Contribution by Alexander Dilger
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Plans as Conditional Strategies

A Concept Enabling Cooperation in the Prisoners’ Dilemma

1. Introduction

This paper provides a new (at least partially, see Section 3) and extended interpretation of strategies in games. This interpretation does not constitute a real break with the old one, but rather a straightforward extension, including the old one as a special case. In a way, strategy is a misnomer in traditional game theory, because (pure) strategies designate ultimately the actions from which a player can choose (in extensive games a strategy is a function, assigning actions to any history of past actions, cf. Osborne/Rubinstein 1994, p. 92). Even in traditional game theory, there is one extension of strategies beyond actions, namely mixed strategies (see von Neumann/Morgenstern 1944, p. 146, or Nash 1950). These mean that the player chooses probabilities, and accordingly, a random device determines the final action. Originally and beyond the bounds of game theory, a strategy is regarded as more than and different from an action. A strategy is a major and long-term plan about goal attainment (cf. Chandler 1962). A tactic is a smaller and short-term plan about means to ends. Finally, at the operational level, the concrete actions are selected and carried out.

The new interpretation of strategies in the context games is to regard them as *plans* (although this particular word is not important, other names could be *tactics*, and *conditional* or *virtual strategies*). These can be simple plans like to do a certain action for sure. Accordingly, the differentiation between plans and actions is not important in this case. A plan can also consist of carrying out different actions with certain probabilities. Clearly, there is no important difference between this case and traditional game theory with mixed-strategies. However, a(n infinite) number of other plans are possible. Even though most are not very useful and fail to bring new insights to game theory, there are some that do. To begin with a simple example, a player could plan to do Action A on Mondays and Action B on all other weekdays. This plan is, like all plans, about (intended) actions, but it is conditional on something other than mere probabilities such as mixed strategies or past actions, in this case on the particular day of the week (when the action is to be done). It is easy to see that the existence of such a plan can change the equilibria in a game.

2. First Examples of Plans

To begin with, let us take the example of a coordination game, the Battle of the Sexes, which is depicted in its traditional form (see Luce/Raiffa 1957, pp. 90-94) in Figure 1.

		Player 2	
		Strategy A_2 = Action A	Strategy B_2 = Action B
Player 1	Strategy A_1 = Action A	2, 1	-1, -1
	Strategy B_1 = Action B	-1, -1	1, 2

Figure 1: Battle of the Sexes, traditional form

In this form, this game has three Nash-equilibria, namely both players choosing A, both choosing B or an equilibrium of mixed strategies, in which Player 1 plays Action A with a probability of 3/5 and B with the counter-probability of 2/5, whereas Player 2 does it the other way round, playing A with a probability of 2/5 and B with a probability of 3/5. This latter equilibrium is the least beneficial for both players, whereas Player 1 prefers the first one and Player 2 the second.

Figure 2 adds the plan, of Player 1 to begin with, choosing A on Mondays and B on all other days. The old equilibria remain and there is no new equilibrium, at least not a constant one on all weekdays. On Mondays, the plans M_1 and A_2 would constitute an additional equilibrium, whereas from Tuesdays to Sunday, the combination of the plans M_1 and B_2 is a new equilibrium. There are also new equilibria for the mixed strategies, depending on the day of the week. In any event, Player 2 has no best answer to plan M_1, as long as she limits her plans to traditional action-strategies (strategies to perform an action unconditionally). Her best answer to M_1 is a comparable M_2, that is, to play A on Mondays and B on all other days of the week. Figure 3 shows this extension of the game with M_1 and M_2 as a new equilibrium. This is an equilibrium on any weekday, but with different payoffs on Mondays, compared to other days.

		Player 2	
		Strategy A_2 = Plan A_2: perform Action A in any case	Strategy B_2 = Plan B_2: perform Action B in any case
Player 1	Strategy A_1 = Plan A_1: perform action A in any case	2, 1	-1, -1
	Strategy B_1 = Plan B_1: perform action B in any case	-1, -1	1, 2
	Strategy M_1 = Plan M_1: perform action A on Mondays, otherwise perform action B	2, 1 on Mondays, -1, -1 all other days	-1, -1 on Mondays, 1, 2 all other days

Figure 2: Battle of the Sexes, with one additional plan

		Player 2		
		Strategy A_2 = Plan A_2: perform action in any case	Strategy B_2 = Plan B_2: perform action B in any case	Strategy M_2 = Plan M_2: perform action A on Mondays, otherwise perform action B
Player 1	Strategy A_1 = Plan A_1: perform action A in any case	2, 1	-1, -1	2, 1 on Mondays, -1, -1 all other days
	Strategy B_1 = Plan B_1: perform action B in any case	-1, -1	1, 2	-1, -1 on Mondays, 1, 2 all other days
	Strategy M_1 = Plan M_1:perform action A on Mondays, otherwise perform action B	2, 1 on Mondays, -1, -1 all other days	-1, -1 on Mondays, 1, 2 all other days	2, 1 on Mondays, 1, 2 all other days

Figure 3: Battle of the Sexes, with an additional plan for both sexes

Certainly, there are indefinitely more such plans and equilibria. Consider Tuesdays, 8 o'clock in the morning, rain outside the window, being in Australia and so on. Moreover, such equilibria do not seem particularly important or to justify an extension of the traditional concept of strategies as actions for including plans.

3. Plans Conditional on Actions and Plans

However, there is at least one class of conditions which leads to new results and makes plans an interesting and useful extension of *action-strategies* (the term used in this paper to refer to the strategies of traditional game theory). These conditions are those depending on actions and plans other than external events and circumstances, such as the day of the week. Instead of M_1 and M_2, each player could have a plan E to choose the action of the other player, as shown in Figure 4.

		Player 2		
		Strategy A_2 = Plan A_2: perform action A in any case	Strategy B_2 = Plan B_2: perform action B in any case	Strategy E_2 = Plan E_2: perform action A if player 1 performs A, otherwise B
Player 1	Strategy A_1 = Plan A_1: perform action A in any case	2, 1	-1, -1	2, 1
	Strategy B_1 = Plan B_1: perform action B in any case	-1, -1	1, 2	1, 2
	Strategy E_1 = Plan E_1: perform action A if player 2 does A, otherwise B	2, 1	1, 2	?

Figure 4: Battle of the Sexes, with plans conditional on the actions of the other party

However, the combination of plans E_1 and E_2 reveals a problem of plans which does not apply to actions or action-strategies. It is possible that plans, if conditional on other plans, are not well-defined (Howard 1966a, 1966b, 1971 and 1976, following an idea by von Neumann/Morgenstern 1944, constructs meta-games with meta-strategies, a well-defined subset of the plans proposed here, see also Rapoport 1967 and for a critique Harris 1969). Plans E_1 and E_2 together do not reveal whether both players will perform A or B. The plans, or at least the plan of one player, need(s) to be somewhat more sophisticated in order to yield a clear result, as in Figure 5.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Player 2</td></tr>
<tr><td>Strategy A_2 = Plan A_2: action A in any event</td><td>Strategy B_2 = Plan B_2: action B in any event</td><td>Strategy E_2 = Plan E_2: action A if player 1 performs A, otherwise action B</td></tr>
<tr><td rowspan="3">Player 1</td><td>Strategy A_1 = Plan E_1: action A in any event</td><td>2, 1</td><td>-1, -1</td><td>2, 1</td></tr>
<tr><td>Strategy B_1 = Plan E_1: action B in any event</td><td>-1, -1</td><td>1, 2</td><td>1, 2</td></tr>
<tr><td>Strategy S_1 = Plan S_1: action A if player 2 performs A unconditionally, otherwise action B</td><td>2, 1</td><td>1, 2</td><td>1, 2</td></tr>
</table>

Figure 5: Battle of the Sexes, with one more sophisticated conditional plan

If player 2 has an analogous plan S_2 to perform action A if player 1 performs A unconditionally, otherwise to do action B, then the combination of plans S_1 and S_2 is the same as that of plans S_1 and E_2, namely that both perform B and player 1 obtains one unit of utility and player 2 obtains two. This is also an equilibrium, as is the combination of S_1 and E_2. Certainly, there are an infinite number of possible plans, some pairs without a defined solution, such as the combination E_1 and E_2 before. Many more combinations simply do not result in an equilibrium. Finally, the original coordination problem of the Battle of the Sexes is not really solved. Although this problem does not exist for all given plan-equilibria, there are many more equilibria than before and choosing between them is therefore no easier.

4. Plans in the Prisoners' Dilemma

As before, with plans conditional on weekdays and similar external circumstances, if all games were like the Battle of the Sexes, then the introduction of plans other than actions and mixed strategies would not be worthwhile to research by game theorists. Yet, there are games in which plans make a real difference and this can be so strong as to seem quite uncanny. For example, in a Prisoners' Dilemma, cooperation becomes possible and is even the preferred equilibrium of really rational players. For a start, in a traditional Prisoners' Dilemma (see Luce/Raiffa 1957, pp. 94-97, with different payoffs), the naive plans N are added to defect (D) as long as the other player does indeed defect unconditionally and cooperate (C) otherwise. This is shown in Figure 6, now without labelling all plans as strategies, because both always refer to the same thing.

		Player 2		
		Plan C_2: C (cooperate) in all cases	Plan D_2: D (defect) in all cases	Plan N_2: D if player 1 performs D unconditionally, otherwise C
Player 1	Plan C_1: C (cooperate) in all cases	2, 2	0, 3	2, 2
	Plan D_1: D (defect) in all cases	3, 0	1, 1	1, 1
	Plan N_1: D if player 2 performs D unconditionally, otherwise C	2, 2	1, 1	2, 2

Figure 6: Prisoners' Dilemma, with naive plans for conditional cooperation

If both players must choose between only these three plans C, D and N, there would be two equilibria instead of one without the concept of plans and therefore without plan N. The combination of D_1 and D_2 remains an equilibrium, but N_1 and N_2 together form another equilibrium now and a more advantageous one for both players. Therefore, rational players should choose this second equilibrium (at least as long as other plans are not considered, see Section 6 below).

Nonetheless, this seems strange. How is cooperation possible in this standard case of the rationality of inefficient defection (given the many invalid attempts to demonstrate the contrary)? The concept of plans makes conditional cooperation possible. As soon as one's own cooperation is allowed, subject to the condition of cooperation by the other player, this cooperation becomes a rational option, and one that is even better than defection. This latter point is well known and nothing new (see e. g. Shubik 1970). A binding contract or any other enforcement mechanism by a third party (such as a norm by the mafia to punish traitors) is a standard "solution" to the Prisoners' Dilemma, by transforming the structure of the game and its pay-offs. The difference here is the lack of a third party (whose existence and incentives are often dubious, such that the dilemma remains). Plans can be effective, taking the form of an implicit contract that enforces itself. The punishment for defection is the *immediate* defection by the other player, *not* in later rounds, as is possible in repeated games of traditional game theory (with the problem of a last round and backward-induction as long as the number of repetitions is finite, see e. g. Selten 1978 or Milgrom/Roberts 1982, p. 283, for counter-arguments see Pettit/Sudgen 1989 and Sobel 1993). This is possible through the conditionality of plans. According to plan N, defection is answered by defection and cooperation by cooperation.

What about someone who plans to cooperate (or pretends to do so), but then defects at the last moment? Would this not be the best possible alternative, deceiving the other player into cooperating and gaining even more by one's own defection? However, if a player planned this switch from the beginning, then his plan is and was to defect from the start and the other player's plan would be to respond by defection. Even if a player originally plans to cooperate and changes his plan later, the result is the same, because plan N does not include the timing and in particular, is not conditional on the first plan someone has had, but on the effective plan or the real action.

5. Possible Theoretical Problems and Discussion

The issue discussed in the above paragraph raises another question. How do the players know each other's plans and subsequent actions? However, this is a problem associated with game theory in general, also of the traditional kind, and not only of the new concept of plans. For example, in the Battle of the Sexes, reaching one of the two equilibria through pure strategies depends on knowing, or at least guessing, the actions of the other player. That does not limit the essential truth that A is the best answer, given that the other player also performs A. It is the same with plans – N_1 is the best answer to N_2 and vice versa.

Admittedly, there is an additional twist associated with these plans, namely that not only the strategy or plan depends on that of the other player, but also the action taken in the context of the plan. In traditional game theory, there is no (relevant) difference between strategies and actions, at least not for pure strategies, whereas mixed strategies depend only on chance, and not on the strategy of the other player. Nevertheless, following a plan does not require any other knowledge than that required to choose a plan that qualifies as an equilibrium. In the worst case, it is simply not possible to follow a particular plan. If one has no idea what plan the other player may follow, than a plan like N may not be feasible. It is then as if such a plan does not exist and the strategy space for such a player is limited, in the extreme, to actions and their random mixture, as in traditional game theory. However, this does not mean that the strategy space is always that limited. Traditional game theory is only a special case of the much wider possibilities offered by plans.

It is neither irrational nor impossible to have plans that depend on the actions or even plans of others. Theoretically, models can be built in which this scenario is simply assumed. Even hybrid models are possible and perhaps of particular interest, in which some players can correctly predict the plans of others, while other players lack this ability, but recognise it in others, while still other players stubbornly refuse to accept the concept of plans. Whether or not real people can recognise the plans of others or make at least educated guesses about them, is an empirical question. The answer is most probably a positive one, at least for some people in some circumstances. Anonymous play could be insufficient for this recognition to occur. Possibly, the players need to talk to one another or to gaze into each other's eyes before choosing their actions.

In many versions of the Prisoners' Dilemma, prior communication is forbidden, but this is an unnecessary restriction. The logic of the game is not changed by cheap talk beforehand, as long as the decisions are made simultaneously and unobserved by the other player(s). In order to coordinate plans and form beliefs about the plan followed by the other player or simply about the type of other player, cheap talk could in fact be very valuable. That means that it could be rational for the same players to defect without communication and to cooperate with communication. To sum up, cooperation in a Prisoners' Dilemma is rational under the condition that the other player cooperates conditionally on one's own cooperation.

Nevertheless, the ability to act according to such conditional plans supposes something more than mere rationality (even in a broad sense), that is, the ability to recognise the plan of the other player or her type (what kind of player she is, as the basis for her planning). This means that if comprehensive rationality, which includes an understanding of the concept of plans, is common knowledge, then cooperation becomes possible. At least, it is not irrational to agree on cooperation and to fulfil the agreement, although one has to remain careful not to trust an untrustworthy person, whereas in traditional game theory, this kind of trust and trustworthiness are irrational. Given that the other player believes only in traditional game theory, the best answer is to defect, also according to plan N.

In a finitely repeated Prisoners' Dilemma, the interaction of an adherent of traditional game theory (player T) and someone who understands the concept of plans (player P), is more interesting theoretically. Two players of type T can only defect (all the time) as is well known, whereas two players of type P can cooperate constantly, as they can do so in any individual game, and therefore also the last one. Furthermore, cooperation in many games is simpler than in only one, because one gets to know the other player through the process of reputation building. Whether a player of type P can cooperate in most rounds with a player of type T, depends on one more characteristic besides her rationality and ability to recognise the other player's plan or type (given type T, all actions follow deterministically), namely the ability to commit to her plan.

6. A Conditionally Cooperative Equilibrium

Plans N_1 and N_2 in Figure 6 do not really form an equilibrium and truly rational players need more sophisticated plans. The reason for this scenario is simply that N_1 is not the best response to N_2 (and vice versa). Given N_2, player 1 could gain by switching from N_1 to another conditional plan such as N_{1+}: "Do C if Player 2 does C unconditionally, otherwise do D." Then, he does not defect unconditionally, such that Player 2 will cooperate according to N_2, but at the same time, he can respond to this cooperation with defection. This is not in the interest of Player 2 and she has an incentive to switch her own plan. Certainly, N_{1+} and an analogous N_{2+} are not equilibria either.

An equilibrium consists of mutual best responses, such that no player has an incentive to change his or her strategy. This means that in a Prisoners' Dilemma, both players have to make their own cooperation contingent on the cooperation of the other player and not on something else, such that the other player could defect and still reap the benefits of cooperation. What about strategy O_1 ("Do C if player 2 does C, otherwise do D.") and the corresponding strategy O_2? In such a situation, the same problem prevails as in Figure 4, the actions are undetermined by this pair of strategies, because both players could cooperate and defect together. A more sophisticated, conditionally cooperative plan could be P_1: "Do C if Player 2 does C subject to the condition of doing C myself, otherwise do D." Player 2 can have a symmetric plan P_2: "Do C if player 1 does C under the condition of doing C myself, otherwise do D." The strategies P_1 and P_2 together, constitute a cooperative equilibrium, because the conditions are fulfilled, such that both will cooperate and no one has an incentive to change his or her plan, provided there is no better alternative. However, symmetry is not decisive for an equilibrium; for example P_1 and O_2 form an equilibrium with asymmetric plans.

7. Conclusion

Certainly, D_1 and D_2 remain an equilibrium, although one that is worse for both players. Defection is no longer (as in a world with only action-strategies) a dominant strategy or plan, as there are no nontrivial dominant plans. Trivial dominance means that a player is indifferent between all alternatives that other players can bring about. Without trivial dominance, for any plan Q, there is a plan Anti-Q from the other player(s), which ensure the player of Q the worst possible result under Q (in the Prisoners' Dilemma, this would be mutual defection) and promises the best possible alternative for any other plan (in the Prisoners' Dilemma, the switch from Q would be rewarded with cooperation by the other player, even if the plan other than Q means defection by the first player). The existence of Anti-Q destroys any nontrivial dominance, meaning one best response to all plans of the other player(s).

Conversely, there are not only infinitely many different plans without any clear order, there is also a limitless set of equilibria. New ones can be constructed at will. Their existence (perpetual) and uniqueness (never) are not important issues in the context of plans. The search for efficiency, robustness or some other relevant properties of plan-equilibria and their comparison in these contexts, are issues that are more worth of analysis and investigation. In any case, contingent cooperation is an advantageous and rational plan in a Prisoners' Dilemma or in comparable collective-good problems with more players.

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