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# A combined Classical – Evolutionary model to explain inter- and inner-sectoral spillover effects of technical change

Andreas Rainer[1]

[1] Graz Schumpeter Centre

University of Graz

RESOWI-Zentrum, F4

A-8010 Graz, Austria

**Abstract**

Inter- and inner-sectoral effects of technical change are investigated in a multi-sector economy. The underlying modeling framework is a combination of classical economics and of evolutionary game theory. The special case of one sector is elaborated at length, leading to insights into economic and social dynamics in the presence of technical change. This includes (1) re-switching of the profitability of processes, (2) technologically induced transitional wage inequality, and (3) a changing wage share due to technical change. For the case of multiple sectors, general properties concerning stability and changing profitability are investigated: technical change in one sector is guided by technical progress in another sector. In a two-sector setting, this property of changing profitability is demonstrated for the case of one basic and one non-basic sector as well as for two basic sectors.

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Inter- and inner-sectoral effects of technical change are investigated in a multi-sector economy. The underlying modeling framework is a combination of *classical economics* and of *evolutionary game theory*. The special case of one sector is elaborated at length, leading to insights into economic and social dynamics in the presence of technical change. This includes (1) re-switching of the profitability of processes, (2) technologically induced transitional wage inequality, and (3) a changing wage share due to technical change. For the case of multiple sectors, general properties concerning stability and changing profitability are investigated: technical change in one sector is guided by technical progress in another sector. In a two-sector setting, this property of changing profitability is demonstrated for the case of one basic and one non-basic sector as well as for two basic sectors.

## 1 Introduction

Technical progress is a characteristic pattern of Western civilization of the past centuries since the onset of the industrial revolution (e.g. Mokyr, 2005). Already identified as driving force of economic growth in Adam Smith's (1723–1790) *Inquiry into the Nature and Causes of the Wealth of Nations* (1776), technical change induces specific effects which need explanation. Inter-sectoral spillover effects, transitional wage inequality, changing wage share and changing profitability are tackled in this article.

David Ricardo (1772–1823) in his book *Principles of political economy and taxation* (1821) made the *choice-of-technique* problem explicit. He also asked for the economic and social causes and effects concerning the use of different modes of production. His thinking about the *long-period position* of an economy was formalized by Piero Sraffa (1898–1983) in his book *Production of Commodities by Means of Commodities* (1960). A further formalization of his ideas can be found in Kurz & Salvadori (1995).

A complementary approach dealing with short-run effects of technical change is provided by *evolutionary economics* launched by Nelson & Winter (1982) (see Dopfer, 2005, for a recent stocktaking of this field of research). From an empirical point of view, diffusion research as surveyed by (Rogers, 2003) serves as a source of patterns to be explained in the presented model. It also acknowledges and studies the social consequences of this process, since economic dynamics triggered by technical change necessarily induce structural change within a social system. On the theoretical side, the analysis of Joseph A. Schumpeter (1883–1950) concerning the economic effects of innovations adds insights into this field of research. His focus is on the role of the *entrepreneur* introducing innovations into the economic system, and on *creative destruction* since firms applying innovations can gain extra profits and therefore accelerate their growth of market shares.

This article adds to the existing literature on economic and social causes and consequences of the diffusion of innovations by setting up a theoretical framework to study inter-sector spillover effects of the emergence of *process innovations*. It connects the concept of a *long-period position* (Kurz & Salvadori, 1995) with evolutionary ideas provided by the concept of *replicator dynamics* as utilized by *evolutionary game theory* (Weibull, 1997; Metcalfe, 1998). The stated problem is closely related to Schumpeter's (1934) dynamic approach to economic development. It merges classical and evolutionary thinking as suggested by Kurz (2008). The discrete-time one-sector model of Steedman & Metcalfe (2011) provides the intellectual starting-point for this article, whereas the presented framework introduces an extension to multiple sectors in a time-continuous setting.

The proposed model is capable of explaining economic and social effects of the diffusion of innovations. This includes the clarification of causes of technologically induced wage inequality, of a changing wage share due to technical change as well as the demonstration of inner- and inter-sectoral spillover effects inducing changing profitabilities of different production processes. Even more, declining output as a result of Schumpeter's *creative destruction* is an outcome of the proposed model. The article proceeds in three steps. Firstly, the multi-sector framework with heterogeneous labor is introduced in Section 2. Price- and quantity-determination in the tradition of Sraffa (1960) and Kurz & Salvadori (1995) is combined with evolutionary aspects provided by the replicator dynamic approach of evolutionary game theory (Weibull, 1997). Next, in Section 3 the one-sector case is studied in detail. In Section 4, general properties of the multi-sector economy are derived and exemplified by an analysis of the two-sector case. Finally, Section 5 concludes and provides an outlook towards future research.

# 2 Replicator dynamics in a multi-sector economy

An evolutionary model to study diffusion processes of technical change in a multi-sector economy is presented in two steps. Firstly, in Subsection 2.1 notational issues and a view onto the long-period position of the economy are introduced. This prepares the ground for the explicit construction of the replicator dynamics in Subsection 2.2.

## 2.1 The multi-sector economy

In an $N$ sector-economy, each sector $n \in [1, N]$ is defined by the homogeneous good it produces. Circulating and fixed capital as well as different skills of labor are non-substitutable input factors of production. Let $\bar{f}_{nm}$ denote the stock of fixed capital provided by sector $m$ necessary to produce one unit of output of good $n$. Then $\bar{a}_{nm} = \delta_m \bar{f}_{nm}$ is the amount of fixed capital $m$ which dissolves according to the rate of deterioration $\delta_m \in (0, 1]$ within one period of production ($\delta = 0$ is excluded, since this would imply infinitely durable capital goods). If sector $m$ provides circulating capital, then $\bar{a}_{nm} \in [0, 1]$ for $m, n = 1, \ldots N$ denotes the quantity of good $m$ which is used to produce one unit of good $n$. From a formal point of view circulating capital equals fixed capital with $\delta_m = 1$. The matrices $\bar{A} \in [0, 1]^{N \times N}$ with $[\bar{A}]_{nm} = \bar{a}_{nm}$ and $\bar{F} \in [0, 1]^{N \times N}$ with $[\bar{F}]_{nm} = \bar{f}_{nm} = \bar{a}_{nm}/\delta_m$ characterize the utilized technology.

The coefficients $x_n$ of the output vector $\mathbf{x} \in \mathbb{R}^N_+$ are the quantities produced in sector $n$. In case of linear technologies, hence if the production side of the economy is represented by a *Leontief* production function, the *viability* constraint

$$0 \leq \mathbf{x}^T \left( (\mathbb{I} + G)\bar{A} + G\bar{F} \right) \leq \mathbf{x}^T \tag{1}$$

has to hold. $G = \mathrm{diag}(g_1, \ldots, g_N) \in \mathbb{R}^{N \times N}$ is the diagonal matrix of sectoral growth rates $g_n = \dot{x}_n / x_n$. Defining the vector $\mathbf{y} = (y_1, \ldots, y_N)^T \in \mathbb{R}^N$ of final consumption, *market clearing* holds if

$$\mathbf{y} = \left(\mathbb{I} - (\mathbb{I} + G)\bar{A} - G\bar{F}\right)^T \mathbf{x} \geq 0.$$

Labor input is heterogeneous, differing in skill and remuneration. $K$ different skill levels exist, and each skill $k \in [1, K]$ pays some wage $w_k \geq w_{k-1}$. The overall wage level $w$ is defined by $\mathbf{w} = w\mathbf{u}$ with vector $\mathbf{u} \in \mathbb{R}_+^K$ denoting the relative wages $u_k \geq u_{k-1}$ of different skill levels. Persistent wage differentials exist due to various reasons, with scarcity on the labor market and different costs of qualification being two out of a number of causes extensively discussed in Kurz & Salvadori (1995, ch. 11.1). The coefficients $\bar{l}_{nk}$ of the unit production labor input matrix $\bar{L} \in \mathbb{R}_+^{N \times K}$ denote the amount of labor of skill $k$ necessary to produce one unit of good $n$. Demand for skill $k$-labor therefore adds up to $s_k = \sum_{n=1}^N x_n \bar{l}_{nk}$.

Acknowledging the total number $\Omega = \sum_{k=1}^K s_k$ of employed workers and defining the aggregate income of all workers up to wage-level $k$ by $\mu_k = w \sum_{i=1}^k u_i s_i$ with $\mu_0 = 0$, the GINI index as a measure of wage-inequality reads

$$GINI = 1 - \frac{1}{\Omega \mu_K} \sum_{k=1}^K s_k \left(\mu_k + \mu_{k-1}\right). \tag{2}$$

*Proof of (2).* Using $Z_k = s_k(\mu_k + \mu_{k-1})/2$, the GINI coefficient can be derived from its definition $GINI = \left(\frac{1}{2}\Omega\mu_K - \sum_{k=1}^K Z_k\right) \Big/ \left(\frac{1}{2}\Omega\mu_K\right)$. □

Price $p_n$ of good $n$ equals unit cost of production in case of free competition (if no entry and exit costs exist, Kurz & Salvadori, 1995, p. 1). Freely moving capital implies a uniform normal rate of profits $r$ across all sectors. Neglecting distributional issues within the working class, the aggregate labor input coefficients $\bar{l}_n = \sum_{k=1}^K u_k \bar{l}_{nk}$ of sectors $n \in [1, N]$ are the components of the unit production labor vector $\bar{\mathbf{l}} = (\bar{l}_1, \ldots, \bar{l}_N)^T \in \mathbb{R}^N$. Hence, the price vector $\mathbf{p} = (p_1, \ldots, p_N)^T \in \mathbb{R}_+^N$ is determined by the $N$-dimensional linear system

$$\mathbf{p} = (1 + r)\bar{A}\mathbf{p} + w\bar{\mathbf{l}}. \tag{3}$$

This price equation is build on the assumption that growth of fixed capital is financed by means of savings out of past normal profits or by loans with credit rates paid back by savings out of future normal profits. Fixed capital therefore does not further need to be considered in the remainder of this article, which primarily is concerned with prices (3) and not with quantities constraint by expression (1).

Introducing a numéraire basket $\mathbf{d} \in \mathbb{R}_+^N$ and thus determining a price level according to $\mathbf{d}^T\mathbf{p} = 1$, the real wage level $w$ can be calculated from expression (3) to

$$w = \frac{1}{\mathbf{d}^T \left(\mathbb{I} - (1+r)\bar{A}\right)^{-1} \bar{\mathbf{l}}}. \tag{4}$$

This defines some $w - r$ relationship for a given technology determined by $\bar{A}$ and $\bar{\mathbf{l}}$. The position of the system on this $w - r$ curve determines the distribution of the surplus between workers and capitalists, since $W = w\mathbf{x}^T\bar{\mathbf{l}}$ is the total sum of wages, and $P = r\mathbf{x}^T\bar{A}\mathbf{p}$ the sum of normal profits which is gained by capital owners. This gives rise to the *wage share* $W/(W+P)$ as a measure of income distribution between wage earners and capital owners. An example of the evolution of the wage share in course of the diffusion process of some innovation will be discussed in Section 3.1 and is depicted in Figure 1.

### 2.2 Multiple technologies and replicator dynamics

Heretofore the technology utilized within sector $n$ is characterized by the unit production input coefficients $\bar{\mathbf{a}}_n = (\bar{a}_{n1}, \ldots, \bar{a}_{nN})^T$ and $\bar{\mathbf{f}}_n = (\bar{f}_{n1}, \ldots, \bar{f}_{nN})^T \in [0,1]^N$ denoting the $n$-th row of $\bar{A}$ and $\bar{F}$ respectively. The unit production labor input coefficients $\hat{\mathbf{l}}_n = (\bar{l}_{n1}, \ldots, \bar{l}_{nK})^T \in \mathbb{R}^K$ on the other hand constitute the $n$-th row of $\bar{L}$. These quantities can be looked upon as being an aggregate of input factors of different processes: if in sector $n$ a number $I_n$ of different production processes exist, then process $i_n \in [1, I_n]$ is characterized by some capital input vectors $\mathbf{a}_n^{i_n}, \mathbf{f}_n^{i_n} \in \mathbb{R}_+^N$ and labor input vector $\hat{\mathbf{l}}_n^{i_n} \in \mathbb{R}_+^K$. The quantities $l_n^{i_n} = \mathbf{u}^T\hat{\mathbf{l}}_n^{i_n} \in \mathbb{R}_+$ define the labor input of the process weighted by the respective wage differentials.

Total output produced by process $i_n$ in sector $n$ is denoted by $x_n^{i_n}$ and hence total output of sector $n$ adds up to $x_n = \sum_{i_n=1}^{I_n} x_n^{i_n}$. Defining the share $q_n^{i_n} = x_n^{i_n}/x_n$ of output of sector $n$ produced by process $i_n$, the sector-specific input coefficients $\bar{\mathbf{a}}_n, \bar{\mathbf{f}}_n$ and $\bar{l}_n$ can be interpreted as characterizing the *average technology* of sector $n$, since

$$\begin{aligned}
\bar{\mathbf{a}}_n &= \textstyle\sum_{i_n=1}^{I_n} q_n^{i_n}\mathbf{a}_n^{i_n}, \\
\bar{\mathbf{f}}_n &= \textstyle\sum_{i_n=1}^{I_n} q_n^{i_n}\mathbf{f}_n^{i_n}, \\
\bar{l}_n &= \textstyle\sum_{i_n=1}^{I_n} q_n^{i_n} l_n^{i_n}.
\end{aligned}$$

For a given growth rate $g_n^{i_n}$, the quantity $x_n^{i_n}$ of good $n$ produced by process $i_n$ evolves according to

$$\dot{x}_n^{i_n} = g_n^{i_n} x_n^{i_n}.$$

Then from $\dot{x}_n = \sum_{i_n=1}^{I_n} \dot{x}_n^{i_n}$ the growth rate $g_n = \dot{x}_n/x_n$ of sector $n$ follows. Furthermore, differentiating of $x_n q_n^{i_n} = x_n^{i_n}$ with respect to time leads to the system

$$\dot{q}_n^{i_n} = q_n^{i_n}\left(g_n^{i_n} - g_n\right) = q_n^{i_n} \sum_{j_n=1}^{I_n} q_n^{j_n}\left(g_n^{i_n} - g_n^{j_n}\right) \tag{5}$$

of ordinary first order differential equations.

(5) resembles the *replicator dynamics* of *evolutionary game theory* (Weibull, 1997), which is based on the notion of the *fitness* of some *population*. In the biological analogy, a process $i_n$ can be viewed as a population within sector $n$. Technologies (processes) compete for market shares $q_n^{i_n}$, which evolve according to relative growth $g_n^{i_n} - g_n$ with respect to the whole sector $n$. Growth $g_n^{i_n}$ of some process can be interpreted as the *fitness* of some process: the fitter a population, the higher its rate of reproduction (and hence its growth of market shares). The ability of process output growth (which equals the ability of the process to replicate itself successfully) is accomplished by extra profits $\rho_n^{i_n}$ determined by its unit costs of production

$$c_n^{i_n} = (1+r)\mathbf{p}^T\mathbf{a}_n^{i_n} + wl_n^{i_n}.$$

Implicitly, extra profits are then defined by

$$\left(1 + r + \rho_n^{i_n}\right)\mathbf{p}^T\mathbf{a}_n^{i_n} + wl_n^{i_n} = p_n. \tag{6}$$

Different processes generally differ in their cost structure, but only one single price $p_n$ prevails for each good $n$. The price vector $\mathbf{p}$ itself can be determined by *average costs*

$$p_n = \sum_{i_n=1}^{I_n} q_n^{i_n} c_n^{i_n}$$

as opposed to the minimum cost principle in the long-period position (Kurz & Salvadori, 1995). This approach is also indirectly applied by D'Agata & Mori (2012) and can be argued for by at least three reasons. Firstly, one can look at the firms as hosts of the respective production processes. New technologies are seldom introduced at once within some firm, but in a stepwise fashion. Intra-firm diffusion of new technologies as for example described by Mansfield (1968, ch. 9) and Stoneman (2002) gives evidence that one firm utilizes different processes. The product then will be sold on the market at some average price. Two further arguments follow from the assumptions that prices are sticky in the short run and that in the long run nevertheless the minimum price approach of the classical long period position prevails. The former short-run argument is not compatible with the minimum cost approach, but with the average cost approach; the latter claim is nevertheless met by the average cost approach as will become apparent in the following two sections.

One preliminary result characterizing the system can be found by multiplying (6) with $q_n^{i_n}$. Summation over $i_n = 1, \ldots, I_n$ then leads to the observation that the absolute quantities $\rho_n^{i_n} \mathbf{p}^T \mathbf{a}_n^{i_n}$ of extra profits level out in each sector, i.e. that

$$\sum_{i_n=1}^{I_n} q_n^{i_n} \rho_n^{i_n} \mathbf{p}^T \mathbf{a}_n^{i_n} = 0. \tag{7}$$

This result will facilitate the understanding of possible output slumps after the introduction of some innovation as exemplified in Section 4.2 and depicted in Figure 5.

*Proof of (7).* (6) implies

$$\sum_{i_n=1}^{I_n} q_n^{i_n} \left[ \left(1 + r + \rho_n^{i_n}\right) \mathbf{p}^T \mathbf{a}_n^{i_n} + w l_n^{i_n} - p_n \right] = \underbrace{(1+r)\, \mathbf{p}^T \bar{\mathbf{a}}_n + w \bar{l}_n}_{= p_n} - p_n + \rho_n^{i_n} \mathbf{p}^T \bar{\mathbf{a}}_n = 0.$$

□

# 3 Diffusion of innovations in one sector

Preceding the investigations of the multi-sector model in Section 4, the special case of one sector is studied. Firstly, the diffusion of some innovation is solved analytically in Subsection 3.1. Then, a third technology is introduced revealing the possible non-monotonic behavior opposed to the case of two processes. Important general properties in case of an arbitrary number of processes are finally discussed in Subsection 3.2.

## 3.1 Two competing processes

Two competing processes are characterized by the input coefficients $(a_i, l_i, f_i)$ for $i = 1, 2$. Hence, following (6), extra profits $\rho_i$ are implicitly determined by

$$(1 + r + \rho_i) a_i + w l_i = 1. \tag{8}$$

Growth rates $g_i$ in case of a linear investment function equal extra profits times savings rate $s \in [0, 1)$. With $q$ denoting the share of the second, innovative technology, the replicator system (5) is given by

$$\dot{q} = sq\,(1-q)\,(\rho_2 - \rho_1)\,. \tag{9}$$

In relative market shares $z \equiv q/(1-q)$ the solution then reads

$$z(t)\,(\mu_1 + \mu_2 z)^D = Ce^{ts\mu_1/l_1}, \qquad D = \frac{\mu_1 l_2}{\mu_2 l_1} - 1, \qquad C = z_0(\mu_1 + \mu_2 z_0)^D, \tag{10}$$

with initial condition $z_0 = z(0)$ and auxiliary parameters $\mu_i = \alpha l_i + \beta\,(1-(1+r)a_i)$.

*Proof of (10).* Inserting $\rho_i$ from (8) into (9), the evolution of $q$ is determined by the differential equation

$$\frac{\dot{q}(t)}{q\,(1-q)} = s\left[\underbrace{\left(\frac{1}{a_2} - \frac{1}{a_1}\right)}_{\alpha} + w\underbrace{\left(\frac{l_1}{a_1} - \frac{l_2}{a_2}\right)}_{\beta}\right]. \tag{11a}$$

From the $w-r$ relationship (4), wages are determined by

$$w = \frac{1-(1+r)\bar{a}}{\bar{l}(t)} = \frac{1+z(t)-(1+r)[a_1+z(t)a_2]}{l_1+z(t)l_2}. \tag{11b}$$

Differential equation (11a) then becomes

$$\frac{\dot{z}(t)}{z(t)}\frac{l_1+z(t)l_2}{\mu_1+z(t)\mu_2} = s.$$

Finally, integration proves the result. □

To give a first intuition of the diffusion process in case of one sector and two processes, fixed capital is neglected and the special cases of labor saving technical progress by simultaneously using more capital ($a_1 < a_2$ and $l_1 > l_2$) is illustrated in Figure 1 (for demonstrative purposes, the following quantities were chosen: $s = 1$, $a_1 = 0.3$, $a_2 = 0.4$, $l_1 = 0.3$, $l_2 = 0.2$, $r = 0.1$, $q(0) = 0.01$). The time path of $q(t)$ with its slow start and sudden take-off including the flattening at the end of the diffusion process resembles the diffusion pattern found in many cases of diffusion research (Rogers, 2003).

An additional feature, which is owed to the labor saving and capital enhancing technical change in this example, is the declining wage share $W/(W+P)$, defined as the percentage of total income devoted to labor as defined at the end of Section 2.1. Generally, labor saving and capital enhancing technical change leads to a decline of the wage

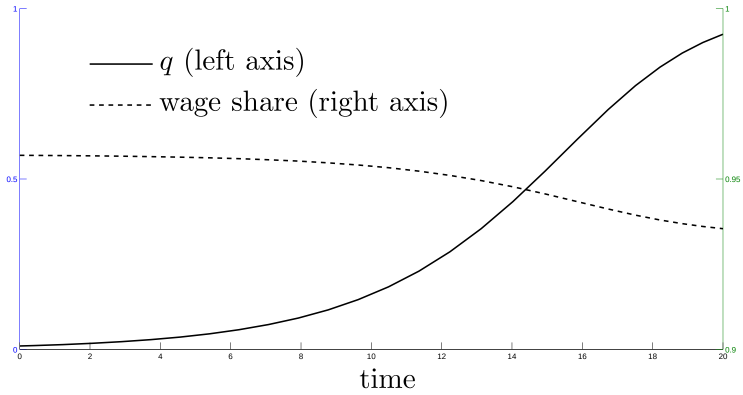


Figure 1: Changing wage share due to technical change

share in this modeling framework. The outcome is ambiguous in case of both capital- and labor saving technical progress.

What gets apparent from Figure 1 is the asymptotic behavior at the beginning and at the end of the process as well as the monotone behavior during the whole diffusion process. This monotonicity property holds true in general in case of two processes:

**Proposition 1.** *For one sector with two distinct processes, the market share of each process is a monotone function of time. It is constant over time if and only if either* $q(0) \in \{0, 1\}$ *or*

$$1 \leq 1 + r = \frac{l_1 - l_2}{l_1 a_2 - a_1 l_2} \leq \min \left\{ \frac{1}{a_i} \right\}. \tag{12}$$

*Proof.* The statement is obviously true for $q(0) \in \{0, 1\}$. Now assume non-monotonicity of $q(t)$ if $q(0) \notin \{0, 1\}$. Then, due to (9), $\dot{q} = 0$ holds for some strictly positive $q \in (0, 1)$ if and only if $\rho_1 = \rho_2$. Acknowledging (8) this is equivalent to $w(a_1 l_2 - a_2 l_1) = a_1 - a_2$. But $a_1 l_2 = a_2 l_1$ implies $a_1 = a_2$ leading to $l_1 = l_2$ and hence to two equal processes. This situation indeed implies $\dot{q} = 0$ but is excluded by assumption. Hence,

$$w = \frac{a_1 - a_2}{a_1 l_2 - a_2 l_1} = [1 - (1 + r)\bar{a}]/\bar{l}$$

by (8) and (11b), leading to $\rho_1 a_1 = q(\rho_1 a_1 - \rho_2 a_2)$. For $\rho_1 = \rho_2 \neq 0$ this implies the contradiction $q = a_1/(a_1 - a_2) \in (-\infty, 0) \cup (1, \infty)$. $\rho_1 = \rho_2 = 0$ then straightforwardly leads to (12). Finally, $1/a_i - 1$ is the maximum rate of profits if only process $i$ is operated, determined by $w = 0$ in the respective $w - r$ relation. □

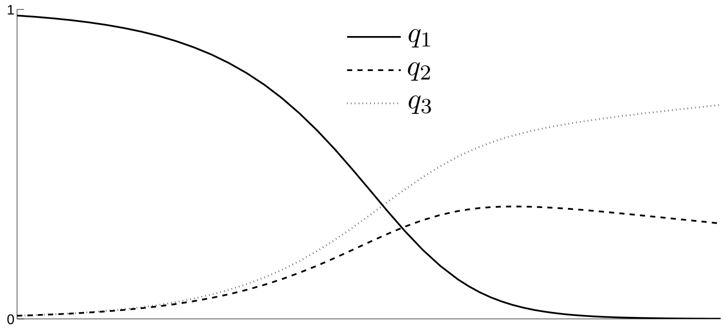


Figure 2: Non-monotonic diffusion

This monotonicity-property does not hold in general for more than two processes. Figure 2 shows an example for non-monotonicity in the presence of a third process characterized by $a_3 = 0.41$, $l_3 = 0.19$ and $q_2(0) = q_3(0) = 0.01$. Both innovations (processes 2 and 3) first grow according to their advantage in comparison with the incumbent process. Finally process 2 is outperformed by process 3.

The special case of two out of the three processes commonly fulfilling condition (12) leads, if they exhibit lower costs of production than the third process, to a long-run equilibrium with both processes simultaneously used. The relative share $q_1/q_2$ for $t \to \infty$ depends on the third process and in case of multiple processes $I > 3$ may even show ambiguous dynamic properties (non-monotonicity), depending on initial conditions. Dependence on initial conditions is exemplified in Figure 3, simulating the evolution of three processes defined by $(a_1, l_1) = (0.3, 0.3)$, $(a_2, l_2) = (0.4, 0.6 \cdot 0.3/0.7)$, $(a_3, l_3) = (0.45, 0.25)$ and $r = 0$. Processes 1 and 2 therefore meet condition (12) and hence both survive in the long run, since both outperform process 3. With which market shares they end up depends on the initial conditions $\mathbf{q}(0)$. Not only initial conditions, also the third process itself influences the long-run outcome: to give a numerical example, $a_3 = 0.5$ leads to $\lim_{t\to\infty} \mathbf{q}(t) \approx (0.752, 0.248, 0)$ in contrast to $\lim_{t\to\infty} \mathbf{q}(t) \approx (0.745, 0.255, 0)$ for $a_3 = 0.45$.

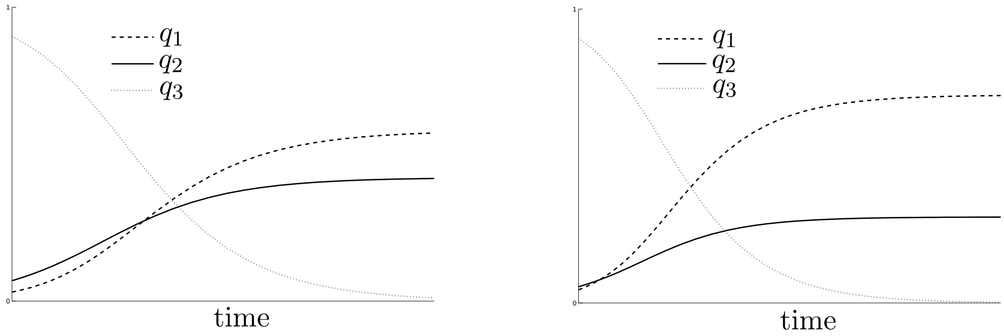


Figure 3: Dependence on initial conditions

### 3.2 An arbitrary number of processes

In the general case of $I$ processes within one sector, possible equilibria and their stability properties can be derived. The evolution of the system is described by

$$\dot{q}_i = q_i \sum_{k=1}^{I} q_k \left(\rho_i - \rho_k\right) = q_i \left(\rho_i - \bar{\rho}\right) \quad \text{with} \quad \bar{\rho} = \sum_{k=1}^{I} q_k \rho_k \tag{13}$$

according to (5), if extra profits are entirely invested into growth. The unit vectors

$$\mathbf{e}_i^I = (0, \dots, 0, \underbrace{1}_{i}, 0, \dots, 0) \in [0,1]^N$$

are steady states of (13), indicating that the system is in rest if only one process is operated. Without loss of generality, let process $I$ be operated, i.e. $q_I = 1$ and $q_i = 0$ for $i = 1, \dots, I-1$. Due to the constraint $\sum_{i=1}^{I} q_i = 1$ the system is described by (13) for $i = 1, \dots, I-1$. Local stability of this equilibrium $\mathbf{q}^* = \mathbf{e}_I^I$ can be calculated by looking at the *Jacobian matrix*

$$\mathcal{J}(\mathbf{q}^*) = \operatorname{diag}\left[\rho_1(\mathbf{q}^*), \dots, \rho_{I-1}(\mathbf{q}^*)\right]. \tag{14}$$

Its eigenvalues are exactly the extra profits of the single processes. Consequently, the stability properties of the equilibria $\mathbf{e}_i^I$ are obtained by looking at the extra profits of those processes which are not yet in use. Negativity implies stability, whereas positivity indicates instability.

*Proof of (14).* To show the structure of the Jacobian as stated in equation (14), the following sequence of calculations can be conducted (with $\delta_{ij}$ denoting the *Kronecker-delta* with $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$):

$$
\begin{aligned}
[\mathcal{J}(\mathbf{q}^*)]_{ij} &= \frac{\partial}{\partial q^j} q_i \sum_{k=1}^{I} q_k \left(\rho_i - \rho_k\right)\Big|_{\mathbf{q}^*} \\
&= \delta_{ij} \sum_{k=1}^{I} \delta_{kI}(\rho_i - \rho_k) + \underbrace{\delta_{iI}}_{=0} \sum_{k=1}^{I} \delta_{kI} \frac{\partial}{\partial q^j} \left(\rho_i - \rho_k\right) + \\
&\quad \underbrace{\delta_{iI}}_{=0} \sum_{k=1}^{I} \delta_{kj} \left(\rho_i - \rho_k\right) \\
&= \delta_{ij} \big(\rho_i(\mathbf{q}^*) - \underbrace{\rho_I(\mathbf{q}^*)}_{=0}\big)
\end{aligned}
$$

□

Next, from Proposition 1 one can conclude that mixed equilibria are possible if and only if condition (12) holds pairwise for two or more processes. Let $I_0 \subset [1, I]$ be the set of processes satisfying (12) in pairs. Then each feasible $\mathbf{q}$ with $q_j = 0$ for all $j \in [1, I] \backslash I_0$ is a mixed equilibrium with $\rho_i = 0$ for all $i \in I_0$. These mixed equilibria are prone to bifurcation, since the slightest deviation of the input coefficients from constraint (12) leads to a change of the characteristic of the system.

No equilibrium position prevails if $\mathbf{q} \neq \mathbf{e}_m^I$ for any $m \in [1, I]$, or if condition (12) does not hold for all processes $i, j$ with $q_i, q_j > 0$. Then an evolutionary process is launched as described by the replicator dynamics (5). One characteristic is growing productivity, which is characterized by rising real wage rates as stated in the following proposition.

**Proposition 2.** *(1) Real wages $w$ are monotonically increasing, i.e. $\dot{w} \geq 0$; extra profits are monotonically decreasing, i.e. $\dot{\rho}_i \leq 0$. (2) $\dot{w} = 0$ holds if and only if the system is in equilibrium, i.e. if and only if $\dot{\mathbf{q}} = 0$.*

*Proof.* (1) From (4), real wages in case of one sector are given by

$$w = \frac{1 - (1 + r)\bar{a}}{\bar{l}}.$$

Differentiation with respect to time and inserting (13) leads to

$$-\,\bar{l}\dot{w} = -\sum_{i=1}^{I} q_i(\rho_i - \bar{\rho})\rho_i a_i \overset{(7)}{=} -\sum_{i=1}^{I} q_i \rho_i^2 a_i \leq 0 \tag{15}$$

hence verifying $\dot{w} \geq 0$. $\dot{\rho}_i \leq 0$ then follows from (8), which provides the definition of $\rho_i$ also for an arbitrary number of processes.

(2) Since $w = w(\mathbf{q}(t))$, necessarily $\dot{\mathbf{q}} = 0$ implies $\dot{w} = 0$. The other way round, from (15) it follows that $q_i \rho_i = 0$, since $q_i \rho_i^2 a_i \geq 0$. Hence, either $q_i = 0$ which implies $\dot{q}_i = 0$ from (13), or $\rho_i = 0$. Let $\mathcal{I} = \{j \in [1, I] : q_j > 0\}$; then $\rho_j = 0$ for all $j \in \mathcal{I}$, also indicating $\dot{q}_j = 0$ by (13), since in this case $\rho_j = \bar{\rho} = 0$. □

# 4 The multi-sector economy

General properties of the multi-sector model are derived in Subsection 4.1. In Subsection 4.2 the case of two sectors is analyzed in three steps. Firstly, the diffusion of an innovative process which emerged after some product innovation is studied; next, the special case of technical change in one of the two sectors is scrutinized; and finally spillover effects of technical change in both sectors are demonstrated.

## 4.1 General properties

As in the case of one sector in the previous section it is assumed that extra profits are entirely invested into growth. This implies that growth rates and extra profits coincide, and from (5) one gets the replicator equations

$$\dot{q}_n^{i_n} = q_n^{i_n} \left( \rho_n^{i_n} - \bar{\rho}_n \right) \tag{16}$$

with $i_n = 1, \ldots, I_n$ for all $n = 1, \ldots, N$. Introducing the vector $\mathbf{q} = (\mathbf{q}_1^T, \ldots, \mathbf{q}_N^T)^T \in [0,1]^I$ of possible market shares $q_i$ with $i = 1, \ldots, I = \sum_{n=1}^N I_n$, this system can be chalked down as a *generalized Lotka-Volterra system*

$$\dot{q}_i = q_i \left[ Q(\mathbf{q}) \mathbf{q} \right]_i \qquad \text{for all} \qquad i = 1, \ldots, I. \tag{17}$$

Initial conditions are given by $\mathbf{q}(0) = \mathbf{q}_0 \geq 0$ with $\sum_{i=1}^{I_n} q_n^{i_n}(0) = 1$ for all $n \in [1, N]$. $Q \in \mathbb{R}^{I \times I}$ is a skew-symmetric block diagonal matrix:

$$Q(\mathbf{q}) = \begin{bmatrix} Q_1(\mathbf{q}) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Q_N(\mathbf{q}) \end{bmatrix}$$

Each sub-matrix $Q_n \in \mathbb{R}^{I_n \times I_n}$ represents one sectors $n$: its coefficients $[Q_n]_{i_n j_n} = \rho_n^{i_n} - \rho_n^{j_n}$ are the differences of extra profits of processes $i_n$ and $j_n$. Rows 1 to $I_1$ of $Q$ belong to sector 1 (starting with index $z_1 = 1$), the next $I_2$ rows contain the processes of sector

2 with fist index $z_2 = I_1 + 1$ to $z_2 + I_2 - 1$. Inductively, $z_n = z_{n-1} + I_{n-1}$ is the index of $\mathbf{q}$ which denotes the market share of the first process of sector $n$.

The following existence-result imposes a restriction onto the capital input coefficients $a^{i_n}_{nm}$. It states that each process necessarily has to use at least some capital input[1]:

**Proposition 3.** *System (17) admits a unique solution path* $\{\mathbf{q}(t)\}_{t\geq 0}$, *if* $\|\mathbf{a}^{i_n}_n\|_1 > 0$ *for all* $n \in [1, N]$ *and* $i_n \in [1, I_n]$.

*Proof.* Applying the existence and uniqueness theorem of Picard-Lindelöf it suffices to show that the function $q^i_n \left(\rho^i_n - \bar{\rho}_n\right)$ is Lipschitz continuous. This holds as a consequence of the boundedness of the extra profits in case of $\|\mathbf{a}^{i_n}_n\|_1 > 0$ (which is not true for $\|\mathbf{a}^{i_n}_n\|_1 = 0$). □

From now on the sufficiency condition $\|\mathbf{a}^{i_n}_n\|_1 > 0$ shall hold, i.e. each single production process needs at least some capital input. This restriction is reasonable, since extra profits $\rho^{i_n}_n$ are defined with respect to some capital input and would diverge in case of $\mathbf{a}^{i_n}_n = 0$. As a consequence of Proposition 3 the interpretation of the $q^{i_n}_n$ as the market share of process $i_n$ in sector $n$ can be shown to hold for all times as a consequence of the skew-symmetry of $Q$ in (16):

**Proposition 4.** $\|\mathbf{q}_n(t)\|_1 = 1$ *holds for all* $n \in [1, N]$ *and* $t \geq 0$.

*Proof.* $\|\mathbf{q}_n(0)\|_1 = 1$ is true per definition of the initial values. By acknowledging (16), one gets

$$\frac{d}{dt}\|\mathbf{q}_n(t)\|_1 = \sum_{i_n=1}^{I_n} \dot{q}^{i_n}_n(t) = \sum_{i_n=1}^{I_n} q^{i_n}_n(t) \left(\rho^{i_n}_n - \bar{\rho}_n\right) = \bar{\rho}_n\bigl(1 - \|\mathbf{q}_n(t)\|_1\bigr).$$

With $\|\mathbf{q}_n(t)\|_1 = 1 + y$, this expression is equivalent to the initial value problem $\dot{y} = -\bar{\rho}(t)y$ and $y(0) = 0$ with solution $y(t) = 0$ (i.e. $\|\mathbf{q}_n(t)\|_1 = 1$) for all $t > 0$. This proves the proposition since $\bar{\rho}_n(t)$ is bounded for all $n$ due to Proposition 3. □

Extending the one-sector discussion of steady states, again corner solutions can be identified as equilibria. They are characterized by only one process in each sector generating positive output. Without loss of generality, let $i_n = 1$ be the incumbent process in sector $n$. For $\mathbf{e}^{I_n}_1 = (1, 0, \ldots, 0)$, this equilibrium is therefore given by

$$\mathbf{q}^* = \left((\mathbf{e}^{I_1}_1)^T, \ldots, (\mathbf{e}^{I_N}_1)^T\right)^T. \tag{18}$$

[1] In the following, $\|\cdot\|_1$ denotes the 1-norm defined by summation of absolute values of the vector coefficients.

*Proof of (18).* That $\mathbf{q}^*$ in (18) is a steady state of the systems becomes clear by acknowledging (17), because $\dot{\mathbf{q}}_i(\mathbf{q}^*) = 0$ if either $[\mathbf{q}^*]_i = 0$ (which is true for $i \notin \{z_1, \dots, z_N\}$) or $[Q(\mathbf{q}^*)\ \mathbf{q}^*]_i = 0$ for $i \in \{z_1, \dots, z_N\}$. The latter holds due to the skew-symmetry of $Q$, since $[Q(\mathbf{q}^*)\ \mathbf{q}^*]_{z_n} = \left[Q_n(\mathbf{q}^*)\mathbf{e}_1^{I_n}\right]_1 = [Q_n(\mathbf{q}^*)]_{11} = 0$. □

Similar to the one-sector case (14), the Jacobian can be calculated to evaluate stability properties. For simplicity of notation and since a generalization can be accomplished straightforwardly, stability is investigated in case of $N$ sectors with only two processes in each sector. System (16) is then given by

$$\dot{q}_n = q_n(1 - q_n)\left(\rho_n^1 - \rho_n^2\right)$$

as a generalization of (9) for one sector and two processes. $q_n$ in this case denotes the share of the incumbent process, and $\rho_n^{i_n}$ the extra profits of process $i_n$ in sector $n$. The equilibrium position to be scrutinized is therefore given by $\mathbf{q}^* = (1, \dots, 1)^T$ with Jacobian

$$\mathcal{J}(\mathbf{q}^*) = \begin{bmatrix} \rho_1^2 & & 0 \\ & \ddots & \\ 0 & & \rho_N^2 \end{bmatrix} \tag{19}$$

which can be derived by calculating $[\mathcal{J}(\mathbf{q}^*)]_{nm} = \partial \dot{q}_n / \partial q_m|_{\mathbf{q}^*}$. The eigenvalues of the Jacobian at some corner equilibrium are therefore again, similar to the one-sector analysis, the extra profits of the disused processes.

Off equilibrium, the system is prone to evolutionary forces. Similar to the one-sector case, real wages are non-decreasing – but now they do not necessarily indicate an equilibrium if constant:

**Proposition 5.** *(1) $w(t)$ is non-decreasing, i.e. $\dot{w} \geq 0$. (2) $\dot{w} = 0$ is a necessary condition for $\dot{\mathbf{q}} = 0$, i.e. $\dot{\mathbf{q}} = 0 \Rightarrow \dot{w} = 0$.*

*Proof.* Statement (1) is true as consequence of Proposition 2: if technical change in each single sector leads to non-decreasing wages, then this also holds in case of multiple sectors.

(2) The argument for $\dot{\mathbf{q}} = 0 \Rightarrow \dot{w} = 0$ is the same as in Proposition 2 in case of one sector. A counterexample for sufficiency is given in the example of technical progress in the non-basic sector in Subsection 4.2 by expression (25). There $\dot{w} = 0$ holds throughout the diffusion process, since the numéraire is chosen to be the good produced in the non-innovative sector, which additionally is non-basic. □

Another difference to the one-sector economy scrutinized in Proposition 2 is the observation that in case of multiple sectors extra profits are not necessarily non-increasing (since relative prices $\mathbf{p}$ change). An example for this case will be provided in the next section by an example depicted in Figure 7. There, technical change in one sector induces an increase of extra profits as a result of spillover-effects of technical change in the other sector.

## 4.2 Two-sector economies

### Product innovation implies process innovation

A one-sector economy reproducing itself is the point of departure to analyze two-sector economies. The prevailing technology is characterized by capital input coefficient $a_1^1$ and labor input coefficient $l_1^1$. One possibility to expand the one-sector economy to two sectors is the introduction of some new product which is produced by means of the good produced in sector 1. Taking good 1 as numéraire, price $p$ of the new product is determined by

$$(1 + r)a_2 p + wul_2 = p \tag{20}$$

with capital input $a_2$ and labor input $l_2$. $u > 1$ indicates that high skill labor is used. Occasionally, a new product can imply the existence of new processes. So at time $t = 0$ when the new product is introduced into the system, in sector 1 a new process characterized by capital and labor input coefficients $(a_{11}^2, a_{12})$ and $l_1^2$ respectively can be launched. It pays extra profits $\rho_2$ defined by

$$(1 + r + \rho_2)(a_{11}^2 + a_{12}p) + wul_1^2 = 1 \tag{21}$$

if the new process also needs high skilled labor. Extra profit (respectively losses) $\rho_1$ of the incumbent technology are then given by

$$(1 + r + \rho_1)a_{11}^1 + wl_{11} = 1. \tag{22}$$

With $q$ denoting the market share of the innovation, the replicator dynamics equation $\dot{q} = q(1 - q)(\rho_2 - \rho_1)$ together with (20-22) determine the dynamics of the system. For the special case of $(a_{11}^1, l_{11}) = (0.3, 0.3)$ and $(a_{11}^2, a_{12}, l_{12}) = (0.4, 0.1, 0.2)$ for the incumbent and innovative process in sector 1, and $(a_2, l_{22}) = (0.1, 0.1)$ for the process of sector 2, the diffusion process is depicted in Figure 4. What can be demonstrated

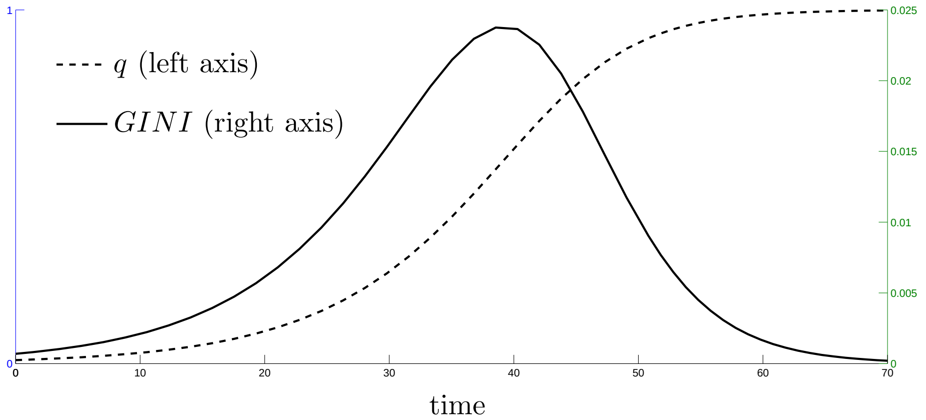


Figure 4: Wage inequality due to technical change

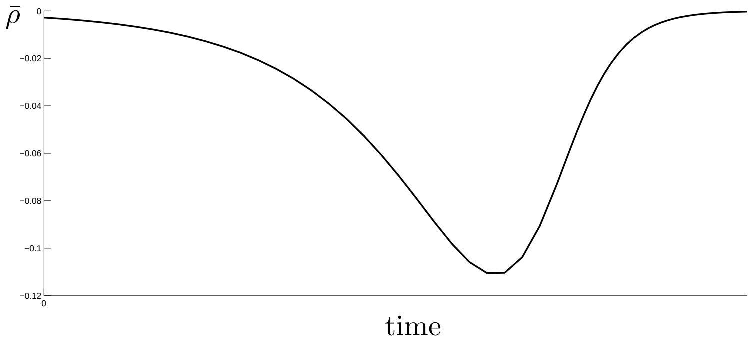


Figure 5: Negative growth induced by technical change

in this example is the transitional wage inequality. It is a consequence of the different remuneration of skills. Equality with vanishing GINI coefficient can be observed at the beginning of the diffusion process (only low-skilled labor is employed) and at the end (only high-skilled labor is employed). Formally this pattern is based on expression (2) for the GINI-coefficient in case of two different skills.

Another interesting feature is the negative growth of output after introducing the innovative process as depicted in Firgure 5 for the cited example. This is a consequence of Schumpeter's *creative destruction*, which leads to a decline of output of the incumbent process due to losses which cannot be compensated by the growth of market shares of the innovation. Technical progress in this respect only indirectly promotes economic growth by facilitating subsequent capital accumulation, something which is not considered in the proposed theoretical framework. But it is in line with empirical observations, especially concerning so-called *general purpose technologies*, which cause an output slump proceeding subsequent economic growth (Helpman, 1998). A further discussion in the context of the introduced evolutionary economic framework is elaborated by Strohmaier & Rainer (2013). Negative growth is no general characteristic of the model, but depends on the capital input coefficients as indicated by expression (7).

**Technical progress in one sector**

The case of process innovation in the wake of a product innovation formally resembles a one-sector diffusion scenario as studied in Section 3.1. To show that this is the case whenever an innovation occurs only in one of the two sectors is the aim of the following remarks.

In case of two sectors with one skill, the price system is given by

$$\begin{aligned} (1+r)(\bar{a}_{11}+\bar{a}_{12}p)+w\bar{l}_1 &= 1, \\ (1+r)(\bar{a}_{21}+\bar{a}_{22}p)+w\bar{l}_2 &= p. \end{aligned} \tag{23}$$

To consider only one kind of labor does not lessen the significance of the following calculations, since heterogeneity of labor (and respective wages) is of importance only if wage inequality is studied.

For sector 2 being non-basic (i.e. sector 1 does not need any input from sector 1, $\bar{a}_{12}=0$), system (23) reduces to

$$(1+r)\bar{a}_{11}+w\bar{l}_1 = 1, \tag{24a}$$

$$(1+r)(\bar{a}_{21}+\bar{a}_{22}p)+w\bar{l}_2 = p. \tag{24b}$$

If only sector 1 is innovative then the diffusion process in this sector is equivalent to the one-sector case investigated in Section 3. On the other hand, if technical progress takes place only in sector 2, then

$$w = \frac{1-(1+r)a_{11}}{l_1} \tag{25}$$

is constant. This expression provides the counterexample for the proof of Proposition 5 that wages can stay constant throughout the diffusion process in case of multiple sectors. If also the innovative sector 2 is non-basic, hence if the two sectors are decoupled, $\bar{a}_{21}=0$ holds and (24b) reduces to

$$(1+r)\bar{a}_{22}+\frac{w}{p}\bar{l}_2 = 1,$$

resembling the single-sector economy studied in Section 3. There, in (11b), $w$ is replaced by $w/p$. The same is true in the more general case of $\bar{a}_{21} \geq 0$, i.e. if sector 2 is allowed to be (but not necessarily is) a basic sector. Then (24b) can be rearranged to

$$(1+r)\bar{a}_{22} + \frac{1}{p}\left[(1+r)\bar{a}_{21} + w\bar{l}_2\right] = 1,$$

which again is formally equivalent to the already studied one-sector economy of Section 3 with $w$ in (11b) replaced by $1/p$ and $\bar{l}$ replaced by $(1+r)\bar{a}_{21} + w\bar{l}_2$.

Also if both sectors are basic, without loss of generality only sector 1 being innovative, the system is equivalent to some one-sector economy: from (23) it follows that price $p$ is given by

$$p = \frac{wl_2 + (1+r)a_{21}}{1-(1+r)a_{22}} = \alpha w + \beta$$

with parameters $\alpha = l_2/[1-(1+r)a_{22}]$ and $\beta = (1+r)a_{21}/[1-(1+r)a_{22}]$. The first equation of (23) consequently is again given by the single-sector equation (11b) with $\bar{a} = \bar{a}_{11} + \beta\bar{a}_{12}$ and $\bar{l} = \bar{l}_1 + (1+r)\alpha\bar{a}_{12}$.

**Technical progress in both sectors**

Things get more involved if technical change takes place in both sectors. For the simpler case of one non-basic sector as indicated by (24a-24b), the diffusion process of sector 1 is similar to the process investigated in Section 3, since sector 2 has no influence on sector 1. The other way round, sector 2 is described by

$$(1+r)\bar{a}_{22} + \frac{1}{p}\left[w(t)\bar{l}_2 + (1+r)\bar{a}_{21}\right] = 1.$$

This price equation can be interpreted similar to equation (11b) with $w$ replaced by $1/p$ and $\bar{l}$ by $w(t)\bar{l}_2 + (1+r)\bar{a}_{21}$. Hence, if $w(t)$ increases (which is the case due to Proposition 2), this formally implies increasing labor input coefficients. This spillover from sector 1 under certain conditions (but not necessarily) under certain conditions induces a re-switching of profitability of the two processes of sector 2.

An example for this special case is depicted in Figure 6 for two processes in each sector. Capital input coefficients are given by $a_{11}^1 = a_{11}^2 = 1$, $a_{21}^1 = 0.35$, $a_{22}^1 = 0.4$, and $a_{21}^2 = a_{22}^2 = 0.45$; labor input coefficients are given by $l_1^1 = 0.4$, $l_1^2 = 0.2$, $l_2^1 = 0.5$ and $l_2^2 = 0.41$. Furthermore, $r = 0.01$ and finally $q_n$ denotes the share of the second process in sector $n$. One can observe that in Figure 6 the second process of sector 2 first looses market shares (decreasing $q_2(t)$). As the innovation of sector 1 gains ground ($q_1(t)$

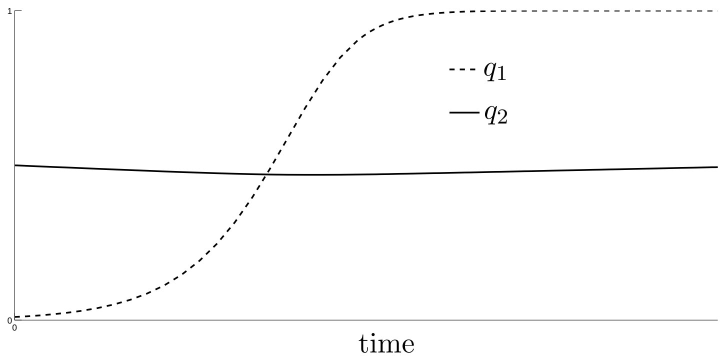


Figure 6: Re-switching in the basic sector

takes off), profitability of the second process in sector 2 increases and eventually leads to rising market shares. This can be explained by the costs of capital input of sector 1 into sector 2, which gets cheaper as time goes by and therefore increases profitability of process 2 in sector 2 such that it finally dominates process 1. Since extra profits, from a mathematical point of view, continuously depend on technical coefficients, this just described re-switching can also happen in case of two basic sectors.

For the general case of two basic sectors with two processes, stability properties of the corner equilibria $\mathbf{q}^* = (1, 0, 1, 0)^T$ can be calculated. To this end, the replicator dynamics is written as

$$\dot{q}_1 = q_1(1 - q_1)\left(\rho_1^2 - \rho_1^1\right),$$
$$\dot{q}_2 = q_2(1 - q_2)\left(\rho_2^2 - \rho_2^1\right).$$

The Jacobian at $\mathbf{q}^*$ is then given by

$$\mathcal{J}(\mathbf{q}^*) = \begin{pmatrix} \rho_1^2(\mathbf{q}^*) & 0 \\ 0 & \rho_2^2(\mathbf{q}^*) \end{pmatrix} \tag{26}$$

as a special case of (19). Hence, it only depends on the extra profits of the not operated process in the respective sector whether $\mathbf{q}^*$ is evolutionary stable or not: negative extra profits induce stability, since an entry of the respective process does not harm the incumbent technology. Positive extra profits of the disused process on the other hand suggest that an introduction of this innovation would lead to creative destruction of the incumbent process.

If $\rho_1^2 > 0$, an entrepreneur can gain extra profits by introducing process 2 as innovation into sector 1, leading to a new equilibrium $\mathbf{q}^{**} = (0, 1, 1, 0)$. It is now possible that

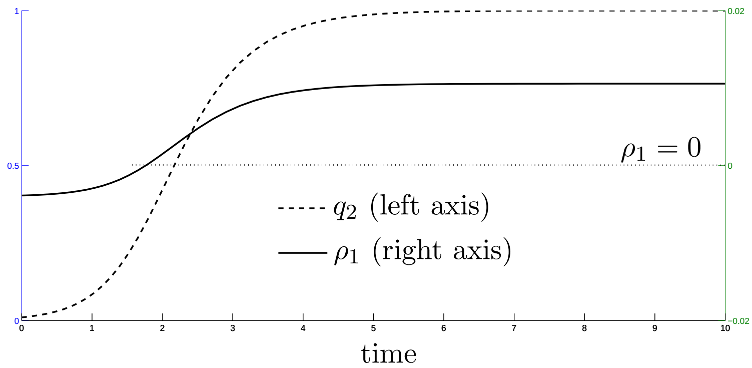


Figure 7: Changing profitability due to inter-sectoral spillover

in sector 2 the unprofitable process 2 with $\rho_2^2(\mathbf{q}^*) < 0$ due to the changing economic environment gets profitable, i.e. $\rho_2^2(\mathbf{q}^{**}) > 0$. This is exemplified in Figure 7 for $a_{11}^1 = 0.3$, $a_{11}^2 = 0.235$, $a_{12}^1 = 0.2$, $a_{12}^2 = 0.21$, $a_{21}^1 = a_{22}^1 = 0.2$ and $a_{21}^2 = a_{22}^2 = 0.1$, and with labor input coefficients $l_1^1 = 0.4$, $l_1^2 = 0.45$, $l_2^1 = 0.5$ and $l_2^2 = 0.3$. Normal profits are taken to be $r = 0.1$. The innovation in one sector therefore may help a disused process in the other sector to become profitable.

## 5 Conclusions

The general framework introduced in this article combines the formalism of replicator dynamics of evolutionary game theory with the concept of extra profits of Classical economics. The proposed multi-sector setting aims at investigating spillover effects between sectors in the presence of technical change. It is set up in the most general case of an arbitrary number of sectors and different processes. Special focus is put on the two-sector setting: different cases of combined non-basic and basic sectors as well as the case of two basic sectors are studied in detail. Also the special case of process innovation induced by a product innovation is investigated.

Some basic features of the model are transitional wage inequality as well as negative growth as a consequence of technical change. The re-switching of profitability in one sector as a consequence of technical progress in another sector is the important aspect investigated both in a dynamic setting as well as by means of steady state stability analysis. In the latter case, it is demonstrated that the extra profits of disused technologies exactly resemble the eigenvalues of the Jacobian matrix calculated at the respective steady states. The just stated results are supported by a thorough analysis of the one-sector case. There, the sigmoid-shaped diffusion pattern, which is often found for the diffusion innovations, is reconstructed. Also general formal properties are derived to facilitate the theoretical understanding of the model.

It is the strength of this model to be presented in a general multi-sector, multi-process setting. Therefore various applications are possible, including the combination of empirical and theoretical analysis. Strohmaier & Rainer (2013) for example empirically investigate the diffusion patterns of general purpose technologies, which can be reconstructed by the presented modeling framework. Hence, the theory is aligned with empirical evidence, especially concerning wage inequality and output decline in the aftermath of the introduction of some new general purpose technology. Since an opening of international markets can be interpreted as the coming into existence of different processes and skills, international trade theory and development patterns can also be investigated by this evolutionary multi-sector modeling framework. Palan & Rainer (2013) in this context look at the empirical evidence of the decline of the textile industry in Europe, which is simulated by a two-country version of the presented model.

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