Abteilung für integratives Management
produktionswirtschaftlicher und logistischer Systeme

# Musterlösungen 3. Übungsblatt
# aus Masterkurs Produktion und Logistik

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**Solution Exercise 1:**

a) Optimal policy and TC:

d = {20,10,15,10,15,20}
k = 60
h = 2 (pm)
- $d$ not constant ⇒ dynamic model
- no uncertainty concerning $d$ ⇒ **Wagner-Within model**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 60 | – | – | – | – | – |
| 2 | 80 | 120 | – | – | – | – |
| 3 | 140 | 150 | 140 | – | – | – |
| 4 | 200 | 190 | 160 | 200 | – | – |
| 5 | 320 | 280 | 220 | 230 | 220 | – |
| 6 | 520 | 440 | 340 | 310 | 260 | 280 |

$q_t = \{30, 0, 25, 0, 35, 0\}$ and $TC = 260$

b) Inventory 15 units (pm):

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 60 | – | – | – | – | – |
| 2 | 80 | 120 | – | – | – | – |
| 3 | – | 150 | 140 | – | – | – |
| 4 | – | – | 160 | 200 | – | – |
| 5 | – | – | – | 230 | 220 | – |
| 6 | – | – | – | – | – | 280 |

$q_t = \{30, 0, 25, 0, 15, 20\}$ and $TC = 280$

c) h=1:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 60 | – | – | – | – | – |
| 2 | 70 | 120 | – | – | – | – |
| 3 | 100 | 135 | 130 | – | – | – |
| 4 | 130 | 155 | 140 | 160 | – | – |
| 5 | 190 | 200 | 170 | 175 | 190 | – |
| 6 | 290 | 280 | 230 | 215 | 210 | 230 |

$q_t = \{55, 0, 0, 0, 35, 0\}$
$TC = 210$

$\Rightarrow$ If -$\Delta$TC > costs of the software $\Rightarrow$ implementation advantagous

**Solution Exercise 2:**

$\Rightarrow$ Wagner-Whitin model

| Period | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 60 | - | - | - |
| 2 | 100 | 120 | - | - |
| 3 | 120 | 130 | 160 | - |
| 4 | $120 + 3 \cdot d_4 \cdot h$ | $130 + 2 \cdot d_4 \cdot h$ | $160 + 1 \cdot d_4 \cdot h$ | 180 |

$\Rightarrow d_4 = x$

a) first period:
$120 + 3x * 2 \leq 130 + 2x * 2 \Rightarrow x \leq 5$
$120 + 6x \leq 160 + 2x \Rightarrow x \leq 10$
$120 + 6x \leq 180 \Rightarrow x \leq 10$
$\Rightarrow x \leq 5 = w.A.$

b) second period:
$130 + 4x \leq 160 + 2x \Rightarrow x \leq 15$
$130 + 4x \leq 180 \Rightarrow x \leq 12.5$
$130 + 4x \leq 120 + 6x \Rightarrow x \geq 5$
$\Rightarrow 5 \leq x \leq 12,5 = w.A.$

c) third period:
$160 + 2x \leq 180 \Rightarrow x \leq 10$
$160 + 2x \leq 120 + 6x \Rightarrow x \geq 10$
$160 + 2x \leq 130 + 4x \Rightarrow x \geq 15$
$\Rightarrow x \leq 10 \wedge x \geq 15 = f.A.$

d) fourth period:
$180 \leq 120 + 6x \Rightarrow x \geq 10$
$180 \leq 130 + 4x \Rightarrow x \geq 12.5$
$180 \leq 160 + 2x \Rightarrow x \geq 10$
$\Rightarrow x \geq 12.5 = w.A.$

**Solution Exercise 3:**

a) Newsvender model:

p = 10
c = 5
v = 0

| d | P(D=d) | P(D≤d) |
|---|---|---|
| 10 | 0.2 | 0.2 |
| 20 | 0.2 | 0.4 |
| **30** | **0.2** | **0.6** |
| 40 | 0.2 | 0.8 |
| 50 | 0.2 | 1 |

$q^* = min\{q|F_D(q) \geq \frac{p-c}{p-v}\}$
$F_D(q^*) \geq \frac{p-c}{p-v} = 0.5 \Rightarrow q^* = 30$
$E[G(q)] = \sum_{d_{min}}^{d_{max}} g(q,d) \cdot f_D(d)$
$E[G(q=30)] = (-5*30) + 10*(10*0.2 + 20*0.2 + 30*0.2 + 30*0.2 + 30*0.2) = 90$
$\Rightarrow E[G(q=30)] = 90$
$\Rightarrow$ service level = 0.6

b) service level = 0.9:

$\Rightarrow$ service level = 1
$\Rightarrow q^* = 50$
$E[G(q=50)] = (-5*50) + 10*(10*0.2 + 20*0.2 + 30*0.2 + 40*0.2 + 50*0.2) = 50$

c) $c_{max}$:

$q^* = 30 \Leftrightarrow 0.4 < \frac{p-c}{p} \leq 0.6$
$\Rightarrow 0.6 \geq \frac{10-c}{10} > 0.4$
$\Rightarrow 4 \leq c < 6$