Financial Integration, intra-EMU and Global External Imbalances in a Three-Country OLG Model

Abstract

EMU’s current account imbalances during the pre-crisis period up to 2008 are traditionally explained by (i) financial integration and convergence expectations and (ii) by “over-optimism” and excessive real appreciation in the periphery. While not questioning these traditional explanations, Chen et al. (2013) present new stylized facts regarding the trade linkages between euro zone’s periphery (and core) and the rest of the world, in particular China, the CEECs and oil exporters. Acknowledging these empirical facts this paper uses a Diamond (1965)-Buiter (1981) three-country (EMU, Asia, US), two-region (EMU core, EMU periphery) OLG model to show which differences in economic fundamentals between northern and southern EMU countries and between the latter and the rest of the world were transformed into the observed external imbalances (current and financial account) when financial integration after the inception of the common currency occurred.

Keywords: Current Account Imbalances, European Economic and Monetary Union, Overlapping Generations, Three-Country Model

JEL Code: F34, F36
Introduction and Motivation

The current account imbalances of the European Economic and Monetary Union (EMU) during the pre-crisis period up to 2008 are empirically well documented (e.g. Lane and Pels 2012). The huge external deficits in southern (including Ireland) (= “periphery”) EMU countries are traditionally explained by (i) financial integration and expectation of convergence within the common currency area and (ii) by “over-optimism” and excessive real appreciation in the periphery (e.g. Lane 2006, Coeurdacier and Martin 2009, Lane and Milesi-Feretti 2008). While not denying the relevance of these traditional explanations, Chen et al. (2013) present new stylized facts regarding intra-EMU current account imbalances and the rest of the world. Among them the trade linkages between the EMU subareas and the rest of the world, in particular China, the CEECs and oil exporters figure prominently, the rough balance of the whole euro area current account notwithstanding. Periphery’s current account deficit while financed mostly by capital inflows from the core increased not vis-à-vis the core but vis-à-vis the rest of world, and similarly for the current account surpluses of the core. Acknowledging these new stylized facts it is natural to ask whether the current account imbalances with respect to the rest of the world and the intra-EMU financial account imbalances can be explained by an intertemporal current account model (Ca’ Zorzi and Rubaszek 2012) for the EMU and the rest of the world.

As is well-known, after the inception of the euro in 1999, northern and center euro countries (Austria, Belgium, Finland, Germany, Netherlands, France), in particular Germany, started to run current account surpluses while the southern and western periphery (= PIIGS: Portugal, Ireland, Italy, Greece and Spain) accumulated huge external deficits accompanied by a dramatic loss of international competitiveness due to striking increases in their wages and prices compared to the northern countries. Moreover, there was a significant divergence in the dynamics of private debt between northern and southern countries (Pisany-Ferry 2012, Figure 4): Up to the outburst of the global financial crisis southern debt boomed, mainly in order to finance housing investment while in the aftermath of the crisis government debt was substituted for private debt.

While the contribution of financial integration to the emergence of intra-EMU external imbalances is empirically largely undisputed, it remains an open theoretical question how divergent current account imbalances can be related to financial integration in an intertemporal general equilibrium model of a heterogeneous currency area. Among the few who address this question are Fagan and Gaspar (2008) who use a two-good, two-country...
overlapping generations pure exchange model without public debt à la Yaari (1965) and Blanchard (1985) to compare the pre-euro financial autarky steady state to euro-related financial integration between southern and northern euro countries. Fagan and Gaspar (2008) find that the evolution of intra-EMU external imbalances can be traced back to North-South differences in time preference. However, Fagan and Gaspar (2008) neglect both production and capital accumulation and the trade linkages between euro core respective periphery and the rest of the world.

In view of the euro-related dynamics of housing investment in Spain and Ireland Farmer (2013, 2014) models production and capital accumulation within Buiter’s (1981) two-country overlapping generations (OLG) economy, one of the two seminal contributions to the intertemporal equilibrium approach to external imbalances. In view of the rather modest intra-EMU trade of goods and services (Chen et al. 2013), Buiter’s one-good setting is appropriate to model the financial account imbalances across EMU’s core and periphery in the run up towards the global financial crisis. Farmer (2014) finds that the financial account deficits of EMU periphery and the financial account surpluses of EMU core can be traced back not only to core-periphery differences in time preference but also to differences in the production technology (capital production share) and government expenditure shares.

While the one-good approach is conducive to model intra-EMU financial account imbalances in line with the traditional explanation of euro-related financial integration, it is not appropriate to model intra-EMU current account imbalances which can be attributed to trade linkages between EMU’s subareas and the rest of the world. Moreover, since the EMU is a large open economy with potential impacts of intra-EMU developments on the other large trading areas and vice versa, the international interdependences among EMU, Asia and USA cannot be neglected. To this end, at least a three-good, three-country intertemporal equilibrium model is needed. To the best of this author’s knowledge of the literature, this three-country, two-region version of the seminal Buiter OLG model does not exist so far.

Thus, there are two main objectives of the paper: First, to present stylized macro facts regarding current and financial account imbalances between EMU’s core and periphery and the rest of the world in order to motivate the model set-up. Secondly, to develop a three-country (EMU, Asia, USA), two-region (EMU core and periphery) OLG model in order to figure out how EMU’s core-periphery external imbalances can be attributed both to financial integration due to the common currency and to core’s respective periphery’s evolving trade linkages to the rest of the world.
The new model specification features roughly main stylized macroeconomic facts of northern and southern EMU countries before the advent and after the introduction of the common currency until the outburst of the global financial crisis. Among them looms prominently the fact that the public debt-to-GDP ratios of several countries in southern euro area did not rise but declined (Lane 2012, p. 51). Acknowledging moreover the fact that the debt to GDP ratios of northern EMU countries increased only slightly, the modeling exercise assumes for simplicity that both northern and southern debt to GDP ratios stay constant over time.

The most obvious manifestation of the creation of the EMU was the convergence of high nominal (short- and long-term) interest rates in southern Europe towards the relatively low northern (German) rates. The main research question addressed by the following model analysis is whether and how the empirically observed international macroeconomic divergence between both northern and southern EMU countries and Asia and USA in the beginning 2000s can be attributed to the convergence of different pre-EMU interest rates (financial autarky). We suggest that differences in economic fundamentals like saving rates and capital production shares existing between northern and southern pre-EMU countries and between Asia and USA were transformed into the empirical observed macroeconomic divergences during the course of EMU interest rate convergence.

The paper is organized as follows. In the next section, main stylized macroeconomic facts, existing both before financial integration and during the EMU integration up to the outburst of the financial crisis in 2008 are assembled. In the following section, the first-order conditions (FOC) for constrained intertemporal utility and temporal profit maxima and the market-clearing conditions are separately specified for pre-euro financial autarky and financial integration after euro inception. After deriving the respective equilibrium dynamics, the existence and dynamic stability of steady-state solutions before and after financial integration are investigated. With these results at hand, it is then shown how the southern EMU and US current account deficit through EMU’s financial integration can be traced back to the lower saving rate in EMU’s South and in the USA and the relatively large capital production share in EMU’s South and in Asia. Concluding remarks in the final section summarize key results of the modeling approach.

**Stylized Macroeconomic Facts: Financial Autarky versus EMU’s Financial Integration**

In order to guide the design of the three-good, three-country and two-region OLG model, some stylized facts with respect to the macroeconomic performance of the EMU members,
Asia and the USA and the evolution of the current account and the net foreign asset position in the EMU, Asia and the USA before the launch of the euro in 1999 and up to 2008 are gathered in this section. Following Fagan and Gaspar (2008, p. 9), the EMU countries are separated into two groups based on the criteria of relative short-term real interest rates in the late 1990s, i.e. before the euro launch. The first group, usually denoted as the “core” countries, comprises low interest rate countries: Austria, Belgium, France, Germany and the Netherlands.\(^1\) The second group denoted as “periphery” or converging countries, consists of countries which had relatively high interest rates before the introduction of the euro (see figure 1).

**Fig. 1 Real short-term interest rates 1995-2008**

![Real short-term interest rates 1995-2008](image)

Legend: --- periphery, --- core. Source: Fagan and Gaspar (2008, p. 34) and own calculations using AMECO.

Figure 1 reveals that in contrast to the pre-EMU situation (before 1999), there is a sizeable convergence of real interest rates between EMU core and periphery thereafter.\(^2\)

**Fig. 2 Personal savings ratios in EMU core and periphery**

![Personal savings ratios in EMU core and periphery](image)

\(^1\) Nowadays Finland is included within core countries. Fagan and Gaspar (2008) exclude Finland from core countries since in the 1990s the Finnish economy was distorted by special factors after the collapse of the Soviet Union. We follow Fagan and Gaspar (2008).

\(^2\) Remaining differences in the real interest rates are due to inflation rate differences across EMU core and periphery.
Regarding differences in economic fundamentals, figure 2 portrays a substantially lower personal saving ratio (= household savings as percent of disposable income) in EMU’s periphery than in the core. Similarly, figures 3 and 4 reveal that US personal saving rates are substantially lower than Asian rates, particularly in the 1990s and 2000s.

Fig. 3 US personal saving rates 1960-2008.

![Graph showing US personal saving rates 1960-2008.](image)

Source: FRED Data.

Fig. 4 Asian personal saving rates 1990-2008.

![Graph showing Asian personal saving rates 1990-2008.](image)

Source: CEIC Data and author’s own calculations.

Fig. 5 Housing investment (as percent of GDP) in euro periphery and core 1995-2005

![Graph showing housing investment in euro periphery and core.](image)

Figure 5 portrays housing investment (as percent of GDP) in EMU periphery and core. While housing investment rose significantly in the periphery it declined in EMU’s core. Starting from a significantly lower personal saving ratio in EMU periphery relative to the core, housing investment expenditures in the periphery experienced a boom, while housing investment declined in the core countries. In view of the sharp increase in private domestic expenditures in the periphery and the muted response of output (Fagan and Gaspar 2008), macroeconomic equilibrium had to be established through changes in the external balances of these countries. As figure 6 shows, this resulted in significant current account deficits in the periphery.

**Fig. 6 Current account balances (as percent of GDP) in EMU periphery and core 1995-2008**

![Graph showing current account balances](image)


Not surprisingly, EMU periphery’s current account deficits led to the accumulation of a significant net foreign debtor position as shown in figure 7 below.

**Fig. 7 Net foreign assets (as percent of GDP) in EMU core and periphery 1994-2008**

![Graph showing net foreign assets](image)

Again, there is a similarly different evolution of current account and net foreign asset position ratios of major Asian countries and of the USA as figures 8 and 9 show.

**Fig. 8 Current account to GDP ratios in China, Japan, USA and UK**

![Current account to GDP ratios](image)

**Source:** ODS

**Fig. 9 US net foreign asset position (absolute and in percent to GDP)**

![US net foreign asset position](image)

**Source:** Wikimedia Commons

**Basic Model**

Consider an infinite-horizon model economy consisting of three areas (“countries”) of the world economy, namely (i) the EMU, comprising two regions, named North (indexed by $N$) representing EMU’s core and South (indexed by $S$) representing EMU’s periphery countries, (ii) the countries characterized by a current account surplus outside the EMU (indexed by $A$)
representing Asia and oil-exporting countries, and (iii) the advanced current-account deficit countries (indexed by $U$) representing mainly the USA. In each country one commodity, representing the aggregate of thousands of goods and services is produced. This can be used for the purpose of consumption as well as for investment. The EMU specializes completely in the production of good $X$, “Asia” in the production of good $Y$, and the “USA” in the production of good $Z$. Perfectly competitive firms in EMU’s South and North, in Asia and in the USA employ in every period $t=1,2,...$ labor services $N^i_t, i=S,N,A,U$ and capital services $K^i_t, i=S,N,A,U$ using the Cobb-Douglas (CD) production function $M^i(a,N^i_t)^{1-a} (K^i_t)^a, i=S,N,A,U$ to produce southern (northern) EMU aggregate output $X^S_t (X^N_t)$, Asia’s aggregate output $Y_t$ and US aggregate output $Z_t$ where $M^i > 0, i=S,N,A,U$ denote total factor productivity in EMU’s South (North), in Asia and in the USA, respectively. $a_t$ is the common labor productivity and $0 < \alpha^i < 1, i=S,N,A,U$ with $\alpha^U \approx \alpha^N < \alpha^S < \alpha^A$ are the capital production shares in EMU’s South (North), in Asia and in the USA.

One-period profit maximization by firms in EMU’s South (North), in Asia and in the USA implies the following FOCs:

$$w^i_t = (1-\alpha^i) M^i a_t \left( \frac{K^i_t}{a_t N^i_t} \right)^{\alpha^i}, i=S,N,A,U, \quad (1)$$

$$q^i_t = \alpha^i M^i \left( \frac{K^i_t}{a_t N^i_t} \right)^{\alpha^i-1}, i=S,N,A,U. \quad (2)$$

whereby $w^i_t$ denotes the real wage rate in region respective country $i=S,N,A,U$. $q^i_t, i=S,N,A,U$ denotes real unit capital user costs in region respective country $i=S,N,A,U$.\(^3\)

As usual in a Diamond (1965) type OLG framework, two generations of homogeneous individuals overlap in each period $t$. At date $t$, a new generation of size $L^i_t$ enters the economy of country (region) $i=S,N,A,U$. For simplicity we assume that $L^S_t = L^N_t = L_t$ for

---

\(^3\) In view of stylized facts presented in previous section a purely real model is clearly unable to explain all relevant empirical facts regarding EMU, Asia’s and US imbalances.
all \( t = 1, 2, \ldots \) and that the population growth factors of all countries (regions) are identical and equal to \( G^L \). In view of the empirically rather similar GDP growth rates in southern and northern EMU countries (Fagan and Gaspar 2008) we assume moreover that the respective growth factors of labor productivities \( G^{a^y} \) and \( G^{a^x} \) are equal in EMU’s South and North, an assumption which applies rather well also to the USA but lesser so to current-account surplus countries like China, India and other Asian countries. However, acknowledging the catch-up growth component in emerging countries’ GDP growth rates the simplifying assumption \( G^{a^y} = G^{a^x} = G^{a^y} = G^{a^x} \) seems to be less premature. This implies that the natural growth factor \( G^a = G^a G^L \) is the same in all countries.

Each generation lives for two periods, working during the first when young, and retiring in the second when old. The choice variables of each generation, when young, are denoted by superscript 1, and, when old, they are denoted by superscript 2. Each member of the generation entering the economy in period \( t \) supplies one unit of labor in-elastically to firms since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across the three countries and across EMU core and periphery is now addressed. Regarding the three countries, we assume that each country has its own currency and before the inception of the EMU South and North had their own currency, too. To mimic the introduction of the common currency in 1999 we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999 EMU’s South and North were financially autarkic while after the launch of the common currency EMU’s South and North became fully financially integrated. In contrast, the financial integration across the EMU and the other two countries remains incomplete: in spite of international mobility of governments bonds emitted by EMU’s southern and northern, by the Asian and the US governments EMU’s and foreign real interest rates do not converge along the intertemporal equilibrium path.

With regard to the trade linkages we assume that after the inception of the common currency the trade linkages between EMU’s North (South) and the rest of the world strengthened while before the euro the EMU and the foreign countries were autarkic. This strong assumption mimics the fact that in the decade after the launch of the euro Germany developed a significant trade surplus vis-à-vis Asia, in particular vis-à-vis China and oil exporters while the trade balance of Greece, Italy and Spain worsened vis-à-vis these
countries, Central and Eastern European countries (CEECs) included (see Chen et al. 2013 for empirical details).

Not surprisingly, both financial integration within the EMU and the evolving trade linkages between the EMU and Asia impact on the choice sets and constraints of younger households as well as on market clearing conditions. In order to work out the consequences of intra-EMU financial integration and the trade developments with non-euro countries as clearly as possible, the optimization problems of (younger) households and firms as well as the market clearing conditions are now described separately for the two cases of real and financial autarky and intra-euro financial integration and ROW-trade of EMU core and periphery.

**Pre-Euro Real and Financial Autarky**

In order to facilitate the modeling of the pre-euro situation as real and financial autarky, we first recall that large real interest rate differences existed between the core (North) and the periphery (South) of the later EMU. As figure 1 shows, southern real interest rates were sizeable larger than the corresponding northern rates. Second, in contrast to the later financial integration in the EMU, in the 1990s South (with the exception of Portugal) did not run large current account deficits (as percent of GDP). Hence, when modeling the period before the euro start it is not unrealistic to assume that both the current account and the net foreign asset position of South and North were zero. In contrast, in the 1990s Asia (including oil exporters) ran a current account surplus (as percent of GDP) roughly equivalent in size to the current account deficit of the USA (Engler 2009, p. 2). However, since at this time the US net foreign asset position was only moderately negative and China and other emerging Asian countries did not contribute much to the imbalance, we assume for the sake of analytic simplicity that the USA and Asia were financially autarkic as South and North were. Third, in contrast to the current post-crisis situation where huge differences in government debt to GDP ratios exist between EMU periphery and core, in the late 1990s the un-weighted average debt to GDP ratio of EMU periphery was not that different from the corresponding EMU core value which is also true for the US federal debt to GDP ratio. In contrast, the Asian public debt to GDP ratio was and is far below the EMU and US ratios. Moreover, the EMU North-South debt to GDP differences did not widen until the outburst of the global financial crisis which is also true for the US and Asia’s debt to GDP ratios. Since the objective of the modeling is to explain the effects of the intra-EMU evolution before the financial crisis it is appropriate to assume that the government debt to GDP ratios in all countries of the model economy remain
constant over time. Additionally, as figure 2 above shows the personal saving rate in South was persistently lower than in North. From figure 4 we know that Asia’s personal saving rate is significantly higher than the corresponding northern EMU rate, while the US personal saving rate is slightly below the southern EMU personal saving rate (see figure 3 and 2). Finally, in view of the differential development of labor compensations costs across EMU core and periphery, it is natural to assume4 corresponding differences in southern and northern production technologies. While US wage compensation cost develop similarly to northern EMU and hence similar production technologies can be applied, Asia’s production technology features a much higher capital production share than in the US or southern EMU (Bai and Qian 2010).

Against this empirical background of stylized facts the intertemporal utility maximization problem in later EMU’s South (North) before euro inception reads as follows:

\[ \max \to \ln x_{t+1}^{j,1} + \beta^j \ln x_{t+2}^{j,2} \]

s. t.:

(i) \[ x_{t+1}^{j,1} + s_t^j = w_t^j (1 - \tau_t^j), \quad s_t^j = \frac{K_t^{j,j}}{L_t^j} + \frac{B_t^{j,j}}{L_t^j}, \]

(ii) \[ x_{t+2}^{j,2} = \left(1 + i_{t+1}^j \right) s_t^j, \quad j = S, N, \]

where 0 < \beta^j \leq 1, j = S, N denotes the time discount factor of (later) EMU’s region \( j \) younger generation, \( x_{t+1}^{j,1}, j = S, N \) is the consumption per capita of the commodity produced in EMU’s region \( j \), \( s_t^j, j = S, N \) is EMU’s region \( j \) per-capita savings, \( \tau_t^j, j = S, N \) denotes region \( j \) flat wage tax rate5, \( x_{t+2}^{j,2}, j = S, N \) is old-age consumption per capita of the commodity produced in region \( j \), \( K_t^{j,j} / L_t^j, j = S, N \) is the real capital produced in region \( j \) which the region \( j \) younger household wants to hold at the beginning of the retirement period, \( i_{t+1}^j, j = S, N \) denotes the real interest rate on region \( j \) government bonds and

\[ \frac{K_t^{j,j}}{L_t^j} + \frac{B_t^{j,j}}{L_t^j} \]

4 This is tantamount to assume that the labor compensation cost differentials are not solely due to differences in output prices and national fiscal instruments.

5 The assumption of flat wage taxes clearly clashes with European tax code reality. However, since this paper does not focus on taxation for the sake of analytical simplicity a constant wage tax rate is assumed.
$B_{j}^t / L_{j}^t, j = S, N$ stands for the region $j$ government bonds the region $j$ younger household wants to hold at the beginning of its retirement period. Constraint (i) depicts the working period budget constraint while constraint (ii) represents the retirement period budget constraint.

After having described the intertemporal optimization problem of later EMU young households, we turn now to the intertemporal choice problems of Asian and US young households. By identifying Asia in the 1990s with Japan and oil exporting countries, we are not entitled to assume trade autarky with respect to Asia and the USA. Consistent with empirical facts, Asia and the United States exchanged before euro launch production goods in addition to the consumption of domestic products and investment in domestic products and domestic government bonds.

The Asian young household before euro launch solves the following intertemporal optimization problem:

Max $\zeta^y \ln y_{t}^{A1} + \zeta^z \ln z_{t}^{A1} + \beta^t \left[ \zeta^y \ln y_{t+1}^{A1} + \zeta^z \ln z_{t+1}^{A1} \right]$

s.t. (i) $y_{t}^{A1} + \frac{z_{t}^{A1}}{e_t} + s_t^{A} = w_t^{A} \left(1 - \tau_t^{A} \right)$, $s_t^{A} = \frac{K_{t+1}^{A1}}{L_t^{A}} + \frac{B_{t+1}^{A1}}{L_t^{A}}$,

(ii) $y_{t+1}^{A1} + \frac{z_{t+1}^{A1}}{e_{t+1}} = \left(1 + i_{t+1}^{A1}\right)\left(\frac{K_{t+1}^{A1}}{L_t^{A}} + \frac{B_{t+1}^{A1}}{L_t^{A}}\right)$,

where again $0 < \beta^t < 1$ denotes the Asian time discount factor, $0 < \zeta^y < 1$ ($0 < \zeta^z < 1$) is the utility elasticity of consuming the Asian (US) product, $y_{t}^{A1} (z_{t}^{A1})$ represents consumption of the domestic (foreign) good by the Asian young household during the working period, $e_t$ denotes the terms of trade of the Asian product (= units of the US good per unit of the Asian good), $s_t^{A}$ represents household real savings, i.e. in terms of the Asian product, $w_t^{A}$ is the Asian real wage rate and $\tau_t^{A}$ is the Asian wage tax rate. Household’s savings are invested in Asian real capital $K_{t+1}^{A1} / L_t^{A}$ and in Asian government bonds $B_{t+1}^{A1} / L_t^{A}$ which the Asian young household plans to hold at the beginning of period $t+1$. Obviously, domestic real capital and domestic government bonds are perfectly substitutable from the perspective of the Asian young household. In the retirement period, the then old Asian household uses the proceeds from the return on investment in domestic real capital and domestic government bonds, $(1 + i_{t+1}^{A1})(K_{t+1}^{A1} / L_t^{A} + B_{t+1}^{A1} / L_t^{A})$ with $i_{t+1}^{A1}$ representing the real Asian interest rate in period rate, in order finance the retirement consumption of the domestic good, $y_{t+1}^{A}$, and the US good, $z_{t+1}^{A}$.
Finally, the US young household before euro launch solves the following intertemporal optimization problem:

$$\text{Max} \to \zeta^y \ln y_t^{U,1} + \zeta^z \ln z_t^{U,1} + \beta^z \left[ \zeta^y \ln y_{t+1}^{U,2} + \zeta^z \ln z_{t+1}^{U,2} \right]$$

s.t.: (i) $e_t y_t^{U,1} + z_t^{U,1} + s_t^{U} = w_t^{U} \left(1 - \tau_t^{U}\right)$, $s_t^{A} = \frac{K_{t+1}^{U,U} + B_{t+1}^{U,U}}{L_t^U}$,

(ii) $e_t y_t^{U,2} + z_t^{U,2} = \left(1 + i_t^{U,2}\right) \left[\frac{K_{t+1}^{U,U} + B_{t+1}^{U,U}}{L_t^U} + \frac{B_{t+1}^{U,U}}{L_t^U}\right]$,

where all parameters and variables have an analogous interpretation as in the optimization problem of the Asian young household.

The government of each country (region) $i = S, N, A, U$ taxes labor income and uses the proceeds from additional borrowing to finance the interest costs on existing government debt and government expenditures. The government budget constraint of country (region) $i$ reads as follows:

$$B_{t+1}^{i} - B_{t}^{i} + \tau_t^{i}w_t^{i}L_t^{i} = \delta_t^{i}B_t^{i} + \Gamma_t^{i}, i = S, N, A, U,$$  \hspace{1cm} (3)

where $\Gamma_t^{i}$ denotes real government expenditures and $B_t^{i}$ is the level of real government debt in country (region) $i = S, N, A, U$ at the beginning of period $t$. In line with Diamond (1965), we assume that government expenditures are unproductive.

In addition to the restrictions imposed by household and firm optimization and by the above government budget constraints, markets for labor have to clear in all countries (regions) and in all periods.

$$N_t^{i} = L_t^{i}, i = S, N, A, U, \hspace{0.5cm} t = 0,1,2,...$$  \hspace{1cm} (4)

Since the market for financial assets is competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds in all countries (regions):

$$1 + i_t^{i} = q_t^{i} + 1 - \delta, i = S, N, A, U, \hspace{0.5cm} t = 0,1,2,...,$$  \hspace{1cm} (5)

whereby $0 < \delta \leq 1$ depicts the common fixed depreciation rate of private capital (period by period) in country (region) $i$.

Regarding clearing of product and asset markets we have to distinguish the two regions of the later EMU from Asia and US.

The asset market clearing conditions in the later EMU regions read as follows:
\[ L_i^j = K_{\tau+1}^i + B_{\tau+1}^i, \quad j = S, N, t = 0,1,2, \ldots, \quad (6) \]
\[ B_i^t = B_i^{t,j}, K_i^t = K_i^{t,j}, \quad j = S, N, t = 0,1,2, \ldots \]

In accordance with Walras’ Law, the clearing condition for the product market is irrelevant in region \( j = S, N \).

Clearing of government bond and real capital markets in Asia and US requires:
\[ B_i^A = B_i^{A,t}, K_i^A = K_i^{A,t}, \quad t = 0,1,2, \ldots \]
\[ (8) \]
\[ B_i^U = B_i^{U,t}, K_i^U = K_i^{U,t}, \quad t = 0,1,2, \ldots \]

(9)

Finally, we have the conditions for the clearing of the product markets:
\[ Y_i = L_i^1 y_i^A + L_i^2 y_i^{A,2} + L_i^3 y_i^{A,3} + L_i^4 y_i^{A,4} + \Gamma_i^A + K_i^A, \quad t = 0,1,2, \ldots \]
\[ (10) \]
\[ Z_i = L_i^1 z_i^U + L_i^2 z_i^{U,2} + L_i^3 z_i^{U,3} + L_i^4 z_i^{U,4} + \Gamma_i^U + K_i^U, \quad t = 0,1,2, \ldots \]

(11)

In order to be able to model the fact of time-stationarity of country (region) \( i \)’s public debt to GDP ratios between 1999 and 2008 we transform total outstanding government debt in country (region) \( i \)’s government budget constraint into debt to GDP ratios. This is achieved by dividing both sides of (3) by \( X_i \) for \( i = S, N \), by \( Y_i \) for \( i = A \), by \( Z_i \) for \( i = U \) and by defining the debt to GDP ratios as
\[ b_i^t = B_i^t / X_i, \quad i = S, N, \quad b_i^A = B_i^A / Y_i, \quad b_i^U = B_i^U / Z_i \]
and we obtain for country (region) \( i \):
\[ G_i^{X,t} b_i^{t+1} = \left( 1 + i_i^X \right) b_i^t + \gamma_i^X - \tau_i^X \left( 1 - \alpha_i^X \right), \quad \text{with} \quad G_i^{X,t} = X_{\tau+1}^i / X_i^t, \quad \gamma_i^X = \Gamma_i^X / X_i^t, \quad w_i^X L_i^X = 1 - \alpha_i^X, \quad i = S, N \]
\[ (12) \]
\[ G_i^{Y,t} b_i^{t+1} = \left( 1 + i_i^A \right) b_i^t + \gamma_i^A - \tau_i^A \left( 1 - \alpha_i^A \right), \quad \text{with} \quad G_i^{Y,t} = Y_{\tau+1}^i / Y_i^t, \quad \gamma_i^A = \Gamma_i^A / Y_i^t, \quad w_i^A L_i^A = 1 - \alpha_i^A \]
\[ (13) \]
\[ G_i^{Z,t} b_i^{t+1} = \left( 1 + i_i^U \right) b_i^t + \gamma_i^U - \tau_i^U \left( 1 - \alpha_i^U \right), \quad \text{with} \quad G_i^{Z,t} = Z_{\tau+1}^i / Z_i^t, \quad \gamma_i^U = \Gamma_i^U / Z_i^t, \quad w_i^U L_i^U = 1 - \alpha_i^U \]
\[ (14) \]

Dividing the asset market clearing condition (6) on both sides by \( X_i^t, \quad i = S, N \) and using the definition of the capital output ratio \( \nu_i^t \equiv K_i^t / X_i^t, \quad i = S, N \), (6) can be rewritten as follows:
In view of the C-D production function and noting
\[ G^*_{i,j} = \left( K_{i}^{*}\right)^{\alpha} \left( a_{i} L_{i}\right)^{1-\alpha} = \left( a_{i} L_{i}\right) \left( K_{i+1}^{*}/a_{i} L_{i+1}\right)^{\alpha}, i = S, N, \]

it turns out that \( G^*_{i,j} = G^n \left( v_i^{i}/v_{i+1}^{i} \right)^{\sigma/(1-\sigma)}. \)

Acknowledging the empirical fact that the pre-crisis public debt to GDP ratios in all countries (regions) remained roughly constant over time we assume time-stationary public debt to GDP ratios:

\[
\frac{B_l^i}{X_i^t} = B_{l+1}^i > 0, i = S, N, \frac{B_{l+1}^A}{Y_{t+1}} = B_{l+1}^A > 0, \frac{B_{l+1}^U}{Z_{t+1}} = B_{l+1}^U, B^U > 0, \forall t. \tag{16}
\]

Moreover, we assume time-stationary government expenditure shares:

\[ \gamma_i = \gamma_{i+1}, \forall t, 0 < \gamma_i < 1, i = S, N, A, U. \tag{17} \]

The government budget constraints (12-14) together with (16) and (17) yield \( 1 - \tau^i_t \) as follows:

\[
1 - \tau^i_t = \frac{1}{1-\alpha} \left[ G^*_{i,j} - (1+\tau^i_t) \right], i = S, N, 1 - \tau^A_t = \frac{1-\alpha^A - \gamma^A_t}{1-\alpha} G^A_{t} - (1+\tau^A_t), 1 - \tau^U_t = \frac{1-\alpha^U - \gamma^U_t}{1-\alpha^U} G^U_{t} - (1+\tau^U_t), \tag{18}
\]

Using the Cobb-Douglas production function it is easily seen that

\[ K_i^*/X_i^t \equiv v_i^t = (1/M^i)[K_i^*/(a,N_i^t)]^{1-\alpha}, i = S, N, K_i^A/Y_i^A \equiv v_i^A = (1/M^A)[K_i^A/(a,N_i^A)]^{1-\alpha^A}, K_i^U/Y_i^U \equiv v_i^U = (1/M^U)[K_i^U/(a,N_i^U)]^{1-\alpha^U}. \]

Thus, the FOC for profit maximizing capital service input (2) can be equivalently written as follows:

\[ \frac{\alpha^i}{v_{i+1}^i} = q_i^i = i_t^i + \delta, i = S, N, A, U. \tag{19} \]
In order to simplify the algebra, we assume $\delta = 1$. Then, acknowledging (19) in (18) and considering $G_t^X = G^n \left( v_{i+1}^f / v_i^f \right)^{\alpha / (1-\delta)}$, $i = S, N$ ($G_t^Y = G^n \left( v_{i+1}^A / v_i^A \right)^{\alpha / (1-\delta)}$, $G_t^Z = G^n \left( v_{i+1}^U / v_i^U \right)^{\alpha / (1-\delta)}$) yields:

$$1 - \tau_i = \frac{1 - \alpha' - \gamma'}{1 - \alpha'} + \frac{b}{1 - \alpha'} \left[ G^n \left( v_{i+1}^f / v_i^f \right)^{\alpha / (1-\delta)} - \alpha' \right] = \frac{1 - \alpha' - \gamma'}{1 - \alpha'} + \frac{b' G^n \left( v_{i+1}^f / v_i^f \right)^{\alpha / (1-\delta)}}{1 - \alpha'}$$

$$- \frac{\alpha' b'}{(1 - \alpha') v_i^f}, i = S, N. \quad (20)$$

The intertemporal equilibrium dynamics of the capital-output ratio in later EMU South (North) is obtained by inserting (20) into (15):

$$G^n \left( v_{i+1}^f / v_i^f \right)^{\alpha / (1-\delta)} \left( v_{i+1}^f + b' \right) = \sigma' \left[ 1 - \alpha' - \gamma' + b' G^n \left( v_{i+1}^f / v_i^f \right)^{\alpha / (1-\delta)} - \frac{\alpha' b'}{v_i^f} \right], i = S, N. \quad (21)$$

or:

$$\left( v_{i+1}^f \right)^{\alpha / (1-\delta)} + b' \left[ 1 - \sigma' \right] \left( v_{i+1}^f \right)^{\alpha / (1-\delta)} = \sigma' \left[ 1 - \alpha' - \gamma' - \frac{\alpha' b'}{v_i^f} \right] \left( v_i^f \right)^{\alpha / (1-\delta)}, i = S, N \quad (22)$$

As usual, a steady-state intertemporal equilibrium is defined as a fixed point of the difference equation in (22): $v_{i+1}^f = v_i^f$, $i = S, N$. Evaluating (22) at a steady state yields:

$$(v_i^f)^2 + \frac{G^n b' - \beta' (1 - \alpha' - \gamma')}{G^n (1 + \beta')} v_i^f + \frac{\alpha' \beta' b'}{G^n (1 + \beta')} = 0, i = S, N. \quad (23)$$

**Proposition 1** (Existence of steady solutions in South and North)

Suppose that $0 < b' \leq \bar{b}' < \beta' (1 - \alpha' - \gamma') / G^n$, $i = S, N$ while $\bar{b}'$ solves $\beta' (1 - \alpha' - \gamma') - G^n \bar{b}' = 2 \sqrt{\alpha' / (1 + \beta')} G^n \bar{b}'$. Then, there are exactly two strictly positive steady state solutions as follows:
\[ \nu_i' = \frac{\beta'(1 - \alpha' - \gamma') - b'G^n - \sqrt{\beta'(1 - \alpha' - \gamma') - b'G^n}^2 - 4\alpha' \beta'(1 + \beta')b'G^n}{2(1 + \beta')G^n}, \quad (24) \]

\[ \nu_i'' = \frac{\beta'(1 - \alpha' - \gamma') - b'G^n + \sqrt{\beta'(1 - \alpha' - \gamma') - b'G^n}^2 - 4\alpha' \beta'(1 + \beta')b'G^n}{2(1 + \beta')G^n}, \quad i = S, N. \]


Since there are two steady-state solutions (local) dynamic stability needs to be investigated which is done in proposition 2.

Proposition 2 (Dynamic stability of steady solutions in South and North)

Suppose that \( 0 < b' < \beta', i = S, N \). Then, the steady-state solution \( \nu_i' \) in (24) is asymptotically unstable while the steady-state solution \( \nu_i'' \) in (24) is asymptotically stable.


Knowing that the larger steady state solution in (24) is asymptotically stable we use it to attribute the empirically observed pre-euro North-South differences with respect to the real interest rates (and real wage rates) to North-South differences regarding fundamentals including private saving rates, governments’ expenditure ratios and capital production shares. To this end, we first try to find out how the fundamental parameters impact the steady-state value of the capital-output ratio in (24). Second, we need information about the relative magnitudes of the saving rates and capital production shares in pre-euro North and South.

Doing the first step, it is helpful to re-write the larger steady-state solution in (24) by using the definition of the saving rates \( \sigma^i \equiv \beta^i / (1 + \beta^i), i = S, N \) as follows:

\[ \nu_i^S = (2G^n)^{-1} [(1 - \alpha^S - \gamma^S)\sigma^S - (1 - \sigma^S)b^S G^n + \sqrt{[(1 - \alpha^S - \gamma^S)\sigma^S - (1 - \sigma^S)b^S G^n]^2 - 4\alpha^S b^S G^n \sigma^S}], (25a) \]

\[ \nu_i^N = (2G^n)^{-1} [(1 - \alpha^N - \gamma^N)\sigma^N - (1 - \sigma^N)b^N G^n + \sqrt{[(1 - \alpha^N - \gamma^N)\sigma^N - (1 - \sigma^N)b^N G^n]^2 - 4\alpha^N b^N G^n \sigma^N}], (25b) \]

Comparing the right-hand side of (25a) to that of (25b) we are led to the following proposition 3.

Proposition 3. Suppose for simplicity that \( b^N = b^S \). Moreover, assume that \( b' < \beta', i = S, N \).

If \( \alpha^S > \alpha^N \), \( \gamma^S \geq \gamma^N \) and \( \sigma^S < \sigma^N \), then \( \nu_i^S < \nu_i^N \) implying \( i^S > i^N \) and \( w^S < w^N \).


The second step is to ensure that the assumptions of proposition 3 are empirically warranted with respect to northern and southern candidate countries for EMU in the late
1990s. The simplifying assumption \( b^N = b^S \) is not warranted (Lane 2012, p. 51), however, the better fitting assumption \( b^S > b^N \) would only enforce the claim in proposition 3 as can be numerically verified. \( \alpha^S > \alpha^N \) is empirically warranted since the southern EMU countries were (are) less developed (lower GDP per capita) than the northern countries and there are prominent empirical examples for the fact that the capital production share is higher in catching-up than in advanced countries (see Bai and Quian (2010) for the high Chinese capital production share of nearly 50% and Caselli and Feyrer (2007) for the much lower US capital production share of 30%). The opposite holds with respect to the government expenditure quota: less developed countries exhibit lesser expenditure quotas than highly developed countries. Since, however, large-economy Italy belongs to the southern bloc \( \gamma^N = \gamma^S \) is rather close to reality which implies that proposition 3 remains relevant. Finally, in view of the empirical evidence provided by figure 2 above it is natural to assume that \( \sigma^S < \sigma^N \), i.e. the saving rate of the southern EMU countries is less than that of northern countries.

Proposition 3 says that the relatively high capital production share and the low saving rate in South imply under financial autarky that the steady-state capital output ratio in South is lower than in North, and is associated with a higher real interest and a relatively low real wage rate. This claim is intuitively plausible. A low saving rate implies for a given capital output ratio low savings thus driving the capital output ratio down to ensure asset market clearing. The capital output ratio is also depressed by a relatively high capital income share since this implies a relatively low labor income share associated with low per capita savings. Due to decreasing marginal productivity of capital the lower capital output ratio is associated with a higher interest rate and a lower real wage rate.

Not surprisingly, under financial autarky both the southern (northern) current account \( CA^i_t = (1 - \gamma^i)X^i_t - K^i_t - L^i_t x^i_t - L^i_{t-1} x^{i-1}_t, i = S, N \) and the respective net foreign asset position \( \Phi^i_{r+1} = L^i_s - a_{r+1} L^i_{r+1} (M^i)^{\delta_l} (v^{i+1}_t)^{\delta_l} (v^{i-1}_t + b^i), i = S, N \) are zero, i.e. no international borrowing and lending takes place in spite of the interest rate differential across countries. Obviously, the costs associated with shifting capital from low-yielding North to profitable South are prohibitively large. When modeling the advent of the common currency we assume that these international capital mobility costs are completely removed over night while the structural parameters of both economies remain as assumed in proposition 3. Before exploring the consequences of completely removing international capital transaction costs in the next
section, we check the existence and dynamic stability of steady-state solutions of the intertemporal equilibrium dynamics in the Asian-US economy.

The intertemporal equilibrium dynamics of the Asian US capital-output ratio is obtained as in North and South:

\[
\left( v^A_{t+1} \right)^{1-\alpha^A} + b^A \left[ 1 - \sigma^A \right] \left( v^A_t \right)^{\alpha^A} = \frac{\sigma^A}{G^A} \left[ 1 - \alpha^A - \gamma^A - \frac{\alpha^A b^A}{v^A_t} \right] \left( v^A_t \right)^{\alpha^A}, \quad j = A, U. \tag{26}
\]

In order to determine the Asian terms of trade we form the ratio of the US product market clearing condition (13) to the Asian product market clearing (12):

\[
\frac{(1-\gamma^U)Z_t - K^U_{t+1}}{(1-\gamma^A)Y_t - K^A_{t+1}} = \frac{L^U_{t+1} z_t^U + L^U_{t+1}z^U_{t+2} + L^A_{t+1}z_t^A + L^U_{t+2}z_t^A}{L^A_t y_t^A + L^A_t y_{t+1}^A + L^U_t y_{t+1}^U + L^A_t y_{t+2}^A}. \tag{27}
\]

From the solution of the Asian respective US intertemporal utility maximization problem we obtain under the simplifying assumption \( \zeta^A + \zeta^U = 1 \) the following consumption functions:

\[
y^A_t = \frac{\zeta^x w^A_t (1 - \tau^A_t)}{(1 + \beta^A)}, \tag{28}
\]

\[
z^A_t = \frac{\zeta^z w^A_t (1 - \tau^A_t)e_t^A}{(1 + \beta^A)}, \tag{29}
\]

\[
y^A_{t+1} = \frac{\zeta^x (1 + i^A_{t+1})\sigma^A w^A_t (1 - \tau^A_t)}{(1 + \beta^A)}, \tag{30}
\]

\[
z^A_{t+1} = \frac{\zeta^z (1 + i^A_{t+1})e_{t+1}\sigma^A w^A_t (1 - \tau^A_t)}{(1 + \beta^A)}, \tag{31}
\]

\[
y^U_t = \frac{\zeta^x w^U_t (1 - \tau^U_t)}{e_t^U (1 + \beta^U)}, \tag{32}
\]

\[
z^U_t = \frac{\zeta^z w^U_t (1 - \tau^U_t)}{(1 + \beta^U)}, \tag{33}
\]

\[
y^U_{t+1} = \frac{\zeta^x (1 + i^U_{t+1})\sigma^U w^U_t (1 - \tau^U_t)}{e_{t+1}}, \tag{34}
\]

\[
z^U_{t+1} = \frac{\zeta^z (1 + i^U_{t+1})\sigma^U w^U_t (1 - \tau^U_t)}{(1 + \beta^U)}. \tag{35}
\]
Using Asian and US production functions, the ratio of US to Asian GDP turns out to be as follows:

$$\frac{Z_t}{Y_t} = \frac{L^U_{t}}{L^A_{t}} \mu \left( \frac{v_{t}^U}{v_{t}^A} \right)^{\frac{\alpha^U}{\alpha^A - 1}}, \mu \equiv \frac{(M^U)^{(1-\alpha^U)}}{(M^A)^{(1-\alpha^A)}}. \quad (36)$$

Inserting (28)-(35) into equation (27), dividing the denominator of (27) on both sides by $Z_t$, dividing the numerator on both sides of (27) by $Y_t$, we obtain after simplifying and rearranging:

$$e_t = \frac{\xi^\gamma \mu L^U_t [1 - \gamma^U - v_{t+1}^U G^\alpha (v_{t+1}^U/v_{t+1}^U)^{\alpha^U/(1-\alpha^U)}] (v_{t+1}^U)^{\alpha^U/(1-\alpha^U)}}{\zeta^\alpha L^A_t [1 - \gamma^A - v_{t+1}^A G^\alpha (v_{t+1}^A/v_{t+1}^A)^{\alpha^A/(1-\alpha^A)}] (v_{t+1}^A)^{\alpha^A/(1-\alpha^A)}}. \quad (37)$$

A steady state intertemporal equilibrium is now defined as: $v_{t+1}^A = v_t^A$, $v_{t+1}^U = v_t^U$, $e_{t+1} = e_t = e$. Evaluating (26) and (37) at a non-trivial steady state yields:

$$(v_j)^2 + \frac{G^b_j - \beta^j (1 - \alpha^j - \gamma^j)}{G^a (1 + \beta^j)} v_j + \frac{\alpha^j \beta^j b^j}{G^a (1 + \beta^j)} = 0, \quad j = A, U, \quad (38)$$

$$e = \frac{\xi^\gamma \mu L^U_t [1 - \gamma^U - v_{t+1}^U G^\alpha (v_{t+1}^U/v_{t+1}^U)^{\alpha^U/(1-\alpha^U)}] (v_{t+1}^U)^{\alpha^U/(1-\alpha^U)}}{\zeta^\alpha L^A_t [1 - \gamma^A - v_{t+1}^A G^\alpha (v_{t+1}^A/v_{t+1}^A)^{\alpha^A/(1-\alpha^A)}] (v_{t+1}^A)^{\alpha^A/(1-\alpha^A)}}. \quad (39)$$

It is immediate that propositions analogous to propositions 1 and 2 above can be formed that ensure the existence and dynamic stability of steady-state solutions for the Asian-US economy. Also proposition 3 can be analogously applied to the Asian-US economy.

**International Equilibrium under intra-Euro Financial Integration and trade with ROW**

To mimic financial integration arising through the set-up of the EMU we assume in line with Buiter (1981) and Lin (1994) that both physical capital and government bonds can be freely traded across southern and northern Home without incurring any transaction costs. In view of the higher interest rate in South, northern younger households will use their savings to invest in southern physical capital and buy the bonds emitted by the southern government until the southern real interest rate declines as much as there is no longer an incentive to shift northern savings towards the South.

Since the same composite commodity is produced in North and South, financial integration does not induce any commodity trade between EMU core and periphery. Thus, while younger households in South cannot choose between consumption of the domestic and

---

6 To mimic the facts presented in Figure 5 above we assume that physical capital is mainly accumulated by housing investment.
of the northern commodity, they can after euro launch choose between investing their savings in domestic or northern real capital and domestic or northern government bonds. Moreover, trade in goods and services between EMU core and periphery and ROW is now possible, since Asia respective the US specializes on other composite goods than EMU. Now, southern households can buy foreign good in addition to the domestic commodity. The budget constraint (in real and per-capita terms) of the household living in South, when young is:

\[ x_{t+1}^{S,1} + (1/e_t^A) y_{t+1}^{S,1} + (1/e_t^U) z_{t+1}^{S,1} + s_t^S = w_t^N (1- \tau_t^S), \text{ with } s_t^S = \frac{K_t^{S,N}}{L_t} + \frac{B_t^{S,S}}{L_t}. \quad (40) \]

Now the southern EMU young household buys \( y_{t+1}^{S,1} \) from Asia at the relative price of \( 1/e_t^A \) and \( z_{t+1}^{S,1} \) from the USA at the relative price of \( 1/e_t^U \). Now \( e_t^A \) denotes the units of the Asian good per unit of EMU good, while \( e_t^U \) portrays the units of the US good per unit of EMU good. In line with pre-crisis empirical reality, the southern EMU young household invests its savings only in domestic real capital and government bonds.

When old the budget constraint of period-\( t \) young household in southern EMU is:

\[ x_{t+1}^{S,2} + (1/e_{t+1}^A) y_{t+1}^{S,2} + (1/e_{t+1}^U) z_{t+1}^{S,2} = q_{t+1}^{S} \left( \frac{K_{t+1}^{S,N}}{L_t} \right) + (1+i_{t+1}^S) \left( \frac{B_{t+1}^{S,S}}{L_t} \right). \quad (41) \]

In line with the new stylized facts about euro area imbalances (Chen et al. 2013), southern EMU households attribute utility not only to consumption of the domestic good but they also benefit from consuming Asian and US goods. Thus, each younger household in southern EMU maximizes its utility function \( \zeta^x \ln x_{t+1}^{S,1} + \zeta^y \ln y_{t+1}^{S,1} + \zeta^z \ln z_{t+1}^{S,1} + \beta^S (\zeta^x \ln x_{t+1}^{N,2} + \zeta^y \ln y_{t+1}^{N,2} + \zeta^z \ln z_{t+1}^{N,2}) \) with \( \zeta^z = 1 - \zeta^x - \zeta^y \) subject to the budget constraints defined by equations (40) and (41).

Analogously, the intertemporal utility maximization problem of the typical northern EMU household reads as follows:

Max \( \zeta^x \ln x_{t+1}^{N,1} + \zeta^y \ln y_{t+1}^{N,1} + \zeta^z \ln z_{t+1}^{N,1} + \beta^N (\zeta^x \ln x_{t+1}^{N,2} + \zeta^y \ln y_{t+1}^{N,2} + \zeta^z \ln z_{t+1}^{N,2}) \)

s.t.:

\[ (i) \quad x_{t+1}^{N,1} + y_{t+1}^{N,1} e_t^A + z_{t+1}^{N,1} e_t^U + s_t^N = w_t^N (1- \tau_t^N), \quad s_t^N \equiv K_{t+1}^{N,N} / L_t + K_{t+1}^{N,S} / L_t + \frac{B_{t+1}^{N,N}}{L_t} + \frac{B_{t+1}^{N,S}}{L_t}, \quad (42) \]

\[ (ii) \quad x_{t+1}^{N,2} + y_{t+1}^{N,2} e_t^A + z_{t+1}^{N,2} e_t^U = q_{t+1}^{N} \left( \frac{K_{t+1}^{N,N}}{L_t} \right) + (1+i_{t+1}^N) \left( \frac{B_{t+1}^{N,S}}{L_t} \right) + (1+i_{t+1}^S) \left( \frac{B_{t+1}^{S,N}}{L_t} \right). \]

Here, \( y_{t+1}^{N,1} \) (\( z_{t+1}^{N,1} \)) stands for the purchases of Asian (US) goods by the northern EMU young household, while \( K_{t+1}^{N,N} / L_t \) and \( B_{t+1}^{N,S} / L_t \) denote the respective stocks of southern real capital and government bonds which the northern EMU young household wants to hold at the
beginning of period $t+1$. Since physical capital and government bonds in each EMU region are perfectly substitutable, and since within the monetary union both assets can be assumed to be perfectly mobile across South and North, the following international Fisher equation ($=$ real international interest parity condition) holds in addition to the national Fisher equations (5):

$$1 + i^{S}_{t+1} = 1 + i^{N}_{t+1}. \tag{43}$$

The typical Asian young household solves the following optimization problem:

$$\text{Max} \rightarrow \zeta^{x} \ln x^{A,1}_{t} + \zeta^{y} \ln y^{A,1}_{t} + \zeta^{z} \ln z^{A,1}_{t} + \beta^{A}(\zeta^{x} \ln x^{A,2}_{t+1} + \zeta^{y} \ln y^{A,2}_{t+1} + \zeta^{z} \ln z^{A,2}_{t+1})$$

s.t.:

$$(i) e^{A}_{t} x^{A,1}_{t} + y^{A,1}_{t} + e^{A}_{t} z^{A,1}_{t} + s^{A}_{t} = w^{A}_{t}(1 - \tau^{A}_{t}), \quad s^{A}_{t} \equiv \frac{K^{A,A}_{t+1}}{L^{A}_{t}} + \frac{B^{A,A}_{t+1}}{L^{A}_{t}} + \frac{e^{A}_{t} B^{A,1}_{t+1}}{L^{A}_{t}}$$

$$(ii) e^{A}_{t+1} x^{A,2}_{t+1} + y^{A,2}_{t+1} + \frac{e^{A}_{t+1} z^{A,2}_{t+1}}{e^{A}_{t+1}} = q^{A}_{t+1} \left( \frac{K^{A,A}_{t+1}}{L^{A}_{t}} + \frac{B^{A,1}_{t+1}}{L^{A}_{t}} \right) + \left( 1 + i^{A}_{t+1} \right) \frac{e^{A}_{t+1} B^{A,1}_{t+1}}{L^{A}_{t}} \right). \tag{44}$$

Here, $x^{A,1}_{t}$ stands for the purchases of EMU goods by the Asian young household at the relative price of $e^{A}_{t}$, while the purchase of the US product by the Asian young household occurs at the relative price $e^{U}_{t} / e^{A}_{t}$, i.e. units of the Asian product per unit of the US good. $B^{U,1}_{t+1} / L^{U}_{t}$ denotes the stock of US government bonds which the Asian young household wants to hold at the beginning of period $t+1$. In line with pre-crisis reality the Asian young household does not hold EMU government bonds.

Finally, the typical US young household solves the following optimization problem:

$$\text{Max} \rightarrow \zeta^{x} \ln x^{U,1}_{t} + \zeta^{y} \ln y^{U,1}_{t} + \zeta^{z} \ln z^{U,1}_{t} + \beta^{U}(\zeta^{x} \ln x^{U,2}_{t+1} + \zeta^{y} \ln y^{U,2}_{t+1} + \zeta^{z} \ln z^{U,2}_{t+1})$$

s.t.:

$$(i) e^{U}_{t} x^{U,1}_{t} + \frac{e^{U}_{t} y^{U,1}_{t}}{e^{A}_{t}} + z^{U,1}_{t} + s^{U}_{t} = w^{U}_{t}(1 - \tau^{U}_{t}), \quad s^{U}_{t} \equiv \frac{K^{U,U}_{t+1}}{L^{U}_{t}} + \frac{B^{U,U}_{t+1}}{L^{U}_{t}} + e^{U}_{t} \frac{B^{U,1}_{t+1}}{L^{U}_{t}}$$

$$(ii) e^{U}_{t+1} x^{U,2}_{t+1} + \frac{e^{U}_{t+1} y^{U,2}_{t+1}}{e^{A}_{t+1}} + z^{U,2}_{t+1} = q^{U}_{t+1} \left( \frac{K^{U,U}_{t+1}}{L^{U}_{t}} + \frac{B^{U,1}_{t+1}}{L^{U}_{t}} \right) + \left( 1 + i^{U}_{t+1} \right) \frac{e^{U}_{t+1} B^{U,1}_{t+1}}{L^{U}_{t}} \right). \tag{45}$$

Here $x^{U,1}_{t}$ stands for US young household’s purchases of the EMU product while $e^{U}_{t} / e^{A}_{t}$ indicates now the units of the US product per unit of the Asian product. Again in line with pre-crisis empirical reality, the US young household does hold only northern EMU government bonds.

In order to ensure arbitrage-free terms of trade, the following international real interest parity conditions in addition to (43) ought to hold:
\[
1 + i^{A}_{t+1} = \frac{e^{A}_{t+1}}{e^{A}_{t}} (1 + i^{N}_{t+1}), \quad \forall t = 0,1,2,\ldots , \tag{46}
\]

\[
1 + i^{U}_{t+1} = \frac{e^{U}_{t+1}}{e^{U}_{t}} (1 + i^{N}_{t+1}), \quad \forall t = 0,1,2,\ldots . \tag{47}
\]

The markets for southern and northern EMU and Asian and US real capital clear according to:

\[
K^{S}_{t+1} = K^{S,N}_{t+1} + K^{N,N}_{t+1}, \quad \forall t = 0,1,2,\ldots . \tag{48}
\]

The markets for southern and northern EMU, Asian and US government bonds clear according to:

\[
B^{S}_{t+1} = B^{S,N}_{t+1} + B^{N}_{t+1}, \quad \forall t = 0,1,2,\ldots . \tag{49}
\]

The international real interest parity conditions (43), (46) and (47) ensure that the worldwide amount of savings equals the worldwide supply of assets from southern and northern EMU, Asia and the US:

\[
L^{S}_{t} + L^{N}_{t} + L^{A}_{t} + L^{U}_{t} = K^{S}_{t+1} + B^{S}_{t+1} + B^{N}_{t+1} + B^{A}_{t+1} + B^{U}_{t+1}, \quad \forall t = 0,1,2,\ldots . \tag{50}
\]

Finally, the product markets in EMU, Asia and US clear for all \( t = 0,1,2,\ldots \) according to the following conditions:

\[
X^{S}_{t} + X^{N}_{t} = L^{A}_{t} x^{A,1}_{t} + L^{A}_{t-1} x^{A,2}_{t} + \Gamma^{S}_{t} + \Gamma^{N}_{t} + L^{N,1}_{t} x^{N,1}_{t} + L^{N,2}_{t} x^{N,2}_{t} + L^{U}_{t} y^{U,1}_{t} + L^{U}_{t-1} y^{U,2}_{t}, \tag{51}
\]

\[
Y_{t} = L^{A}_{t} y^{A,1}_{t} + L^{A}_{t-1} y^{A,2}_{t} + \Gamma^{A}_{t} + L^{A}_{t} y^{A,1}_{t} + L^{A}_{t-1} y^{A,2}_{t} + L^{N,1}_{t} y^{N,1}_{t} + L^{N,2}_{t} y^{N,2}_{t} + L^{U}_{t} y^{U,1}_{t} + L^{U}_{t-1} y^{U,2}_{t}, \tag{52}
\]

\[
Z_{t} = L^{U}_{t} z^{U,1}_{t} + L^{U}_{t-1} z^{U,2}_{t} + \Gamma^{U}_{t} + K^{U}_{t+1} + L^{U}_{t-1} z^{U,2}_{t} + L^{N,1}_{t} z^{N,1}_{t} + L^{N,2}_{t} z^{N,2}_{t} + L^{A,1}_{t} z^{A,1}_{t} + L^{A,2}_{t} z^{A,2}_{t}. \tag{53}
\]

Having described the optimization problems of households and firms as well as the market clearing conditions, the intertemporal equilibrium dynamics can now be derived.

From (19) and from the international Fisher equations (43), (46) and (47) the following relationships between southern EMU, Asian, US and northern EMU capital output ratios and the terms of trade result:

\[
v^{S}_{t+1} = \frac{\alpha^{S}}{\alpha^{N}} v^{N}_{t+1}, \quad t = 0,1,2,\ldots , \tag{54}
\]
\[ e_{r+1}^t = e_t^t \frac{\alpha^t N^{r+1}}{\alpha^N}, t = 0, 1, 2, \ldots \]  
(55)

\[ e_{r+1}^U = e_t^U \frac{\alpha^U N^{r+1}}{\alpha^N}, t = 0, 1, 2, \ldots \]  
(56)

Dividing (50) on both sides by \( X_i^N \) and introducing the definitions of the capital output ratios as well as the debt output ratios, the asset market clearing condition (50) can be rewritten as follows:

\[
\frac{L_t^S}{X_t^N} X_t^S + \frac{L_t^S}{X_t^N} + \frac{L_t^S}{X_t^N} Y_t^{e_t} + \frac{L_t^S}{X_t^N} Z_t^{e_t} + \frac{v_{r+1}^S + b_{r+1}^S}{e_t^t} G_t^{e_t} X_t^S + \frac{v_{r+1}^N + b_{r+1}^N}{e_t^t} G_t^{e_t} X_t^N, t = 0, 1, 2, \ldots
\]  
(57)

Using the production functions, the ratios of southern EMU’s, Asia’s and US’s to northern EMU’s GDP turn out to be as follows:

\[
\frac{X_t^S}{X_t^N} = \frac{L_t^S}{L_t^N} \left( \frac{M_t^S}{M_t^N} \right)^{\frac{1}{(1-\sigma^S)}} \left( \frac{\alpha^S}{\alpha^N} \right)^{\frac{1}{(1-\sigma^S)(1-\sigma^N)}}, \]  
(58)

\[
\frac{Y_t}{X_t^N} = \frac{L_t^A}{L_t^N} \left( \frac{M_t^A}{M_t^N} \right)^{\frac{1}{(1-\sigma^A)}} \left( \frac{v_t^A}{v_t^N} \right)^{\frac{1}{(1-\sigma^A)}} \left( v_t^N \right)^{\sigma^A/(\sigma^A - 1)}, \]  
(59)

\[
\frac{Z_t}{X_t^N} = \frac{L_t^U}{L_t^N} \left( \frac{M_t^U}{M_t^N} \right)^{\frac{1}{(1-\sigma^U)}} \left( \frac{v_t^U}{v_t^N} \right)^{\frac{1}{(1-\sigma^U)}} \left( v_t^N \right)^{\sigma^U/(\sigma^U - 1)}. \]  
(60)

Acknowledging (57)-(60), the definitions of the GDP growth rates, the optimal savings functions resulting from household’s utility maximization problems (40)-(42), (44)-(45) and (18) in (57) yield:

\[
\left\{ \sigma^S \left[ 1 - \sigma^S - \sigma^N - \frac{\sigma^S b^S}{v_t^S} \right] - G^S \left( \frac{v_{r+1}^S}{v_t^S} \right)^{\sigma^S/(1-\sigma^S)} \left( v_{r+1}^S + b^S (1 - \sigma^S) \right) \right\} \left\{ \left( \frac{M_t^S}{L_t^N} \right)^{\frac{1}{1-\sigma^S}} \left( \frac{\alpha^S}{\alpha^N} \right)^{\frac{1}{1-\sigma^S}} \left( v_t^N \right)^{\sigma^S/(1-\sigma^S)} \right\} = -\frac{1}{e_t^t} \left\{ \sigma^S \left[ 1 - \alpha^S - \gamma^S - \frac{\alpha^S b^S}{v_t^S} \right] - G^S \left( \frac{v_{r+1}^S}{v_t^S} \right)^{\sigma^S/(1-\sigma^S)} \left( v_{r+1}^S + b^S (1 - \sigma^S) \right) \right\}
\]  

\[
+ \sigma^N \left[ 1 - \alpha^N - \gamma^N - \frac{\alpha^N b^N}{v_t^N} \right] - G^N \left( \frac{v_{r+1}^N}{v_t^N} \right)^{\sigma^N/(1-\sigma^N)} \left( v_{r+1}^N + b^N (1 - \sigma^N) \right) \right\} = -\frac{1}{e_t^t} \left\{ \sigma^A \left[ 1 - \alpha^A - \gamma^A - \frac{\alpha^A b^A}{v_t^A} \right]
\]  

25
\[-G^a \left( \frac{v_i^L}{v_i^R} \right) ^{\alpha^U} \left[ v_i^L + b_i(1 - \sigma^L) \right] L_i^A \left( M^A \right)^{\beta(1 - \alpha^A)} \left( v_i^L \right)^{\alpha^A} \left( v_i^N \right)^{\alpha^N} \left( \frac{\sigma^U}{e^U} \right) \left[ 1 - \alpha^U - \gamma^U - \frac{\alpha^A b_i^A}{v_i^R} \right] \]

\[-G^a \left( \frac{v_i^U}{v_i^R} \right) ^{\alpha^U} \left[ v_i^U + b_i^U(1 - \sigma^U) \right] L_i^U \left( M^U \right)^{\beta(1 - \alpha^U)} \left( v_i^U \right)^{\alpha^U} \left( v_i^N \right)^{\alpha^N} \left( \frac{\sigma^U}{e^U} \right) \left[ 1 - \alpha^U - \gamma^U - \frac{\alpha^A b_i^A}{v_i^R} \right]. \tag{61} \]

Equation (61) represents the fourth equation of the intertemporal equilibrium dynamics of the capital-output ratios, $v_i^L, v_i^N, v_i^A, v_i^U$, and the terms of trade $e_i^A$ and $e_i^U$ of the three-good, three-country OLG model under EMU financial integration.

The two remaining dynamic equation we obtain by dividing Asia’s good respective the US good market clearing condition (52) respective (53) by the combined EMU good market clearing condition (51):

\[
\frac{(1 - \gamma^A)Y_i - K_i^A}{(1 - \gamma^N)X_i^N - K_i^N} = \frac{L_i^A y_i^{A1} + L_i^A y_i^{A2} + L_i^S y_i^{S1} + L_i^S y_i^{S2} + L_i^N y_i^{N1} + L_i^N y_i^{N2}}{L_i^S x_i^{S1} + L_i^S x_i^{S2} + L_i^N x_i^{N1} + L_i^N x_i^{N2} + L_i^A x_i^{A1} + L_i^A x_i^{A2}} + \frac{L_i^U y_i^{U1} + L_i^U y_i^{U2}}{L_i^x x_i^{x1} + L_i^x x_i^{x2}}, \tag{62} \]

\[
\frac{(1 - \gamma^U)Z_i - K_i^U}{(1 - \gamma^N)X_i^N - K_i^N} = \frac{L_i^U z_i^{U1} + L_i^U z_i^{U2} + L_i^S z_i^{S1} + L_i^S z_i^{S2} + L_i^N z_i^{N1} + L_i^N z_i^{N2}}{L_i^S x_i^{S1} + L_i^S x_i^{S2} + L_i^N x_i^{N1} + L_i^N x_i^{N2} + L_i^A x_i^{A1} + L_i^A x_i^{A2}} + \frac{L_i^U z_i^{U1} + L_i^U z_i^{U2}}{L_i^x x_i^{x1} + L_i^x x_i^{x2}}. \tag{63} \]

Upon inserting the utility maximizing consumption functions resulting from household’s utility maximization problems (40)-(42), (44)-(45) into the right hand side of equation (62) and (63), it fortunately turns out that (62) respective (63) can be rewritten as follows:

\[
\frac{(1 - \gamma^A)Y_i - K_i^A}{(1 - \gamma^N)X_i^N - K_i^N} = \frac{1}{\zeta^A} e_i^A, \tag{64} \]

\[
\frac{(1 - \gamma^U)Z_i - K_i^U}{(1 - \gamma^N)X_i^N - K_i^N} = \frac{1}{\zeta^N} e_i^U. \tag{65} \]

Dividing the numerator on both sides of (64) by $Y_i$ and the denominator on both sides of (64) by $X_i^N$ and taking account of (58) and (59), we obtain the fifth equation of the intertemporal equilibrium dynamics:
\[ 1 - \gamma^A - G^a \left( \frac{V_{t+1}^a}{V_t^a} \right)^{\alpha^a - 1} \] \[ V_{t+1}^a = \frac{\xi^y}{\xi^a - 1} \left( \frac{1}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{L_1}{L_2} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right) \left( 1 - \gamma^S - G^a \right) \left( \frac{V_{t+1}^a}{V_t^a} \right)^{\alpha^a - 1} \] \[ \times \left( \frac{L_2}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right)^{\alpha^a - 1} \left( 1 - \gamma^N - G^a \right) \left( \frac{V_{t+1}^a}{V_t^a} \right)^{\alpha^a - 1} \] \[ \times \left( \frac{L_2}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right)^{\alpha^a - 1} \left( 1 - \gamma^N - G^a \right) \left( \frac{V_{t+1}^a}{V_t^a} \right)^{\alpha^a - 1} \] \[ \times \left( \frac{L_2}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right)^{\alpha^a - 1} \left( 1 - \gamma^N - G^a \right) \left( \frac{V_{t+1}^a}{V_t^a} \right)^{\alpha^a - 1} \]

Dividing the numerator on both sides of (65) by $Z_t$ and the denominator on both sides of (65) by $X_t^N$ and taking account of (58) and (60), we obtain the sixth equation of the intertemporal equilibrium dynamics:

\[ 1 - \gamma^U - G^a \left( \frac{v_{t+1}^U}{v_t^U} \right)^{\alpha^U - 1} v_{t+1}^U = \frac{\xi^\alpha}{\xi^a - 1} \left( \frac{1}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{L_1}{L_2} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right) \left( 1 - \gamma^S - G^a \right) \left( \frac{v_{t+1}^U}{v_t^U} \right)^{\alpha^U - 1} \] \[ \times \left( \frac{L_2}{L_1} \right)^{1-(\alpha^a - 1)} \left( \frac{v_t^N}{V_t^a} \right)^{\alpha^a - 1} \left( 1 - \gamma^N - G^a \right) \left( \frac{v_{t+1}^U}{v_t^U} \right)^{\alpha^U - 1} \]

In a steady state with $v_{t+1}^S = v_t^S$, $v_{t+1}^N = v_t^N$, $v_{t+1}^A = v_t^A$, $v_{t+1}^U = v_t^U$, $e_{t+1}^A = e_t^A$, $\epsilon_{t+1}^U = \epsilon_t^U$, the system of first-order difference equations collapses on the following system of steady state equations:

\[ v_t^S = (\alpha^S/\alpha^N) v_t^N, \] \[ v_t^A = (\alpha^A/\alpha^N) v_t^N, \] \[ v_t^U = (\alpha^U/\alpha^N) v_t^N, \]

\[ \phi^S = \phi^S(v_t^N)^{\gamma_{S,N}^S} + \phi^N = -e_t^A - (\gamma^A/G^a v_t^A) v_t^N, \] \[ (1 - \gamma^A - G^a v_t^A)(\phi^A)^{\gamma_{A,N}^A} = e_t^A [(\xi^A/\xi^S)](1 - \gamma^S - G^a v_t^S) v_t^N, \] \[ (1 - \gamma^U - G^a v_t^U)(\phi^U)^{\gamma_{U,N}^U} = \epsilon_t^U [(\xi^U/\xi^S)](1 - \gamma^S - G^a v_t^S) v_t^N. \]

Inserting (68)-(70) into (71)-(73), then solving (73) for $\epsilon^U$ and inserting the result into (71), we obtain two equations of $e^A$ as function of $v_t^N$. Proposition 4 tells us under which conditions non-trivial steady state solutions exist.

**Proposition 4** (Existence of non-trivial steady states under EMU financial integration)

Let \( \omega = (\alpha^S, \alpha^N, \alpha^A, \alpha^U, \beta^S, \beta^N, \beta^A, \beta^U, \gamma^S, \gamma^N, \gamma^A, \gamma^U, \xi^S, \xi^N, \xi^A, \xi^U, L^S, L^N, L^A, L^U, M^S, M^N, M^A, M^U, b^S, b^N, b^A, b^U, G^n) \) be the parameter vector and \( \Omega = [0,1]^{14} \times [0,1]^4 \) be the parameter space in the EMU-Asia-US steady-state market equilibrium with EMU financial integration. For any admissible parameter combination, \( \omega \in \Omega \), there exist some \( \bar{b}^S \in (0, \infty), \bar{b}^N \in (0, \infty), \)
\( \overline{b}^A \in (0, \infty) \) and \( \overline{b}^U \in (0, \infty) \) such that for \( b^S \in (0, \overline{b}^S) \), \( b^N \in (0, \overline{b}^N) \), \( b^A \in (0, \overline{b}^A) \) and \( b^U \in (0, \overline{b}^U) \), there are two nontrivial steady state solutions \((e^A_L, e^U_L, v^S_L, v^N_L, v^A_L, v^U_L) \gg 0 \) and \((e^A_H, e^U_H, v^S_H, v^N_H, v^A_H, v^U_H) \gg 0 \). For \( b^S = b^N = b^A = b^U = 0 \), there is only one non-trivial steady state.

Proof. See the appendix A4 in Farmer (2013, pp. 23-24).

Since there are in general two steady state solutions, we had to investigate the local dynamic stability of each steady state by calculating the eigenvalues of the Jacobian matrix of the intertemporal equilibrium dynamics (54)-(56), (61) and (66)-(67) in a small neighborhood of the steady states. Due to the analytical complexity of the six-dimensional dynamical system we are not able to prove dynamic stability in general but can only show saddle-point stability of the larger steady state solution for certain numerically specified parameter sets.

Proposition 5 (Saddle-point stability of the larger steady state solution)

Suppose that the conditions in proposition 4 hold. Moreover, assume that the following numerical values are attributed to model parameters mentioned in proposition 4: \( G^n = 1.6, \zeta^x = 1/3, \zeta^y = 1/3, \zeta^z = 1/3, \beta^s = 0.45, \beta^N = 0.55, \beta^A = 0.8, \beta^U = 0.4, \alpha^s = 0.26, \alpha^N = 0.2, \alpha^A = 0.31, \alpha^U = 0.2, \gamma^s = 0.14, \gamma^N = 0.23, \gamma^A = 0.15, \gamma^U = 0.2, M^s = 2, M^N = 2.5, M^A = 2, M^U = 3.5, L = 75, L^A = 1300, L^U = 140, b^S = 0.027, b^N = 0.023, b^A = 0.02, b^U = 0.025 \). Given this parameter set, the steady state \((e^A_L, e^U_L, v^S_L, v^N_L, v^A_L, v^U_L) \) is saddle-point stable while \((e^A_H, e^U_H, v^S_H, v^N_H, v^A_H, v^U_H) \) is saddle-point unstable.

Financially Integrated versus Financially Autarkic Steady State

On knowing from positions 4 and 5 that the larger steady-state solution under financial integration is unique and dynamically stable, proposition 6 below provides an answer to the main question whether financial integration across southern and northern EMU, i.e. the convergence of northern and southern EMU real interest rates, contribute to the divergence of southern and northern EMU current account and net foreign asset positions. Proposition 6 provides the answer to this question.

Proposition 6 (EMU current account and net foreign asset position effects of EMU financial integration)

Suppose that the assumptions of proposition 3 hold, i.e. the southern EMU financial autarky (FA) interest rate, \((i^S)^{FA}\), is larger than the northern EMU financial autarky interest rate, \((i^N)^{FA}\). Then, after financial integration (FI), the ratio of the southern current account and
the net foreign asset position to southern GDP is negative while the respective northern ratios become larger than zero, i.e. \((ca^S)^{FI} < 0\) and \((\phi^S)^{FI} < 0\) and \((ca^S)^{FI} > 0\) and \((\phi^S)^{FI} > 0\).

Proof. By assumption, we have \((i^S)^{FA} > (i^N)^{FA}\). Thus, \(1 + (i^S)^{FA} = \alpha^A/((v^S)^{FA} > 1 + (i^N)^{FA} = \alpha^N/(v^N)^{FA}\). Financial integration means that the positive differential between southern and northern EMU autarky interest rates diminishes as the southern interest rate declines and the northern interest rate rises. Due to decreasing marginal productivity of capital the decline in southern interest rate is associated with a rise in southern capital output ratio and vice versa in North. Next, note that the southern current account to GDP ratio taking into account southern trade with Asia and US reads as follows: \((ca^S) = 1 - \gamma^S - G^S v^S \left[1 + \beta^S \alpha^S \right]/\left(G^S v^S [1 - \alpha^S - \gamma^S + b^S (G^S - \alpha^S/v^S)]/(1 + \beta^S) + (\alpha^S/v^S - 1) \phi^S/G^S\right].\) It is easy to see that \((ca^S)^{FI}\) decreases as \(v^S\). Since under trade autarky \((ca^S)^{FA} = 0\) it follows that \((ca^S)^{FI} < 0\). Remembering the definition of the southern EMU net foreign asset position in steady state as \(\phi^S (v^S) = \sigma^S (1 - \alpha^S - \gamma^S - \alpha^S b^S/v^S) - G^S [v^S + b^S (1 - \alpha^S)],\) differentiation of \(\phi^S\) with respect to \(v^S\) yields \(\phi^S (v^S) = \alpha^S \sigma^S b^S/(v^S)^2 - G^S).\) From the proof of proposition 2 we know that there is a small neighborhood of the southern EMU autarky steady state with the larger capital output ratio in which \(\phi^S ((v^S)^{FA}) < 0\) holds. Hence, the southern EMU net foreign asset position deteriorates with rising southern capital output ratio. Since at the autarky value of \(v^S\) the southern EMU net foreign asset position is zero, and since the southern EMU net foreign asset position declines with rising capital output ratio, at \((v^S)^{FI}\) the southern EMU net foreign asset position is smaller than zero, i.e. \(\phi^S ((v^S)^{FI}) < 0\). On the other hand, since northern EMU real interest rate rises during financial integration and the higher interest rate is associated with lower capital output ratio, the northern EMU current account and net foreign asset to GDP ratio becomes larger than zero in response to the lower northern capital output ratio. ■

Proposition 7 (Asian and US current account and net foreign asset position effects of EMU financial integration)

Suppose that US financial autarky (FA) interest rate, \((i^U)^{FA}\), is larger than the Asian financial autarky interest rate, \((i^A)^{FA}\). Then, after worldwide financial integration (FI), the ratio of US current account and US net foreign asset position to US GDP is negative while the respective Asian ratios become larger than zero, i.e. \((ca^U)^{FI} < 0\) and \((\phi^U)^{FI} < 0\) and \((ca^A)^{FI} > 0\) and \((\phi^A)^{FI} > 0\).

Proof. In general, the proof of proposition 7 is similar to that of proposition 6. However, notice that through steady-state interest parity conditions (68)-(70) EMU financial integration impacts Asian and US current account and net foreign asset positions.
Concluding Remarks

This paper explores, within a three-good, three-country OLG model with production, capital accumulation and public debt, the emergence of external imbalances (current and financial account) both among EMU core and EMU periphery and Asia respective US after the inception of the common currency and before the outburst of the global financial crisis in 2008. It models the pre-euro situation as real and financial autarky and the EMU before the onset of the global financial crisis as financial integration characterized by complete convergence of real interest rates. The financial integration across the ROW, i.e. Asia and USA and the EMU after euro launch is incomplete due to different currencies.

After assuring the existence and dynamic stability of financial autarky steady states, a lower saving rate, an equal government expenditure quota and a higher capital production share in South were shown to imply the empirically observed southern high real interest rate and low real wage rate associated with no external imbalances between South and North before the advent of the common currency. Symmetrically, the higher saving rate and government expenditure quota and the lower capital production share in North implied a northern low real interest rate and a high real wage. After the inception of the common currency, free capital mobility between South and North induced immediate international real interest parity leading to a quick fall in southern interest rates and to rising northern interest rates. Simultaneously, relatively high initial southern interest rates led the northern core to invest their wealth in southern housing and residential objects. Thus, both northern and southern external balances widen: EMU periphery’s current account relative to Asia and USA becomes negative while it simultaneously incurs a net foreign debtor position vis-à-vis EMU core. On the other hand, EMU core runs a current account surplus vis-à-vis ROW and simultaneously becomes a net foreign creditor for EMU periphery. Moreover, intra-EMU real interest convergence contributes also to the widening of Asian and US external imbalances measured by current and financial account imbalances.

We may thus conclude that the three-good, three-country OLG model is capable of reproducing qualitatively, the main stylized facts presented above: a converging real interest rate across southern and northern EMU countries, rising southern EMU and US current account deficits and northern EMU and Asian current account surpluses associated with larger external debt of the former and larger external credit of the latter. Proposition 3, 6 and 7 corroborate the claim that the emergence of external imbalances between both northern and southern EMU and the Asian and US external imbalances after EMU financial integration can
be traced back to fundamental North-South and East-West differences in saving rates, government expenditure quotas and capital production shares.

References


