Do Productivity and Accounting Quality Increase with More Conservative Performance Measures?

Ralf Ewert

and

Alfred Wagenhofer

Abstract

We examine how a manager can be induced to improve the accounting quality (AQ) of financial reports that are used for measuring management performance. We find that a more conservative accounting system increases incentives to invest in AQ and always strictly increases firm value. When the incentives are too low for the manager to exert AQ effort, conservatism reduces the expected compensation while keeping productive incentives intact. In the optimum, it is optimal to increase compensation and elicit more productive effort. When the manager does exert AQ effort, then the owner reduces compensation with more conservatism and foregoes some productivity gains, but firm value still increases due to the increased AQ. Overall, AQ incentives and firm value strictly increase with conservatism; total welfare is typically maximal at the degree of conservatism where the regimes switch. We also show that AQ is more important in firms with greater productivity.

Keywords: Accounting quality; management incentives; conservatism; productivity

We thank Jeremy Bertomeu, Robert Göx, Christian Hofmann, David Johnstone, Christian Laux, Stefan Schantl, Ulf Schiller, and participants at the EIASM Workshop on Accounting and Economics, MEAFA at the University of Sydney, and at workshops at the University of Hannover, Vienna University of Economics and Business, University of Munich, and University of Zurich for helpful comments.

Ralf Ewert
University of Graz, Universitaetsstrasse 15, A-8010 Graz, Austria
Tel.: +43 (316) 380 7168, Email: ralf.ewert@uni-graz.at

Alfred Wagenhofer
University of Graz, Universitaetsstrasse 15, A-8010 Graz, Austria
Tel.: +43 (316) 380 3500, Email: alfred.wagenhofer@uni-graz.at

Revised, September 2019

1. Introduction

This paper examines how to incentivize managers to improve the accounting quality (AQ) of financial reports in an agency setting when the reports are used for their performance measurement. Investment in AQ include, for example, implementation of high-quality accounting processes and more effective internal controls over financial reporting and are primarily the responsibility of management. We provide insights whether – and under which conditions – it is in the best interest of shareholders to induce management to take AQ enhancing activities, how an optimal management compensation looks like, and what role characteristics of the firm's accounting system play. Apparently, shareholders would prefer choosing AQ investments themselves, if they could, over allowing management to make this decision because delegation creates a control loss and its resolution interferes with management's incentives to increase productive efficiency. Yet AQ investments are typically in the realm of management.[1]

We develop an agency model featuring an owner who writes a compensation contract and a manager who can provide both productive and AQ effort. A higher AQ effort decreases errors in the accounting system proportionally. Since the owner's only control variable is managerial compensation, productive effort and AQ effort are chosen together. We show that the incentives depend strongly on the bias in the accounting system, and we find that a more conservative accounting system is strictly preferred by the owner. A conservative bias reduces the β-error, i.e., a high signal occurs although the outcome is low, at the cost of increasing the α-error, i.e., a low signal occurs although the outcome is high. The manager exerts AQ effort only when it increases the probability to receive a bonus. This is precisely the case if the underlying accounting system is more conservative. Conservatism biases the performance measure against the manager, and higher AQ reduces this effect.

[1] See, e.g., the appendix of the PCAOB's standard AS2201 about audits of internal controls: "Internal control over financial reporting is a process designed by, or under the supervision of, the company's principal executive and principal financial officers, or persons performing similar functions, and effected by the company's board of directors, management, and other personnel, to provide reasonable assurance regarding the reliability of financial reporting …"

We show that there exist two regimes with distinctly different optimal solutions, contingent on the accounting system's bias. If conservatism is relatively low, it does not pay to induce the manager to exert AQ effort, so only productive effort is induced, foregoing the benefits (net of the cost) of AQ effort. In this regime 1, we show that increasing conservatism reduces the owner's expected cost needed to implement a particular productive action, which again allows the owner to increase the bonus and to motivate productive effort. Because of that, firm value and total welfare strictly increase in conservatism. We consider total welfare as a productive efficiency measure besides firm value because in our model the manager is protected by limited liability and earns a rent under the optimal contract, and the compensation also reallocates welfare between owner and manager.

If the bonus grows higher for a given level of conservatism, the manager will also exert AQ effort, which fundamentally changes the underlying trade-offs for the owner. We show that in this regime 2, the owner takes advantage of the higher accounting quality that mitigates the control problem and optimally *reduces* the bonus with greater conservatism. With the lower bonus the manager decreases productive effort. AQ effort directly increases with conservatism, although the lower bonus attenuates this effect. Firm value again strictly increases with conservatism, but total welfare strictly decreases mainly due to the lower productive effort.

We then derive the overall optimal contract, which includes a switch from a regime 1 contract (with no incentives for AQ effort) to a regime 2 contract (with strict incentives for AQ effort) for greater conservatism and for higher productivity. At the point of the contract switch the bonus jumps upwards, which induces an upward jump of the AQ effort, and total welfare jumps upwards as well. Firm value exhibits a kink at the point of the regime switch, but remains monotone increasing in conservatism.

Finally, we derive the optimal level of conservatism for three different objectives. First, if the owner can choose the conservative bias of the accounting system, she chooses the maximum conservatism because firm value always increases with conservatism. Second, a regulator may be interested in increasing productivity and value generation in the agency; it would then choose an interior level of conservatism in settings where regime 1 and regime 2 contracts are both present.

Third, an accounting standard setter may be most interested in maximizing the quality of financial reports, which is the informativeness of the accounting signal; since quality also increases with conservatism, maximum conservatism is optimal.

Our paper contributes to our understanding of economic effects of conservatism in several ways. First, we provide a novel channel by which conservatism shapes the optimal contracts and establish its impact on different objectives, which are relevant to assess effects of conservatism. Second, our results are informative to regulatory debates. Because conservatism introduces a downward bias, the International Accounting Standards Board and the U.S. Financial Accounting Standards Board reject it in favor of neutrality (IASB 2018, FASB 2010).[2] Yet decision usefulness is an elusive objective because it depends on the decision context in which the information is used. In our model, which is in the stewardship realm, conservatism increases decision usefulness through providing higher incentive of management to invest more in accounting quality, which overall increases informativeness. We also demonstrate that this goes along with reduced productivity. Our paper further speaks to the effects of the Sarbanes-Oxley Act (SOX) of 2002, which requires firms to maintain efficient internal controls and managers and auditors to disclose material weaknesses. For example, Goh and Li (2011) find a positive association between conditional conservatism and the quality of internal controls. We show that reducing or eliminating conservatism in accounting impairs the objective to induce firms to enhance internal controls.

The setting we employ is similar to that in Kwon, Newman, and Suh (2001), who study a binary agency model with a risk-neutral manager protected by limited liability. They establish that conservatism increases firm value by decreasing the probability but increasing the precision of a favorable signal, which reduces the expected compensation needed to implement the desirable productive effort. Kwon (2005) extends this model by allowing for continuous productive effort and shows that conservative accounting also has a positive productive effect. The results for our

[2] The IASB (2018) reintroduced conservatism in the form of (symmetric) caution in the face of uncertainty.

regime 1 contract are fully consistent with Kwon (2005). Our novel feature is the introduction of AQ effort, which leads to new insights into the trade-offs involved in regime 2.

Precursors of our model are multi-action agency models (e.g., Feltham and Xie 1994) that study effects of changing characteristics of accounting systems on incentives. Liang and Nan (2014) model a manager who exerts productive effort and can improve the precision of the performance measurement system. They find that it may not be in the owner's best interest to use a performance measure whose precision the manager controls when the costs of the two tasks interact. Friedman (2014, 2016) examines a setting in which the CFO controls the precision of the performance measure but the CEO can put pressure on the CFO to bias this measure based on which both are evaluated. Like the present paper, these works consider management's control over the quality of the accounting system, but their use of a LEN agency model precludes an analysis of accounting bias.

Other papers consider settings in which the owner, rather than the manager, determines accounting quality. Drymiotes (2011) examines a setting in which the firm's owner can increase the precision of the accounting system and finds that making the accounting system more precise can increase earnings management. Chan (2016) examines the effect of the SOX requirement to disclose internal control weaknesses on investment in internal controls and audit effort. He finds that the disclosure of internal control weaknesses increases audit effort, but can increase or decrease investment in internal controls. Neither of these papers considers conservatism.

In a direct extension of Kwon, Newman, and Suh (2001), Bertomeu, Darrough, and Xue (2017) allow the manager to engage in earnings management. They find an optimal interior optimum of conservatism. Chen, Hemmer, and Zhang (2007) analyze the relation between conservatism and earnings management in an agency setting where the owner sells the firm. They find that at least a small degree of conservatism is beneficial, but in their approach the owner, not the manager, biases earnings. Caskey and Laux (2017) show that the strength of board governance, through mitigating earnings management, increases the benefits of accounting conservatism in an investment setting under asymmetric information.

Ewert and Wagenhofer (2019) study an agency model with productive effort and earnings management and show that increasing enforcement can crowd out auditing, although it still mitigates earnings management. Accounting quality declines if earnings management is "good" in the sense that it corrects more understatement errors in the accounting system than it aggravates overstatement errors. Glover and Levine (2019) consider asymmetric information about measurement quality and also show that earnings management can be "good" in that it reduces understatement. "Good" earnings management in these papers is akin to our conditions that motivate AQ investment. We briefly examine ex post earnings management in an extension of our model and find that it neutralizes some benefits of conservatism.

Providing incentives to enhance accounting quality bears resemblance to controlling managers' actions to control risk in the production process. A less risky production process has similar effects as an AQ effort as both increase the informativeness of the accounting signal. For example, Meth (1996) and Chen, Mittendorf, and Zhang (2010) study agency models in which the manager takes two actions, one to increase the average outcome and the other to influence its variance or spread. In particular, Chen et al. find that conservatism is useful to motivate the mean-increasing action, but a liberal bias is necessary to induce the spread-reducing action.

Finally, accounting quality and internal controls are also relevant in an audit and a governance context. For example, Nelson, Ronen, and White (1988), Smith, Tiras, and Vichitlekarn (2000), Pae and Yoo (2001), and Patterson and Smith (2007, 2016) consider strategic interactions between the manager's or the owner's implementation of internal controls and the auditor's audit activities and focus on the role of auditor liability regimes. These papers do not explicitly consider optimal contracting.

2. Model

We consider a firm featuring a risk neutral owner and a manager in a one-period agency model. The manager decides on two unobservable and privately costly actions: productive effort and effort to improve the quality of the accounting system.

The firm owns a production technology and installs an accounting system to track performance. Output is a binary random variable $X \in \{0, x\}$ with $x > 0$. Higher productive effort a increases the probability p of the high output x at a decreasing rate:

$$p(a) = 1 - \exp(-a).$$

The manager incurs a private cost $V^A a$ of exerting effort a, where $V^A > 0$ is sufficiently small that the manager can be motivated to exert any positive effort, specifically, $x > V^A$.[3]

The owner designs a compensation contract $s(\cdot)$ written on a single observable signal, $y \in \{y_L, y_H\}$ ("earnings"), where $y_L < y_H$, which is informative about the output X. The realized output X is unobservable over the period covered by the contract, for example, because it is a long-term profit or includes non-financial benefits. The optimal compensation maximizes the owner's expected utility, consisting of the expected output less the expected compensation,

$$E[U^O] = px - \left(\Pr(y_L)s(y_L) + \Pr(y_H)s(y_H)\right). \tag{1}$$

Figure 1: Production and accounting systems

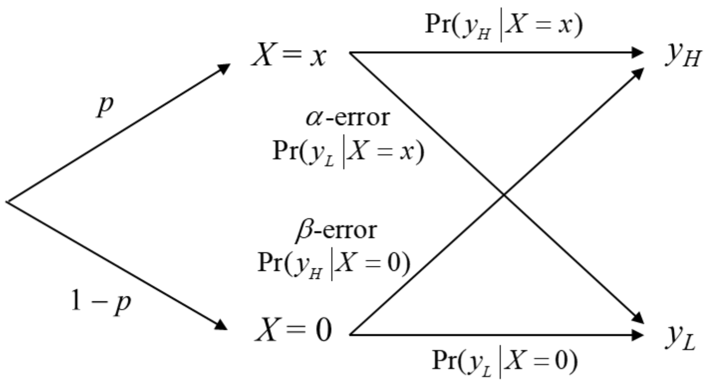


Figure 1 depicts the accounting system. It is characterized by an overstatement error $\Pr(y_H | X = 0)$ and an understatement error $\Pr(y_L | X = x)$, resulting from underlying α- and β-

---

[3] This assumption avoids the explicit consideration of uninteresting cases with accounting systems and conservatism thresholds in which the owner would never hire a manager.

errors, the degree δ of conservatism, and the manager's accounting quality effort c. We describe each element in turn.

The accounting system is imperfect and features random errors α and β, where α and $\beta < 0.5$. These errors result from bookkeeping and other accounting processes, e.g., inventory sampling, misrecording book entries, double-booking, not booking transactions or events, individual mistakes, and misjudgments. Following prior literature, we distinguish two basic features of the accounting system, informativeness and conservatism.[4] Let the sum of the conditional error probabilities be $\varphi \equiv \alpha + \beta$, then informativeness is defined as

$$\Pr\left(y_H \middle| X = x\right) + \Pr\left(y_L \middle| X = 0\right) = \left(1-\alpha\right) + \left(1-\beta\right) = 2 - \varphi.$$[5]

That is, reducing either α or β or both increases informativeness.[6] To vary the degree of bias, we parameterize the two errors by δ such that the accounting errors are

$$\alpha_\delta = \alpha + \delta\,,\ \beta_\delta = \beta - \delta\,.$$

Given $\{\alpha, \beta\}$, an accounting system is more conservative the higher is δ, therefore we use δ as conservatism parameter. Note that a variation of δ leaves informativeness ($2 - \varphi$) constant. We require that α and $\beta \in (0, 0.5)$ and

$$0 \le \delta \le \min\left\{.5 - \alpha, \beta\right\} \equiv \bar{\delta}$$

to ensure that $\Pr(y_L \,|\, X = x)$ and $\Pr(y_H \,|\, X = 0) \in (0, 0.5)$. Thus, the support of δ depends on the size of the original α- and β-errors.

---

[4] See Gigler and Hemmer (2001) and Venugopalan (2004); a more general formulation is in Gigler, Kanodia, Sapra, and Venugopalan (2009). Our notation reconciles with that of Venugopalan (2004) as follows. $\Pr(y_L \,|\, X = x) = 1 - \lambda - \delta,\ \Pr(y_H \,|\, X = 0) = \delta,$ appropriately chosen, where his λ is informativeness and δ is negative conservatism. His δ is our β, and since we want to use a positive conservatism variable, we use ($\alpha + \delta$) and ($\beta - \delta$) where δ is a positive conservatism measure.

[5] This is analogous to Bertomeu, Darrough and Xue (2017), p. 256.

[6] We do not study an individual variation of α or β because that directly affects the informativeness. We briefly discuss this in Section 4.

In our formalization, greater δ makes higher earnings more and lower earnings less precise, which is akin to empirical measures of conditional conservatism, such as requiring a higher degree of verifiability for the recognition of gains than for losses or a timelier recognition of losses than gains (Basu 1997, Watts 2003).[7] In our subsequent analysis, we focus on the economic effects of a variation of the conservative bias of the accounting system through the parameter δ. Note that a reduction of δ is equivalent to making the accounting system more aggressive, so our results speak in both directions.

The third factor that determines the accounting system is the manager's accounting quality (AQ) effort c to improve the informativeness of the accounting system. The manager decides on c before y is realized. For example, he can invest in the quality of the accounting processes or the internal controls to increase the precision of the accounting system. The manager incurs a private cost of AQ effort, $V^C c$, where $V^C > 0$ but sufficiently small that there arise situations in which the manager indeed exerts AQ effort. Greater AQ effort c mitigates the errors by the same percentage $\exp(-c)$, where $\exp(-c)$ is convex decreasing in c. This specification ensures that $\exp(-c) \in (0,1]$ for any $c \geq 0$. Note that c reduces the errors α_d and β_d, which includes the parameter δ as our instrument to vary the bias of the accounting system. For example, improved processes of inventory taking, bad debt accounting, or provisioning reduces both errors.

The resulting ex ante probabilities of the signals y_i are

$$\begin{aligned}\Pr(y_L|a,c) &= p\alpha_\delta \exp(-c) + (1-p)\left(1-\beta_\delta \exp(-c)\right)\\ &= p(\varphi \exp(-c)-1) + \left(1-\beta \exp(-c)\right) + \delta \exp(-c)\end{aligned}$$

and

$$\begin{aligned}\Pr(y_H|a,c) &= p\left(1-\alpha_\delta \exp(-c)\right) + (1-p)\beta_\delta \exp(-c)\\ &= p\left(1-\varphi \exp(-c)\right) + (\beta-\delta)\exp(-c).\end{aligned}$$

The manager is risk neutral and protected by limited liability that precludes payment of negative compensation (as a penalty), hence $s(y_L)$ and $s(y_H) \geq 0$, and we assume the manager's

[7] Gigler et al. (2009) and Nagar, Rajan, and Ray (2018) discuss the consistency with empirical measures of asymmetric timeliness.

reservation utility is zero. Since the manager can always choose efforts of 0, so the disutility is 0 as well, the participation constraint is always satisfied and is ignored in the following analysis. The manager's expected utility is

$$E[U^M | a,c] = \Pr(y_L | a,c)s(y_L) + \Pr(y_H | a,c)s(y_H) - V^A a - V^C c. \tag{2}$$

As compensation is bound to be non-negative, the owner optimally pays a bonus s if y_H is reported because $\Pr(y_H)$ increases in a, and no bonus otherwise, i.e., $s \equiv s(y_H) > s(y_L) = 0$.

Figure 2: Timeline

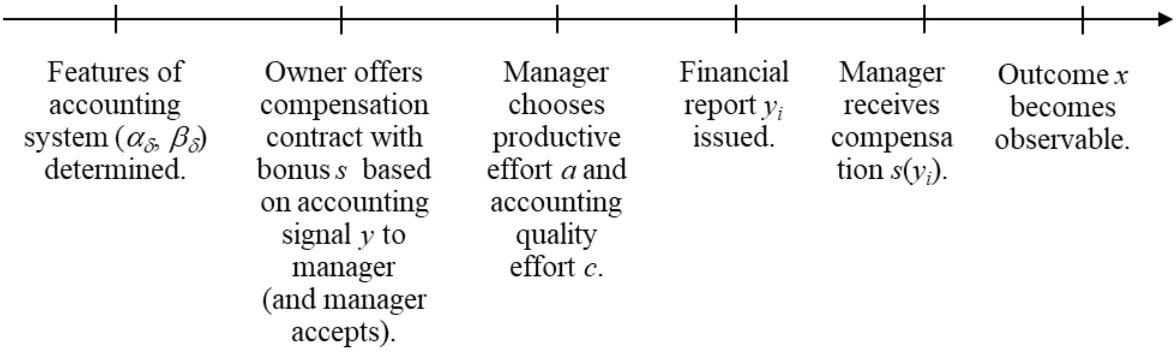


Figure 2 depicts the timeline. We take the basic features of the accounting system α, β, as given and vary the bias through δ. The variation of δ can result from decisions by the owner, by a regulator or by a standard setter. These institutions are likely to have different objectives. Therefore, we consider the effects of δ on three measures. First, we determine firm value (*FV*), which equals the expected utility of the owner, where

$$FV \equiv E[U^O] = px - \Pr(y_H)s, \tag{3}$$

Second, we consider total welfare (*TW*), which is the sum of the expected utilities of the owner and the manager, which is

$$\begin{aligned} TW &= E[U^O] + E[U^M] \\ &= px - V^A a - V^C c. \end{aligned} \tag{4}$$

Expected compensation cancels out in total welfare, so *TW* comprises the expected outcome less the agent's effort costs to provide the two actions. Firm value and total welfare do not coincide in our setting because the manager earns a rent from employment in the firm due to the limited liability constraint.

Third, we use financial reporting quality (*FRQ*) as the measure of the informativeness of the accounting signal,

$$\begin{aligned} FRQ &= \Pr\left(y_H \,|X = x\right) + \Pr\left(y_L \,|X = 0\right) \\ &= \left(1 - \alpha_\delta \exp(-c)\right) + \left(1 - \beta_\delta \exp(-c)\right) \\ &= 2 - \left(\alpha + \delta\right)\exp(-c) - \left(\beta - \delta\right)\exp(-c) \\ &= 2 - \varphi \exp(-c). \end{aligned} \tag{5}$$

Which of these two measures is more relevant depends on who can determine the level of conservatism of the accounting system. The firm's owner is interested in maximizing firm value and takes the manager's reservation utility as given. As we show, AQ effort can help reallocate rents from the manager to the owner, thus increasing firm value but without a productive effect. We do not endogenize the managerial labor market (i.e., the reservation utility), which can lead to an adjustment for non-transitory managerial rents. Rather, we consider total welfare that reflects the joint productive effects of conservatism on the agency consisting of the owner and the manager and generated total welfare is distributed to the owner and the manager. Furthermore, from a regulator's perspective, total welfare encompasses the full economic surplus of the agency. Accounting standard setters can influence the level of conservatism through the standards, and they are generally interested in increasing the informativeness of the resulting financial reports, although not for performance evaluation but for decision usefulness in capital markets.

3. Analysis

3.1. Manager's choice of accounting quality effort

Given the compensation contract with bonus *s*, the manager chooses productive effort *a* and AQ effort *c* simultaneously to maximize his expected utility,

$$E[U^M] = \Pr(y_H \,|a, c)s - V^A a - V^C c,$$

where the probability of receiving s is

$$\begin{aligned}\Pr(y_H \mid a,c) &= 1-\Pr(y_L \mid a,c)\\ &= p-\left(p\alpha-(1-p)\beta+\delta\right)\exp(-c).\end{aligned}$$

The factor with which AQ effort c enters the expected utility is

$$T \equiv p\alpha-(1-p)\beta+\delta. \tag{6}$$

It determines whether the manager has an incentive to choose a positive AQ effort at all. If $T < 0$, then $\Pr(y_H \mid a,c)$ decreases in c. Conversely, if $T > 0$, the manager experiences an incremental benefit of AQ effort and chooses c to equate the marginal benefit and the marginal cost.

The term T increases in p and α, it decreases in β, and increases in conservatism δ. Intuitively, the likelihood of earning the bonus s increases with an overstatement error of an actual low outcome, which occurs with probability $(1-p)(\beta-\delta)$, and it decreases with an understatement error of an actual high outcome, that is, $p(\alpha+\delta)$. Therefore, if

$$(1-p)(\beta-\delta) > p(\alpha+\delta),$$

which is exactly the condition $T < 0$, the manager has no incentive to exert AQ effort because that would in fact decrease the probability of earning the bonus. In contrast, if the understatement error dominates, the manager has an incentive to exert AQ effort to reduce the disadvantage. The following result states this condition and the effects of conservatism on managerial efforts. All proofs are in the appendix.

Lemma 1: (i) A *necessary condition that the manager exerts AQ effort is*

$$T \equiv p\alpha-(1-p)\beta+\delta > 0. \tag{7}$$

Higher conservatism δ increases the domain in which (7) *holds.*

(ii) *Holding the bonus s constant, a higher δ induces a higher AQ effort and a higher productive effort* (*if $c > 0$*).

The proof contains explicit upper bounds for the cost parameter V^C to ensure that $c > 0$ is possible. Since T increases in δ, the first part follows immediately. Part (ii) states that higher conservatism makes it more likely that $c > 0$ and increases c, creating a complementarity between a and c as long as s is held fixed. The optimal c strictly increases in conservatism δ because

$$\Pr(y_L \mid a,c) = (1-p) + \left(p\varphi - \beta + \delta\right)\exp(-c)$$

increases in δ, and this increase can be mitigated by higher c, which lowers $\exp(-c)$. Finally, ceteris paribus, the manager chooses a higher a for higher δ because the marginal benefit of $p(a)$ increases with $1-\varphi\exp(-c)$, and this probability increases in c.

3.2. Locally optimal regimes

The owner chooses the optimal bonus s to maximize firm value,

$$FV = px - \Pr(y_H)s$$

subject to the manager's participation constraint, which is always satisfied. We show that the properties of the optimal contract differ fundamentally whether or not the owner wants to induce the manager to improve accounting quality. This depends on the conservatism δ. We refer to settings with $c = 0$ as regime 1 and settings with $c > 0$ as regime 2 and use the indexes "1" and "2" for the functions of FV and TW and the optimal choices s and a in the respective regimes.

The following proposition states a strict benefit of conservative accounting if the manager is not incentivized to exert AQ effort ($c = 0$), which arises if δ is low or if the cost of AQ effort V^C is high.

Proposition 1 (regime 1): *Let $c = 0$. The optimal contract includes a bonus s_1 and induces productive effort a_1 of*

$$s_1 = \sqrt{\frac{V^A x}{\left(1-(\alpha+\delta)\right)(1-\varphi)}} \text{ and } a_1 = \ln\left(\frac{(1-\varphi)s_1}{V^A}\right). \tag{8}$$

Increasing conservatism has the following effects:

(i) *The bonus strictly increases.*

(ii) *Productive effort strictly increases.*

(iii) *Firm value strictly increases.*

(iv) *Total welfare strictly increases.*

To understand the results, consider the first-order condition of the manager's expected utility with respect to a for a given bonus s,

$$p'(1-\varphi)s - V^A = 0 .$$

Conservatism δ has no direct effect on the manager's productive effort choice. Yet there is an indirect effect on a because δ impacts s, and we state in the proposition that the optimal s_1 increases in δ. To see this, consider the firm value

$$FV_1 = px - \left(p(1-\varphi) + \beta - \delta \right) s .$$

Conservatism reduces the probability that y_H occurs and, thus, the probability with which the manager earns the bonus. Holding s constant, firm value strictly increases in δ because it reduces the probability $\Pr(y_H)$ with which the bonus must be paid. The lower expected compensation does not change production incentives, but simply diminishes the manager's rent. Yet because it becomes less costly to induce productive effort, the owner optimally increases the bonus s. Both the lower probability of paying the bonus and the increase of the bonus to extract more productivity increase firm value. These results are similar to those shown in Kwon (2005).

Proposition 1 (iv) states that total welfare strictly increases in δ as well. Total welfare equals

$$TW_1 = px - V^A a .$$

The increase of TW_1 stems from the higher productive effort a; it is unaffected by the redistribution of wealth between the owner and the manager.

We now turn to regime 2 in which we study the properties of the optimal contract that induces the manager to choose an AQ effort $c > 0$.

Proposition 2 (regime 2): *Let* $c > 0$. *Increasing conservatism has the following effects:*

(i) *The bonus strictly decreases.*

(ii) *Productive effort strictly decreases.*

(iii) *Accounting quality effort strictly increases.*

(iv) *Firm value strictly increases.*

(v) *Total welfare strictly decreases.*

The optimal bonus s_2, productive effort a_2 and AQ effort c are implicitly defined in the proof. The results are markedly different from those presented in Proposition 1. The only, but arguably most important, result that carries over is that firm value strictly increases in conservatism (part (iv)). But even this result is a consequence of different countervailing effects.

To understand the results, note that in regime 2, the manager's incentive to exert AQ effort is to mitigate the negative impact of more conservatism on the probability of earning the bonus. It is *not* the desire to improve the quality of the accounting system per se, which is in the interest of the owner. Despite these diverging objectives, the AQ effort improves the informativeness of the signal, which benefits the owner because it alleviates the agency problem. Because of that, the owner can *reduce* the bonus s and still keep productive incentives intact. We show that firm value strictly increases in δ.

According to Lemma 1 (ii), holding s constant, productive effort and AQ effort (if $c > 0$) increase in δ. The reduction of s introduces a countervailing effect. We state in Proposition 2 that with the optimal s_2, AQ effort strictly increases in δ, whereas productive effort strictly decreases in δ. This result is particularly interesting because the effects are unambiguous but converse. The reason is that conservatism has a much stronger effect on c than on a. The manager chooses the optimal a_2 and c to maximize the expected utility as follows:

$$a_2 = \ln\left(\frac{\left(1-\varphi\exp(-c)\right)s_2}{V^A}\right),$$

$$c = \ln\left(\frac{\left(p\varphi-\beta+\delta\right)s_2}{V^C}\right).$$

Conservatism affects a_2 only indirectly through s_2 and the complementary impact on c, whereas δ has a direct increasing effect on AQ effort c, besides the effect of δ through s_2. Thus, the productive effort is entirely governed by indirect effects where the decrease of the optimal bonus s_2 eventually dominates the positive complementary effect of a larger c. In contrast, the direct positive effect of δ on AQ effort c dominates the indirect effects through s_2 and a, implying that AQ effort ultimately increases in conservatism.

Since $TW_2 = px - V^A a - V^C c$, the decrease of both *p* and *a* leads to a welfare loss, which is further increased by the disutility of the increasing AQ effort. Therefore, total welfare strictly declines in conservatism. Since $E[U_2^M] = TW_2 - FV_2$, the manager's expected utility strictly decreases in δ, hence, conservatism also leads to a reallocation of wealth from the manager to the owner.

3.3. Optimal contract choice

The key determinant for which regime emerges is whether the contract induces the manager to exert AQ effort or not. This choice depends first of all on the sign of $T = p\alpha - (1-p)\beta + \delta$, where $T < 0$ induces $c = 0$. Yet T is endogenous because it depends on the probability p that is determined by the productive effort a, which again is induced by the bonus s. Indeed, $T \geq 0$ is necessary but not sufficient to induce $c > 0$ because s has also to be high enough to lift the manager's marginal benefits of injecting c above the cost V^C, i.e. $T\exp(-c)s \geq V^C$, which is $Ts = V^C$ at $c = 0$. Therefore, we restate the condition of c greater than or equal to 0 in terms of the bonus s that induces c.

Lemma 2: *For each $\delta \in [0, \bar{\delta}]$ there exists a threshold for the bonus,*

$$\hat{s}(\delta) = \frac{1}{\alpha + \delta}\left(V^C + \frac{V^A \varphi}{(1-\varphi)}\right), \tag{9}$$

such that $s \leq \hat{s}(\delta)$ induces $c = 0$ and $s > \hat{s}(\delta)$ induces $c > 0$. The threshold strictly declines in δ.

Note that $\hat{s}$ depends only on exogenous parameters. It does not depend the outcome x because that is relevant for the owner but not for the manager. If s equals $\hat{s}$ for some δ, then the manager's optimal productive action a_1 in regime 1 (see Proposition 1) induces a probability p that exactly satisfies $T(\delta)s = V^C$. Hence, for any feasible δ, if $s_1 \leq \hat{s}$ then we have a regime 1 solution with $c = 0$ and if $s_2 \geq \hat{s}$ a regime 2 solution. The threshold $\hat{s}$ strictly declines in conservatism, which indicates that regime 1 is less likely to arise for larger δ.

The bonus s is the owner's sole instrument to control the manager's actions and she determines the regime through the choice of $s \in \{s_1, s_2\}$. According to Lemma 2, the owner can always induce $c > 0$ for each δ by setting a bonus that exceeds $\hat{s}(\delta)$. However, this is not necessarily desirable because paying this bonus can be too expensive. The owner may be better off to set a smaller $s_1 \le \hat{s}$, which induces c = 0 but reduces expected compensation. Consider the constrained optimization problem

$$\operatorname*{Max}_s FV_2 \text{ subject to } s \ge \hat{s}(\delta),$$

then a necessary condition for $s_2 > \hat{s}(\delta)$ (and $c > 0$) to occur is that

$$\frac{\partial FV_2}{\partial s}\bigg|_{s=\hat{s}(\delta)} > 0.$$

If this condition is satisfied, a regime 2 contract with $s_2 > \hat{s}(\delta)$ is a candidate for the global optimal contract; we call this a "relevant" regime. If, for the same δ, the owner alternatively optimizes under the assumption c = 0 and the resulting optimal bonus is $s_1 < \hat{s}(\delta)$, then this regime 1 contract is also a candidate for the global optimal contract.[8] The next proposition states when the two regimes are relevant and which of them is optimal for the owner for a given δ.

Proposition 3: (i) *A regime* 1 *contract is relevant only for* $\delta \in [0, \delta_1]$, *and a regime* 2 *contract is relevant only for* $\delta > \delta_2$ *where* $\delta_2 < \delta_1$.

(ii) *If* $\delta_1, \delta_2 \in [0, \bar{\delta}]$, *there exists a threshold* $\hat{\delta} \in (\delta_2, \delta_1)$ *where* $FV_1(s_1(\hat{\delta})) = FV_2(s_2(\hat{\delta}))$ *holds. For* $\delta < \hat{\delta}$, *the optimal contract is a regime* 1 *and for* $\delta > \hat{\delta}$ *a regime* 2 *contract.*

Part (i) of Proposition 3 establishes intervals of δ for which optimal regime 1 and regime 2 contracts are relevant. A regime 1 contract is relevant for relatively low δ, whereas a regime 2 contract is relevant for relatively large δ. Moreover, there exists an interval of $\delta \in [\delta_2, \delta_1]$ in which both contracts are relevant.

Part (ii) states that if there is a region of δ in which both contracts are relevant, there is a threshold $\hat{\delta}$ strictly within (δ_2, δ_1) such that a regime 1 contract dominates for $\delta < \hat{\delta}$ and a regime 2 contract dominates for $\delta > \hat{\delta}$. Note that for $\delta = \hat{\delta}$ both contracts are relevant and they induce the same firm value with the regime 2 contract leading to a positive AQ effort $c > 0$ so the owner

---

[8] The owner can implement a regime 1 contract also for higher δ, but then the optimal bonus is $\hat{s}(\delta)$. Such a contract is always dominated by a regime 2 contract, and we do not further consider such a constrained regime 1 contract.

is indifferent between the two (we describe the behavior of the strategies at the threshold $\hat{\delta}$ in more detail in Corollary 2 below). She strictly prefers an optimal bonus $s = s_1$ for $\delta < \hat{\delta}$, and $s = s_2$ for $\delta > \hat{\delta}$, where $s_2 > s_1$.

We next explore how the optimal contract $s \in \{s_1, s_2\}$ depends on the fundamental characteristic of the firm's productivity, in our model reflected by the outcome *x*. The following result provides sufficient conditions on the outcome *x* for the existence of only a regime 1 or a regime 2 optimal contract.

Corollary 1: (i) *If* $x < \hat{s}(\bar{\delta}) = \dfrac{V^A + \frac{\varphi}{1-\varphi} V^C}{\alpha + \bar{\delta}}$ *then only a regime 1 contract is relevant.*

(ii) *If* $x > x_1(\delta) = \dfrac{\left(1-(\alpha+\delta)\right)(1-\varphi)}{V^A}\left[\dfrac{1}{\alpha+\delta}\left(V^C + \dfrac{V^A \varphi}{(1-\varphi)}\right)\right]^2$ *then only a regime 2 contract is relevant.*

The two bounds for *x* in Corollary 1 depend only on exogenous parameters, including the marginal effort costs and the errors of the accounting system. In the proof, we first establish that in either regime, the optimal bonus *s* increases in the outcome *x* for any feasible δ. A higher *s* induces an increase of the productive effort *a* and the AQ effort *c* (if $c > 0$). This is intuitive because a greater outcome *x* increases the expected marginal benefit of productive effort; hence, the owner increases the bonus.

Part (i) states a lower bound of *x* that must be satisfied in order to make a regime 2 contract relevant and, therefore, optimal. Otherwise, only a regime 1 contract can exist. This boundary term is the minimum bonus *s* that a regime 2 contract requires. If *x* is less than this minimum required bonus, the owner would make a loss and, therefore, she offers no regime 2 contract.

If *x* exceeds the upper bound in part (ii), then only a regime 2 solution is relevant and also optimal. The bound $x_1(\delta)$ is derived from the threshold $\hat{s}(\delta)$ that limits a regime 1 contract. Therefore, if $x > x_1(\delta)$ the owner would want to increase $s_1(\delta)$ over $\hat{s}(\delta)$ because the outcome is so high that more productive effort is desirable, but this is not implementable under a regime 1 contract because the manager would then choose $c > 0$. In settings with intermediate outcomes, $x \in (\hat{s}(\bar{\delta}), x_1(0)]$, both regimes can be relevant.

Figure 3: Implementable and relevant regimes conditional on the outcome *x*

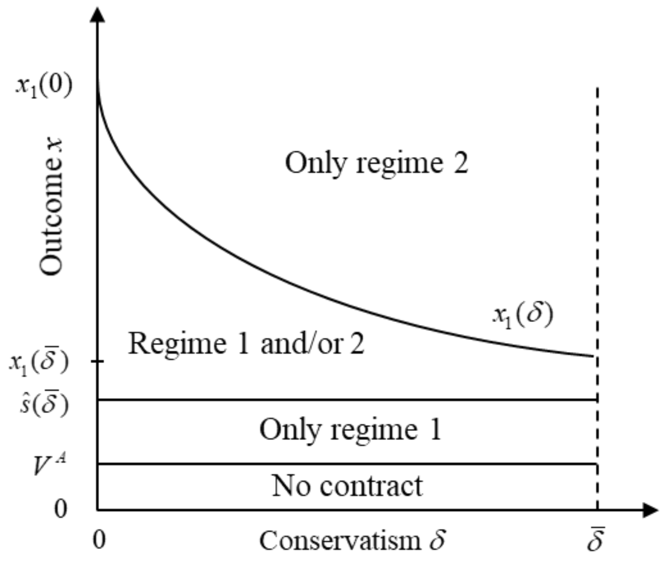


Figure 3 illustrates the results in Corollary 1 and shows the threshold $x_1(\delta)$ line that separates the regimes. That is, if a regime 1 (2) contract is relevant for some $\{\delta, x\}$ combination, the optimal s_1 (s_2) must be less (greater) than $\hat{s}(\delta)$. For example, consider some x within this intermediate interval. For relatively low δ, a regime 1 contract is relevant; if δ is relatively high, a regime 1 solution becomes increasingly unattractive and a regime 2 contract more attractive. Therefore, holding x fixed, when δ increases further, then only a regime 2 contract becomes relevant. Figure 3 does not explicitly show a particular optimal regime in this intermediate interval; that depends on the specific parameters. For completeness, the figure also depicts another lower bound, $x = V^A$, which we assume is always satisfied because otherwise the agency problem is so severe that the owner would never hire the manager.

3.4. Properties of the optimal contract

In the following, we specifically consider the functional forms of the strategies and the welfare measures at the threshold $\delta = \hat{\delta}$, which is the point where the regimes shift.

Corollary 2: *At the conservatism threshold,* $\delta = \hat{\delta}$*, the following holds.*

(i) *The optimal bonus discretely jumps to a higher level, that is,* $s_2(\hat{\delta}) > s_1(\hat{\delta})$.

(ii) *The optimal productive effort discretely jumps to a higher level, that is,* $a_2(\hat{\delta}) > a_1(\hat{\delta})$.

(iii) *The optimal AQ effort discretely jumps from* 0 *to* $c(\hat{\delta}) > 0$.

(iv) *Firm value is equal under both regimes but exhibits a kink at* $\delta = \hat{\delta}$.

(v) *Total welfare discretely jumps to a higher level, that is,* $TW_2(\hat{\delta}) > TW_1(\hat{\delta})$.

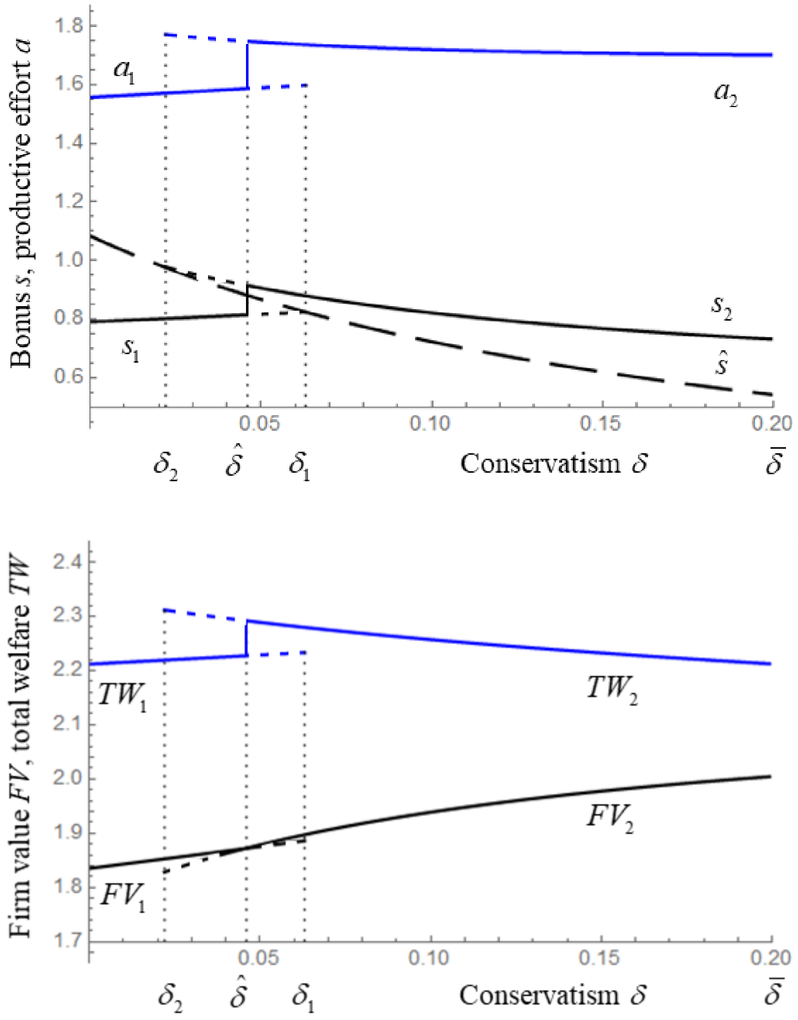
While the functions of all strategies and welfare measures are continuous within each regime (see Proposition 1 and 2), the shift from regime 1 to regime 2 at $\delta = \hat{\delta}$ induces discontinuities and discrete jumps of the functions. As we state in Proposition 3, $\hat{\delta}$ is defined as the degree of conservatism at which the firm values generated in the two regimes are just equal. Yet, the *FV* function shows a kink because the slope of FV_1 is less than that of FV_2 at $\delta = \hat{\delta}$.

To induce a shift from regime 1 to regime 2 at $\delta = \hat{\delta}$, the owner must offer a significantly higher bonus to induce regime 2 at $\delta = \hat{\delta}$ because it requires a bonus that is greater than the threshold bonus $\hat{s}(\hat{\delta})$ to implement a regime 2 solution. In fact, because $\hat{\delta} \in (\delta_2, \delta_1)$, where both boundaries lie on the threshold line (i.e., $s_1(\delta_1) = \hat{s}(\delta_1)$ and $s_2(\delta_2) = \hat{s}(\delta_2)$), it must be that $s_1(\hat{\delta}) < \hat{s}(\hat{\delta}) < s_2(\hat{\delta})$.

A discrete upward jump of *s* induces a discrete upward jump of the productive effort, $a_2(\hat{\delta}) > a_1(\hat{\delta})$, and – since total welfare is predominantly determined by productive effort – also on total welfare. Finally, at the boundary of regime 2, δ_2, $c = 0$. That is, the AQ effort discretely jumps from zero to $c > 0$ at the regime shift at $\delta = \hat{\delta}$. Such a discrete jump is consistent with the view that AQ investments need to have a minimum size in order to be effective or with a fixed cost of AQ investments, although in our model there is no such assumption.

Figure 4 presents a numerical example to illustrate the functional forms of the optimal strategies $\{a_1, s_1\}$ in regime 1 and $\{a_2, s_1\}$ in regime 2 in the upper panel. It also includes the threshold $\hat{s}$ as a dashed line. The optimal AQ effort *c* in regime 2 is not included: it has a value of $c = 0$ and increases, starting with δ_2, up to around 0.5. The lower panel shows the resulting firm values $\{FV_1, FV_2\}$ and total welfares $\{TW_1, TW_2\}$. Both panels illustrate the optimal functional forms with the discontinuities stated in Corollary 2 at $\hat{\delta}$, where the regimes shift.

Figure 4: Optimal strategies and welfare measures for different levels of conservatism (underlying parameters: $\alpha = .2$; $\beta = .2$; $x = 3$; $V^A = 0.1$, and $V^C = 0.15$)



3.5. Optimal conservatism

Up to now, we assumed an exogenous δ that varies over the feasible range $\delta \in [0, \bar{\delta}]$, where $\bar{\delta} = \min\{.5 - \alpha, \beta\}$. The larger α or the lower β, the smaller is the range of δ. We next consider the case that δ is a choice variable, so our analysis provides insights into the optimal degree of conservatism.

We show results for three measures, firm value *FV*, total welfare *TW*, and the quality of the financial report. Recall that $TW = FV + E[U^M]$. In the optimal solution the manager earns a rent due to limited liability; therefore, $E[U^M] > 0$ and $TW > FV$. If the owner can choose conservatism, she will choose the contract to maximize *FV*. A regulator interested in productivity and value generation in the agency chooses the degree of conservatism to maximize *TW*. An accounting standard setter is interested in maximizing the quality of financial reports *FRQ*, where according to (5),

$$FRQ = 2 - \varphi \exp(-c).$$

Proposition 4: *The optimal conservatism δ^* depends on the objective and is as follows.*

(i) *Maximizing firm value: $\delta^* = \bar{\delta}$.*

(ii) *Maximizing total welfare: $\delta^* = \hat{\delta}$ if both regimes are relevant within $[0, \bar{\delta}]$; $\delta^* = \bar{\delta}$ if only regime 1 is relevant; $\delta^* = 0$ if only regime 2 is relevant.*

(iii) *Maximizing quality of financial reports: $\delta^* = \bar{\delta}$; if regime 1 is optimal, any other δ is optimal as well.*

If the owner can choose δ, she always prefers maximum conservatism. The reason is that both FV_1 and FV_2 strictly increase in δ.

In contrast, maximizing total welfare has either a unique interior maximum at $\delta^* = \hat{\delta}$ or leads to a corner solution, depending on which of the two regimes is relevant. If the two values $\hat{\delta}$ and δ_1 lie in $(0, \bar{\delta})$, which is the case if the optimal contract shifts from regime 1 to regime 2 in this interval, then an interior optimal degree of conservatism maximizes *TW*. In that case, $TW_2(\delta)$ for $\delta > \hat{\delta}$ is less than $TW_2(\hat{\delta})$ according to Proposition 2 (v). If only one regime governs the interval of feasible δ, then the optimum lies at a boundary. If regime 1 is not implementable (e.g., because *x* is sufficiently high; for a sufficient condition see Corollary 1), then the optimal $\delta^* = 0$, that is conservatism is strictly undesirable in regime 2. Conversely, if regime 2 is not relevant, then the maximum conservatism, $\delta^* = \bar{\delta}$, is optimal in a regime 1 contract. Thus, maximum conservatism is not always desirable, particularly not when the optimal contract does induce the manager to strictly exert AQ effort. The reason that *TW* reacts differently from *FV* regarding the desirable degree of conservatism is that it captures the productivity of the agency in total, whereas *FV* does not include the manager's rent.

Finally, the δ that maximizes *FRQ* is again $\delta^* = \bar{\delta}$. This solution is unique if regime 2 governs because *FRQ* depends on conservatism δ exclusively through the AQ effort *c*. The greater

c, the greater is *FRQ*. If the optimal contract at $\delta = \overline{\delta}$ is a regime 1 contract, then any degree of conservatism is optimal because *FRQ* does not depend on δ in regime 1 as $c = 0$ for all δ.

We conclude that the accounting system becomes (weakly) more informative the greater is the conservatism. Thus, conservatism has a positive effect on accounting quality, and the greater the conservative bias the larger is the gain in accounting quality. Therefore, the view held by accounting standard setters such as the FASB (2010) and the IASB (2018) that conservatism impairs accounting quality, does not apply generally.[9] It does not consider that the accounting system can induce managers to make accounting quality investments, such as more effective internal controls, which increases the informativeness of the financial report. In fact, if the owner could choose the conservatism in our model, she would make the same choice as the standard setter because it is also in her best interest.

4. Discussion and extensions

In this section, we discuss key assumptions of our economic setting and some extensions.

Modeling conservatism. We define conservative accounting as the increase in the probability of an understatement of earnings and a decrease in the probability of an overstatement by an equal amount. This is a reduced-form representation used in much of the theoretical literature, particularly in binary settings. There are other ways to incorporate conservatism into formal models, which may capture particular accounting standards or econometric properties. We also do not consider individual adjustments of the understatement or overstatement errors. In that case, increasing the overstatement error is always undesirable, whereas increasing the understatement error can induce some of the advantages we show for conservatism.

Like much prior literature, we do not consider multi-period effects of accounting choices, for example, that an understatement of earnings reverses in future periods, which arises under a clean surplus property. For example, Christensen, Feltham, and Şabac (2005) study characteristics

[9] However, financial reporting quality can be defined in various ways. We use a specific definition, which depends only on the sum of the errors.

of earnings in a dynamic agency setting. Glover and Lin (2018) consider a model in which conservatism reverses in the second period. In such a model there must be some additional economic link between the two periods to give rise to interesting findings. It would be interesting to extend our setting to a multi-period model and explicitly include inter-period accounting processes.

Modeling the cost of efforts. Our agency model employs strictly concave benefit functions and linear cost functions for both productive and AQ effort. This makes the model tractable and allows explicit solutions in certain parts of the model (e.g., regime 1) and explicit thresholds for our results. An alternative formalization is to use effort as the probabilities directly and combine this with strictly convex cost functions. One might question whether some of our findings, for example, the jumps at the switch of the regimes, are primarily driven by the linear cost because it implies a strictly positive marginal cost at the point of zero effort. Alternatively, consider quadratic effort costs, i.e., $V^A a^2/2$ and $V^C c^2/2$. This changes the expressions for the optimal solutions, but it retains the existence of two regimes 1 and 2 because the necessary condition $T = p\alpha - (1-p)\beta + \delta > 0$ (Lemma 1) must still hold to motivate the manager to exert $c > 0$ (in fact, $T > 0$ is now also sufficient to induce $c > 0$). Thus, as long as p is not large enough to satisfy at least $T = 0$, we have $c = 0$ and a solution as in regime 1. This implies that there is again a threshold $\hat{s}(\delta)$ (analogous to Lemma 2) such that $c > 0$ only occurs for $s > \hat{s}(\delta)$. One can easily show that the threshold strictly declines in conservatism ($\partial\hat{s}/\partial\delta < 0$), and the owner's choice of the bonus is still determined by the size of the outcome x. Thus, the basic structures of our solution regarding the switch of regimes carry over to other cost structures.

Owner can control accounting quality effort. In our model, the owner is dependent on the manager who chooses the AQ effort. The owner may have another control variable besides compensation based on the single accounting signal, which she could use to induce productive or AQ effort specifically and thus make different effort allocations implementable. Alternatively, consider the case in which the owner could prohibit managerial AQ effort, i.e., set $c = 0$. The optimal contract is then always a regime 1 contract, and the owner can increase s_1 above $\hat{s}$ as the manager cannot choose a $c > 0$. Our main result in Proposition 1 then describes the optimum over the full range of

δ; in particular, more conservatism is always preferable. Yet, for sufficiently high δ, when the manager would prefer inducing $c > 0$ if it was available, the owner loses by restricting herself to a regime 1 contract because we show that at the point of switching to regime 2 $(\delta = \hat{\delta})$, the increase of FV_2 is greater than that of FV_1, implying that a regime 2 contract generates a higher firm value than a regime 1 contract in an area of δ above $\hat{\delta}$.

Ex post earnings management. We do not consider earnings management by the manager. Suppose the manager can engage in ex post earnings management (EM), i.e., he observes the signal y and then can bias the accounting report m at a personal cost. A truthful report has $m_j = y_j$. The manager would only overstate a y_L (thus report m_H) signal but not understate a y_H signal to maximize the probability of earning the bonus. Conservative accounting can be beneficial because it runs counter upwards earnings management (Chen, Hemmer, and Zhang 2007, Drymiotes 2011). For concreteness, let $b \geq 0$ be the EM effort that increases the probability $(1-\exp(-b))$ that the upward bias is successful and $V^B b$ with $V^B > 0$ the cost of EM. The manager's expected utility at the time of EM is $E[U^M | y_L] = \Pr(m_H | a, c, y_L)s - V^B b = (1-\exp(-b))s - V^B b$, which implies an optimal EM effort of $b = \ln(s / V^B) > 0$ assuming $s > V^B$.

Corollary 3: *In regime 1 (c = 0), the bonus and productive effort are independent of conservatism. Firm value still strictly increases and total welfare strictly decreases in δ.*

Somewhat surprisingly, in regime 1 the effect of conservatism on the bonus and productive effort is exactly neutralized by upwards EM. The reason is that with EM, the owner pays the bonus not only when y_H obtains but also for y_L when EM is successful, which occurs with probability $1-\exp(-\ln(s / V^B)) = 1-(V^B / s)$. Since the manager benefits more from EM the higher is the bonus, this probability is strictly increasing in s, implying that given the low earnings y_L the owner pays an expected compensation $s(1-(V^B / s)) = s - V^B$. Therefore, the owner's total expected compensation amounts to

$$\Pr(y_H)s + \Pr(y_L)(s - V^B) = s - \Pr(y_L)V^B.$$

For setting the bonus, the owner trades off the expected benefit of increasing the bonus (i.e., the increase in the productive effort) against the marginal increase in expected compensation.

With EM the marginal expected compensation is independent of δ, and since the manager's optimal productive effort choice for a certain bonus does not depend on δ either, both the bonus and the productive effort are unaltered by conservatism. Yet the owner still benefits from more conservatism because δ strictly reduces the probability $\Pr(m_H)$ of paying the bonus because $\Pr(y_L) = p\alpha + (1-p)(1-\beta) + \delta$ strictly increases in δ. Thus, firm value strictly increases in conservatism, and the manager's expected compensation declines by the same amount.[10] While this reallocation of compensation does not change total welfare, expected earnings management increases in δ (because $\Pr(y_L)$ increases in δ such that EM occurs more frequently) and so does its cost, which decreases total welfare.

A regime 2 contract is more complicated because EM has an effect on incentives for both productive and AQ effort. One can show that, similar to our main results, the bonus strictly decreases in δ and firm value strictly increases. However, due to the multiple trade-offs involved in the optimal solution, there are no unambiguous results for the effect of δ on productive and AQ effort as well as for total welfare. Therefore, our main results continue to hold when adding ex post earnings management opportunities.

More general risk aversion. We assume a risk neutral manager who is protected by limited liability. This assumption reflects a specific strong form of risk aversion, which is particularly convenient for tractability. Kwon (2005) considers a risk-averse manager and his results are similar to those of papers that use the limited-liability assumption instead. Risk aversion might have an additional effect in our model because it captures a risk-mitigating effect of accounting quality effort that is not present under risk neutrality. Introducing risk aversion can therefore increase the benefit of accounting quality.

5. Summary

In an agency model, we analyze conditions in which a manager can be induced to invest in accounting quality (AQ) that benefits the owner but not necessarily the manager. Higher AQ

---

[10] This finding is in line with Bertomeu, Darrough, and Xue (2017) and Caskey and Laux (2017).

increases the informativeness of the financial report by mitigating errors in the accounting system, for example, through improving accounting processes and implementing more effective internal controls. The owner's control of the agency problem is the manager's compensation, which affects productive and AQ effort together. Greater conservatism reduces the β-error, i.e., a high accounting signal occurs although the outcome is low, at the cost of increasing the α-error, which reduces the probability of the manager earning a bonus, while generally keeping marginal incentives intact.

We find that a more conservative accounting system facilitates the owner to elicit AQ effort from the manager and it strictly increases firm value whether or not AQ effort can be induced. While the setting in which no AQ effort can be induced (our regime 1) produces similar results as in Kwon (2005), the underlying economics change significantly once the manager exerts AQ effort (regime 2), which occurs for greater productive outcomes. Specifically, the owner increases the bonus in regime 1 and reduces it in regime 2 for higher conservatism, which induces more (regime 1) and less (regime 2) productive effort, respectively. We establish that firm value strictly increases in conservatism in both regimes, but total welfare (equal to firm value plus the manager's rent) increases in regime 1 and decreases in regime 2. Financial reporting quality strictly increases in regime 2 and is unaffected by conservatism in regime 1. Our analysis establishes that biasing accounting information towards more conservatism is useful to induce the manager to enhance accounting quality, which attenuates the agency problem and in fact decreases the manager's expected utility, by better aligning the manager's with the owner's objectives.

If the owner can choose the degree of conservatism, she maximizes firm value through maximum conservatism. If a benevolent regulator can determine conservatism, there is typically an interior degree of conservatism that maximizes total welfare, which occurs at a regime switch. Finally, if a standard setter wants to maximize the informativeness of the financial report, again maximum conservatism is the optimal choice. Thus, whether conservatism is desirable depends on the objective of the institution that determines the bias.

References

Basu, S. 1997. The conservatism principle and the asymmetric timeliness of earnings. *Journal of Accounting and Economics* 24(1), 3–37.

Bertomeu, J., M. Darrough, and W. Xue (2017). Optimal Conservatism with Earnings Manipulation. *Contemporary Accounting Research* 34(1), 252–284.

Caskey, J. and V. Laux (2017): Corporate Governance, Accounting Conservatism, and Manipulation. *Management Science* 63(2), 424–437.

Chan, D. K. (2016). The Effects of Internal Control Reporting Regulation on Control Quality, Compensation and Audit Quality. Working paper, University of Hong Kong.

Chen, Q., T. Hemmer, and Y. Zhang (2007). On the Relation Between Conservatism in Accounting Standards and Incentives for Earnings Management. *Journal of Accounting Research* 45(3), 541–565.

Chen, Q., B. Mittendorf, and Y. Zhang (2010). Endogenous Accounting Bias when Decision Making and Control Interact. *Contemporary Accounting Research* 27(4), 1063–1091.

Christensen, P. O., G. A. Feltham, and F. Şabac (2005). A Contracting Perspective on Earnings Quality. *Journal of Accounting and Economics* 39(2), 265–294.

Drymiotes, G. C. (2011). Information Precision and Manipulation Incentives. *Journal of Management Accounting Research* 23, 231–258.

Ewert, R., and A. Wagenhofer (2011). Earnings Management, Conservatism, and Earnings Quality. *Foundations and Trends in Accounting* 6(2), 65–186.

Ewert, R., and A. Wagenhofer (2019). Effects of Increasing Enforcement on Financial Reporting Quality and Audit Quality. *Journal of Accounting Research* 57(1), 121–168.

FASB (2010). Concepts Statement No. 8 – Conceptual Framework for Financial Reporting—Chapter 1, The Objective of General Purpose Financial Reporting, and Chapter 3, Qualitative Characteristics of Useful Financial Information. Norwalk, CT.

Feltham, G. A., and J. Xie (1994). Performance Measure Congruity and Diversity in Multi-Task Principal/Agent Relations. *The Accounting Review* 69(3), 429–453.

Friedman, H. L. (2014). Implications of Power: When the CEO Can Pressure the CFO to Bias Reports. *Journal of Accounting and Economics* 58(1), 117–141.

Friedman, H. L. (2016). Implications of a Multi-Purpose Reporting System on CEO and CFO Incentives and Risk Preferences. *Journal of Management Accounting Research* 28(3), 101–132.

Gigler, F. B., and T. Hemmer (2001). Conservatism, Optimal Disclosure Policy, and the Timeliness of Financial Reports. *The Accounting Review* 76(4), 471–493.

Gigler, F., C. Kanodia, H. Sapra, and R. Venugopalan (2009). Accounting Conservatism and the Efficiency of Debt Contracts. *Journal of Accounting Research* 47(3), 767–797.

Glover, J., and C. Levine (2019). Information Asymmetries about Measurement Quality. *Contemporary Accounting Review* 36(1), 50–71.

Glover, J. C., and H. H. Lin (2018). Accounting Conservatism and Incentives: Intertemporal Considerations. *The Accounting Review* 93(6), 181–201.

Goh, B. W., and D. Li (2011). Internal Controls and Conditional Conservatism. *The Accounting Review* 86(3), 975–1005.

IASB (2018). Conceptual Framework for Financial Reporting. London.

Kwon, Y. K. (2005). Accounting Conservatism and Managerial Incentives. *Management Science* 51(11), 1626 –1632.

Kwon, Y. K., D. P. Newman, and Y. S. Suh (2001). The Demand for Accounting Conservatism for Management Control. *Review of Accounting Studies* 6(1), 29–51.

Liang, P. J., and L. Nan (2014). Endogenous Precision of Performance Measures and Limited Managerial Attention. *European Accounting Review* 23(4), 693–727.

Meth, B. (1996). Reduction of Outcome Variance: Optimality and Incentives. *Contemporary Accounting Research* 13(1), 309–328.

Mora, A., and M. Walker (2015). The Implications of Research on Accounting Conservatism for Accounting Standard Setting. *Accounting and Business Research* 45(5), 620–650.

Nagar, V., M. V. Rajan, and K. Ray (2018). An Information-based Model for the Differential Treatment of Gains and Losses. *Review of Accounting Studies* 23(2), 622–653.

Nelson, J., J. Ronen, and L. White (1988). Legal Liabilities and the Market for Auditing Services. *Journal of Accounting, Auditing & Finance* 3(3), 255–285.

Pae, S., and S.-W. Yoo (2001). Strategic Interaction in Auditing: An Analysis of Auditors' Legal Liability, Internal Control System Quality, and Audit Effort. *The Accounting Review* 76(3), 333–356.

Patterson, E., and J. R. Smith (2007). The Effects of Sarbanes-Oxley on Auditing and Internal Control Strength. *The Accounting Review* 82(2), 427–455.

Patterson, E. R, and J. R. Smith (2016). The Strategic Effects of Auditing Standard No. 5 in a Multi-Location Setting. *Auditing: A Journal of Practice and Theory* 35(1), 119–138.

Smith, R., S. Tiras and S. Vichitlekarn (2000). Internal Control Assessment and Substantive Testing in Audits for Fraud. *Contemporary Accounting Research* 17(2), 327–356.

Venugopalan, R. (2004). Conservatism in Accounting: Good or Bad? Working paper, University of Chicago.

Watts, R. (2003). Conservatism in Accounting Part I: Explanations and Implications. *Accounting Horizons* 17(3), 207–221.

Appendix

Proof of Lemma 1

Since the manager determines a and c simultaneously, the two first-order conditions of maximizing his expected utility are:

$$\frac{\partial E[U^M]}{\partial a} = p'\left(1-\varphi\exp(-c)\right)s - V^A = 0, \tag{A1}$$

$$\frac{\partial E[U^M]}{\partial c} = \left(p\varphi - \beta + \delta\right)\exp(-c)s - V^C = 0. \tag{A2}$$

A necessary condition that (A2) has a positive solution, $c > 0$, is

$$V^C < \underbrace{\left(p\varphi - \beta + \delta\right)}_{=T} s\,.$$

This condition requires that $T > 0$. Since $p < 1$, α and $\beta \in \left(0, 0.5\right)$, $\delta \le \min\left\{0.5-\alpha, \beta\right\}$ we have $\left(p\varphi - \beta + \delta\right) < \frac{1}{2}$ which results in $2V^C < s$ for $c > 0$ to occur.

The second-order conditions for a maximum are

$$\begin{aligned}\frac{\partial^2 E[U^M]}{\partial a^2} &= p''\left(1-\varphi\exp(-c)\right)s \\ &= -p'\left(1-\varphi\exp(-c)\right)s \\ &= -V^A < 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 E[U^M]}{\partial c^2} &= -\left(p\varphi - \beta + \delta\right)\exp(-c)s \\ &= -V^C < 0.\end{aligned}$$

The third second-order condition for a maximum is that the following expression is positive,

$$SOC \equiv \frac{\partial^2 E[U^M]}{\partial a^2} \cdot \frac{\partial^2 E[U^M]}{\partial c^2} - \left(\frac{\partial^2 E[U^M]}{\partial a \partial c}\right)^2 > 0,$$

where the cross derivative is

$$\begin{aligned}CR \equiv \frac{\partial^2 E[U^M]}{\partial a \partial c} &= p'\varphi\exp(-c)s = \frac{V^A \varphi\exp(-c)}{1-\varphi\exp(-c)} \\ &= \frac{\varphi V^A}{\exp(c)-\varphi},\end{aligned}$$

after substituting $s = \dfrac{V^A}{p'\left(1-\varphi\exp(-c)\right)}$ from (A1). Inserting yields

$$SOC = V^A\left(V^C - V^A\left(\frac{\varphi}{\exp(c)-\varphi}\right)^2\right).$$

This condition must be positive (otherwise the solution is a saddle point), which requires

$$\frac{V^C}{V^A} > \left(\frac{\varphi}{\exp(c)-\varphi}\right)^2.$$

The RHS strictly decreases in *c*, hence, a sufficient condition for a maximum is

$$\frac{V^C}{V^A} > \left(\frac{\varphi}{1-\varphi}\right)^2.$$

Given this condition holds, the maximum is unique: Both second-order partial derivatives are negative everywhere, implying that any stationary point of $E[U^M]$ is a maximum. Multiple maxima cannot exist because that would imply existence of at least one minimum, but that is not possible.

Applying the implicit function theorem, the comparative statics with respect to any parameter *j* are determined as follows:

$$\begin{aligned}\frac{\partial^2 E[U^M]}{\partial a\partial j} + \frac{\partial^2 E[U^M]}{\partial a^2}\frac{\partial a}{\partial j} + CR\frac{\partial c}{\partial j} = 0\\ \frac{\partial^2 E[U^M]}{\partial c\partial j} + CR\frac{\partial a}{\partial j} + \frac{\partial^2 E[U^M]}{\partial c^2}\frac{\partial c}{\partial j} = 0\end{aligned} \Rightarrow \begin{bmatrix}\dfrac{\partial a}{\partial j}\\ \dfrac{\partial c}{\partial j}\end{bmatrix} = \begin{pmatrix}-V^A & CR\\ CR & -V^C\end{pmatrix}^{-1}\begin{bmatrix}-\dfrac{\partial^2 E[U^M]}{\partial a\partial j}\\ -\dfrac{\partial^2 E[U^M]}{\partial c\partial j}\end{bmatrix}.$$

The inverse matrix is

$$\begin{pmatrix}-V^A & CR\\ CR & -V^C\end{pmatrix}^{-1} = \frac{1}{SOC}\begin{pmatrix}-V^C & -CR\\ -CR & -V^A\end{pmatrix}.$$

To prove the effect of an increase in δ, holding *s* constant, we have

$$\frac{\partial^2 E[U^M]}{\partial a\partial\delta} = 0 \text{ and } \frac{\partial^2 E[U^M]}{\partial c\partial\delta} = \exp(-c)s > 0,$$

which results in

$$\frac{\partial a}{\partial \delta} = \frac{CR}{SOC}\exp(-c)s > 0 \quad \text{and} \quad \frac{\partial c}{\partial \delta} = \frac{V^A}{SOC}\exp(-c)s = \frac{\partial a}{\partial \delta}\frac{V^A}{CR} > 0. \qquad \square$$

Proof of Proposition 1

If $c = 0$, the first-order condition of the manager's expected utility is

$$\frac{\partial E[U^M]}{\partial a} = p'(1-\varphi)s - V^A = 0.$$

Because $p'' = -p' < 0$, the manager's utility is strictly concave in a (given s), and there is a unique maximum. With respect to any parameter j we have

$$\frac{\partial a}{\partial j} = -\left(\frac{\partial^2 E[U^M]}{\partial a \partial j}\right)\left(\frac{\partial^2 E[U^M]}{\partial a^2}\right)^{-1} \quad \Rightarrow \quad \text{sign}\left(\frac{\partial a}{\partial j}\right) = \text{sign}\left(\frac{\partial^2 E[U^M]}{\partial a \partial j}\right)$$

It immediately follows that $\frac{\partial a}{\partial \delta} = 0$ and

$$\frac{\partial a}{\partial s} = -\frac{p'(1-\varphi)}{p''(1-\varphi)s} = \frac{p'(1-\varphi)}{p'(1-\varphi)s} = \frac{1}{s}.$$

Using $p = 1 - p'$, firm value $FV_1 = px - \left(p(1-\varphi) + \beta - \delta\right)s$ can be rewritten as

$$\begin{aligned} FV_1 &= px - \left(p(1-\varphi) + \beta - \delta\right)s \\ &= p\left(x - s\right) + \left(p\varphi - \beta + \delta\right)s \\ &= p\left(x - s\right) + \left((1-p')\varphi - \beta + \delta\right) \\ &= p\left(x - s\right) + \underbrace{\left(-p'\varphi + (\alpha + \delta)\right)}_{=T} s. \end{aligned}$$

The first-order condition is

$$\begin{aligned}
\frac{\partial FV_1}{\partial s} &= p' \frac{\partial a}{\partial s}(x-s) - p - p''\varphi \frac{\partial a}{\partial s} s + \left(-p'\varphi + (\alpha+\delta)\right) \\
&= p' \frac{\partial a}{\partial s}(x-s) - p + p'\varphi \frac{\partial a}{\partial s} s + \left(-p'\varphi + (\alpha+\delta)\right) \\
&= p' \frac{1}{s}(x-s) - p + p'\varphi \frac{1}{s} s - p'\varphi + (\alpha+\delta) \\
&= p' \frac{1}{s}(x-s) - p + (\alpha+\delta) \\
&= p' \frac{1}{s}(x-s) - (1-p') + (\alpha+\delta) \\
&= p'\left(1 + \frac{1}{s}(x-s)\right) - 1 + (\alpha+\delta) \\
&= p' \frac{x}{s} - 1 + (\alpha+\delta) = 0,
\end{aligned}$$

where we used $p'' = -p'$ and $\frac{\partial a}{\partial s} = \frac{1}{s}$. The second-order condition is satisfied because

$$\frac{\partial^2 FV_1}{\partial s^2} = p'' \frac{\partial a}{\partial s} \frac{x}{s} - p' \frac{x}{s^2} < 0 \,,$$

hence, hence, s_1 is unique. Solving the first-order-condition $p' = \frac{V^A}{s(1-\varphi)}$ yields $\frac{V^A x}{s^2(1-\varphi)} = 1 - (\alpha+\delta)$ and the optimal bonus

$$s_1 = \sqrt{\frac{V^A x}{\left(1-(\alpha+\delta)\right)(1-\varphi)}}.$$

We have $\partial s_1 / \partial \delta > 0$ and $\frac{\partial^2 s_1}{\partial \delta^2} = -\frac{1}{2\left(1-(\alpha+\delta)\right)^{3/2}} \sqrt{\frac{V^A x}{1-\varphi}} < 0$.

Using s_1, the optimal productive effort a_1 becomes

$$p' = \exp(-a_1) = \frac{V^A}{(1-\varphi)s_1},$$

implying $a_1 = \ln\left(\frac{(1-\varphi)s_1}{V^A}\right)$. We again have $\frac{\partial a}{\partial s} = \frac{1}{s} > 0$ and $\frac{\partial^2 a}{\partial s^2} = -\frac{1}{s^2} < 0$. Moreover, $da_1 / d\delta > 0$.

Applying the envelope theorem,

$$\frac{dFV_1}{d\delta}=\frac{\partial FV_1}{\partial\delta}+\underbrace{\frac{\partial FV_1}{\partial s}}_{=0}\frac{\partial s_1}{\partial\delta}=\frac{\partial FV_1}{\partial\delta}=\underbrace{\frac{\partial a}{\partial\delta}}_{=0}p'\left(x-V^A\right)+s=s>0\,.$$

The productive effort is less than the first-best effort. To see this, note that using $FV_1=px-\left(p(1-\varphi)+\beta-\delta\right)s$, the principal's first-order condition can be written as

$$\frac{\partial FV_1}{\partial s}=p'\frac{\partial a}{\partial s}\left(x-(1-\varphi)s\right)-\left(p(1-\varphi)+\beta-\delta\right)=0\,.$$

Inserting the agent's first-order condition yields

$$\frac{\partial FV_1}{\partial s}=\frac{\partial a}{\partial s}\left(p'x-V^A\right)-\left(p(1-\varphi)+\beta-\delta\right)=0\,,$$

which implies $p'x>V^A$ (the first-best effort requires $p'x=V^A$). It follows that

$$\frac{dTW_1}{d\delta}=\left(\underbrace{\frac{\partial a}{\partial\delta}}_{=0}+\frac{\partial a}{\partial s}\frac{\partial s}{\partial\delta}\right)(p'x-V^A)>0$$

because $\partial a/\partial s>0$ and $\partial s/\partial\delta>0$. □

Proof of Proposition 2

If c > 0, applying (A1) and (A2) yields

$$\frac{\partial^2E[U^M]}{\partial a\partial s}=p'\left(1-\varphi\exp(-c)\right)>0,$$
$$\frac{\partial^2E[U^M]}{\partial c\partial s}=\left(p\varphi-\beta+\delta\right)\exp(-c)>0.$$

$$\begin{aligned}\frac{\partial a}{\partial s}&=\frac{V^C}{SOC}p'\left(1-\varphi\exp(-c)\right)+\frac{CR}{SOC}\left(p\varphi-\beta+\delta\right)\exp(-c)\\&=\frac{1}{s\cdot SOC}V^C\left(V^A+CR\right)>0,\end{aligned}\tag{A3}$$

$$\begin{aligned}\frac{\partial c}{\partial s}&=\frac{CR}{SOC}p'\left(1-\varphi\exp(-c)\right)+\frac{V^A}{SOC}\left(p\varphi-\beta+\delta\right)\exp(-c)\\&=\frac{1}{s\cdot SOC}V^A\left(CR+V^C\right)=\frac{\partial a}{\partial s}\cdot\frac{V^A\left(CR+V^C\right)}{V^C\left(V^A+CR\right)}>0.\end{aligned}\tag{A4}$$

CR and SOC are defined in the proof of Lemma 1. Firm value is

$$FV=px-\left(1-\Pr(y_L)\right)s=px-s+\left((1-p)+\left(p\varphi-\beta+\delta\right)\exp(-c)\right)s\,.$$

Rewrite FV_2 as $FV_2 = p\left(x-s\right)+V^C$ using (A2). The first-order condition with respect to *s* yields

$$\frac{\partial FV_2}{\partial s} = \frac{\partial a}{\partial s} p'(x-s) - p = 0 . \tag{A5}$$

The second-order condition,

$$\frac{\partial^2 FV_2}{\partial s^2} = \left(\frac{\partial a}{\partial s}\right)^2 p''\left(x-s\right) + \frac{\partial^2 a}{\partial s^2} p'\left(x-s\right) - 2\frac{\partial a}{\partial s} p' ,$$

must be negative. To show this, first recall that $p'' < 0,$ hence the first term on the RHS of the second-order condition is negative. Second, using $p's = V^A + CR$ (from rewriting (A1)) we obtain from (A3)

$$\frac{\partial a}{\partial s} = \frac{1}{s \cdot SOC} V^C \left(V^A + CR\right) = \frac{1}{s \cdot SOC} V^C p's = \frac{V^C p'}{SOC}. \tag{A6}$$

Applying $\dfrac{\partial SOC}{\partial c} = -2CR\dfrac{\partial CR}{\partial c} = 2CR^2 \dfrac{\exp\left(c\right)}{\exp\left(c\right)-\varphi} = 2CR^2 \dfrac{V^A + CR}{V^A} > 0$ yields

$$\begin{aligned} \frac{\partial^2 a}{\partial s^2} &= \frac{V^C p''}{SOC}\frac{\partial a}{\partial s} - \frac{1}{SOC}\frac{V^C p'}{SOC}\frac{\partial SOC}{\partial c}\frac{\partial c}{\partial s} \\ &= -\frac{V^C p'}{SOC}\left(\frac{\partial a}{\partial s} + \frac{1}{SOC}\frac{\partial SOC}{\partial c}\frac{\partial c}{\partial s}\right) \\ &= -\frac{\partial a}{\partial s}\left(\frac{\partial a}{\partial s} + 2CR^2\frac{V^A + CR}{V^A}\frac{\partial c}{\partial s}\right) < 0. \end{aligned} \tag{A7}$$

The third term in $\partial^2 FV_2 \big/ \partial s^2$ is clearly negative due to $p' > 0$ and $\partial a / \partial s > 0$. Thus, we have $\dfrac{\partial^2 FV_2}{\partial s^2} < 0$ and the bonus *s* defined by the first-order condition (A5) is a unique maximum. The second-best effort is again smaller than the first-best one. To see this, note that (A1) can be rewritten as $p's = V^A + p'\varphi \exp(-c)s = V^A + CR$, and inserting this expression into (A5) gives

$$\frac{\partial FV_2}{\partial s} = \frac{\partial a}{\partial s} p'(x-s) - p = \frac{\partial a}{\partial s}\left(p'x - V^A - CR\right) - p = 0 ,$$

which implies $p'x - V^A > 0$.

Taking the total derivative of FV_2 with respect to δ at the point of the optimal *a* and *c* yields

$$\frac{dFV_2}{d\delta}=\frac{\partial FV_2}{\partial\delta}+\underbrace{\frac{\partial FV_2}{\partial s}}_{=0}\frac{\partial s}{\partial\delta}=\frac{\partial FV_2}{\partial\delta}=\underbrace{\frac{\partial a}{\partial\delta}}_{>0}p'(x-s)>0\,,$$

which proves part (iv).

The effect of δ on s is implicitly determined by

$$Z\equiv\frac{\partial^2 FV_2}{\partial s\partial\delta}+\frac{\partial^2 FV_2}{\partial s^2}\frac{\partial s}{\partial\delta}=0\,,$$

which implies $\operatorname{sign}\left(\frac{\partial s}{\partial\delta}\right)=\operatorname{sign}\left(\frac{\partial^2 FV_2}{\partial s\partial\delta}\right)$.

$$\frac{\partial^2 FV_2}{\partial s\partial\delta}=\underbrace{\frac{\partial a}{\partial s}\frac{\partial a}{\partial\delta}p''(x-s)}_{<0\text{ due to }p''<0}+\frac{\partial\left(\partial a/\partial s\right)}{\partial\delta}p'(x-s)-p'\underbrace{\frac{\partial a}{\partial\delta}}_{>0}\,.$$

Using (A6), we obtain

$$\begin{aligned}\frac{\partial\left(\frac{\partial a}{\partial s}\right)}{\partial\delta}&=\frac{V^C p''}{SOC}\frac{\partial a}{\partial\delta}-\frac{1}{SOC}\frac{V^C p'}{SOC}\frac{\partial SOC}{\partial c}\frac{\partial c}{\partial\delta}\\&=-\frac{\partial a}{\partial s}\frac{\partial a}{\partial\delta}-\frac{\partial a}{\partial s}\frac{1}{SOC}\frac{\partial SOC}{\partial c}\frac{\partial c}{\partial\delta}\\&=-\frac{\partial a}{\partial s}\left(\frac{\partial a}{\partial\delta}+2CR^2\frac{V^A+CR}{V^A}\frac{\partial c}{\partial\delta}\right)<0,\end{aligned}\tag{A8}$$

which implies $\frac{\partial^2 FV_2}{\partial s\partial\delta}<0$ and $\frac{\partial s}{\partial\delta}<0$. This proves part (i).

Since *s* decreases in δ, whereas *a* and *c* increase in δ directly, but decrease with lower *s*, the total effect depends on the relative magnitude of the effects. We prove that the optimal solution has

$$\frac{dc}{d\delta}=\frac{\partial c}{\partial\delta}+\frac{\partial c}{\partial s}\frac{\partial s}{\partial\delta}>0\text{ and }\frac{da}{d\delta}=\frac{\partial a}{\partial\delta}+\frac{\partial a}{\partial s}\frac{\partial s}{\partial\delta}<0.$$

First solve (A1) for *a* giving

$$a=\ln\left(1-\varphi\exp\left(-c\right)\right)+\ln\left(s\right)-\ln\left(V^A\right).$$

Totally differentiating *a* with respect to δ leads to

$$\begin{aligned}\frac{da}{d\delta} &= \frac{1}{1-\varphi\exp(-c)}\varphi\exp(-c)\frac{dc}{d\delta}+\frac{1}{s}\frac{\partial s}{\partial\delta}\\ &= \frac{\varphi}{\exp(c)-\varphi}\frac{dc}{d\delta}+\frac{1}{s}\underbrace{\frac{\partial s}{\partial\delta}}_{<0}.\end{aligned} \tag{A9}$$

From (A9) it directly follows that if $dc/d\delta < 0$ then it must be that $da/d\delta < 0.$ We now show that $dc/d\delta > 0$. To see this, rewrite the owner's first-order condition (A5) as

$$s = x - \frac{p}{p'}\left(\frac{\partial a}{\partial s}\right)^{-1} = x - \frac{p}{(1-p)}\frac{s\cdot SOC}{V^C\left(V^A+CR\right)},$$

using (A3) and $p' = 1 - p$. Solving for *x* yields

$$s\left(1+\frac{p}{(1-p)}\frac{SOC}{V^C\left(V^A+CR\right)}\right) = x. \tag{A10}$$

Since *x* is a given constant, the LHS of (A10) must remain constant as well. Since *s* strictly decreases in δ according to part (i), the term in brackets must strictly increase in δ. Recall that *CR* strictly decreases in *c* and *SOC* strictly increases in *c*, implying that $SOC/(V^C(V^A+CR))$ strictly increases in *c*. Hence, should $dc/d\delta < 0$ in fact hold, then an increase of $p/(1-p)$ is required, which implies $da/d\delta > 0$. However, as demonstrated above, this is impossible. Thus, $dc/d\delta > 0$, which proves (iii).

To establish that *a* strictly decreases in δ, we demonstrate that given the implicit equation

$$Z \equiv \frac{\partial^2 FV_2}{\partial s\partial\delta}+\frac{\partial^2 FV_2}{\partial s^2}\frac{\partial s}{\partial\delta} = 0,$$

from which we derived $\partial s/\partial\delta < 0$, it is impossible to have $da/d\delta > 0$. To show this, we use the explicit expressions for $\partial^2 FV_2/\partial s\partial\delta$ and $\partial^2 FV_2/\partial s^2$ as given above, then we employ (A6), (A7) and (A8), insert all these into *Z* and collect terms to eventually arrive at

$$Z = -\underbrace{2p'(x-s)\frac{\partial a}{\partial s}\frac{da}{d\delta}}_{\equiv Z_1} - \underbrace{p'(x-s)\frac{\partial a}{\partial s}\frac{2CR^2\left(V^A+CR\right)}{SOC\cdot V^A}\frac{dc}{d\delta}}_{\equiv Z_2>0} - \underbrace{p'\frac{da}{d\delta}}_{\equiv Z_3} - \underbrace{p'\frac{\partial a}{\partial s}\frac{\partial s}{\partial \partial}}_{\equiv Z_4<0} = 0.$$

Suppose $da/d\delta > 0$ were to hold, then Z_1, Z_2 and Z_3 are strictly positive, implying that their negative sum is strictly negative. This requires that Z_4 must be sufficiently negative to satisfy Z = 0. This cannot be the case. To see this, insert

$$x - s = \frac{p}{p'}\left(\frac{\partial a}{\partial s}\right)^{-1} = \frac{p}{p'}\frac{s \cdot SOC}{V^C\left(V^A + CR\right)}$$

into Z_2 to obtain

$$Z_2 = p'\frac{\partial a}{\partial s}\frac{2CR^2 \cdot s}{V^A V^C}\frac{p}{p'}\frac{dc}{d\delta}.$$

Solving (A9) for $dc/d\delta$ leads to

$$\frac{dc}{d\delta} = \frac{da}{d\delta}\frac{\exp(c) - \varphi}{\varphi} - \frac{1}{s}\frac{\partial s}{\partial \delta}\frac{\exp(c) - \varphi}{\varphi} = \frac{da}{d\delta}\frac{V^A}{CR} - \frac{1}{s}\frac{\partial s}{\partial \partial}\frac{V^A}{CR}.$$

Inserting this expression into Z_2 yields

$$\begin{aligned} Z_2 &= p'\frac{\partial a}{\partial s}\frac{2CR^2 s}{V^A V^C}\frac{p}{p'}\left(\frac{da}{d\delta}\frac{V^A}{CR} - \frac{1}{s}\frac{\partial s}{\partial \partial}\frac{V^A}{CR}\right) \\ &= p'\frac{\partial a}{\partial s}\frac{2CR \cdot s}{V^C}\frac{p}{p'}\frac{da}{d\delta} - p'\frac{\partial a}{\partial s}\frac{p}{p'}\frac{2CR}{V^C}\frac{\partial s}{\partial \delta}. \end{aligned}$$

Further inserting this into Z leads to

$$\begin{aligned} Z &= -Z_1 - \left(p'\frac{\partial a}{\partial s}\frac{2CR \cdot s}{V^C}\frac{p}{p'}\frac{da}{d\delta} - p'\frac{\partial a}{\partial s}\frac{p}{p'}\frac{2CR}{V^C}\frac{\partial s}{\partial \delta}\right) - Z_3 - p'\frac{\partial a}{\partial s}\frac{\partial s}{\partial \partial} \\ &= -Z_1 - p'\frac{\partial a}{\partial s}\frac{2CR \cdot s}{V^C}\frac{p}{p'}\frac{da}{d\delta} - Z_3 + p'\frac{\partial a}{\partial s}\frac{\partial s}{\partial \delta}\underbrace{\left(\frac{p}{p'}\frac{2CR}{V^C} - 1\right)}_{\equiv H}. \end{aligned}$$

If $H > 0$, it follows that $Z < 0$ if $da/d\delta > 0$ is assumed. Observe that

$$H = \frac{2pCR - p'V^C}{p'V^C} = \frac{2CR(1 - p') - p'V^C}{p'V^C} = \frac{1}{p'V^C}\left(\underbrace{CR(1 - p')}_{>0} + CR - p'\left(CR + V^C\right)\right).$$

Rewrite T as

$$T = p\varphi - \beta + \delta = p(\alpha + \beta) - \beta + \delta = (1 - p')(\alpha + \beta) - \beta + \delta = -p'\varphi + \alpha + \delta.$$

Inserting this in (A2) yields

$$-p'\varphi\exp(-c)s+(\alpha+\delta)\exp(-c)s=V^C \Rightarrow (\alpha+\delta)\exp(-c)s=V^C+CR.$$

Inserting this in H leads to

$$\begin{aligned} CR-p'\left(CR+V^C\right) &= p'\varphi\exp(-c)s-p'(\alpha+\delta)\exp(-c)s \\ &= p'\exp(-c)s(\beta-\delta)\geq 0. \end{aligned}$$

Thus, $H > 0$, implying that $Z < 0$ if $da/d\delta$ were to hold; this is a contradiction. Therefore, $da/d\delta < 0$, proving (ii).

Finally, total welfare is $TW_2 = px - V^A a - V^C c.$ Increasing conservatism δ leads to

$$\frac{dTW_2}{d\delta} = \underset{<0}{\frac{da}{d\delta}}\underbrace{\left(p'x-V^A\right)}_{>0} - \underset{>0}{\frac{dc}{d\delta}}V^C < 0,$$

which proves (v). □

Proof of Lemma 2

The threshold $\hat{s}(\delta)$ is the bonus for a given δ such that the manager chooses the optimal productive effort a_1 with $c = 0$ and the first-order condition (A2) for the optimal quality effort c in regime 2 is exactly satisfied at $c = 0$. Under regime1, from (A1) for any given s and $c = 0$, we have

$$p's(1-\varphi)-V^A=0 \Rightarrow p'=\frac{V^A}{s(1-\varphi)}.$$

(A2) must also hold, and using the expression for T as in the proof of Proposition 2, this implies $(-p'\varphi+\alpha+\delta)s=V^C.$ Substituting for p' yields

$$\left(-\frac{V^A}{s(1-\varphi)}\varphi+\alpha+\delta\right)s=-\frac{V^A\varphi}{(1-\varphi)}+(\alpha+\delta)s=V^C \Rightarrow \hat{s}(\delta)=\frac{1}{\alpha+\delta}\left(V^C+\frac{V^A\varphi}{(1-\varphi)}\right),$$

and $\dfrac{\partial\hat{s}}{\partial\delta}=-\dfrac{\hat{s}}{\alpha+\delta}<0$ follows immediately. □.

Proof of Proposition 3

To prove part (i), consider first regime 1. Since we assume V^A is sufficiently low to induce an $a_1 > 0$ if $\delta = 0$, we know from Proposition 1 that $ds_1/d\delta > 0$ and $da_1/d\delta > 0$. From Lemma 2 we know that $\hat{s}(\delta)$ strictly decreases, so there exists a δ for which $s_1(\delta)=\hat{s}(\delta)$. Denote this δ by

δ_1. Within the interval $\delta \in [0, \delta_1]$ there exists a regime 1 solution with an optimal choice of $s_1(\delta) \leq \hat{s}(\delta)$ with $FV_1\left(s_1(\delta)\right)$.

If δ is such that $s_1(\delta) > \hat{s}(\delta)$, then $s_1(\delta)$ is not a feasible solution in regime 1 because it would induce $c > 0$. The best contract with $c = 0$ is then the one with $s = \hat{s}(\delta)$. Note however that $\hat{s}(\delta)$ decreases in δ, as does a. Therefore, such a constrained contract performs worse than the infeasible, unconstrained contract for $\delta > \delta_1$, but both contracts reach the same firm value at $\delta = \delta_1$. We establish below that for all $\delta \geq \delta_1$, a constrained contract can never be optimal since it is dominated by a regime 2 contract with $c > 0$ already at $\delta = \delta_1$, that is $FV_2\left(s_2(\delta_1)\right) > FV_2\left(\hat{s}(\delta_1)\right) = FV_1\left(s_1(\delta_1)\right)$. By continuity, this implies that there must exist $\delta_2 < \delta_1$ such that a regime 2 contract becomes relevant for $\delta \geq \delta_2$ which completes the proof of part (i). Moreover, it implies that a regime 2 contract also dominates the constrained contract for all $\delta > \delta_1$ as we know $dc/d\delta > 0$ from Proposition 2 (iii), which requires $s_2(\delta) > \hat{s}(\delta)$ for $\delta > \hat{\delta}$.

To establish that there is an interior regime 2 solution at δ_1, i.e., $s_2(\delta_1) > \hat{s}(\delta_1) = s_1(\delta_1)$, requires that the owner's first-order condition at δ_1 is positive, implying

$$\frac{\partial FV_2}{\partial s} = p' \frac{\partial a}{\partial s}\left(x - \hat{s}(\delta_1)\right) - p = p' \frac{V^C\left(V^A + CR\right)}{\hat{s}(\delta_1) SOC}\left(x - \hat{s}(\delta_1)\right) - p > 0,$$

where $\partial a / \partial s$ is calculated according to (A3). To show this, first observe that at δ_1 we also have the regime 1 solution s_1 with

$$\frac{\partial FV_1}{\partial s} = p' \frac{1}{s_1}\left(x - s_1\right) - p + \left(\alpha + \delta_1\right) = 0.$$

Rewriting this condition as in the proof of Proposition 1 yields

$$\begin{aligned} p' \frac{1}{s_1}\left(x - s_1\right) - p + \left(\alpha + \delta_1\right) &= p' \frac{1}{s_1}\left(x - s_1\right) - \left(1 - p'\right) + \left(\alpha + \delta_1\right) \\ &= p'\left(1 + \frac{1}{s_1}\left(x - s_1\right)\right) - 1 + \left(\alpha + \delta_1\right) \\ &= p' \frac{x}{\hat{s}} - 1 + \left(\alpha + \delta_1\right) = 0, \end{aligned}$$

due to $s_1 = \hat{s}$.

Observe that $\frac{V^C\left(V^A+CR\right)}{\hat{s}SOC}=\frac{1}{\hat{s}}+\frac{1}{\hat{s}}\left(\frac{CR\left(V^C+CR\right)}{SOC}\right)$. Inserting this into $\partial FV_2/\partial s$ and rearranging yields

$$\begin{aligned}\frac{\partial FV_2}{\partial s}&=p'\frac{V^C\left(V^A+CR\right)}{\hat{s}SOC}\left(x-\hat{s}\right)-p\\&=p'\left(\frac{1}{\hat{s}}+\frac{1}{\hat{s}}\frac{CR\left(V^C+CR\right)}{SOC}\right)\left(x-\hat{s}\right)-\left(1-p'\right)\\&=p'\frac{x}{\hat{s}}-1+p'\frac{1}{\hat{s}}\frac{CR\left(V^C+CR\right)}{SOC}\left(x-\hat{s}\right)\\&=-\left(\alpha+\hat{\delta}\right)+p'\frac{1}{\hat{s}}\frac{CR\left(V^C+CR\right)}{SOC}\left(x-\hat{s}\right).\end{aligned}$$

Since this holds at $\hat{s}$, we have c = 0 and $CR=V^A\frac{\varphi}{(1-\varphi)}$. It follows that $V^C+CR=V^C+V^A\frac{\varphi}{(1-\varphi)}=\hat{s}\left(\alpha+\hat{\delta}\right)$. Inserting this term yields

$$\begin{aligned}\frac{\partial FV_2}{\partial s}&=-\left(\alpha+\delta_1\right)+p'\frac{1}{\hat{s}}\frac{CR\left(V^C+CR\right)}{SOC}\left(x-\hat{s}\right)\\&=-\left(\alpha+\delta_1\right)+p'\frac{1}{\hat{s}}\frac{CR\left(\alpha+\delta\right)\hat{s}}{SOC}\left(x-\hat{s}\right)\\&=\frac{\left(\alpha+\delta_1\right)}{SOC}\underbrace{\left(p'CR\left(x-\hat{s}\right)-SOC\right)}_{\equiv K}.\end{aligned}$$

We next establish that K > 0. At δ_1 we must have $x=\frac{\hat{s}^2\left(1-\left(\alpha+\delta_1\right)\right)\left(1-\varphi\right)}{V^A}$ and $p'=\frac{V^A}{s_1\left(1-\varphi\right)}=\frac{V^A}{\hat{s}\left(1-\varphi\right)}$. Using this and the identity $SOC=V^AV^C-CR^2$ leads to

$$\begin{aligned}K&=p'CR\left(x-\hat{s}\right)-SOC\\&=CR\frac{V^A}{\hat{s}\left(1-\varphi\right)}\frac{\hat{s}^2\left(1-\left(\alpha+\delta_1\right)\right)\left(1-\varphi\right)}{V^A}-CR\frac{V^A}{\hat{s}\left(1-\varphi\right)}\hat{s}+CR^2-V^AV^C\\&=CR\hat{s}\left(1-\left(\alpha+\delta_1\right)\right)-CR\frac{V^A}{\left(1-\varphi\right)}+CR^2-V^AV^C\\&=CR\left[\hat{s}\left(1-\left(\alpha+\delta_1\right)\right)-\frac{V^A}{\left(1-\varphi\right)}+\frac{V^A\varphi}{\left(1-\varphi\right)}-V^C\frac{\left(1-\varphi\right)}{\varphi}\right]\\&=CR\left[\hat{s}\left(1-\left(\alpha+\delta_1\right)\right)-V^A-V^C\frac{\left(1-\varphi\right)}{\varphi}\right].\end{aligned}$$

Rewrite

$$V^A + V^C \frac{(1-\varphi)}{\varphi} = \left(V^C + V^A \frac{\varphi}{(1-\varphi)} \right) \frac{(1-\varphi)}{\varphi} = \hat{s}\left(\alpha + \delta_1\right) \frac{(1-\varphi)}{\varphi}.$$

Inserting into K yields

$$\begin{aligned} K &= CR\left[\hat{s}\left(1-\left(\alpha+\delta_1\right)\right) - V^A - V^C \frac{(1-\varphi)}{\varphi} \right] \\ &= CR\left[\hat{s}\left(1-\left(\alpha+\delta_1\right)\right) - \hat{s}\left(\alpha+\delta\right) \frac{(1-\varphi)}{\varphi} \right] \\ &= \hat{s}CR\left(1 - \frac{\left(\alpha+\delta_1\right)}{\varphi} \right) \geq 0 \end{aligned}$$

and $K > 0$ if $\delta_1 < \bar{\delta} = \min\{.5-\alpha; \beta\}$. The positive sign of the inequality is due to the fact that if δ_1 is an interior solution, we always have $\alpha + \delta_1 < \varphi$. To see this, consider two cases:

$$\begin{aligned} &\bar{\delta} = .5-\alpha \Rightarrow .5-\alpha < \beta \Rightarrow .5 < \alpha+\beta \Rightarrow \alpha+\delta_1 < \alpha+\beta, \\ &\bar{\delta} = \beta \Rightarrow \beta < .5-\alpha \Rightarrow \alpha+\beta < .5 \Rightarrow \alpha+\delta_1 < \beta. \end{aligned}$$

Thus, it follows that for $\alpha + \delta_1 < \varphi$, the owner prefers a bonus $s_2(\delta_1) > \hat{s}(\delta_1) = s_1(\delta_1)$, implying an interior solution in regime 2 with $FV_2\left(s_2(\delta_1)\right) > FV_1\left(s_1(\delta_1)\right).$ Due to continuity, there must exist a $\delta < \delta_1$ where the owner shifts to regime 2 instead of the threshold $\hat{s}$.

To prove part (ii), observe that for all $\delta < \hat{\delta}$, $\hat{s}(\delta) > s_1(\delta)$ so that

$$FV_1\left(\hat{s}\left(\delta\right)\right) < FV_1\left(s_1\left(\delta\right)\right)$$

because c = 0 for either bonus and the threshold $\hat{s}(\delta)$ induces a productive effort exceeding the one the owner would optimally choose for δ in regime 1. At $\delta_2 < \delta_1$, we still have

$$FV_1\left(\hat{s}\left(\delta_2\right)\right) < FV_1\left(s_1\left(\delta_2\right)\right)$$

but for all $\delta > \delta_2$, there is a regime 2 contract $s_2(\delta) > \hat{s}(\delta)$ such that

$$FV_2\left(s_2(\delta_1)\right) > FV_1\left(s_1(\delta_1)\right) = FV_2\left(s_1(\delta_1)\right) = FV_2\left(\hat{s}(\delta_1)\right).$$

Thus, there must exist $\hat{\delta} \in \left(\delta_2, \delta_1\right)$ such that $FV_2\left(s_2(\hat{\delta})\right) = FV_1\left(s_1(\hat{\delta})\right)$ and $FV_2\left(s_2(\delta)\right) > FV_1\left(s_1(\delta)\right)$ for $\delta > \hat{\delta}.$ □

Proof of Corollary 1

We first establish that a higher *x* leads to a higher optimal *s* in either regime. In regime 1, according to Proposition 1, $s_1 = \sqrt{\dfrac{V^A x}{\left(1-(\alpha+\delta)\right)(1-\varphi)}}$, which strictly increases in *x*. Since $a_1 = \ln\left(\dfrac{(1-\varphi)s_1}{V^A}\right)$, a_1 strictly increases in s_1 because s_1 increases in *x*.

In regime 2, *x* enters neither (A1) nor (A2) directly, hence there is no direct impact of *x* on both *a* and *c* (i.e., $\partial a/\partial x = 0$ and $\partial c/\partial x = 0$). Conservatism induces a change of *s* according to

$$\text{sign}\left(\frac{\partial s}{\partial x}\right) = \text{sign}\left(\frac{\partial^2 FV}{\partial s \partial x}\right).$$

Since $\dfrac{\partial^2 FV}{\partial s \partial x} = \dfrac{\partial a}{\partial s}\underbrace{\dfrac{\partial a}{\partial x}}_{=0} p''(x-s) + \underbrace{\dfrac{\partial\left(\frac{\partial a}{\partial s}\right)}{\partial x}}_{=0} p'(x-s) + p'\dfrac{\partial a}{\partial s} - p'\underbrace{\dfrac{\partial a}{\partial x}}_{=0} = p'\dfrac{\partial a}{\partial s} > 0$, where $\partial\left(\frac{\partial a}{\partial s}\right)\big/\partial x = \partial\left(\frac{p'V^C}{SOC}\right)\Big/\partial x = 0$, we have $\partial s/\partial x > 0$.

We next prove the two bounds for *x*. Consider the upper bound δ_1 of the interval in which a regime 1 solution is implementable, which is defined by $s_1(\delta_1) = \hat{s}(\delta_1)$. From Proposition 1, we have

$$\hat{s}(\delta_1) = \sqrt{\frac{V^A x}{\left(1-(\alpha+\delta_1)\right)(1-\varphi)}}.$$

Solving this equation for *x* yields

$$x_1(\delta_1) = \hat{s}(\delta_1)^2 \frac{\left(1-(\alpha+\delta_1)\right)(1-\varphi)}{V^A}.$$

Because $\hat{s}(\delta) = \dfrac{1}{\alpha+\delta}\left(V^C + \dfrac{V^A\varphi}{(1-\varphi)}\right)$ depends only on exogenous parameters (except *x*), $x_1(\delta)$ also depends only on exogenous parameters. Therefore, for $x > x_1(\delta)$ a regime 1 solution is not implementable, so only a regime 2 solution with $c > 0$ can occur. The reason is that *x* is so high that the owner prefers setting a bonus that exceeds $\hat{s}(\delta)$ for a particular δ even under regime 1, which precludes its implementation.

We have excluded low V^A, specifically $x \le V^A$, because then the owner cannot induce an $a > 0$ because that would require a bonus $s \ge V^A > x$; in that case, the optimal solution is a degenerate one with $s = 0$, which induces $a = 0$ and an expected outcome of 0.

To arrive at relevant lower threshold for x, consider $\hat{s}(\overline{\delta}),$ which is the lowest bound of the threshold dividing the bonus in regimes 1 and 2. If $x \le \hat{s}(\overline{\delta})$, then only a regime 1 solution can exist because an implementable regime 2 solution requires $s_2 \ge \hat{s}(\overline{\delta}) = \left(V^A + V^C \tfrac{\varphi}{(1-\varphi)}\right)\left(\alpha + \overline{\delta}\right)^{-1} > x$, which cannot be an optimum. □

Proof of Corollary 2

We begin with part (iv). By definition, at $\delta = \hat{\delta}$ we must have $FV_1(\hat{\delta}) = FV_2(\hat{\delta})$. We have shown in Proposition 3 that $0 < \frac{dFV_1}{d\delta} < \frac{dFV_2}{d\delta}$ at $\delta = \hat{\delta}$, so the FV function has a kink at $\delta = \hat{\delta}$.

To prove part (i), note that because $\hat{\delta} < \delta_1$ and $s_1(\delta_1) = \hat{s}(\delta_1)$, we must have $s_1(\hat{\delta}) < \hat{s}(\hat{\delta})$. But since there exists a solution in regime 2, $\hat{s}(\hat{\delta}) < s_2(\hat{\delta}),$ where the strict inequality results from the fact that $\hat{s}(\delta_2) = s_2(\delta_2)$ and $\delta_2 \le \hat{\delta}.$

Part (ii) follows because a strictly increases in s in both regimes; therefore, the jump in s from s_1 to s_2 leads to $a_1(\hat{\delta}) < a_2(\hat{\delta}).$

Part (iii) follows from the fact that by definition $c = 0$ at $\delta = \delta_2$ in regime 2. Hence, if $\hat{\delta} > \delta_2,$ $c(\hat{\delta})$ must jump to a value greater than some bound $\underline{c} > 0$ due to continuity of $c(\delta)$.

Finally, to prove (iv), note that

$$TW_1 = FV_1 + E[U_1^M] = px - V^A a \text{ and } TW_2 = FV_2 + E[U_2^M] = px - V^A a - V^C c.$$

By definition of $\hat{\delta},$ $FV_2(\hat{\delta}) = FV_1(\hat{\delta})$. Therefore,

$$TW_2(\hat{\delta}) - TW_1(\hat{\delta}) = E[U_2^M(\hat{\delta})] - E[U_1^M(\hat{\delta})].$$

The manager maximizes $E[U_1^M(\hat{\delta})]$ under the bonus $s_1(\hat{\delta})$, resulting in $a_1(\hat{\delta})$. In the regime 2 contract, $s_2(\hat{\delta}) > s_1(\hat{\delta}),$ which directly increases the manager's expected utility ceteris paribus. The manager could still choose $c = 0$ and $a_1(\hat{\delta})$, but the optimal choice is $c(\hat{\delta}) > 0$ and $a_2(\hat{\delta}) > a_1(\hat{\delta})$. Therefore,

$$E[U_2^M(\hat{\delta})\left|a_2(\hat{\delta}), c(\hat{\delta})>0\right] > E[U_2^M(\hat{\delta})\left|a_1(\hat{\delta}), c=0\right] > E[U_1^M(\hat{\delta})\left|a_1(\hat{\delta})\right],$$

proving $TW_2(\hat{\delta}) > TW_1(\hat{\delta})$. □

Proof of Proposition 4

Part (i) follows immediately from Proposition 1 (iii) and Proposition 2 (iv).

To prove (ii), note that according to Corollary 2 (iv) $TW_2(\hat{\delta}) > TW_1(\hat{\delta})$. Since $TW_2(\delta)$ strictly decreases in δ and a regime 1 solution is not chosen by the owner for $\delta > \hat{\delta}$, the δ that maximizes *TW* must be $\delta^* = \hat{\delta}$ if the threshold $\hat{\delta} \in \left(0, \bar{\delta}\right)$. The statements about the corner solutions follow from the results in Proposition 1 (iv) and Proposition 2 (v).

Part (iii) follows from the fact that $FRQ = 1-\exp(-c)\varphi,$ which depends only on *c*. In regime 1, *c* = 0, so *FRQ* does not change with δ and any feasible δ is (weakly) optimal. In regime 2, Proposition 2 (iii) states that *c* strictly increases in δ, thus *FRQ* is maximal at $\delta^* = \bar{\delta}$. □

Proof of Corollary 3

Using $b = \ln(s/V^B)$ if $s > V^B$, the manager's expected utility is

$$\begin{aligned} E[U^M\left|a,b\right] &= \Pr(m_H\left|b\right)s - V^A a - \Pr(y_L\left|a\right)V^B b \\ &= s - \left(p\alpha + (1-p)(1-\beta) + \delta\right)\left(1+\ln\left(s/V^B\right)\right)V^B - V^A a. \end{aligned}$$

The manager's optimal choice of *a*, given *s*, is determined by

$$\frac{\partial E[U^M]}{\partial a} = p'\left(1-\varphi\right)\left(1+\ln\left(s/V^B\right)\right)V^B - V^A = 0.$$

Due to $p'' < 0$ this maximum is unique. We have $\operatorname{sign}\left(\frac{\partial a}{\partial j}\right) = \operatorname{sign}\left(\frac{\partial^2 E[U^M]}{\partial a \partial j}\right)$. Hence, $\frac{\partial a}{\partial s} = -\frac{p'\left(1-\varphi\right)\frac{V^B}{s}}{p''\left(1-\varphi\right)\left(1+\ln\left(s/V^B\right)\right)V^B} = \frac{1}{s\left(1+\ln\left(s/V^B\right)\right)} > 0$ and $\frac{\partial a}{\partial \delta} = 0$. Firm value is

$$\begin{aligned} FV &= px - \Pr(m_H)s \\ &= px - s + \left(p\alpha + (1-p)(1-\beta) + \delta\right)V^B. \end{aligned}$$

The total derivative with respect to δ is $\frac{dFV}{d\delta} = \frac{\partial FV}{\partial \delta} + \frac{\partial FV}{\partial s}\frac{\partial s}{\partial \delta}$, where by optimality of *s*, the first-order condition is

$$\frac{\partial FV}{\partial s} = p' \frac{\partial a}{\partial s} x - 1 - p' \frac{\partial a}{\partial s}(1-\varphi)V^B$$
$$= \frac{1}{s\left(1+\ln(s/V^B)\right)} p'\left(x-(1-\varphi)V^B\right) - 1 = 0.$$

Assuming $x > \max\left\{V^A; V^B; V^C\right\}$ we have $x > (1-\varphi)V^B$, hence, the second-order condition $\partial^2 FV / \partial s^2 < 0$ holds. Note that $\partial FV/\partial s$ does not directly depend on δ, and since $\partial a/\partial s = 0$, we have $\frac{\partial^2 FV}{\partial s \partial \delta} = 0$, implying that $\partial s/\partial \delta = 0$. Moreover,

$$\frac{\partial FV}{\partial \delta} = \underbrace{\frac{\partial a}{\partial \delta}}_{=0} p'\left(x-(1-\varphi)V^B\right) + V^B = V^B > 0$$

implies $\frac{dFV}{d\delta} = \frac{\partial FV}{\partial \delta} > 0.$ Total welfare is

$$TW = px - V^A a - \left(p\alpha + (1-p)(1-\beta) + \delta\right)V^B \ln\left(s/V^B\right)$$

and
$$\frac{dTW}{d\delta} = \left(\underbrace{\frac{\partial a}{\partial \delta}}_{=0} + \frac{\partial a}{\partial s} \underbrace{\frac{\partial s}{\partial \delta}}_{=0}\right)\left[\left(p'x - V^A\right) + p'(1-\varphi)V^B \ln\left(s/V^B\right)\right] - V^B \ln\left(s/V^B\right)$$
$$= -V^B \ln\left(s/V^B\right) < 0.$$

□