Optimal control with L^1 constraints

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Goals

During the last decade, PDE-constrained optimal control problems with sparsity promoting objectives such as the L^1 norm have received a great deal of attention. They model, for instance, the optimal placement of actuators and sensors as well as the energy-minimal steering of a system towards a desired state. This can, e.g., be used to design earthquake-resistant buildings or to control chemical reactions. Using this problem class it is, furthermore, possible to obtain controls that have a significantly simpler structure than controls for differentiable objective functions. In a practical application this could translate to an automated vehicle that takes passengers from A to B in a time-efficient manner while maintaining constant speed during long periods of the trip (as opposed to frequent de- and acceleration).

In contrast to sparsity promoting *objectives*, sparsity promoting *constraints* have received little attention so far. This project focuses on the investigation of optimal control problems with an L^1 norm constraint. Since semismooth Newton methods have been applied very successfully to optimal control problems with sparsity promoting objectives, we aim, in particular, at extending these methods to optimal control problems with sparsity promoting constraints. This will result in highly efficient algorithms to solve such problems.

Problem formulation and methods used

We investigate a generalization of the control problem

$$\min_{u\in L^2(\Omega)} \frac{1}{2}\left\|y(u)-y_d\right\|^2_{L^2(\Omega)}+\frac{\alpha}{2}\left\|u\right\|^2_{L^2(\Omega)} \quad \text{s.t.} \quad \|u\|_{L^1(\Omega_1)}\leq\gamma,\ a\leq u\leq b \text{ a.e. in } \Omega_2,$$

where $\Omega\subset\mathbb{R}^d$ is a bounded domain and $\Omega_1,\Omega_2\subset\Omega$ are open sets, the constants α and γ are positive, there holds $a,b\in L^2(\Omega_2)$, and $y=y(u)$ solves

$$\begin{cases} -\Delta y+y^3=u & \text{in } \Omega,\\ y=0 & \text{on } \partial\Omega. \end{cases}$$

Particularly interesting is the case where $\Omega_1 \cap \Omega_2$ has positive measure, e.g., $\Omega_1 = \Omega_2 = \Omega$. On $\Omega_1 \cap \Omega_2$ the control is constrained both pointwise and with respect to its distribution. In consequence, the Lagrange multipliers are not unique, which is challenging.

Optimal controls for this problem have certain sparsity properties that we characterize completely. Moreover, we provide a complete description of the Lagrange multipliers. Considering the Lagrange multiplier of the norm constraint as an implicit function of the control, we are able to show local superlinear convergence of the resulting semismooth Newton method in an infinite-dimensional setting by use of a new semismooth implicit function theorem. Numerical results confirm that the novel approach is very efficient.

Numerical results

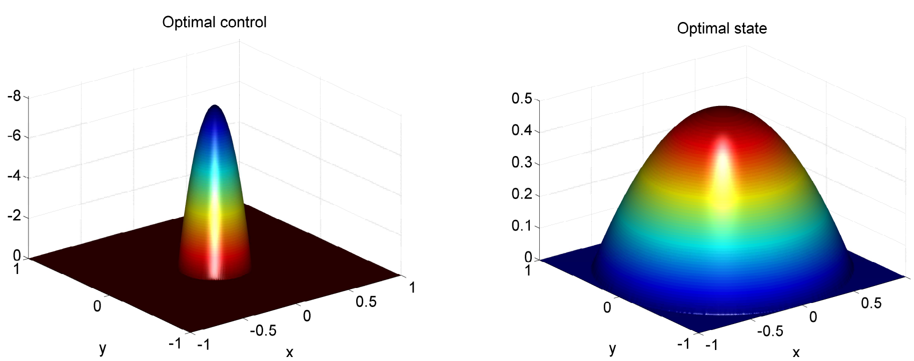

Figure 1: Solution of an L^1 constrained control problem on the unit disc