Session 4

Winning all or losing everything

- Zero-Sum Games -

What you will learn today: Our objectives

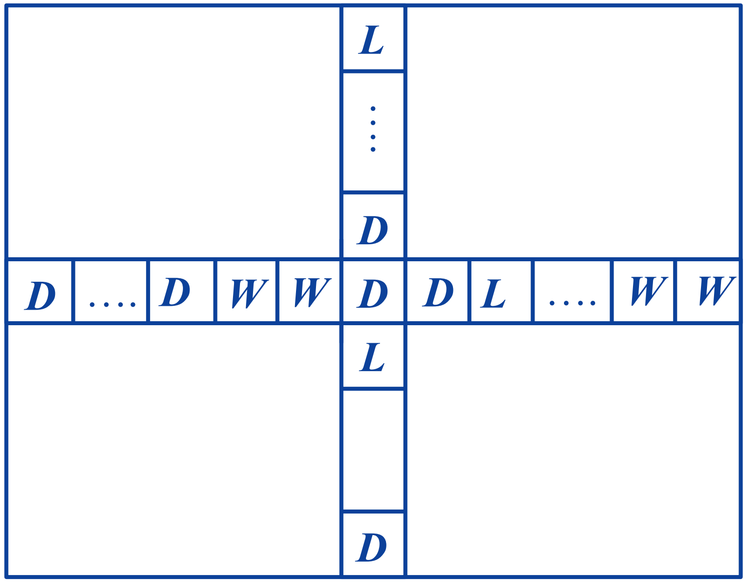
- What should we do in situations in which the interests of the other parties are diametrally opposed to ours such that their gain is our loss? What happens if each party attempts to minimize his opponent's maximal payoff?
  - The (non-) classical game and its equilibrium
- How can we use these insights to explain how people behave in more complex situations?
  - Extensions
- What can we do to win in such situations of pure conflict? Which strategic moves are beneficial?
  - Strategic moves

Our path to succeed: Course outline for today

- The “(non-) classical” game and its equilibrium
    - The Battle of the Bismarck Sea
    - Playing the Joker and the ambiguity of behavior
- Extensions: Drug dealers, Rock-Paper-Scissors, and food control
    - Sherlock Holmes and “The Final Problem”
    - Joan of Arcadia and some behavioral models for RPS
    - The goalkeeper's fear of the penalty
- Strategic moves: Improve yourself, move second, and salami tactics

The classical zero-sum game: Chess

And its strategic form:



Definition: A **zero-sum game** is a game in which one player’s gain is the other player’s loss. That is, the sum of payoffs is zero whatever strategies they choose.

The non-classical zero-sum game: American football

- In American football the offense of a team attempts to advance the ball down the field by running with or passing it, either short, medium or long. The role of the defense is to prevent the offense from scoring, either by a run or pass defense or by a blitz of the quarterback.
- Each play is a zero-sum game: the offense tries to gain yardage while the defense tries to prevent them from doing so.
- The average gained yardage by the offense that could be expected under different strategy combinations is as follows:

Defense

Offense

	run	*pass*	*blitz*
run	2,-2	5,-5	13,-13
short pass	6,-6	5.6,-5.6	9.5, -9.5
medium pass	6,-6	4.5,-4.5	1,-1
long pass	10,-10	3,-3	-2,2

How to behave in American football: Use maximin strategies

	defense			
offense	*run*	*pass*	*blitz*	
run	2,-2	5,-5	13,-13	min = 2
short pass	6,-6	5.6,-5.6	9.5, -9.5	min = 5.6
medium pass	6,-6	4.5,-4.5	1,-1	min = 1
long pass	10,-10	3,-3	-2,2	min = -2
	max = -10	max = -5.6	max = -13	

Definition:

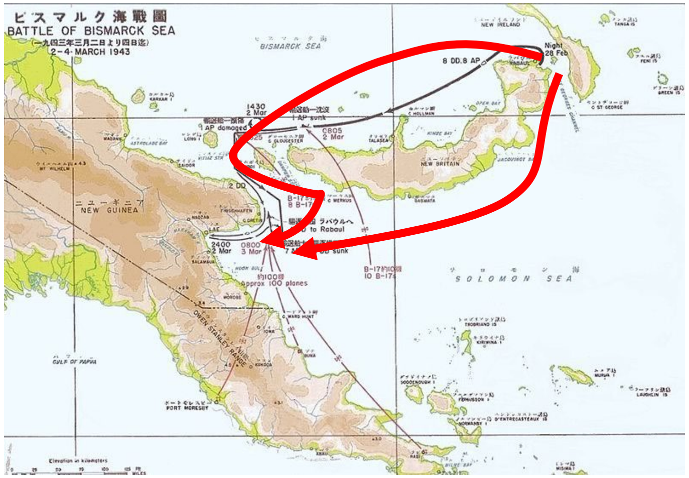
A **maximin strategy** maximizes a player’s minimal gain. This is identical to his **minimax strategy** which minimizes the player’s maximal loss.

A **maximin equilibrium** is an equilibrium if both players use a maximin strategy.

Insight:

For every two-person zero-sum game the maximin equilibrium is a Nash equilibrium.

Case study: The Battle of the Bismarck Sea - World War II, 1943



	Kimura	
	north	south
Kenney north	2,-2	2,-2
Kenney south	1,-1	3,-3

The history:

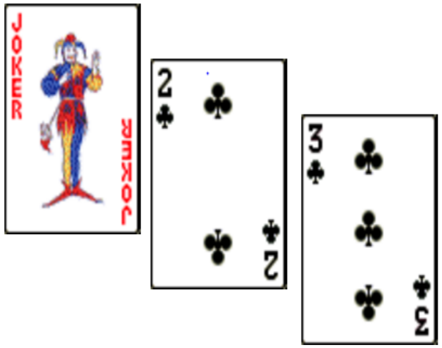
In 1943, Japanese Admiral Kimura was ordered to transport Japanese troops across the Bismarck Sea to New Guinea. U.S. Admiral Kenney was ordered to bomb the troop transport with his carrier-based bombers. Kimura had to choose among taking a rainy northern route, or taking a sunnier southern route. Once set out, he could not recall his ships. Kenney had to decide whether to send all his planes along the northern or southern route. His planes returned to their carriers at the end of the day and could be reassigned on the following day.

The rest of the story:

Kimura took the northern route, Kenney send the bombers northwards, and two days of bombing inflicted serious damage on Japanese ships.

Case study: Playing the Joker and the ambiguity of behavior

- Consider the following two player card game: Player 1 and Player 2 hold three cards, a joker, a two and a three. Each of them selects a card from his hand and places it face down on a table. They then simultaneously turn their card face up.



- The winner of the game is the player who selected a number card given the other player choose a joker. In all other cases there is a draw:

Player 2

Player 1

	two	*joker*	*three*
two	0,0	1,-1	0,0
joker	-1,1	0,0	-1,1
three	0,0	1,-1	0,0

Insight:

In zero-sum games many equilibria might exist and players' behavior is ambiguous

Extending the classical game: Drug dealers and patrols on the beat

- To crack down drug trade, additional police officers are put out on patrol to disrupt business of drug dealers. A drug dealer can work out his trade either on a street corner or in the park. Each day, he decides where to set up shop, knowing that word about his location will travel among users – but not to the police. The police officer on the beat then needs to decide whether he will patrol the park or the street corner.

- Without disruption by the police, 100 trades will occur in the park , but only 90 on the street corner due to patrolling police cars. If both are on the street corner, only 20 trades occur. If both end up in the park, 40 trades occur due to the size of the park.

Dealer

Officer

	street corner	*park*
street corner	80,20	0,100
park	10,90	60,40

Extending the classical game: Drug dealers and how to turn constant-sum into zero-sum games

Definition: A constant-sum game is a game in which the sum of payoffs is constant whatever strategies they choose.

Transformation: A constant-sum game can always be normalized into a zero-sum game by constant subtraction or multiplication.

Dealer

	street corner	*park*
street corner	80,20	0,100
park	10,90	60,40

Officer

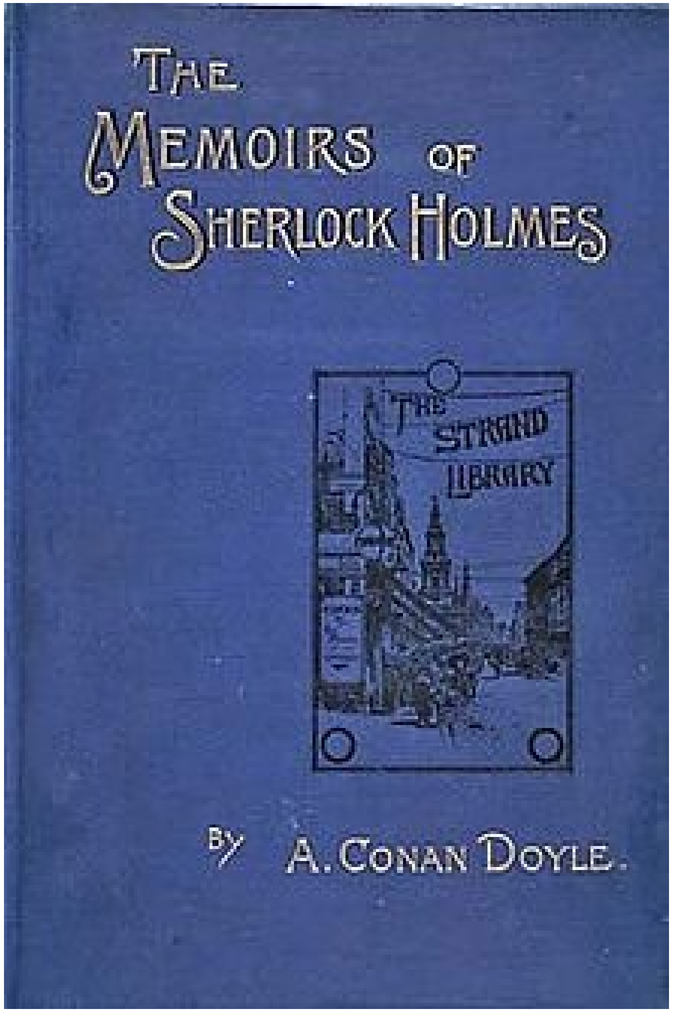
-50,-50

Dealer

	street corner	*park*
street corner	30,-30	-50,50
park	-40,40	10,-10

Officer

Case study: Sherlock Holmes and "The Final Problem"



The previous plot:

Holmes has been tracking Professor Moriarty, the genius behind a secret criminal force, for months. Now Moriarty is out to thwart Holmes's plan. After having survived three murder attempts that day, Holmes wants to escape by taking the train from London to Dover and then fleeing to the Continent.

As the train pulls out, Holmes spots Moriarty on the platform. He infers that Moriarty, who has also seen Holmes, will secure a special train to overtake him.

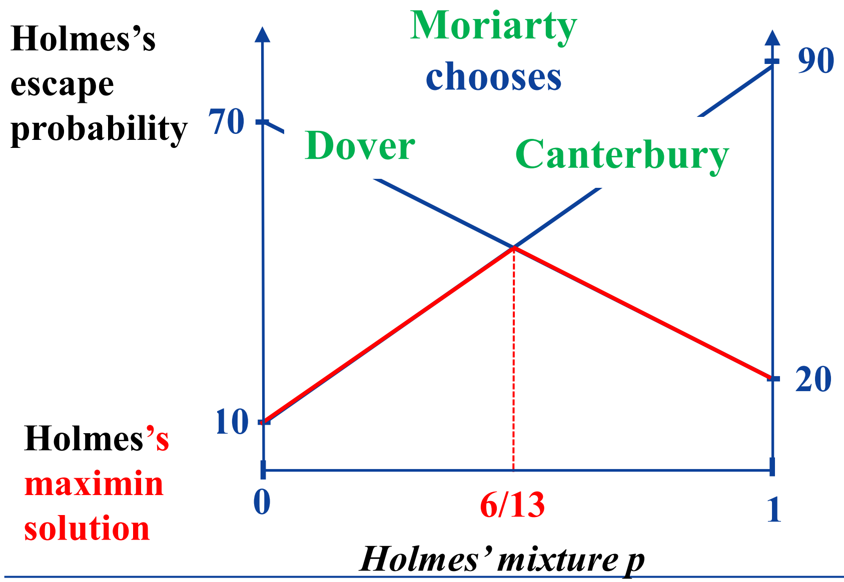
Holmes is faced with the decision of either going to Dover or disembarking at Canterbury, the only intermediate station. Moriarty, equally intelligent, has the same options. Holmes believes he is likely to be killed if they should find themselves on the same platform. If Holmes, however, reaches Dover unharmed, he can safely escape to the Continent.

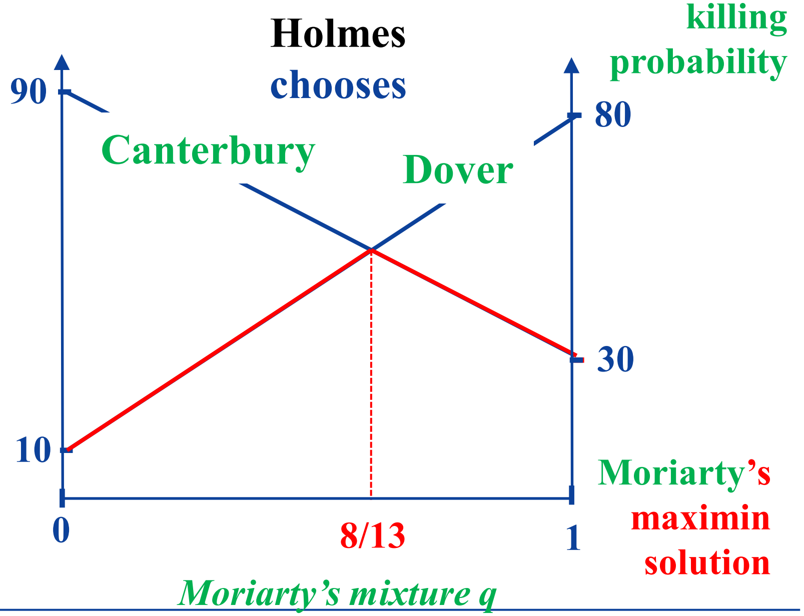
Sherlock Holmes and “The Final Problem”: Dover or Canterbury?

Sherlock Holmes

Moriarty

	Dover	*Canterbury*	
Dover	**20,80**	**90,10**	*p*
Canter-bury	**70,30**	**10,90**	**1-*p***
	q	**1-*q***	





Sherlock Holmes and “The Final Problem”: The final solution

Holmes and Moriarty fighting over the Reichenbach Falls

The final plot:

Holmes changes his planned route and gets off at Canterbury. As he is waiting for another train, a special train roars through Canterbury – it contains Moriarty.

Holmes escapes death, but fails to get to the Continent.

What happened next:

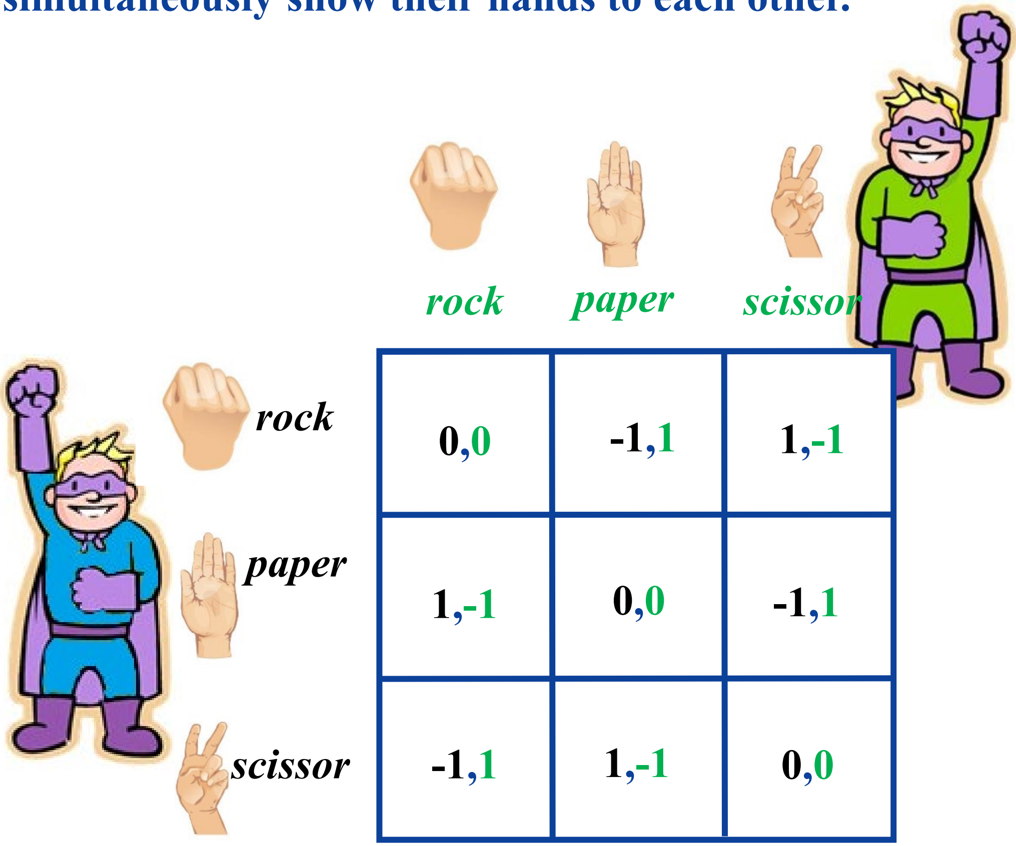
Having heard that most of Moriarty’s gang has been arrested in England, Holmes escapes to the Continent. His journey takes him to Switzerland.

As he takes a walk at the Reichenbach Falls, a natural wonder, Moriarty appears. While locked in mortal combat, Holmes and Moriarty both seem to fall to their deaths down the gorge…

Extending the classical game: Rock-Paper-Scissors

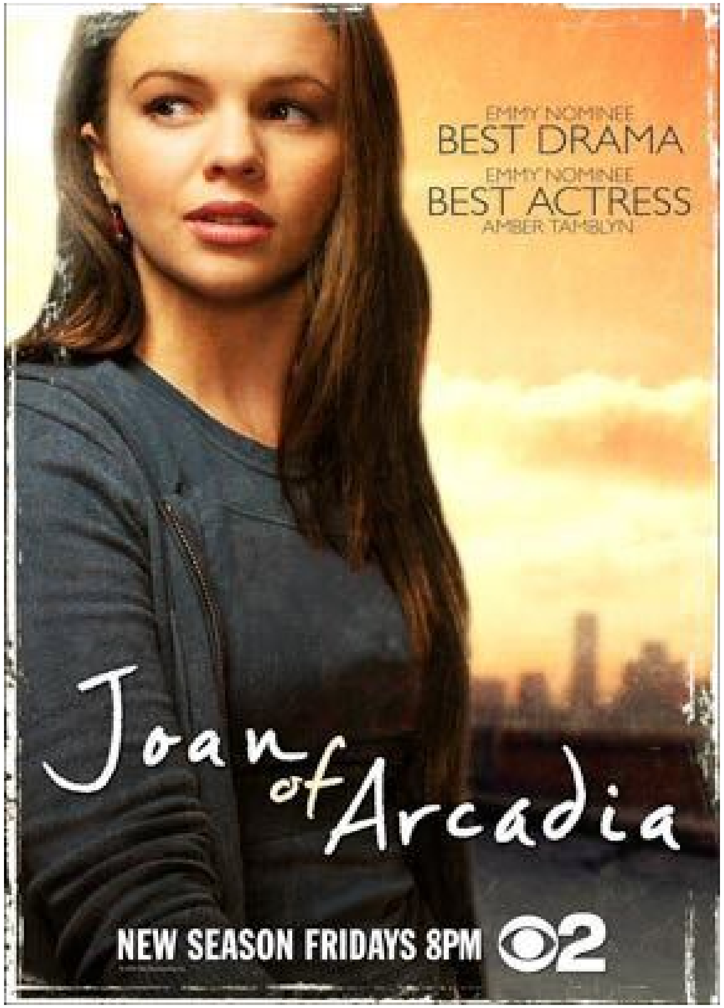
- Rock-Paper-Scissors is a children’s game that requires players to form their hands into one of the three shapes, and to simultaneously show their hands to each other.

- There can be only ties, wins, or loses: Paper “covers” rock, so a player showing paper wins over a player showing rock; scissors “cut” paper, so a player showing scissor wins over a player showing paper; and rock ”breaks” scissors, so a player showing rock wins over a player showing scissor. Ties occur when both players show the same item.



	rock	paper	scissor
rock	0,0	-1,1	1,-1
paper	1,-1	0,0	-1,1
scissor	-1,1	1,-1	0,0

Case study: Joan of Arcadia and some behavioral models for RPS



Film poster for Joan of Arcadia, retrieved from impawards.com, copyright is believed to belong to Barbara Hall Production

Some of the reasoning:

Glynis: Perhaps it's the psychology she's using, not algebra. You know, rock is id, paper--ego, scissor--superego.

Grace: You people are seriously warped. It's a game. It's about luck.

Luke: No, grace. Games are never about luck.

⋮

Glynis: You have to find a way to take psychology out of the game.

Friedman: Following a predetermined gambit like the avalanche-- rock rock rock or the scissor sandwich, paper scissors paper.

Luke: Which takes the decision-making away from the player, giving us the advantage in a zero sum situation.

Extending the classical game: Rock-Paper-Scissors and the minimax equilibrium in mixed strategies

	rock	*paper*	*scissors*	
rock	0,0	-1,1	1,-1	p_1
paper	1,-1	0,0	-1,1	p_2
scissors	-1,1	1,-1	0,0	$1-p_1-p_2$

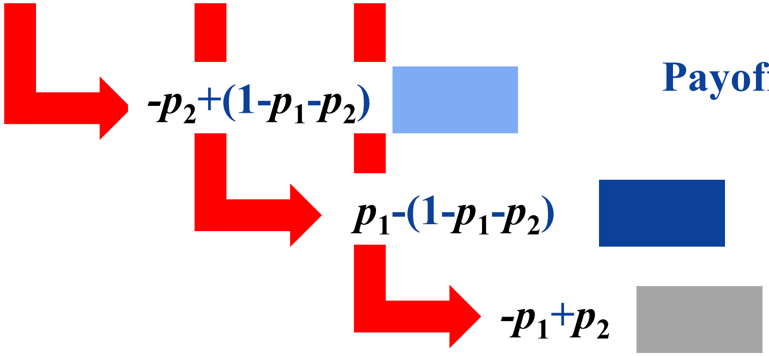
His indifference implies:

$$-p_2+(1-p_1-p_2) = p_1-(1-p_1-p_2)$$

$$-p_2+(1-p_1-p_2) = -p_1+p_2$$

$$\Rightarrow \quad p_1^* = p_2^* = 1/3$$

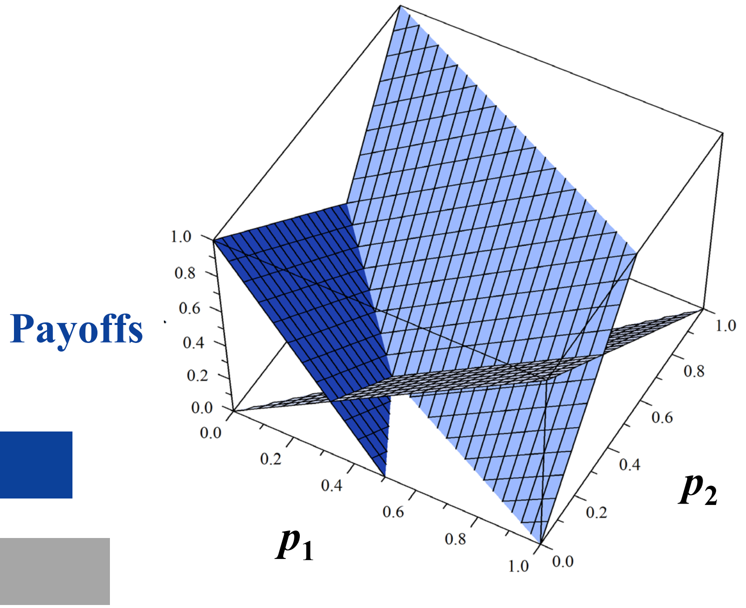
Payoffs:



$-p_2+(1-p_1-p_2)$ 🟦

$p_1-(1-p_1-p_2)$ ■

$-p_1+p_2$ ▩



Case study: Professionals play minimax in penalty kicks

- In soccer, a penalty kick is awarded against a team which commits a punishable offense inside its own penalty area while the ball is in play. The rules that govern this play are described in detail in the *Official Laws of the Game* (FIFA, 2000).

Chelsea's David Luiz scores his penalty kick during the Champions League final soccer match between Bayern Munich and Chelsea, May 2012, retrieved from news.com.au, copyright belongs to AP

Case study: The goalkeeper's fear of the penalty

Modelling a penalty kick as a game

Goalkeeper

Kicker

	left	center	right	
left	65,35	95,05	95,05	k_l
center	95,05	0,1	95,05	k_c
right	95,05	95,05	65,35	k_r

based on data from 1417 penalty kicks during 1995-2000 from professional games in Spain, Italy, and England.*

Mixed equilibrium:

$k_l^* = k_r^* \approx 43\%, k_c^* \approx 14\%$

➡ Probability of goal ≈ 82%

Actual strategy distribution:*

Goal-keeper

Kicker

	left	center	right
left	19.6	0.9	21.9
center	3.6	0.3	3.6
right	21.7	0.9	27.6

and actual scoring probability if kicker kicked in

- natural direction ≈ 82.68%
- unnatural direction ≈ 81.11%

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* Palacios-Huerta (2003): “Professionals play Minimax” RES

Extending the classical game: Food control and inspections over time

- Consider the unscrupulous restaurant "Zum Anker" that is determined to discharge a pollutant into the river in the next week. The local health authority will learn immediately when the river is polluted because of the telephone complaints it will receive from local residence. However, to obtain conviction, the authority has to catch the restaurant red-handed. Unfortunately, the agency can only afford to dispatch an inspector to the site on one of the seven days.

Restaurant owner

Inspector

	wait	*act*	
wait	$v_n, -v_n$	-1,1	p
act	-1,1	1,-1	$1-p$

where v_n is the inspector's payoff if n days remain

Equilibrium analysis:

His indifference requires

$$-pv_n + 1*(1-p) = 1*p - 1*(1-p)$$

⇒ $$p^* = \frac{2}{3+v_n}$$

⇒ $$v_{n+1} = 1 - 2p = \frac{v_n - 1}{v_n - 3}$$

⇒ with $v_0 = 1$: $v_n = -1 + 2/n$

⇒ $1 - p^* = 1/n$

Improve yourself as strategic move in tennis

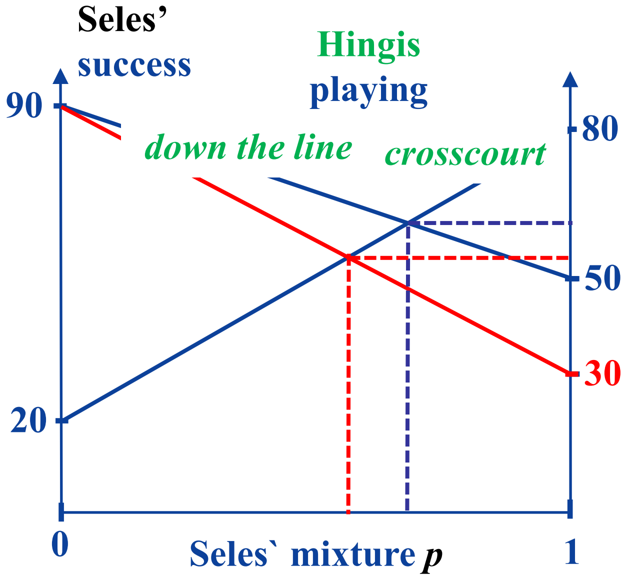
- Consider a tennis match between two professionals, Monica Seles and Martina Hingis. Hingis at the net had just volleyed a ball to Seles on the baseline, and Seles is about to attempt a passing shot. She can try to send the ball either down the line or crosscourt. Hingis must likewise prepare to cover one side or the other. Seles's success percentages are as follows:

- Suppose that Hingis works on improving her skills covering down the line to the point where Seles's success drops form 50% to 30%.

	Hingis		
	down the line	*cross-court*	
down the line	**50**	**80**	*p*
cross-court	**90**	**20**	1-*p*

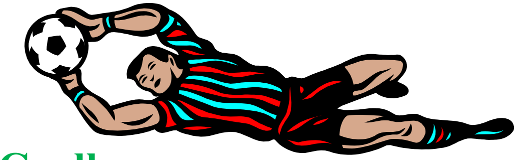
Seles

Seles` success rate



Moving second as strategic move in soccer

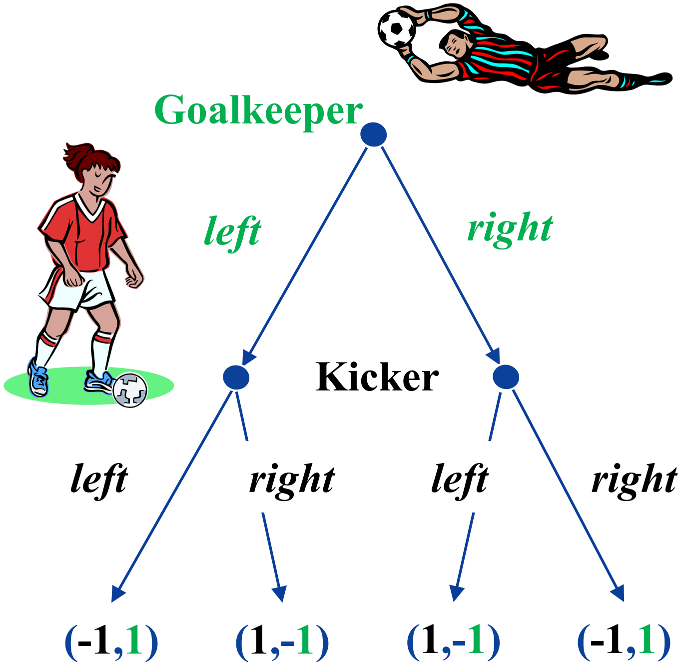
- **Consider a penalty kick and suppose that the kicker can disguise her intended shoot until the very last moment. Assume, that her movement as she goes to hit the ball belies her shot intentions so that the goalkeeper reacts and move first.**



Goalkeeper

Kicker

	left	*right*
left	-1,1	1,-1
right	1,-1	-1,1



20 yards to go in American Football…

- **Consider two teams in American football and suppose that the offense has two more plays left, the third and fourth down, to gain 20 yards, in which case it wins; otherwise it loses. The offense's coach can either cover 10 yards first and with the fourth down the remaining yards, or directly 20 yards. Success probabilities for each down are as follows:**

Defense

Offense

	10 yards	*20 yards*	
10 yards	4/5	1	p
20 yards	1	1/2	$1-p$

Offense's success probabilities

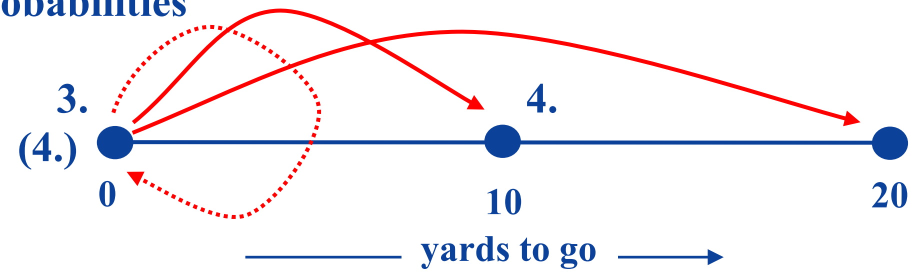
Defense is indifferent if

$$0.8p + (1-p) = p + 0.5\,(1-p)$$

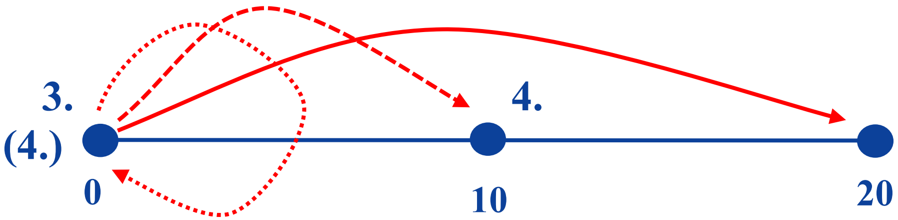
⇒ $p^* = 5/7$

⇒ **Payoff = 6/7**

Possible plays



… and the use of salami tactics as strategic move



Offense

	10 yards	20 yards
10 yards	4/5	1
20 yards	1	1/2

Defense

Solution by backwards induction:

Payoff for fourth down

- when 20 yards are to go: ½
- when 10 yards are to go: 6/7

Payoff for third down

- when both choose (10,10): 4/5*6/7 + 1/5*½
- when both choose (10,20): 1*6/7
- when both choose (20,10): 1
- when both choose (20,20): ½*1 + ½*½

	10 yards	20 yards	
10 yards	11/14	6/7	p
20 yards	1	3/4	1-p

$p^* = 7/9$ and payoff = 5/6 > ½ + ½ ½ =3/4

Zero-Sum Games: What we learned today

- A player’s maximin strategy maximizes his own payoff when he believes that whatever strategy he chooses, the other player will act in the worst way possible for him. In a zero-sum game the minimax equilibrium, whereby both players use a minimax strategy, is also a Nash equilibrium.
- In real life, most economic, political or social situations are different from zero-sum games such as football or poker. Pervasive, however, are constant-sum games in which a pie is distributed between different eaters. Unfortunately, constant-sum games are also games of pure conflicts.
- To win in a zero-sum game, the best strategic move is to wait what the opponent will do and then react accordingly. Improve yourself or salami tactics are also good strategic moves.

Further readings

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