable at the time of calculation will also remain fixed in relation to the initial level.

On national level loans in general, e.g., loans to non-financial institutions, are regulated in the Austrian Civil Code (ABGB). In contrary to consumer loans, the effective interest rate does not have to be declared in credit agreements with companies. Additional regulations for consumer loans are found in the Verbraucherkreditgesetz (VKrG), Hypothekar- und Immobilienkreditgesetz (HIKrG) and Bankwesengesetz (BWG §33 Abs. 6). These regulations refer to the EU Directives 2008/48/EC and 2014/17/EU. In Germany, regulations concerning loans in general are in the German Civil Code (BGB). Analogous to Austria the effective interest rates do not have to be declared in credit agreements where the debtor is not a consumer. Additional regulation for consumer loans regarding the calculation of the effective interest rates and its declaration in loan offers are found in the Preisangabenverordnung (PAngV).

In addition to that, different effective interest rates have to be calculated whenever a loan allows variations in the underlying interest rate. Banks in the member states have to make sure that the debtor is aware of these possible changes and their impacts on the amounts payable. This is done in the European Standard Information Sheet (ESIS) where an additional effective interest rate is calculated to illustrate the impact of rising reference rates concerning the loan repayments.

3.2 Typical loan offers

To illustrate how typical loan offers and the resulting key figures are calculated based on the EU regulations we discussed in Section 3.1, we take the following simple example for a fixed and variable rate loan. The face value of 100,000 and a term of 15 years are identical for both loans. The disbursement discounts differ from each other and amount to 2% for the fixed rate loan and 3% for the variable rate loan. The interest rate for the fixed rate loan is 2.10% p.a., for the variable rate loan the interest rate is linked to the 6M Euribor (- 0.268% per August 3, 2018) and in addition, a risk premium of 1.50% p.a. is charged. As the variable rate loan allows variation in the lending rate it is assumed (as proposed in EU directive) that the initial interest rate will remain fixed in relation to the initial rate. Interest payments and principal repayments are due monthly. The disbursement discounts lead to credit payouts at t = 0 of 98,000 for the fixed rate loan and 97,000 for the variable rate loan. To show the impact of different repayment agreements we calculate our decision criteria for three different type of repayments, namely lump sum, constant principal and annuity repayment.

		Fixed rate	Variable rate
	Face value	100,000	100,000
	Payout at t = 0	98,000	97,000
	Future reference rates	Initial	Initial
Type of repayment	**Key figures**	**Fixed rate**	**Variable rate**
Lump sum	Effective interest rate	2.28% p.a.	**1.46% p.a.**
	Total repayments	131,500.00	**118,465.00**
	(Average) monthly repayments	730.56	**658.14**
Constant principal	Effective interest rate	2.42% p.a.	**1.68% p.a.**
	Total repayments	115,837.50	**109,283.79**
	(Average) monthly repayments	643.54	**607.13**
Annuity	Effective interest rate	2.41% p.a.	**1.66% p.a.**
	Total repayments	116,662.27	**109,567.61**
	(Average) monthly repayments	648.12	**608.71**

Table 1: Typical loan offers: results

Based on the assumptions made in the EU Directives for calculating the effective interest rate and the total repayments the variable rate loan is advantageous independent of the type of repayment. Although in practice decision are made based on these assumptions there are some critiques we want to highlight. First, if a decision is made based on the effective interest rate or on total (or average monthly) repayments better estimates on future reference rates are essential to capture the risk of rising interest rates in the future, which lead to higher future interest payments of the variable rate loan. This fact leads to the problem that a decision based on the effective interest rates can be false in a financial perspective for variable rate loan although this criterion secures a true decision for fixed rate loans.[1] In addition, in order to make an optimal decision better estimates of future reference rates, calculations have to be done for the same initial payoff of the loans if the decision is based on total (or average monthly) repayments. Otherwise the loan with the lower face value will be chosen given all other data is constant. Therefore, adjustments in the face value of (both) loans have to be done to secure the condition of matching loan payouts at t = 0. If the decision is based on effective interest rates, calculations have to be done for the same initial payoffs if costs that are independent of the (outstanding) face value exist, e.g., account management charges or costs for valuation of property. Otherwise, if all payments are proportional to the face value, different initial payoffs have no impact on the effective interest rate. In our case, all costs are proportional to the face value.

To deal with the first drawback of non-representative future interest rates, expected future reference rates have to be forecasted. There exist numerous possible ways to forecast future interest rates. As a first method, expert opinions can be brought in to forecast future interest rates. Secondly, macroeconomic analysis can be done to predict the future path of term structure (see, e.g., Kucera, 2017). Inflation, GDP growth or unemployment rate can be taken into account as macroeconomic variables in order to forecast future interest rates. Stochastic Diffusion Processes (see, e.g., Fischer and Zechner, 1984) and more advanced time series analysis (see, e.g., Radha and Thenmozhi, 2006) offer a wider range of possibilities to forecast interest rates. As a special form of stochastic diffusion process, the LIBOR Market Model (see, e.g., Brace, Garek and Musiela, 1997) can be used. Another possibility is to forecast future spreads that can be used to

[1] For explanation, see Fischer (2001).

calculate various future interest rates (see, e.g., Campbell and Shiller, 1991 or Dbouk, Jamali and Kryzanowksi, 2015). As for the euro zone, Euro TED spreads, EONIA-Swap spreads or base rate spreads can be predicted. In addition to that, implied future reference rates from future or option prices (see, e.g., Cesari and Sevini, 2004 or Gebbia, 2016 and ECB, 2018) are suitable to estimate future interest rates. The problem with these implied future reference rates it that they can be forecasted only for a short time horizon because of the short-term maturity of futures and options. For predicting interest rates for a longer time horizon, interest rate swap curves, forward swaps or swaptions can be used. These three methods are widely used in practice as they allow prediction of interest rates for longer time horizons (see, e.g., Danske Bank, 2018).

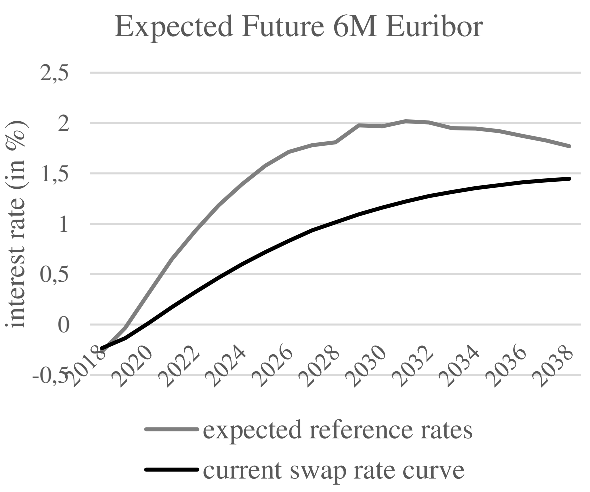
In our analysis, we form predictions for future reference rates based on the current interest rate swap curve. The expected future reference rates are calculated based on the pure expectation hypothesis (PEH). The hypothesis postulates that the expected future one-period spot rate equals the current one-period forward rate (Sangvinatsos, 2010). Based on the PEH in the discrete time case, we extract expected future reference rates from current interest rate swap rates.

$$(1 + IRS_t)^{\,t} = (1 + IRS_{t-1})^{\,t-1}(1 + Ref_{t-1})$$

Our estimates for future reference rates are:

$$Ref_{t-1} = \frac{(1+IRS_t)^t}{(1+IRS_{t-1})^{t-1}} - 1,$$

where IRS_t is the interest rate swap with maturity t. The reference rate Ref_{t-1} is then used to adjust the variable interest rate for period t. By taking the swap curve, we can calculate future expected reference rates over a long-time horizon, which is necessary in order to calculate future interest rates of variable rate loans. This is essential as loans, especially loans for housing purposes tend to have longer terms ranging from ten to thirty years. Our calculation allows us to predict these reference rates for the longer-term in comparison to other forecasting methods, which can be used only in the short run. For calculations we take the current swap rate curve (August 3, 2018) and extract expected future reference rates based on the PEH.



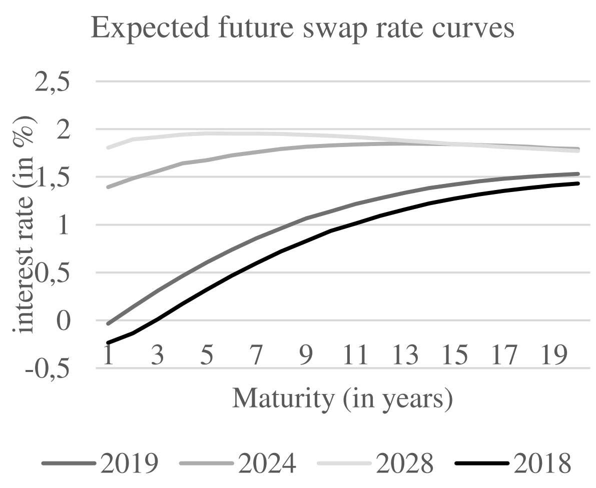


Figure 4: Expected future 6M Euribor and expected future swap rate curves

Which forecasting methods the decision maker uses in the end can vary from our method since we only provide two different criteria that can be combined with any forecasting method for future reference rates.

4 The optimal choice for different payback agreements

In the course of our analysis, we examine the three different repayment types lump sum repayment, constant principal repayment and constant annuity repayment to show the effect of changing interest on these three loan types. As mentioned before we assume that the same amount is paid out at t = 0 for both loans which is equivalent to

$$\left(1-d^{fix}\right)Nom^{fix} = \left(1-d^{var}\right)Nom^{var} \Leftrightarrow Nom^{var} = \frac{1-d^{fix}}{1-d^{var}}Nom^{fix},$$

where d^{fix} (d^{var}) is the disbursement discount of the fixed (variable) rate loan that lowers the payout at t = 0. This assumption allows us to write the face value of the variable rate loan in terms of the face value of the fixed rate loan. Furthermore, we assume that both loans are uncallable so no prematurity compensation payment is taken into account. In addition to that, the initial costs are given and summarized in the disbursement discount. Furthermore, no additional costs occur during the term of the loans.

Repayment type	**Lump sum repayment**	Case I
	Constant principal repayment	Case 2
	Constant annuity repayment	Case 3

Table 2: Overview of possible cases

We use our two proposed key figures to decide whether the fixed or the variable rate loan is advantageous. According to the exact method the decision is made based on the effective interest rates. Therefore, we calculate the effective interest rates for both loans and then compare them straightforward. As mentioned in Section 3.1, the effective interest rate is the discount rate, which ensures equality between the present value of the payments from the creditor and the present value of the payments to the creditor. Payments from the creditor are in this case only the payout at t = 0. Payments to the creditor include the principal repayments as well as interest payments by the debtor. To calculate the effective interest rate i_{eff} we solve the following equation:

$$(1-d)Nom = \sum_{t=1}^{T} \frac{C_t}{(1+i_{eff})^{t}},$$

where C_t represent the total cashflows paid by the debtor due to a specific loan agreement. In this context, the variable rate loan is advantageous if

$$i_{eff}^{fix} > i_{eff}^{var}$$

is valid and vice-versa. The second method is an approximation and is based on the total repayments or average payments of the fixed and variable rate loan. The variable rate loan is advantageous if

$$\frac{1}{T}\sum_{t=1}^{T} C_t^{fix} > \frac{1}{T}\sum_{t=1}^{T} C_t^{var}$$

is valid, where C_t^{fix} and C_t^{var} are the total cashflows, which arise from both loans.

4.1 Case I: Lump sum repayment

In Case I, we compare a fixed and a variable rate loan each with lump sum repayment. Periodic interest payments and a single repayment at maturity characterize this type of repayment. As no repayments are made during the term and thus the outstanding nominal value does not change over the term, increases in the interest rate for variable loans have the greatest impact. For the fixed rate loan interest payments amount to

$$Z^{fix} = i^{fix} * Nom^{fix}$$

and are constant over the loan term. The amount of interest payments of the variable loan changes with the future expected reference rates and result in

$$Z_t^{var} = i_t^{var} * \frac{1-d^{fix}}{1-d^{var}} Nom^{fix},$$

with

$$i_t^{var} = Ref_{t-1} + RP^{var}.$$

The variable interest rate i_t^{var} is composed of an underlying reference rate Ref_{t-1} and a risk premium RP^{var}, which is fixed at the beginning. Since periodic adjustments to the current reference rate are made, the variable interest rate is not constant over the loan term. As current reference rates are negative, we have to ensure that the interest rate for the variable rate loan is non-negative.

$$Z_t^{var} = \max(0; i_t^{var}) * \frac{1-d^{fix}}{1-d^{var}} Nom^{fix}$$

An upper cap for the variable interest rate is not specified in this matter. In order to calculate our two proposed key figures for choosing a loan we first determine all relevant payments.

	t = 0	t = 1	t = 2	…	t = T
Fixed rate	$(1-d^{fix})Nom^{fix}$	$-Z^{fix}$	$-Z^{fix}$		$-Z^{fix} - Nom^{fix}$
Variable rate	$(1-d^{\mathrm{var}})Nom^{\mathrm{var}}$	$-Z_1^{var}$	$-Z_2^{var}$		$-Z_T^{var} - Nom^{\mathrm{var}}$
Variable - fixed	0	$Z^{fix} - Z_1^{var}$	$Z^{fix} - Z_2^{var}$		$Z^{fix} - Z_T^{\mathrm{var}} - \frac{\mathrm{d^{var}} - \mathrm{d^{fix}}}{1-\mathrm{d^{var}}} \mathrm{Nom^{fix}}$

Table 3: Lump sum repayment loan cash flows

Based on the effective interest rate i_{eff}^{fix} and i_{eff}^{var} the fixed rate loan is advantageous if $i_{eff}^{fix} < i_{eff}^{var}$ is true. In order to calculate these two effective interest rates we solve the two following equations:

$$\left(1-d^{fix}\right)Nom^{fix} = \sum_{t=1}^{T} \frac{Z_t^{fix}}{\left(1+i_{eff}^{fix}\right)^t} + \frac{Nom^{fix}}{\left(1+i_{eff}^{fix}\right)^T}$$

and

$$(1-d^{var})Nom^{var} = \sum_{t=1}^{T} \frac{Z_t^{var}}{\left(1+i_{eff}^{var}\right)^t} + \frac{\frac{1-d^{fix}}{1-d^{var}} Nom^{fix}}{\left(1+i_{eff}^{var}\right)^T}.$$

The approximate solution is based total or average cashflows from both loans. In order to make a decision we calculate the difference of the monthly payments of both loans as can be seen in Table 3. The fixed rate loan is advantageous based on the average cashflows if the average difference is negative.

$$Z^{fix} - \frac{1}{T}\sum_{t=1}^{T} Z_t^{\mathrm{var}} - \frac{1}{T}\frac{\mathrm{d^{var}} - \mathrm{d^{fix}}}{1-\mathrm{d^{var}}} \mathrm{Nom^{fix}} \leq 0$$

That can be simplified to

$$i^{fix} \leq \frac{1-d^{fix}}{1-d^{var}} Ref_{\emptyset} + \frac{1-d^{fix}}{1-d^{var}} RP^{var} + \frac{1}{T} \frac{d^{var}-d^{fix}}{1-d^{var}},$$

with

$$Ref_{\emptyset} = \frac{1}{T} \sum_{t=1}^{T} Ref_{t-1}.$$

This equation allows us to calculate the critical fixed interest rate for which the debtor is indifferent between a fixed and variable rate loan. This critical interest rate can be used as a basis for loan negotiations. For better illustration, we take the same simple numerical example we used in Section 3.2 to choose between a fixed rate and variable rate loan.

As the approximation requires equal credit payments at t = 0 we change the face value of both loans in order to secure this correspondence. The adjusted face values Nom^{adj} are calculated with

$$Nom^{adj} = \frac{Nom}{1-d},$$

where Nom is the (old) face value that we used in Section 3.2. The adjustment is not required when a consumer makes his/her decision based on the effective interest rate and all costs are proportional to the face value, which is true in our example. The term of both loans is equal and amounts to 15 years. The interest payments are made monthly and the variable interest rate is adjusted annually to the 6M Euribor at the beginning of the adjustment period. As mentioned, the costs at disbursement differ and amount to 2% for the fixed rate loan and 3% of the face value for the variable rate loan. The interest rate of the fixed rate loan is determined at 2.10% at the beginning of the loan. The interest rate of the variable rate loan is bound to the development of the 6M Euribor and an additional risk premium of 1.50%, which represents the creditworthiness of the debtor, is charged. Based on the assumption that future reference rates equal the initial reference rate over the loan term the effective interest rate of the variable rate loan (1.46%) is lower than the effective interest rate of the fixed rate loan (2.28%). In addition to that, also the total repayments from the fixed rate loan (134,183.67) are higher than the ones of the variable rate loan (122,128.87). Based on our forecast for expected future reference rates the effective interest rate is 3.06% for the variable rate loan and again 2.28% for the fixed rate loan as this one is not influenced by future interest rates. Based on the assumption of fixed reference rates a false decision would be made and the consumer would choose the variable rate loan as it seems to be more advantageous. This false decision would be taken based on typical loan offers since exactly this assumption regarding future reference rates are made in these offers when calculating the effective interest rate as laid out in detail in Section 3.1. In contrary, if the decision is made based on our proposed assumptions of future reference rates, the consumer would make the right decision. Since the effective interest rate of the fixed rate loan is smaller than the effective interest rate of the variable rate loan, the fixed rate loan is more advantageous. Based on the approximate solution the same decision is proposed since the average monthly repayments arising from the fixed rate loan (745.46) are lower than the average monthly cashflows of the variable rate loan (818.60). Both criteria lead in our example to the same decision but as the term of the loan gets longer we found out that based on the effective interest rates a variable rate loan is more advantageous sooner than based on the static approximation.

4.2 Case 2: Constant principal repayments

In Case 2, we examine the payback type of constant principal repayments. These loans are characterized by constant principal repayments that amount to $\frac{1}{T}Nom$ in each repayment period.

	t = 0	t = 1	t = 2	…	t = T
Fixed rate	$(1-d^{fix})\ Nom^{fix}$	$-\frac{1}{T}Nom^{fix}-Z_1^{fix}$	$-\frac{1}{T}Nom^{fix}-Z_2^{fix}$		$-\frac{1}{T}Nom^{fix}-Z_T^{fix}$
Variable rate	$(1-d^{var})Nom^{var}$	$-\frac{1}{T}Nom^{\mathrm{var}}-Z_1^{var}$	$-\frac{1}{T}Nom^{\mathrm{var}}-Z_2^{var}$		$-\frac{1}{T}Nom^{\mathrm{var}}-Z_T^{var}$
Variable - fixed	0	$Z_1^{fix}-Z_1^{var}$ $-\frac{1}{T}\frac{d^{var}-d^{fix}}{1-d^{var}}Nom^{fix}$	$Z_1^{fix}-Z_2^{var}$ $-\frac{1}{T}\frac{d^{var}-d^{fix}}{1-d^{var}}Nom^{fix}$		$Z_T^{fix}-Z_T^{var}$ $-\frac{1}{T}\frac{d^{var}-d^{fix}}{1-d^{var}}Nom^{fix}$

Table 4: Constant principal repayments loan cash flows

Interest payments Z_t^{fix} are calculated in each period as a percentage of the outstanding nominal amount of the loan and amount to

$$Z_t^{fix} = i^{fix} * Nom_{t-1}^{fix}.$$

Since constant principal repayments reduce the outstanding nominal amount in each period, interest payments decrease in each period by $\frac{1}{T}i_t * Nom^{fix}$. The outstanding face value in period t is therefore

$$Nom_{t-1}^{fix} = Nom^{fix} - \frac{1}{T}(t-1)Nom^{fix}$$

for both the fixed rate loan and the variable rate loan. The interest payments of the variable rate not only depend on the outstanding face value but also once again on the reference rate. The interest payments of the variable rate loan are

$$Z_t^{var} = i_t^{var} * Nom_{t-1}^{var}$$

with

$$i_t^{var} = \max(0; Ref_{t-1} + RP^{var}).$$

As in Section 4.2 we once again calculate the effective interest rate of both loans in order to make a decision based on the exact criterion. The approach to calculate these effective interest rates is the same as it was in the previous section and following equations have to be solved:

$$(1-d^{fix})Nom^{fix} = \sum_{t=1}^{T}\frac{Z_t^{fix}+\frac{1}{T}Nom^{fix}}{\left(1+i_{eff}^{fix}\right)^t}$$

and

$$(1-d^{var})Nom^{var} = \sum_{t=1}^{T}\frac{Z_t^{var}+\frac{1}{T}\frac{1-d^{fix}}{1-d^{var}}Nom^{fix}}{\left(1+i_{eff}^{var}\right)^t}.$$

Based on the static criterion of average monthly payments the fixed rate loan is chosen if

$$\frac{1}{T}\sum_{t=1}^{T}(Z_t^{fix}-Z_t^{var}) - \frac{1}{T}\frac{d^{\mathrm{var}}-d^{fix}}{1-d^{\mathrm{var}}}Nom^{fix} \leq 0,$$

which can be simplified to

$$i^{fix} \leq \left(\frac{2T}{T+1}\right)\left[-\frac{1}{T}\frac{d^{\text{var}}-d^{fix}}{1-d^{\text{var}}} + \frac{1-d^{fix}}{1-d^{\text{var}}}Ref_{\emptyset} - RP^{var}\frac{1-d^{fix}}{1-d^{\text{var}}}\left(\frac{T-1}{2T}-1\right) - \frac{1}{T^2}\frac{1-d^{fix}}{1-d^{\text{var}}}\sum_{t=1}^{T}(t-1)Ref_{t-1}\right].$$

That inequation allows a debtor to calculate the critical interest rate for a fixed rate loan. For illustration, we use again the same loan specification as in the previous section. The only difference is the repayment type, which switches from lump sum repayment to constant principal repayments. The effective interest rate for the fixed rate loan amounts to 2.42% p.a. and is higher than it was for the lump sum repayment. That can be explained because the effective interest rate takes into account the chronological order of cashflows and the highest cashflow, namely the principal repayment arises in the last period for a loan with lump sum repayment. The effective interest rate of the variable rate loan under the assumption that future reference rates equal our forecast is 2.89%. In contrary, the effective interest rate that would be advertised is only 1.68% p.a. that would lead to a false decision. Based on our proposed criterion and assumptions the fixed rate loan is advantageous. That would not be the case if only the advertised effective interest rates are available. Based on the criterion of average monthly payments also the fixed rate loan is more advantageous as total payments arising from the fixed rate loan (656.68) are smaller than the ones from the variable rate loan (679.25).

4.3 Case 3: Annuity repayments

Loans with annuity repayments as payback agreement are characterized by constant annuity payments that are the sum of interest and principal payments. The annuity for a fixed rate loan is constant over the loan term since the interest rates are constant. For the fixed rate loan, the annuity is calculated with the annuity factor and the face value:

$$Ann^{fix} = \frac{i_t(1+i_t)^T}{(1+i_t)^T-1} * Nom^{fix},$$

where $\frac{i_t(1+i_t)^T}{(1+i_t)^T-1}$ represents the annuity factor.

Interest payments Z_t are equal to

$$Z_t^{fix} = i^{fix} * Nom_{t-1}^{fix}$$

with

$$Nom_t^{fix} = Nom_{t-1}^{fix} - (Ann^{fix} - Z_t^{fix})$$

The principal repayments are the difference between the annuity and the interest payment in each period. For the fixed rate loan, we get constant annuity repayments as the interest rate is fixed. For the variable rate loan, a new annuity has to be calculated due to changing interest rates after an adjustment to the current reference rate to ensure full principal repayment until the end of the loan. If no new annuity is calculated and the variable interest rate rises the share of interest payment rises in relation to the principal repayment and therefore the repayment of the open nominal amount cannot be done during the term of the loan. This makes periodic adjustments of the annuity payments essential. The annuity of the variable rate loan equals:

$$Ann_t^{var} = \frac{i_t^{var}(1+i_t^{var})^{T-t+1}}{(1+i_t^{var})^{T-t+1}-1} * Nom_{t-1}^{var}$$

with

$$i_t^{var} = \max(0; Ref_{t-1} + RP^{var}),$$

$$Z_t^{var} = i_t^{var} * Nom_{t-1}^{var}$$

and

$$Nom_t^{var} = Nom_{t-1}^{var} - (Ann_t^{var} - Z_t^{var}).$$

	t = 0	t = 1	t = 2	…	t = T
Fixed rate	$(1-d^{fix})Nom^{fix}$	$-Ann^{fix}$	$-Ann^{fix}$		$-Ann^{fix}$
Variable rate	$(1-d^{\mathrm{var}})Nom^{var}$	$-Ann_1^{\mathrm{var}}$	$-Ann_2^{var}$		$-Ann_T^{\mathrm{var}}$
Variable - fixed	0	$Ann^{fix}-Ann_1^{\mathrm{var}}$	$Ann^{fix}-Ann_2^{\mathrm{var}}$		$Ann^{fix}-Ann_T^{var}$

Table 5: Annuity repayment loan cash flows

In order to make a decision based on the exact criterion we again calculate the effective interest rates of both loans which are the solutions to the following equations:

$$(1-d^{fix})Nom^{fix} = \sum_{t=1}^{T} \frac{\mathrm{Ann}^{fix}}{\left(1+i_{eff}^{fix}\right)^{t}}$$

and

$$(1-d^{var})Nom^{var} = \sum_{t=1}^{T} \frac{Ann_t^{var}}{\left(1+i_{eff}^{var}\right)^{t}}.$$

Based on the approximate criterion the fixed rate loan is advantageous if

$$\frac{1}{T}\sum_{t=1}^{T} Ann_{\mathrm{t}}^{fix} - \frac{1}{T}\sum_{t=1}^{T} Ann_t^{var} \leq 0$$

is valid. This inequation can be simplified to

$$Ann^{fix} \leq \frac{1}{T}\sum_{t=1}^{T} Ann_{\mathrm{t}}^{\mathrm{var}}.$$

Because interest rates of the variable rate loan are not constant, a new annuity has to be calculated whenever a new reference rate is applied. Therefore, a simple comparison between the annuity of the fixed rate loan and that of the variable rate loan, which is calculated in the first period, cannot be made to choose the optimal loan.

As in Case 1 and Case 2 we once more use our simple example to illustrate our proposed criteria to choose the optimal loan. As in the prior two cases the loan contract specifications are identical. Only the repayment type is in this case an annuity repayment for both loans. The results for this type of payback agreement is similar to the constant principal repayment agreement. The effective interest rate based on our forecast for expected future reference rates is 2.41% p.a. (2.90% p.a.) for the fixed rate (variable rate) loan. Based on this key figure the debtor will choose the fixed rate loan over the variable rate loan. The same decision will be made based on the average monthly payments as the payments of the fixed rate loan (661.35) are lower than the ones of the variable rate loan (686.66).

4.4 Sensitivity analysis

We also perform a sensitivity analysis to calculate the annual percentage point Δ by which the expected future reference rates have to change. If the fixed rate loan is advantageous beforehand, we calculated the annual percentage point Δ by which the expected future reference rates have to decrease so that a variable rate loan becomes more advantageous. If the variable rate is advantageous beforehand, we calculated the annual percentage point Δ by which the expected future reference rates can increase so that a variable rate loan remains advantageous.

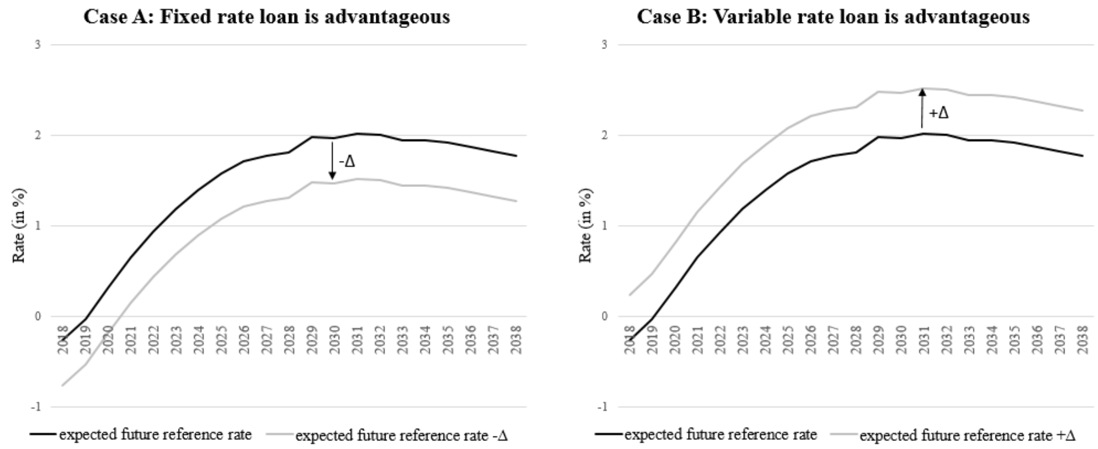


Figure 5: Sensitivity analysis two possible cases

As for Case 1 the expected future reference rates have to decrease by 77 bp so that the variable rate loan becomes more advantageous than the fixed rate loan based on the effective interest rates. This shift is equivalent to a very low risk premium of 0.73% p.a. that lies below commonly charged risk premia in practice. In comparison the reference rates have to decrease by 45 bp (47 bp) in Case 2 (Case 3).

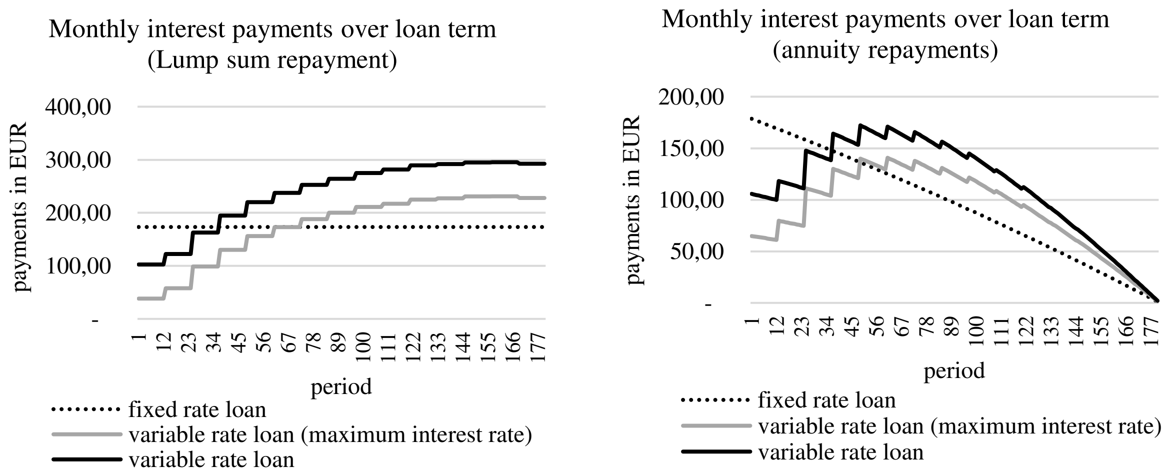


Figure 6: Sensitivity analysis: results

These shift are equivalent to risk premia of 1.04% p.a. and 1.02% p.a. The difference in the three shifts can be explained by the fact that the outstanding amount does not decrease in the case of a loan with a lump sum repayment. The shift of future reference rates for a lump sum agreement leads to a parallel shift of monthly interest payments as the (outstanding) face value remains the same. Because the fixed interest rate does not change, interest payments are constant over the loan term. In contrary, a shift of future reference rates for an annuity payback agreement has different consequences as the (outstanding) face value changes due to changing annuities over the loan term. Figure 6 shows that at first interest payments increase for a variable rate loan until they finally decrease. This fact can be explained due to the two conflicting influences of rising reference rates and a decreasing (outstanding) face value.

5 The effects of data modifications and the payback to payout ratio

In this section, we show the effects of data modification regarding the future reference rates as well as the face value.

		Initial data		Revised data	
	Face value	100,000	100,000	102,040.82	103,092.78
	Payout at t = 0	98,000	97,000	100,000	100,000
	Future reference rates	Initial	Initial	Our forecast	Our forecast
Type	**Key figures**	**Fixed rate**	**Variable rate**	**Fixed rate**	**Variable rate**
Lump sum	Effective interest rate	2.28% p.a.	1.46% p.a.	2.28% p.a.	3.06% p.a.
	Total repayments	131,500.00	118,465.00	134,183.67	147,348.14
	(Average) monthly repayments	730.56	658.14	745.46	818.60
Constant principal	Effective interest rate	2.42% p.a.	1.68% p.a.	2.42% p.a.	2.89% p.a.
	Total repayments	115,837.50	109,283.79	118,201.53	122,264.62
	(Average) monthly repayments	643.54	607.13	656.68	679.25
Annuity	Effective interest rate	2.41% p.a.	1.66% p.a.	2.41% p.a.	2.90% p.a.
	Total repayments	116,662.27	109,567.61	119,043.13	123,598.29
	(Average) monthly repayments	648.12	608.71	661.35	686.66

Table 6: Effects of different payouts and of our reference rates forecasts

First, we present the effects of our proposed assumptions of future expected reference rates in comparison to the assumptions made in typical loan offers. In addition to that, we simultaneously changed the face value of both loans in order to secure the condition of equal payouts at t = 0.

As can be obtained from Table 6 the effective interest rates do not react to changes in the (outstanding) face value since all costs are proportional to the (outstanding) face value of the loan. The total repayments that consist of all principal and interest payments over the loan term, on the other hand, vary with the face value and cannot serve as a basis for a decision if two loans with different credit payments at t = 0 are compared. The same critique can be applied to the average monthly repayments of the loans. Table 6 shows that a decision based on the effective interest rates that are declared in loan offer leads to a false decision since the possibility of rising reference rates in the (near) future is excluded. We also introduce another key figure, namely the payback to payout ratio for choosing the optimal loan. We show that if our forecasted expected future reference rates are used when calculating the effective interest rate as well as the total repayments, no adjustments in the face value have to be done in order to choose the optimal loan. For illustration, we again use our simple numerical example and once again calculate our key figures.

		Initial data		Revised data	
	Face value	100,000	100,000	102,040.82	103,092.78
	Payout at t = 0	98,000	97,000	100,000	100,000
	Future reference rates	Our forecast	Our forecast	Our forecast	Our forecast
Type	**Key figures**	**Fixed rate**	**Variable rate**	**Fixed rate**	**Variable rate**
Lump sum	effective interest rate	2.28% p.a.	1.46% p.a.	2.28% p.a.	3.06% p.a.
	Total repayments	131,500.00	118,465.00	134,183.67	147,348.14
	(average) monthly repayments	730.56	658.14	745.46	818.60
	Payback to payout ratio	1.34	1.47	1.34	1.47
Constant principal	effective interest rate	2.42% p.a.	1.68% p.a.	2.42% p.a.	2.89% p.a.
	Total repayments	115,837.50	109,283.79	118,201.53	122,264.62
	(average) monthly repayments	643.54	607.13	656.68	679.25
	Payback to payout ratio	1.18	1.22	1.18	1.22
Annuity	Effective interest rate	2.41% p.a.	1.66% p.a.	2.41% p.a.	2.90% p.a.
	Total repayments	116,662.27	109,567.61	119,043.13	123,598.29
	(average) monthly repayments	648.12	608.71	661.35	686.66
	Payback to payout ratio	1.19	1.24	1.19	1.24

Table 7: The effect of different payouts for our reference rates forecast

Table 7 shows that the effective interest rates as well as the payback to payout ratio do not change with the face value of the loan. Total repayments and average monthly repayments, on the other hand, are influenced by the face value of the loan. However, this is only valid if all payments are proportional to the (outstanding) face values. Whenever independent payments exist, face values of both loans have to be adjusted to make an optimal choice.

6 Conclusion

The decision whether to take out a fixed rate or variable rate loan can be difficult due to the current interest rate landscape as short-term interest rates, which serve as reference rates for variable rate loan are negative and therefore make variable rate loans to appear more advantageous. Nevertheless, when choosing a loan expectations about future interest rates are also essential as they have an impact on the total repayments. The effective interest is the most common key figure to choose between fixed rate loan offers. The calculation of the effective interest rate is harmonized in the EU to secure comparability between loan offers and to offer a decision variable to debtors. However, if a loan contains interest rates that are linked to a reference rate that can vary over the loan term assumptions regarding this future interest rates have to be made. According to EU Directives effective interest of variable rate loans are calculated based on the assumption that future reference rates equal the initial reference rate. Furthermore, effective interest rates or total repayments announced by banks only account for payments between creditor and debtor and ignore all payments from or to third parties. In addition, if not all payments are proportional to the (outstanding) face value, the effective interest rate and the total/average repayments depend on the face value of the loan. We propose two different key figures based on which a debtor should choose between a fixed rate or variable rate loan. Therefore, we propose that for variable interest rate loans more representative forecast for the future reference rates have to be applied in order to choose the right loan. We calculate expected future reference rates based on the pure expectation hypothesis but also other forecasting methods are possible as we offer an approach that can be carried out with different expected future interest rates. To compare different loans, identical initial payouts for all loans have to be assumed and therefore their face values have to be adjusted accordingly. The adjustment of the face value can only be omitted if no fixed payments, which are independent of the face value, are involved. In this case, the effective interest rate is independent of the face value and the appropriate approximate criterion is the payback to payout ratio.

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