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Diffusion in a simple classical model
Micro decisions and macro outcomes

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Abstract

The paper explores the interface between evolutionary and modern classical thinking by posing the question of how a long-period position is reached if multiple methods of production are simultaneously in use. Firm decisions on investment and on technology provide the basis of two possible mechanisms of convergence: differential growth and imitation. Both mechanisms rely on the concept of 'extra profits' and imply that during a period of disequilibrium economically superior methods of production gradually supersede inferior ones. The model reproduces the stylized fact of sigmoid diffusion curves and shows that diffusion leads to uneven growth with ambiguous long term effects, a change of income distribution and of the industry structure.

1 Introduction

Joseph A. Schumpeter defines economic evolution as "changes in the economic process brought about by innovation, together with all their effects, and the response to them by the economic system" (Schumpeter, 1939, p. 37). Based on Schumpeter's contribution, contemporary evolutionary economics explores "the sources

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of innovative novelties in economic practice, and the adaptation of the economic system to the potential contained in those novelties." (Metcalfe, 2008, p. 24) This strand of thinking views the economy as a disequilibrium process shaped by the interplay between variety generation and competitive selection.

In evolutionary economics, two basic models of evolution are in use. In the two-stage model, generation of variety and competitive selection do not act at the same time. Variety of behavior is taken to be given and fixed and the question of how innovations arise from within the system is set aside. The focus therefore is on the selection mechanism and its macro consequences. This view postulates convergence towards equilibrium: Since economic evolution depends on the presence of variety, and since selection destroys variety, evolution "consumes its own fuel. [...] Unless this variety is replenished, evolution will come to an end." (Foster & Metcalfe, 2001, p. 9) This 'selectionist view' on economic change does not account for endogenous variety creation. Including this aspect leads to the three-stage model of evolution in which variety generation and variety destruction act simultaneously and are mutually interdependent (Foster & Metcalfe, 2001).

Although the three-stage model is more sophisticated and comprehensive, viewing the generation of variety and selection as two distinct steps in the analysis of economic change allows one to study each of these two aspects of economic change in isolation. By this separation, economic change is grasped as a stylized sequence of three successive steps: Invention, Innovation and Diffusion. The third item of this 'Schumpeterian trilogy', diffusion of innovations, is connected to the evolutionary mechanism of selection (Stoneman, 2007). This scientific route also underlies Schumpeter's analysis. Schumpeter (1934) admits that it is the interplay between the 'creative construction' of 'energetic' men and the passive 'hedonistic' mass that puts economic evolution at work and that the internal source of variation always will disturb any tendency to restore equilibrium. But in his analysis Schumpeter distinguishes between "*definite periods* in which the system embarks upon an excursion away from equilibrium and *equally definite periods* in which it draws toward equilibrium" (Schumpeter, 1939, p. 63; emphasis added). The first 'definite period' is shaped by invention, innovation and creative behavior; in the second ‘definite period’ adaptive behavior, which restores equilibrium, is seen as dominating. The system will, after having been disrupted by innovation, settle in a circular flow, in which its evolutionary potential is exhausted.

The circular flow is a state of the system in which no agent has an incentive to change his position. It is the bridge between evolutionary and modern classical thinking. Two papers explore the interface between the two schools of thought: Kurz (2008) discusses innovation within a classical two sector model and uses the classical long-period method to study the effects of new methods of production on prices and distribution. In his analysis, Kurz interprets a process innovation as a change in the data and evaluates the consequences for the economic system by comparing the long-period positions (hereinafter *LPP*) before the innovation enters the system and after the innovation has been fully absorbed. Steedman & Metcalfe (2011) argue that a full account of economic transformation also has to explain how new methods of production replace old ones. Within a one-sector framework they explore the process of adaptation taking place out of equilibrium and explain how competitive selection moves the system towards an LPP.

We add to this literature about how a long-period position is established after an innovative impulse has disrupted the circular flow by studying how the nature of the innovation effects aggregate growth and income distribution. What are the forces that lead the system towards a new position of rest? It is the aim of this paper to explore possible mechanisms of convergence and to study the disequilibrium dynamics for different kinds and intensities of technical change. The study of disequilibrium paths is relevant for two reasons: First, if disequilibrium prevails for a long time, understanding the dynamics outside equilibrium is crucial. Secondly, it adds to our understanding of how equilibria form and it illuminates phenomena characterizing disequilibrium, which do not appear in long-run equilibrium.

The paper proceeds in three steps. Section 2 presents basic concepts and assumptions. In Section 3 the core mechanisms of convergence, differential accumulation and imitation are formalized. Section 4 explores the macro regularities initiated by diffusion. Section 5 concludes.

2 The economy out of equilibrium

In a one-commodity world, a homogeneous good is produced by means of homogeneous labor and by the good itself. Production functions are of the Leontief-type and returns to scale are constant. There are no barriers to growth as labor is available in abundance. Take the economy to be in an LPP, "characterized by a uniform rate of profit and uniform rates of remuneration for each particular kind of 'primary' input in the production process" (Kurz & Salvadori, 1995, p. 1). This definition of an LPP implies that in terms of market shares only one method of production is in use, if one abstracts from the possibility that at the given level of wages and for a given normal rate of profit various methods of production just break even. An LPP is an equilibrium position, which is understood as the outcome of a disequilibrium process. As (Knell, 2008, p. 39) notes, "the uniform rate of profit describes the outcome of the competitive behavior of heterogeneous actors in the market, whereas profit-seeking entrepreneurs [...] minimize the cost of production because of the competitive process". The mechanism which leads to such a state of uniformity relies on the assumption that markets are characterized by free competition. By definition, this implies the absence of substantial barriers of entry and exit and therefore allows both capital and labor to be fully mobile across sectors. Furthermore, free competition demands firms to have access to all known methods of production and that the availability of these methods is independent of firm size (Kurz & Salvadori, 1995, p. 17). Given these conditions, profit-seeking firms will look for the method of production which yields the highest rate of profit given current prices.

For a new method of production, which is superior in the sense that at the prevailing wage rate it has a cost advantage, it is reasonable to assume that it is not adopted instantaneously by all firms. Due to numerous constraints it is rather a gradual process by which new technological knowledge is absorbed. Several reasons can be found to explain this time lag of adoption: First, the pioneer might succeed in keeping the innovative method of production a secret for some time. Secondly, due to a lack of information about technical characteristics and experience, firms face uncertainty about the innovation's superiority. Thus the basis for a decision on technology is blurred and, given limited knowledge, an immediate adoption might be seen as involving high risk. Third, there will be limits to the ability to adopt novel business practices due to organizational and financial frictions. (Stoneman, 2002; Rogers, 2003; Baptista, 1999; Nelson *et al.*, 2004) Given that the speed at which an innovation spreads is not infinitely large, there is a period of disequilibrium in which multiple methods of production co-exist and rates of profit vary across firms.

Consider the case of two methods of production co-existing ($i = 1, 2$), the first being established and the second invading the system. Take the produced good as the numéraire and assume that on the output and on the labor market the law of one price holds. Wages are paid ex post. For some real wage rate w, the rate of profit $r_i(w)$ of production process i is then given by

$$(1 + r_i(w))a_i + wl_i = 1 \tag{1}$$

with a_i and l_i denoting the input of capital and labor respectively necessary to produce one unit of output. With x_i denoting the output produced by means of process i, let $q = x_2/x$ be the share of total output x produced by the innovative process 2, and $1 - q = x_1/x$ the market share of the incumbent process 1. Given the prevailing distribution of production methods across firms, the average amount of capital and the average amount of labor needed to produce one unit of output is computed as

$$\bar{a} = (1 - q)a_1 + qa_2 \quad \text{and} \quad \bar{l} = (1 - q)l_1 + ql_2.$$

The rate of profit $\bar{r}(w)$ of the average production process $(\bar{a}, \bar{l})$ – which in general does not coincide with the average rate of profit $(1 - q)r_1 + qr_2$ – is then given by

$$(1 + \bar{r})\bar{a} + w\bar{l} = 1. \tag{2}$$

The average production process is an abstract measure of the prevailing state of technical knowledge at a given moment of time. It reflects the normal conditions of production. By comparing r_i and $\bar{r}$, hence by measuring the distance of this method from the average method, the relative economic superiority of the method of production i is determined. To measure this relative economic superiority, the concept of *extra profits* is used. Defining the rate of extra profits by $\rho_i = r_i - \bar{r}$, equation (1) reads

$$(1 + \bar{r} + \rho_i)a_i + wl_i = 1. \quad (3)$$

Methods of production which yield positive (negative) extra profits have a cost advantage (disadvantage) compared with the average or normal conditions of production and are hence economically superior (inferior). Figure 1 illustrates the wage-profit relationship corresponding to equations (2) and (3).

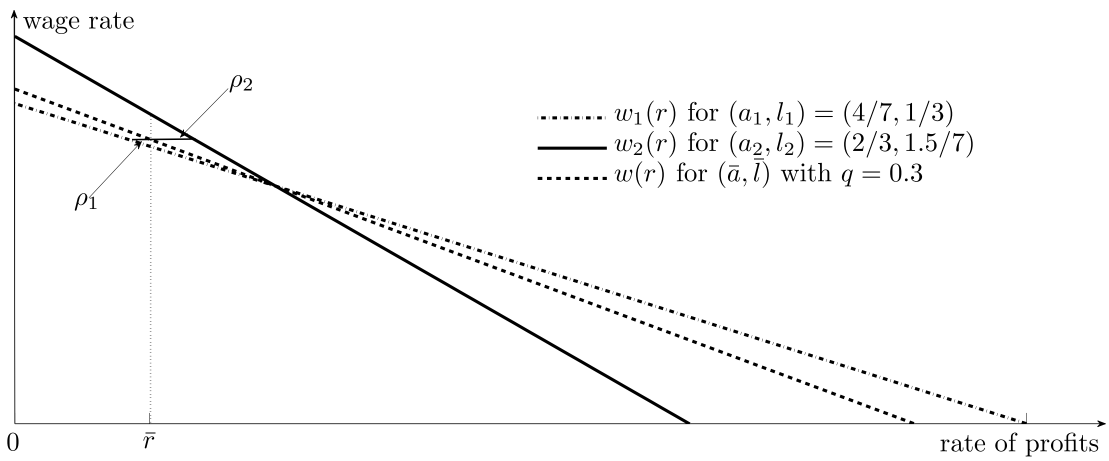

Figure 1: Wage-profit curves for different processes

Disequilibrium, a situation in which different methods of production co-exist and rates of extra profits are non-zero, is our starting point for the characterization of the economy's path towards an LPP. The system reaches a new LPP via two inter-related adjustments. First, an adjustment concerning quantities takes place. Its essence is that the relative significance of different methods of production changes over time: Superior methods gain ground, whereas inferior methods loose ground. The diffusion of an innovation is based on two mechanisms: *differential accumulation* and *imitation*. *Differential accumulation* relies on the idea that those firms which generate higher profits than others can expand at a higher rate. If there is a functional relation between past profits and firm growth, the relative significance of a cost-saving innovation will increase over time. *Imitation* is the adoption of a new method of production by a non-innovator and can be understood as the outcome of a choice-of-technique problem on the firm level. Both mechanisms are formalized in Section 3.

The second adjustment concerns income distribution. Since a superior method allows for a higher surplus of production, there is the question of how this surplus is distributed amongst capitalists and workers. Before the innovation enters the system with technology (a_1, l_1) and some exogenously given wage rate w_0, the (normal) rate of profit is given by $r(w_0)$. To account for the theoretical argument that profit due to innovation is transitory and acknowledging the stylized fact that the economy-wide profit rate has no long-term trend, we assume that in disequilibrium average conditions of production just generate the normal rate of profits and hence $\bar{r} = r$. This assumption implies that the wage rate adjusts according to average labor productivity. As a result of equation (2), the wage rate is then determined by

$$w = \frac{1 - (1 + r)\bar{a}}{\bar{l}}. \tag{4}$$

The wage rate is therefore endogenously determined by the normal conditions of production, influencing extra profits ρ_i by equation (3), which is now given by

$$(1 + r + \rho_i)a_i + wl_i = 1. \tag{5}$$

The adjustment of the wage rate due to a change in the relative significance of the methods of production in use thus feeds back on the diffusion process and implies a competitive pressure on firms: In order to generate the normal rate of profit a firm has to produce with the average conditions of production. Further it follows that notwithstanding profit-seeking behavior, the more widespread the use of a novel method the less profitable it becomes.

3 Mechanisms of diffusion

In this section we formalize the two mechanisms which account for the gradual pervasion of a process innovation. Diffusion by differential accumulation relies on assumptions on investment behavior and is studied in Subsection 3.1. In Subsection 3.2 the second mechanism, diffusion by imitation, is formalized. For both mechanisms extra profits are pivotal.

3.1 Firm growth

Firm growth as the driver of diffusion can be investigated by abstracting from the possibility that firms can change their current method of production. The possible strategy is therefore firm-specific investment, which is financed by past profits. According to equation (5), the output of a firm using i at time t is divided into wage payments, into capital investment to maintain the output-level and into profits. Output $x_{i,t}$ is produced by process i and hence divides into

$$x_{i,t} = w_t l_i x_{i,t} + a_i x_{i,t} + (r + \rho_{i,t}) a_i x_{i,t}.$$

To determine the next period's output $x_{i,t+1}$ produced by process i, the following variation of the classical investment hypothesis formulated at the level of firms is adopted: Let $s \in (0, 1]$ be the propensity to invest in case of a positive rate of profit $r + \rho_{i,t}$ and let $C_{i,t} = (1 - s)\,(r + \rho_{i,t})\,a_i x_{i,t}$ denote consumption out of profit; there are no savings out of wages. Because the economy is out of equilibrium three cases have to be distinguished:

Case 1: $(r + \rho_{i,t}) > 0$. In this case the firm accumulates and hence total output produced by process i increases: $x_{i,t+1} \geq x_{i,t}$. The net-output $x_{i,t} - w l_i x_{i,t}$ which remains after paying wages is split up into investment $s(r + \rho_{i,t}) a_i x_{i,t}$ and capitalist consumption $C_{i,t} \geq 0$.

Case 2: $-1 < (r + \rho_{i,t}) \leq 0$. Using the same rule as in Case 1 would imply that firms using process i shrink at rate $s(r + \rho_{i,t})$ and that capitalist consumption

$C_{i,t}$ would turn negative. Sticking to the assumption that no savings out of wages exist, the whole output $(1 + r + \rho_{i,t})a_i x_{i,t}$ which is left over after paying wages is invested, implying $C_{i,t} = 0$. Total output produced by process i in this case declines, since $1 + r + \rho_{i,t} < 1$.

Case 3: $(r + \rho_{i,t}) \leq -1$. Since now $wl_i x_{i,t} \geq x_{i,t}$, firms using process i fail to be able to pay the total wage bill. This can happen because in period t the wage rate is determined after production has taken place. In this case it is assumed that the firm pays its laborers as far as it can and then leaves the market.

Summing up, output growth is given by

$$g_{i,t} = \frac{x_{i,t+1} - x_{i,t}}{x_{i,t}} = \begin{cases} s\,(r + \rho_{i,t}) & \text{in Case 1: } r + \rho_{i,t} > 0 \\ r + \rho_{i,t} & \text{in Case 2: } -1 < r + \rho_{i,t} \leq 0 \\ -1 & \text{in Case 3: } r + \rho_{i,t} \leq -1 \end{cases} \tag{6}$$

and illustrated in Figure 2. Comparing Cases 1 and 2 illustrates the asymmetry

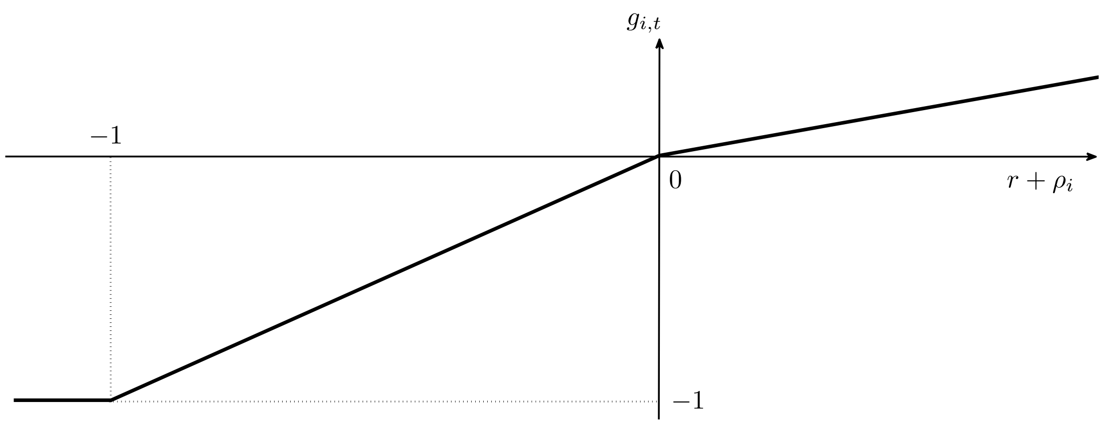

Figure 2: Kinked investment function for $s = 0.4$

between firm growth and decline: The firm in the first case decides how much to grow, depending on its propensity s to invest; only if $s = 1$ it realizes its full growth potential. The firm in the second case has to de-accumulate if there is no capital injected from outside, i.e if $C_{i,t}$ is non-negative. From equation (6) it follows that the market share $q_t = x_{2,t}/x_t$ of the innovation evolves according to

$$q_{t+1} = \begin{cases} \frac{1+s(r+\rho_{2,t})}{1+s(r+\bar{\rho}_t)} q_t & \text{in Case 1: } r + \rho_{1,t} > 0 \\ \frac{[1+s(r+\rho_{2,t})]q_t}{1+s(r+\bar{\rho}_t)+(1-s)(r+\rho_{1,t})(1-q_t)} & \text{in Case 2: } -1 < r + \rho_{1,t} \leq 0 \\ 1 & \text{in Case 3: } r + \rho_{1,t} \leq -1 \end{cases} \quad (7)$$

with $\bar{\rho}_t = (1 - q_t)\rho_{1,t} + q_t\rho_{2,t}$ denoting average extra profits. In Case 2, the diffusion path takes place faster than for Case 1. This can be seen formally by acknowledging the negative term $(1 - s)(r + \rho_{1,t})(1 - q_t)$ in the denominator. An innovation meeting the condition of Case 3 leads to an extinction of the incumbent process as opposed to the asymptotic behavior of the diffusion curve for Cases 1 and 2. Two examples of diffusion paths of some specific innovative processes, which replace the incumbent process, are exemplified in Figure 3. They mimic the stylized fact of S-shaped curves of diffusion processes (see for example Stoneman, 2002).

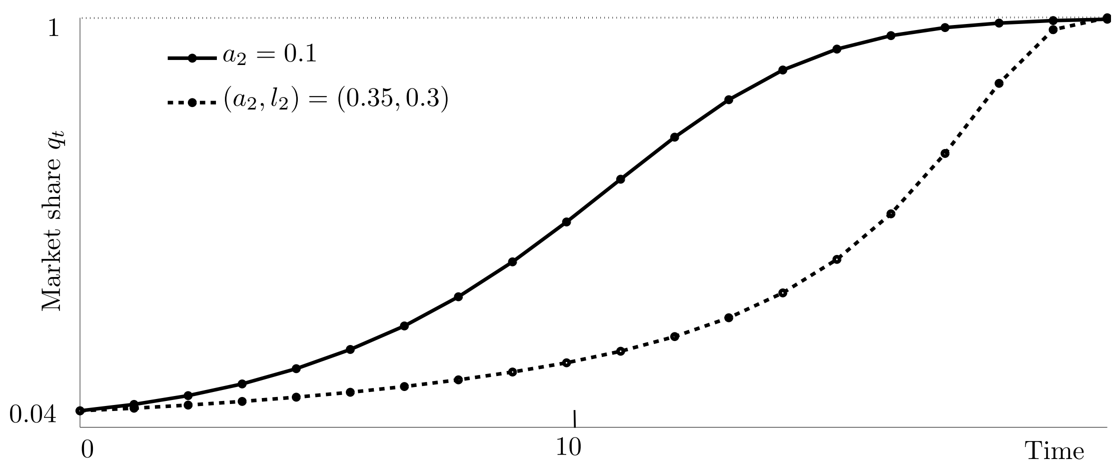

Figure 3: Examples of firm growth diffusion paths for $(a_1, l_1) = (0.2, 0.5)$, $r = 0.1$ and $s = 0.4$

The market share dynamics described by equation (7) is related to the model of Steedman & Metcalfe (2011), who assume that the propensity to invest is one ($s = 1$). Then the first two cases of (7) would coincide. By excluding Case 3, one gets

$$\frac{q_{t+1} - q_t}{q_t} = \frac{(1 - q_t)(\rho_{2,t} - \rho_{1,t})}{1 + r + \bar{\rho}_t}. \tag{8}$$

Extra profits ρ_i gained by process i therefore evolve over time according to the prevailing real wage rate w defined by equation (4). Steedman & Metcalfe (2011) determine prices by the marginal firm, hence by unit costs of production of the incumbent process. This implies that the original wage rate w_0, according to equation (5), leads to constant extra rates of profit $\rho_1 = 0$ and $\rho_2 = (1 - w_0 l_2)/a_2$. Since in this case $\bar{\rho}_t = q_t \rho_2$, equation (8) is identical to the replicator equation

$$\frac{q_{t+1} - q_t}{q_t} = \frac{(1 - q_t)\rho_2}{1 + r + q_t \rho_2}$$

of Steedman & Metcalfe (2011). There, the feedback of changing wages is excluded from the analysis, leading to sustained positive extra profits of the innovative firms in case of unrestricted labor supply.[1] In contrast, in the present model with adapting real wages, guided by the wage setting mechanism (4), extra profits decrease and finally vanish.

3.2 Imitation

In the previous section firms were assumed to stick to their current method of production irrespective of its performance in terms of profits. Therefore diffusion takes place by differential growth alone. If, on the other extreme, firms are assumed not to grow, but to be concerned with choosing amongst available methods of production, diffusion is the outcome of imitative behavior of non-innovators. In contrast to the diffusion-by-growth mechanism, this approach involves interaction between firms to bring about knowledge transfer. To isolate the mechanism of

[1] Steedman & Metcalfe (2011) propose a fixed supply of labor to bring the diffusion process to a halt as soon as all workers are headhunted by the innovative firm from the incumbent firms by infinitesimally larger wages (which hence do not influence extra profits). At the end of this diffusion process, a discontinuous jump of the wage rate towards the new level is assumed to restore the exogenously given original normal rate of profit.

imitation, take each firm to produce exactly one unit of output, not shrinking or growing in size regardless of the profits or losses it incurs.

At time t the state of firm k is given by $f_t(k) = i$, with $i \in \{1, 2\}$ depending on whether the firm is still using the incumbent process or if it has already switched to the new one. Let N be the fixed number of firms and n_t the number of firms using the innovative process. The market share of the innovation is then given by $q_t = n_t/N$ and, accordingly, the market share of the incumbent process is $1 - q_t = (N - n_t)/N$. Firms use the following behavioral rule: At each step in time, firms which use the incumbent process, and hence earn negative extra profits, decide on whether to imitate or not. Firms are myopic and have no a-priori knowledge about the innovation but only learn from some other firm, which is randomly drawn from the set of all firms. If the chosen firm also uses the incumbent process, nothing will change; if it already uses the innovation, the firm using the old process will switch to the new process with probability $\mathbb{P}_t$. This probability includes two aspects, *choice* and *capability*: It might be the case that one knows a superior process, but for whatever reason, for instance due to vested interests, the firm decides not to change its currently employed method; if a firm decides to adopt the innovation, obstacles such as a lack of financial resources, human capital (skills) or tacit knowledge may render the attempt to imitate unsuccessful.

The evolution of the expected number $\hat{n}_t$ of firms using the innovation is therefore given by

$$\hat{n}_{t+1} - \hat{n}_t = \mathbb{P}_t \cdot (N - \hat{n}_t)\hat{n}_t/N. \tag{9}$$

$N - \hat{n}_t$ is the number of firms using the old process and $\hat{n}_t/N$ is the probability that this firm choses an innovative firm with which to compare its process. The expected market share $\hat{q}_t$ of the innovation due to (9) is then determined by

$$\hat{q}_{t+1} - \hat{q}_t = (1 - \hat{q}_t)\hat{q}_t\mathbb{P}_t. \tag{10}$$

In a first approximation, one can take the probability that the innovation is adopted to be given by an exponential distribution, with the adoption probability negatively influenced by some parameter λ and positively influenced by the profit differential $\rho_{2,t} - \rho_{1,t} > 0$:

$$\mathbb{P}_t = 1 - e^{-\lambda(\rho_{2,t} - \rho_{1,t})}$$

For $\lambda \to \infty$ firms adopt the superior method whenever they get in contact with a firm already using it. Thus, equation (10) reduces to the logistic equation and diffusion becomes a pure epidemic process. In the other extreme, for $\lambda = 0$, no firm ever switches.

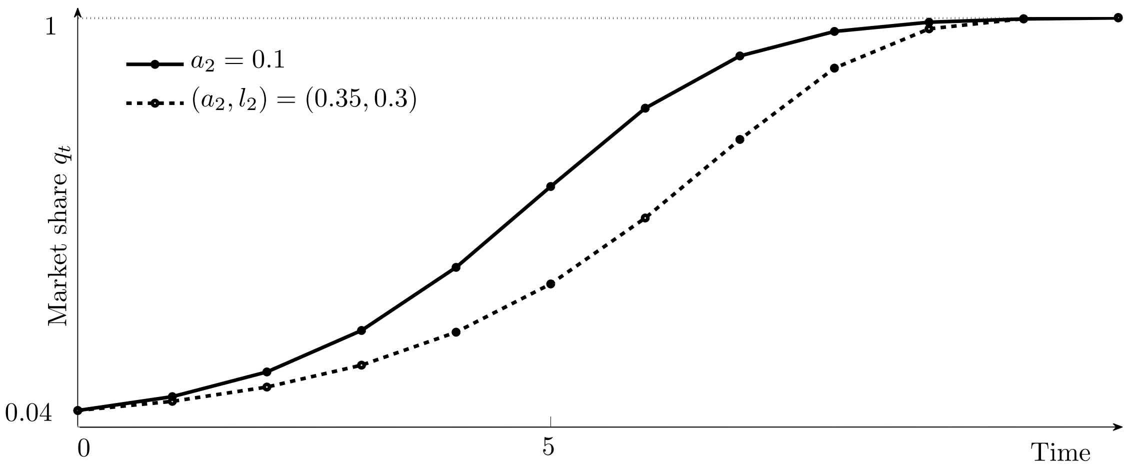

Figure 4: Examples of diffusion paths by imitation for $(a_1, l_1) = (0.2, 0.5)$, $r = 0.1$ and $\lambda = 2$

As shown in Figure 4, similar to the case of firm growth in Subsection 3.1, an S-shaped diffusion path emerges as indicated by the structure of equation (10). The latter resembles the *logistic equation* with some variable diffusion-factor $\mathbb{P}$, which serves as a measure for the *diffusion velocity*.

4 Disequilibrium dynamics

Based on the formalization of diffusion mechanisms, this section explores the aggregate dynamics of disequilibrium: What are the implications of the diffusion of a new method of production for the economy as a whole? As a preparatory step, in

Subsection 4.1 we subdivide the factor space. In Subsections 4.2 and 4.3 we then analyze the dynamics of aggregate growth and the change of income distribution. Finally, in Subsection 4.4 we look at the industry structure.

4.1 The factor space

With reference to the incumbent method of production 1, any innovative process 2 is characterized in terms of the relative change of the capital and labor input coefficients

$$\Theta_a = \frac{a_2 - a_1}{a_1} \quad \text{and} \quad \Theta_l = \frac{l_2 - l_1}{l_1}.$$

For a given r and maximum rate of profit

$$R_1 = \frac{1 - a_1}{a_1}$$

of the incumbent process, the factor space is subdivided along two dimensions: First, according to different kinds of technical change, listed in Table 1; secondly, according to the degree of technical change, determining whether case 1, 2 or 3 of the investment function (6) applies, listed in Table 2.

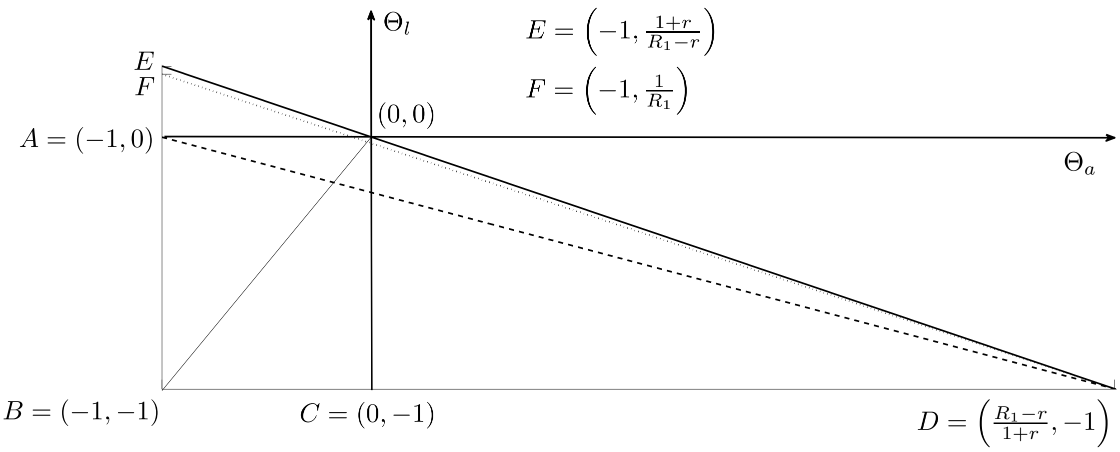

Figure 5: Partition of the factor space for an incumbent process for $R_1 = 4$ and $r = 0.1$

In Figure 5, the coefficients of an incumbent process are represented by the

Kind of technical change	**technical coefficients**	**in Figure 5**
capital saving and labor using	$\Theta_a < 0\ ,\ \Theta_l > 0$	ΔOEA
labor saving and capital using	$\Theta_a > 0\ ,\ \Theta_l < 0$	ΔOCD
pure capital saving	$\Theta_a < 0\ ,\ \Theta_l = 0$	$\overline{OA}$
pure labor saving	$\Theta_a = 0\ ,\ \Theta_l < 0$	$\overline{OC}$
combined factor saving	$\Theta_a < 0\ ,\ \Theta_l < 0$	$\Delta OABC$
neutral	$\Theta_a = \Theta_l < 0$	$\overline{OB}$
dominantly capital saving	$\Theta_a < \Theta_l < 0$	ΔOAB
dominantly labor saving	$0 > \Theta_a > \Theta_l$	ΔOBC

Table 1: Partition of the factor space I

Interval of total profits in the new LPP	**in Figure 5**
Case 1: $r \geq r + \rho_1\|_{q=1} \geq 0$	ΔDEF
Case 2: $0 \geq r + \rho_1\|_{q=1} \geq -1$	ΔDFA
Case 3: $-1 \geq r + \rho_1\|_{q=1}$	ΔDAB

Table 2: Partition of the factor space II

origin $(\Theta_a, \Theta_l) = (0, 0)$. Θ_l and Θ_a are bounded from below by -1. The line $B - C - D$ is characterized by $\Theta_l = -1$, i.e. by $l_2 = 0$. Similarly, $\Theta_a = -1$ holds for the line $B - A - F - E$, indicating that $a_2 = 0$.

The downward sloping *iso-profit-rate line* $\overline{DE}$, given by

$$\overline{DE}: \quad \Theta_l = -\frac{1 + r}{R_1 - r}\Theta_a,$$

defines the set of all methods of production (a_2, l_2) which have the same unit costs of production as the incumbent process. $\overline{DE}$ is plotted for some given positive normal rate of profit r in Figure 5. It divides superior methods – potential innovations – below the line from inferior ones lying above it. A method of production which lies above $\overline{DE}$ is not able to pervade the system since it exhibits higher unit costs of production. This line, which separates innovations from economically inferior methods of production, gets steeper for increasing r, with $(0, 0)$ as fixed point. If $r = R_1$, the iso-profit rate line is a vertical through the origin in Figure 5, implying that any method of production which is capital saving has a cost advantage irrespective of its labor coefficient.

Secondly, the line identified by separating innovations of cases 2 and 3 in Table 2 is defined by $r + \rho_1|_{q=1} = 0$: The line

$$\overline{DF}: \quad \Theta_l = -\frac{r}{R_1} - \frac{1+r}{R_1}\Theta_a$$

defines the set of all innovations for which the incumbent process generates exactly zero total profits in the *new* LPP. Note that for a new method of production within the triangle ΔDEF, in the new LPP the incumbent process yields a positive rate of profit. Nevertheless, the market share of the innovation asymptotically approaches 1. Despite positive absolute output growth, in relative terms the market share of the incumbent process vanishes.

Finally, the area ΔDFA contains innovations where the total rate of profit of the incumbent process lies within -1 and 0 in the new LPP. Any combination of a_2 and l_2 below

$$\overline{AD}: \quad \Theta_l = -\frac{1+r}{1+R_1}(1+\Theta_a)$$

implies a rate of profit smaller than -1 in the new LPP. The three lines intersect at point D.

This discussion shows that for reasonable values of r the wedge of innovations which leave the inferior method with a positive rate of profit is very narrow. Thus the case in which profitability of inferior method turns negative is decisive in this model. In the following investigation we focus on this case and abstract from cases below $\overline{AD}$ in order to hold the number of firms in disequilibrium constant.

4.2 Aggregate growth

In this section we deal with the dynamics of the economy in disequilibrium. Both the nature of the invading method and the degree of technical change, together with the assumptions on investment behavior, play a role. Aggregate output is given by $x_t = x_{1,t} + x_{2,t}$. In the LPP, the aggregate growth rate is determined by

$g_{LPP} = sr$. The aggregate growth rate g_t is given by

$$g_t = \frac{x_{t+1} - x_t}{x_t} = (1 - q_t)g_{1,t} + q_t g_{2,t}$$

with the growth rate $g_{i,t}$ of a firm using process i determined by equation (6). As long as $r + \rho_{1,t} > 0$ it follows that

$$g_t = (1 - q_t)s(r + \rho_{1,t}) + q_t s(r + \rho_{2,t}) = s\left(r + \bar{\rho}_t\right), \tag{11}$$

where $\bar{\rho}_t$ denotes the average rate of extra profit. Thus, the transient growth rate deviates from the long-run growth rate whenever $\bar{\rho}_t$ does not equal zero. Before we explore this effect, which we call *technology effect*, the second effect is introduced. The second effect, which renders aggregate growth uneven, emerges from the kink in the investment function. If the invading method lies within the area ΔDAF, equation (11) is replaced by

$$g_t = (1 - q_t)(r + \rho_{1,t}) + q_t s(r + \rho_{2,t}) \tag{12}$$

as soon as the profitability of the inferior method turns negative. This *investment effect* starts to work at some $q_t = q_0$, where $r + \rho_{1,t} = 0$, that is when the ruling wage rate equals the maximum wage rate $(1 - a_1)/l_1$ process 1 can pay. Given equation (4), q_0 is determined by

$$q_0 = \frac{r}{-(1 + r)\Theta_a - R_1\Theta_l}.$$

The technology effect: To rule out the investment effect, consider the case $s = 1$, in which the kink in the investment function and thus the asymmetry between firm growth and decline vanishes. The aggregate growth rate then is $g_t = r + \bar{\rho}_t$. From equations (4) and (5) it follows that $(1 - q_t)a_1\rho_{1,t} + q_t a_2 \rho_{2,t} = 0$, i.e. that

$$\bar{\rho}_t = -q_t \rho_{2,t} \Theta_a. \tag{13}$$

In disequilibrium it holds that q_t and $\rho_{2,t}$ are strictly positive. Therefore, the sign of $\bar{\rho}_t$ only depends on the sign of Θ_a. This implies that the sign of $\bar{\rho}_t$ remains constant. Three cases can be distinguished:

1. $\bar{\rho}_t < 0$ holds for labor saving and capital using technical change ($\Theta_a > 0$).

2. $\bar{\rho}_t = 0$ holds for pure labor saving technical change ($\Theta_a = 0$).

3. $\bar{\rho}_t > 0$ holds for capital saving technical change ($\Theta_a < 0$).

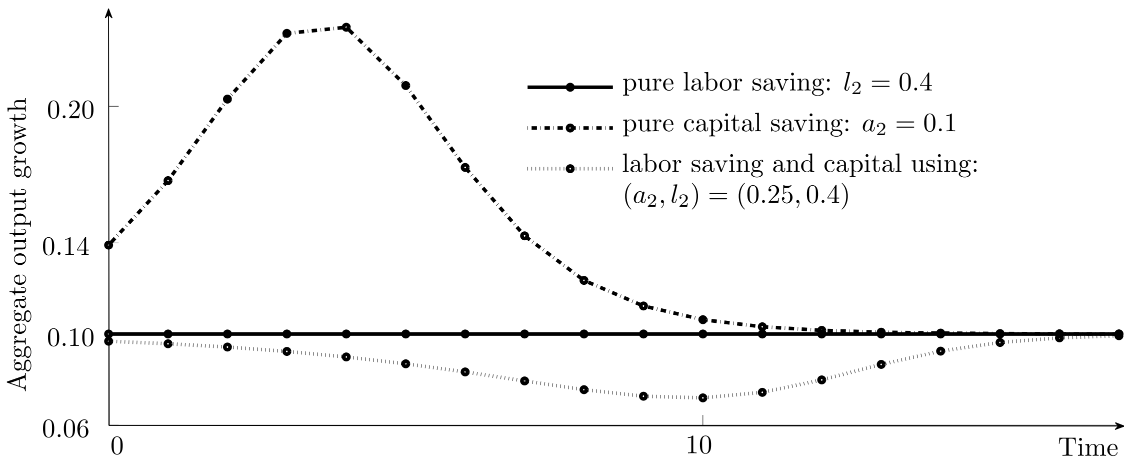

Figure 6: Technology effect for $(a_1, l_1) = (0.2, 0.5)$ and $r = 0.1$

Figure 6 illustrates the three possible patterns arising due to the technology effect. Whereas the diffusion of the pure capital saving method accelerates aggregate growth, labor saving and capital using technical change slows down economic growth; only the diffusion of the pure labor saving method shows no effect on aggregate growth.

The investment effect: The second important determinant of aggregate transient growth is the behavioral parameter s. For $s < 1$ and $q_t \in (q_0, 1)$, the investment effect is at work and g_t is determined by equation (12). The investment effect implies that during the diffusion the decline of the incumbent firm dominates growth, leading to a negative aggregate growth effect. The investment effect is illustrated in Figure 7 for different values of s. To isolate the investment effect, pure labor saving technical change is considered, because this combination of parameters does not harm steady growth for $s = 1$. The value of q_0 and the length of the period of disequilibrium are negatively correlated with the propensity s to invest. Also, the slow-down of aggregate growth is more pronounced for smaller values of s. For example, for $s = 0.25$ output declines while the superior method of production supersedes the inferior one.

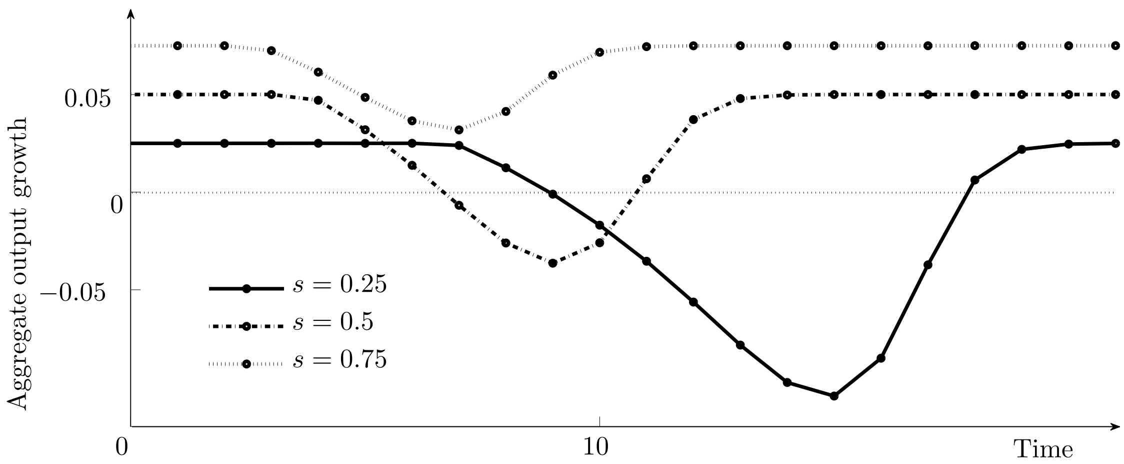

Figure 7: Investment effect for $(a_1, l_1) = (0.2, 0.5)$ and $(a_2, l_2) = (0.2, 0.4)$, with $r = 0.1$

Interference of the technology and the investment effect: The diffusion of a new method of production with $\Theta_a < 0$ small enough to turn the profit rate of the inferior method negative at some q_0 leads to a wave-like path of the aggregate growth rate for the following reasons: First, all firms experience a positive rate of profit and thus the technology effect accelerates growth. Yet, as soon as the profit rate of firms using the old method turns negative, aggregate growth is dampened due to the investment effect. Figure 8 provides an illustration.

Short term and long term effects: Transitional growth due to diffusion can be evaluated along two dimensions: the short term and the long term effect on

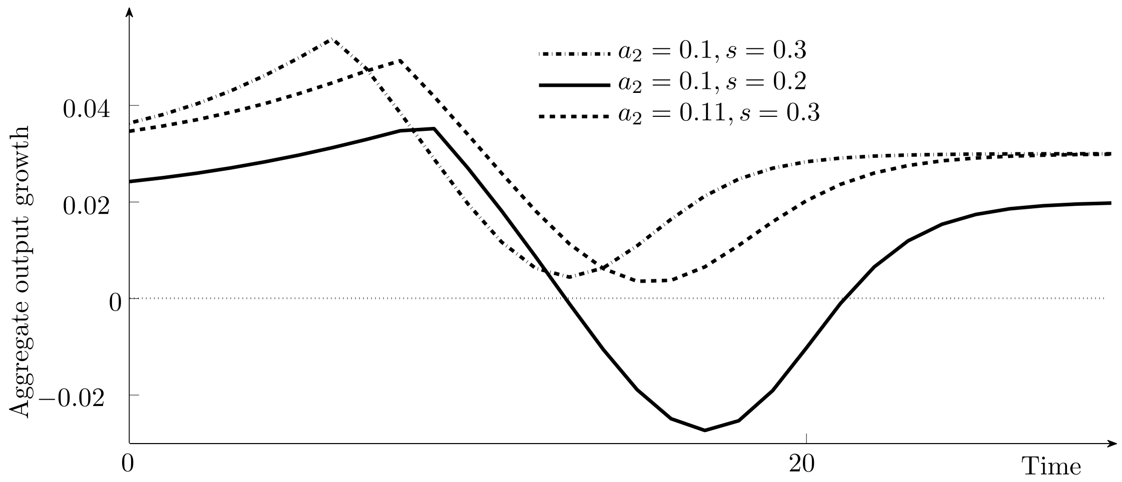

Figure 8: Interference of the technology and the investment effect for $(a_1, l_1) = (0.2, 0.5)$ and $r = 0.1$

output. The short term effect relates to the extent of the output slump after initially accelerated growth. A comparison of the first two examples given in Figure 8 with same capital input $a_2 = 0.1$ and different investment propensities s shows that a lower s implies a less pronounced upswing and a deeper downturn: The differences between the maximum growth rate and the minimum growth rate are 0.049 for $s = 0.3$ and 0.063 for $s = 0.2$ respectively.

The long term effect is the deviation of the output path from the *business-as-usual* (BAU) scenario, the hypothetical output path without diffusion taking place. This deviation is calculated as follows: For initial output x_0 and propensity to save s, BAU output at time T is given by $\tilde{x}_T = (1 + rs)^T x_0$. The relative deviation of the diffusion output from the BAU output in the long run is calculated as follows:

$$\Delta_s = \lim_{T\to\infty} \sqrt[T]{\prod_{t=1}^{T} \frac{1 + g_t}{1 + rs}} - 1 \tag{14}$$

This product series provides an assessment of the long term impact of uneven growth caused by diffusion. For the first two examples of Figure 8 with $a_2 = 0.2$, one gets $\Delta_{0.3} \approx -0,033$ and $\Delta_{0.2} \approx -0,232$.[2] Thus for $s = 0.3$ ($s = 0.2$) long-term output is 3.3% (23.2%) smaller than BAU output as shown in Figure 9. The analysis and numerical examples lead to the following observations: First, although technical change does not change the long run growth rate $g_{LPP} = sr$, short-term fluctuations due to diffusion in general have long-run implications on the level of output. Secondly, although an innovation may boost growth by accumulation via the technology effect, due to the investment effect the economy might end up with a lower output level compared to the business-as-usual scenario without diffusion.

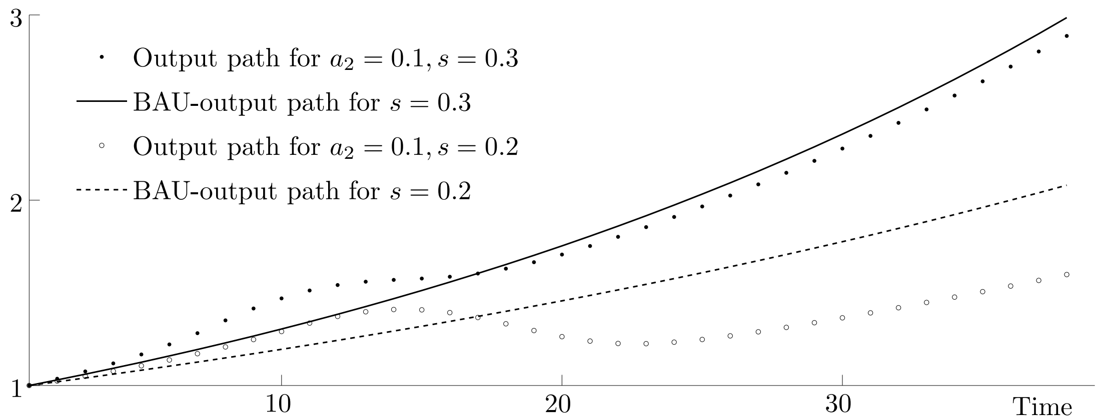

Figure 9: Long-term effects on the output level for $r = 0.1$

Summing up, the study of disequilibrium growth reveals that *creative destruction* (Schumpeter, 1954, chapter VII), the replacement of old and inferior methods of production by new and superior ones and its consequences, manifests itself in different ways. In our model three intensities of creative destruction can be distinguished: (1) Asymptotic diffusion together with *relative decline* of firms using the inferior method yields a rather smooth growth path determined by technology only. Because the profit rate of the inferior methods remains positive, firms using it are not forced to exit but they co-exist with innovative firms even in the long run. (2) Asymptotic diffusion together with *absolute decline* of firms using the inferior method changes the growth regime as soon as profitability of the incumbent

[2]The approximation arises from the truncation of power series in (14) at $T = 38$.

production process turns negative. For some kinds of technical change aggregate growth follows a wave-like path. As firms which use the inferior method gradually decline, in the long run only firms using the innovative process survive. (3) Diffusion in finite time due to *firms going bust*, which is the strongest evidence of creative destruction in out setting, is the third possible case.

4.3 Income distribution

In this section we explore the change of the income distribution due to the diffusion of a process innovation. To this end, the wage share ω is defined as $\omega = W/(W+P)$ with W denoting total wage payments and P total profits. In the LPP with method of production (a_i, l_i) being used, the wage share is given by

$$\omega_i = \frac{w_i l_i x}{w_i l_i x + r a_i x} = 1 - \frac{r}{R_i}. \tag{15}$$

with $w_i = [1-(1+r)a_i]/l_i$ denoting the wage rate process i can pay given the rate of normal profits r. $R_i = (1-a_i)/a_i$ is the maximum rate of profit of process i.

From equation (15) it follows that technical change influences income distribution, since the real wage rate rises due to the diffusion of a new method of production. A comparison of the wage share ω_1 before the innovative process enters with the wage share ω_2 after diffusion is complete shows the following:

1. If the innovation is capital using ($\Theta_a > 0$), $R_2 < R_1$ and the wage share falls: $\omega_2 < \omega_1$.

2. If the innovation is pure labor saving ($\Theta_a = 0$), $R_2 = R_1$ and the wage share does not change: $\omega_2 = \omega_1$.

3. If the innovation is capital saving ($\Theta_a < 0$), $R_2 > R_1$ and the wage share increases: $\omega_2 > \omega_1$.

Because the difference between the two maximum rates of profit is given by $R_2 - R_1 = -\Theta_a/a_2$, there is a symmetry between the technology effect on growth and the change in income distribution: Pure labor saving technical change neither affects aggregate growth nor income distribution, whereas capital using technical change dampens aggregate growth and reduces the wage share. All other forms of technical change increase both aggregate growth and the wage share. But, whereas the technology effect is related to the average rate of extra profits, the effect on income distribution arises from the change in the maximum rate of profit alone. Even in disequilibrium, income distribution is not influenced by the dynamics of extra profits but evolves according to

$$\omega_t = 1 - \frac{r}{R_t}$$

with $R_t = (1 - \bar{a}_t)/\bar{a}_t$. The income distribution in disequilibrium therefore only depends on the exogenously given normal rate of profit and on the change in the normal conditions of production due to the diffusion. Even if the average *rate* of extra profit is non-zero, equation (13), which is equivalent to

$$\rho_{1,t} a_1 x_{1,t} + \rho_{2,t} a_2 x_{2,t} = 0,$$

implies that total extra profits always sum up to zero. It follows that extra profits only redistribute income within the group of capitalists but that they do not have any direct effect on the wage share. Indirectly, extra profits act on ω_t via its impact on q_t. This result is a consequence of the wage setting rule given by equation (4).

4.4 Industry structure

In this section we explore a combination of differential accumulation and imitation to evaluate the change of the industry structure as a consequence of the diffusion process. In the diffusion-by-growth model of Section 3.1, firms using different methods of production experience different growth histories. More precisely, two growth paths exist, one for the group of innovators and one for the group of non-innovators. Non-imitating firms gradually go out of business, and only innovating firms survive. Abstracting from the entry of new firms, the market structure in the new LPP depends on how many firms have innovated at the beginning. In the pure imitation model there is no growth. Firm size is taken to remain constant in order to isolate the effect of imitation on the diffusion process (see Section 3.2).

If diffusion is the outcome of both investment and adoption decisions, each firm k at time t is in a state $f_t(k) \in \{1,2\}$ producing output x_t^k by means of process $i \in 1,2$. To calculate x_{t+1}^k according to the respective imitation and investment behavior, the case of changing capital demand for unit production has to be taken into account. For some firm k, equation (6) is replaced by

$$\frac{x_{t+1}^k - x_t^k}{x_t^k} = \frac{a_{f_t(k)}}{a_{f_{t+1}(k)}} \cdot \begin{cases} s\left(r + \rho_{f_t(k),t}\right) & \text{in Case 1: } r + \rho_{f_t(k),t} > 0 \\ r + \rho_{f_t(k),t} & \text{in Case 2: } -1 < r + \rho_{f_t(k),t} \leq 0 \\ -1 & \text{in Case 3: } r + \rho_{f_t(k),t} \leq -1. \end{cases}$$

Hence, firm output changes from period t to $t+1$ due to accumulation and is rescaled due to a change of the method of production if $\Theta_a \neq 0$. Irrespective of this rescaling effect, adding imitative behavior to the growth model speeds up the diffusion process. But, if firms both grow and imitate, the output path of the single firm and its long-run market share depend on its timing of imitation and on how much of the growth potential is left, which in turn depends on the investment and adoption decisions of all firms. A hint on the micro growth dynamics and

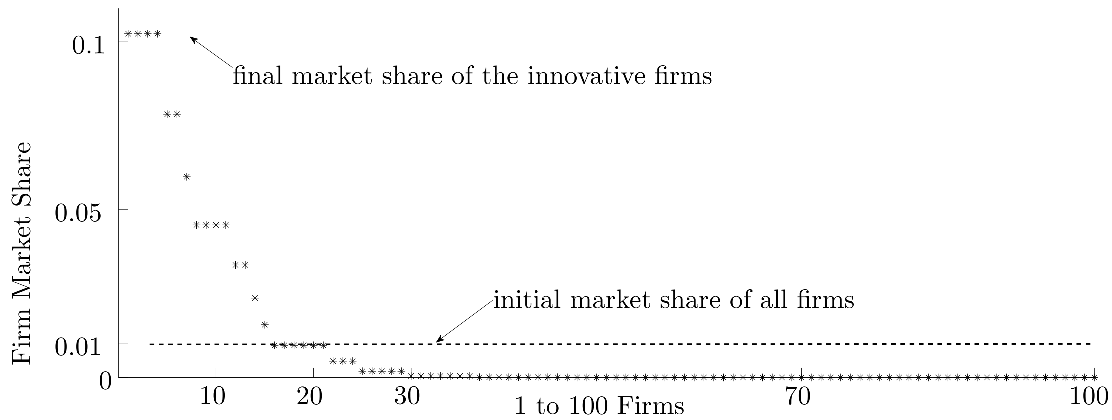

Figure 10: Firm size distribution in the new LPP for $s = 0.4$, $\lambda = 0.5$, $(a_1, l_1) = (0.2, 0.5)$ and $(a_2, l_2) = (0.2, 0.4)$. The initial firm market share equals 0.01

the resulting industry structure is given for the following specification illustrated in Figure 10. Starting with 100 firms of equal size, in the new LPP the four initial innovators control about 40 % of the market. As a result some dimension of heterogeneity among firms persists in the long run.

5 Conclusions

Building on Kurz (2008) and Steedman & Metcalfe (2011) the paper explores the question of how a long-period position is established when multiple methods of production are simultaneously in use within a simple classical model. Whereas the LPP is characterized by uniformity of technology and profitability, in disequilibrium different methods of production co-exist and profit rates of firms differ. Our model is based on the concept of extra profits. This measure relates activated methods of production to the average method of production, with the latter reflecting the normal condition of production.

The dynamics of the model depends on the change in the prevailing significance of actual methods in use. Firm decisions on investment and on technology provide the basis for two mechanisms driving the diffusion process. On the one hand, differential growth relies on autonomous investment decisions of firms. Because the profitability of inferior methods of production might turn negative, an asymmetry between firm growth and firm decline arises. Whereas growth is the outcome of a purposeful firm decision, decline is a consequence of producing below the normal conditions of production. Thus, one can speak of a potential to grow, but not of a potential to shrink. Imitation on the other hand relies on direct interaction between firms. To explore the relation between differential growth and imitation, a simple imitation mechanism has been proposed relying on epidemics and on a comparison of profitability determining the probability to adopt. Both mechanisms are based on the concept of 'extra profits' and account for the stylized fact of sigmoid diffusion curves. Because of the assumed wage adjustment mechanism, for both mechanisms it holds that the more widespread the innovation the less profitable it is. This non-intended effect of individual profit-seeking behavior is at the heart of the convergence argument.

We examine uneven growth patterns, which arise due to the interference of technology and investment effects. The former relates to the kind of technical change, the latter is due to the degree of technical change, which has an effect on firm investment. The path towards the new LPP is studied for different kinds of technical change. Furthermore, the degree of technical change influences whether the innovation takes over the market asymptotically or in finite time, and it determines whether firms using the incumbent process stay in the market, implying different intensities of Schumpeterian *creative destruction*. Finally, uneven growth results in ambiguous results in long-term changes of the output level, revealing the permanent effect of technical change on economic performance.

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