Session 1

Theory of

Games & Moves

What you will learn today: Our objectives

- What characterizes a situation as a game in which we do not know what the others are doing or have done? And how should we act in those situations?
  - Put yourself in your rival's shoes: Simultaneous-move games
- What characterizes a situation as a game in which we know that others will take our decisions as given? And how can we think about such situations in a well-structured way?
  - Look ahead: Sequential-move games
- How can we influence the behavior of the following parties? And how can we change others' belief about our own behavior?
  - Manipulate the game: Strategic-move games

Our path to succeed: Course outline for today

- Put yourself in your rival's shoes: Simultaneous-move games
  - Lance Armstrong and doping in professional sport
  - Princess Bride and the proof of the pudding…
- Look ahead: Sequential-move games
  - Galileo Galilei and the interrogation by the inquisition
  - Memento and the lost recall
- Manipulate the game: Strategic-move games
  - Charlie Brown and his playing with Lucy
  - Dr. Strangelove and the premier who loves surprises

Definition of simultaneous-move games in their strategic form

Definition: The strategic form of a simultaneous-move game specifies

1. **the players in the game: $i = 1,\ldots,n$,**

2. **the strategies $a_i \in A_i$ available to each player i,**
   - where A_i denotes player i's action space, and

3. **the payoff $u_i(a_1,\ldots,a_n)$ received by each player i for each possible strategy profile $(a_1,\ldots,a_n) \in A_1 \times \ldots \times A_n$.**

Illustrating simultaneous-move games by game matrices

Example of a two-player-two-action-game, called 2x2 game:

- Two players: $i \in \{1,2\}$
- Each player has two possible strategies: $a_i \in A_i = \{a_{i1}, a_{i2}\}$
- The payoffs are given by functions $u_i : A_1 \times A_2 \rightarrow \Re$

column player

cells/ payoffs

row player

		Player 2: a_{21}	Player 2: a_{22}
Player 1	a_{11}	$u_1(a_{11},a_{21})$, $u_2(a_{11},a_{21})$	$u_1(a_{11},a_{22})$, $u_2(a_{11},a_{22})$
	a_{12}	$u_1(a_{12},a_{21})$, $u_2(a_{12},a_{21})$	$u_1(a_{12},a_{22})$, $u_2(a_{12},a_{22})$

Best responses and the Nash equilibrium

Definition: A strategy b_i of player i is his **best response** to a given strategy profile $a_{-i} \in A_{-i} = A_1 \times \ldots \times A_{i-1} \times A_{i+1} \times \ldots \times A_n$ of the other players if for all $a_i \in A_i$

$$u_i(b_i, a_{-i}) \geq u_i(a_i, a_{-i})$$

Definition: The **best response correspondence** $b_i^*: A_i \rightarrow A_i$ of player i assigns a best response to every possible strategy profile of the other players, that is $b_i^*(a_{-i})$ is a best response to a_{-i} for all $a_{-i} \in A_{-i}$.

Definition: A strategy profile $s^* = (s_1^*, \ldots, s_i^*, \ldots, s_n^*)$ is a **Nash equilibrium** if each player's strategy is a best response correspondence, that is, if for all $a_i \in A_i$ and for all $i = 1, \ldots, n$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(a_i, s_{-i}^*)$$

Case study: Lance Armstrong and doping in professional sport

Case study: Lance Armstrong and doping in professional sport – a short history of his doping allegations

"I've said it for longer than seven years: I have never doped. I can say it again. But I've said it for seven years; it doesn't help. But the fact of the matter is I haven't."

Lance Armstrong, August 26, 2005

- **2004: Armstrong's former masseuse and Motorola teammate Steve Swart claim that he and other cyclists, including Armstrong, began using drugs in 1995**
- **2010: A former U.S. Postal teammate accuses Armstrong of doping in 2002 and 2003**
- **2011: Former teammate Tyler Hamilton claims that they had together taken EPO before and during the 1999, 2000, and 2001 Tours de France**
- **2012: A documentary reveals Armstrong's doping since the beginning of his career**
- **2012: The U.S. Anti-Doping Agency states he was part of "the most sophisticated, professionalized and successful doping program that sport has ever seen"**
- **2013: Armstrong confesses that he has used banned performance-enhancing drugs throughout much of his cycling career and falsified results of drug tests**

Case study: Lance Armstrong and doping in professional sport – the doping game

- **Consider a setting with three road racing cyclists, Lance, Jan and Tristan. They differ in both innate skill and their propensity to take steroids: Lance is faster than Jan and Jan is faster than Tristan. As the propensity to take steroids, Tristan is more inclined to use them than Jan and Jan is more inclined than Lance. More specifically, Tristan will take steroids regardless of whether Lance and Jan do. Jan will not take steroids if no one else does, but in order to remain competitive, he'll take them if either Lance or Tristan does so. Lance won't take steroids unless both Jan and Tristan do so.**

Tristan

chooses steroids

Lance \ Jan chooses	*steroids*	*no steroids*
chooses steroids	2,3,3	3,1,5
chooses no steroids	1,4,5	5,2,6

chooses no steroids

Lance \ Jan chooses	*steroids*	*no steroids*
chooses steroids	3,4,1	4,2,2
chooses no steroids	5,5,2	6,6,4

Case study: Lance Armstrong and doping in professional sport – best responses and the Nash equilibrium

Tristan

chooses steroids

Lance	Jan chooses *steroids*	Jan chooses *no steroids*
chooses steroids	2,3,3	3,1,5
chooses no steroids	1,4,5	5,2,6

chooses no steroids

Lance	Jan chooses *steroids*	Jan chooses *no steroids*
chooses steroids	3,4,1	4,2,2
chooses no steroids	5,5,2	6,6,4

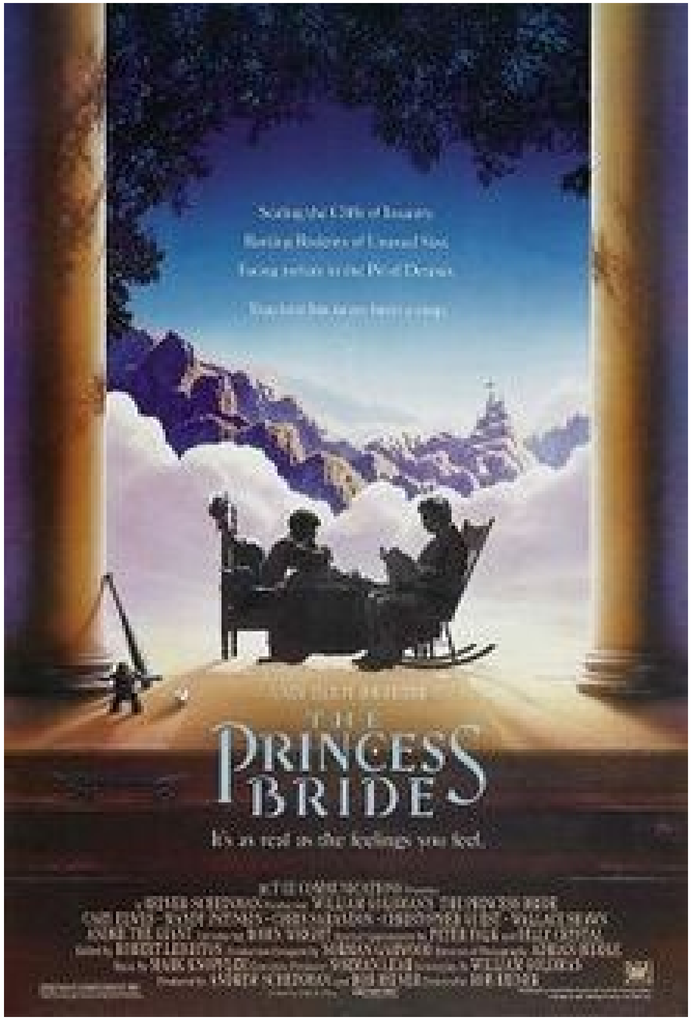
Best response correspondence of Tristan: always choose steroids.

Best response correspondence of Jan: always choose steroids if Tristan does so. If Tristan chooses no steroids, choose steroids if and only if Lance does so.

Best response correspondence of Lance: choose steroids only if both Jan and Tristan do so.

Nash equilibrium: all three use steroids.

Case study: The Princess Bride and the proof of the pudding…



Film poster for The Princess Bride, retrieved from en.Wikipedia.org, copyright 1987, 20th Century Fox

The previous plot:

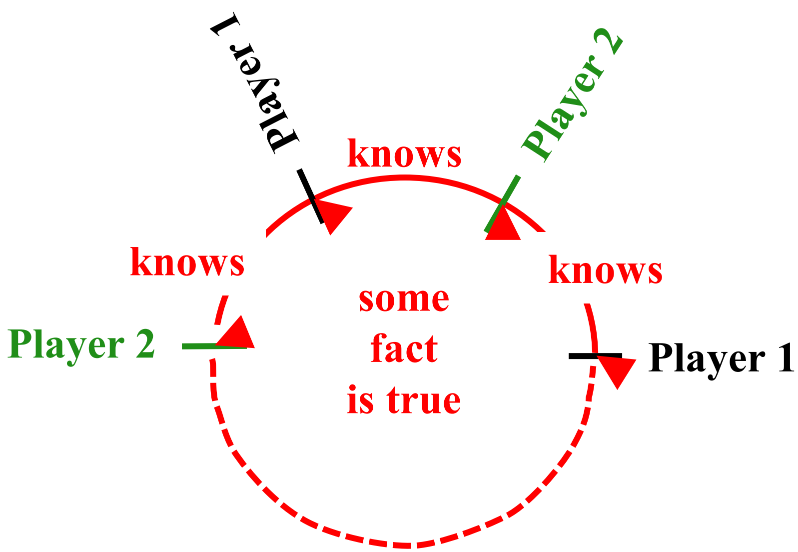
Buttercup, a young woman living on a farm in the countryside, agrees to marry Prince Humperdinck, heir to some throne. Before the wedding, she is kidnapped by a Sicilian boss named Vizzini and two other outlaws. All three were hired by Prince Humperdinck to kidnap her and kill her.

Besides a complement of soldiers, a masked man dressed in black pursues the outlaws. He catches up to the outlaws, defeats two of them and is finally in a battle with Vizzini….

Case study: The Princess Bride and the proof of the pudding… - Validity of the Nash equilibrium concept

Underlying assumptions:

- All players are rational
- Rationality and the game is common knowledge for all players



Man in black \ Vizzini	*take left*	*take right*
poison left	1,-1	-1,1
poison right	-1,1	1,-1

The proof of the pudding is in the eating:

- Case studies
- Experiments

The beauty contest experiment

- For this experiment, you need:
  - 2 pieces of paper
  - a pen
- Choose a number between 0 and 100, write it on a piece of paper, and bring it upfront

Case study: The Princess Bride and the proof of the pudding…
- Validity of the Nash equilibrium concept

Is Vizzini a rational player?

Man in black \ Vizzini	*take left*	*take right*
poison left	1,-1	-1,1
poison right	-1,1	1,-1

"But it's so simple. All I have to do is divine from what I know of you. Are you the sort of man who would put the poison into his own goblet, or his enemy's? Now, a clever man would put the poison into his own goblet, because he would know that only a great fool would reach for what he was given. I'm not a great fool, so I can clearly not choose the wine in front of you. But you must have known I was not a great fool; you would have counted on it, so I can clearly not choose the wine in front of me."

Is the game common knowledge?

Definition of sequential-move games in their extensive form

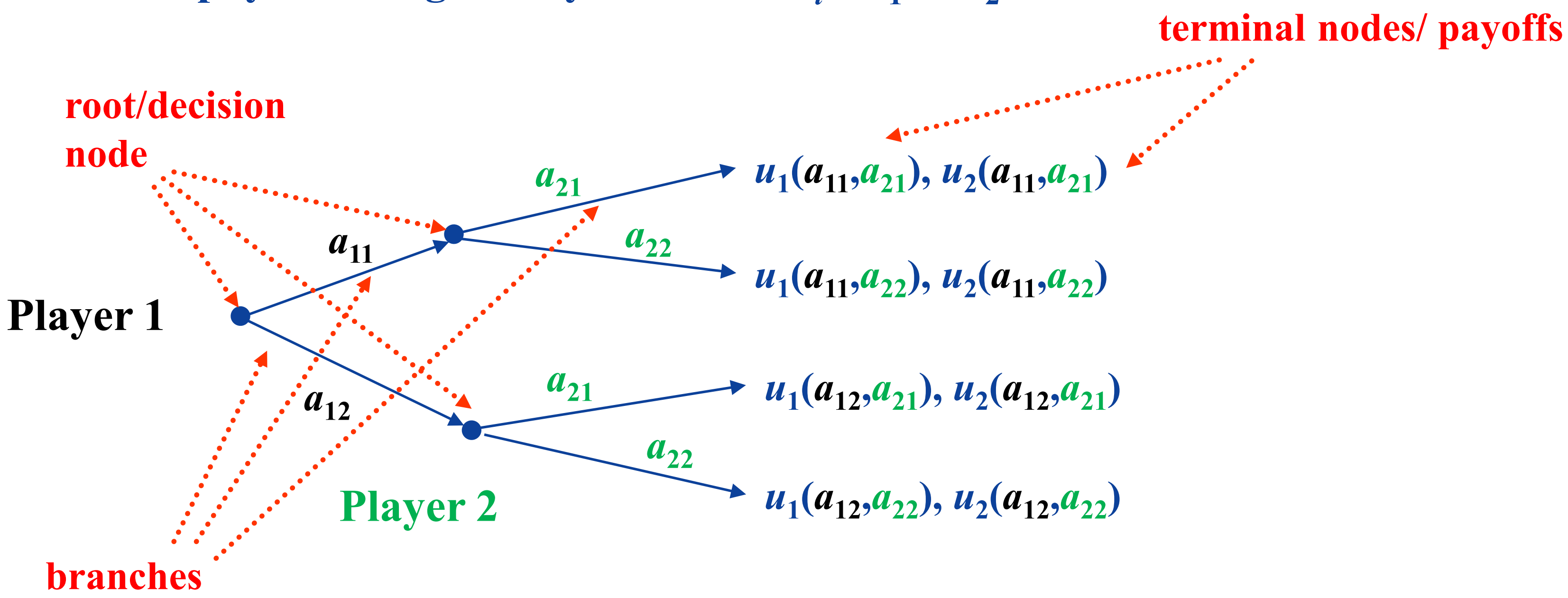
Definition: The **extensive form** of a sequential-move game specifies

1. the players i =1,..,n
2. when each player has the move
3. the possible actions available to a player at each of his or her opportunities to move
4. the payoff u_i for each player i for each possible sequence of actions

Illustrating sequential-move games by game trees

Example of a two-player-two-action game:

- Two players: $i \in \{1,2\}$, player 1 chooses first
- Each player has two possible actions: $a_i \in A_i = \{a_{i1}, a_{i2}\}$
- The payoffs are given by functions $u_i : A_1 \times A_2 \rightarrow \Re$



Strategies, best responses, and backwards induction equilibrium

Definition: A **strategy** of a player is a complete plan of actions which specifies a feasible action for every contingency in which the player might be called to act.

Definition: An action $b_i^* \in A_i$ of player i is a **best response** at stage k given the history h_k of previous actions if $b_i^*(h_k)$ maximizes his payoff.

Definition: The outcome $(a_1^*, b_2^*(a_1^*), \ldots, b_n^*(a_1^*, b_2^*(a_1^*), \ldots))$ is a **backwards induction equilibrium** of an n -stage game, if for all players $i = 1, \ldots, n$, $b_i^*(h_k)$ is a best response for player i.

Case study: Galileo Galilei and the interrogation by the inquisition

Case study: Galileo Galilei and the interrogation by the inquisition – a short history of his affair

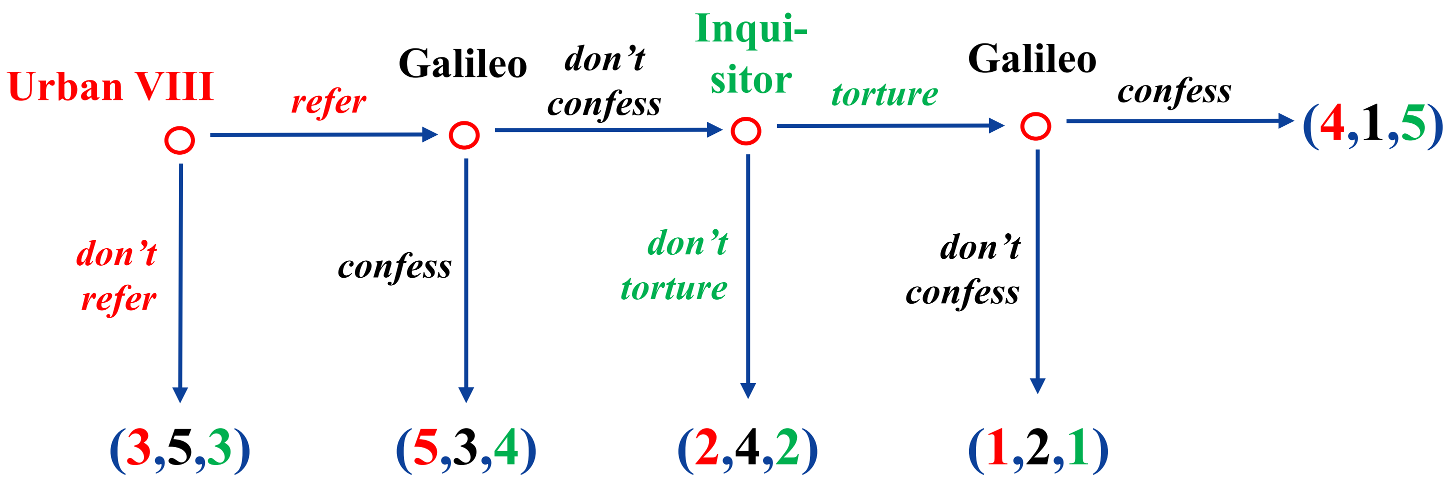
"My dear Kepler, I wish that we might laugh at the remarkable stupidity of the common herd. What do you have to say about the principal philosophers of this academy who are filled with the stubbornness of an asp and do not want to look at either the planets, the moon or the telescope…"

Galileo Galilei, August 19, 1610

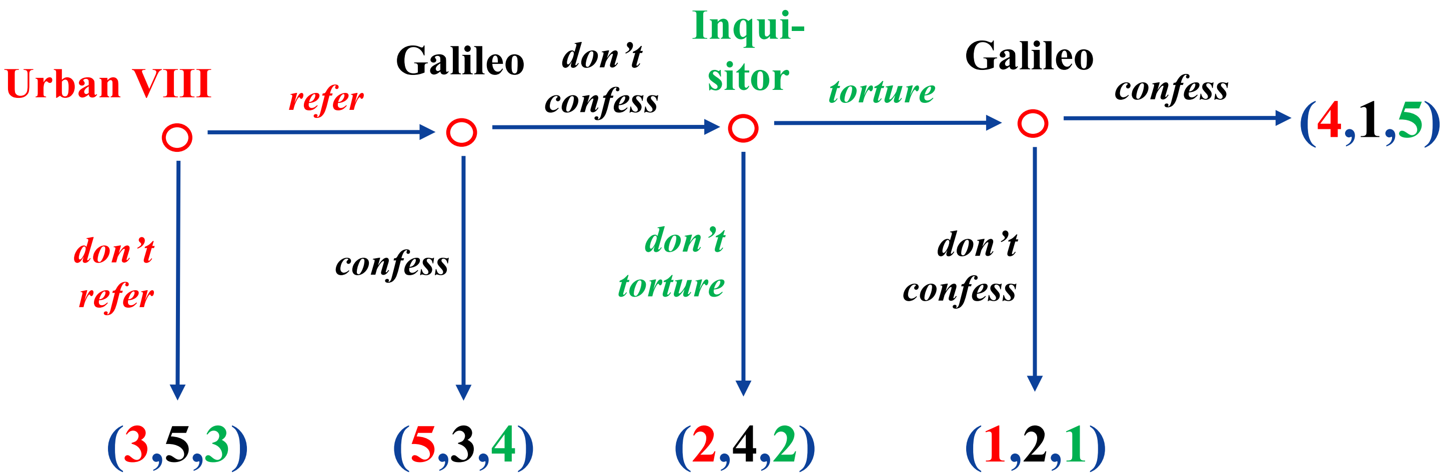
- 1610: Galileo published his *Sidereus Nuncius* in which he promotes the heliocentric theory of Copernicus and, thereby, came into conflict with the Catholic Church
- 1616: The Roman Inquisition declares heliocentrism to be heretical, bans Copernican books, and prohibits Galileo from elaborating his heliocentric ideas
- 1632: Galileo, however, defends heliocentrism in his *Dialogue Concerning the Two Chief World Systems*

Case study: Galileo Galilei and the interrogation by the inquisition – the inquisition game

- Consider a setting with three actors, Urban VIII, Galileo, and the Inquisitor. Pope Urban initially has to decide whether to refer Galileo's case to the Inquisition. If he declines to do so, the affair is over. If he does refer the case, Galileo is brought to the Inquisition, at which time he must decide whether to confess that he did indeed support the model of Copernicus too strongly. If he confesses, then he is punished and the game is over. If he does not confess, then the Inquisitor decides whether to torture Galileo. If he decides not to torture him, Galileo has won. Otherwise, Galileo must decide whether to confess. Preferences over outcomes are as follows:



Case study: Galileo Galilei and the interrogation by the inquisition – Equilibrium and history



The actual history:

- April 12, 1633: Galileo was brought to trial after Pope Urban VIII refers Galileo to the Inquisition, Galileo confessed that he had gone too far in supporting the Copernican theory.
- June 22, 1633: Final hearing by the Inquisition after he was shown the instruments of torture. Galileo complied in every way, was convicted and sentenced to life imprisonment.

Case study: Memento and the lost recall

Film poster for Memento, retrieved from en.Wikipedia.org, copyright is believed to belong to Summit Entertainment

The previous plot:

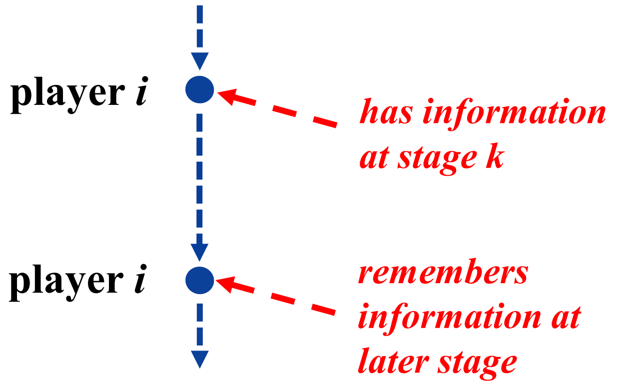
Leonard Shelby suffers from an inability to store recent memories due to psychological trauma associated with an attack by two men. They broke into his house, raped and strangled his wife who died. Leonard killed one of the attackers, but the second clubbed him and escaped. Since the police did not believe that there was a second attacker, Leonard conducts his own investigation.

Unable to create long-term memories, Leonard develops a system for recollection using hand-written notes, tattoos, and Polaroid photos. Whatever information is not written down will be forgotten.

Case study: Memento and the lost recall - Validity of the backwards induction equilibrium concept

Underlying assumptions:

- All players are rational
- Rationality and the game is common knowledge for all players
- Perfect recall by all players

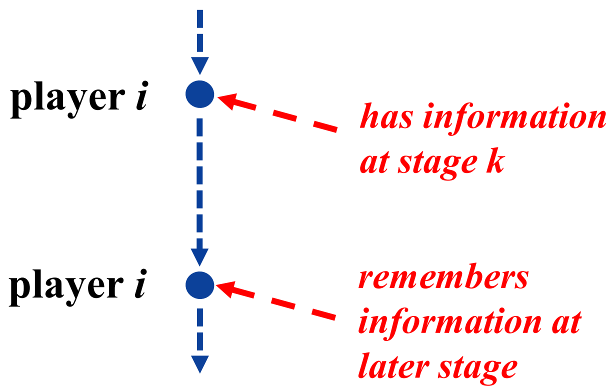


Is Leonard's imperfect recall plus memorizing identical to perfect recall?

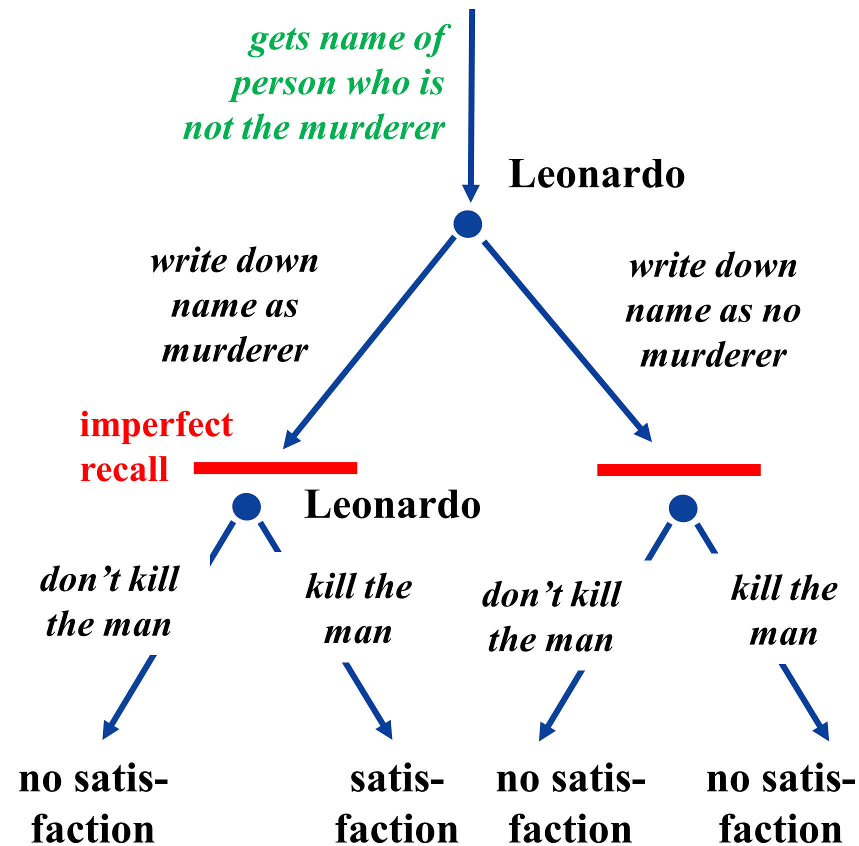
Case study: Memento and the lost recall - Validity of the backwards induction equilibrium concept

Underlying assumptions:

- All players are rational
- Rationality and the game is **common knowledge** for all players
- **Perfect recall** by all players



Using imperfect recall strategically:



Definition and classification of strategic moves

Definition: For a given game, a **strategic move** is a tactic of a player trying to change the other players' believes about her behavior in order to correct an otherwise adverse course of interaction to her advantage. A game in which strategic moves are possible is called **strategic-move game**.

Classification:

- Unconditional strategic moves:

  Taking the initiative – first-mover-advantage

- Active conditional strategic moves:

  Announcing behavior (promising and threatening) – fixing reactions

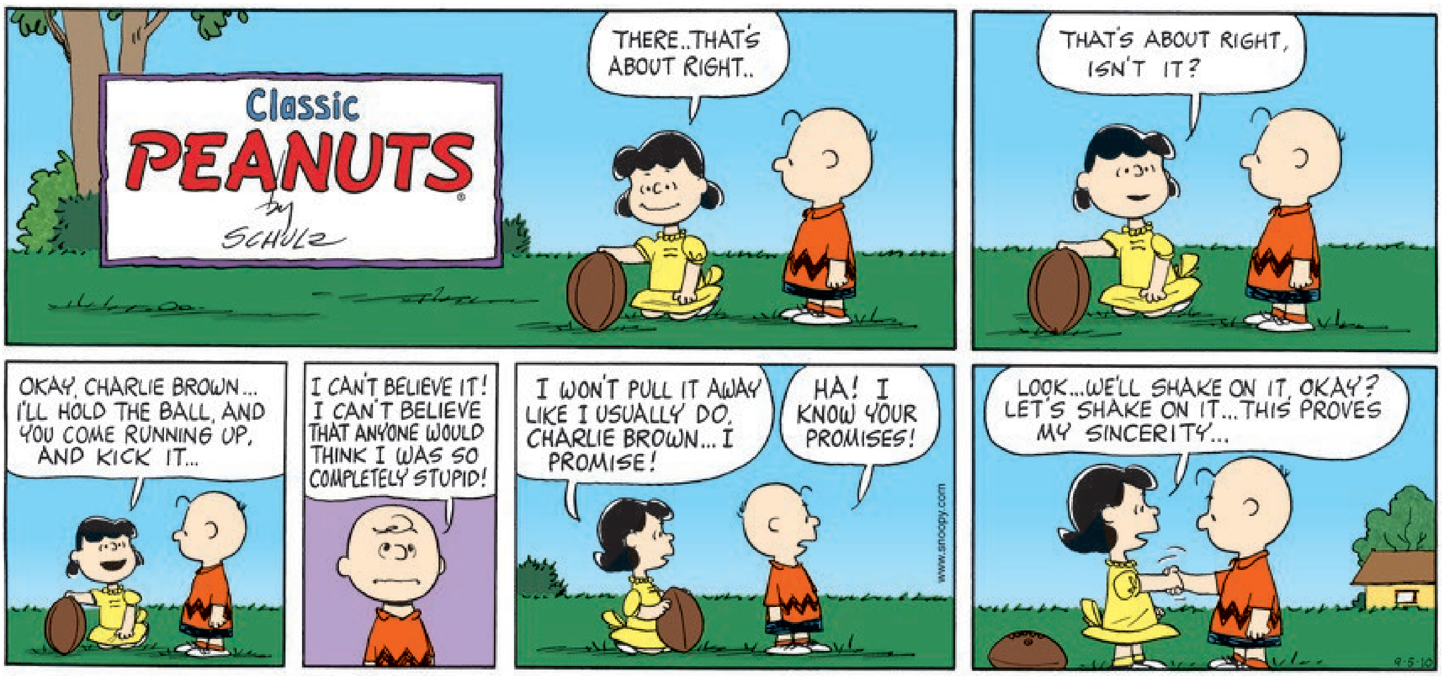
- Passive conditional strategic moves:

  Waiting for the other party to move – second-mover-advantage

Case study: Charlie Brown and his playing with Lucy – The previous history

Peanuts comic strip, retrieved from: www.peanuts.com, PEANUTS © 2005, United Feature Syndicate, Inc.

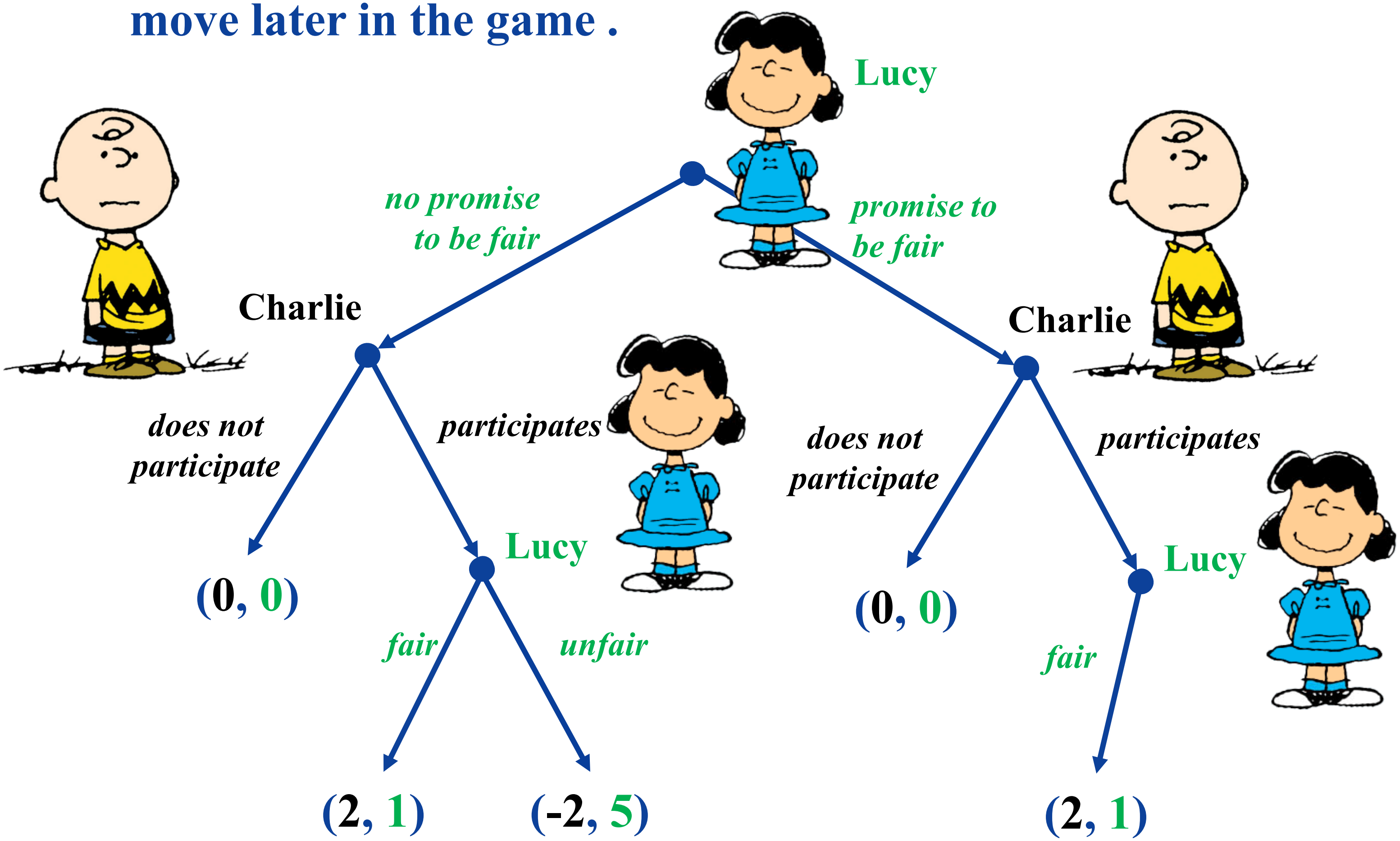
Case study: Charlie Brown and his playing with Lucy – change the game by using a strategic move



Peanuts comic strip, retrieved from: www.peanuts.com, PEANUTS © 2005, United Feature Syndicate, Inc.

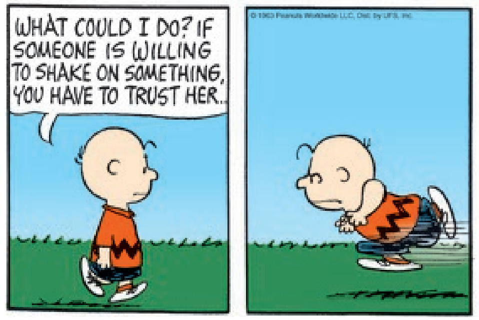
Case study: Charlie Brown and his playing with Lucy – the case of an irreversible strategic move

Definition: A strategic move is **irreversible** if that player cannot alter his move later in the game .



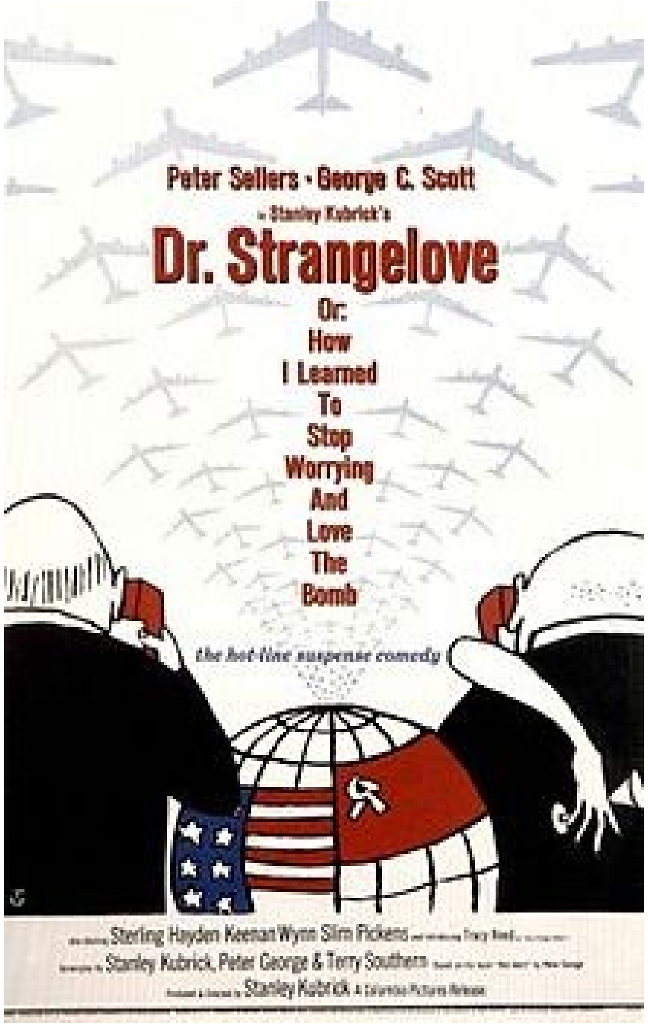
Case study: Charlie Brown and his playing with Lucy – the role of irreversibility

Is Lucy's promise really irreversible?



Peanuts comic strip, retrieved from: www.peanuts.com, PEANUTS © 2005, United Feature Syndicate, Inc.

Case study: Dr. Strangelove and the doomsday device



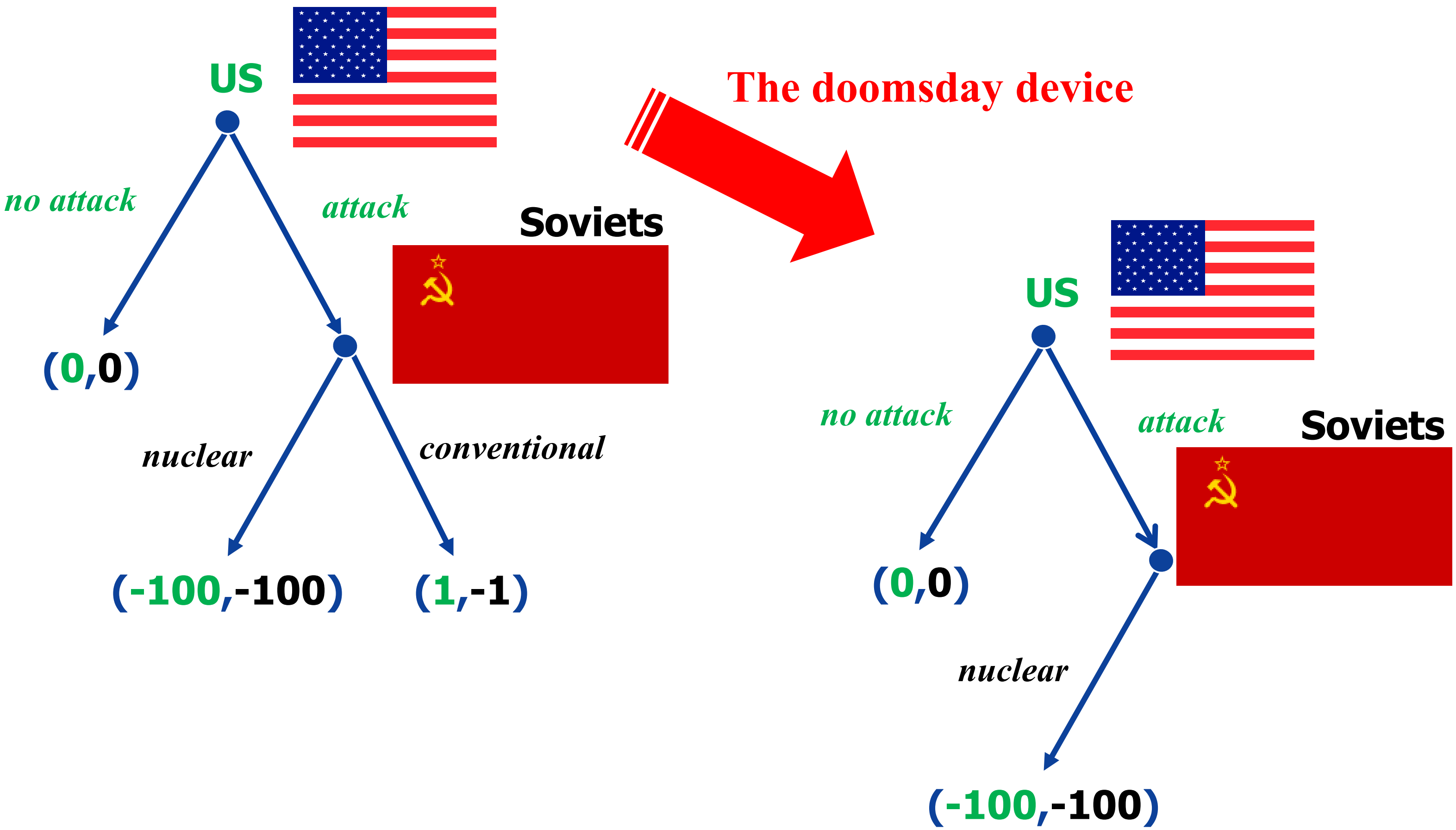
Film poster for Dr. Strangelove, retrieved from en.Wikipedia.org, copyright is believed to belong to Tomi Ungerer

The previous plot:

General Jack D. Ripper is commander of an US Air Force Base, which houses a bomb equipped bombers. The bombers are on airborne alert, in flight just hours from the Soviet border.

At the Pentagon's "War Room", General Turgidson briefs President Muffley and several other top officers including the scientific advisor Dr. Strangelove. He attempts to convince Muffley to let the attack continue, as their first strike on the Soviets would wipe out the majority of the Soviet missiles, and the few remaining would only cost a few million American lives. Muffley refuses, and instead brings Soviet ambassador Sadeski into the War Room.

Case study: Dr. Strangelove and the premier who loves surprises



Every strategic move requires credibility, that is, observability and irreversibility

Possibilities to create credibility of strategic moves:

- Signing a contract
- Building a reputation
- Building confidence in small steps
- Delegating one‘s reactions
- Relying on team pressure
- Making up-front investments
- Burning bridges
- Cutting off communication
- ……

Thinking Strategically - Theory of Games & Moves: What we learned today

- Put yourself in your rival's shoes in simultaneous move game situations; look for best responses and anticipate that your opponent will recognize them as well!
- Look ahead in sequential move game situation; think about how the following parties will behave and take their behavior into account when you choose your own strategy!
- Try to change your opponent's belief about your behavior, but avoid strategic moves by your opponents. Use your strategic move selectively: Sometimes it's beneficial to be first-mover, sometimes being second-mover is better. And make your strategic move credibly!

Further readings

- Jost, P.-J. & U. Weitzel, 2007. Strategic Conflict Management. Edward Elgar: Chapters 2.2.3, 3.2.1., 3.3.
- Dixit, A. & B. Nalebuff, 1993. Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life. Norton: Chapters 2, 3, 5.
- Gibbons, R., 1992, A Primer in Game Theory. Financial Times Prent.: Chapters 1, 2.
- Dixit, A. & S. Skeath, 1999.Games of Strategy. Norton: Chapters 3, 4, 9.
- Harrington, J., 2008. Games, Strategies and Decision Making. Worth Publishers: Chapters 4, 8.
- Schelling, T. C., 1990. The Strategy of Conflict. Harvard University Press: Chapter 5.