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The legacy of Veblen and Schumpeter in interpreting mathematical aspects of economic diffusion processes

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Abstract

The history of economic thought as well as contemporary economic theorizing witness several attempts to explain the diffusion of innovations within an economic system. This article surveys some prominent and important strands of economic and mathematical literature dealing with this topic. Veblen and Schumpeter serve as examples for verbal theorizing, whereas the theory of Markov chains and evolutionary game theory are the complementary formal approaches. The focus is on working out the similarities between institutionalism (Veblen) and mathematical theories of decision making in economic systems on the one hand, and between Schumpeter's theory of innovations and evolutionary game theoretic concepts on the other hand.

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The history of economic thought as well as contemporary economic theorizing witness several attempts to explain the diffusion of innovations within an economic system. This article surveys some prominent and important strands of economic and mathematical literature dealing with this topic. Veblen and Schumpeter serve as examples for verbal theorizing, whereas the theory of Markov chains and evolutionary game theory are the complementary formal approaches. The focus is on working out the similarities between institutionalism (Veblen) and mathematical theories of decision making in economic systems on the one hand, and between Schumpeter's theory of innovations and evolutionary game theoretic concepts on the other hand.

Keywords: diffusion research, Schumpeter, Veblen, Markov chains, evolutionary game theory

1 Introduction

Technological progress is one of the driving forces of economic growth, notably since the first and second industrial revolution in the 18th and 19th century (Landes 2003; Mokyr 1990). This ongoing change within economic systems both on the local as well as on the global (world) scale poses chances and difficulties for economic agents, who

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are time and again confronted with decisions that influence their present and future (or: the present value of their) utility. One important catalyst to promote or hinder technological progress is the design of institutions, which are either capable of reducing the just mentioned risk or of fixing inherited behavior as a response to the unpredictable, complex economic environment. Institutions in this context are meant in a very broad sense (Hodgson G.M. 1998; North 1990), and the respective design might be deliberately (in case of *formal* institution) or unconsciously (in case of *informal* institutions). Hence, the literature on institutional economics–beginning with Thorstein Veblen (1857–1929; see Veblen 1924)–is of interest and exemplified in this article as a support for proper theoretical modeling in economics. The appropriate mathematical toolkit to deal with decisions under uncertainty in the presence of technological change is the theory of *Markov chains*. This article aims at combining two complementary strands of economic reasoning. On the one hand, aspects of Veblen's thinking are outlined to make it accessible to contemporary economic theory. This is of importance since formal economic theories to some extent lack the interpretative strength and meaningful applicability to real-world problems, as mathematics is a structural science not concerned with reality at first sight. A proper understanding of the underlying economic features as well as of the mathematical formalism applicable to some specific problem–which is the theory of diffusion processes in economic systems in this paper–are necessary to cope with the complex problems of economic systems.

There is another aspect concerning the diffusion of innovations, which is not related to decision processes, but to economic growth. In case of product innovations, different technologies enable firms to gain different levels of extra profits due to diverging unit costs of production. These profits can be invested into growth of the respective firm, leading to unequal growth of firms depending on which technology they apply. At the heart of this concept is the idea of *creative destruction* (Schumpeter 1912) by Joseph A. Schumpeter (1883–1950), which is surveyed in this article to provide an idea of the heuristics leading to the formal framework of evolutionary economics (Nelson and Winter 1982) and evolutionary game theory (Weibull 1997). Similar to the complementary account of Veblen and Markov chains in case of decision under uncertainty, growth issues in this context are also presented in a complementary approach by surveying Schumpeter's *theory of innovations* and suggesting an appropriate theoretical framework in terms of the *replicator dynamics* of evolutionary game theory.

Summarizing, the theory of economic diffusion processes is presented in a twin representation of verbal (Veblen and Schumpeter) and formal (Markov chains and replicator dynamics), distinguishing between aggregate diffusion patterns arising on the one hand from individual decision making under uncertainty, and on the other hand on different growth rates. From a formal perspective, firstly the state space of the system can be viewed as finite (or at least countable) in case of decision processes of single firms. This leads to the concept of Markov chains. And secondly, in case of replicator dynamics the state space is continuous.

This paper proceeds as follows. To get a basic idea of how technological progress evolves over time within an economic system, in section 2 the findings of *diffusion research* (Rogers 2003) are outlined and complemented by contemporary macroeconomic approaches. Subsequently, sections 3 and 4 study microeconomics and mesoeconomic foundations of economic diffusion processes respectively as mentioned above. Finally, section 5 concludes.

2 Diffusion of innovations

Preparing grounds for the main part of this article, namely the micro-and mesoeconomic foundation of diffusion processes of process innovations (sections 3 and 4), macroeconomic findings of, as well as econometric approaches towards, diffusion research are surveyed in this section. After the introduction of the formal modeling framework in subsection 2.1, the main results of the field of diffusion research are reported in subsection 2.2. Then, in subsection 2.3 the econometric benchmark-model (the *Bass-model*) is introduced in brief.

2.1 Basic framework

The model economy to be kept in mind throughout this paper is the following. A finite (or at least countable) number N of firms exist, which at time t have two different technologies $i = 1, 2$ at their disposal to produce some homogeneous good. Technologies are characterized by their respective unit costs, which under the prevailing price system imply certain extra profits ρ^i $(i = 1, 2)$. Formally, in the presence of some normal rate of profit r and a prevailing wage rate w, extra profits are implicitly determined by

$$(1 + r + \rho^i(t))a^i + w(t)l^i = 1, \tag{1}$$

if a^i is the material input and l^i the labor input for producing one unit of output used for process $i = 1, 2$ (Kurz and Salvadori 1995). Extra profits are losses, if negative. The state space $\mathcal{S}$ of the economy is given by $\mathcal{S} = \{n/N\}_{n=1,\dots,N} \subset \mathbb{Q} \cap [0,1]$, if $x \in \mathcal{S}$ is the share of firms applying process $i = 2$. Alternatively, $\mathcal{S} = [0,1]$, if $x \in \mathcal{S}$ denotes the share of output produced by technology $i = 2$, or if a continuum of infinitesimally small firms is considered. The time series $\{x(t)\}_{t \geq 0}$ then characterizes the diffusion pattern of the innovative process $i = 2$ within the system, if it enters the system at time $t = 0$ to compete with the incumbent technology $i = 1$.

2.2 Diffusion Research

If at time $t = 0$ of the economy a process innovation (a^2, l^2) emerges, not all firms will immediately adopt this new process, even if it is unambiguously advantageous. For the aggregate outcome, this is similar to the observation that in the presence of technological progress several technologies are operated parallel to each other, as many firms stick to the incumbent technology (a^1, l^1). *Diffusion research* (Rogers 2003) in this context deals with the question how the diffusion of an innovation through the economic system looks like as time goes by. In our formal setting, diffusion research investigates the time-path (and its underlying forces) of the share $x(t)$ of the innovation, defining an *aggregate technology*

$$\begin{aligned} \bar{a}(t) &= (1 - x(t))a^1 + x(t)a^2 \\ \bar{l}(t) &= (1 - x(t))l^1 + x(t)l^2. \end{aligned} \tag{2}$$

From (2), and because commodity prices are used as numéraire in (1), real wages $w(t)$ are implicitly determined by

$$(1 + r)\bar{a}(t) + w(t)\bar{l} = 1, \tag{3}$$

if prices are driven by the unit production costs of the aggregate technology $(\bar{a}, \bar{l})$.

On the level of the single firm, the following time-line of an *innovation-developement process* and of an *innovation-decision process* can be constructed (Rogers 2003): At first, only the old technology exists. In Schumpeterian terms (Schumpeter 1912), this is the *circular flow* or *steady state*. Then, basic as well as applied research (also in form of R&D activities) is conducted, or new inventions are forged by chance. New products or new processes may be looked for intentionally or they are found as an unintended by-product of some research. An invention only becomes an innovation, if it is put into practice by members of the economic system. Rogers (2003: ch. 4) identifies six stages of the innovation-development process, namely basic research, applied research, development (to "*meet the needs of ... potential adopters*", Rogers 2003: p. 146), commercialization, diffusion and consequences. The whole process is characterized by fundamental uncertainty about the outcome of the respective activities, an observation which led Schumpeter to his characterization of the entrepreneur as someone not guided by purely rational behavior. Nevertheless there is some rationality behind the agents which are the driving force behind the innovation-development process. On the one hand, a need is identified and therefore an invention is required. On the other hand, the development of new products or better production processes increases the chance of some successful firm to stay in the market, also in the long run. Thus, the extra profits gained by some firm can be used for further research and development then even extending its lead.

As a new invention enters the system and becomes an innovation, one can ask for the diffusion of information about the new innovation. What is called awareness-knowledge by Rogers (2003) diffuses through the system by means of different communication channels, the most important being mass media on the one hand and interpersonal communication on the other hand. Knowledge is a precondition of later adoption of some new technology, hence the communication infrastructure and cultural habits concerning the use of mass media as well as interpersonal communication networks influence the knowledge diffusion process. But awareness-knowledge is not enough to observe adoption of the new technology, even if it objectively (for reasons of profit, for example) would be advantageous. This observation is related to the question of formal and informal institutions as discussed in the introduction, and points at institutionalism and Veblen's *habits of thought* (Veblen, 1924) as discussed in section 3.1.

With respect to the diffusion of information, at least two aspects are of importance for the present article, addressing the homogeneity of the system. First, there is a bias concerning the information which is accepted (for example, if some new technology deviates too much from established habits). It is also common practice to be part of communication networks with members who are alike, with similar knowledge and similar interests. This is what Rogers (2003: p. 306) explains by *homophily*, defined as the similarity of two communication partners, hindering diffusion. *Heterophily* on the other side, defined as the difference between two persons concerning certain communication-related attributes such as competence, status, and language or beliefs, would encourage the spread of new information, but is only seldom found within communication networks. Nevertheless these links exist, and they are a driving force concerning the spread of information as indicated by Granovetter's (1973) *strength-of-weak-ties* theory (see also Rogers 2003: ch. 8).

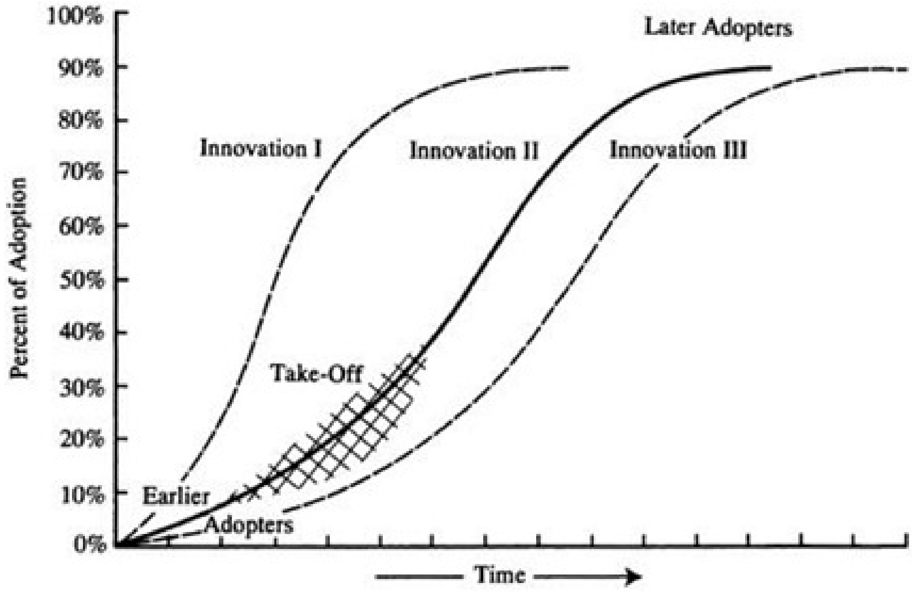

Figure 1: Successful diffusion process (Source: Rogers 2003: fig. 5.2)

Secondly, the uncertainty concerning the success of the innovation has to be considered. There are different ways to cope with this situation. Schumpeter (1912) for instance introduced the *entrepreneur* as an outstanding personality opposed to the hedonic agent who is highly risk-averse (see section 4.1). Rogers (2003), on the other hand, defines five different adopter categories which are outlined below, recognizing the influence of both personal characteristics of the adopters as well as the structure of the economic systems.

Historically, the paper of Ryan and Gross (1943) on the diffusion of hybrid seed corn in Iowa is credited to be the point of departure of modern diffusion research (even if preliminary efforts existed, see Rogers 2003: ch. 2). Ryan and Gross (1943) revealed the typically S-shaped diffusion pattern of successful innovations, as depicted in figure 1. The beginning and the end of the diffusion processes can be regarded as points of rest in the respective economic (sub-) system, comparable with Schumpeter's circular flow (Schumpeter 1912). As an invention emerges, a new dynamic can be observed. First preconditions of a diffusion process to take place are awareness-knowledge and the availability of the respective input factors. Schumpeter then would tell his story of the entrepreneur, who is a risk taker looking for new opportunities to produce corn more effectively. Assuming farmers to be profit-maximizers, a new technology always poses some risk as the full capacity of the innovation is not yet clear-cut for the first adopters. Thus, these decision makers who are the first to apply a new technology (the hybrid seed corn) cannot be sure about the personal and social consequences.

Diffusion research explains the consecutive diffusion by identifying five adopter categories. Following the innovator, Rogers (2003) introduces the early adopter, the early majority, the late majority and the laggards as depicted in the diagram of figure 2. Regarding them as evenly distributed on a continuous scale of innovativeness, he introduces a model of heterogeneous agents. To each adopter category, special personal characteristics are attributed: Innovators, for instance, are venturesome, and additionally not too much rooted in the respective social system; hence, they are free to deviate from traditional thinking and norms. An important part in course of the diffusion process is incurred by the early adopters, who are identified to be the most respected persons within the community. They have on the one hand the capability (both financially and intellectually) to adopt the innovation in an early state and similarly have the authority to convince others of the profitability of the technology; they are *opinion leaders* of the social system and the pace-makers of the diffusion process, inducing the take-off indicated in figure 1. Then, the early majority includes all further adopters up to fifty percent of the total system, characterized by a deliberate decision for the utilization of the new technology as they are able to judge the outcomes of the innovators and early adopters. Next, the late majority is to some extent forced to adopt, as now more than half of the community already uses the innovation. Finally, laggards as the fifth adopter category are on the lower end of the social ladder, either for financial or for cultural reasons delaying the adoption of the innovation.

As just mentioned, not only personal characteristics count but also economic reasons are of importance. The adoption of an innovation is connected to money in two respects. Firstly, there is uncertainty about realized (extra) profits. The risk of an investment into a new technology implies the necessity of some reserve funds of the innovator, which is not necessary for later adopters, who already know from experience of the early adopters that the new technology is more profitable. Additionally, a new technology is often accompanied by huge investments for example in new machinery, which is easier to organize by rich producers. This coincides with Schumpeter's (1912) reference to the financial sector as an important player in the presence of technological progress, as the implementation of innovations demands investments which can (if the innovation is successful) be repaid by means of the extra profits earned by the innovator, who enjoys a kind of monopoly as long as no further adopters exist. These monopoly profits are the

Figure 2: Adopter categories

Figure 3: Stages of the innovation-decision process

driving force of the diffusion model as developed in section 4.2.

Following Rogers (2003: ch. 7) one can argue that an originally homogeneous group by chance gets diversified into different adopter categories, as the first one to succeed by introducing a more profitable technology has an advantage (both financially as well as by means of self-consciousness) if the next innovation comes along. The late majority for example is to some extent forced by economic reasons to wait until the technology is available at cheaper costs. This can be a consequence of micro-inventions or due to growing tacit knowledge (Mokyr 1990). Also the uncertainty concerning the possible profits decreases as time goes by, hence a better risk assessment is possible. Each adopter is to some extent caught in his category by this feedback effect.

Summarizing, each adopter perambulates a five-step process as depicted in the diagram of figure 3. The implementation stage can be delayed for example by credit restrictions. As Schumpeter (1912) pointed out, the entrepreneur needs a credit to finance the investment into innovations. Profits therefore cause profits (for a discussion see e.g. Kurz 2012). In the presence of restricted creditworthiness, this leads to a delay of investments and therefore to different innovation-decision periods of possible adopters, even if they are persuaded at the same time that a possible adoption is beneficial to them.

2.3 Econometric aspects of diffusion processes

Diffusion research as outlined in subsection 2.2 is chiefly concerned with data collecting and evaluation. The S-shaped pattern as indicated by figure 1 leads to a *logistic* or *cumulative normally distributed* approach (Griliches 1950). For mathematical convenience usually the logistic approach is chosen (Mansfield 1961).

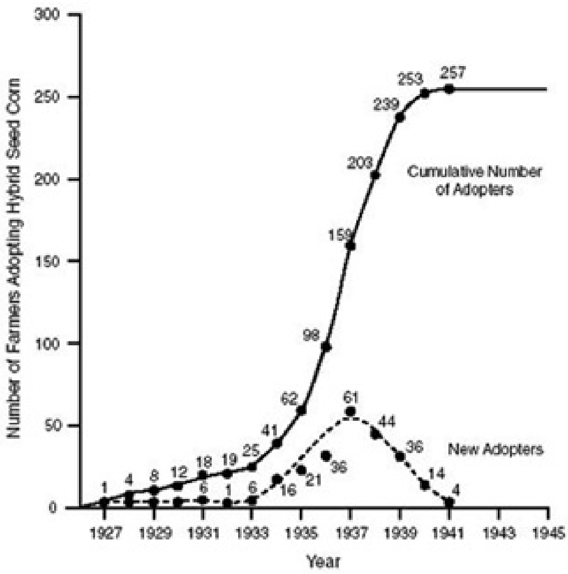

Figure 4: The diffusion process of the Hybrid Seed Corn in Iowa (Source: Ryan and Gross 1943)

The respective equation for one isolated diffusion process to be calibrated then reads

$$\dot{x}(t) = q \cdot x(t)[K - x(t)], \tag{4}$$

with share $x(t)$ of adopters at time t, coefficient of imitation q and ceiling K of the maximum share of adopters. The microeconomic rationale is one of infection, as two populations exist and as an innovator meets a non-innovator, both learn from each other and finally the better technology is adopted by both. Thus, the respective models are called *epidemic models* (Stoneman 2002: sec. 3.2); the underlying interpretation is word-of-mouth communication within a group of possible adapters. A ceiling, i.e. that not all firms adopt the superior technology, might arise for instance as a consequence of firm-heterogeneity as considered by *probit models* (Stoneman 2002: sec. 3.3). An alternative microeconomic foundation will be provided in section 4.2 by means of relative growth rates of firms.

Equation (4) contains three parameters to estimate, namely $K \in (0, 1]$, q and the share x_0 of adopters at time $t = 0$. At least one problem arose from this simple econometric analysis: diffusion data are by no means symmetrically distributed, as figure 4 shows for the Iowa Corn Study or Ryan and Gross (1943). This asymmetry asks for extensions of the symmetric logistic approach, and further effects can be included into the

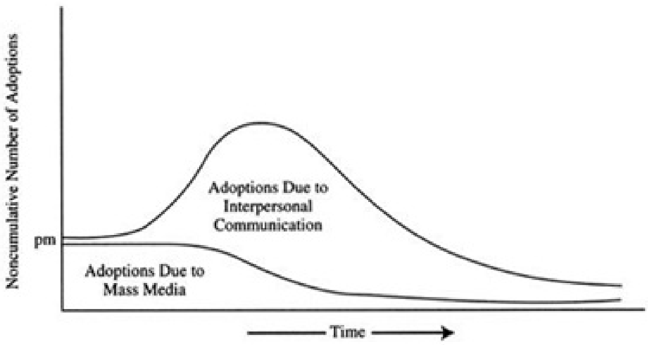

Figure 5: Sources of persuasion for adoption (Source: Rogers 2003: fig. 5-2)

analysis. In a seminal paper, Bass (1969) formalizes the separation of diffusion channels into interpersonal (as just considered) and external effects (mass media), augmenting the logistic equation (4) for $K = 1$ to

$$\dot{x}(t) = q \cdot x(t)[1 - x(t)] + r[1 - x(t)]. \tag{5}$$

Figure 5 indicates the different channels of communication at work. The second term on the right-hand-side in (5) indicates the influence of the mass media (i.e. possibly symmetric information for all agents) by means of the parameter r. This aspect will be re-investigated in the decision model developed in section 3.2. Further extensions of the Bass-model can be found for example in Mahajan et al. (1990). Peres et al. (2010) provide an updated survey of the respective literature, including models which allow for inter-market dependencies. Technological details concerning the econometric estimation of diffusion models are provided for example by Putsis and Srinivasan (2000). There, also the inclusion of stochastic elements is discussed, as advanced time series analysis incorporate stochastic elements by admitting uncertainty and indeterminacy. Overall, contemporary econometric modeling approaches (as presented in Putsis and Srinivasan 2000) are a sophistication of diffusion research of the second half of the past century as outlined in subsection 2.2.

3 Diffusion of innovations as the result of individual choices

In the presence of uncertainty concerning the outcome of some process innovation, it is not clear whether a single agent, namely the firm or producer of some commodity, will adopt the new process. In the simple setting of two processes at a firm's disposal, each economic agent faces a *binary decision process*. Decisions about adoption or non-adoption are influenced by several different parameters such as profit, trust, experience, similarity to already utilized processes, etc. Therefore not only calculable expectations count but also social and economic external effects play a role.

Preceding the explanation of Markov chains in subsection 3.2 as mathematical tool capable of dealing with the evolution of some system, based on individual decisions, institutional influences in the tradition of Veblen concerning individual decision processes are outlined in subsection 3.1.

3.1 Sluggish behavior: Veblen, habits of thought and institutions

Economics deals with the aggregate behavior of economic systems, which are a result of the individual behavior of single economic agents. Hence the question arises: Can or should macroeconomic phenomena be explained by means of the microeconomic behavior of individuals? A positive answer was prominently advocated by Max Weber (1864–1920), who claimed that social phenomena must be explained accordingly (Weber 1922: p. 1). Schumpeter coined the term *methodological individualism* for this special methodological approach, as will be discussed in section 4.1.

Veblen in his approach was not as strict as Weber and Schumpeter. In his view, economic action is teleological on the individual level, as "...men always and everywhere seek to do something." (Veblen, 1898, p. 391) But in the aggregate, economics as a science should not be teleological in its methodology, but evolutionary. Veblen defines evolution as "... a process of selective adaption of temperament and habits of thought under the stress of circumstances of associated life." (Veblen 1924: ch. 9) Non-evolutionary, or pre-Darwinian, scientific forms of economics are defined as an approach emphasizing the teleological perspective, as development processes tend to move towards some equilibrium. Veblen's understanding of an evolutionary process is characterized by continuous endogenous change and adaption of the underlying economic system. This can be explained as an uneven dynamic path of economic development, whereby change in every moment causes new change to occur. The system will never come to rest in equilibrium, as new change is immediately driven by old change. With Veblen, the evolution of an economic system is interpreted as a causal sequence. An economic transformation process is an unfolding sequence of consecutive change (Veblen 1919: ch. 2). Further details concerning Veblen's understanding of the meaning of evolutionary processes can be found in his book "The Theory of the Leisure Class: An Economic Study of Institutions" (Veblen 1924: ch. 8). There he describes an evolutionary process as a perpetual and continuous process with change occurring in the system, thereby again producing ongoing change: "When a step in development has been taken, this step itself constitutes a change of situation which requires a new adaption; it becomes the point of departure for a new step in the adjustment..." (Veblen 1924: p. 93). In this context, *process* refers to the dynamics, whereby divergence of the former status-quo appears. Veblen's approach to the explanation of an evolutionary process is characterized by the use of specific terms which originate from evolutionary biology: Darwin and the evolution of biological systems serves as a role model for his modeling approaches of economic systems: "The life of man in society, just like the life of other species, is a struggle for existence and therefore it is a process of selective adaption. The evolution of social structure has been a process of natural selection of institutions." (Veblen 1924: p. 92).

To outline the contributions of Veblen to the understanding of economic diffusion processes, three concepts are to be discussed: Firstly, *instincts* play a major role in his explanation of the development of an economic system; secondly, *institutions* to a large extent shape the development path of an economic system; and thirdly, this process is characterized by men's *habits of thought*. These concepts are closely connected to each other. Habits of thought make up the individual's judgment about facts and incidents, depending on the social and cultural environment (Veblen 1919). The former can be interpreted in some animistic sense, inherent to an individual, and not so much shaped by external factors as by habits of thought. Instincts also include psychological elements, which make up the character of a person and therefore might also be referred to as some kind of "natural behavior". Habits of thought are developed as a causal sequence and constitute both the motives of individual action and the reactions of an individual to any situation. Habits of thought and instincts are related to institutions, which are defined by North (1990: p. 3) as "... the humanly devised constraints that shape human interaction." Hence, institutions can be seen as all formal and informal norms which are in force in a society. North's definition approximately reflects Veblen's notion of institutions: an institution evolves over the course of time out of the habits of thought, the interactions of humans in a society and the environmental influences. Moreover, institutions are dynamic entities, which change over time. The present institutional landscape of an economic system represents some past state of habits of thought and other former characteristic elements. Additionally, there are feedback effects at work between institutions, habits of thought and instincts. Institutions are linked to habits of thought and instincts such that they shape the individual behavior. The evolution of an economy in the course of time is therefore greatly formed by the institutional landscape of the system. Since institutions according to Veblen reflect some past state and "...are therefore never in full accord with the requirements of the present." (Veblen 1924: p. 93), they might hinder development of a society. Thus, certain institutional settings might be antagonistic to progress and hence to the diffusion of innovations.

In the context of all these just described dynamic forces, another prominent perception of Veblen, namely *workmanship*, is of importance. This term, focusing on economic aspects of evolution, corresponds to the collective seeking and will for promoting wealth in pursuing certain objectives. It is reflected in aggregate behavior, mirrored at the lowest level in a single individual's action. Strongly related to Veblen's recognition of *workmanship* is the occurrence of technological change. Hence, workmanship results in the application of new knowledge in differential ways, and in developing mechanisms of how to allocate and transform resources to a meaningful use. *Meaningful* in this context is understood as supporting collective will and, hence, progress. Workmanship therefore leads to the development of different techniques and induces technological change.

As a consequence, at each point in time there is a certain *scheme of technology* (Veblen 1918: p. 39) established in a society, which corresponds to the application of useful knowledge ('useful' must be understood as supporting men's pursuit of objectives) evolving out of collective actions and habits and being governed by institutions. Veblen emphasizes that the scheme of technology in a society experiences change over time (which is captured in present days as technological change) in that new elements might be added to this scheme of technology (through innovations). In contrast to that, other elements of this scheme of technology get affluent and this can be expressed in Schumpeter's words as *creative destruction*. Additionally, the extent of change of this scheme of technology is contingent upon some specific members of the leisure class, namely the *captains of industry* (Veblen 1924: p. 177). These individuals indirectly promote the creation of new knowledge and technological progress. The standard of living these men adopt–which is a rather outstanding one, compared to the rest of society–serves as benchmark for individuals belonging not to the leisure class, and this is then reflected in a reformulation of their objective end and results in changing behavior. Likewise, this procedure leads to an adaption in workmanship, which again reinforces changes of the scheme of technology. To sum up, the described forces are the conditions for ongoing progress and this is aptly described by Veblen as follows:

> It is only as an outcome of this discipline that comes with the routine of

group life, and by help of the commonplace knowledge diffused through the community, that any of its members are enabled to make any new move that may in this way be traceable to their individual initiative. Any new technological departure necessarily takes its rise in the workmanlike endeavours of given individuals, but it can do so only by force of their familiarity with the body of knowledge which the group already has in hand. (Veblen 1918: p. 104)

3.2 Markov chains and economic diffusion

On the individual level, decisions about innovative or established behavior shape the aggregate outcome of the economy. Formally, in case of discrete time, for time $t < 0$ the system is in state $x(t) = 0$ as the innovation emerges at time $t = 0$. As stated in the introduction, two technologies $i = 1, 2$ are at the firm's disposal to produce some specific commodity. Uncertainty about the real input factors, about demand or about prevailing prices indicate a stochastic approach. Given $w(t) = [1 - (1 + r)\bar{a}(t)] / \bar{l}(t)$ from (3) and expected extra profits

$$\rho^i(t) = \frac{1}{a^i}\left(1 - a^i - w(t)l^i\right)$$

from (1), the difference $\Delta\rho(x(t)) = \rho^2(x(t)) - \rho^1(x(t))$ can approximately be assumed to be normally distributed with mean value $\Delta\rho(x(t))$ and variance $\sigma(x)$ (Aoki and Yoshikawa 2007: ch. 3). This introduces heterogeneous firms, which differ with respect to their beliefs and decisions. State $x(t)$ of the economy, defined as the share of firms utilizing technology $i = 2$, is some externality, indicating that the state of the system influences the decisions of the agents both by hard facts, since prices change with x (indicated by changing real wages $w(t)$), or by soft facts, since trust in the innovation may grow as more colleagues utilize it. This dependence also includes the idea that normative behavior plays a role, which is strongly bounded to inherited strategies and informal institutional settings as discussed in case of Veblen's *habits of thought* (as outlined in subsection 3.1).

From $\Delta\rho(x(t)) \sim \mathcal{N}[\Delta\rho(x(t)), \sigma(x)]$, the probability p that some firm switches from technology $i = 1$ to $i = 2$ can be derived as

$$p(x(t)) = \mathbb{P}[\Delta\rho(x(t)) \geq 0] = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\Delta\rho(x(t))}{\sqrt{2}\sigma(x(t))}\right)\right], \tag{6}$$

with

$$\text{erf}(y) \equiv \frac{2}{\sqrt{\pi}}\int_0^y e^{-z^2}dz$$

denoting the *error function*. As a result, for each pair $(i, j) \in \mathcal{S} \times \mathcal{S}$ some transition probability $P_{ij} \equiv \mathbb{P}(X_t = j | X_s = i)$ can be given, which is the entry of the i-th row and j-th column of the *transition matrix* P. For convenience of reading and by some abuse of mathematical notation, the first row/column of P has index zero. P is a *stochastic matrix*, as its entries are non-negative and the row sum equals one.

Remark 1 *The sequence* $\{X_n\}$ *of random variables is a* $(\mathbf{e}_1, P)$-Markov chain. *This is a consequence of the assumption that decisions are only influenced by the present state* x *of the system, and therefore the* Markov property

$$\mathbb{P}[X_{t+1} = x(t+1) | X_t = x(t)] = \mathbb{P}[X_{t+1} = x(t+1) | X_t = x(t), \ldots, X_0 = x(0)]$$

with the initial state $x(0) = 0$ *holds. As a consequence, for* $n \in \mathbb{N}_+$ *the* Chapman-Kolmogorov-Equation

$$\mathbb{P}[X_{t+s} = x(t+n) | X_t = x(t)] = P^s, \tag{7}$$

holds true fro all $s \geq 0$. *Considering the initial state* $\mathbb{P}[X_0 = 0] = 1$, *one can calculate the probability distribution at time* $t \geq 0$ *to*

$$\mathbb{P}[X_t = i] = \left[\mathbf{e}_1^T P^t\right]_i \tag{8}$$

with unit vector $\mathbf{e}_1 = (1, 0, \ldots, 0)^T \in \mathbb{R}^{N+1}$. *(Meintrupp and Schäffler 2005: ch. 9)*

P can be calculated explicitly for the case of normally distributed beliefs about the profit-difference $\Delta\rho$: (6) yields the probability that a firm switches to the new technique. Additionally, in a first approach it is reasonable to assume in this two-technique setting that no return to the old process is possible, as for example investments are necessary for each change of the production process. Consequently, P is an upper triangular matrix with $P_{NN} = 1$. Let $n(t) \leq N$ be the number of firms which already implemented the innovation at time t, therefore $x(t) = n(t)/N \in \mathcal{S}$. Hence $N - n(t)$ firms face the choice-of-technique decision at time t and stick to the old process with probability $1 - p(x)$. State $m/N \in \mathcal{S}$ with $m \geq n$ will be attained after the next time step if and only if exactly $m - n$ out of $N - n$ firms adopt the innovation. The probability that a specific combination of $m - n$ firms switches is $p(x)^{m-n}$. For combinatorial reasons, this leads to

$$P_{nm} = \begin{pmatrix} N-n \\ m-n \end{pmatrix} p(x)^{m-n} (1 - p(x))^{N-m} \text{ for all } m \in [n, N]$$

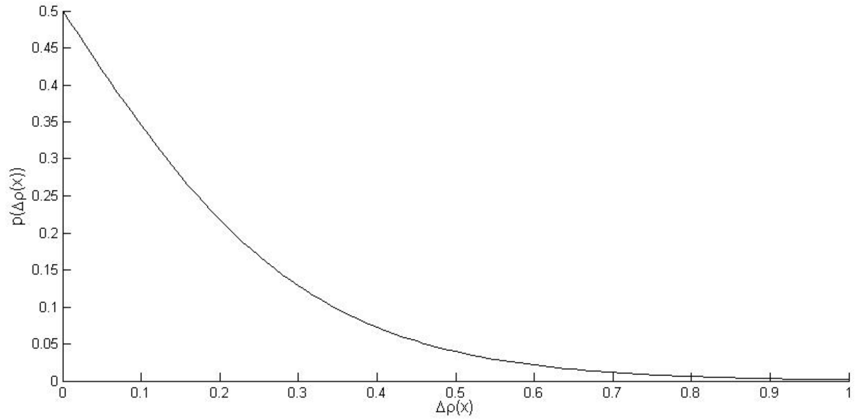

Figure 6: Boltzmann-Gibbs distribution

as exactly $m-n$ firms have to switch, but $N-m$ firms must not switch. From (8), the expected time path

$$\tilde{x}(t) \equiv \mathbb{E}[x(t)|x(0)=0] = (0, 1/N, \ldots, (N-1)/N, 1) \cdot \mathbf{e}_1^T P^t \tag{9}$$

can be calculated for all $t > 0$.

Remark 2 *$p(x)$ can be approximated by the* Boltzmann-Gibbs *distribution (Aoki and Yoshikawa 2007: ch. 3)*

$$p(x) = \frac{e^{-\sqrt{\frac{2}{\pi}}\frac{\Delta\rho(x)}{\sigma(x)}}}{e^{\sqrt{\frac{2}{\pi}}\frac{\Delta\rho(x)}{\sigma(x)}} + e^{-\sqrt{\frac{2}{\pi}}\frac{\Delta\rho(x)}{\sigma(x)}}} \text{ with} \tag{10}$$

$$\Delta\rho(t) = \left(\frac{1}{a^2} - \frac{1}{a^1}\right) - w(t)\left(\frac{l^2}{a^2} - \frac{l^1}{a^1}\right) \tag{11}$$

The assumption of transition probabilities (6) therefore imply a fast increasing diffusion process, which slows down as more firms utilize the innovative process. This can be argued by observing that the term $\Delta\rho(x(t))/\sigma(x(t))$ in (10) is a somewhat ambiguous term, as uncertainty–represented by σ–and profit differences $\Delta\rho$ are both decreasing, since wages rise as a result of increasing productivity and acknowledging equation (3). Constant variance σ can be interpreted as the absence of learning effects, whereas a decreasing function $\sigma(x)$ indicates the reduction of uncertainty as more adopters utilize the innovation. Hence p can be assumed to be roughly constant in a first approximation, leading to the constant-probability curve in figure 6 for $p = 0.5$ (i.e. $\Delta\rho = 0$; both technologies are of the same quality). There is an immediate take-off and as the number of non-adopters decreases, also the rate of adoption slows down. This effect is related to the mass-media influence in equation (5), as all firms have the same information, but different beliefs. The Boltzmann-Gibbs approach described so far is capable of simulating decision making under uncertainty, including the consequences of network effects as late adopters can learn from early adopters, which increases their probability to adopt the innovation.

Next, the S-shaped diffusion pattern, represented by the first term on the right-hand-side of equation (5), is a result of network-externalities. It is reasonable to argue that an innovation is more likely to be adopted by some firm, if it is already utilized by others, leading to a probability $p(x)$ of a non-adopter to adopt, monotonically increasing with respect to the state x of the economy. Apart from the microeconomic argumentation above, normative effects of the social network, in which the decision maker is embedded, play a key role. This is indicated by Veblen's *habits of thought* explained in subsection 3.1. Norms may hinder the acceptance of some new technique, something which can only be overcome as time goes by and more agents adopt the innovation. This situation in a first attempt can be approximated by a linear approach

$$p(x) = \underline{p} + (\bar{p} - \underline{p})x, \tag{12}$$

where $\underline{p} \in (0, 1)$ denotes some baseline-probability which is valid for the first adopter, and $\bar{p} \in (\underline{p}, 1]$ indicates the upper limit of this kind of social effect. The result is depicted in figure 7 by the state-adjusted probability curve, which reproduces the S-shaped diffusion patterns of section 2.2 for $\underline{p} = 0.01$, and $\bar{p} = 1$.

4 Evolutionary aspects of diffusion processes

The previous section modeled the stochastic behavior of agents facing a choice-of-technique problem and acting according to their expected probabilities. This was exemplified by uncertainty about outcomes (or heterogeneity of firms) in section 3.2. One important aspect were normative pressure and network externalities, which revealed that state-dependent transition probabilities were necessary to get the S-shaped diffusion pattern as depicted in figure 7. Nevertheless, these two aspects are not exclusive as the success of the *Bass model* as introduced in section 2.3 shows. There, in equation (5), the first term corresponds to the S-shaped pattern and the second term represents the influence of

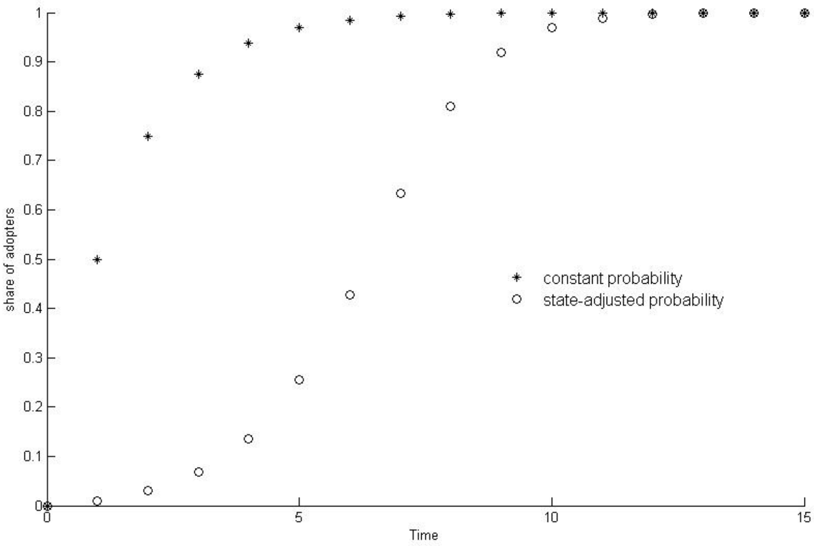

Figure 7: Markov-Chain Diffusion processes

mass media, which can be interpreted as the coice-of-technique decision process guided by the Boltzmann-Gibbs distribution (10).

The S-pattern in section 3 was explained by normative forces of the economic environment on the single agent. Hence, the epidemic interpretation given in the macroeconomic approach in section 2.3 to some extent can be brought in line as personal networks play an important role in shaping the decisions of individuals (Rogers 2003: ch. 8). Alternatively, on can assume *growth* as the driving force of the diffusion of innovations (Schumpeter 1912). In the former setting information was mobile and symmetrically distributed within the system. The growth-approach on the other hand assumes that the allocation of technologies (i.e. the set or firms which utilize the innovation) remains fixed. Information about new processes is therefore either not available for the non-innovators, or economic (e.g. necessary investments) respectively legal (e.g. patent laws) settings prevent firms from being innovative. Hence, in this case the innovative technology is utilized by one subset of all firms, and the incumbent process is hosted by the complement subset. No conscious decision is modeled, but firms grow according to their fitness. The biological term *fitness* in the present setting can be associated with extra profits, which are re-invested, leading to capital accumulation. If some technology earns negative extra profits, and therefore incurs losses, the respective share of output produced by this process diminishes. What remains to be solved in this context is the initial condition, as no invention can create growth as long as no firm utilizes it and therefore becomes an innovation. One way to overcome this problem is the combination of growth-induced and decision-induced diffusion processes. Another approach was chosen by Schumpeter (2012) by means of the notion of the entrepreneur as the one character within the economic system, who brings about technological change. To underpin the subsequent application of evolutionary game theory and replicator dynamics in subsection 4.2, an outline of Schumpeters theory of innovations is given in the following subsection 4.1.

4.1 Schumpeter's Theory of Innovations

Schumpeter‘s theory of innovations is concerned with the transition of an economy between two *circular flows*–or equilibria–of the system. In the formal setting introduced at the beginning of section 2, a circular flow can be identified with a situation, where $(\bar{a}, \bar{l}) = (a^i, l^i)$ and therefore only one technology is utilized. This provides the starting point for his analysis of the structural change of economic system from within (Schumpeter 1912). He considers equilibrium as a theoretical concept, which creates the basis for a subsequent consistent analysis of dynamic processes. A circular flow is characterized by free and (nearly) unrestricted markets, with perfect competition prevailing. His conception includes the description of the behavior of a static economic system, based on an a-priori determined construct of data and conditions. Hence, he combines empirical evidence (Schumpeter 1939) and axiomatic theorizing on verbal grounds. The actions and decisions of agents in a circular flow are based on past experiences and expected future outcomes. Schumpeter distinguishes between producers and demanders. Objective of the former is the maximization of profits, and the latter seek optimally to satisfy their wants and needs. The behavior of agents in a circular flow is of a constant and persistent character. Hence, they do not seek for change in their behavior, as they prefer to act in an already ready-made fashion. Schumpeter in this context coins the term *hedonic agents* to circumscribe this passive behavior. Producers generate output (the social product). Its magnitude is determined by the availability of resources, technological conditions and aggregate demand.

The process of production involves the combination of existing forces and inputs. Two factors of production are necessary, labor and land, whereby labor is assumed to be heterogeneous. Schumpeter abstracts from capital as a sole factor of production by arguing that capital can be subsumed under the term productive goods. Since he assumes a ranking of goods, productive goods in the end can again be attributed to the employment of land and labor. Therefore productive goods are just intermediates, which have no specific functioning in the production process of a circular flow. In this setting, Schumpeter's price theory rests on the assumption that prices result from the value of labor and land embodied in the production of a certain good. Hence, prices are determined by costs of production. Concerning distributive issues, the social product flows to workers and landowners in the form of wages and rents respectively. The level of wages and the rent of land are determined by the marginal productivity of these two factors. Since there is no use of capital in the production of goods, profits are absent in the circular flow. Producers receive a wage for providing their productive goods. Additionally, the technological and economic risk, which is related to the production of goods, is compensated in form of a risk premium serving as an incentive for producers to employ their means. Summarizing, the state of a circular flow can be described as one in which all actions of agents take place in a usual manner. Conditions of the economic system can be identified of being constant and mechanical, and the system permanently reproduces itself over the course of time. In this equilibrium state there is no room for development or dynamic change from within.

To describe the diffusion of innovations, dynamic changes within the respective system have to be considered. Schumpeter (1912) distinguishes two main categories, which allow for changes in an economic system. Firstly, *external factors* occur in a circular flow and do not allow for a development of the economy. The system reacts passively to their occurrence, and these factors are acting over and all the economic sphere. Examples are population growth, economic policies as well as weather conditions. To understand dynamic changes of an economic system, the second category, namely *internal factors*, have to be considered. They are inherent to the economic system and imply a fundamental structural change of the economy. Internal factors trigger the changes of the data, which form the basis of the economic system, leading to a change of the prevailing quantity and price system. Examples are changes in the quantity or quality of goods and resources, and also changes in taste. Changing quantities or qualities of goods and resources include the implementation of new goods, the development of new markets, institutional and/or organizational changes as well as changes in the production processes. The consideration of these internal factors allows Schumpeter to advance from static to dynamic economic analysis.

Schumpeter's theory of innovation paves the way for an understanding of diffusion processes within an economy. Three terms are of importance in this context: *Entrepreneur*, *invention* and *innovation*. The entrepreneur, as distinct from the hedonic agent, is responsible for penetrating the circular flow. Characterized by a high degree of motivation and energy, an entrepreneur looks for change, revolutionizing the system from within. Thus, emergence of structural change is strongly tied to this special type of agents. The tasks and obligations of the entrepreneur in Schumpeter's notion cannot simply be restricted to the management of a firm in a usual sense and further it is not just a conventional profession. In fact, the entrepreneur has other objectives than the hedonic agents. It is incumbent to him to boost change and lead the economy to a new future. The entrepreneur is interested in changing the data and strategies with are prevalent in a circular flow. He sets the evolutionary process of an economic system in motion, and hence pushes the economy out of its static circular flow position. Schumpeter (1912) shares the view that the entrepreneur can fund his plans either by his own financial means or by borrowing. Here, one more class of agents comes into play, namely the financier. He enables the entrepreneur to realize his ideas and at the same time takes over part of the risk of failure. In this context, the banking sector is an essential element of the development process of an economic system, which is strongly tied to the availability of credit lending.

Secondly, inventions identify the pure availability of new combinations and methods of production, whereas innovations express the successful introduction of new combinations of factors of production for commodity supply. It is the responsibility of the entrepreneur to transform an invention into an innovation. At this moment, the existence of profits is possible. The innovation allows the entrepreneur to reap extra profits, a kind of (temporary) monopoly rent used for repayment of the loans and probably also for further innovation purposes. The existence of extra profits attracts competing firms to enter the market, and as a consequence implies that the extra profit in the innovating market tends to vanish. As a further consequence of some innovation, supplementary innovations emerge. Whilst some of these subsequent innovations comprise just marginal adaption to the initial innovation, the assumption of competitive markets leads to imitation as well as to completely different goods or processes. Firms, which do not adapt to the new innovation, are crowded out of the market (*creative destruction*, see Schumpeter, 1947, p.83). If only one single innovation occurs, the system would sooner rather than later return to its static circular flow position. But as a result of the market environment, innovations additionally lead to feedback effects, encouraging further research and development activities. The resulting mechanism of sequential innovations causes ongoing development of the system and keeps the system in permanent motion, resembling Veblen's (1898) idea of an ever continuing evolutionary process.

4.2 Output growth induces diffusion patterns

Formally, the growth approach to this leads to a continuous state space as each frequency $x \in [0, 1]$ can be produced by technology $i = 2$. As in section 3, time can be assumed to be discrete or continuous. The former can be found in parts of evolutionary economic modeling (Nelson and Winter, 1982; recently Metcalfe and Steedman, 2011) A first idea of the respective dynamics can be gained by assuming that two processes yielding extra profits $\rho^i(x(t))$, which can be re-invested to induce growth of output $y^i(t)$ of the respective technology. Hence, $y^i(t+1) = (1 + \rho^i(x(t)))y^i(t)$. As $x(t) = y^2(t)/(y^1(t) + y^2(t))$, one can conclude that

$$x(t+1) = \frac{(1+\rho^2(x(t)))y^2(t)}{(1+\rho^1(t))y^1(t) + (1+\rho^2(t))y^2(t)}$$

and therefore

$$\frac{x(t+1)}{x(t)} = \frac{1+\rho^2(x(t))}{1+\bar{\rho}(x(t))}. \tag{13}$$

As long as profits are above the average profits $\bar{\rho} = (1 - x(t))\rho^1 + x(t)\rho^2$ of a sector, the share of output of the respective technology therefore grows, and vice versa profits below average naturally lead to a decline of the respective technology.

In a time-continuous setting, again the growth rate of output $y^i(t)$ of technology i is determined by the extra profits $\rho^i(x)$–depending on the share of output $x(t)$–produced by the respective technology. Formally (Weibull 1997: p. 72f),

$$\dot{y}^i(t) = \rho^i(x(t))y^i(t)$$

and therefore, as $y(t)\dot{x}(t) = \dot{y}^2 - \dot{y}(t)x(t)$ (from $y(t)x(t) = y^2(t)$), one gets

$$\dot{x}(t) = x(t)\left(\rho^2(t) - \bar{\rho}(x(t))\right) = \Delta\rho \; x(t)(1 - x(t)) \tag{14}$$

with $\Delta\rho$ defined in equation (11).

Equation (14) denotes the *replicator dynamics* for the case of two processes and again–as in equation (13) of discrete time–identifies the difference between actual and average extra profits as the driving force of the diffusion process of the innovation. According to (3),

$$w(t) = \frac{1 - (1+r)\bar{a}(t)}{\bar{l}(t)}. \tag{15}$$

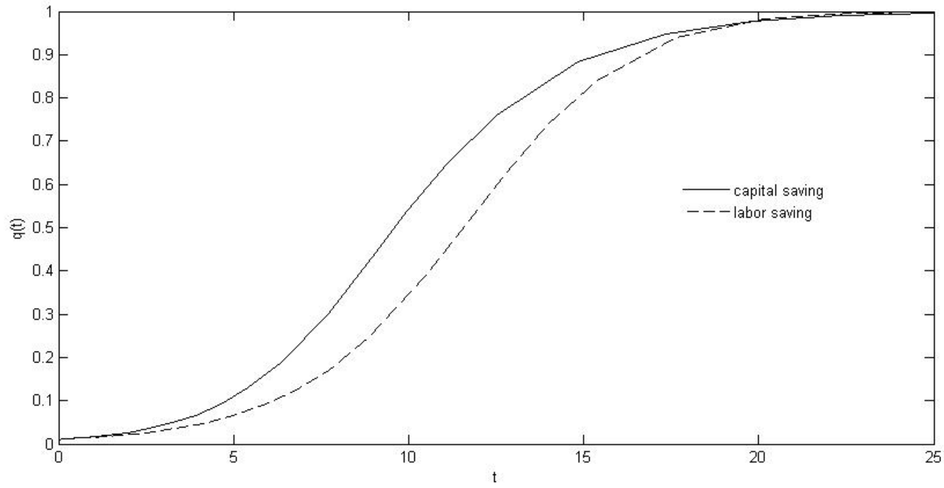

Figure 8: The diffusion of capital and labor saving technological progress

Inserting (11) into (14) leads to the first order differential equation

$$\dot{q}(t) = q(t)\,(1 - q(t)) \left[\left(\frac{1}{a^2} - \frac{1}{a^1} \right) + w(t) \left(\frac{l^1}{a^1} - \frac{l^2}{a^2} \right) \right], \tag{16}$$

which can be solved for some initial condition $q(0) = q_0 > 0$. (14) ensures that the right-hand-side of (16) is positive if and only if the innovation yields higher extra profits than the incumbent technology.

To give a first intuition of the diffusion process, the special cases of pure labor saving technological progress and of pure capital saving technological progress are calculated from (16) and is depicted in figure 8 for the special values $a^1 = 0.3$, $a^2 = 0.2$, $l^1 = 0.3$, $l^2 = 0.2$, $r = 0.01$ and $q(0) = 0.01$. The respective differential equations, obtained from (16), read

$$\dot{q}(t) = q(t)\,(1 - q(t)) \left(\frac{1}{a^2} - \frac{1}{a^1} \right) \tag{17}$$

in case of capital saving technological progress, i.e. $a^1 > a^2$ and $l^2 = l^1$, and

$$\dot{q}(t) = q(t)\,(1 - q(t))\,w(t)\frac{l^1 - l^2}{a^1} \tag{18}$$

in case of labor saving technological progress, i.e. $l^1 > l^2$ and $a^2 = a^1$.

Some straightforward conclusions concerning the speed of the diffusion process, characterized by the right-hand-side of (17) and (18), can be drawn. Firstly, technological progress always leads to a strongly monotonic diffusion pattern $\dot{q}(t) > 0$. Next, in

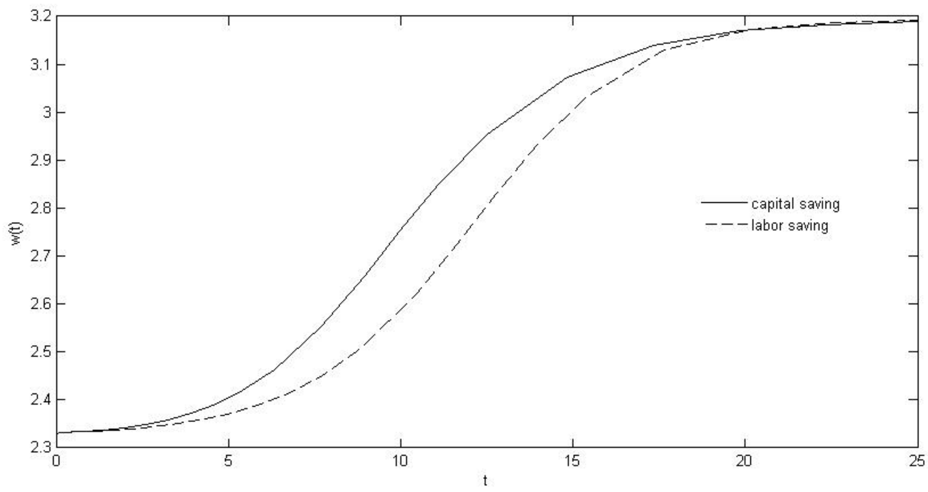

Figure 9: Wage dynamics in the presence of capital-saving and labor-saving technological progress

case of labor saving technological progress, (18) indicates that the amount of wages $w(t)\left(l^1 - l^2\right)$, determined both by real wages $w(t)$ and the amount of saved labor $l^1 - l^2$, is positively related to the speed of the diffusion process. Higher capital input a reduces the speed of diffusion, as in this case he share of wages is of less importance. Analogously, in case of capital saving technological progress, the difference $1/a^2 - 1/a^1$ of *capital productivity* determines the process of aggregate adoption.

To complete the short analysis in this article on this simplest case of diffusion within a one sector economy, the wage dynamics, as a consequence of (4.2), reads

$$w(t) = \frac{1 - (1+r)\left[(1-q(t))a^1 + q(t)a^2\right]}{(1-q(t))l^1 + q(t)l^2}.$$

and is also depicted in figure 9 for both labor and capital saving technological progress. As expected, wages increase as a consequence of higher productivity. This is true for both labor savings and capital saving technological progress.

Summarizing, the replicator dynamics approach reflects Schumpeter's concept of *creative destruction*, since the non-innovative technology yields losses and therefore shrinks over time. Also the *entrepreneur* can be found again in terms of the initial condition x_0, which is the output of the entrepreneur at time $t = 0$. *Monopoly profits* in terms of Schumpeter, which are the extra profits in the present formal setting, enable innovative firms to grow.

5 Conclusions

This article introduced selected aspects of diffusion processes within economic systems. At the beginning, the characteristics of diffusion processes were outlined, followed by two complementary approaches to interpret them. The first was based on individual choices of economic agents, and the second dealt with growth processes in the presence of technological progress.

These three parts were themselves sub-divided, as each was approached from a verbal as well as a formal perspective. The underlying idea of this article therefore is an appreciation of the multiplicity of various complementary approaches to one specific field of economic research. It connects the history of economic thought (Veblen and Schumpeter) with neo-Ricardian thinking and contemporary mathematically sophisticated economic theorizing (Markov chains, evolutionary game theory). Additionally, it is accentuated in which respect these different fields of research interact and profit from each other.

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