Feedback control for reaction diffusion systems

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Objectives

Many problems of practical interest often cannot be modeled exactly but rather include uncertainties in form of noise or process disturbances. In this case, feedback-based, or closed-loop, control laws often are more robust than open loop controls. The optimal feedback law is known to be given in terms of the gradient of the value function satisfying the Hamilton-Jacobi-Bellman (HJB) equation. Due to the curse of dimensionality, solving the HJB equation however is tractable only for very limited examples. The development of suboptimal feedback laws is the focus of this project. A particular application of interest will be the monodomain and bidomain equations arising in electric cardiophysiology.

Problem Formulation and Methods Used

We will study abstract Cauchy problems of the form

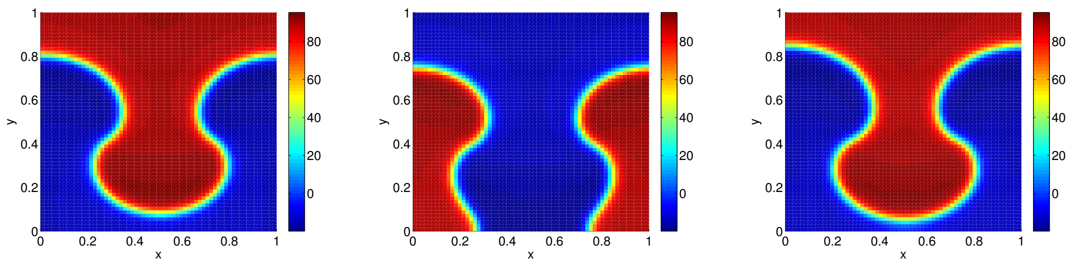
$$\dot{y}(t) = Ay(t) + F(y(t)) + Bu(t), \quad y(0) = y_0,$$

where $A : \mathcal{D}(A) \subset Y \to Y$ is the infinitesimal generator of a strongly continuous semigroup on the Hilbert space Y, B is a control operator and F is a nonlinear operator resulting from a non-monotone nonlinearity. Particularly relevant will be the case where B is an unbounded operator, typically arising for boundary control problems. We plan to investigate the applicability of linear feedback laws $u(t) = Ky(t)$ such that the closed-loop system

$$\dot{y}(t) = (A + BK)y(t) + F(y(t)), \quad y(0) = y_0,$$

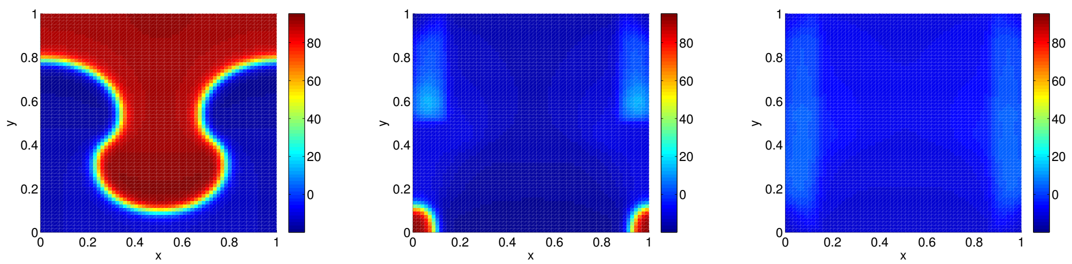
is locally stable around a given unstable stationary solution. This requires the investigation of controllability and stabilizability properties of the linearized system. For the specific needs of feedback and observer design we also investigate the scope of model reduction techniques.

Results



(a) $t = 0$. (b) $t = 40$. (c) $t = 80$.

Figure 1: Uncontrolled reentry wave



(a) $t = 0$. (b) $t = 40$. (c) $t = 80$.

Figure 2: Feedback-controlled reentry wave