Karl-Franzens-University Graz
Faculty of Social and Economic Sciences

When Do Loan and Bond Prepayments Pay Off?

Edwin O. Fischer, Ines Wöckl



Working Paper 2018-02

August 3, 2018

Subsequently published as:
Fischer, E. O., Wöckl, I., "Debt Restructuring: When Do Loan and Bond Prepayments Pay Off?", Österreichisches Bankarchiv 67(1), 2019, 39-49.

An electronic version of the paper may be downloaded from:

University of Graz: sowi.uni-graz.at/forschung/working-paper-series/
RePEc: ideas.repec.org/s/grz/wpsses.html

Working Paper Series
Faculty of Social and Economic Sciences
Karl-Franzens-University Graz
ISSN 2304-7658
sowi.uni-graz.at/forschung/working-paper-series/
sowi-wp@uni-graz.at

UNIVERSITY OF GRAZ

When Do Loan and Bond Prepayments Pay Off?

Edwin O. Fischer[a], Ines Wöckl[a]*

Working Paper 2018-02

August 3, 2018

Abstract

For ten years interest rates in the Eurozone have been declining. This has created a situation where loan or bond prepayments and subsequent refinancing transactions are potentially beneficial for debtors. The advantageousness depends on the costs induced. We analyze the favorability of debt restructuring using the method of differential investment and provide critical limits for the nominal interest rate of the new loan up to which prepayment is optimal. The calculations address both fixed and variable rate loans and consider whether the debt agreement is repaid at maturity or in annuities.

Keywords: debt restructuring, loan prepayments, fixed and variable rate loans

Subsequently published as:
Fischer, E. O., Wöckl, I., "Debt Restructuring: When Do Loan and Bond Prepayments Pay Off?", Österreichisches Bankarchiv 67(1), 2019, 39-49.

JEL: G21, G31, G32

[a] University of Graz, Institute of Finance, Universitätsstraße 15/G2, 8010 Graz, AUSTRIA

[*] Corresponding author. Tel.: +43(316)380-3514, E-Mail ines.woeckl@uni-graz.at.

Working Paper Series
Faculty of Social and Economic Sciences
Karl-Franzens-University Graz
ISSN 2304-7658
sowi.uni-graz.at/forschung/working-paper-series/
sowi-wp@uni-graz.at

Debt Restructuring: When Do Loan and Bond Prepayments Pay Off?

Edwin O. Fischer*
Ines Wöckl*,†

August 2018

*University of Graz, Institute of Finance, Universitätsstrasse 15/G2, A-8010 Graz
†Corresponding author: Tel. +43 (0) 316 380 3514, E-mail ines.woeckl@uni-graz.at.

Debt Restructuring:
When Do Loan and Bond Prepayments Pay Off?

Abstract

For ten years interest rates in the Eurozone have been declining. This has created a situation where loan or bond prepayments and subsequent refinancing transactions are potentially beneficial for debtors. The advantageousness depends on the costs induced. We analyze the favorability of debt restructuring using the method of differential investment and provide critical limits for the nominal interest rate of the new loan up to which prepayment is optimal. The calculations address both fixed and variable rate loans and consider whether the debt agreement is repaid at maturity or in annuities.

Keywords: debt restructuring, loan prepayments, fixed and variable rate loans

JEL Codes: G21, G31, G32

1 Introduction

Many debtholders, whether private households, companies or states, are caught up in high-interest long-term loans. At the same time, economic developments over the past few years have created a low-interest environment in which prepaying an existing loan and simultaneously refinancing into a new loan can be advantageous from the borrower's point of view. By redeeming an existing loan before maturity and refinancing into a loan with a lower interest rate, the amount of interest owed to the lender can be reduced significantly, potentially saving thousands of euros in the long term. Similar considerations hold for prepaying callable bonds since these are in fact simply a kind of securitized loan.

Intuitively debt restructuring seems advantageous whenever the nominal interest rate of the new loan is lower than that of the old loan. However, prepayment considerations are more complex since debt restructuring entails transaction costs. These include a possible penalty for the early redemption of the existing loan, called a *prepayment penalty*, as well as credit charges and a possible loan disbursement fee for taking out a new loan. Analogously, exercising the right to call a callable bond and simultaneously issuing a new bond will also lead to transaction costs. These costs need to be factored in when deliberating the advantageousness of a prepayment.

At the moment, interest rates in the Euro area are at an all-time low and have been so for quite some time. However, the market seems to anticipate that interest rates, and in consequence lending rates, will begin to rise in the (near) future, so an analysis on whether or not to restructure one's debt should be undertaken now in order to lock in the lowest possible interest rate for the new debt contract.

Although this topic is without doubt highly relevant and affects companies, states and private individuals alike, academic literature is scarce to almost non-existing. Mainstream media do seem to have caught onto the subject; however, these articles fail to provide theoretic foundations for their assertions and recommendations.

On these grounds, we examine in detail under which circumstances loan and bond prepayments make sense for debtholders, providing an intuitive solution concept for determining an upper limit on the nominal interest rate of the new loan up to which prepaying the old loan and refinancing into a new low pays off. The article is structured as follows: Section 2 describes the circumstances that have led to this favourable environment. In Section 3 we provide an exact solution as well as an easy-to-use approximation for the most common debt restructuring scenarios, taking into account the different payment modalities of the debt instruments as well as how the transaction costs incurred are to be financed. Section 4 concludes.

2 Background

The economic development over the past decade, fuelled by the unconventional monetary policies of the world's largest central banks, has pushed lending rates on loans and coupon rates on bonds to an all-time low. However, market indicators show that this sustained low-interest situation is not set to last. In the Unites States, the Federal Reserve (Fed) is far along its path to a more "normal" interest rate environment, having increased its key interest rate, the Federal Funds Rate, five times since the beginning of 2017. While its European counterpart, the European Central Bank (ECB), has jet to signal a specific date for its first interest rate hike, the market has already begun pricing in possible rate hikes.

2.1 Past Economic Development

The current beneficial situation for prepaying high-interest debt has arisen due to the economic developments of the past decade. For almost ten years Europe and large parts of the rest of the world have experienced a decline in interest rates, with central banks eager to keep their main

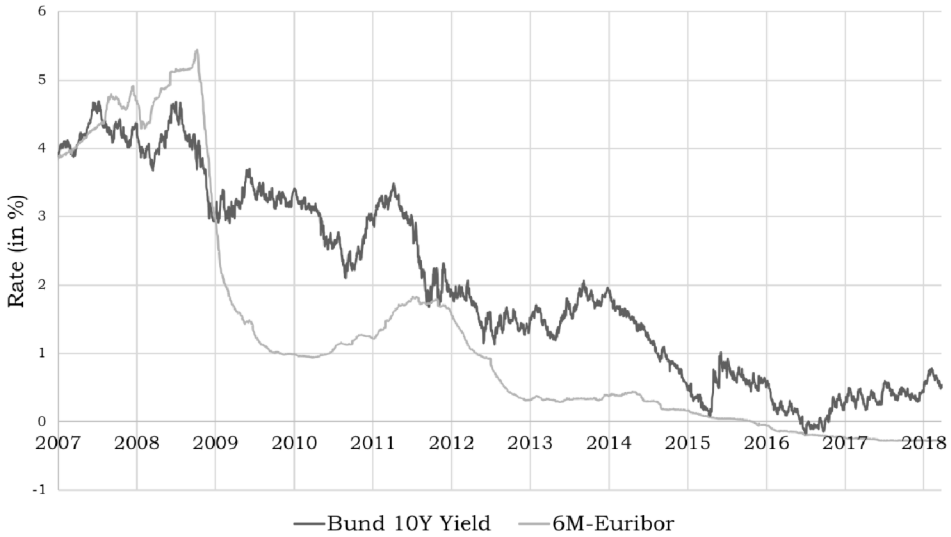


Figure 1: Euribor and Bund Yield Development
Source: Thomson Reuters Datastream

refinancing rates close to zero while simultaneously purchasing vast quantities of bonds in the hope of refuelling economic development and growth and returning inflation rates to target levels.

In Europe, the downwards development of interest and lending rates was primarily driven by the effort of the ECB to reduce its key interest rate in an effort to jump-start the faltering European economy. Mid-2008 the interest rate was at 4.25 %, from where the central bank has reduced it continuously over the past years. In March 2016 the interest rate was eventually reduced to 0 % where it has remained ever since.

Since private individuals, corporations, and states cannot take out loans at the ECB refinancing rate, we take a closer look at the yield of German government bonds with a maturity of ten years (10Y-Bund yield) and the 6-month Euribor (6M-Euribor). These provide good indications of the interest rate development on common debt contracts and bonds. The development of these two rates from 2007 to mid-2018 is shown in Figure 1.

The *6M-Euribor* is often used as the underlying reference interest rate for variable rate loans and therefore provides a good indication of the development of the lending rates for variable rate loans. It experienced an upward trend during 2007 and most of 2008, followed by a pronounced drop from its peak at above 5 % in late 2008 due to the full impact of the financial crisis. By the end of 2009 it had dropped below 1 %. The interest rate recovered moderately from 2010 to mid-2011 but has shown a steady decline since then. In November 2015 the 6M-Euribor fell below zero for the first time in history, where it still remains. The *10Y-Bund yield* is often used as a reference point for the development of lending rates on fixed rate loans and the coupons on bonds. Bunds are guaranteed by the German government and therefore involve very small risk. Interest rates on fixed rate loans to corporations, private individuals, and other states will typically involve more risk and therefore have a higher interest rate. While the 10Y-Bund yield does not show as pronounced a decline as the 6M-Euribor, the continuous downward trend is clearly observable. The yield drifted into negative territory for a short period of time in mid-2016 and is currently trending below 0.5 %.

In addition to the development of an interest rate for a specific maturity over time, also the interest rates for different maturities at a single point in time are of interest when evaluating debt prepayment. Figure 2 shows the term structures of interest rates, or spot curves, for the years 2007 to 2018, with lighter lines indicating term structures that lie further back in time. Term structures are constructed from AAA-rated, EUR-denominated government bonds of different maturities and give an overview of the prevailing interest rate level and the current state of the economy. From

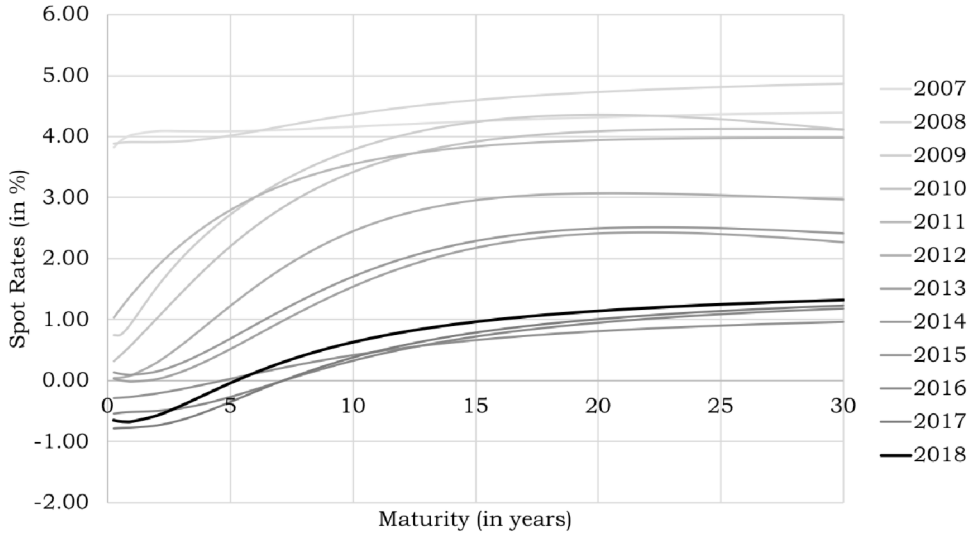


Figure 2: Spot Curves (Term Structures)
Source: Thomson Reuters Datastream (May 3, 2018)

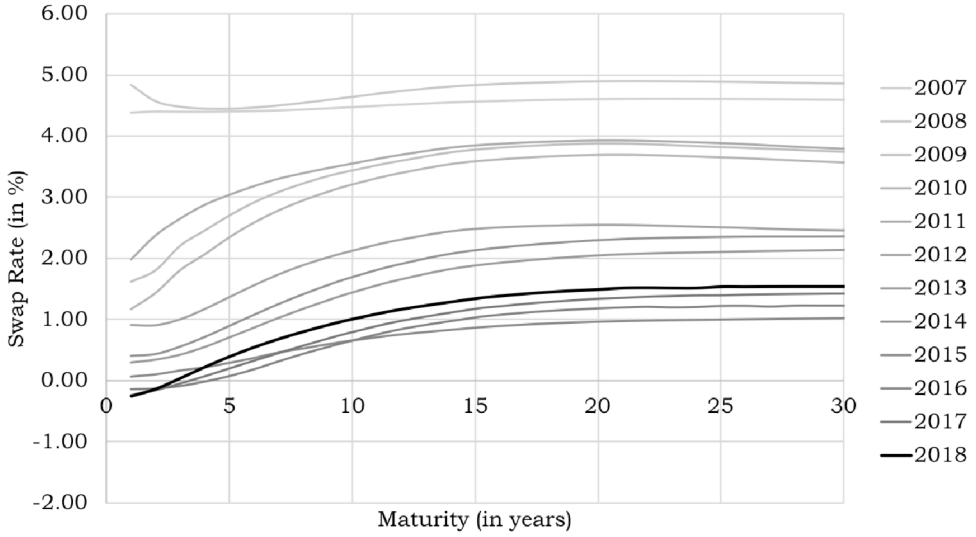


Figure 3: Swap Curves (6M-Euribor Interest Rate Swaps)
Source: Thomson Reuters Datastream (May 3, 2018)

Figure 2 it is clearly discernible that the general interest rate level in the Euro area has decreased significantly over the past decade. Similar insights can be derived from Figure 3, which plots the swap curves from 2007 to 2018 constructed from 6M-Euribor interest rate swaps.

2.2 Lending Rates

Of even greater interest than the indicative interest rates is the development of lending rates of European banks over the past decade. Figure 4 shows the development of lending rates of German banks on loans for consumption to households, housing loans to households, and loans to non-financial corporations over 1,000,000 €, for fixed rate and variable rate loans each.

The interest rates on loans issued to non-financial corporations, whether fixed or variable, have decreased significantly since late 2008. Non-financial corporations can currently acquire loans of one million Euro or more with a fixed nominal interest rate of below 2 %. Variable interest rates for loans over one million Euro to non-financial corporations may even lie below 1 %.

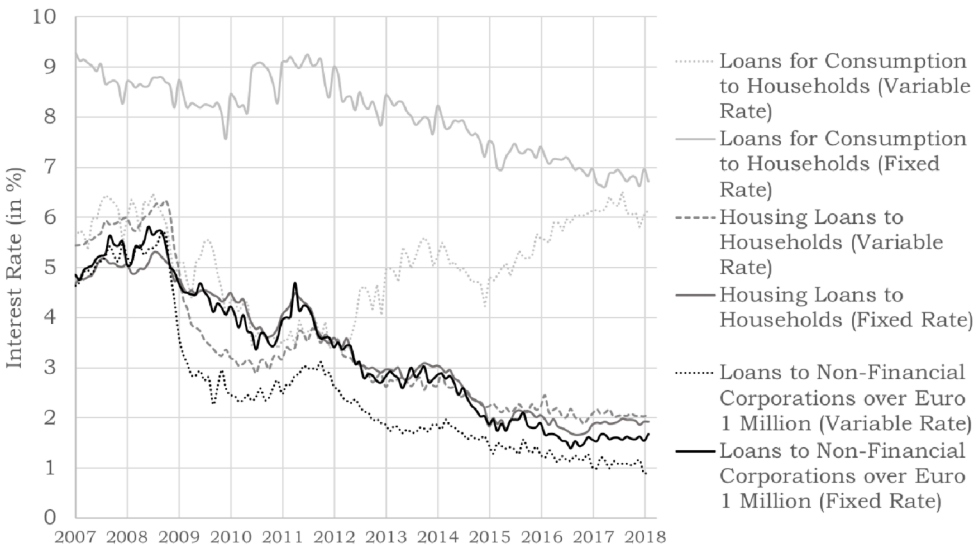


Figure 4: Lending Rates of German Banks (New Business)
Source: Deutsche Bundesbank

Fixed and variable rates on housing loans to households are also historically low, trending sideways at close to or slightly above 2 %. From Figure 4 it can also be seen that the lending rates on fixed rate loans for consumption to households have continuously decreased over the past years. Merely variable rates on consumer loans seem not to fit the pattern. Lending rates on variable rate loans for consumption to households did drop from late 2008 until the beginning of 2012. However, they have been steadily increasing since then. This phenomenon is due to the fact that the interest rate on variable rate loans is composed of two parts: a reference rate and a risk premium, where the height of the latter depends on the creditworthiness of the borrower. While reference rates have been consistently decreasing, risk premia have been increasing disproportionately since the beginning of 2012 due to a decrease in the creditworthiness of households over the past years, resulting in a increase of the overall interest rate.

Figures 1 and 4 show that interest and lending rates in the Eurozone have been very low for a significant amount of time. At the same time the inflation rate in the Euro area has been gradually recovering since 2012 and is now trending at levels close to 2 %. This gives rise to speculations that the ECB might begin reducing its monetary stimulus in the near future which would cause opportunities for advantageous debt prepayment to diminish.

2.3 Current Interest Rate Landscape and Market Expectations

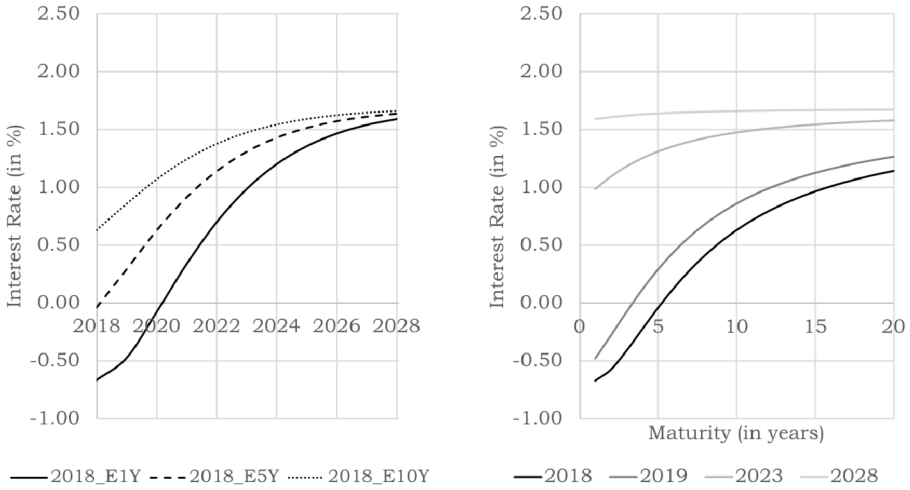
Expectations of the future development of the interest and lending rates also play a significant role in evaluating whether or not debt prepayment pays off. Expectations can be derived based on a multitude of different theories.

One of the simplest and most straight-forward approaches for deriving market expectations of future interest rates is the *pure expectations hypothesis* (PEH). It postulates that the current term structure fully incorporates all information on the future development of interest rates, so future rates can be derived as the geometric mean of the current interest rates. Using the PEH and the spot-forward relation, future nominal spot rates and future t-year spot rates can be forecast from the spot curve and subsequently used as reference points on the possible development of the lending rates on fixed interest loans. The swap curve, on the other hand, can be used to forecast future reference interest rates used in variable rate debt contracts.

Figure 5 shows the expected future spot rates and expected future term structures derived from the spot rates on May 3, 2018. It can be seen that the market expects rising interest rates based on the PEH. One-year, five-year, and ten-year spot rates are expected to increase continuously over

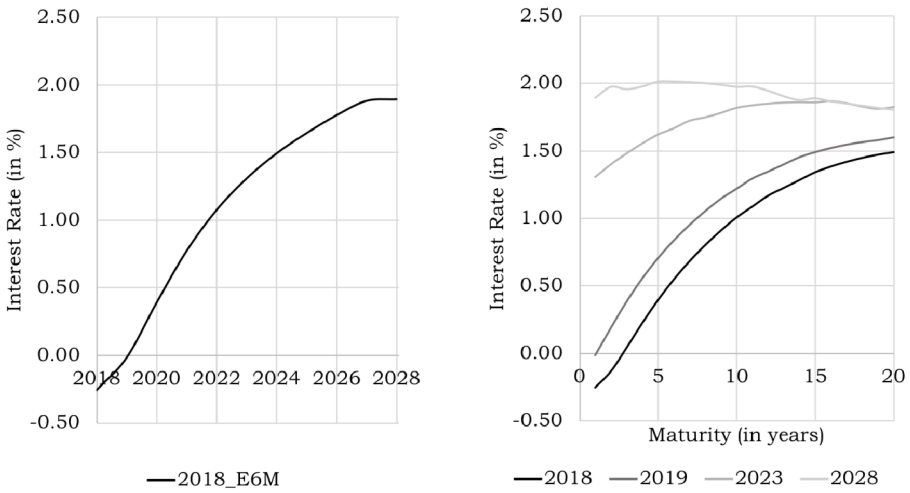
the upcoming years (Figure 5(a)). Also the general interest rate level is expected to rise; however, the spot curve is expected to flatten in due course (Figure 5(b)). Figure 6 shows that similar expectations are priced into the swap market. Figure 6(b) suggests that the swap curve may even become inverted in the long run.

The PEH has been criticized as a method for predicting future rates, especially since it neglects the time-varying nature of the risk premium.[1] Alternative theories that can be used to forecast future interest rates include the *Liquidity Preference Theory*, the *Market Segmentation Theory*, and the *Preferred Habitat Theory*. As mentioned above, the PEH is one of the most straight-forward methods; however, which theory is ultimately used is left to the user. Generally, it would even be conceivable to use subjective interest rate expectations to derive future rates.



(a) Expected Future Spot Rates (b) Expected Future Term Structures

Figure 5: Spot Rate Expectations
Source: Thomson Reuters Datastream (May 3, 2018) and own calculations



(a) Expected Future 6M-Euribor (b) Expected Future 6M-Euribor Swap Curves

Figure 6: Swap Rate Expectations
Source: Thomson Reuters Datastream (May 3, 2018) and own calculations

[1]For an extensive overview see Sangvinatsos (2010).

2.4 Prepayment Regulations

Having stated that the past economic development and the future expectations of the development of interest rates have created an environment optimal for prepaying outstanding debt, it remains to be considered whether or not a borrower is actually permitted to prepay his or her loan. Generally, prepayment is only permitted if the debt contract specifications contain provisions regarding this aspect. Otherwise prepayment is contingent on negotiations with the creditor. Many loans issued to private individuals by banks do indeed specify rules that apply to the early repayment of the loan.

In the European Union (EU) special regulations have been put in place in order to harmonize the laws concerning loans issued to consumers across the member states. In consequence, considerations on debt prepayment must always take into account whether the debtor is a consumer or not. More precisely, Directive 2008/48/EC on credit agreements for consumers specifies that for these loans the right to early repayment cannot be excluded and that consumers may at any time discharge their obligations fully or partially. Additionally, it specifies that the creditor is in return entitled to a fair and objectively justified compensation for possible costs directly linked to the early repayment of the loan. This compensation, however, may only be charged provided that the prepayment falls within a period for which the borrowing rate is fixed. Directive 2014/17/EU grants the EU member states the right to also implement these regulations for consumer loans relating to residential immovable property. Both directives have been translated into national law in all EU countries. In Germany the regulations can be found in the German Civil Code (BGB). In Austria Directives 2008/48/EG and 2014/17/EU are implemented in two separate acts; i.e., *Verbraucherkreditgesetz* (VKrG) and *Hypothekar- und Immobilienkreditgesetz* (HIKrG), respectively.

As mentioned above, loan prepayments may entail a so-called *prepayment penalty*; i.e., a compensation to the creditor for losses incurred through the early repayment of the outstanding debt. When national law or the debt contract specification does not specify any upper limits, the prepayment penalty may even exceed the present value of the remaining interest payments. For consumer loans the EU has implemented upper boundaries on the penalty. Directive 2008/48/EC specifies that the compensation may not exceed 1 % of the amount of debt repaid early if the period of time between the prepayment and the agreed termination of the loan exceeds one year. If the period does not exceed one year, the compensation may not exceed 0.5 %. In any case, the directive limits the prepayment penalty to the amount of interest the consumer would have paid during the period between the prepayment and the official maturity date.[2]

For callable bonds prepayment regulations can be found in the bond indenture. The indenture may also specify a *call protection period*; i.e., an initial period during which the bond cannot be repaid. Additionally, it specifies the *call premium*, which is the equivalent of the prepayment penalty on loans. The call premium may decrease with the remaining term between the prepayment and scheduled maturity.

3 Calculations

The legitimacy of our considerations lies in the fact that transaction costs are incurred when prepaying and subsequently restructuring debt. Were debt refinancing not to induce these transaction costs, it would be sensible to refinance whenever the interest rate of the new loan is lower than the interest rate of the old loan. In reality, prepayment often does entail a prepayment penalty and refinancing into a new loan usually involves additional costs. The bank may, for example, charge a *loan disbursement fee*, due to which the borrower receives less than the nominal value of the loan at disbursement. However, interest payments are still calculated based on the full nominal amount. Acquiring a new loan will typically also involve various other credit charges, such as lender fees, attorney fees, closing fee, etc.

[2] In Austria the boundaries on the prepayment penalty only apply to loan agreements signed after June 11, 2010.

		new loan	
		fixed rate	variable rate
old loan	fixed rate	Case I	"Reverse" Case II
	variable rate	Case II	n.a.

Table 1: The Four Switching Scenarios

Time	0	1	...	T
Project X (cheap)	$-I^X$	C_1^X		C_T^X
Project Y (expensive)	$-I^Y$	C_1^Y		C_1^Y
$Y - X$	$-I^Y + I^X$	$C_1^Y - C_1^X$		$C_T^Y - C_T^X$

Table 2: Method of Differential Investment

Since loans may either be conceptualized as fixed rate or variable rate loans, four "switching" scenarios can be distinguished (Table 1). Our focus lies on Cases I and II; i.e., switching from a fixed rate loan to a fixed rate loan and from a variable rate loan to a fixed rate loan, respectively. Switching from a fixed rate to a variable rate loan can be seen simply as the "reverse" of Case II and replacing a variable rate loan by a new variable rate loan is merely an analysis of a change in the debtor-specific risk premium which is not the focus of our analysis. Also, Cases I and II are those for which action is implied by the current interest rate landscape and the corresponding market expectations.

Furthermore, prepayment considerations require a differentiated approach based not only on the interest rate agreement of the old and new loan, but also the repayment modalities of the debt contracts. Repayment modalities include *lump sum repayment* at the end of maturity, constant principal payments, and constant annual instalments (*annuity repayment*). The repayment modality of a loan significantly influences the amount of debt outstanding at a specific point in time, so prepayment may in some situations make sense for annuity loans but not for interest-only loans and vice-versa. For this reason, we further distinguish our calculations between loans with lump sum repayment and such that provide for annuity repayment and therefore provide different advantageousness criteria for each kind.

Our solution concept is based on the *Method of Differential Investment*, a method used for evaluating the advantageousness of two alternative investment projects.[3] Simply put, there are two alternative projects of which *exactly one* must be invested into. The initial investment costs, I, of one project exceed those of the alternative. In return, the more expensive project results in higher cash flows in future periods, C_t. When deciding upon the superiority of one project over the other, first of all, the difference between the cash flows of the two projects is calculated in each period by subtracting the cash flow of the cheaper project from that of the more expensive one. An exemplary cash flow overview is illustrated in Table 2. Then, the net present value (NPV) of the differential payments is compiled. A positive NPV implicates an investment in the more expensive project, whereas a negative NPV implies investing in the cheaper project. An NPV of zero signals indifference between the two projects.

We propose two solution approaches: (a) an *exact approach* (dynamic criterion) based on the NPV of the differential investment where the cash flows are discounted by the effective interest rate of the new loan, i_{eff}, and (b) an *approximation* (static criterion) based on the average profit from the differential investment where the average profit is defined as

$$average\,profit = average\,cash\,flow - imputed\,depreciation - imputed\,interest. \tag{1}$$

As mentioned above, the discount rate used in the exact solution is the effective interest rate

[3] Fischer (2008), 52-55.

of the new loan, i_{eff}. We use this rate since it most accurately reflects the current interest rate landscape and the market expectations of future interest rates. In case of lump sum repayment it is calculated from

$$(1-d) \cdot Nom^{new} = \sum_{t=1}^{T} \frac{Z_t^{new}}{(1+i_{eff})^t} + \frac{Nom^{new}}{(1+i_{eff})^T}, \tag{2}$$

where Nom^{new} is the nominal value, Z_t^{new} are the annual interest payments, and d is the loan disbursement discount in each case of the new loan. T is the term of the new loan in years and is equivalent to the remaining term of the old loan.

In case of annuity repayment, i_{eff} is derived from

$$(1-d) \cdot Nom^{new} = \sum_{t=1}^{T} \frac{Ann_t^{new}}{(1+i_{eff})^t} \tag{3}$$

and Ann_t^{new} are the annual payments, or annuities, of the new loan.

Another aspect that must be considered when restructuring debt is how the transaction costs incurred are to be financed. As noted above, transaction costs include the prepayment penalty of the old loan as well as the loan disbursement fee and possible additional credit charges of the new loan. In general, two alternatives for financing transaction costs are conceivable:

1. Transaction costs can be covered via *debt financing*. In this case the nominal value of the new loan not only covers the outstanding debt of the old loan, Nom, but also incorporates all transaction costs; i.e., the prepayment penalty, p, and the loan disbursement fee, d.

$$Nom^{new} = \frac{1+p}{1-d} \cdot Nom$$

2. Alternatively, transaction costs can be covered by *equity financing*. Consequently, the nominal value of the new loan simply equals the outstanding nominal value of the old loan.

$$Nom^{new} = Nom$$

It can be proven that borrowers are indifferent between debt and equity financing of transaction costs, so we limit the illustration of our formulas to the debt financing case.[4]

3.1 Case I – Fixed Rate Loan to Fixed Rate Loan

First of all, we illustrate Case I; i.e., prepaying an existing fixed rate loan and refinancing the debt with a new fixed rate loan. In Section 3.1.1 we provide the formulas for the case where both loans have lump sum repayment (Case I.A). Section 3.1.2 covers the case of annuity loans (Case I.B).

3.1.1 Case I.A – Lump Sum Repayment

In Case I.A both the old and the new loan are fixed rate debt contracts with lump sum repayment. They therefore feature constant annual interest payments calculated as

$$Z_t = Z = i \cdot Nom \tag{4}$$

for the old loan and

$$Z_t^{new} = Z^{new} = i^{new} \cdot \frac{1+p}{1-d} \cdot Nom, \tag{5}$$

for the new loan. i and i^{new} are the nominal interest rates of the old loan and the new loan, respectively.

For the exact solution, in the first step, the structure of the differential payments must be determined. As can be seen from Table 3, transaction costs are fully covered by the new loan which means that no own funds need to be employed in $t = 0$ when debt restructuring is carried out.

[4] The proof is available on request.

Time		0	1	2	...	T
without cancellation	old loan		$-Z$	$-Z$		$-(Z+Nom)$
with cancellation	old loan	$-(1+p)\cdot Nom$				
	new loan	$+(1+p)\cdot Nom$	$-Z^{new}$	$-Z^{new}$		$-\left(Z^{new}+\frac{1+p}{1-d}\right)\cdot Nom$
difference	with – without	0	$Z-Z^{new}$	$Z-Z^{new}$		$Z-Z^{new}-\frac{p+d}{1-d}\cdot Nom$

Table 3: Cash Flow Overview for Case I.A

In the second step, the NPV of the differential payments is calculated using

$$NPV_0 = \left[PVF_{i_{eff},T}\cdot\left(i-\frac{1+p}{1-d}\cdot i^{new}\right)-\frac{1}{(1+i_{eff})^T}\cdot\frac{p+d}{1-d}\right]\cdot Nom \geq 0, \tag{6}$$

with present value factor of annuity (PVF)

$$PVF_{i_{eff},T} = \frac{(1+i_{eff})^T-1}{i_{eff}\cdot(1+i_{eff})^T}. \tag{7}$$

The NPV must be larger than zero for switching to the new loan to be preferable over keeping the old loan. Solving the equation for i_{new} leads to the following dynamic advantageousness condition

$$i^{new} \leq \frac{1}{1+p}\cdot\left[(1-d)\cdot i-\frac{i_{eff}}{(1+i_{eff})^T-1}\cdot(p+d)\right], \tag{8}$$

which defines the upper limit for the nominal interest rate of the new loan. When the equation is binding, i^{new} is the nominal interest rate of the new loan for which the borrower is indifferent between switching to a new loan and keeping the old loan.

Effectively, the dynamic solution concept involves simultaneously solving a system of equations with two unknowns, namely i_{eff} and i^{new}. For a quicker, albeit not as exact, solution approach we offer an approximative solution based on the average annual profit that can be made from the differential investment. The average annual profit must be larger than or equal to zero for a prepayment to pay off.

$$\left(i-\frac{1+p}{1-d}\cdot i^{new}\right)\cdot Nom-\frac{1}{T}\cdot\frac{p+d}{1-d}\cdot Nom \geq 0. \tag{9}$$

The static advantageousness condition reads as follows

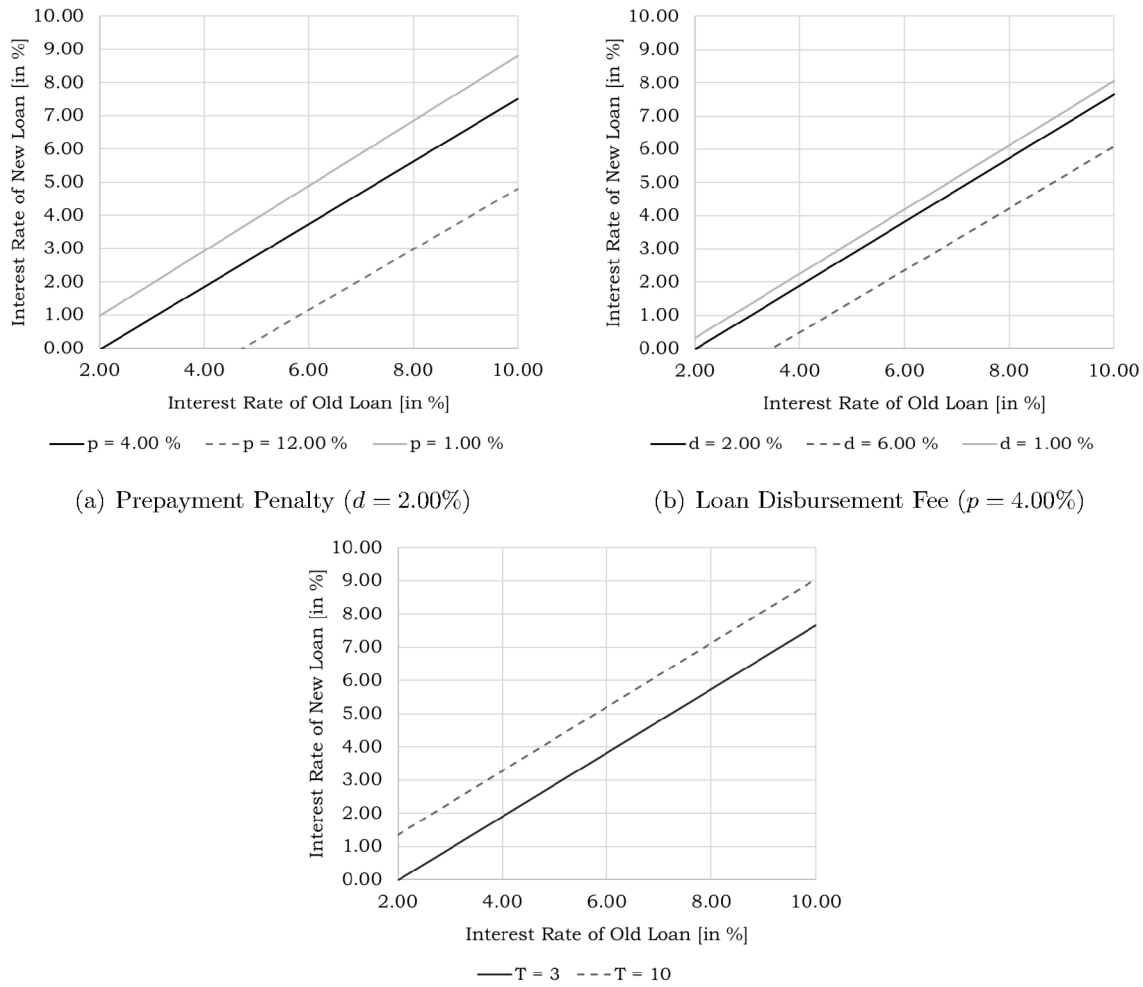
$$i^{new} \leq \frac{1}{1+p}\cdot\left[(1-d)\cdot i-\frac{p+d}{T}\right], \tag{10}$$

where $\frac{p+d}{T}$ is the annual depreciation of the additional expenditure due to the new loan.

A borrower generally has two options how to use our formulas. He can either insert the parameters of an existing new loan offer into the formulas in order to evaluate whether refinancing his old loan with the credit offer at hand makes sense or he can calculate an interest rate upper limit for any new loan which could serve as orientation in negotiations with potential creditors.

Take the following simple numerical example: A company currently has a ten-year loan with a nominal value of 100,000€, a fixed nominal interest rate of 4 % (interest paid yearly in arrears), and lump sum repayment at maturity. The loan was taken out seven years ago and the company can now either keep the existing loan for the remaining three years or prepay it and take on a new three-year fixed rate loan.[5] In case of prepayment the penalty would amount to 4 % of the

[5]The past term of the loan has no influence on the prepayment considerations.



(a) Prepayment Penalty ($d = 2.00\%$)

(b) Loan Disbursement Fee ($p = 4.00\%$)

(c) Remaining Term ($p = 4.00\%$, $d = 2.00\%$)

Figure 7: Sensitivity Analyses for Case I.A

nominal value of the old loan while the loan disbursement fee and all other credit charges for the new loan are estimated to amount to 2 % of the its nominal value. Since all transaction costs are to be covered via debt financing, the nominal value of the new loan must amount to 106,122 €.

Simultaneously solving the system of equations for the nominal and effective interest rates of the new loan leads to an upper boundary on i^{new} of 1.895 % for prepaying and subsequently refinancing to pay off. A quicker solution can be obtained by simply inserting the above-mentioned parameters of the two loans into the static advantageousness condition in Equation (10). The upper limit for i^{new} is then 1.846 %.

The approximation becomes less accurate the higher the interest rate of the old loan, but it always lies below the exact solution. Thus, borrowers will avoid a false positive; i.e., prepaying an existing loan even though it is not advantageous. However, there remains a small risk of being too conservative and foregoing a beneficial prepayment.

We perform a series of sensitivity analyses to examine the effect of changes in the individual parameters on the dynamic advantageousness condition. Figure 7 consists of three graphs, each depicting the interest rate of the new loan as a function of the interest rate of the old loan for different levels of the influencing parameters.[6] More precisely, the graphs show the effect on i^{new} of changing (a) the prepayment penalty, (b) the loan disbursement fee, or (c) the remaining term while keeping all other parameters fixed. We use the initial parameters of our numeric example;

[6] Although hardly discernible, the relationship between i and i^{new} is slightly concave.

Time		0	1	2	...	T
without cancellation	old loan		$-Ann$	$-Ann$		$-Ann$
with cancellation	old loan	$-(1+p)\cdot Nom$				
	new loan	$+(1+p)\cdot Nom$	$-Ann_{new}$	$-Ann_{new}$		$-Ann_{new}$
difference	with – without	0	$Ann-Ann_{new}$	$Ann-Ann_{new}$		$Ann-Ann_{new}$

Table 4: Cash Flow Overview for Case I.B

i.e., a prepayment penalty of 4 %, a loan disbursement fee of 2 %, and a remaining term of 3 years.

The relationship between the prepayment penalty, p, and i^{new} is inverse. The same holds true for the loan disbursement fee, d. This means that an increase in either of these two parameters causes a downward shift in the graph. The shift, however, is not parallel. Rather, there is a twist in the graph meaning that the line becomes flatter and flatter as either the prepayment penalty or the loan disbursement fee increases. The influence of the remaining term, T, on the exact solution is not inverse, on the other hand. Instead an increase in T leads to a higher limit for i^{new}. One might argue that for all debtholders besides consumers the prepayment penalty increases with the remaining term, as it is closely related to the present value of the future interest payments.[7] Our sensitivity analyses, however, are *ceterius paribus* deliberations, so only one parameter is modified at a time.

3.1.2 Case I.B – Annuity Repayment

Unlike the interest only debt contracts mentioned in Section 3.1.1 where the nominal value of the debt is repaid as a lump sum at maturity, the amount of debt outstanding for annuity repayment loans decreases with the loan's lifetime. Case I.B focuses on the analysis of prepaying a fixed rate loan with annuity repayment and refinancing into a new such loan.

The annual payments, or annuities, of the old loan, which consist of both interest and nominal repayments, are

$$Ann = AF_{i,T} \cdot Nom, \tag{11}$$

where the annuity factor (AF) is

$$AF_{i,T} = \frac{i \cdot (1+i)^T}{(1+i)^T - 1}. \tag{12}$$

The annual payments of the new loan are also constant,

$$Ann^{new} = AF_{i^{new},T} \cdot \frac{1+p}{1-d} \cdot Nom, \tag{13}$$

with the corresponding annuity factor,

$$AF_{i^{new},T} = \frac{i^{new} \cdot (1+i^{new})^T}{(1+i^{new})^T - 1}. \tag{14}$$

The exact solution is once again derived from the NPV of the cash flows of the differential investment which are depicted in Table 4.

$$NPV_0 = PVF_{i_{eff},T} \cdot (Ann - Ann^{new}) \geq 0. \tag{15}$$

[7] As mentioned in Section 2.4, the prepayment penalty on consumer loans is capped at 1 % respectively 0.5 %.

Since the NPV must be larger than or equal to zero for the loan prepayment and subsequent refinancing to pay off, the dynamic advantageousness condition is

$$Ann^{new} \leq Ann, \tag{16}$$

or

$$AF_{i_{new},T} \leq \frac{1-d}{1+p} \cdot \frac{Ann}{Nom}. \tag{17}$$

For the approximation the average annual profit of the differential payments,

$$Ann - Ann^{new} \geq 0, \tag{18}$$

is calculated, resulting in the same advantageousness conditions as for the exact solution (see Equations (16) and (17)).

Reconsider the company from the numeric example in Section 3.1.1. In Case I.B the company has an existing loan with the same conditions as specified above. The only difference is that the loan now has annuity instead of lump sum repayment. This means that after the seven years that have passed, the outstanding amount of debt of the old loan has been reduced to 20,419.61 €. Provided that all expenses are to be covered by debt financing the nominal value of the new loan must be 21,669.79 €. Equation (16) states that the annuity of the new loan must not exceed the annuity of the old loan. So the upper limit on the new annuity is 7,358.18 € which implies an upper limit on the nominal interest rate of the new loan of 0.931 %.

Figure 8 shows the sensitivity analyses for Case I.B. The interpretation is the same as for Case I.A; i.e., the higher the prepayment penalty or the loan disbursement fee, the lower the upper limit on i^{new}. In other words, the higher any one of these two parameters, the more stringent is the upper limit on the nominal interest rate of the new loan. The opposite counts for the remaining term of the old loan, where a longer term implies more leeway for i^{new}.

3.2 Case II – Variable Rate Loan to Fixed Rate Loan

This section shows the calculations for Case II; i.e., replacing an existing variable rate loan with a fixed rate loan. Once again we first cover refinancing an existing loan with lump sum repayment into a new loan with lump sum repayment (Case II.A) and subsequently focus on the case of annuity repayment loans (Case II.B).

3.2.1 Case II.A – Lump Sum Repayment

In Case II.A both loans feature lump sum repayment. The annual interest payments of the old loan are

$$Z_t = \max(0, Ref_{t-1} + RP) \cdot Nom, \tag{19}$$

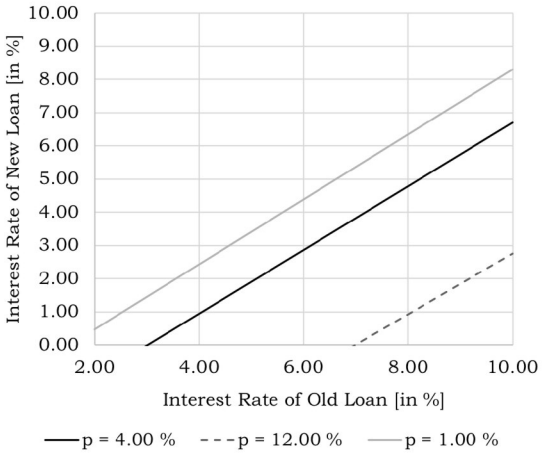
where Ref_{t-1} is the reference interest rate in the tth-period $[t-1, t]$ and RP is the risk premium (or quoted margin). The estimates for the future reference interest rates, Ref_t, are calculated using

$$Ref_{t-1} = \frac{(1+IRS_t)^t}{(1+IRS_{t-1})^{t-1}} - 1, \tag{20}$$

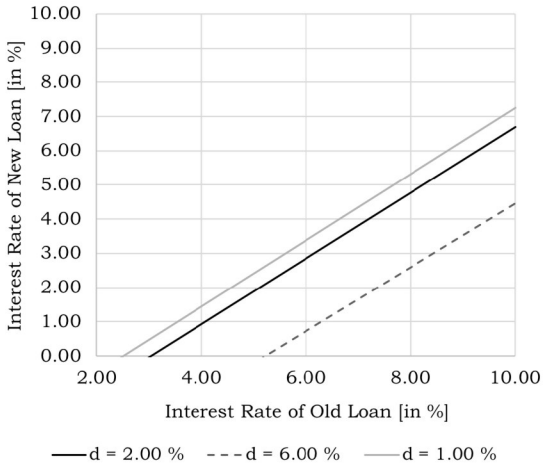
where IRS_t is the interest rate swap with maturity t. The annual interest payments of the new loan are calculated in the same manner as in Case I.A using Equation (5).

As in Case I, the exact solution is derived by setting up a table of payments (Table 5) and subsequently calculating the NPV of the differential investment using i_{eff} as the calculation interest rate. The resulting dynamic advantageousness condition is

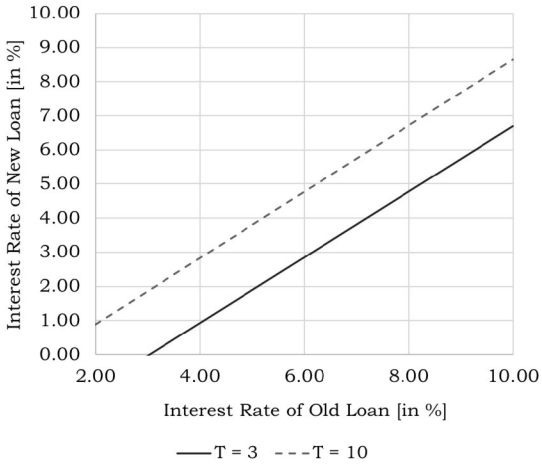
$$i^{new} \leq \frac{1}{1+p} \cdot \left[(1-d) \cdot \left(AF_{i_{eff},T} \cdot \sum_{t=1}^{T} \frac{Ref_{t-1}}{(1+i_{eff})^t} + RP\right) - \frac{i_{eff}}{(1+i_{eff})^T - 1} \cdot (p+d)\right]. \tag{21}$$



(a) Prepayment Penalty ($d = 2.00\%$)



(b) Loan Disbursement Fee ($p = 4.00\%$)



(c) Remaining Term ($p = 4.00\%$, $d = 2.00\%$)

Figure 8: Sensitivity Analyses for Case I.B

Time		0	1	2	...	T
without cancellation	old loan		$-Z_1$	$-Z_2$		$-(Z_T + Nom)$
with cancellation	old loan	$-(1+p)\cdot Nom$				
	new loan	$+(1+p)\cdot Nom$	$-Z^{new}$	$-Z^{new}$		$-\left(Z^{new} + \frac{1+p}{1-d}\right) \cdot Nom$
difference	with – without	0	$Z_1 - Z^{new}$	$Z_2 - Z^{new}$		$Z_T - Z^{new} - \frac{p+d}{1-d} \cdot Nom$

Table 5: Cash Flow Overview for Case II.A

As can be seen from Table 5, the annual interest payments of the old loan can now differ from one period to another since they are dependent on the stochastic reference rate. For this reason, the interest differential can no longer be calculated using the PVF. Instead, the constant nominal interest rate i from Equation (8) is replaced by the *actuarial* average of the future variable interest rates, where the term

$$AF_{i_{eff,T}} \cdot \sum_{t=1}^{T} \frac{Ref_{t-1}}{(1+i_{eff})^t}$$

is the average future reference rate.

The approximation for Case II.A is again calculated using the average annual profit from the differential investment,

$$\left(\frac{1}{T} \cdot Ref_{t-1} - \frac{1+p}{1-d} \cdot i^{new}\right) \cdot Nom - \frac{1}{T} \cdot \frac{p+d}{1-d} \cdot Nom \geq 0, \tag{22}$$

resulting in the static advantageousness condition

$$i^{new} \leq \frac{1}{1+p} \cdot \left[(1-d) \cdot i_{\varnothing} - \frac{p+d}{T}\right]. \tag{23}$$

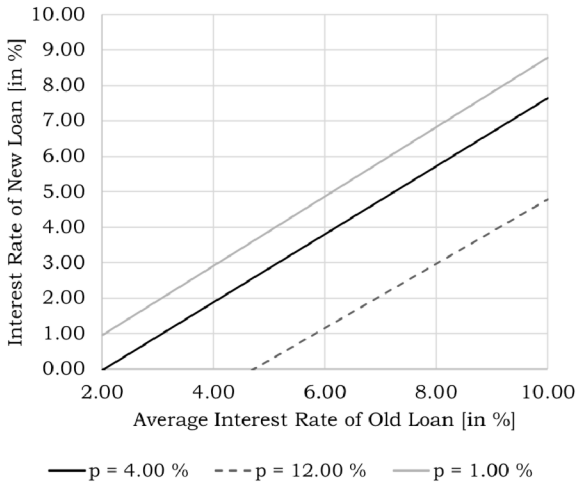
Here, $i_{\varnothing}$ is the *arithmetic* average of the expected future reference rates adjusted by the risk premium.

$$i_{\varnothing} = \frac{1}{T} \cdot \sum_{t=1}^{T} Ref_{t-1} + RP \tag{24}$$

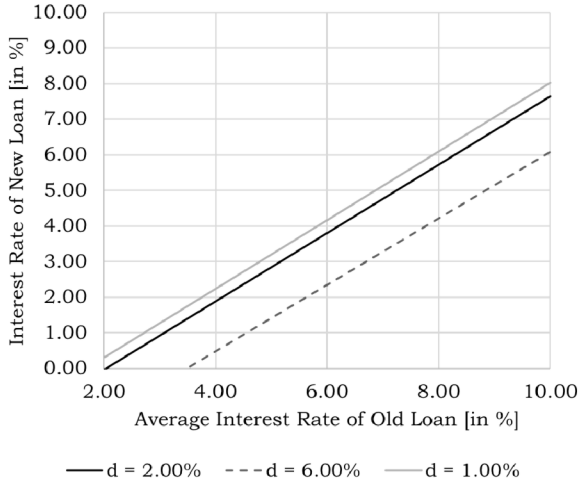
Once again a numeric example should serve as illustration. As in Case I.A, a company has a ten-year loan with nominal value of 100,000 €, lump sum repayment, yearly interest payments in arrears, and a remaining term of three years. However, interest payments are now variable, with the 6M-Euribor at the beginning of each period used as the reference rate adjusted. The quoted margin is 3.5 %. The company can once again decide between keeping the existing loan for the remaining three years or prepaying it and taking out a new three-year fixed rate loan. The prepayment penalty is unchanged at 4 % of the nominal value of the old loan while the transaction costs associated with the new loan amount to 2 % of its nominal value. In this case, since the old loan is a variable rate loan, the expectations of the future reference rates play a key role. Using the interest rate swaps on May 3, 2018, the following reference rates for the upcoming three periods are derived:

$$Ref_0 = -0.26\%,\ Ref_1 = -0.01\%, Ref_2 = 0.39\%.$$

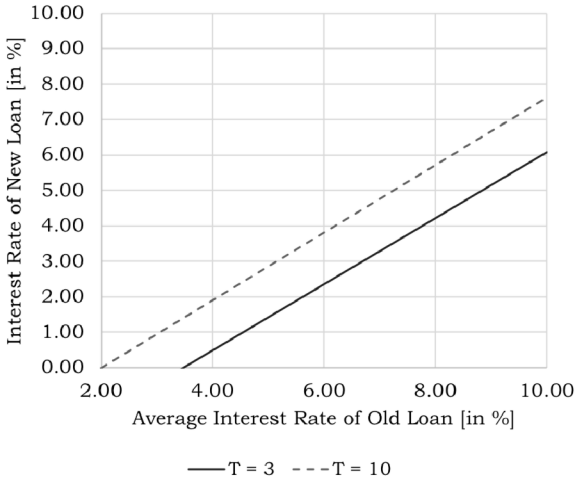
Since all transaction costs are to be covered by debt financing, the nominal value of the new loan must again be 106,122 €. The exact solution then delivers 1.449 % as the upper limit for the nominal interest rate of the new loan while the approximation leads to 1.413 %. This is significantly



(a) Prepayment Penalty ($d = 2.00\%$)



(b) Loan Disbursement Fee ($p = 4.00\%$)



(c) Remaining Term ($p = 4.00\%$, $d = 2.00\%$)

Figure 9: Sensitivity Analyses for Case II.A

lower than the results achieved in the numeric example for Case I.A. The difference is due to the fact that the market is currently only expecting a very moderate increase in the reference rate over the next three years, so the nominal interest rate of the new loan must be very low for a prepayment to pay off. Generally, the steeper the expected increase for the upcoming periods, the higher the limit on i^{new} in Case II. Figure 9 shows the sensitivity analyses for Case II.A. From Figure 9(c) it can be seen that also an increase in the remaining term implies more leeway for i^{new}. The interpretation of the impact of the remaining parameters apply here in an analogous manner as in Case I.A.

Also for Case II the quality of the approximation deteriorates the higher the average future interest rate of the old loan. Analogously to Case I, the approximation lies below the corresponding exact solution, so a false positive decision can once again be ruled out, but a beneficial prepayment may be missed out on.

Time		0	1	2	...	T
without cancellation	old loan		$-Ann_1$	$-Ann_2$		$-Ann_T$
with cancellation	old loan	$-(1+p)\cdot Nom$				
	new loan	$+(1+p)\cdot Nom$	$-Ann_{new}$	$-Ann_{new}$		$-Ann_{new}$
difference	with – without	0	$Ann_1 - Ann_{new}$	$Ann_2 - Ann_{new}$		$Ann_T - Ann_{new}$

Table 6: Cash Flow Overview for Case II.B

3.2.2 Case II.B – Annuity Repayment

Case II.B focuses on with switching from a variable rate to a fixed rate loan when both loans are annuity loans. The annuity of the old loan is

$$Ann_t = AF_{i_t,T} \cdot Nom_{t-1}, \tag{25}$$

with annuity factor

$$AF_{i_t,t,T} = \frac{i_t \cdot (1+i_t)^{T-t+1}}{(1+i_t)^{T-t+1} - 1}. \tag{26}$$

The annual payments are again stochastic because the interest component depends on the reference rate at the beginning of each period.

$$Z_t = i_t \cdot Nom_{t-1} = \max(0, Ref_{t-1} + RP) \cdot Nom_{t-1} \tag{27}$$

The outstanding nominal value is

$$Nom_t = Nom_{t-1} - (Ann_t - Z_t) \tag{28}$$

and the annual payments of the new fixed rate loan are constant and are calculated using Equation (12).

Table 6 shows the cash flows of the two alternatives and the resulting differential investment. Debt prepayment and restructuring is once again advantageous when the NPV of the differential payments is larger than or equal to zero.

$$NPV_0 = \sum_{t=1}^{T} \frac{Ann_t}{(1+i_{eff})^t} - PVF_{i_{eff},T} \cdot Ann^{new} \geq 0 \tag{29}$$

This leads to the exact advantageousness condition

$$Ann^{new} \leq \sum_{t=1}^{T} \frac{Ann_t}{(1+i_{eff})^t} \cdot AF_{i_{eff},T} \tag{30}$$

or

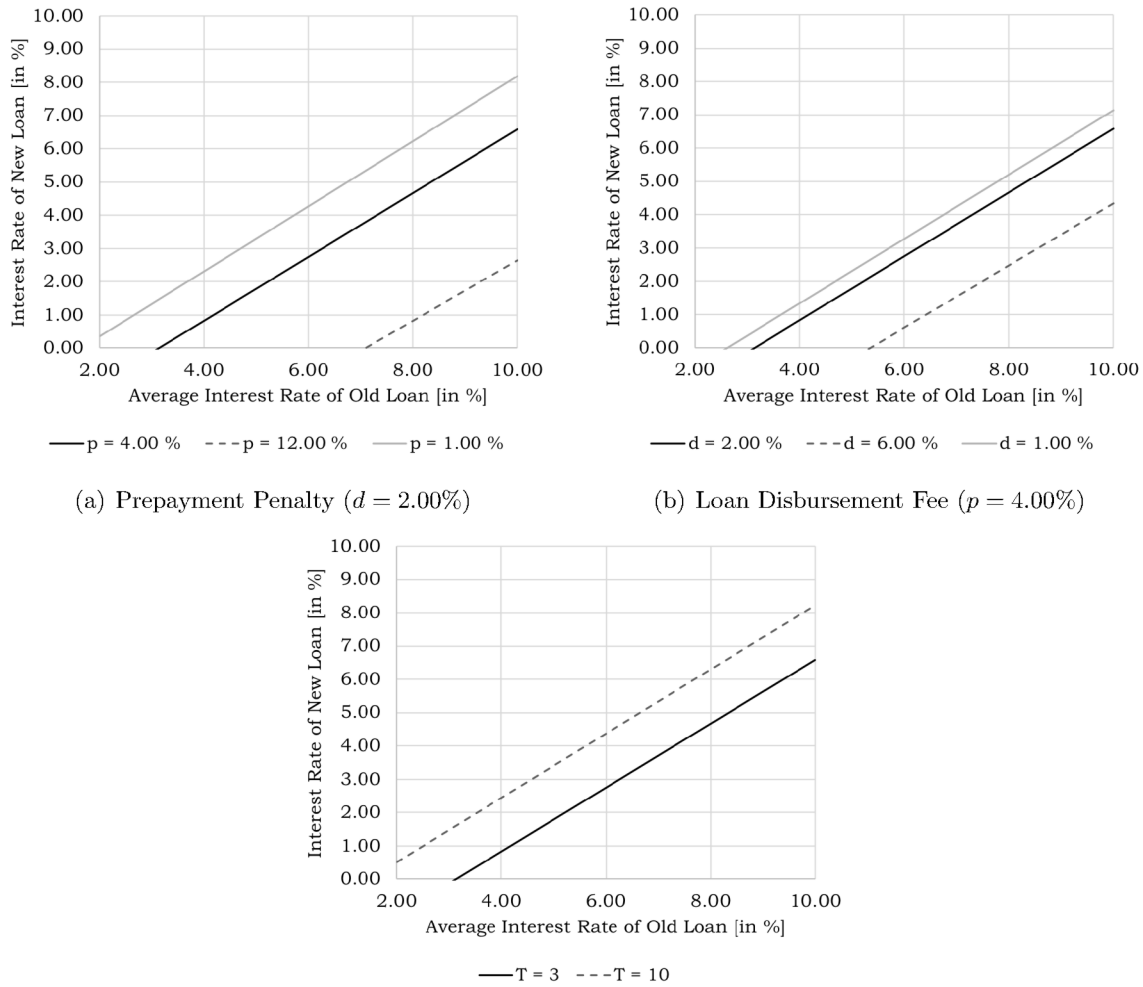
$$AF^{i_{new},T} \leq \sum_{t=1}^{T} \frac{Ann_t}{(1+i_{eff})^t} \cdot \frac{AF_{i_{eff},T}}{Nom} \cdot \frac{1-d}{1+p}. \tag{31}$$

The approximation is derived from the average annual profit from the differential investment,

$$Ann_{\varnothing} - Ann^{new} \geq 0 \tag{32}$$

where

$$Ann_{\varnothing} = \frac{1}{T} \cdot \sum_{t=1}^{T} Ann_t.$$



(a) Prepayment Penalty ($d = 2.00\%$)

(b) Loan Disbursement Fee ($p = 4.00\%$)

(c) Remaining Term ($p = 4.00\%$, $d = 2.00\%$)

Figure 10: Sensitivity Analyses for Case II.B

The static advantageousness condition is

$$Ann^{new} \leq Ann_{\varnothing}. \tag{33}$$

or consequently

$$AF_{i_{new},T} \leq \frac{1-d}{1+p} \cdot \frac{Ann_{\varnothing}}{Nom}. \tag{34}$$

We once again illustrate the application of these formulas by continuing the numeric example from subsection 3.2.1. The debt contract specifications of both the old and the new loan are the same except for the repayment terms. In this case both loans feature annuity repayment. Seven years have passed since the original loan was taken out, so the residual nominal value is 21,882.45 €.[8] All transaction costs are debt financed so the nominal value of the new loan must be 23,222.19 €. The upper limit on the annuity of the new loan is the annuity of the old loan; i.e., 7,800.76 €. This means that the nominal interest rate of the new loan must not exceed 0.386 % for prepayment to pay off. Based on the expected future reference rates this would require the new loan to charge a risk premium of maximum 0.3452 %. Such a low quoted margin will hardly be obtainable even with excellent creditworthiness, hence prepayment will most likely not be favorable in this case.

This section concludes with the sensitivity analyses for Case II.B shown IN Figure 10. Again, increasing either the prepayment penalty or the loan disbursement fee will lower the acceptable

[8]The outstanding debt is calculated based on past the past reference rates from May 3 of each year.

nominal interest rate of the new loan while increasing the remaining term will raise the maximum i^{new}, all other parameters held constant. It must be noted that in Case II.B the approximation may lead to a higher upper limit on i^{new} than the exact solution. This means that a debtor may carry out a refinancing transaction based on the static criterion that in fact is not optimal judging by the dynamic advantageousness criterion. However, it lies in the nature of an approximation that it can lead to false positive decisions.

4 Conclusion and Final Remarks

Loan prepayment and subsequent debt refinancing is currently advantageous under specific circumstances due to the favourable interest rate landscape as well as market expectations of rising interest rates. However, the advantageousness of such an action depends on different factors. First and foremost, for all debt agreements besides loans issued to consumers, prepayment is contingent on the consent of the creditor. If the debt contract specifications of the existing loan do not specify any prepayment modalities, it lies within the creditor's discretion whether and under which conditions a prepayment is possible. Generally, debt restructuring causes transaction costs. In most cases the creditor will charge a prepayment penalty, designed to compensate the losses incurred through the early repayment of the outstanding debt. This penalty can amount to the present value of the remaining interest payments or may even be higher if nothing is defined in the debt contract specifications. In the EU, regulations exist that cap the prepayment penalty. For consumer loans, depending on whether the remaining term of the old loan exceeds one year or not, the penalty is limited 1 % and 0.5 % of the amount of debt prepaid, respectively. Finally, if prepayment is carried out in combination with refinancing, also the terms of the new loan must be taken into account when evaluating the advantageousness.

We propose a solution approach based on the method of differential investment and provide both an exact solution for calculating the upper bound of the nominal interest rate of the new loan as well as an approximation that, although not as precise, provides reasonable indications of whether to restructure or not. The exact solution is derived by applying a net present value concept to the differential investment while the approximation focuses on the average profit of the differential payments.

Finally, we have to remark that profitable debt restructuring with loan prepayment might be falsely omitted by the decision-makers because it results in an increase of the current debt level. The agent may want to avoid restructuring existing debt even if the present value of future interest savings is larger than the debt level increase. This is especially plausible if his or her reputation or performance is negatively connected to this key figure, or if the agent is near the end of his or her term of office and does not benefit from interest savings in the future. This leads to a principal-agent-problem of omitted debt restructuring.

References

Fischer, Edwin O. (2008). Finanzwirtschaft für Anfänger, 5. Aufl., Oldenbourg, München Wien.
Staab, Helmut und Peter Staab (2014). Kreditvertrags- und Kreditsicherungsrecht – Die Rechtsprechung des BGH, Springer Gabler, Wiesbaden.
European Parliament, European Council (22.05.2008). Directive 2008/48/EC on credit agreements for consumers, OJ L 133, 66-92.
European Parliament, European Council (28.02.2014). Directive 2014/17/EU on credit agreements for consumers relating to residential immovable property, OJ L 60, 34-85.
Sangvinatsos, Antonios (2010). Expectations Hypothesis, In: Cont, Rama (Ed.). Encyclopedia of Quantitative Finance, Wiley Finance, Hoboken.