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Abstract

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Keywords: closed-loop supply chain, product acquisition management, manufacturing/remanufacturing strategies, two-period newsvendor model

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Abstract

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Keywords: Closed-loop Supply Chain, Product Acquisition Management, Manufacturing/Remanufacturing Strategies, Two-period Newsvendor Model

1 Introduction

An observable trend in supply chain management is the change from a view on forward logistic activities including disposal logistics to reverse logistics and reverse production, and finally, to closed-loop supply chains (CLSC) integrating coordinated forward and reverse processes. Competition, resource scarcity, and legislation (e.g., the well-known WEEE [29]) force companies to act resource efficient. Besides these enforcements incentives can support the implementation of reusing and reprocessing concepts in companies. In detail, an increased profitability can attract companies to put reprocessing activities into practice, as indicated by case studies [2] or scientific work [10].

Hewlett-Packard (HP) as well as IBM established various reprocessing activities. In 2011, HP enabled direct reuse of 26,700 tons of electronic products, and additionally collected 133,900 tons of used electronic products for recycling [11]. Similar to HP, IBM reprocessed around 37,950 tons of used electronic equipment in 2011. In detail, 6.6% of the collected products could be reused, 38.6% were sold to reproducers, and 52.4% recycled to receive reusable raw materials [12].

The requirements of acting sustainable and environment-conscious also led to an increased attention within the scientific community. Guide and Van Wassenhove [9] describe the activities of reprocessing as acquisition of a used product, testing, sorting, and quality grading of an acquired core. Moreover, explicit

disposition options are remanufacturing, repair, parts recovery, material recycling, and disposal. Traditional approaches of operations research (OR) consider in many cases single isolated activities for research. In the field of reverse logistics and closed-loop supply chains, the optimization of inventory systems ([26], [28]), the design of closed-loop supply networks ([6]), or production lot sizing ([24]) are examples applying OR-methods on single processes. Considering such a single sub-problem does not allow a holistic view on closed-loop supply chains with interrelated sub-processes. To overcome this issue, reprocessing activities can be combined to integrated models with interdependent processes. For example, an integrated model must consider that the acquisition process has complex effects on the manufacturing and remanufacturing decisions, as the quantity and quality of the available used cores determines the optimal disposition decision. Consequently, there is need for integrated models including interdependent sub-processes within a closed-loop context to explore these effects.

In this paper, we present a stochastic two-period model with a joint consideration of an active effort-dependent acquisition process and a manufacturing-remanufacturing disposition process. The acquisition process is a crucial factor, as it ensures the supply with used cores for remanufacturing and, in consequence, affects the manufacturing-remanufacturing strategy. In the first period new products are manufactured, depending on the uncertain demand. The sales of the first period can be acquired for remanufacturing in the second period by spending some costly effort. Thus, the demand in the second period can either be fulfilled by producing new items or remanufacturing used products. The costly effort is motivated both by giving incentives to customers (e.g., discounts, when customers buy a new product) and costs, for instance to establish and operate a collection network.

The optimal acquisition, manufacturing, and remanufacturing strategy is determined in different scenarios. The main trade-off in the model is the interplay between sales in the first period and the spent acquisition effort to obtain an optimal pool of used products to acquire for remanufacturing in the second period. There are two options to control the supply with used products. On the one hand, the producer can stimulate sales by excess production in the first period. On the other hand, the acquisition quantity can be increased by spending a higher effort on the acquisition in the second period. As a result, the conditions for excess production in period one and the optimal acquisition effort can be determined. The base model is extended by the possibility to store excess production produced in the first period. Therewith, another trade-off between the costs for inventory in the first period and the discounted costs for acquisition and remanufacturing of used products in the second period is implemented. Numerical analyses provide insights into the profitability, the acquisition effort, and the manufacturing and remanufacturing quantities of the models.

One of the main results obtained is the necessity to increase/decrease the first period new production and the acquisition effort simultaneously in the model without the option to store first period new production. In this model without inventory, both the new production quantity in the first period and the acquisition effort increase with rising savings from remanufacturing. Compared with this, increasing remanufacturing savings do not trigger the same behavior in the model with the possibility to store first period new overproduction: numerical analyses show deviating results where the first period new production increases, then decreases, and finally, increases again with rising remanufacturing savings, while the acquisition effort increases in all cases. Finally, we find that the optimal acquisition/manufacturing/remanufacturing(/inventory)-strategy is highly depending on the efficiency of the acquisition process.

The first model without inventory represents two similar, but not equal generations of one product. The sold products of the first generation can be collected and remanufactured after their end-of-use. There are several possible fields of application, e.g., concerning smart-phones, gambling machines, or PC's. Regarding the model with the possibility to store excess production, the practical usage is on an operational

production-planning level, for example, the production planning of printer cartridges.

The remainder of the paper is organized as follows. Section 2 presents the literature related to our model and the research gap which we work on with this article. The formal model is introduced and related analytical results are shown in section 3. Section 4 gives insights into the extension of the model to store excess production and the according analytical results. Numerical analyses regarding the model without and with the option to store first period production are presented in section 5. Finally, section 6 includes a summary and an outlook on further research opportunities.

2 Related Work

Over the last years, scientists paid high attention on closed-loop supply chains. The acquisition of used products for reprocessing is studied intensively, often in combination with different activities within a closed-loop supply chain.

The joint consideration of active acquisition and a quality grading process is presented in [7] and [8]. In [7], a bundle of used products with known quality distribution is acquired. Afterwards, a grading process determines the qualities of the acquired products and in further consequence the quality-dependent remanufacturing costs. The optimal acquired quantity of used products may exceed the consumer demand, as the increased acquisition quantity allows a stricter selectivity which items to remanufacture. This procedure results in a less costly remanufacturing process. A similar model is presented in [8], but in this paper the quality distribution of the acquired products is unknown. The authors analyze several models with different quality levels and cost functions, whereby the results found in [7] are confirmed.

The combination of an acquisition process and a disposition process can be found in [10], [13], [15], [16], [17], [19], and [22]. [10] and [13] use deterministic single-period models, while [19] and [22] consider stochastic ones. In [10], the quantity and quality of returned used products is affected by an acquisition price. The authors determine the optimal acquisition prices, acquisition quantities, and sales prices. Optimal acquisition and sales prices are also determined in [13]. An original equipment manufacturer and a remanufacturer are competitors or cooperate in different scenarios. The analyses are performed in a model environment where the prior sales restrict the supply with used products. In [19], the focus is on the optimal acquisition prices and quantities of a consolidation center, which acquires used products with different quality levels from collection centers. While the orders of the remanufacturer for each quality level are deterministic, the supply with used cores is an uncertain process. Optimal production and remanufacturing quantities and optimal prices for the acquisition of used items are derived in [22]. The authors present a production planning-model with a joint capacity constraint for a product portfolio. The supply with used products is price-sensitive, and both the supply as well as the demand are stochastic. In [15], the optimal product acquisition prices are derived under a stochastic sales price. By setting incentives, the acquisition rate can be influenced actively. A single-period and a finite multi-period model to obtain the optimal strategy concerning acquisition price and manufacturing and remanufacturing quantities under deterministic demand is studied in [16]. Moreover, storing the acquired used cores is possible in the multi-period model. A dynamic optimization approach is developed, and some heuristics are tested. In [17], the authors use a multi-period model solved with mixed-integer linear programming to obtain optimal procurement, remanufacturing, stocking, and salvaging decisions. The acquired used products need different remanufacturing efforts due to the heterogeneous qualities of the items.

Integrated models with acquisition, grading, and disposition sub-processes are presented in [1], [21], [23], [25], and [27]. In [1], a model with both price-sensitive supply of used cores and price-sensitive demand is analyzed. By increasing the acquisition price, the return rate can be raised, but the remanu-

facturer faces a stochastic yield. The optimal acquisition and sales prices are derived. The consequences of the option to remanufacture are explored in a one-period newsvendor-like setting in [21]. The authors compare two models with and without the option to remanufacture. Depending on the uncertain quality of the acquired items, the products can either be resold directly, or undergo a remanufacturing process to make them saleable. Additionally, high quality items can also be sold in the low-quality-market, but not vice versa. The remanufacturing option reduces the quantities which have to be acquired to optimize the profits, what leads to higher overall profits due to reduced costs. The design of a supply chain with centralized/decentralized grading processes is studied in [23]. The multi-period model includes uncertain demands and deterministic yield in the grading process. Considering settings without, with centralized, or with decentralized sorting, the optimal supply chain-design and the value of a sorting procedure can be determined. In [25], a model containing uncertain quality of returned products and uncertain demand is presented. The authors determine optimal acquisition and production strategies, whereby the quality information coming from a grading process always raises profits compared to the case without any quality information. A model including a product recovery facility facing deterministic demand is analyzed in [27]. Two out of four scenarios consider a passive return of used items, and two further scenarios allow an active acquisition process with price incentives paid to customers. The scenarios with an active acquisition process outperform the passive ones in terms of costs.

Sales-dependent acquisition processes can be found in [14] and [26]. [14] considers stochastic returns, and the returned quantity for reusing is restricted by the sales in previous periods. As loss of products over time is assumed, new production may appear. In [26], the sales in past periods are the base for the return flow of used products. The model contains two sources of uncertainty, on the one hand the return delay and on the other the return probability of used cores.

Summarizing the literature, our model extends the current work in some points. Unlike [1], [7], [8], [10], [15], [19], [21], [23], [25], and [27], we use a closed-loop supply chain with joint manufacturing-remanufacturing decisions instead of approaches with pure reverse logistics. In [13], [16], [17], and [27] deterministic demands are assumed, while we consider stochastic demands. In contrast to the single-period-approach in [22], we use a time horizon of two periods to explore intertemporal effects. In [14] and [26], the returns are received passively, without the option to control the acquisition process. The restriction of the maximum acquisition quantity to sales in previous periods does not appear in any of the mentioned papers, except for [13], [14], and [26].

3 The Model

The model is based on [20], which is motivated by a model presented by Ferrer and Swaminathan [4]. It deals with the optimal planning decisions concerning the manufacturing of new products, and the acquisition and remanufacturing of used cores in a two-period environment with stochastic, newsvendor-like demands. The production of new items is possible in both periods. Additionally, in the second period products sold in the first period can be acquired and remanufactured at lower costs than producing new ones, while the demand in the first period must be covered by new production. In the second period new and remanufactured products can fulfill the demand, as they are assumed to be perfect substitutes. The manufacturer is considered as a price-taker, so overproduction is possible in the first period to stimulate the expected sales. In contrast to [20], where the return rate is exogenously given and therefore fixed, the core collection rate can be controlled actively by spending a certain acquisition effort. In this way, an increased per-unit acquisition effort leads to an increased return rate, and consequently to a raised acquisition quantity. The core collection rate increases in the average acquisition effort due to efficiency reasons, but at a

decreasing rate. Thus, the acquired quantity depends on both the effort-dependent core collection rate and the sales in the first period. Logically, the sales of new products in period one restricts the quantity that can be acquired in the second period.

In periods $t = 1, 2$, the new production quantities q_t , the acquisition effort c_r , and the remanufacturing quantity \hat{q}_2 are optimized in order to maximize the expected profit π over both periods. $0 \leq \beta \leq 1$ represents the discount factor for profits in the second period. p_t, c_t are the per-unit selling prices and production costs for a new unit, respectively. In the case that a product is remanufactured instead of newly manufactured, $\delta > 0$ is the obtained saving per remanufactured item, i.e., remanufacturing is more efficient than new production ($c_2 - \delta < c_2$). The acquisition effort c_r directly influences the concave, continuous core collection yield function $\gamma(c_r)$. Without loss of generality, we assume that $\gamma(0) = 0$. Additionally, the resulting costs from the acquisition effort may not exceed the savings from remanufacturing, as then new production would be more efficient than remanufacturing (therefore, $c_r \leq \delta$). D_t are uncertain demands, following known probability density functions $f_{D_t}(\cdot)$ and cumulative distribution functions $F_{D_t}(\cdot)$, respectively ($E(D_t) > 0$). At last, $S_{D_t}(q) = \int_0^q u f_{D_t} du + q_t (1 - F_{D_t}(q))$ are the expected sales in period t . It is obvious that the offered quantities are $q_t = q_1$ in the first period and $q_t = q_2 + \hat{q}_2$ in the second period.

3.1 Formulation of the Model

We can formulate the optimization model including the defined requirements as:

$$\begin{aligned} \max_{q_1, q_2, \hat{q}_2, c_r} \quad & \pi = -c_1 q_1 + p_1 S_{D_1}(q_1) + \\ & + \beta [-c_2 (q_2 + \hat{q}_2) + \delta \hat{q}_2 + p_2 S_{D_2}(q_2 + \hat{q}_2) - c_r \gamma(c_r) S_{D_1}(q_1)] \end{aligned} \quad (1)$$

s.t.

$$\hat{q}_2 \leq \gamma(c_r) S_{D_1}(q_1) \quad (2)$$

$$\gamma(c_r) \leq 1 \quad (3)$$

$$q_1, q_2, \hat{q}_2, c_r \geq 0 \quad (4)$$

Constraint (2) ensures that the remanufactured quantity does not exceed the number of acquired used items, while constraint (3) restricts the core collection rate to a maximum of 1. The concavity of the objective function and the convexity of the constraints allow the application of Karush-Kuhn-Tucker-conditions (KKT) to the model.

3.2 Analytical Results

The unconstrained single-period newsvendor quantity $q_1^{NV} = F_{D_1}^{-1}\left(\frac{p_1 - c_1}{p_1}\right)$ serves as a reference value to measure a possible quantity deviation in the first period. Therefore, excess production occurs in the case when q_1 exceeds q_1^{NV} .

Note that the remanufacturing constraint (2) is always binding ($\hat{q}_2 = \gamma(c_r) S_{D_1}(q_1)$, $\lambda_1 > 0$). In the case that the acquired quantity is greater than the remanufactured quantity, an adjustment of the acquisition quantity to the remanufacturing quantity would always lead to higher profits due to acquisition cost savings.

Depending on the shadow price λ_1 , the optimal acquisition and manufacturing-remanufacturing strategy is characterized by the following two scenarios. In both scenarios, excess production in the first period and

acquisition and remanufacturing of used products takes place.

Scenario M1.1: Excess production in period 1, acquisition & remanufacturing of used products in period 2, no new production.

If $\beta\delta > \lambda_1 > \beta c_r$, the following equations define the optimal production scenario:

$$q_1 = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r)} \right) \quad (5)$$

$$\hat{q}_2 = \gamma(c_r) S_{D_1}(q_1) \geq F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) \quad (6)$$

$$q_2 = 0 \quad (7)$$

$$\left(c_r + \frac{\gamma(c_r)}{\frac{\partial}{\partial c_r} \gamma(c_r)} \right) = \frac{1}{\beta} \lambda_1 \quad (8)$$

Scenario M1.2: Excess production in period 1, acquisition & remanufacturing of used products and new production in period 2.

If $\lambda_1 = \beta\delta$, the following equations define the optimal production scenario:

$$q_1 = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\delta - \beta c_r)}{p_1 + \gamma(c_r)(\delta - \beta c_r)} \right) \quad (9)$$

$$\hat{q}_2 = \gamma(c_r) S_{D_1}(q_1) < F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) \quad (10)$$

$$q_2 = F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) - \hat{q}_2 \quad (11)$$

$$\lambda_1 = \beta\delta \quad (12)$$

$$\left(c_r + \frac{\gamma(c_r)}{\frac{\partial}{\partial c_r} \gamma(c_r)} \right) = \delta \quad (13)$$

New production in the second period only appears in the second scenario M1.2 presented below, while in both scenarios remanufacturing of used acquired cores occurs. Equation (8) shows that c_r increases/decreases with λ_1 , and vice versa. Furthermore, this property can also be detected concerning the first period new production q_1 , as it rises with c_r and λ_1 (see equation (5)). With respect to these properties, the optimal profit-maximizing value for λ_1 can be found by a numerical search method, e.g., the bisection method.

Regarding scenario M1.2, in which λ_1 equals the discounted savings of remanufacturing $\beta\delta$, a similar approach is used. The crucial point is to numerically solve equation (13) to get the optimal value for c_r , and subsequently obtain the optimal solution for the second scenario M1.2.

As mentioned above, an intuitive result is that all acquired used products are remanufactured, as con-

straint (2) is binding in any case. Interestingly, there is always excess production in period 1. This can be explained by the remanufacturing savings: a larger supply induces increased first period sales, which lead to higher availability of used products for acquisition in the second period, and, in consequence, to higher second period profits due to less production costs.

New production increases in the first period as long as the difference between optimal and decreased 1st period profits is less than the raised profits in the second period.

As the analytical findings shown above are based on a general core collection function, using a specific acquisition function leads to more explicit analytical results. Therefore, a base case acquisition function is introduced:

$$\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}. \quad (14)$$

The parameter x allows to control the slope of the acquisition cost functions, and consequently, to increase the variability of possible cost functions. Constraint (3), $\gamma(c_r) \leq 1$, combined with the acquisition functions (14) leads to the requirement $\frac{c_r}{c_2 x} \leq 1$. δ is assumed to be greater than c_r and less than c_2 , and therefore, $c_r < c_2$. Furthermore, as x is assumed to be greater or equal than 1, constraint (3) always holds for this acquisition function.

Using (14), the optimal scenarios M1.1/M1.2 can be further specified. Note that $\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$, $\gamma(c_r)' = \frac{1}{2c_2 x \sqrt{\frac{c_r}{c_2 x}}}$, $\frac{\gamma(c_r)}{\gamma(c_r)'} = 2c_r$, and consequently, $\lambda_1 = 3\beta c_r$ (see equation (8)) and $c_r = \frac{\delta}{3}$ (see equation (13)), respectively.

Scenario MIS.1: Excess production in period 1, acquisition & remanufacturing of used products in period 2, no new production.

If $\beta\delta > \lambda_1 > \beta c_r$ and $\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$, the following equations define the optimal production scenario:

$$q_1 = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r)} \right) = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \sqrt{\frac{c_r}{c_2 x}}(2\beta c_r)}{p_1 + \sqrt{\frac{c_r}{c_2 x}}(2\beta c_r)} \right) \quad (15)$$

$$\hat{q}_2 = \sqrt{\frac{c_r}{c_2 x}} S_{D_1}(q_1) \geq F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) \quad (16)$$

$$q_2 = 0 \quad (17)$$

$$c_r = \frac{\lambda_1}{3\beta} < \frac{\beta\delta}{3\beta} = \frac{\delta}{3} \quad (18)$$

Scenario MIS.2: Excess production in period 1, acquisition & remanufacturing of used products and new production in period 2.

If $\lambda_1 = \beta\delta$ and $\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$, the following equations define the optimal production scenario:

$$q_1 = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\delta - \beta c_r)}{p_1 + \gamma(c_r)(\delta - \beta c_r)} \right) = F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \sqrt{\frac{c_r}{c_2 x}} (\delta - \frac{\beta \delta}{3})}{p_1 + \sqrt{\frac{c_r}{c_2 x}} (\delta - \frac{\beta \delta}{3})} \right) \quad (19)$$

$$\hat{q}_2 = \sqrt{\frac{c_r}{c_2 x}} S_{D_1}(q_1) < F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) \quad (20)$$

$$q_2 = F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) - \hat{q}_2 \quad (21)$$

$$c_r = c_r^{max} = \frac{\delta}{3} \quad (22)$$

One interesting effect concerning c_r and q_1 appears: both variables are always optimized simultaneously, but not only the acquisition effort or the new production in period 1. This effect can be explained by analyzing equations (15) and (19), respectively. In both cases an increase/decrease of c_r directly leads to a raised/reduced first period production quantity q_1 , and vice versa. This ensures the optimal balance of the trade-off between raised costs in the first period and gainings from acquisition/remanufacturing of used products in the second period.

In scenario M1S.1, the acquisition effort c_r is below the maximum acquisition effort c_r^{max} . A rise in c_r directly increases the shadow price λ_1 up to a specific maximum value. In the case that this maximum value is reached, the optimal scenario switches to M1S.2. As it is determined by the acquisition effort function, the specific maximum effort is $\frac{\delta}{3}$ when $\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$. Interestingly, this is far below the limit of $c_r < \delta$.

3.3 Numerical Analysis

In this section, we present some numerical analyses to support the insights obtained from the analytical results. First of all, a numerical base case is introduced. The first period demand $D_1 \sim U(a_1, b_1)$, $a_1 = 25$, $b_1 = 75$ is the same as the second period demand $D_2 \sim U(a_2, b_2)$, $a_2 = 25$, $b_2 = 75$. The discount factor is $\beta = 0.9$, and both prices $p_1 = p_2 = 10$ and costs $c_1 = c_2 = 8$ are the same in the first and second period. Remanufacturing savings amount to $\delta = 4$. Insights into the sensitivity of the model can be gained by varying the parameter δ , which directly leads to different remanufacturing conditions. δ is varied in steps of 0.5, beginning from 0.5 to 7.5, to obtain the results in the whole range from a rather cost-intensive remanufacturing process up to high savings from remanufacturing used items. Subsequently, the resulting scenarios are analyzed. The base case includes a value for x of 1. Regarding the effort-dependent acquisition function, equation (14) is implemented.

The analyses considering different savings from remanufacturing (δ) are presented in Figure 1. One main effect observable is the increasing profitability of remanufacturing with a raising δ . In the case $D_2 \sim U(25, 75)$ on the left side, new production (q_2) always occurs in the second period (optimal scenario M1.2). Rising cost savings from remanufacturing lead to a higher possible acquisition effort, an increased excess production in the first, and higher remanufacturing rates in the second period, as acquisition and remanufacturing of used products becomes more attractive. The remanufacturing cost savings have a strong positive impact on the profits.

Next, we consider the case of a market decline for the product in the second period ($D_2 \sim (5, 55)$), shown in the right part of Figure 1. This decline can be driven by macro-economical factors, as ob-

served in the semiconductor industry (see, e.g., [30]). Furthermore, micro-economical influences may result in this effect. For example, innovation in high-technology industries leads to substitution of first-generation products by products of the second generation, and subsequently, to reduced market potential for first-generation products (e.g., smartphones, microchips). Clearly, an OEM tries to find the optimal manufacturing-remanufacturing strategy under these conditions. Hence, the question when new production in the second period should take place and when a pure remanufacturing strategy suffices, can be answered: for a sufficiently low expected demand in the second period (e.g., in this example $D_2 \sim (5, 55)$) and a relatively high δ (e.g., in this case $\delta \geq 4$), remanufacturing is the only source of supply in the second period. The reason for this characteristic is the rising profitability of remanufacturing with a more efficient remanufacturing process. In detail, for $\delta \geq 4$ the acquisition efforts are below the maximum limits compared to the base case ($c_r < c_r^{max}$), and the second period new production is zero. Even acquisition efforts which are lower than the potential maximum acquisition efforts ensure a sufficient supply with used products, and no new production is necessary. With respect to the analytical results presented in section 3.2, this can be explained by the transition from scenario M1.2 to scenario M1.1 due to the increased savings from remanufacturing in an environment with a low demand level. As shown in the analytical results, excess first period production appears in all scenarios ($q_1 > q_1^{NV}$).

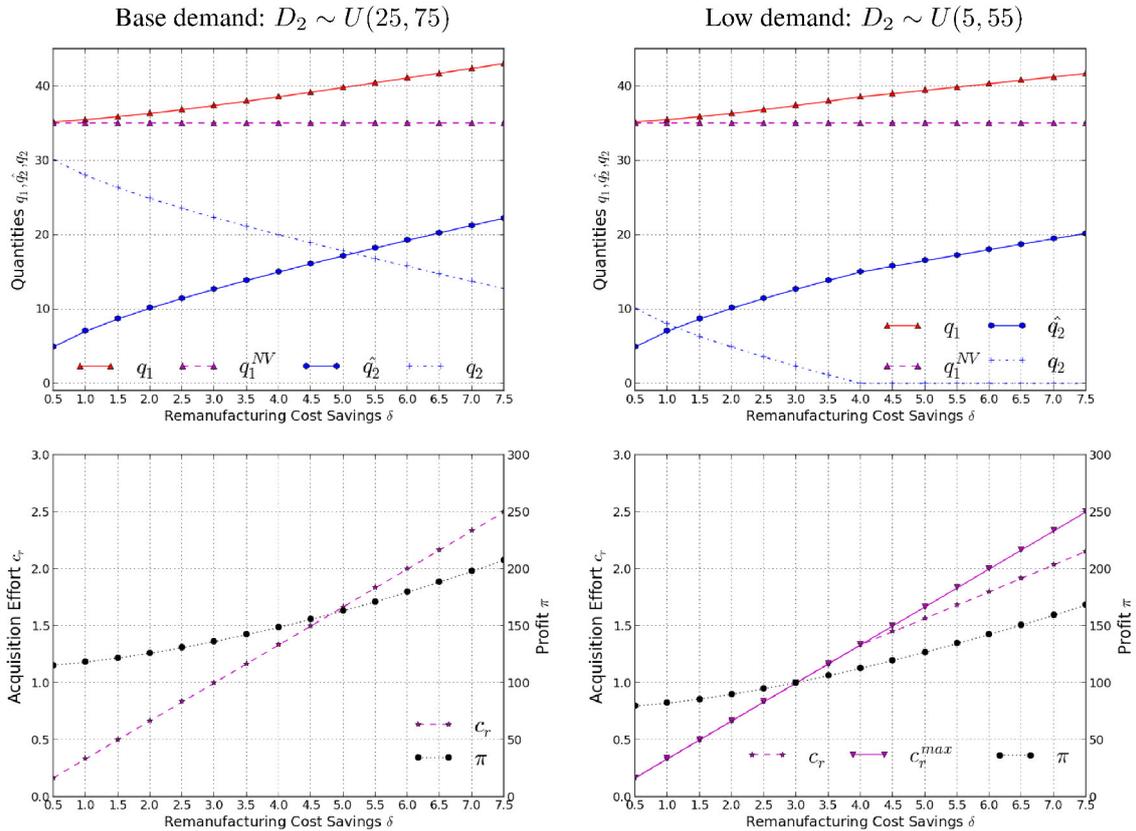


Figure 1: Sensitivity analyses with varying δ , $D_2 \sim U(25, 75)$ and $D_2 \sim U(5, 55)$

4 The Model with Inventory Carry-over

In this section, the model presented in Section 3 is extended by the possibility to store excess first period new production. Due to demand uncertainty, there may remain some unsold items in the first period, which

can be stored and sold in the second period. Consequently, inventory can act as a third source of supply, next to new production and remanufacturing. This model represents a production planning problem, considering the production/remanufacturing of the same product types in two sequenced periods. The inventory I causes costs of h in the first period, while it can be used for sales in period 2 at no charge. A new constraint is introduced in equation (27): Logically, the stored production quantity I can not exceed the expected remaining quantity $q_1 - S_{D_1}(q_1)$. We refer to the remaining quantity $q_1 - S_{D_1}(q_1)$ as expected inventory $I_{D_1}(q_1)$. Another assumption concerns the holding costs $0 \leq h \leq \beta c_2$. In the case of $h > \beta c_2$, storing can never be optimal, because then new production in period 2 would always be favored over holding inventory due to lower costs. To ensure that producing a quantity greater than the maximum demand in period 1 and storing these products can not appear, we assume $c_1 + h \geq \beta c_2$. Clearly, the supply quantity for the second period demand is given by $q_2 + \hat{q}_2 + I$, so in consequence the expected sales are $S_{D_2}(q_2 + \hat{q}_2 + I)$.

4.1 Formulation of the Model

Again, the concave objective function and the convex constraints allow the application of Karush-Kuhn-Tucker-conditions. Considering the requirements mentioned above, the model is defined by:

$$\begin{aligned} \max_{q_1, q_2, \hat{q}_2, c_r, I} \pi = & -c_1 q_1 + p_1 S_{D_1}(q_1) - h I + \\ & + \beta [-c_2 (q_2 + \hat{q}_2) + \delta \hat{q}_2 + p_2 S_{D_2}(q_2 + \hat{q}_2 + I) - c_r \gamma(c_r) S_{D_1}(q_1)] \end{aligned} \quad (24)$$

s.t.

$$\hat{q}_2 \leq \gamma(c_r) S_{D_1}(q_1) \quad (25)$$

$$\gamma(c_r) \leq 1 \quad (26)$$

$$q_1 - S_{D_1}(q_1) - I \geq 0 \quad (27)$$

$$q_1, q_2, \hat{q}_2, I, c_r \geq 0 \quad (28)$$

4.2 Analytical Results

Depending both on the costs for acquisition/remanufacturing of used products ($\beta(c_2 - \delta)$) as well as storing first period production (h), either remanufacturing or holding is the primary supply for the second period. The two different resulting manufacturing - remanufacturing - inventory strategies are determined by the shadow prices λ_1 (remanufacturing constraint (25)) and λ_2 (inventory constraint (27)). In both cases regarding the preferred first supply option for the demand in period 2, four different production scenarios define the optimal manufacturing - remanufacturing - inventory strategy. The strategy is presented below: in the left column, storing excess production is the preferred primary supply ($h < \beta(c_2 - \delta)$). On the right hand side, the case when remanufacturing acquired used products is the preferred primary supply is shown ($h > \beta(c_2 - \delta)$). Beginning from the third scenario, the optimal strategy is the same for both cases.

Optimal manufacturing - remanufacturing - inventory strategy

$$h < \beta(c_2 - \delta)$$

Storing excess production is preferred

Scenario M2.1.A:

- No excess production in period 1
- Limited use of inventory
- No acquisition/remanufacturing of used products

$$\begin{aligned}\lambda_1 &= 0 \\ \lambda_2 &= 0 \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1}{p_1} \right) \\ I &= F_{D_2}^{-1} \left(\frac{\beta p_2 - h}{\beta p_2} \right) < I_{D_1}(q_1) \\ \hat{q}_2 &= 0 \\ q_2 &= 0 \\ c_r &= 0\end{aligned}$$

Scenario M2.2.A:

- Excess production in period 1
- Full use of inventory
- No acquisition/remanufacturing of used products

$$\begin{aligned}\lambda_1 &= 0 \\ 0 < \lambda_2 &\leq \beta(c_2 - \delta) - h \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1}{p_1 - \lambda_2} \right) \\ I &= F_{D_2}^{-1} \left(\frac{\beta p_2 - h - \lambda_2}{\beta p_2} \right) = I_{D_1}(q_1) \\ \hat{q}_2 &= 0 \\ q_2 &= 0 \\ c_r &= 0\end{aligned}$$

Scenario M2.3.A/M2.3.B:

- Excess production in period 1
- Full use of inventory
- Acquisition/remanufacturing of used products

$$\begin{aligned}\lambda_1 &= \beta \left(\frac{\gamma(c_r)}{\gamma(c_r)^{\gamma c_r}} + c_r \right) < \beta \delta \\ \beta(c_2 - \delta) - h < \lambda_2 &= \beta \left(c_2 + c_r - \delta + \frac{\gamma(c_r)}{\gamma(c_r)^{\gamma c_r}} \right) - h < \beta c_2 - h \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r) - \lambda_2} \right) \\ \hat{q}_2 &= \gamma(c_r) S_{D_1}(q_1) \\ I &= I_{D_1}(q_1) \\ q_2 &= 0\end{aligned}$$

Scenario M2.4.A/M2.4.B:

- Excess production in period 1
- Full use of inventory
- Acquisition/remanufacturing of used products
- New production in period 2

$$\begin{aligned}\lambda_1 &= \beta \delta \\ \lambda_2 &= \beta c_2 - h \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r) - \lambda_2} \right) \\ \hat{q}_2 &= \gamma(c_r) S_{D_1}(q_1) \\ I &= I_{D_1}(q_1) \\ q_2 &= F_{D_2}^{-1} \left(\frac{p_2 - c_2}{p_2} \right) - I - \hat{q}_2\end{aligned}$$

$$h > \beta(c_2 - \delta)$$

Remanufacturing acquired used products is preferred

Scenario M2.1.B:

- Excess production in period 1
- No use of inventory
- Acquisition/remanufacturing of used products

$$\begin{aligned}\lambda_1 &= \beta \left(\frac{\gamma(c_r)}{\gamma(c_r)^{\gamma c_r}} + c_r \right) < \beta(\delta - c_2) + h \\ \lambda_2 &= 0 \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r)} \right) \\ \hat{q}_2 &= F_{D_2}^{-1} \left(\frac{p_2 - c_2 + \delta}{p_2} - \frac{\lambda_1}{\beta p_2} \right) \\ I &= 0 \\ q_2 &= 0\end{aligned}$$

Scenario M2.2.B:

- Excess production in period 1
- Limited use of inventory
- Acquisition/remanufacturing of used products

$$\begin{aligned}\lambda_1 &= \beta \left(\frac{\gamma(c_r)}{\gamma(c_r)^{\gamma c_r}} + c_r \right) = \beta(\delta - c_2) + h \\ \lambda_2 &= 0 \\ q_1 &= F_{D_1}^{-1} \left(\frac{p_1 - c_1 + \gamma(c_r)(\lambda_1 - \beta c_r)}{p_1 + \gamma(c_r)(\lambda_1 - \beta c_r)} \right) \\ \hat{q}_2 &= \gamma(c_r) S_{D_1}(q_1) \\ I &= F_{D_2}^{-1} \left(\frac{\beta p_2 - h}{\beta p_2} \right) - \hat{q}_2 < I_{D_1}(q_1) \\ q_2 &= 0\end{aligned}$$

First of all, the shadow prices of constraints (25) and (27), λ_1 and λ_2 , have to be found by applying a search method.

Compared to the analytical results of the model without the possibility to store first period excess production, acquisition and remanufacturing of used products do not occur in each of the scenarios. Concerning the first strategy, when storing first period production is the primary supply option, the first and second scenario cover the second period demand only by inventory. In the first scenario ($\lambda_1 = \lambda_2 = 0$), the new production in period 1 equals the optimal unconstrained single-period newsvendor quantity, and the second period demand can be fulfilled by the difference between the production quantity in period 1 and the related sales. Starting from scenario 2, excess production occurs in period 1 ($\lambda_1 = 0, \lambda_2 > 0$). While the shadow price λ_2 is less or equal than $\beta(c_2 - \delta) - h$, the second period demand still can be fulfilled by the stored first period production $I_{D_1}(q_1)$. The excess production in the first period, which is triggered by λ_2 , is solely used for inventory, until increased excess production becomes more costly than acquiring and remanufacturing used products. At this point, the optimal strategy is the third scenario; as the third and fourth scenarios are equal in both strategies, they are discussed below.

The second strategy, when remanufacturing used acquired products is the preferred first supply option, starts with excess production in period 1 ($\lambda_1 > 0, \lambda_2 = 0$). The optimal first period production quantity is controlled by λ_1 . Acquisition and remanufacturing of used products is the only supply for the expected demand in the second period in scenario 1. With a rising acquisition effort c_r to acquire more products, λ_1 increases, and storing production becomes more and more attractive as a second supply option. When λ_1 reaches $\beta(\delta - c_2) + h$, a further raised acquisition effort is more costly than storing excess production. At this stage, additional demand in period 2 is satisfied by inventory $I < I_{D_1}(q_1)$, and the optimal scenario is the second one, including acquisition and remanufacturing and storing first period production quantity ($\lambda_2 = 0$).

In the third scenario, both inventory and acquisition/remanufacturing of used items occur. The shadow prices $\beta(\delta - c_2) + h < \lambda_1 < \beta\delta$ as well as $\beta(c_2 - \delta) - h < \lambda_2 < \beta c_2 - h$ increase with the acquisition effort c_r , so the inventory and the acquisition/remanufactured quantity increase simultaneously. In case that the limits $\lambda_1 = \beta\delta$ and $\lambda_2 = \beta c_2 - h$ are reached (when $c_r + \frac{\gamma(c_r)}{\gamma(c_r)c_r} = \delta$), the optimal scenario switches to the fourth, in which new production q_2 in the second period appears.

An illustrative example concerning the different optimal scenarios is presented in Figure 2, based on acquisition function (14). The complete optimal strategy appears with rising second period demand on the x-axis and increasing holding costs ($0 < h < \beta c_2$) on the y-axis. In the area with holding costs between $h = 0 < h < \beta(c_2 - \delta)$, storing is the preferred supply option. While all of the demand in period 2 can be satisfied by storing some or all of the items remaining from producing the newsvendor quantity q_1^{NV} in area A (M2.1.A), excess production in period 1 is necessary in area B (M2.2.A) to cope with the increased demand. Still, inventory is the sole supply option satisfying the second period demand, but with rising holding costs acquisition/remanufacturing becomes more attractive compared to holding new production. On that account, the area B narrows with increasing holding costs.

Clearly, when costs for holding exceed the cost for remanufacturing, $h > \beta(c_2 - \delta)$, the preferred supply option is acquisition and remanufacturing of used products. Considering a relatively low demand in the second period, it can be satisfied by solely acquisition and remanufacturing of used products (area C: M2.1.B). With rising demand, the shadow price λ_1 increases. As soon as $\lambda_1 = \beta(\delta - c_2) + h$, the optimal strategy switches to area D (M2.2.B), and the second period demand is fulfilled both by acquisition/remanufacturing and some inventory. Increased holding costs make remanufacturing more attractive compared to storing excess production. Consequently, the area C of second period demands, when M2.1.B is optimal, enlarges with rising holding costs. The same can be observed for area D/M2.2.B: as better con-

ditions for remanufacturing increase first period new production, the potential inventory quantity increases, too. Therefore, more excess production can be stored to be used in the second period.

According to the analytical results, with a further increase of the second period demand at first the area E (M2.3.A/M2.3.B) and afterwards area F (M2.4.A/M2.4.B) become optimal in all cases. The decrease of the optimal area E (M2.3.A/M2.3.B) with rising holding costs can be referred to the conditions of holding excess production. Clearly, in the case that the conditions for inventory deteriorate, stored first period new production is substituted by new production in period 2, and a transition to the area F (M2.4.A/M2.4.B) takes place at a lesser second period demand.

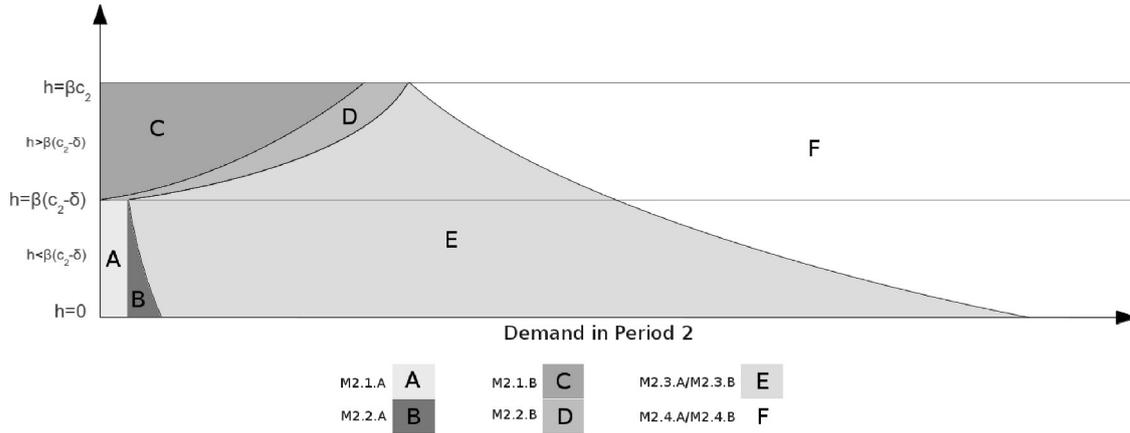


Figure 2: Optimal strategy space of the model with inventory

4.3 Numerical Analysis

A similar setting as presented in section 3.3 is used for the numerical study, extended by the possibility to store first period production. The holding costs are introduced with values of $h = 1, 2, \dots, 6, 7$ to perform a sensitivity analysis over different costs for storing, whereby $h = 2$ is considered as the base case value. Certainly, the proposed holding costs comply with the constraints defined in the analytical section, $0 \leq h \leq \beta c_2$ and $c_1 + h \geq \beta c_2$.

Table 1 contains the comparison between different scenarios: one without the possibility to store excess production and two scenarios with possible inventory. These models differ in the holding costs, representing scenarios with low-cost ($h = 2$) and high-cost inventory ($h = 7$). Additionally, both a base-demand and a low-demand scenario are presented. Manufacturing - remanufacturing quantities (q_1, \hat{q}_2, q_2), the acquisition effort c_r , the stored 1st period production I , and the profit π are shown in the table.

Regarding the base demand-scenarios, one obvious difference between the different presented scenarios is the higher first period production quantity in the setting with low-cost inventory: despite the risen costs in period 1 due to increased excess production, the higher production quantity in period 1 increases the profits. This effect occurs because of the more favorable conditions in the second period, in detail, the additional supply option with stored products combined with the raised pool of used products for acquisition. In the case of $\delta = 7.5$, second period new production still occurs in the settings without storing and with high-cost inventory, but not in the model with possible low-cost inventory. The supply with stored and remanufactured products suffices to cover the second period expected demand. As second period new production is only the third supply option, no new production occurs. Concerning the high-cost inventory case, the difference to the scenario without holding new production is smaller compared to low-cost inventory. Still, the first period

production quantities and the remanufacturing quantities are higher, but due to the rather cost-intensive holding less of the second period new production quantity can be substituted by inventory. Nevertheless, both the low as well as the high-cost inventory increases the profit in all cases.

| Base case: $D_2 \sim (25, 75)$ | | | | | | | | | | | | | | | | | | |
|--------------------------------|---------------|-------------|-------|-------|-----|--------|-----------------------|-------------|-------|-------|------|--------|-----------------------|-------------|-------|-------|------|--------|
| δ | w/o Inventory | | | | | | w/ Inventory: $h = 2$ | | | | | | w/ Inventory: $h = 7$ | | | | | |
| | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π |
| 0.5 | 35.17 | 4.93 | 30.07 | 0.17 | - | 115.48 | 46.09 | 6.01 | 24.54 | 0.17 | 4.45 | 126.63 | 35.38 | 4.95 | 28.97 | 0.17 | 1.08 | 115.69 |
| 1.5 | 35.88 | 8.67 | 26.33 | 0.50 | - | 121.73 | 47.14 | 10.56 | 19.54 | 0.50 | 4.90 | 134.25 | 36.10 | 8.72 | 25.05 | 0.50 | 1.23 | 121.97 |
| 2.5 | 36.85 | 11.44 | 23.56 | 0.83 | - | 130.82 | 48.51 | 13.87 | 15.60 | 0.83 | 5.53 | 145.30 | 37.08 | 11.50 | 22.04 | 0.83 | 1.46 | 131.10 |
| 3.5 | 37.97 | 13.86 | 21.14 | 1.17 | - | 142.22 | 50.01 | 16.71 | 12.04 | 1.17 | 6.25 | 159.09 | 38.21 | 13.93 | 19.33 | 1.17 | 1.75 | 142.56 |
| 4.5 | 39.19 | 16.10 | 18.90 | 1.50 | - | 155.71 | 51.55 | 19.27 | 8.68 | 1.50 | 7.05 | 175.29 | 39.44 | 16.18 | 16.74 | 1.50 | 2.09 | 156.12 |
| 5.5 | 40.46 | 18.22 | 16.78 | 1.83 | - | 171.16 | 53.06 | 21.63 | 5.50 | 1.83 | 7.87 | 193.71 | 40.73 | 18.31 | 14.21 | 1.83 | 2.47 | 171.65 |
| 6.5 | 41.75 | 20.27 | 14.73 | 2.17 | - | 188.49 | 54.50 | 23.83 | 2.46 | 2.17 | 8.70 | 214.18 | 42.03 | 20.36 | 11.74 | 2.17 | 2.90 | 189.06 |
| 7.5 | 43.04 | 22.24 | 12.76 | 2.50 | - | 207.62 | 55.66 | 25.79 | 0.00 | 2.49 | 9.40 | 236.56 | 43.33 | 22.34 | 9.30 | 2.50 | 3.36 | 208.28 |

| Low demand: $D_2 \sim (5, 55)$ | | | | | | | | | | | | | | | | | | |
|--------------------------------|---------------|-------------|-------|-------|-----|--------|-----------------------|-------------|-------|-------|------|--------|-----------------------|-------------|-------|-------|------|--------|
| δ | w/o Inventory | | | | | | w/ Inventory: $h = 2$ | | | | | | w/ Inventory: $h = 7$ | | | | | |
| | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π |
| 0.5 | 35.17 | 4.93 | 10.07 | 0.17 | - | 79.48 | 46.09 | 6.01 | 4.54 | 0.17 | 4.45 | 90.63 | 35.38 | 4.95 | 8.97 | 0.17 | 1.08 | 79.69 |
| 1.5 | 35.88 | 8.67 | 6.33 | 0.50 | - | 85.73 | 46.93 | 10.39 | 0.00 | 0.49 | 4.81 | 98.25 | 36.09 | 8.71 | 5.06 | 0.50 | 1.23 | 85.97 |
| 2.5 | 36.85 | 11.44 | 3.56 | 0.83 | - | 94.82 | 46.45 | 12.39 | 0.00 | 0.70 | 4.60 | 108.52 | 37.08 | 11.49 | 2.05 | 0.83 | 1.46 | 95.10 |
| 3.5 | 37.97 | 13.86 | 1.14 | 1.17 | - | 106.22 | 46.26 | 14.16 | 0.00 | 0.92 | 4.52 | 120.48 | 38.02 | 13.71 | 0.00 | 1.14 | 1.70 | 106.54 |
| 4.5 | 39.00 | 15.76 | 0.00 | 1.45 | - | 119.63 | 46.26 | 15.80 | 0.00 | 1.15 | 4.52 | 133.97 | 38.91 | 15.61 | 0.00 | 1.43 | 0.50 | 119.65 |
| 5.5 | 39.88 | 17.27 | 0.00 | 1.68 | - | 134.50 | 46.40 | 17.33 | 0.00 | 1.37 | 4.58 | 148.88 | 39.88 | 17.27 | 0.00 | 1.68 | 0.00 | 134.50 |
| 6.5 | 40.78 | 18.74 | 0.00 | 1.92 | - | 150.71 | 46.62 | 18.78 | 0.00 | 1.60 | 4.67 | 165.13 | 40.78 | 18.74 | 0.00 | 1.92 | 0.00 | 150.71 |
| 7.5 | 41.70 | 20.19 | 0.00 | 2.15 | - | 168.23 | 46.91 | 20.17 | 0.00 | 1.84 | 4.80 | 182.66 | 41.70 | 20.19 | 0.00 | 2.15 | 0.00 | 168.23 |

Table 1: Sensitivity analyses with varying δ , different h , and different demand

The lower part of Table 1 includes the settings with low demand in the second period. All different types of scenarios (M2.1.B, M2.2.B, M2.3.A/M2.3.B, M2.4.A/M2.4.B) of an optimal strategy can be observed with rising δ under holding cost of $h = 7$ and low demand. Up to $\delta = 2.5$, all three possible supply options occur. Beginning from $\delta = 3.5$, new production in the second period disappears (M2.4.A/M2.4.B to M2.3.A/M2.3.B). A further increase of remanufacturing cost savings to $\delta = 4.5$ leads to another transition of the optimal scenario (M2.3.A/M2.3.B to M2.2.B), in which the potential inventory is only partly used. Another raise of δ ($\delta > 4.5$) results in the reduction of the used supply options to a single one, in particular acquisition and remanufacturing of used products (scenario M2.1.B). Clearly, this case of using only remanufacturing as second period supply yields the same quantities, acquisition effort, and profit as the model without inventory.

Another interesting effect can be found in Table 1 when the cost for inventory is relatively low ($h = 2$). With rising remanufacturing cost savings δ , both the first period production quantity q_1 and the inventory I firstly increase, then decrease, and finally increase again. This non-trivial result can be explained by the different sources of supply and the structure of the acquisition effort function. The first increase is caused by three properties: firstly, remanufacturing becomes more profitable with a rising δ , and consequently, a higher acquisition effort can be chosen. Secondly, storing first period excess production is rather cheap. Thirdly, increasing q_1 raises both the possibly stored first period production and the pool for acquisition and remanufacturing of used products. Therefore, the better conditions for remanufacturing and the rather beneficial increased stored first period new production supersede the new production in the second period. In detail, a transition within the optimal strategy from scenario M2.4.A/M2.4.B, including all three possible sources of supply, to scenario M2.3.A/M2.3.B without new production in period 2, takes place. At this transition point, λ_1 changes from $\beta\delta$ to $\lambda_1 < \beta\delta$, and accordingly, $\lambda_2 < \beta c_2 - h$ due to the fact that the acquisition effort c_r deviates from the maximum possible value $c_r < c_r^{max} = \frac{\delta}{3}$. The following decrease of q_1 and I is initiated by the acquisition effort function: as the acquisition effort is still at a rather low level,

a small increase in c_r raises the return rate highly (see Figure 3). Therefore, the first period new production can even be decreased, as the raised c_r with the related high increase of the return rate ensures the supply with used products. Additionally, the rising savings from remanufacturing improve the conditions for remanufacturing. In consequence, the remanufacturing quantity rises with δ . Beginning from $\delta = 4.5$, both the first period new production quantity and the acquisition effort increase. As the increased remanufacturing saving allows a higher acquisition/remanufacturing quantity, also c_r can be increased. Nevertheless, the marginal utility of c_r is decreasing in the case of a further rise. Thus, a sole increase of c_r does not suffice in those scenarios, so q_1 must be increased to ensure the supply with used products. Concerning the inventory quantities in the example above, the explanation is straightforward: as the possible inventory is fully used in all cases, the inventory quantities directly depend on the first period new production quantity. Therefore, each increase/decrease of q_1 increases/decreases the potential inventory quantity and the stored products.

5 Further Numerical Analyses

In this section, further numerical analyses and insights are provided. Firstly, the consequences of different acquisition functions on both the models with and without possible inventory are analyzed in section 5.1. Furthermore, as the analytical results suggest a joint optimization of the acquisition effort c_r as well as the first period new production q_1 , the implications of a fixation of one of these variables are studied in section 5.2.

5.1 The impact of different acquisition functions on decision making and performance of an OEM

To study the consequences of the acquisition process efficiency on the system, we use different explicit acquisition cost functions. Next to the function used in the base case ($\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$, see equation (14)), the following equations represent two additional types of cost structures regarding the acquisition of used products:

$$\gamma(c_r) = \frac{c_r}{c_2 x}, \quad (29)$$

$$\gamma(c_r) = 1 - e^{-\frac{c_r}{x}}. \quad (30)$$

In Figure 3, exemplary return rates of the effort-dependent acquisition functions (14), (29), (30) with varying acquisition efforts are presented ($x = 1$, $c_r = 0.0, 0.1, \dots, 3.9, 4.0$).

Constraint (3), $\gamma(c_r) \leq 1$, combined with the acquisition functions (14) and (29) leads to the requirement $\frac{c_r}{c_2 x} \leq 1$. δ is assumed to be greater than c_r and less than c_2 , and therefore, $c_r < c_2$. x is assumed to be greater or equal than 1. The exponential function (30) can never violate the restriction as $1 - e^{-\frac{c_r}{x}} < 1$, so constraint (3) holds for all acquisition functions.

The chosen acquisition functions cover a wide range of possible scenarios. For example, the linear function represents an acquisition process where rather high effort is needed to acquire used items, but still may lead to low return rates. An exemplary case is the market of car batteries. Used batteries are collected by several companies, for example, OEM's, companies doing car battery refurbishing, and scrap dealers. To ensure the supply with raw materials or reusable parts, an OEM has to take costly measures like establishing transportation networks or rebuying from scrap dealers to collect/acquire them. Oppositely, the root and the exponential acquisition functions allow to acquire a rather high quantity of used products by spending a reasonable effort. This may occur in, e.g., the market for business copiers. Business copiers can

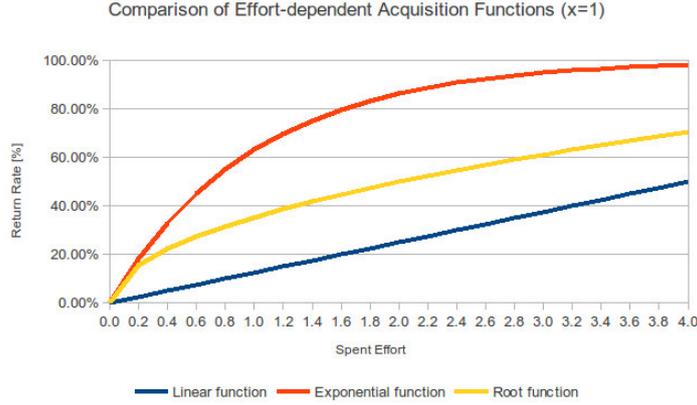


Figure 3: Different acquisition functions with varying acquisition efforts ($x = 1$)

not only be bought but also rented and leased. Consequently, this ensures the availability of used products for acquisition, as they will be returned after use (see, e.g., [3]).

5.1.1 The impact of different acquisition functions on the model without possible inventory

In Table 2, the results of using different acquisition functions (14), (29), (30) in the model without holding inventory considering varying savings from remanufacturing δ are presented. Here, the quantities, acquisition efforts, and profits strongly differ between the functions. Nearly all of the shown acquisition efforts are at a maximum value, and consequently, new production in period 2 arises in most of the presented optimal results (thus, scenario M1.2 of the optimal strategy is applied). The only scenario in which new production in the second period does not occur (and consequently, M1.1 is optimal), is the setting including $\gamma(c_r) = 1 - e^{-\frac{c_r}{x}}$ and $\delta = 7.5$. This relates to the fact that this acquisition function is the most efficient one, and therefore, excess production in period 1, remanufacturing quantities, and profits are the highest. Furthermore, new production in the second period is rather low, because the bigger part or even all of the demand can be satisfied by remanufacturing. Clearly, the more efficient the acquisition process is, the more profitable remanufacturing becomes, and in consequence, higher excess production and remanufacturing quantities appear (see Figure 3 regarding the acquisition function efficiency). Still, even the least efficient acquisition process $\gamma(c_r) = \frac{c_r}{c_2 x}$ yields higher profits than without remanufacturing: the profit of two aggregated single-period newsvendor problems reaches $\pi^{NV} = 114.00$.

| δ | $\gamma(c_r) = \frac{c_r}{c_2 x}$ | | | | | $\gamma(c_r) = 1 - e^{-\frac{c_r}{x}}$ | | | | | $\gamma(c_r) = \sqrt{\frac{c_r}{c_2 x}}$ | | | | |
|----------|-----------------------------------|-------------|-------|-------|--------|--|-------------|-------|-------|--------|--|-------------|-------|-------|--------|
| | q_1 | \hat{q}_2 | q_2 | c_r | π | q_1 | \hat{q}_2 | q_2 | c_r | π | q_1 | \hat{q}_2 | q_2 | c_r | π |
| 0.5 | 35.03 | 1.06 | 33.94 | 0.25 | 114.24 | 35.20 | 7.15 | 27.85 | 0.24 | 115.70 | 35.17 | 4.93 | 30.07 | 0.17 | 115.48 |
| 1.5 | 35.25 | 3.21 | 31.79 | 0.75 | 116.16 | 36.41 | 16.36 | 18.64 | 0.63 | 126.65 | 35.88 | 8.67 | 26.33 | 0.50 | 121.73 |
| 2.5 | 35.69 | 5.40 | 29.60 | 1.25 | 120.03 | 38.15 | 22.20 | 12.80 | 0.94 | 144.17 | 36.85 | 11.44 | 23.56 | 0.83 | 130.82 |
| 3.5 | 36.33 | 7.67 | 27.33 | 1.75 | 125.90 | 40.05 | 26.35 | 8.65 | 1.20 | 166.11 | 37.97 | 13.86 | 21.14 | 1.17 | 142.22 |
| 4.5 | 37.16 | 10.03 | 24.97 | 2.25 | 133.86 | 41.95 | 29.52 | 5.48 | 1.41 | 191.31 | 39.19 | 16.10 | 18.90 | 1.50 | 155.71 |
| 5.5 | 38.14 | 12.52 | 22.48 | 2.75 | 143.99 | 43.75 | 32.04 | 2.96 | 1.59 | 219.06 | 40.46 | 18.22 | 16.78 | 1.83 | 171.16 |
| 6.5 | 39.25 | 15.12 | 19.88 | 3.25 | 156.42 | 45.44 | 34.09 | 0.91 | 1.75 | 248.84 | 41.75 | 20.27 | 14.73 | 2.17 | 188.49 |
| 7.5 | 40.46 | 17.85 | 17.15 | 3.75 | 171.25 | 46.82 | 35.60 | 0.00 | 1.87 | 280.27 | 43.04 | 22.24 | 12.76 | 2.50 | 207.62 |

Table 2: Comparison of different acquisition functions $\gamma(c_r)$ ($D_2 \sim U(25, 75)$, $\delta = 4$)

5.1.2 The impact of different acquisition functions on the model with possible inventory

Similar to the results presented above, the analyses are applied to the model with possible inventory in Table 3. In the base case in the column on the right, the 4th scenario of the optimal strategy, M2.4.A/M2.4.B, is optimal in any case but the one when $\delta = 7.5$, and all three supply sources are used to fulfill the demand (see section 4.3).

The two additional acquisition functions are shown in the first two columns, again for holding cost of $h = 2$ and $h = 7$ and varying over δ . Concerning the linear acquisition function on the left hand side, the optimal scenario is scenario M2.4.A/M2.4.B. Compared to the base case function, the linear acquisition function is less efficient. This leads to reduced excess production and acquisition/remanufacturing of used products due to the increased costs. Clearly, the reduced q_1 also results in a decreased expected inventory. The missing supply from remanufacturing and storing products is substituted by new production q_2 . In detail, the acquisition effort c_r is at the function-specific maximum limit $\frac{\delta}{2}$ in all cases, and in the sequel, λ_1, λ_2 are at their maximum values. Similar as described above, all three sources of supply are used, and due to the higher possible savings from remanufacturing q_1, \hat{q}_2, I, π increase and q_2 decreases with a rising δ . In the mid column of the table, containing an exponential acquisition function, an interesting transition occurs: up to $\delta = 2.5$ in the base case and $\delta = 4.5$ in the low demand scenario, respectively, new production appears in period 2. In the case of a further increase of δ , the production quantity q_2 becomes zero. This effect is based on a switch of the optimal scenario from scenario M2.4.A/M2.4.B to scenario M2.3.A/M2.3.B ($\lambda_1 < \beta\delta$ and $\lambda_2 < \beta c_2 - h$) due to the high efficiency of the exponential acquisition function. Exactly at the point where c_r falls below the critical limit $c_r + \frac{\gamma(c_r)}{\gamma(c_r)^{c_r}} = \delta$, the new production in period 2 disappears. When c_r is below this maximum value, remanufacturing and storing products is more attractive compared to new production, and therefore, $q_2 = 0$. Regarding the profits, the same observation as in the model without the possibility to store first period excess production can be made: the more efficient the acquisition process is, the more profit can be obtained.

Furthermore, an interesting effect occurs concerning the first period new production quantities when $h = 2$. While q_1 is higher in the setting with the exponential acquisition function when $\delta \leq 4.5$ than in the one with the base root function, this flips with increasing savings from remanufacturing. These results can be explained as follows: up to $\delta = 2.5$, both settings use all three possible sources of supply. As the exponential acquisition function is more efficient, only acquisition/remanufacturing and holding excess production appear from $\delta > 2.5$, while the root acquisition function remains in M2.4.A/M2.4.B up to $\delta = 6.5$. As the two supply options are sufficient to fulfill the second period demand in the case with the exponential acquisition function, the values of both q_1 and c_r are not at their maximum possible values but below. Consequently, the increase of q_1 is decelerated. In the root function case, q_1 and c_r still are at their maximum values up to $\delta = 7.5$ to gain from the high increases. Nevertheless, the profits are less than in the exponential function setting due to the decreased acquisition efficiency and consequently, lower remanufacturing quantities.

Finally, another transition from scenario M2.3.A/M2.3.B to M2.2.B can be observed in the scenario with high inventory cost and the exponential acquisition function: not all of the possible inventory is used from $\delta > 5.5$, and therefore, it is even reduced. As acquisition and remanufacturing becomes more and more attractive with a rising δ but holding is still rather expensive, the stored quantity I is decreased and substituted by remanufactured used products.

| Base case: $h = 2$ | | | | | | | | | | | | | | | | | | |
|----------------------------|----------------------------------|-------------|-------|-------|------|--------|--|-------------|-------|-------|------|--------|---|-------------|-------|-------|------|--------|
| δ | $\gamma(c_r) = \frac{c_r}{c_2x}$ | | | | | | $\gamma(c_r) = 1 - e^{-\frac{c_r}{x}}$ | | | | | | $\gamma(c_r) = \sqrt{\frac{c_r}{c_2x}}$ | | | | | |
| | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π |
| 0.5 | 45.87 | 1.30 | 29.34 | 0.25 | 4.36 | 125.13 | 46.13 | 8.73 | 21.80 | 0.24 | 4.46 | 126.91 | 46.09 | 6.01 | 24.54 | 0.17 | 4.45 | 126.63 |
| 1.5 | 46.21 | 3.91 | 26.59 | 0.75 | 4.50 | 127.47 | 47.90 | 19.88 | 9.88 | 0.63 | 5.24 | 140.24 | 47.14 | 10.56 | 19.54 | 0.50 | 4.90 | 134.25 |
| 2.5 | 46.86 | 6.58 | 23.64 | 1.25 | 4.78 | 132.18 | 50.25 | 26.73 | 1.89 | 0.94 | 6.37 | 161.43 | 48.51 | 13.87 | 15.60 | 0.83 | 5.53 | 145.30 |
| 3.5 | 47.78 | 9.32 | 20.49 | 1.75 | 5.19 | 139.33 | 50.79 | 29.82 | 0.00 | 1.13 | 6.65 | 187.18 | 50.01 | 16.71 | 12.04 | 1.17 | 6.25 | 159.09 |
| 4.5 | 48.93 | 12.15 | 17.12 | 2.25 | 5.73 | 148.98 | 50.84 | 31.73 | 0.00 | 1.27 | 6.68 | 214.89 | 51.55 | 19.27 | 8.68 | 1.50 | 7.05 | 175.29 |
| 5.5 | 50.22 | 15.08 | 13.56 | 2.75 | 6.36 | 161.22 | 51.09 | 33.38 | 0.00 | 1.40 | 6.81 | 244.21 | 53.06 | 21.63 | 5.50 | 1.83 | 7.87 | 193.71 |
| 6.5 | 51.62 | 18.09 | 9.82 | 3.25 | 7.09 | 176.14 | 51.49 | 34.82 | 0.00 | 1.53 | 7.02 | 274.91 | 54.50 | 23.83 | 2.46 | 2.17 | 8.70 | 214.18 |
| 7.5 | 53.06 | 21.18 | 5.94 | 3.75 | 7.88 | 193.81 | 51.97 | 36.08 | 0.00 | 1.65 | 7.28 | 306.83 | 55.66 | 25.79 | 0.00 | 2.49 | 9.40 | 236.56 |
| High holding cost: $h = 7$ | | | | | | | | | | | | | | | | | | |
| δ | $\gamma(c_r) = \frac{c_r}{c_2x}$ | | | | | | $\gamma(c_r) = 1 - e^{-\frac{c_r}{x}}$ | | | | | | $\gamma(c_r) = \sqrt{\frac{c_r}{c_2x}}$ | | | | | |
| | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π | q_1 | \hat{q}_2 | q_2 | c_r | I | π |
| 0.5 | 35.23 | 1.07 | 32.88 | 0.25 | 1.05 | 114.44 | 35.41 | 7.20 | 26.72 | 0.24 | 1.08 | 115.91 | 35.38 | 4.95 | 28.97 | 0.17 | 1.08 | 115.69 |
| 1.5 | 35.46 | 3.22 | 30.68 | 0.75 | 1.09 | 116.37 | 36.64 | 16.45 | 17.20 | 0.63 | 1.35 | 126.92 | 36.10 | 8.72 | 25.05 | 0.50 | 1.23 | 121.97 |
| 2.5 | 35.90 | 5.42 | 28.39 | 1.25 | 1.19 | 120.26 | 38.40 | 22.31 | 10.90 | 0.94 | 1.79 | 144.52 | 37.08 | 11.50 | 22.04 | 0.83 | 1.46 | 131.10 |
| 3.5 | 36.56 | 7.70 | 25.96 | 1.75 | 1.34 | 126.16 | 40.32 | 26.48 | 6.17 | 1.20 | 2.35 | 166.57 | 38.21 | 13.93 | 19.33 | 1.17 | 1.75 | 142.56 |
| 4.5 | 37.39 | 10.08 | 23.38 | 2.25 | 1.53 | 134.16 | 42.23 | 29.66 | 2.37 | 1.41 | 2.97 | 191.90 | 39.44 | 16.18 | 16.74 | 1.50 | 2.09 | 156.12 |
| 5.5 | 38.38 | 12.58 | 20.63 | 2.75 | 1.79 | 144.35 | 43.77 | 31.92 | 0.00 | 1.58 | 3.52 | 219.74 | 40.73 | 18.31 | 14.21 | 1.83 | 2.47 | 171.65 |
| 6.5 | 39.51 | 15.20 | 17.70 | 3.25 | 2.11 | 156.83 | 45.08 | 33.67 | 0.00 | 1.72 | 4.44 | 249.18 | 42.03 | 20.36 | 11.74 | 2.17 | 2.90 | 189.06 |
| 7.5 | 40.73 | 17.94 | 14.59 | 3.75 | 2.48 | 171.73 | 46.66 | 35.43 | 0.00 | 1.86 | 6.68 | 280.30 | 43.33 | 22.34 | 9.30 | 2.50 | 3.36 | 208.28 |

Table 3: Sensitivity analyses over δ for different acquisition functions

5.2 Evaluating the importance of a joint optimization of first period new production q_1 and acquisition effort c_r

Based on the analytical insights presented in section 3.2, that a simultaneous optimization of the first period new production and the acquisition effort is necessary to obtain the optimal profit, some more numerical experiments are analyzed. They concentrate on the influence of the trade-off between first period new production q_1 and the acquisition effort on quantities and profits. Either q_1 or c_r are not optimized but fixed, so the consequences of concentrating on only one variable instead of both can be shown. Therefore, two different scenarios are created to obtain the effects of fixed acquisition efforts and a fixed first period production quantity on profits and quantities in the systems.

The first scenario contains a fixed first period new production quantity. The OEM wants to optimize the first period profit and sets the first period production quantity to the optimal newsvendor quantity of the unconstrained single-period newsvendor model, in the base case $q_1 = 35$. As no excess production is possible, the profit is optimal for a single period, but the supply with acquired used cores for remanufacturing is restricted to this quantity. Thus, the acquisition can only be controlled by the acquisition effort c_r . A practical situation, where such a behavior can occur, is the goal incongruence of different departments within a company. While a production department wants to optimize the new production, a remanufacturing department may suffer from this sub-optimized small production quantity due to reduced returns in the following periods. Related to this, another typical example is the short-term production strategy of companies listed on stock markets to optimize the profit in the financial year by reducing production costs.

In the remaining additional scenario, the acquisition effort is specified as a certain value ($c_r = 0.25$). As a consequence, the level of acquisition effort is determined and kept constant. Thus, the only option for the OEM to control the return flow is to change the first period new production quantity q_1 . The optimized result of the base case with the full optimization of q_1 , \hat{q}_2 , q_2 , and c_r including acquisition function (14) serves as a reference solution. This scenario can be considered as a decision of a company to limit the effort spent to a certain, rather low level.

As can be seen in Table 4, fixing q_1 results in a reduction of the remanufacturing quantity compared to

the fully optimized case. The spent acquisition effort stays the same, it is at the maximum limit. Consequently, the only cause for this decrease is the reduced q_1 . Furthermore, the difference between the profits of the optimal and the fixed q_1 scenarios raises with increasing remanufacturing cost savings δ : keeping in mind that c_r is already at its maximum value, the missing supply with used products has to be substituted with the increased second period new production q_2 . New production is less efficient than remanufacturing, and in the sequel, profits are lower. This effect increases with a raising δ due to the increasing loss of efficiency.

Furthermore, the results of considering a fixed acquisition effort c_r are shown. Fixing the acquisition effort has a huge impact on the variables. In the setting $c_r = 0.25$, the first period production quantity q_1 is less or equal than q_1 in the fully optimized base case. The variable q_1 is at a rather low level, as the losses of further excess production in period 1 would exceed the gainings of an increased pool of used products for acquisition and remanufacturing. The remanufacturing quantity \hat{q}_2 is rather low, too. Instead of remanufacturing items, the new production quantity in the second period is raised.

| δ | Base case c_r, q_1 optimized | | | | | $q_1 = q_1^{NV} = 35$ | | | | | $c_r = 0.25$ | | | | |
|----------|--------------------------------|-------------|-------|-------|--------|-----------------------|-------------|-------|-------|--------|--------------|-------------|-------|-------|--------|
| | q_1 | \hat{q}_2 | q_2 | c_r | π | q_1 | \hat{q}_2 | q_2 | c_r | π | q_1 | \hat{q}_2 | q_2 | c_r | π |
| 0.5 | 35.17 | 4.93 | 30.07 | 0.17 | 115.48 | 35.00 | 4.91 | 30.09 | 0.17 | 115.47 | 35.16 | 6.03 | 28.97 | 0.25 | 115.35 |
| 1.5 | 35.88 | 8.67 | 26.33 | 0.50 | 121.73 | 35.00 | 8.50 | 26.50 | 0.50 | 121.65 | 35.78 | 6.12 | 28.88 | 0.25 | 120.82 |
| 2.5 | 36.85 | 11.44 | 23.56 | 0.83 | 130.82 | 35.00 | 10.97 | 24.03 | 0.83 | 130.46 | 36.38 | 6.20 | 28.80 | 0.25 | 126.37 |
| 3.5 | 37.97 | 13.86 | 21.14 | 1.17 | 142.22 | 35.00 | 12.98 | 22.02 | 1.17 | 141.27 | 36.97 | 6.28 | 28.72 | 0.25 | 131.99 |
| 4.5 | 39.19 | 16.10 | 18.90 | 1.50 | 155.71 | 35.00 | 14.72 | 20.28 | 1.50 | 153.75 | 37.53 | 6.36 | 28.64 | 0.25 | 137.68 |
| 5.5 | 40.46 | 18.22 | 16.78 | 1.83 | 171.16 | 35.00 | 16.28 | 18.72 | 1.83 | 167.71 | 38.08 | 6.43 | 28.57 | 0.25 | 143.43 |
| 6.5 | 41.75 | 20.27 | 14.73 | 2.17 | 188.49 | 35.00 | 17.69 | 17.31 | 2.17 | 183.01 | 38.62 | 6.50 | 28.50 | 0.25 | 149.25 |
| 7.5 | 43.04 | 22.24 | 12.76 | 2.50 | 207.62 | 35.00 | 19.01 | 15.99 | 2.50 | 199.53 | 39.14 | 6.57 | 28.43 | 0.25 | 155.13 |

Table 4: Analysis for different values of δ (base case, fixed $q_1 = 35$, fixed $c_r = 0.25$)

Concerning the profits of the presented scenarios, Figure 4 demonstrates the relative profit decline (RPD) over different remanufacturing cost savings. The optimized base case (optimized q_1 and c_r) serves as the reference profit value. In this example, the deviation of the resulting profits considering the scenarios $q_1 = q_1^{NV} = 35$ and $c_r = 0.25$ from the optimal value is presented. In the case that the acquisition effort is fixed to a rather small value, e.g., $c_r = 0.25$ in our example, the decline in profit can reach around 25%. The amount spent for acquisition impedes the profitable acquisition of more used products, and this effect worsens with an increasing δ , as even more acquired used products could be remanufactured profitably.

The profit deviation in the case of a fixed first period quantity $q_1 = 35$ is less dramatic. Nevertheless, it still reaches up to -3.90% in our example. As explained above, the second period new production substitutes the lack of remanufactured products due to less acquired used products. However, this results in lower profits and consequently, in a higher profit deviation.

Structurally identical results can be observed in the case with the possibility to store first period excess production. The detrimental effect of a sub-optimization increases in both cases: while there is only a marginal increase of the RPD in the case of $c_r = 0.25$, the RPD changes dramatically when the first period new production is fixed to $q_1 = 35$. The explanation for this is rather straightforward: the fixed new production in period 1 restricts the expected inventory to $I_{D_1}(q_1) = 1$. Therefore, the potential profits of excessive first period production can not be utilized, neither by inventory nor by acquisition and remanufacturing.

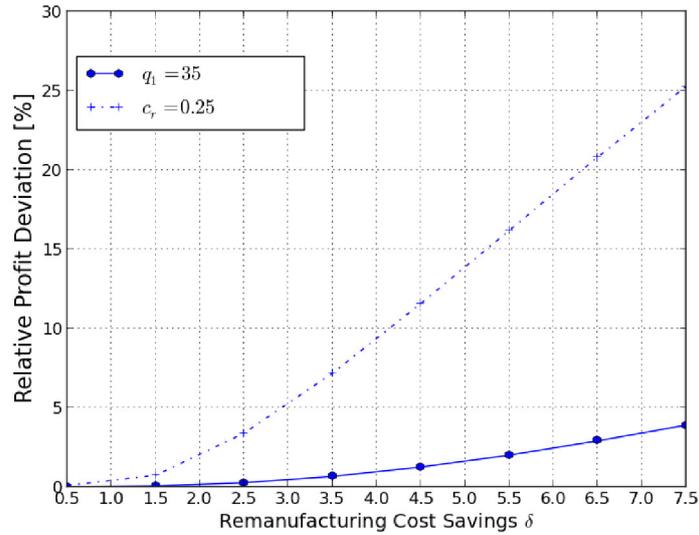


Figure 4: Relative profit deviation of settings $q_1 = 35, c_r = 0.25$ compared to fully optimized model

6 Conclusion and Further Research

In this paper, we presented a two-period model considering a closed-loop supply chain with an actively controlled process for acquiring used products for remanufacturing. Depending on the spent acquisition effort, the return rate can be controlled. The demand is uncertain in both periods, and the expected sales in the first period determine the availability of used products for remanufacturing in the second period. Additionally, an extension to store excess production in the first period and offer the inventory in the second period, was included. Depending on the company's cost structure, we analytically derived the optimal acquisition - manufacturing - remanufacturing - (inventory) strategies for both models.

Moreover, we performed some numerical studies to gain insights into the sensitivity of the models. Particularly, we explored and quantified the effects of different numerical settings on the manufacturing, remanufacturing, and holding quantities, the acquisition effort taken, and the profits. This was performed both for the model without and with the option to store excess first period production.

Some of our key findings lead to interesting managerial insights. First of all, the option to remanufacture may increase profits compared to simple new production. One main result concerns the optimization of the first period new production quantity and the acquisition effort. As they are correlated, a partial optimization of only one variable decreases the possible profits. Therefore, both variables must be optimized simultaneously. Consequently, both the pool of available products for remanufacturing and the effort spent for acquiring those items have to be taken into account. Related to this result, one difference can be discovered between the model without and the model with the option to store excess production. While in the model without inventory the increase (decrease) of savings from remanufacturing leads to an increase (decrease) of both the first period new production quantity and the acquisition effort, this is not necessarily the case in the model with inventory. Actually, we show some numerical results where the new production in period 1 first increases, then decreases, and finally, increases again with rising savings from remanufacturing, while the acquisition effort increases in all cases. Furthermore, we found that a declining market may be covered by a pure acquisition/remanufacturing strategy or acquisition/remanufacturing/inventory, respectively. One of the main influencing factors for this is the efficiency of the acquisition process, as a sufficient supply with used products to remanufacture is a basic requirement to satisfy the occurring demands.

Clearly, there exist some limitations concerning the model. First of all, the newly produced and re-manufactured items are assumed to be perfect substitutes. Related to this, there exists a lack of possible disposition possibilities, as solely as-new products are demanded. Another critical point is the missing consideration of quality, which would have high impact on acquisition and disposition decisions. Finally, the presented model is a heuristic concerning the timing of the decisions, as in the current model all variables for the first and second period are optimized at the beginning of the first period.

Nevertheless, these limitations allow many opportunities for further research: multiple reprocessing options (e.g., refurbishing, recycling) besides manufacturing and remanufacturing can be considered. Additionally, cannibalizational effects may be explored by coupling the markets for new and reprocessed products. Within these extensions, the assumption of perfect substitutes can be eliminated easily. As mentioned, an interesting extension is the inclusion of heterogeneous quality of used products to study the influence on acquisition and production disposition decisions. The implementation of a grading process to determine the quality of acquired cores with unknown quality would enlarge the field of application of the model. Finally, to better reflect the fact that in practice second period decisions can be taken on the basis of information about first period sales, the model could be altered to a two-stage stochastic optimization problem.

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