Session 3

Being unpredictable

- Discoordination Games -

What you will learn today: Our objectives

- What should we do in situations in which one side likes a coincidence of actions while the other wishes to avoid it? How can we be unpredictable in our behavior?
  - ➢ The classical game and its equilibrium
- How can we use these insights to explain how people behave in more complex situations?
  - ➢ Extensions
- Are there any strategic moves we can use to align the actions of others with our intentions?
  - ➢ Strategic moves

Our path to succeed: Course outline for today

- The “classical” game and its equilibrium: The arms’ race
  - Are we as rational as chimpanzees?
  - Invasion of Normandy
- Extensions: The good Samaritan, the race to the moon, and moral courage
  - Kitty Genovese and the bystander effect
- Strategic moves: Taking the initiative, setting incentives, delegating decision making, and awaiting what the other will do

The “classical” story of a discoordination game: The arms control inspection

- In the context of arms control, inspections are procedures designed to provide data with which an agent’s compliance to an agreement can be assessed. There is, potentially at least, a conflict between the inspection authority and the agent required to comply.

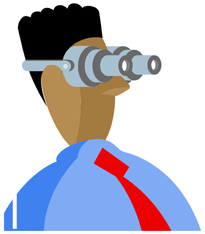
- In a bilateral relationship, an inspectee may choose to comply or violate an arms-control agreement, an inspector may choose to inspect, or not, for a possible violation by the inspectee.

Assumptions:

cost of detected violation:	3
benefit of clean inspection:	1
benefit of violation:	2
cost of inspection:	1
cost of violation:	2
benefit of detected violation:	3

Inspectee

Inspector

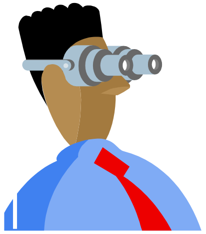


	comply	*violate*
inspect	-1,1	0,-1
don’t inspect	0,0	-2,2

How to behave in the inspection game: Be unpredictable

Inspectee

Inspector



	comply	*violate*
inspect	-1,1	0,-1
don’t inspect	0,0	-2,2

Definition: **A 2x2 discoordination game is a game with no equilibrium in pure strategies. Instead each player wants to be unpredictable in his behavior. That is, there exists a single equilibrium in mixed strategies.**

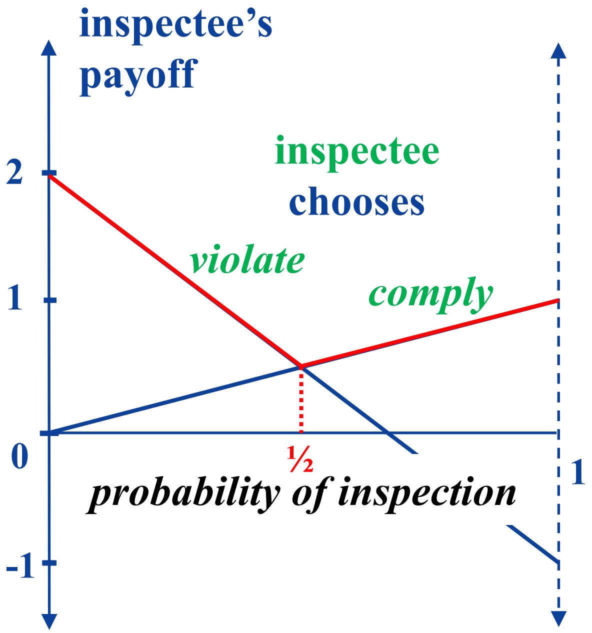
Mixed strategies and mixed strategy equilibrium

Definition: A **mixed strategy** for a player i with n strategies $s_{i1},\dots,s_{in}$ is a probability distribution $p_i = (p_{i1},\dots p_{in})$ where $0 \leq p_{ik} \leq 1$ for k = 1,…,n, and $p_{i1} + \dots + p_{in} = 1$. A pure strategy s_{ik} then is defined by $p_{ik} = 1$.

Definition: In a 2x2 game, a mixed strategy profile (p_1, p_2) is a **Nash equilibrium in mixed strategies** if each player’s mixed strategy is a best response to the other player’s mixed strategy.

The inspectee’s problem and his best responses

Inspectee



	comply	*violate*	
inspect	**-1,1**	**0,-1**	*p*
don’t inspect	**0,0**	**-2,2**	1-*p*

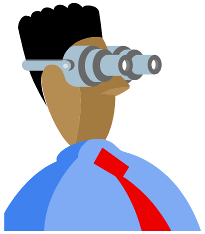
Expected payoff for *compliance*: $p \bullet 1 + (1-p) \bullet 0 = p$

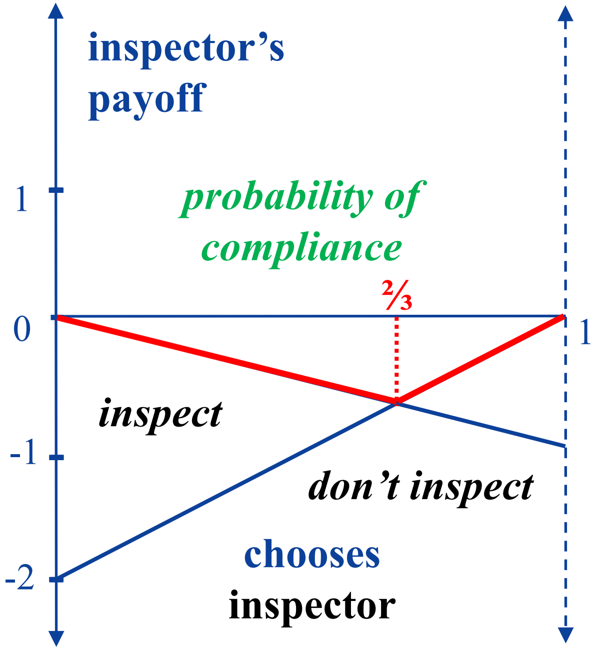
Expected payoff for *violation*: $p \bullet (-1) + (1-p) \bullet 2 = 2-3p$

⇒ Inspectee chooses *compliance* if $p \geq (2-3p)$ ⇒ $p \geq ½$

The inspector’ s problem and his best responses

Inspector





	comply	*violate*
inspect	-1,1	0,-1
don’t inspect	0,0	-2,2
	q	$1-q$

Expected payoff for *inspection*: $q\cdot(-1) + (1-q)\cdot 0 = -q$

Expected payoff for *non-inspection*: $q\cdot 0 + (1-q)\cdot(-2) = -2(1-q)$

⇒ **Inspector** chooses *inspection* if $-q \geq -2(1-q)$ ⇒ $q \leq ⅔$

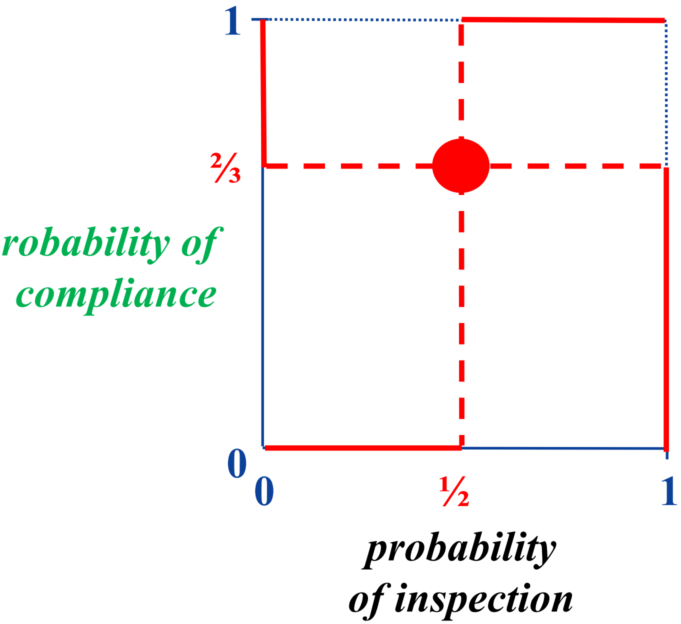
The Nash equilibrium in mixed strategies and the optimal amount of unpredictability

Inspector chooses *inspection* if $q \leq 2/3$

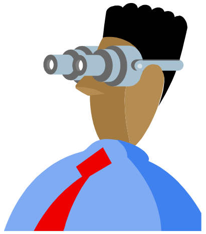
Inspectee chooses *compliance* if $p \geq 1/2$

	comply	*violate*	
inspect	-1,1	0,-1	p
don't inspect	0,0	-2,2	$1-p$
	q	$1-q$	

Inspectee



Inspector



Nash equilibrium in mixed strategies

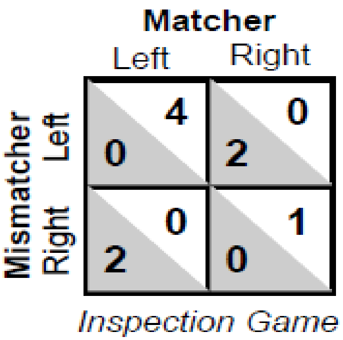
$p^* = 1/2$

$q^* = 2/3$

Expected payoff
for inspector -⅔
for inspectee ½

Peter-J. Jost – Thinking Strategically

Case study: Are we as rational as chimpanzees?

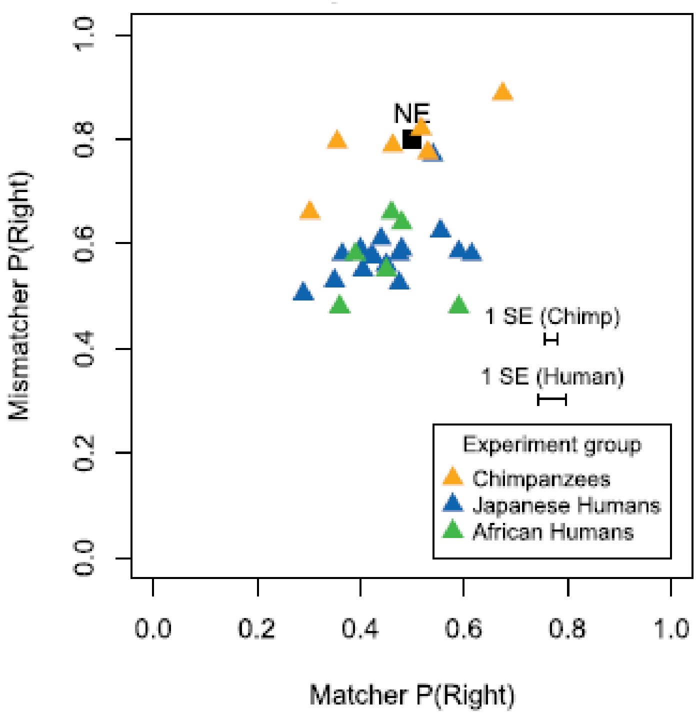


- In a natural experiment, the Nash equilibrium predictions were tested in an inspection game with chimpanzee and human participants

- Subjects made choices on a dual touch-screen panel and earned food or coin rewards

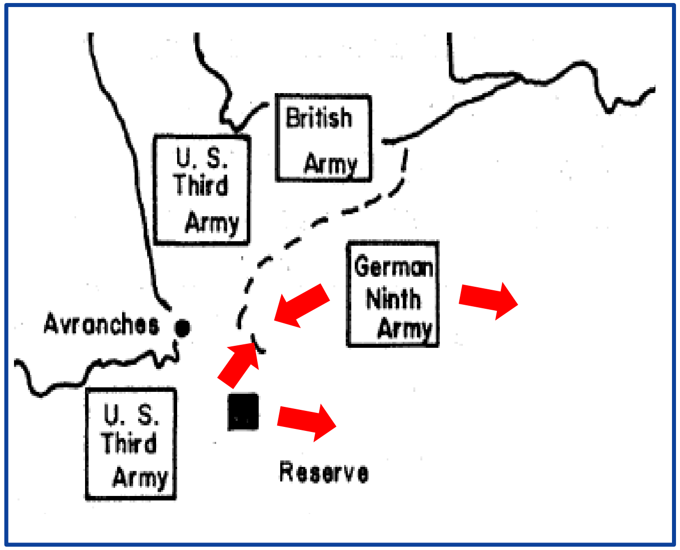
Empirical findings:

- Chimpanzee choices are closer to the equilibrium prediction than human choices and their choices shift with reward changes almost exactly as predicted:



Martin et al (2014): Chimpanzee choice rates in competitive games match equilibrium game theory predictions, Scientific Report 4

Case study: Invasion of Normandy - World War II, 1944



The Avranches Gap Situation

von Kluge

		attack	*withdraw*
Bradley	*reinforce*	3,0	2,3
	eastward	0,5	4,2

The history:

Aug 1944: As part of the invasion of Normandy, a strategic situation arose in the Avranches Gap, between General Bradley and Field Marshall v. Kluge:*

> *“Either we could play safe on the hinge by calling back those last four divisions to … safeguard the lifeline of Brittany forces, or we could … throw those four divisions against his open flank in an effort to destroy the German Seventh Army.”*

> *“The German Command … could [either] withdraw the loose left flank… or he could gamble an Army by striking for Avranches in an effort to close our gap and peg the loose end of his line back to the sea...”*

The rest of the story:

Von Kluge chose to withdraw but was overruled by Hitler. The attack failed, and von Kluge committed suicide.

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*O. Bradley, A Soldier’s Story, 1951, p. 369ff

Extending the classical game: The parable and dilemma of the good Samaritan

The parable of the Good Samaritan

And Jesus said: 'A certain man was going down from Jerusalem to Jericho, and he fell among robbers, who both stripped him and beat him, and departed, leaving him half dead. By chance a certain priest was going down that way. When he saw him, he passed by on the other side. In the same way a Levite also, when he came to the place, and saw him, passed by on the other side. But a certain Samaritan, as he travelled, came where he was. When he saw him, he was moved with compassion, came to him, and bound up his wounds, pouring on oil and wine. He set him on his own animal, and brought him to an inn, and took care of him. On the next day, when he departed, he took out two denarii, and gave them to the host, and said to him, 'Take care of him. Whatever you spend beyond that, I will repay you when I return.'

Luke 10:30–37

The dilemma of the modern-day Samaritan

And the Government said to the pauper: "I wish to aid you if you search for work but not otherwise."

And the pauper said to the Government: "I search for work only if I cannot depend on your aid."

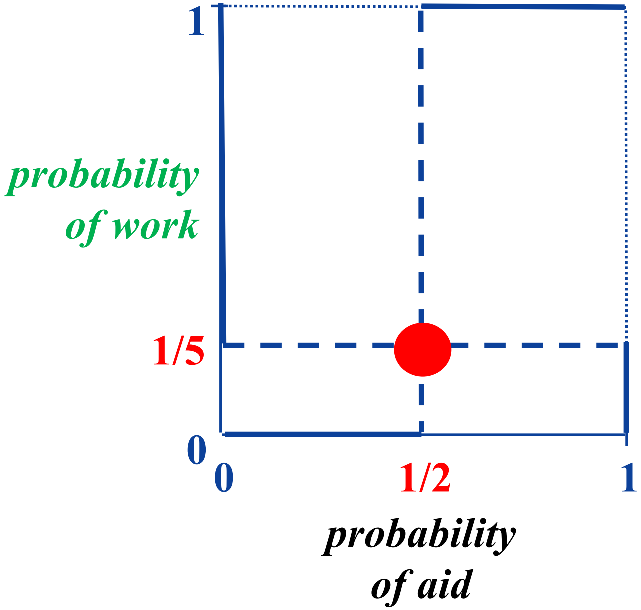
Pauper

Government	*work*	*loaf*
aid	3,2	-1,3
no aid	-1,1	0,0

Extending the classical game: The solution for the good Samaritan

Government	*work*	*loaf*	
aid	**3,2**	**-1,3**	p
no aid	**-1,1**	**0,0**	$1-p$
	q	$1-q$	

Pauper



Government is indifferent to *aid* if

$$3q - (1-q) = -q \Longrightarrow q = 0.2$$

Pauper is indifferent to *work* if

$$2p + (1-p) = 3p \Longrightarrow p = 0.5$$

Nash equilibrium in mixed strategies

$$p^* = 1/2$$

$$q^* = 1/5$$

Extending the classical game: Race to space – Elon Musk or Jeff Bezos?

- SpaceX and Blue Origin are the space exploration companies of Elon Musk and Jeff Bezos. The two billionaires are currently competing who will develop the first fully functional reusable space shuttle with the ultimate goal to eventually colonize other planets. Elon Musk with SpaceX puts a value of USD 50bn on being the first, whereas Jeff Bezos' value is between USD 40bn and 60bn. To improve their chances of success, they can spend a budget of up to USD 40bn on R&D.
- Suppose that their chances are very similar ex ante and that the two companies have equally capable engineers so that the company which invests more money wins the competition.

BLUE ORIGIN

SPACEX

Extending the classical game: The optimal investment into space exploration

The strategic form of their bidding

Jeff Bezos

Elon Musk	*0 Inv*	*20 Inv*	*40 Inv*	
0 Inv	25, $V/2$	0, V-20	0, V-40	p_0
20 Inv	30, 0	5, $V/2$-20	-20, V-40	p_1
40 Inv	10, 0	10, -20	-15, $V/2$-40	$p_2 = 1-p_0-p_1$
	q_0	q_1	$q_2 = 1-q_0-q_1$	

where $V \in [40,60]$ is Jeff Bezos' value

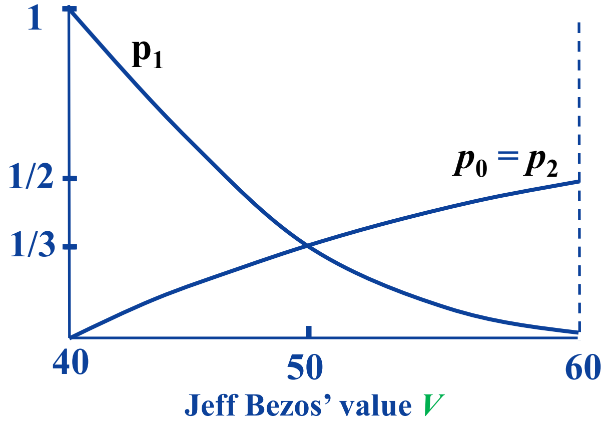
Elon Musk's bidding strategy

Jeff Bezos is indifferent between *Inv* if

$$p_0 V/2 = p_0(V-20) + p_1(V/2-20) - p_2 20$$

$$= p_0(V-40) + p_1(V-40) + p_2(V/2-40)$$

⇒ $p_0 = p_2 = 1-40/V,\ p_1 = 80/V-1$



Case study: Kitty Genovese and the bystander effect

from March 27, 1964 *New York Times* article: "37 Who Saw Murder Didn't Call the Police".

The history:

- Genovese had driven home from work early in the morning and parked about 100 feet (30 m) away from her apartment's door
- As she walked toward the building, Winston Moseley ran after her and stabbed her; Genovese’s scream was not recognized as cry for help
- When one of the neighbors shouted at the attacker, Moseley ran away, but Genovese was out of sight for any witness
- Moseley returned, found Kitty Genovese and further attacked her - Genovese died on the way to the hospital
- Later investigation revealed that a dozen individuals nearby had heard or observed portions of the attack

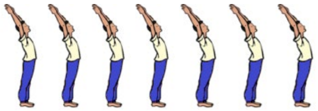
The bystander effect:

- 2 weeks after the murder, the newspaper article “37 Who Saw Murder Didn't Call the Police" reported the lack of neighbors’ reaction and prompted research of the bystander effect
- Contrary to common expectations, larger numbers of bystanders decrease the likelihood that someone will step forward and help a victim

Extending the classical game: Moral courage and the problem of helping

- Suppose there are $n \geq 2$ people who face the decision of whether to help someone in need for assistance. Each simultaneously chooses between helping or ignoring the victim.

- Each person cares about whether the victim is helped, but also that helping is costly:

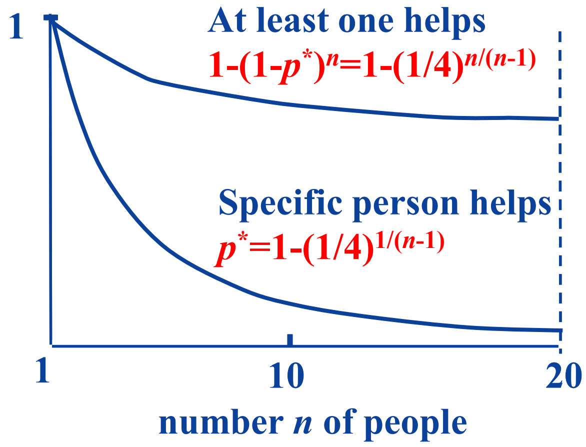


Other people

Specific person	*all ignore*	*at least one helps*	
helps	4	2	p
ignores	1	3	$1-p$

If each person helps with probability p, then:

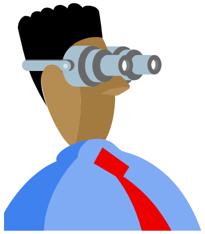
- Expected utility from helping is $(1-p)^{n-1}\cdot 4+[1-(1-p)^{n-1}]\cdot 2$
- Expected utility from ignoring is $(1-p)^{n-1}\cdot 1+[1-(1-p)^{n-1}]\cdot 3$
- Indifference implies



Taking the initiative as strategic move in the inspection game

Inspectee

Inspector



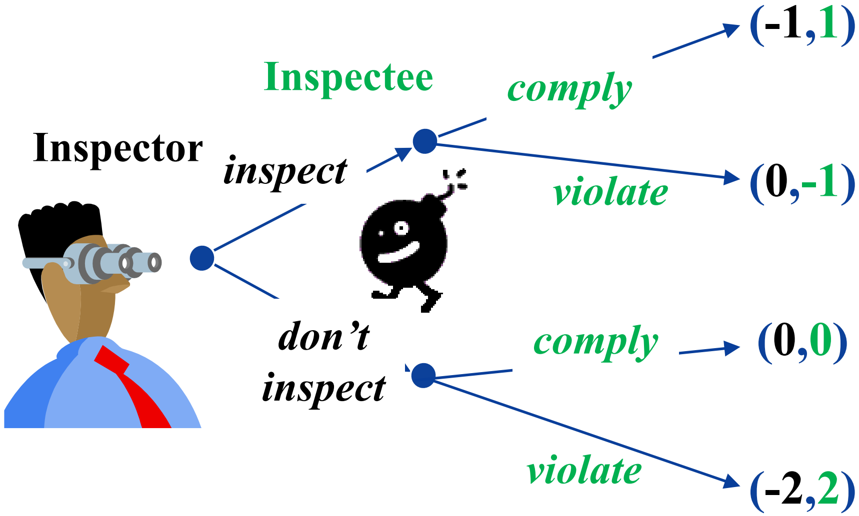
	comply	*violate*	
inspect	-1,1	0,-1	p
don't inspect	0,0	-2,2	$1-p$
	q	$1-q$	

Nash equilibrium in mixed strategies

$p^* = 1/2$

$q^* = 2/3$

Expected payoff
for inspector -2/3
for inspectee 1/2



Optimal *probability of inspection*

Choose p* such that

$$p^* = -p^*+2(1-p^*) \Longrightarrow p^* = 1/2$$

Expected payoff
for inspector -1/2
for inspectee 1/2

Setting incentives as strategic move in the auditing game

- In the context of tax evasion, the Internal Revenue Service IRS must decide whether to audit a certain class of suspect tax returns to discover whether they are accurate or not. The goal is to prevent or catch cheating at minimal costs.
- Suppose the suspect's income is 10 units, tax rate is t, the fine for tax evasion $f \geq 1$, and the cost of auditing 1.

IRS

Suspect

	audit	*trust*	
obey	$10(1-t)$, $10t-1$	$10(1-t)$, $10t$	p
cheat	$10(1-t)-f$, $10t+f-1$	$10,0$	$1-p$
	q	$1-q$	

IRS is indifferent to audit if

$$(10t-1)p + (10t+f-1)(1-p) = 10tp \quad \Rightarrow \quad 1-p = \frac{1}{10t+f}$$

Suspect is indifferent to cheat if

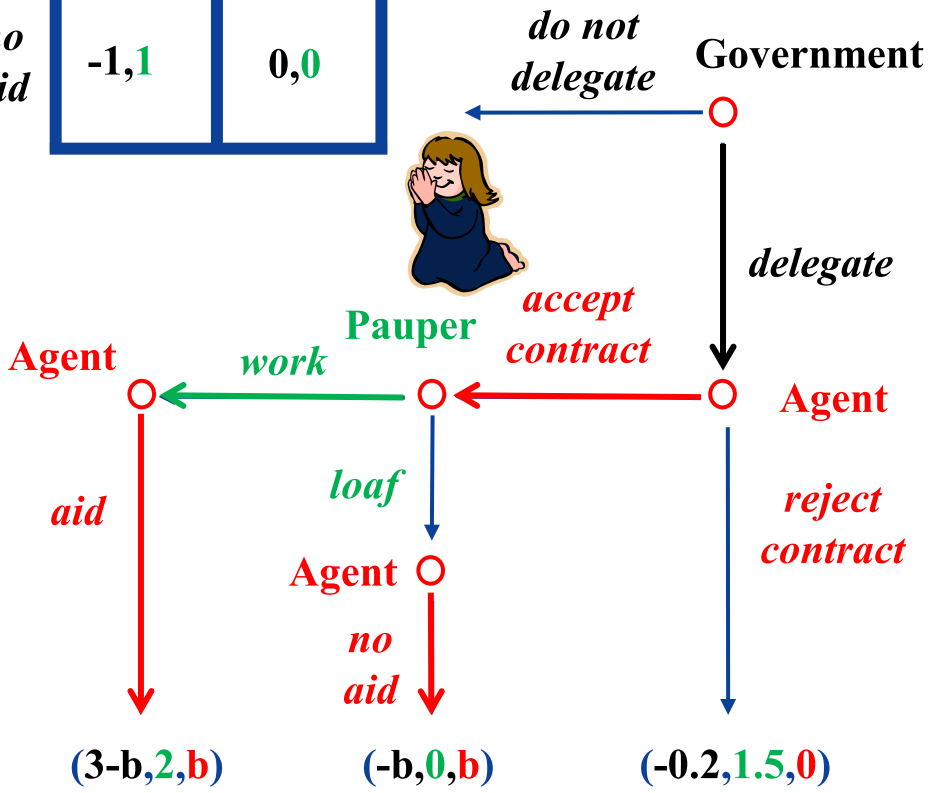
$$(10(1-t)-f)q + 10(1-q) = 10(1-t) \quad \Rightarrow \quad q = \frac{t}{t+f}$$

Cheating and auditing costs are minimal for *t* and *f* as high as possible

Delegating decision making power as strategic move in the Samaritan’s dilemma

- Suppose that helping the pauper means giving him a financial aid of 3€. Suppose the government can delegate its power of decision to an agent as follows: “I pay you a bonus b for watching the pauper’s behavior. If you see him working, help. If you see him loafing, do not help. I entrust you with 3€ at the moment you accept the contract. You get the bonus b after you have done the job.”

	work	*loaf*
aid	3,2	-1,3
no aid	-1,1	0,0



Government

do not delegate

delegate

Agent

accept contract

reject contract

Pauper

work

loaf

Agent

aid

Agent

no aid

(3-b,2,b)

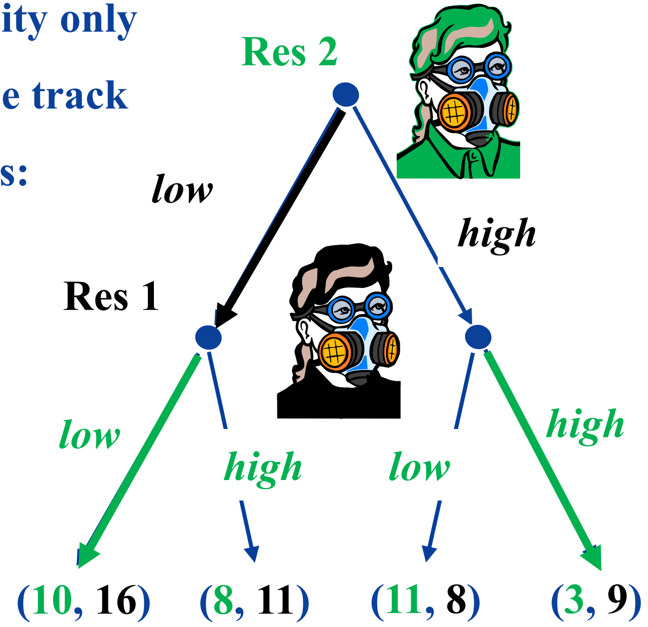
(-b,0,b)

(-0.2,1.5,0)

Waiting what the other will do as strategic move in the awarding research grant game

- **Two researchers submit project proposals to a university committee, which awards research grants of 30 units in total. A researcher can influence the prospects of his proposal by more intensive preparation. High intensity requires 9 units effort, low intensity only 2. The committee awards grants according to the track record of successful projects by both researchers:**

		Res 2	
		low	*high*
Res 1	*low*	**16,10**	**8,11**
	high	**11,8**	**9,3**



		Chances of success for Res 2	
		low	*high*
Chances of success for Res 1	*low*	**3:2**	**1:2**
	high	**2:1**	**1:1**

Discoordination games: What we learned today

- A mixed strategy for a player is a probability distribution over his pure strategies, that is, his actions. Discoordination games are characterized by the fact that there exists only one Nash equilibrium in mixed strategies.
- Situations of discoordination are pervasive in everyday live and arise whenever we want to surprise another party. Here, the best is to surprise ourselves.
- There are several strategic moves a player can use to improve his situation in a discoordination game: Taking the initiative is sometimes advantageous, in other situation, it is better to wait what the other will do. Setting incentives or delegating decision making also works, if possible.

Further readings

- Harrington, J., 2008. Games, Strategies and Decision Making. Worth Publishers: Chapter 7.
- Jost, P.-J. & U. Weitzel, 2007. Strategic Conflict Management. Edward Elgar: Chapter 2.2.4.
- Dixit, A. & S. Skeath, 1999.Games of Strategy. Norton: Chapter 5.
- Dixit, A. & B. Nalebuff, 1993. Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life. Norton: Chapter 7.