Andreas Rainer

Diffusion Processes in Economic Systems

Doctoral Thesis

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1st supervisor: Prof. Dr. Heinz D. Kurz

Institute of Economics
University of Graz

2nd supervisor: Prof. Dipl. Ing. Dr. Klemens Fellner

Institute for Mathematics and Scientific Computing
University of Graz

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Contents

Motivation

Looking at economic growth patterns, one can identify a *take-off* of per-capita gross domestic product at the end of the 18^{th} and the beginning of the 19th century, a time which is labeled *industrial revolution* (Acemoglu, 2008, Ch. 1). The term 'industrial revolution' stems from the observation of a number of important inventions at that time, which fostered the subsequent development of technological knowledge and its application. These activities, including the respective institutional environment, are pivotal in explaining economic growth patterns of modern civilization (see for example Mokyr, 2005). Hence, technical change seems to be one of the driving forces of economic growth.

The relationship between technical change and economic growth was studied by economists since Adam Smith's (1723–1790) book *An Inquiry into the Nature and Causes of the Wealth of Nation* (Smith, 1776). It was David Ricardo (1772–1823) in his book *On the Principles of Political Economy and Taxation* (1821), who made the *choice-of-technique* problem explicit, asking for the economic and social causes and effects concerning the use of different modes of production. His thinking was concerned with the *long-period position* of an economy. Pierro Sraffa (1898–1983) made an attempt to formalize the long-period approach in his book *Production of Commodities by Means of Commodities* (1960); it was further elaborated by Kurz & Salvadori (1995).

Evolutionary economics (Dopfer, 2005) provides a complementary approach, dealing with short-run effects of technical change. This strand of economic literature to a considerable part relies on the analysis of Joseph A. Schumpeter (1883–1950). Schumpeter was concerned with the economic effects of innovations, and Nelson & Winter (1982) introduced the respective modeling approaches. As a consequence of the *bounded rationality* assumption of evolutionary economics, the time-lag between the emergence of some innovation and its adoption throughout the system is explained. On empirical grounds and from a sociological point of view, this effect is investigated by *diffusion research* (Rogers, 2003). There the importance of the institutional setting influencing the emergence of innovations as well as the pace of the diffusion of some innovation is studied. Diffusion research also acknowledges and studies the social consequences of this process, since economic dynamics triggered by technical change necessarily induce structural change within a social system.

My doctoral thesis adds to the existing literature on economic causes and consequences of technical change by introducing a mathematical framework to study intra- and inter-sectoral feedback effects of the diffusion of *innovations.* To focus on the diffusion process in isolation, the generation and the introduction of innovations are excluded from the analysis. The modeling framework connects the concept of a *long-period position* (Kurz & Salvadori, 1995) with evolutionary ideas provided by the concept of *replicator dynamics* as utilized by *evolutionary game theory* (Weibull, 1997). The stated problem is closely related to Schumpeter's (2012 [1934]) dynamic approach to economic development and is accomplished by a fusion of classical and evolutionary thinking as suggested by Kurz (2008).

Survey and results of Part I

Part I of my doctoral thesis provides a thorough discussion of the outlined multi-sector model of economic diffusion. As a preliminary chapter to the model itself, in the Introduction of Part I an overview is given of what diffusion research adds to our understanding of diffusion processes in economic systems. It provides the empirical starting point. In Chapter 1 the basic setting is outlined. There, a transition path towards a new LPP determined by the set of available production processes is studied for a multi-sector economy. Core assumptions are an exogenously given set of Leontief processes; prices are determined by average unit costs of production; and firms invest a given share of their overall rate of profits into output growth. Hence, processes earning an above normal rate of profit can gain in market share relative to less profitable technologies.

Besides the formal properties concerned with existence and stability of diffusion paths, in a two-sector economy possible spillover effects of technical change are demonstrated. Beginning with a single sector, a product innovation opens up a new sector, which influences the production possibilities of the first sector. The order of profitability of different processes is shown to depend on at least two effects: (1) The changing wage rate induces changes of unit costs according to the amount of labor used as input factor of production. In certain circumstances this leads to a change of the order of profitability of processes. (2) Technical change in one sector can also change the order of profitability in some related sector by means of changing unit costs of production, hence by changing relative prices of capital input factors.

This instance of inter-sectoral feedback effects is applied in Chapter 2 to the case of *General Purpose Technologies*. There, the following stylized facts are reconstructed: (1) The emergence of a new GPT sector (by a product innovation) and technical change in a GPT sector (by process innovations) induce changing productivity in related sectors. This includes negative output growth after the emergence of a new GPT, if technical change is capital intensive. (2) By assuming a higher skill level necessary for processes which are related to the GPT sector, transitional wage inequality is demonstrated. (3) The S-shaped diffusion pattern, which prevails for successful innovations, is endogenized, and feedback effects between output growth and prices (respectively wages) are studied.

Next, in Chapter 3 some extensions of the model are provided for the special case of a one-sector economy: (1) Fixed capital is added and (2) some time lag of the price adjustment mechanism resulting from finite periods of production is introduced. Fixed capital leads to a lagged diffusion due to the need for investment into capacity, whereas lagged price adjustment increases the speed of diffusion due to the extended time of larger extra profits for the innovators.

Finally, in Chapter 4 – again for the case of a single sector economy – an in-depth discussion of the two pivotal mechanisms of innovation diffusion is provided. Firm decisions concerning investment and technology provide the basis for two possible mechanisms, which determine the rate and direction of technical change: (1) Differential growth relies on autonomous investment decisions of firms. Because the profitability of inferior methods of production might turn negative, an asymmetry between firm growth and decline arises. Whereas growth is the outcome of a purposeful firm decision, decline is a consequence of producing below the normal conditions of production. (2) Imitation relies on direct interaction between firms. To explore the relation between differential growth and imitation, a simple imitation mechanism is proposed relying on contagion and a comparison of profitability determining adoption probability. One result is the potentially negative long-term impact of aggregate growth due to the uneven growth path during the diffusion process of a new method of production.

Survey and results of Part II

Since diffusion processes are complex phenomena, illustrated by simplifying mathematical models in Part I, I added a discussion of methodological issues of economic theorizing in Part II of my doctoral thesis. In the Introduction of this second part I pose the question whether economic modeling is *science* or *art* to highlight the similarities of these apparently incommensurable aspects of our modern civilization. This provides the starting point for a more complete discussion on the reason and value of mathematics in the realm of economic theorizing in Chapter 5: Based on the observation that in the late 19th century the *Methodenstreit* between the German Historical School and the Austrian School was relevant for economists who thought about the foundations of economics, the writings of Joseph A. Schumpeter (1883–1950) concerning this topic are critically discussed. Schumpeter in his first years as an economist wanted to clarify the fundamental principles as well as the scope of pure economics. Two goals were at least approximately met by his efforts. First, an idea of what pure economics is and what it can accomplish was outlined to strengthen the position of the deductively arguing theoretical economists. Secondly, at the same time he argued in favor of methodological pluralism in the field of economic research and the importance of empirical economics by embedding pure economics into themore general science of economics. Pivotal for Schumpeter in discussing pure economics was the utilization of mathematics and logical rigor. Three arguments were put forth to motivate the introduction of mathematical symbolism in economics: the 'quantity', the 'structure', and the 'complexity' argument. The first argument is related to the observation that economic activities are concerned with quantitative concepts (prices and quantities of commodities), whereas the second argument is concerned with the structure of economic systems.

Calling attention to the limitations of pure economic theory for solving practical problems of economic policy is of fundamental importance to Schumpeter. It is a consequence of both the discussions concerning mathematical methods as well as his plea for methodological pluralism. This to a great extent is a consequence of the static property of pure economics. Schumpeter's statements that static and dynamic economics are something completely different and that pure economics cannot readily be applied to practical economic problems are interconnected by acknowledging that the real economy is not in equilibrium and one simply cannot know exactly what will happen next.

Two topics, which were approached in Chapter 5, are further elaborated in two follow-up chapters. First, in Chapters 6 I discuss the dichotomy between static and dynamic economic theorizing, exemplified by the different views of Schumpeter and Thorstein B. Veblen (1857–1929) on this topic. This methodological discussion is important to evaluate the modeling approaches of Part I: In its contemporary meaning, a theory is called 'dynamic' as soon as it is represented by time dependent variables. Schumpeter as well as Veblen offered a view on economic dynamics, which goes beyond what contemporary economists regard as dynamic. Both held sophisticated views on the interpretation of 'dynamics' and its suitability for uncovering dynamic phenomena. In their dynamic economic theorizing, Veblen relied on anthropology, psychology and biology, whereas Schumpeter's interest was closer to the related sciences of sociology and history. Another demarcation line between these two economists can be found with respect to the appraisal of pure economics.For Schumpeter, pure static economic theorizing provides the basis on which some evolutionary theory can build on, whereas Veblen regarded it as an obstacle. Despite these differences, their basic understandings of dynamic economic systems show striking similarities, especially concerning their notion of change arising from within the system.

This complexity of evolutionary economic systems leads over to the last Chapter 7, where the claim of mathematical economics to provide a scientific foundation for economic policy advice is challenged. It is argued that economics is not capable of properly dealing with data in the sense that reliable forecasts are possible. For policy decision makers and analysts, this has at least two implications. Firstly, extrapolations of economic data-sets for the sake of forecasting the consequences of some specific policy action cannot be conducted with the necessary accuracy.

The capability of economic theories to *predict* the net outcome of some policy action is at best disputable, since fundamental uncertainty in the sense of Frank H. Knight (1885–1972) prevails (Knight, 1921). Secondly, economic theory influences economic policy, and indirectly therefore influences the dynamics of the system. The results may be intended or not, they may be welcomed or not, but the policy action per se cannot be evaluated since the outcomes of possible alternative choices or of some business-as-usual policy (the default-option) are not known.

Part I

Modeling diffusion of innovations

Introduction: Economic diffusion

This introduction of Part I is based on the discussion paper Rainer & Schütz (2012), which was presented at the ESHET conference in St. Petersburg 2012 with a talk on *Diffusion Research and Economics*.

The investigation of diffusion processes in economic systems is one instance of dealing with the complexity of dynamically evolving economic systems. Two concepts are pivotal to understand diffusion processes: the *innovation - development* process and the *innovation-decision* process. The former topic is concerned with the development of new products and new processes as a result of chance or R&D activities, whereas the latter looks at the diffusion of already launched innovations by studying the individual choice process of economic agents. Rogers (2003, Chapter 4) identifies six stages of the innovation-development process: basic research, applied research, development (to "meet the needs of [...] potential adopters", Rogers, 2003, p. 146), commercialization, diffusion and consequences. This process is characterized by fundamental uncertainty about the outcome of the respective activities. If an invention enters the system it is in competition with incumbent products and processes, and economic agents face a decision whether to adopt the innovation or stick to the incumbent product respectively process. The economic agent hence is a possible *adopter* of the innovation who perambulates a five-step innovation-decision process: (1) gaining knowledge about an innovation; (2) persuasion of applying the innovation; (3) the decision is reached and finally (4) the innovation is implemented. The last step of (5) confirmation of the usefulness of the innovation then possibly leads to a revision of the original decision.

But also social and institutional factors matter. Preceding the decision process itself (stages (2-3) of the innovation-decision process) it is necessary to have *awareness-knowledge*. Knowledge diffuses through the system by means of communication channels, the most important being mass media on the one hand and interpersonal communication on the other hand. Knowledge is a precondition of later adoption of some new technology, hence the communication infrastructure and cultural habits concerning the use of mass media and the existence of communication networks are pivotal. Concerning interpersonal communication networks, at least two aspects are of importance. Firstly, there is a bias concerning the information which is accepted (for example, if some new technology deviates too much from established habits). It is also common practice to be part of communication networks with members who are alike, with similar knowledge and similar interests. This is what Rogers (2003, p.306) calls *homophily*, defined as the similarity of two communication partners, hindering the entrance of some innovations. *Heterophily* on the other hand, defined as the difference between two persons concerning certain communication-related attributes such as competence, status, and language or beliefs, encourages the appearance of new information. Even if seldom to be found within established communication networks, these links exist between separated communication networks. They are a driving force, triggering the spread of information as outlined by Mark Granovetter's (1973) *strength-of-weak-ties* theory (see also Rogers, 2003, Chapter 8).

Awareness-knowledge is not enough to adopt some new technology. Also steps (2) and (3) of the innovation-decision process, namely persuasion and decision, have to be taken into account. These steps depend on several economic and social factors influencing the individual agent as discussed throughout the chapters of the firs part of my dissertation. A successful innovation can be defined as an innovation which is adopted by almost all members of the observed system after some reasonable time. What can be observed in case of successful innovations is an S-shaped diffusion pattern. Historically, the article of Ryan and Gross (1943) on the diffusion of hybrid seed corn in Iowa is credited to be the point of departure of modern diffusion research investigating this specific pattern, its underlying forces and different characteristics (even if preliminary efforts existed, see Rogers, 2003, Chapter 2). Rogers explains the diffusion process by identifying five adopter categories (defined on statistical grounds): The *innovator*, the *early adopter*, the *early majority*, the *late majority* and the *laggards*. Special personal characteristics are attributed to each adopter category: Innovators are venturesome and additionally not too much rooted in the respective social system; hence, they are free to deviate from traditional thinking and norms. An important role is assigned to the early adopters, who are identified as the most respected persons within the community. They have on the one hand the capability (both financially and intellectually) to adopt the innovation at an early stage of the diffusion process and similarly they have the authority to convince others of the profitability of the technology; they are opinion leaders of the social system and the pace-makers of the diffusion process. The early majority includes all further adopters up to fifty percent of the total system, characterized by a deliberate decision for the utilization of the new technology, since they are able to judge the success of the innovators and of the early adopters. The late majority is to some extent forced to adopt as a result of social and economic pressure. Finally, laggards as the fifth adopter category are on the lower end of the social ladder, either for financial or for cultural reasons, with a long delay before they adopt the innovation.

Especially in an early stage of the process, *creative responses* to the emerging innovation prevail: "[W]henever [...] some firms in an industry do something [...] that is outside of the range of existing practice, we may speak of *creative response*." (Schumpeter, 1947, p. 150) Creative response implies non-predictability of the diffusion process of some innovation, it therefore "can always be understood *ex post*; but it can practically never be understood *ex ante*; [...] creative response shapes the whole course of subsequent events and their 'long-run' outcome." (Schumpeter, 1947, p. 150) Innovators and Early adopters face higher uncertainty and show creative response by adapting the innovation to their personal situation. The late majority as well as the laggards are closer to what is labeled *adaptive response* by Schumpeter: "Whenever an economy or a sector of an economy adapts itself to a change in its data in the way that traditional theory describes, [...] we may speak of the development as adaptive response." (Schumpeter, 1947, p. 150) And in the later stage of the diffusion process an innovation is no longer an innovation: "It is the process of diffusion and imitation of a new method of production, enforced by competition, that renders what at first was a purely private good a public one." (Kurz, 2008, p. 265) Hence, a new mode of production gets adopted, which is already well established within the system. What once was an innovation gets common knowledge, and economic, social or other normative forces make the late adopters to change their behavior.

Knowing some of the fundamental driving forces of economic diffusion processes of innovations, in this part of my doctoral thesis a mathematical framework is introduced to describe the diffusion of cost-saving innovation by means of growth and imitation. In Chapter 1 a multi-sector economy is introduced. Using a two-sector example, inter-sectoral spillover-effects are discussed. These are further elaborated and applied to the case of General Purpose Technologies in Chapter 2. The core model solely deals with labor and circulating capital as input factors of production. Extensions of the model by including time delays due to longer periods of production and by including fixed capital is provided for the single-sector economy in Chapter 3. Finally, in Chapter 4 time-continuous model of Chapters 1–3 is transformed into a time-discrete model with one sector and two processes to highlight the influence of different kinds and different degrees of technical change.

Chapter 1

A multi-sector model of technical change

This chapter is a re-print of the core-article of my doctoral thesis: *Formal properties of a multi-sector replicator model of technical change* (Rainer, 2014). It was submitted to the journal *Structural Change and Economic Dynamics* and is currently under review. Talks on draft versions of this paper were given at the junior fellow workshop *New Directions in Economic Analysis* of the AK Wien with the title *An Evolutionary Multisector Model to evaluate Monetary and Legal Policy* and at the EAEPE conference 2012 in Krakow with the title *Technical change in a combined Classical - Evolutionary multi-sector economy: Causes, Effects and implications for economic and social policy.*

The diffusion of process-innovations in a multi-sector economy is formalized as a replicator dynamics system. Within the framework of classical economics, this gives the transition path of an economic system towards its long-period position. Prices are determined by average costs of production, and the uniform normal rate of profit is constant over time. Profitability differences, characterized by extra profits above or below the normal rate of profit, trigger the diffusion dynamics. A stability analysis of possible equilibria is carried out and basic properties of the diffusion path are derived. In a two-sector example, inter-sectoral spillover effects are demonstrated: the productivity of some process changes as a consequence of innovative activity in a related sector.

1.1 Introduction

Technical change in economic systems comprises (1) the invention of new products and processes, (2) their introduction into the system, and (3) their diffusion through the system. The literature on these topics deals either with one of these three aspects or with their mutual interdependence. This paper adds to the theoretical literature solely concerned with the diffusion of innovative processes. Point of departure is the long-period position (LPP) of a multi-sector economy (Sraffa, 1960; Kurz & Salvadori, 1995), defined as "a position of rest, given the data of the problem, including the level of the rate of profit." (Kurz & Salvadori, 1995, p. 75) A *technology*, which is the set of utilized processes, defines some downward sloping wage-profit curve. An LPP prevails, if the system settles on the outer envelope of these curves (Kurz & Salvadori, 1995, Ch. 5.5), since then "no producer can obtain a higher rate of profit by operating another process" (Kurz & Salvadori, 1995, p. 74). Adding new processes leads to new technologies, which are more profitable, if they shift the wage-profit frontier in the relevant range outwards. Then some dynamic process towards a new LPP is triggered as a result of "diffusion and imitation [...], enforced by competition, that renders what at first was a purely private good a public one" (Kurz, 2008, p. 265). Competition between firms using different methods of production as well as price competition of firms using the same process therefore lead to an LPP with all firms using the innovation, the incumbent process being superseded.

Keeping technical coefficients fixed, the model describes a *pure selection process* in the tradition of Nelson & Winter (1982, Ch. 10). There, technologies are embodied in differing kinds of capital, with relative input prices determining the profitability of some process. This approach is extended by (1) endogenizing relative prices in a (2) multi-sector setting. For a constant normal rate of profit – which reflects the rough stylized fact that the general rate of profit is a trendless magnitude – the dynamics of the wage rate influences extra profits, respectively losses, generated by some specific method of production. In a single sector economy, in a similar setting Steedman & Metcalfe (2011) take the wage rate to be set by the marginal firm, which hence earns normal profits. Innovative firms thus reap extra profits during the whole diffusion process, which only comes to a halt if limited resources are assumed. In the model of Steedman & Metcalfe (2011) at the end of the diffusion process the wage rate exhibits a jump, which results in a normal rate of profits equal to the one which prevailed before the innovation entered the system. In the presented model, this discontinuity of the wage rate dynamics is avoided by continuously adjusting prices in the course of the diffusion process with recourse to the average technology. The latter changes over time, asymptotically converging towards the technology of the new LPP.

In this chapter I investigate the formal properties of the diffusion process of existing production processes in a multi-sector economy, including local and global stability analyses of equilibria. Fixing the set of available technologies in advance, the model abstracts from endogenous technical change. In Section 1.2 the model is introduced, originating from the framework outlined by Kurz & Salvadori (1995). A replicator dynamics system is derived to simulate the transition towards an LPP, if the system is not in equilibrium. Next, in Section 1.3 properties of the single-sector economy are studied. This paves the way to the analysis of the multi-sector economy, for which formal properties are derived in Section 1.4. Spillover effects between sectors as a consequence of technical change are exemplified in Section 1.5. Finally, Section 1.6 concludes. Mathematical proofs can be found in the Appendices A and B of this chapter.

1.2 Multi-sector replicator dynamics

The model to study diffusion processes of competing technologies in a multi-sector economy is presented in two steps. In Subsection 1.2.1, a formalization of the LPP is introduced. This prepares for the derivation of the replicator system in Subsection 1.2.2.

1.2.1 The multi-sector economy

In an N sector-economy, sector $n \in [1, N]$ is defined by the homogeneous good it produces. The semi-positive vector $\mathbf{a}_n = (a_{n1}, \ldots, a_{nN})^T$ of circulating capital and labor input $l_n > 0$ are non-substitutable input factors of unit production of good n. Hence, the production function is of the Leontief type. Constant returns to scale are assumed to hold, and therefore the technical coefficients are constants. a_{nm} denotes the input of good m necessary to produce one unit of good n. The $N \times N$ matrix A with coefficients a_{nm} and the input vector $\mathbf{l} = (l_1, \ldots, l_N)^T$ of homogeneous labor characterize the current technology. x_n denotes the quantity produced in sector n, defining the output vector $\mathbf{x} = (x_1, \ldots, x_N)^T$.

The normal rate of profit r is taken to be exogenously given and to be the same across all sectors (Kurz & Salvadori, 1995, Chapter 1.2). This is a consequence of the assumption of free competition, allowing for unrestricted entry and exit of firms. With prices equaling unit costs of production, the price vector $\mathbf{p} = (p_1, \ldots, p_N)^T$ is determined by the N-dimensional linear system

$$\mathbf{p} = (1 + r)A\mathbf{p} + w\mathbf{l}. \tag{1.1}$$

Introducing an N-dimensional numéraire commodity basket $\mathbf{d}$ and thus determining a price index by means of $\mathbf{d}^T\mathbf{p} = 1$, the wage rate w can be calculated from expression (1.1):

$$w = \frac{1}{\mathbf{d}^T \left(\mathbb{I} - (1 + r)A\right)^{-1} \mathbf{l}} \tag{1.2}$$

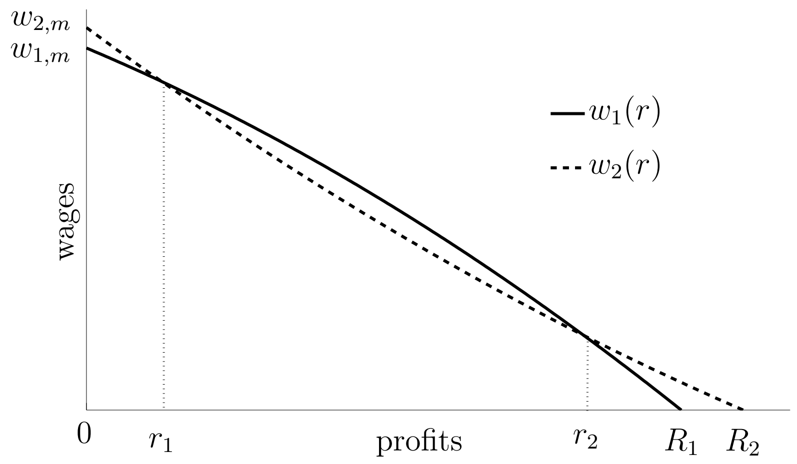

Figure 1.1: $w - r$ relationships with re-switching

This defines the downward sloping $w - r$ relationship for a given technology. Changing technical coefficients A and/or $\mathbf{l}$ lead to a shift of this curve. Since the normal rate of profit r is taken to be a non-negative constant, expression (1.2) can be used to calculate w for a given technology. Positivity of wages indicates that r is bounded from above by the maximum rate of profit R to be calculated for $w = 0$ in equation (1.2). Hence $R = (1 - \lambda)/\lambda$, with λ denoting the dominant eigenvalue of A, and its eigenvector $\mathbf{p}$ determining prices (Kurz & Salvadori, 1995, p. 98).

For illustrative purposes, in Figure 1.1 the $w - r$ relationships $w_i(r)$ for $i = 1, 2$ with technical coefficients

$$A_1 = \begin{bmatrix} 0.05 & 0.3 \\ 0.2 & 0.1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$

and $\mathbf{l}_i = (1, 1)^T$ are depicted. R_i and $w_{i,m}$ denote the maximum rates of profit and the maximum wage rate of technology i. In this example, *re-switching* emerges: The intersections r_1 and r_2 are those $w - r$ relations, for which both technologies are equally profitable. Hence, there is a range of normal profits $[r_1, r_2]$ for which technology 1 is cost minimizing, whereas for $r \leq r_1$ and $r \geq r_2$ technology 2 is cost minimizing.

Those sectors, which produce goods supplying the production of each other sector either directly or indirectly, are *basic sectors*. Its number gives a maximum of switch points. This holds due to the fundamental theorem of algebra, since equation (1.2) defines a polynomial in r at most of order N.

1.2.2 Multiple technologies

The implicit assumption of the preceding section was that each sector has one process at its disposal. In a further step, in case of more than one process being available for producers, the technical coefficients $\mathbf{a}_n$ and l_n can be looked upon as being aggregates of input factors of various processes: In sector n, let a number I_n of production processes be employed. Process $i_n \in [1, I_n]$ of this sector is characterized by some capital input vector $\mathbf{a}_n^{i_n}$ and labor input $l_n^{i_n}$. $x_n^{i_n}$ is the quantity of good n produced by process i_n. Total output of sector n thus adds up to $x_n = \sum_{i_n=1}^{I_n} x_n^{i_n}$. Defining the share $q_n^{i_n} = x_n^{i_n}/x_n$ of output of sector n produced by process i_n, the sectoral input coefficients

$$\mathbf{a}_n = \sum_{i_n=1}^{I_n} q_n^{i_n} \mathbf{a}_n^{i_n} \qquad\qquad l_n = \sum_{i_n=1}^{I_n} q_n^{i_n} l_n^{i_n} \tag{1.3}$$

give the *average technology* of sector n.

Let $g_n^{i_n}$ be the growth rate of $x_n^{i_n}$ and therefore $\dot{x}_n^{i_n} = g_n^{i_n} x_n^{i_n}$. The growth rate g_n of sector n coincides with average growth:

$$g_n = \frac{\dot{x}_n}{x_n} = \sum_{i_n=1}^{I_n} \frac{\dot{x}_n^{i_n}}{x_n} = \sum_{i_n=1}^{I_n} \frac{\dot{x}_n^{i_n}}{x_n^{i_n}} \frac{x_n^{i_n}}{x_n} = \sum_{i_n=1}^{I_n} q_n^{i_n} g_n^{i_n}$$

Differentiation of $x_n q_n^{i_n} = x_n^{i_n}$ with respect to time then leads to a system of differential equations, determining the time path of market shares of different technologies:

$$\dot{q}_n^{i_n} = q_n^{i_n} \left(g_n^{i_n} - g_n \right) = q_n^{i_n} \sum_{j_n=1}^{I_n} q_n^{j_n} \left(g_n^{i_n} - g_n^{j_n} \right) \tag{1.4}$$

This *replicator dynamics system* determines the evolution of the market share $q_n^{i_n}$ of process i_n. The potential of some firm to grow is given by the rate of extra profit $\rho_n^{i_n}$ of process i_n hosted by this firm. Extra profits are determined by unit costs of production $c_n^{i_n} = (1+r)\mathbf{p}^T \mathbf{a}_n^{i_n} + w l_n^{i_n}$, leading to

$$\left(1 + r + \rho_n^{i_n}\right) \mathbf{p}^T \mathbf{a}_n^{i_n} + w l_n^{i_n} = p_n. \tag{1.5}$$

Depending on the price setting rule of the model, the rate of extra profit $\rho_n^{i_n}$ can be calculated from equation (1.5). Sticking to the price equation (1.1) and due to definition (1.3) of average processes, prices $\mathbf{p}$ are equal to average unit production costs:

$$p_n = \sum_{i_n=1}^{I_n} q_n^{i_n} c_n^{i_n} \tag{1.6}$$

It follows from the analysis of Sections 1.3-1.5 that this implies a smooth transition path of prices. Two possible alternatives of how prices are determined are minimum prices or taking the production costs of the marginal firm. This would lead to price discontinuities, contradicting empirical evidence provided for instance by Stoneman (2002, Ch. 2.6). The average pricing rule also overcomes the problem of the marginal firm approach taken by Steedman & Metcalfe (2011), where the rate of extra profit of innovative firms would never vanish. In the present setting of unconstrained growth possibilities, this is a consequence of the asymptotic behavior of the system: Any technology which is once utilized will be part of the production process ad infinitum, even if its market share for the limit $t \to \infty$ converges towards zero. This leads to the situation that the marginal firm (the firm exhibiting the highest costs) will always determine the price by its costs of production. Innovative firms, using cheaper processes, would be able to reap strictly positive extra profits for all times. This contradicts the claim that these extra profits tend towards zero in the course of the diffusion process (Schumpeter, 2012 [1934], p. 131–132). For the case of two sectors, the respective adjustment process is described by Kurz (2008, p. 269–270) as follows:

> [P]rofit-seeking agents from the same or from the other industry will invest in the new method. This involves a change in output proportions, with the commodity in whose production the innovation has taken place becoming relatively abundant. As a consequence, its price relative to that of the other commodity can be expected to fall. In classical political economy the adjustment process under consideration is referred to as the 'gravitation' of 'market' prices to their (new) 'natural' levels. On the premise that the producers of the other commodity do not benefit from the change in prices, the wage rate increases [...], the rent of the innovators will gradually diminish and the producers in the industry, in which the innovation has taken place, who are not imitating will incur losses.

As another heuristic, why prices can be approximated by average costs, consider the case of output growth of some process, resulting from imitation in case of one incumbent and one innovative process. With c_n^1 and c_n^2 denoting the unit costs of production of the incumbent and of the innovative process, an innovative firm will set some price $p_n \in (c_n^1, c_n^2)$; the incumbent firms, which produce at higher costs, are price takers. Then a rising market share q_n^2 of the cheaper process increases the competitive pressure within the set of innovative firms. For small q_n^2, the price ought to be in the neighborhood of c_n^1, since the innovator wants to sell as expensive as possible. The price is therefore approximately defined by the marginal firm. For large q_n^2, the minimum cost principle ought to apply, since competition drives p_n down towards c_n^2. One convenient way to formalize these assumptions is the average-cost price setting rule $p_n = q_n^1 c_n^1 + q_n^2 c_n^2$, which for the case of multiple processes is given by equation (1.6).

1.3 Diffusion of innovations in one sector

Studying the special case of one sector, the diffusion of one and of two innovations is covered in Subsection 1.3.1. General properties of the single sector economy with an arbitrary number of processes are discussed in Subsection 1.3.2.

1.3.1 Two competing processes

Two competing processes are characterized by input coefficients a_i and l_i for $i = 1, 2$. Following (1.5), the rate of extra profit ρ_i of process i is implicitly determined by

$$(1 + r + \rho_i)a_i + wl_i = 1. \tag{1.7}$$

Given capitalist's propensity $s \in [0, 1]$ to accumulate, that is, to save and invest, growth rates are $g_i = s(r + \rho_i)$. With q denoting the market share of the second (innovative) technology, the replicator system (1.4) can be reduced to

$$\dot{q} = s\, q\, (1 - q)\, (\rho_2 - \rho_1)\, . \tag{1.8}$$

This differential equation is analytically solved in Appendix A. Additionally, a monotonicity property of the diffusion path for the case of two processes can be derived (see Appendix B):

Proposition 1. *For one sector with two distinct processes, the market share of each process is a monotonic function of time. It is constant over time if and only if either* $q(0) \in \{0, 1\}$*, or if*

$$1 + r = \frac{l_1 - l_2}{l_1 a_2 - a_1 l_2} \leq \min \left\{ \frac{1}{a_i} \right\}. \tag{1.9}$$

This result shows that in case of a switch point at r (hence if the technical coefficients lead to a $w - r$ curves which cross exactly at the prevailing normal rate of profit r), two processes survive in the LPP.

The solution path is a logistic curve if and only if the difference $\rho_2 - \rho_1$ of rates of extra profit were constant, hence if $l_1/a_1 = l_2/a_2$. From formula (1.2) and expression (1.7) one can see that otherwise there is an additional non-linearity due to the feedback effect of the changing wage rate on the rates of extra profit (and therefore on growth rates). Thus, the diffusion path is in general not a logistic curve, but a *logistic process*:

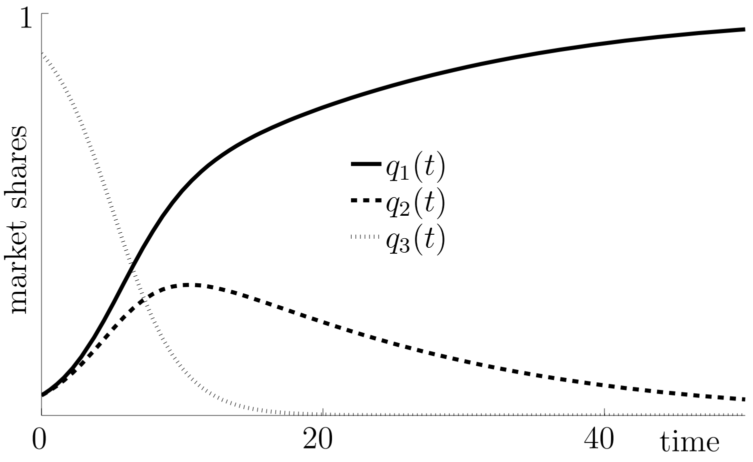

Figure 1.2: Diffusion path for two innovations

> [T]here is no particular reason to expect that the parameters of the logistic process should remain constant over the adoption or diffusion process. Although the underlying process is logistic for given parameters there would be a branch of a different logistic curve for each set of those parameters and so the overall envelope of these branches need not be logistic. (Metcalfe, 2005, p. 173)

This consequence of the non-constancy of the profit differential gets apparent for more than two processes or for multiple sectors, where the monotonicity-property does not hold in general. In the special case of two out of the three processes defining a $w-r$ curve with switch point at r two processes are operated in the new LPP. As another point demonstrating the difference of a logistic curve and a logistic process, not even the monotonicity property of Proposition 1 holds in general for more than two processes. This is exemplified in Figure 1.2 for the incumbent process $(a_3, l_3) = (0.5, 0.25)$ and the two innovative processes $(a_1, l_1) = (0.4, 0.21)$ and $(a_2, l_2) = (0.4, 0.21)$, the latter showing a non-monotonic diffusion path.

In contrast to the time-discrete formulation of the model, where huge losses occasionally lead to an extinction of the old process (see Chapter 4), in the time continuous setting no technology, which once is operated, ultimately vanishes. To understand this difference, compare the continuous time growth condition $\dot{x}_i = g_i x_i$ and the discrete time growth condition $x_i(t+1) = (1+g_i)x_i(t)$ for one sector and two processes. In the latter case, the claim for non-negative output leads to $g_i(t) \geq -1$, with $x_i(t+1) = 0$ if $g_i(t) = -1$; in the time continuous case, no such limit exists and hence $x_i > 0$ for any t if $x(0) > 0$.

1.3.2 An arbitrary number of processes

For I processes, let $\mathbf{q}$ denote the vector of market shares q_i $(i = 1, \ldots, I)$. If a proportion $s \in (0,1)$ of the rate of profit $r + \rho_i$ is invested into growth, the replicator system (1.4) reduces to

$$\dot{q}_i = s\, q_i \sum_{k=1}^{I} q_k \left(\rho_i - \rho_k\right) = q_i \left(\rho_i - \rho\right), \tag{1.10}$$

with $\rho = \sum_{k=1}^{I} q_k \rho_k$ denoting the *average rate of extra profit.* Any I-dimensional unit vector $\mathbf{e}_i^I = (0, \ldots, 0, 1, 0, \ldots, 0)$ with entry 1 at position i is an equilibrium of system (1.10).[1] This indicates that the system is at rest whenever exactly one process is operated. From Proposition 1, one can conclude that mixed equilibria are possible if and only if condition (1.9) holds pairwise for two or more processes, hence if a switch point exists at r. Let $I_0 \subset [1, I]$ be the set of processes satisfying equation (1.9) in pairs. Then each feasible $\mathbf{q}$ with $q_j = 0$ for all $j \in [1, I] \backslash I_0$ is a mixed equilibrium with $\rho_i(\mathbf{q}) = 0$ for all $i \in I_0$.

In case of a single-process equilibrium $\mathbf{e}_m^I$, due to the constraint $\sum_{i=1}^{I} q_i = 1$, the replicator system (1.10) can be reduced by one dimension to

$$\dot{q}_i = s\, q_i \Big(\rho_i - \rho_m + \sum_{k=1}^{I} q_k \left(\rho_m - \rho_k\right)\Big) \quad \text{for} \quad i \neq m. \tag{1.11}$$

Local stability of $\mathbf{e}_m^I$ can be scrutinized by calculating the *Jacobian matrix*

$$\left[\mathcal{J}(\mathbf{e}_m^I)\right]_{ij} = d\dot{q}_i / dq_j(\mathbf{e}_m^I)$$

from equation (1.11):

$$\mathcal{J}(\mathbf{e}_m^I) = \operatorname{diag}\left[\rho_1(\mathbf{e}_m^I), \ldots, \rho_{m-1}(\mathbf{e}_m^I), \rho_{m+1}(\mathbf{e}_m^I), \ldots, \rho_I(\mathbf{e}_m^I)\right] \tag{1.12}$$

This is a diagonal matrix with the rates of extra profits of the not used processes as eigenvalues. Consequently, local stability of $\mathbf{e}_m^I$ can be obtained by looking at the rates of extra profit of unused processes. Negativity implies that the system returns to the initial equilibrium if this process is introduced into the system, whereas positivity indicates a transition to some new equilibrium. An equilibrium $\mathbf{e}_m^I$ is therefore *evolutionary stable* if and only if all eigenvalues of $\mathcal{J}(\mathbf{e}_m^I)$ are negative, hence if the incumbent process cannot be superseded by any of the given alternative processes. A mixed equilibrium exists, if more then one process per sector can be

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[1] The mathematical term *equilibrium* resembles the economic term of an LLP used so far in this paper. These two concepts will henceforth be used synonymously.

operated in the LPP of the system. Its stability properties can be studied by calculating the Jacobian at some equilibrium $\mathbf{q}^* = \sum_{j \in I_0} q_j \mathbf{e}_j^I$ or, equivalently, at $\mathbf{e}_m^I$ with $m \in I_0$, since $\rho_j(\mathbf{q}^*) = 0$ and $\mathcal{J}(\mathbf{q}^*) = \mathcal{J}(\mathbf{q}^{**})$ for all equilibria $\mathbf{q}^*$ and $\mathbf{q}^{**}$. If the system is not in equilibrium, a diffusion process is triggered, guided by the replicator system (1.4). The final LPP is determined independently of the path of the system, since no process totally vanishes and therefore the attained equilibrium is the most effective attainable (there is therefore no path-dependence). It is not possible to hit some equilibrium with some other operated process gaining positive extra profits.

Another property of this process is its increasing wage rate along the transition path of the system (for the proof of Proposition 2 see Appendix B):

Proposition 2. *(1) The wage rate w is monotonically increasing, i.e. $\dot{w} \geq 0$; the rates of extra profit are monotonically decreasing, i.e. $\dot{\rho}_i \leq 0$. (2) $\dot{w} = 0$ holds if and only if the system is in equilibrium, i.e. if and only if $\dot{\mathbf{q}} = 0$.*

The wage rate therefore serves as a *Ljapunov functional*, indicating global stability of evolutionary stable equilibria. From an economic point of view it reveals a decreasing price level, leading to a higher real wage rate as a consequence of technical change.

1.4 The multi-sector economy

For the general case of N sectors, output growth of the quantity of good n produced by process i_n is given by $g_n^{i_n} = s(r + \rho_n^{i_n})$. Then, from equation (1.4), one gets the replicator system

$$\dot{q}_n^{i_n} = s\, q_n^{i_n} \left(\rho_n^{i_n} - \rho_n \right), \tag{1.13}$$

with the average rate of extra profit of sector n defined by $\rho_n = \sum_{i_n=1}^{I_n} q_n^{i_n} \rho_n^{i_n}$. Introducing $\mathbf{q} = (\mathbf{q}_1^T, \ldots, \mathbf{q}_N^T)^T$ with $\mathbf{q}_n$ denoting the vector of market shares of the processes of sector n, equation (1.13) can be written as a *generalized Lotka-Volterra system*

$$\dot{q}_i = s\, q_i \left[Q(\mathbf{q})\mathbf{q} \right]_i \tag{1.14}$$

with the skew-symmetric block-diagonal matrix

$$Q(\mathbf{q}) = \begin{bmatrix} Q_1(\mathbf{q}) & & \emptyset \\ & \ddots & \\ \emptyset & & Q_N(\mathbf{q}) \end{bmatrix}.$$

The sub-matrices $[Q_n]_{i_n j_n} = \rho_n^{i_n} - \rho_n^{j_n}$ represent the differences of rates of extra profit of sector n.

Due to semi-positiveness of all capital input vectors $\mathbf{a}_n^{i_n}$ – stating that each process necessarily uses at least some capital input – Lipschitz continuity of $s\, q_n^i\, (\rho_n^i - \rho_n)$ holds. From the Picard-Lindelöf theorem it therefore follows that system (1.14) admits a unique solution path $\{\mathbf{q}(t)\}_{t \geq 0}$, given the initial conditions $\mathbf{q}(0) = \mathbf{q}_0 \geq 0$ with $\sum_{i=1}^{I_n} q_n^{i_n}(0) = \|q_n^{i_n}(0)\|_1 = 1$. By additionally acknowledging the skew-symmetry of Q in equation (1.13), the interpretation of the $q_n^{i_n}$ as the market share of process i_n in sector n holds (the proof of Proposition 3 is given in Appendix B):

Proposition 3. *$\sum_1^{I_n} q_n^{i_n}(t) = 1$ with $q_n^{i_n}(t) \in [0, 1]$ holds for all $n \in [1, N]$ and $t \geq 0$.*

Extending the single-sector stability discussion, equilibria

$$\mathbf{q}^* = \left((\mathbf{e}_{m_1}^{I_1})^T, \ldots, (\mathbf{e}_{m_N}^{I_N})^T\right)^T$$

with exactly one process m_n being operated in sector n exist. More than one process in some sectors can survive in the LPP – similar to the single-sector case investigated in Proposition 1 – if the $w - r$ curves of different technologies intersect at r. Similar to equation (1.11) for one sector, system (1.13) can be reduced to

$$\dot{q}_n^{i_n} = s\, q_n^{i_n} \left(\rho_n^{i_n} - \rho_n^{m_n} + \sum_{k_n=1}^{I_n} q_n^{k_n} \left(\rho_n^{m_n} - \rho_n^{k_n}\right)\right) \quad \text{for} \quad i_n \neq m_n, \tag{1.15}$$

since $\sum_{j_n}^{I_n} q_n^{j_n} = 1$ holds for all sectors n. The Jacobian of system (1.15) is a diagonal matrix with rates of extra profit $\rho_n^{i_n}(\mathbf{q}^*)$ for $i_n \neq m_n$ as eigenvalues. The stability analysis of the single-sector economy therefore carries over to the multi-sector case: A technology in equilibrium, defined as the set of all utilized processes, is evolutionary stable, if and only if all eigenvalues of the Jacobian are negative. An equilibrium with more than one process in some sector exists, if at least one technology exists, for which at least one entry of the Jacobian is zero. It depends on the initial conditions of the system, which market shares for each of the conjointly existing processes of the new LPP emerge. If some eigenvalues are positive, the system approaches a new equilibrium if one of those processes, for which the assigned eigenvalue is positive, enters the system .

Similar to Proposition 1 for a single-sector economy, the wage rate w is non-decreasing in the multi-sector economy – but, in contrast, not necessarily increasing as shown in Appendix B:

Proposition 4. *(1) $w(t)$ is non-decreasing, i.e. $\dot{w} \geq 0$. (2) $\dot{w} = 0$ is a necessary but not a sufficient condition for $\dot{\mathbf{q}} = 0$, i.e. $\dot{\mathbf{q}} = 0 \Rightarrow \dot{w} = 0$.*

The general Ljapunov property of the wage rate can only be restored either if all entries of the numéraire $\mathbf{d}$ are positive or if all commodities as *basic*: then cost

I	Incumbent process in Sector 1	$(a_{11}^1, a_{12}^1, l_1^1) = (0.7, 0, 0.2)$
II	Product innovation: prototype process	$(a_{21}^1, a_{22}^1, l_2^1) = (0.6, 0, 0.5)$
IIIa	Innovation 1 in sector 1	$(a_{11}^2, a_{12}^2, l_1^2) = (0.2, 0.2, 0.2)$
IIIb	Innovation 2 in sector 1	$(a_{11}^3, a_{12}^3, l_1^3) = (0.4, 0.05, 0.22)$
IV	Innovation in sector 2	$(a_{12}^2, a_{22}^2, l_2^2) = (0, 0.2, 0.5)$

Table 1.1: Collection of processes of Section 1.5

decreasing technical change in any sector influences the wage rate w according to equation (1.2). Otherwise it is possible to get a stagnant wage rate despite changing relative prices, if those sectors, which exhibit cost-reducing technical change, do not contribute – either directly or indirectly – to the wage basket $\mathbf{d}$.

1.5 Inter-sectoral spillover effects

In a multi-sector setting spillover effects, leading to a changing order of profitability of processes, can be observed. Structural change in one sector by means of diffusion of process innovations leads to changing prices of input factors in related sectors. The change of prices in the latter again influences prices of products of other sectors, and so on. This can affect competing processes within one sector, reversing their relative profitability and thus influencing the kind of structural change in innovating sectors. The change of order of competing processes within one sector in terms of their profitability is demonstrated in the following example by the successive introduction of innovations. The respective processes are listed in Table 1.5 according to their order of appearance: (1) First, only one sector and one process I exist. At time $t = 0$, a product innovation enters the system, establishing a second sector. The new product is at first produced only by the prototype process II. (2) In sector 2 a new capital good is produced, which is utilized by processes IIIa and IIIb in sector 1. In a first step, the diffusion of these two innovations is simulated. The changing order of profitability of the two innovations in the course of the diffusion process in this example is a consequence of the increasing wage rate. (3) As an alternative, also in sector 2 technical change in terms of some process innovation IV occurs. The switch of the order of profitabilities of the two innovations in sector 1 is then reversed. These scenarios are discussed one after the other.

(1) Product innovation: At time $t = 0$, good 1 is produced by some process I, characterized by technical coefficients a_{11}^1 and l_1^1. Only good 1 is used as capital input and hence $a_{12}^1 = 0$. The newly introduced good 2 is produced by some prototype process II, using capital a_{21}^1 and labor l_2^1. Only capital from sector 1 is

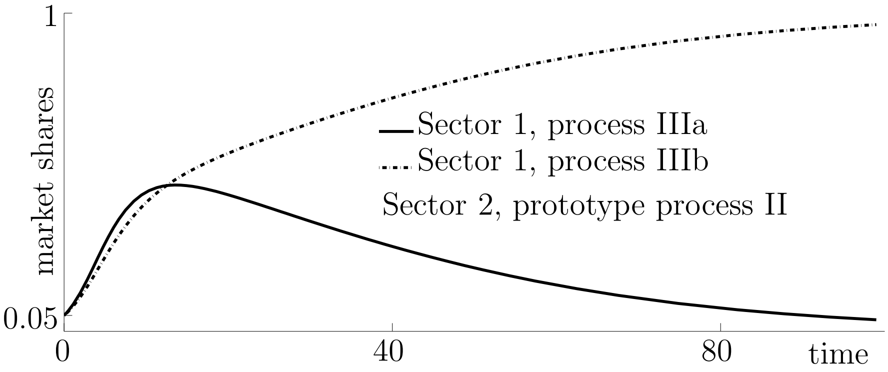

Figure 1.3: Inter-sectoral spillover effect without innovation in sector 2

utilized, since the product of sector 2 was not available at $t = 0$ for production, but had to be produced out of what was available then. Take good 1 as the numéraire, hence $\mathbf{d} = (1, 0)^T$. From (1.2), the price p of commodity 2 is then determined by

$$p = (1 + r)a^1_{21} + wl^1_2.$$

(2) Process innovation in sector 1: According to the assumption that the new product of sector 2 facilitates technical change in sector 1, two innovative processes IIIa and IIIb emerge at $t = 0$ in sector 1 (see Table 1.5). In Figure 1.3, the diffusion path of these two innovations is depicted for given technical coefficients. Feedback-effects induced by changing prices and wages for the given technical coefficients reverse the order of profitability of the innovations in sector 1: Process IIIa is more profitable at the beginning, but finally fails to succeed. This can be seen by comparing the slopes of the two the diffusion paths. That of process IIIa is steeper in the beginning, but then flattens and eventually becomes downward sloping. This results from the rising wage rate (see Proposition 4), leading to counteracting effects: expenses for the direct labor input of process IIIb increase more than those of process IIIa, since $l^3_1 > l^2_1$. Also costs of *indirect labor input* – in terms of the price of input from sector 2 – are rising, which has an even more aggravating influence on unit production costs of process IIIa as opposed to process IIIb, since $a^2_{12} \gg a^3_{12}$. In sum, this leads to a pervasion of sector 1 by process IIIb, superseding process IIIa.

(3) Process innovation in sector 2: Without further innovations, the market share of process IIIb asymptotically approaches 1 as shown in Figure 1.3. But if another cost-reducing process IV becomes available at $t = 0$, a reverse of relative superiority of processes IIIa and IIIb does not take place in the given example – in

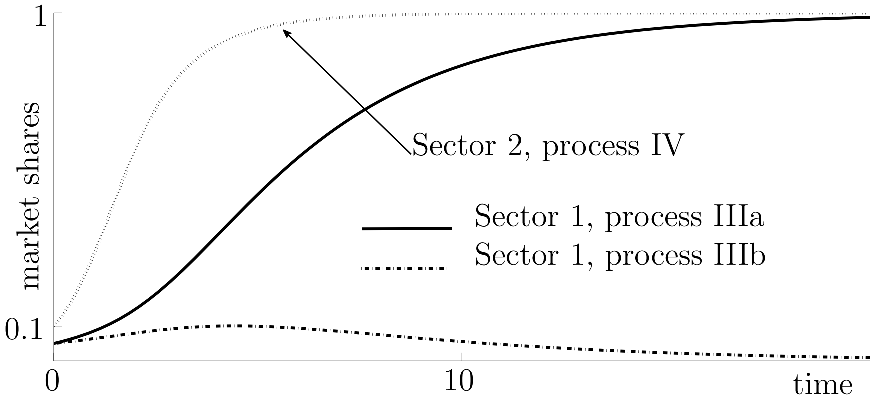

Figure 1.4: Inter-sectoral spillover effect with innovation in sector 2

contrast to the just described situation without technical change in sector 2. For the chosen technical coefficients the system converges towards an LPP with process IIIa instead of process IIIb succeeding in sector 1. This situation is depicted in Figure 1.4. Process IIIa pervades the market, if in sector 2 cost reducing technical change takes place, as opposed to the situation shown in Figure 1.3. Structural change is hence triggered across sectors by innovative behavior:

> *What begins as a small, even insignificant change may end in revolutionizing the entire economic system and compel producers at large to engage in decisions and actions they never planned or anticipated.* This is how the coercive law of competition propels the economy incessantly to transform itself both quantitatively and qualitatively. (Kurz, 2008, p. 274)

1.6 Conclusions

The study of a transition path towards a new LPP determined by the set of available production processes is studied for a multi-sector economy. Core assumptions are an exogenously given set of Leontief processes for each sector; prices are determined by average unit costs of production; and firms invest a share s of their overall rate of profit into output growth. Hence, processes earning an above normal rate of profit can gain in market share relative to less profitable technologies. The wage rate can be shown to serve as a Ljapunov functional, proving global stability of the system.

Besides the formal properties concerned with existence and stability of diffusion paths, in a two-sector economy the spillover effects of technical change are demonstrated. A product innovation, opening up a new sector, influences the production possibilities of the first sector. The order of profitability of different processes is shown to depend on at least two effects: (1) The changing wage rate induces changes of unit costs according to directly and indirectly used labor, which under specific circumstanced leads to a change of the order of profitability of processes. (2) Technical change in one sector can also change the order of profitability in some related sector by means of changing unit costs of production, hence by changing relative prices of capital input factors.

Appendix A: Analytical solution

In relative market shares $z \equiv q/(1-q)$ and with initial condition $z_0 = z(0)$, the solution of equation (1.8) is

$$z\,(\mu_1 + \mu_2 z)^D = C e^{st\mu_1/l_1}, \qquad D = \frac{\mu_1 l_2}{\mu_2 l_1} - 1, \qquad C = z_0(\mu_1 + \mu_2 z_0)^D. \tag{1.16}$$

$\mu_i = \alpha l_i - \beta[1-(1+r)a_i]$ with $\alpha = 1/a_2 - 1/a_1$ and $\beta = l_2/a_2 - l_1/a_1$ are auxiliary parameters.

Proof. Inserting ρ_i from equation (1.7) into equation (1.8), the evolution of q is determined by

$$\frac{\dot{q}(t)}{s\,q\,(1-q)} = \alpha - w\,\beta. \tag{1.17a}$$

With $a = (1-q)\,a_1 + q\,a_2$ and $l = (1-q)\,l_1 + q\,l_2$, acknowledging the $w-r$ relationship (1.2), the wage rate is given by

$$w = \frac{1-(1+r)a}{l} = \frac{1+z-(1+r)(a_1+z\,a_2)}{l_1+z\,l_2}. \tag{1.17b}$$

Since $\dot{z}/z = \dot{q}/[q(1-q)]$, inserting w from equation (1.17b) into equation (1.17a) and collecting terms leads to

$$\frac{\dot{z}}{z}\frac{l_1+z\,l_2}{\mu_1+z\,\mu_2} = s.$$

Finally, integration proves the result. $\square$

Appendix B: Formal proofs

Proof of Proposition 1: The statement is obviously true for $q(0) \in \{0,1\}$. The only other possibility to get an equilibrium is the condition $\rho_1 = \rho_2$, which straightforwardly leads to expression (1.9). The inequality there is motivated by the observation that $1/a_i - 1$ is the maximum rate of profit if only process i is operated. Monotonicity is trivial, since equation (1.8) is an ordinary first order differential equation, hence $\dot{q} = 0$ cannot occur outside equilibrium.

Proof of Proposition 2: As a consequence of the average cost pricing rule (1.6), it holds that

$$\sum_{i=1}^{I} q_i \rho_i a_i = 0. \tag{1.18}$$

This can be seen by multiplying equation (1.7) by q_i and summing up over i. From equation (1.2), the wage rate in case of one sector is given by equation (1.17b). Differentiation with respect to time, acknowledging equation (1.7) and inserting equation (1.10), leads to

$$-l\dot{w} = -s \sum_{i=1}^{I} q_i(\rho_i - \rho)\rho_i a_i \overset{(1.18)}{=} -s \sum_{i=1}^{I} q_i \rho_i^2 a_i \leq 0, \tag{1.19}$$

since $\sum_{i=1}^{I} \dot{q}_i = 0$. This proves $\dot{w} \geq 0$. $\dot{\rho}_i \leq 0$ then follows from equation (1.7), which provides the definition of ρ_i also for an arbitrary number of processes.

From equation (1.19) it follows that $\dot{w} = 0 \Rightarrow q_i \rho_i = 0$, since $q_i \rho_i^2 a_i \geq 0$. Hence, either $q_i = 0$, which implies $\dot{q}_i = 0$ from equation (1.10), or $\rho_i = 0$. Let $\mathcal{I} = \{j \in [1, I] : q_j > 0 \text{ in equilibrium}\}$; then $\rho_j = 0$ for all $j \in \mathcal{I}$, also indicating $\dot{q}_j = 0$ by equation (1.10), since in this case $\rho_j = \rho = 0$. Conversely, since $w = w(\mathbf{q}(t))$, $\dot{\mathbf{q}} = 0$ implies $\dot{w} = 0$.

Proof of Proposition 3: $\sum_{i_n=1}^{I_n} q_n^{i_n}(0) = 1$ is true per definition for all n. By acknowledging equation (1.13), one gets

$$\frac{d}{dt} \sum_{i_n=1}^{I_n} q_n^{i_n}(t) = \sum_{i_n=1}^{I_n} \dot{q}_n^{i_n}(t) = \sum_{i_n=1}^{I_n} q_n^{i_n}(t) \left(\rho_n^{i_n} - \rho_n\right) = \rho_n \left(1 - \sum_{i_n=1}^{I_n} q_n^{i_n}(t)\right).$$

With $y = \sum_{i_n=1}^{I_n} q_n^{i_n}(t) - 1$, this expression is equivalent to the initial value problem $\dot{y} = -\rho y$ and $y(0) = 0$ with solution $y(t) = 0$ for all $t > 0$. This proves Proposition 3, since ρ_n is bounded for all n.

Proof of Proposition 4: Statement (1) is true as a consequence of Proposition 2: if technical change in each sector leads to decreasing prices, then the wage rate is non-decreasing, depending on the choice of the numéraire according to equation (1.2).

(2) The argument for $\dot{\mathbf{q}} = 0 \Rightarrow \dot{w} = 0$ is the same as in Proposition 2 in case of one sector. A counterexample for sufficiency it suffices to take any decoupled two sector economy, with each sector using only its own commodity as input factor of production. With good 1 being the numéraire, technical change and hence changing technical coefficients of sector 2 do not affect equation (1.2), which then reads

$$w = \frac{l_1}{1-(1+r)a_{11}}.$$

Chapter 2

Diffusion of General Purpose Technologies

This chapter is a re-print of the article *Modeling the Diffusion of General Purpose Technologies in an Evolutionary Multi-Sector Framework* which I wrote together with Rita Strohmaier (Rainer & Strohmaier, 2014). It was presented with the title *Economic and social consequences of general purpose technologies - A combined classical and evolutionary approach to the evaluation of economic policy in the presence of technical change* at the 2013 WWWforEurope conference on *Modelling Growth and Socio-ecological Transition.*

General Purpose Technologies (GPTs) are characterized by their pervasive use in the economy. The introduction of a new GPT (product innovation) as well as increasing productivity within a GPT-sector (as a consequence of process innovations) affect the economy in several ways. First, a new GPT offers the opportunity to produce goods by means of cheaper processes; secondly, technical change within the GPT sector influences productivity gains in related sectors. Also social consequences such as changing wage share, technical unemployment and transitional wage inequality can be observed. Finally, the emergence of a GPT often coincides with output decline, preceding economic growth. This paper introduces a multi-sector diffusion model to study these effects by combining classical economics and replicator-dynamics. Empirical evidence is given by the ICT sector in Denmark and its impact on the economic structure from 1966 to 2007.

2.1 Introduction

General Purpose Technologies (GPTs) are basic innovations which change the production structure of the economy via their pervasive use. The steam engine and electricity as well as the information and communication technology (ICT) in the past decades are examples of GPTs. Their emergence (as product innovations) paved the way for process innovations and hence for productivity gains. Intersectoral spillover effects by the introduction of a new GPT and by technical change within a GPT-producing sector implied aggregate economic as well as distributive (and hence social) consequences: A downturn of aggregate output, transitory wage inequality, technical unemployment and changing skills are examples of effects associated with the emergence of a new GPT.

Several formal economic models were set up to facilitate the understanding of economic and social consequences of a new GPT and of technical change in the GPT sector (Helpman & Trajtenberg, 1998a,b; Aghion & Howitt, 1998; Petsas, 2003; Carlaw & Lipsey, 2006). These models are based on assumptions concerning the individual behavior of economic agents, including the rational expectations hypothesis and endogenously modeled technical change due to R&D activities. So far the link between technological change (i.e. product innovations) and technical change (process innovations) has been dealt with insufficiently in this literature; the paper therefore presents a complementary approach based on a classical multisector framework which is merged with the formalism of replicator dynamics, in order to model the dynamics of technical change by means of *diffusion* of innovations.

The static part of the model is based on the approach of Sraffa (1960), in the notation of Kurz & Salvadori (1995). The dynamic part, represented by replicator dynamics, is an offshoot of evolutionary game theory as introduced by Weibull (1997) and utilized by evolutionary economics in the tradition of Nelson & Winter (1982). The following features are incorporated: (1) Different sectors of the economy are related to each other by some unit production input-output matrix. Technical change in one sector can therefore induce productivity changes as well as technical change in other sectors. Product innovations are implemented by increasing the dimension of the technology matrix of the model. (2) Different skill levels with differing remuneration are factored in to model wage inequality. Productivity gains lead to a rising wage rate, and also the investigation of the development of the wage share is conducted to enable discussions concerning distributional issues. (3) The different technologies within each sector are of Leontief-type, assuming instantaneous constant returns to scale. (4) A population view of the economy is introduced. Each sector comprises a population of firms which host the respective processes. A population is therefore defined by the sector respectively by the homogeneous good produced in this sector:

> The obvious candidate for the status of an evolutionary population is an ensemble of business units that differ individually in terms of their behavioral traits, technology, organization, strategic purpose, but are

> members of an evolutionary population by virtue of being subjected to common, market selective processes operating on that population. (Metcalfe, 2008, p. 31)

If several processes exist, this population is subdivided into different *species*, each one characterized by the technical coefficients of the specific process attributed to it. Depending on its profitability for given prices and wage rate, processes exhibit different growth potentials. The resulting growth patterns of technologies imply changes of the cost structure, which in turn lead to altering prices and wages. These again influence the extra profits generated by some technology, hence affecting the growth potential of some species.

Concerning the empirical analysis that accompanies this model, the focus lies on the meso-unit and the changes it triggers off for the economic system, "as it is the population, not the carrier, that evolves"(Dopfer & Potts, 2008, p. 50). The dynamics of the model are placed in juxtaposition to the development of the ICT sector in Denmark and its influence on related sectors. Spanning the period from 1966 to 2007, the analysis covers the time before and after the emergence of important innovations in the field of information and communication technology, such as microcomputers in the 70ies or the Internet in the late 90ies of the last century. The new ICT indeed features the characteristics commonly attributed to GPTs (see for example Jovanovic & Rousseau, 2005a): (1) It has affected virtually all sectors of the economy, (2) persistent improvements in the technology have led to economy-wide productivity gains, and (3) it has spurred inventive activities, also in the development of complementary goods that ensure its widespread use. Other key technologies – such as electricity or the steam engine – share the same characteristics, but to a different extent: In comparison to the preceding GPT, electricity[1], the information technology has been diffusing at a much slower pace and triggered a stronger productivity slowdown upon its arrival (Jovanovic & Rousseau, 2005a,b). Like ICT, electricity also had deep impacts on the labor market: not only were workers replaced by the new technology, electricity also lowered the basic skill level required for formerly skilled jobs (Lipsey *et al.*, 2005). Thus, whereas electricity led to a falling demand for human capital, ICT caused the opposite due to the high knowledge level required for its utilization. Yet it was the former technology that enabled the development of the latter.

Denmark is chosen due to its position as a net-importer of ICT-products[2] and due to the extent of the available data. The first is important in so far as this

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[1] The era of electricity was triggered by the invention of the dynamo in 1867 and spanned from the end of the twentieth century until 1930. This time was characterized by big transformations in the economic system, as new products and industries arose and the industry organization changed from small-scale production to assembly lines.

[2] The only exceptions are central processing units. For a detailed analysis of Denmark's position among Europe with regard to ICT activities see Koski *et al.* (2002).

allows to analyze the effects of ICT predominantly for the production side without needing to consider its impact on economic development via export activities. As regards the latter, Statistics Denmark provides a comprehensive data base that entails annual input-output tables in constant prices and employment data from 1966 to 2007, as well as capital flow tables spanning from 1993 to 2007 (see Strohmaier & Rainer, 2013, for a detailed description of the data handling). The following industrial and service classes comprise the notion of ICT: (1) Mfr. of office machinery and computers, (2) Mfr. of radio and communications equipment etc., (3) Computer activities, Software consultancy and supply[3]. To study the distributive consequences of a GPT on an empirical level, labor input data from Denmark provided by the EU KLEMS database (Edition 2008) is used. This dataset comprises the shares in total hours worked together with the shares in total labor compensation for three different qualification levels, covering a time-span of 26 years (1980–2005).

This chapter proceeds as follows: In Section 2.2 the multi-sectoral model of economic diffusion is introduced. The spread of GPTs as some product innovation, and the subsequent influence of process innovations within the GPT sector, are studied in Section 2.3. A demonstration of wage inequality, changing wage share and output downturn is included. In addition to the model, empirical evidence is provided for the case of the ICT sector in Denmark from 1966 to 2007. Finally, Section 2.4 concludes.

2.2 Multi-sector diffusion model

To study pervasiveness and innovative complementarity as two characteristic properties of GPTs (Helpman & Trajtenberg, 1998a), a multi-sector model is introduced in Subsection 2.2.1. It is augmented by dynamic elements in Subsection 2.2.2, to simulate the transition path in the presence of different processes in each sector.

2.2.1 Mutual dependence of sectors

Let N be the number of different sectors in a closed economy. Within each sector m, an amount x_m of a homogeneous good is produced. This commodity can either be used as input factor of production or for final consumption y_m. $a_{nm} \geq 0$ denotes the quantity of good m, which is *on average* necessary to produce one unit of good n. Let $g_n = \dot{x}_n / x_n$ denote the growth rate of sector n. Final consumption y_n as

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[3] Telecommunication services could not be included, because the classification scheme does not list them as a separate activity.

the net residual of gross production is then given by

$$y_n = x_n - \sum_{m=1}^{N} a_{nm} x_m - \sum_{m=1}^{N} g_m a_{nm} x_m. \tag{2.1}$$

The second term in equation (2.1) is the amount of good n used for productive purposes, whereas the third term is the correcting factor due to sectoral growth. Defining the $N \times N$-matrix A by the technical coefficients a_{nm}, the N equations stated in equation (2.1) are given in matrix notation by

$$\mathbf{y}^T = \mathbf{x}^T \left[\mathbb{I} - (\mathbb{I} + \hat{\mathbf{g}}) A \right]. \tag{2.2}$$

$\mathbb{I}$ is the identity matrix, $\mathbf{y}, \mathbf{x}$ and $\mathbf{g}$ are the column vectors of final demand, of gross output and of the sectoral growth rates, respectively.[4] Equation (2.2) is the *market clearing condition*, which is assumed to hold (changing inventories are neglected).

Concerning labor input, let l_{nk} denote the quantity of labor skill k necessary to produce one unit of good n. The $N \times K$-matrix L of labor input coefficients l_{nk} together with A then characterize the presently used technology. Skill k is remunerated by some wage rate w_k, which are The vector $\mathbf{w} = w\mathbf{u}$ of wage rates for the different skills can – per assumption – be decomposed into some general wage level and the constant vector $\mathbf{u}$ of wage premia. Labor is remunerated ex-post by the prevailing wage rate. For r denoting the given and constant *normal rate of profit* and assuming prices to be given by average unit production costs, the price vector $\mathbf{p}$ is implicitly defined by

$$(1 + r) A \mathbf{p} + w L \mathbf{u} = \mathbf{p}. \tag{2.3}$$

Normalizing prices with respect to some commodity bundle $\mathbf{d}$ (the *numéraire*) by $\mathbf{d}^T \mathbf{p} = 1$. Then from equation (2.3) the wage rate w can be derived to

$$w = \frac{1}{\mathbf{d}^T \left[\mathbb{I} - (1 + r) A \right]^{-1} L \mathbf{u}}. \tag{2.4}$$

The evolution of this $w - r$ relationship provides information about the kind of technical change that takes place. The intersections with the axes determine the maximum wage rate (for $r = 0$), and the maximum rate of profit (for $w = 0$), respectively. Pure capital saving technical change corresponds to an anti-clockwise rotation of the $w - r$ curve. Pure labor saving technical change leads to a clockwise rotation of the $w - r$ curve around its intersection with the abscissa, where the rate of profit is plotted. Neutral technical change leads to a parallel shift of the

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[4] Superscript T denotes transposition and a hat on a vector means the diagonal matrix built from this vector.

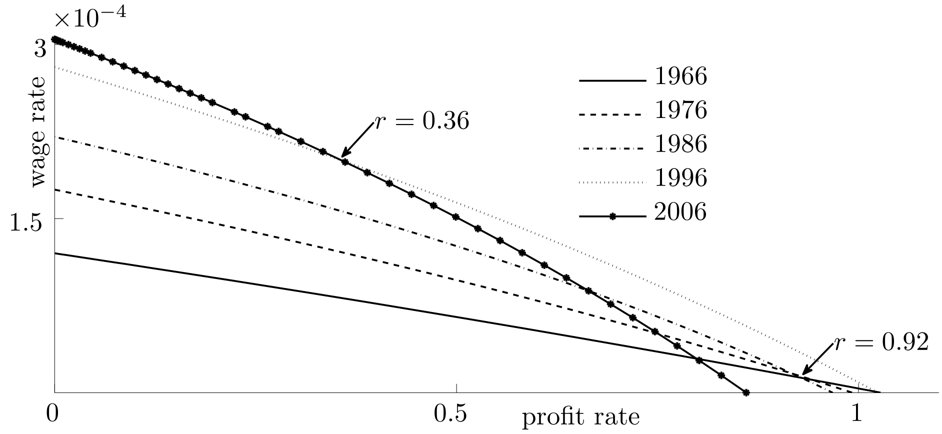

Figure 2.1: Wage-profit curve for Denmark from 1966 to 2006

$w - r$ curve outwards. Figure 2.1 shows the development of the wage-profit curve for Denmark from 1966 to 2006. Until 1986 the curve rotates clockwise around a more or less stable rate of profit[5] in the range of 0.92. Since the aggregate gross profit rate in Denmark has been far lower over this time span – at around 10.5%[6] – one can conclude that in the 20 years between 1966 and 1986 technological change was labor saving. For 1996 the $w - r$ relationship shows unambiguous technical change, because both intersection points moved outwards. Since then, the maximum rate of profit has continuously been decreasing and the curves of 1996 and 2006 intersect at a rate of profit equal to 0.36. Nonetheless, since the actual gross profit rate in Denmark was about 10% on average, technical change turns out to be indeed labor saving and capital using.

2.2.2 Modeling technological diffusion

The technical coefficients A and L are defined as average inputs necessary for unit production. This average either is determined by one single process, or it is the result of a collection of different processes which are operated in this sector. Let the population of sector n be divided into I_n different species, the latter being characterized by some specific process i_n which produces good n. $\mathbf{a}_n^{i_n}$ and $\mathbf{l}_n^{i_n}$ are the respective vectors of circulating capital and of labor, used by process i_n in

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[5] Most studies that deal with $w - r$ curves refer to the profit rate net of depreciation; however, Vaona (2011) showed for Denmark that net and gross profit rate are highly correlated, i.e. accounting for the depreciation of capital does not change the overall trend of the profit rate.

[6] The computation of the aggregate gross profit rate was calculated as the relation between annual gross operating surplus (less mixed income) and real capital stock.

sector n to produce one unit of output. If a fraction $q_n^{i_n}$ of good n is produced by process i_n, then

$$\mathbf{a}_n = \sum_{i=1}^{I_n} q_n^{i_n} \mathbf{a}_n^{i_n}, \quad \mathbf{l}_n = \sum_{i=1}^{I_n} q_n^{i_n} \mathbf{l}_n^{i_n}$$

are the rows of A and L, respectively.

From equation (2.3) it then follows that prices are determined by average costs, since process i_n in sector n occasions unit costs $(1+r)\mathbf{p}^T\mathbf{a}_n^{i_n} + w\mathbf{u}^T\mathbf{l}_n^{i_n}$. The rate of extra profit $\rho_n^{i_n}$ of process i_n in sector n is then implicitly given by

$$(1 + r + \rho_n^{i_n})\mathbf{p}^T\mathbf{a}_n^{i_n} + w\mathbf{u}^T\mathbf{l}_n^{i_n} = p_n.$$

A positive rate of extra profit of some process has different effects on producers: (1) firms get encouraged to invest into growth, (2) new firms get convinced to enter the sector and to use this special process, or (3) firms within the sector change their mode of production and switch to the cheaper process. A negative rate of extra profit (losses) has the reverse effects: They make firms leave either the sector or this specific mode of production (by switching to another, more profitable process), or a firm has to shrink if it further on uses the unprofitable process. Positive extra profits can be earned by firms utilizing an innovation, a new, cheaper method of production. This approach thus shows parallels to Schumpeterian *entrepreneurial profit*. The entrepreneur can reap extra profits by introducing some innovation, which offers the possibility to produce at lower costs (Schumpeter, 2012 [1934], p. 130). Subsequent competition leads to an adaption of prices and wages, causing these profits to decline and finally to vanish (Schumpeter, 2012 [1934], pp. 131–132). The system is then in a new equilibrium position, where the incumbent method of production is replaced by the innovation, hence resembling the Schumpeterian notion of *creative destruction* (Schumpeter, 2010 [1942], Chapter VII). A reconstruction of this pattern is shown in Section 2.3, especially in context of Figure 2.2.

By abstracting from the single firm, which only hosts some process, a *species* is defined by some technology as part of the population, the latter being defined by the respective sector. Each species is characterized by its input coefficients, leading to some *reproductive fitness*. Fitness in this context is a synonym for growth: "By absolute fitness is meant the expansion or contraction over some given time interval of the capacity output of a particular business unit." (Metcalfe, 2008, p. 30) It is influenced by three treats: (1) by rate of extra profit $\rho_n^{i_n}$, which are idiosyncratic for the process; (2) by the overall growth rate g of the economy due to savings; and (3) by the sectoral growth rate Δ_n, which corrects sectoral output according to changes in effective demand due to varying demand for production and final consumption. The growth rate of output $x_n^{i_n}$ produced by means of process i_n in sector n is then given by

$$g_n^{i_n} = \dot{x}_n^{i_n} / x_n^{i_n} = \rho_n^{i_n} + \Delta_n + g.$$

As a consequence of $x_n = \sum_{i_n=1}^{I_n} q_n^{i_n} x_n^{i_n}$, this expression leads to the sectoral growth rate

$$g_n = (\rho_n + \Delta_n + g) \tag{2.5}$$

with average growth $\rho_n = \sum_{i_n=1}^{I_n} q_n^{i_n} \rho_n^{i_n}$ of sector n. By differentiation of $q_n^{i_n} = x_n^{i_n}/x_n$, the dynamics of the system in the presence of technical change is described by the replicator dynamics

$$\dot{q}_n^{i_n} = q_n^{i_n} \left(\rho_n^{i_n} - \rho_n \right). \tag{2.6}$$

Different rates of extra profit of different processes producing the same homogeneous good consequently imply changing market shares. The dynamics of $q_n^{i_n}$ depends on the rate of extra profit and therefore on the price structure $(\mathbf{p}, w)$ and on the technical coefficients $\mathbf{a}_n^{i_n}$ and $\mathbf{l}_n^{i_n}$. Equation (2.6) hence describes a diffusion process, if within one sector several processes with different rates of extra profit are in use. Introducing innovative (cheaper) processes consequently triggers off a diffusion process of this innovation, gradually superseding the incumbent process.

2.3 Diffusion of GPTs

In this section the diffusion of the influence of GPTs is analyzed by means of the just introduced multi-sector diffusion model. In Subsection 2.3.1, a new GPT is introduced as a second sector in a former single-sector economy, making the emergence of one or more new processes in the first sector possible. Then, in Subsection 2.3.2, technical change in the GPT sector is allowed for, and possible consequences on the first sector are investigated.

2.3.1 Introducing a new GPT

A new GPT is invented and introduced into the economic system at time $t = 0$. For $t < 0$ the economy is described by one sector, which reproduces itself with the net-output used up for final consumption. The production process is characterized by the technical coefficients a_{11}^1 and l_{11}^1. Gross production x_1 of this sector equals total production x_1^1 of this process. For $t \geq 0$, a second sector exists, producing a GPT, such that the old technology in sector 1 is now characterized by $\mathbf{a}_1^1 = (a_{11}^1, 0)^T$ and $\mathbf{l}_1^1 = (l_{11}^1, 0)^T$.

GPT as product innovation: In the following, a new GPT will be introduced as a product innovation. In the context of ICT as the latest GPT, one might think in this regard of the development of products such as mainframe and microcomputers that replaced the former office machinery, or the Internet that opened up a new platform for communicating and trading goods and services. The GPT is produced by means of capital input from sector 1. The process utilized in sector 2 is characterized by technical coefficients $\mathbf{a}_2^1 = (a_{21}^1, 0)^T$ and $\mathbf{l}_1^1 = (0, l_{22}^1)^T$. Hence the GPT is produced by high skill labor with wage premium $u > 1$. For $\mathbf{d} = (1, 0)^T$, taking the good of sector 1 as numéraire, price p of the GPT is given by

$$p = (1 + r)a_{21}^1 + wl_{22}^1 u. \tag{2.7}$$

According to equation (2.7) the price of a GPT equals its production costs, i.e. the costs of commodity inputs (including interest, as they need to be available at the beginning of the production year) and the expenses for high skilled labor. An introduction of the GPT sector with the produced good not being used for final consumption ($y_2 = 0$) only pays if similarly in sector 1 a second process is introduced, using the GPT as factor of production. If the GPT enters as circulating capital, the innovative process can be characterized by the technical coefficients $\mathbf{a}_1^2 = (a_{11}^2, a_{12}^2)^T$ and $\mathbf{l}_1^1 = (0, l_{12}^2)^T$. Let $q_1 > 0$ denote the share of the new process in sector 1. From goods market clearing (2.2), which now reads

$$(y_1, 0) = (x_1, x_1^2 q_1 a_{12}^2) \left[\mathbb{I} - (\mathbb{I} + \hat{\mathbf{g}})A\right], \tag{2.8}$$

the growth rate g_2 of the GPT sector is given by $1 + g_2 = q_1(t)(1 + g_1)a_{12}/a_{21}$. From the first equation in (2.8), total output x_1 of the consumption sector exhibits a growth rate $g_1 = \rho_1 + r$ according to equation (2.5) with $g = r$. $\Delta_1 = 0$ holds because forced savings are assumed (y_1 equals net-output) and no substitution of consumption exists due to $y_2 = 0$.

Diffusion of the GPT: The dynamics of the system is driven by the rate of extra profit ρ_1^i of the two processes $i = 1, 2$ in sector 1, implicitly given by

$$\begin{array}{lcccc} (1 + r + \rho_1^1)\ a_{11}^1 & + & wl_{11}^1 & = & 1, \\ (1 + r + \rho_1^2)(a_{11}^2 + a_{12}^2 p) & + & wl_{12}^2 u & = & 1. \end{array} \tag{2.9}$$

From (2.7), p can be replaced in equation (2.9) as well as in the now prevailing price equation

$$(1 + r)[(1 - q_1)a_{11}^1 + q_1(a_{11}^2 + a_{12}^2 p)] + w[(1 - q_1)l_{11}^1 + q_1 l_{12}^2 u] = 1.$$

This problem is formally equivalent to a one-sector economy employing two processes: the first one is the same as above, characterized by technical coefficients

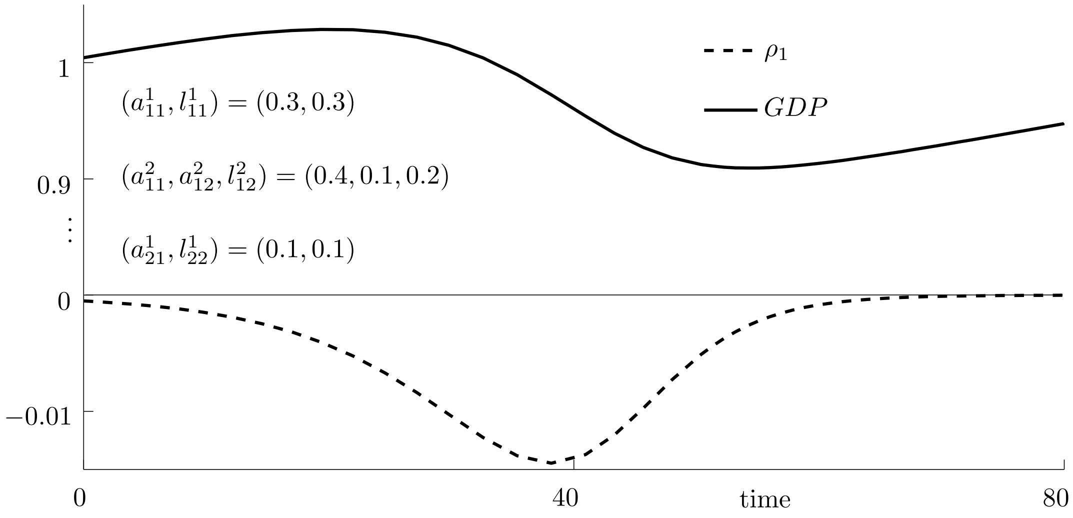

Figure 2.2: Negative growth and output slump

$\tilde{a}_1 = a_{11}^1$ and $\tilde{\mathbf{l}}_1^1 = (l_{11}^1, 0)^T$; the second one is a combination of the GPT sector and the formerly defined second process, characterized by the technical coefficients $\tilde{a}_2 = a_{11}^2 + (1+r) a_{12}^2 a_{21}^1$ and $\tilde{\mathbf{l}}_2 = (0, (1+r) a_{12}^2 l_{22}^1 + l_{12}^2)^T$. In general, each two-sector economy with one innovative sector formally can be reduced to a 1-sector diffusion problem, which is analytically solvable. This solution as well as further discussions of formal properties of the model are derived in Chapter 1. Subsequent simulations are based on numerical solutions of the general replicator equation (2.6), adapted to the given numbers.

Output slump: The development of total output is depicted in Figure 2.2 for the technical coefficients $(a_{11}^1, l_{11}^1) = (0.3, 0.3)$, $(a_{11}^2, a_{12}^2, l_{12}^2) = (0.4, 0.1, 0.2)$ and $(a_{21}^1, l_{22}^1) = (0.1, 0.1)$. Since the average rate of extra profit is negative as a result of the capital using characteristic of the technical change, real GDP exhibits a recessive tendency throughout the diffusion process. Only in the long run the economic growth pattern given by $g = r = 0.01$ is restored. The reason for the output slump after introduction of some GPT is the following: By the market clearing condition (2.2), the growth component $\rho_1 + \Delta_1$ of the sectoral growth rate $g_1 = \rho_1 + \Delta_1 + r$ is obtained by savings of workers. Since $\Delta_1 = 0$, $\rho_1 < 0$ implies forced savings. Consumers therefore accept lower final consumption due to changing circumstances. This downturn cannot be compensated by the rising output of the innovative process and therefore leads to a regression of available

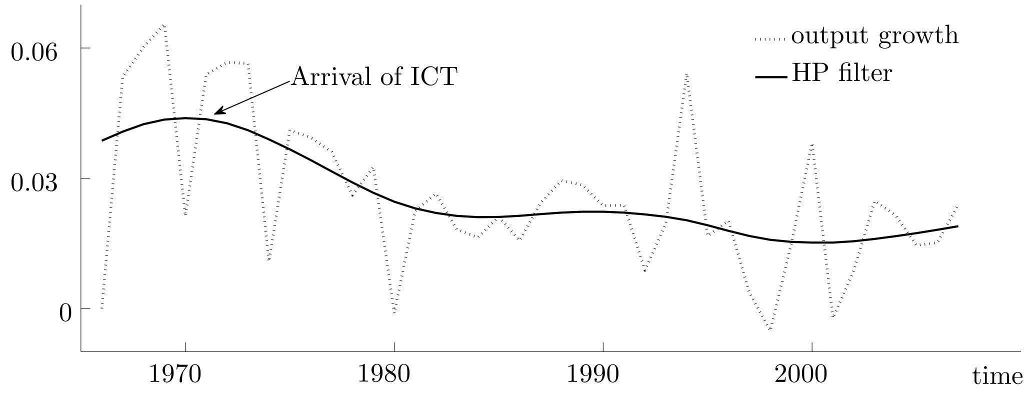

Figure 2.3: Growth of real output per man-hour

goods for final consumption.

From an empirical perspective, looking at the output development over time, Jovanovic & Rousseau (2005b) showed that the emergence of the new information technology in 1971 was not able to reverse the decline of output growth in the US that had been persisting since the 1960ies. In their study, the arrival of IT was dated to 1971, because in this year Intel's 4004 processor came out and revolutionized the market for personal computers (PCs). We undertook a similar analysis for the Danish economy. The date of introduction was also fixed to 1971, as, on the one hand, PCs are the most important ICT product among all import goods; on the other hand, ICT equipment reached 1% of total capital stock[7] of the median sector in this year. Figure 2.3 represents the time series of real output per man-hour[8] in Denmark between 1966 and 2007. The solid line shows the long-term trend as obtained by the Hodrick-Prescott (HP) filter. While output growth shows a falling tendency throughout the whole period under study, Figure 2.3 suggests that the emergence of ICT in 1971 did certainly not mitigate the slump. Differentiating between ICT goods and services, the decline in growth rates after 1985 coincides with the second wave of ICT services that started off after ICT manufacturing products had pervaded the gros of industries (see Strohmaier, 2014).

Wage inequality: Different skills which are differently remunerated imply wage inequality within the class of laborers. For two different skills, as assumed in this

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[7] The corresponding data was retrieved from the EU KLEMS database.

[8] Real output represents deliveries of final goods and services per sector to domestic households, investment, government and nonprofit institutions, as well as net exports to other countries, in constant prices of the year 2000. The total sum equals the gross domestic product.

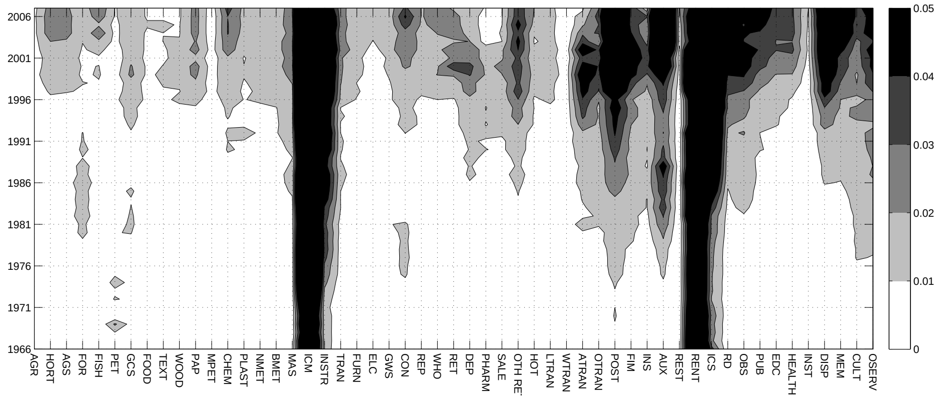

Figure 2.4: The diffusion of ICT products across sectors (intensive use)

example, wage inequality can be estimated by the GINI index[9]

$$GINI = q_h(1 - q_h)\frac{u-1}{1+(u-1)q_h}. \tag{2.10}$$

The share q_h of high skill labor is remunerated by some wage premium $u > 1$ relative to low skill labor. It is given by

$$q_h = \frac{x_1^2 l_{12}^2 + x_2^1 l_{22}^1}{x_1^1 l_{11}^1 + x_1^2 l_{12}^2 + x_2^1 l_{22}^1} = \left[1 + \frac{1-q_1}{q_1}\frac{l_{11}^1}{l_{12}^2 + a_{12}^2 l_{22}^1}\right]^{-1}. \tag{2.11}$$

The last term in equation (2.11) accrues from $x_2 = x_1^2 q_1 a_{12}^2$ and by acknowledging $x_1^i = q_1^i x_1$ for $i = 1, 2$. In this case, the GINI index is independent of sectoral growth patterns, since growth of the GPT sector is coupled to the demand from sector 1.

The diffusion process described by equation (2.6) and the resulting transitional wage inequality calculated by equation (2.10), which is depicted in Figure 2.7, can also be analyzed empirically. The compound direct requirements matrix, which includes not only domestic and imported flows of intermediate products, but also of capital, is used in the following to derive the diffusion pattern of ICT.[10] Figure 2.4 depicts the diffusion of ICT throughout the Danish economy from 1966 until 2007 (the industry classification is listed in the appendix). An input coefficient above 0.01 indicates that the corresponding sector has adopted ICT. The darker the color, the more intensive is the employment of ICT in the respective industry. The

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[9] The derivation of the GINI index for the case of K skills is conducted in Rainer (2012).

[10] Including investment flows is especially important in the case of ICT, as most of these products are of fixed-capital type.

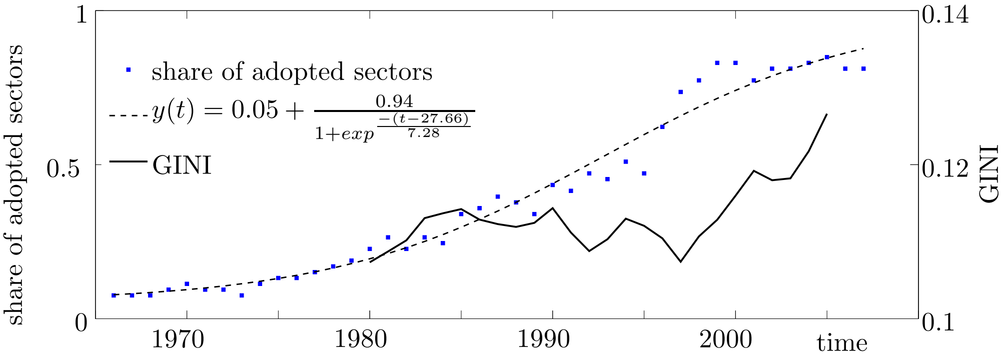

Figure 2.5: The diffusion of ICT products across sectors (left ordinate) and the GINI-coefficient for low and high-skilled labor in ICT using industries (right ordinate)

contour plot shows that ICT goods (ICM) and ICT services (ICS) initially spread over the neighboring industries, such as Mfr. of machinery and equipment n.e.c. (MAS), Mfr. of other electrical, medical and optical equipment (INSTR), as well as Real estate activities (REST) and Renting of machinery and equipment (incl. office computers) n.e.c. (RENT). In the mid 70ies Post and telecommunications (POST) and the Financial markets (FIM) started to utilize ICT. Almost a decade later, one can see the beginning of online sale (OTH RET) and online auctioning (CONS), and the entry of ICT in Research & development (RD). Afterwards, the technology spreads over most sectors in manufacturing and services, with the primary industries as the last sector to adopt it.

Furthermore, Figure 2.5 links the diffusion of ICT to the dynamics of the wage rate[11] of low and high skilled labor in Denmark. The left ordinate presents the share of industries already using ICT, and the right ordinate gives the GINI coefficient as a measure of the dispersion of wages of low and high-skilled labor[12] in the ICT using industries. Figure 2.5 shows that the diffusion path approaches the typical sigmoid curve with the adoption rate increasing around 1985 and again after 1995. After the dot-com crash in 2000 the speed of diffusion slowed down significantly. Since not all industries that produce ICT goods and services could

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[11] In order to take into account self-employed persons, wages and salaries per industry were reestimated by assuming that self-employed and employees have the same wage rate.

[12] For the purpose of this paper, only between low-skilled and higher (i.e. middle and long cycle education)-skilled workers was discriminated. For Denmark, low-skilled labor refers to basic schooling, whereas middle and high skilled labor comprises short, middle and long cycle higher education as well as vocational education and training (for further details on the labor accounts see the EU KLEMS Manual, pp. 24–31).

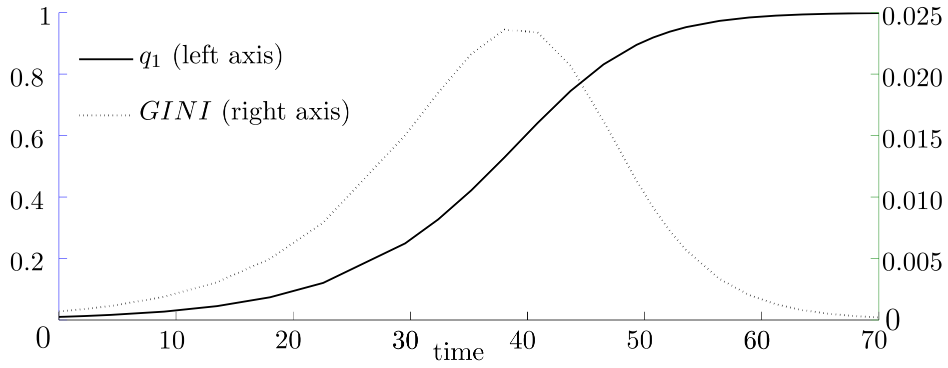

Figure 2.6: The diffusion of an innovative process and the resulting wage inequality

be taken into account (e.g. Telecommunication services), the level of adoption is still below 100 per cent at the end of the period under study, even though the diffusion process has already reached its fade-out phase. Hence by that time, the vast majority of the Danish enterprises had implemented ICT for supporting their business processes, especially for finance and sales management, production and logistics, and human resource management (Statistics Denmark, 2006, p. 30).

With regard to the development of wage differentials, the GINI of ICT using sectors is measured on the right-hand axis of Figure 2.5. While the dispersion of wages and salaries in Non-ICT industries was constantly decreasing between 1966 and 2003 (Strohmaier & Rainer, 2013), ICT using industries exhibit a different pattern: The GINI as an indicator of wage dispersion peaked for the first time when the rate of adoption of ICT was about taking off in the early 1990s. At that time, the demand for qualified IT people was simply not possible to meet. This lack of e-skills, especially from incument employees, has been one of the major barriers to ICT adoption experienced by Danish enterprises (Statistics Denmark, 2006, p. 57). After 1990, the GINI was decreasing, since the labor market could adapt to the new order of skills that were required for efficient ICT usage: In 2000, 69,300 persons (about 2.4 per cent of the labor force) had an ICT related education. Until 2004, this number was rising by 21 per cent to 83,500. From these persons, 83 per cent were employed; this rate is significantly higher than the average employment rate of 76.2 per cent in that year. The upward trend of the GINI since the beginning of this century can be attributed to the rapidly growing importance of ICT services which has been accompanied by a rising demand for persons with high ICT skills (for further details see Strohmaier & Rainer, 2013).

These empirically found diffusion patterns can be reconstructed by the model, as can be seen in Figure 2.6 which, on the basis of equation (2.6), reveals a similar behavior of the share q_1 of the innovative process as suggested by Figure 2.5. What

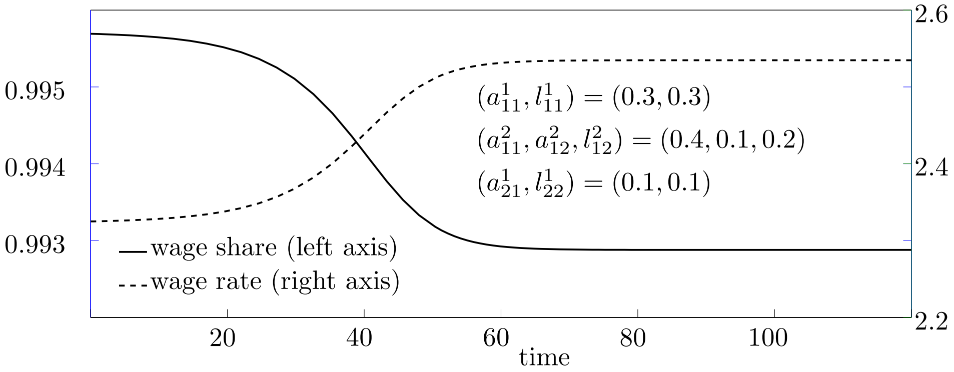

Figure 2.7: Changing wage rates and wage share

gets apparent is the slow start of the diffusion of the innovative process due to the growth process based on the replicator dynamics, which is followed by a *take off* at $t \approx 25$. The respective course of the GINI index, also depicted in Figure 2.6 for $u = 1.1$, can be explained as follows: At the beginning of the diffusion process almost all workers perform low skill labor with wage rate w, whereas near the end of the process almost all workers are high skilled with wage rate $wu > w$. Therefore the GINI index approaches zero at the beginning and towards the end of the process, whereas there is *transitional wage inequality* in between when high and low skill labor is concurrently employed.

Wage share: Another measure touching on inequality and distribution is the wage share $\omega = W/(W + P)$ comprising total wages

$$W = w\mathbf{x}^T L\mathbf{u} = \mathbf{x}^T \left[\mathbb{I} - (1 + r)A\right] \mathbf{p}$$

and total profits $P = r\mathbf{x}^T A\mathbf{p}$. The changing wage share in the present example is depicted in Figure 2.7. It is decreasing as a consequence of the capital *using* and labor *saving* nature of the technical change. An increasing wage rate is a general property of this model, indicating the tendency of the system towards higher labor productivity (see Chapter 1). This, as a result of rising labor productivity, including the decline of the wage share, indicates *technical unemployment* or increasing leisure time.

2.3.2 Technical change in the GPT sector

The model economy of the preceding subsection can be extended to the case of two different processes, which enter the first sector as a consequence of the occurrence

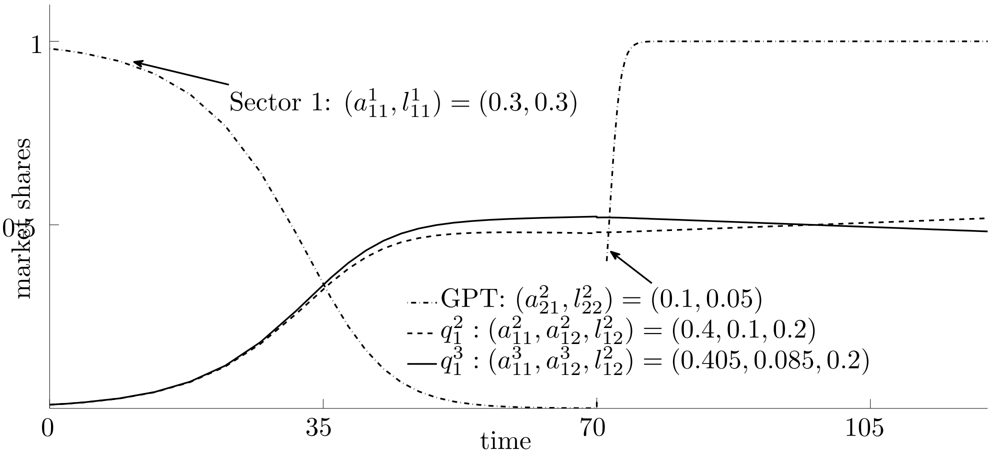

Figure 2.8: Two innovative processes

of the new GPT in sector 2. Process 3 is characterized by the input coefficients $(a^3_{11}, a^3_{12}) = (0.405, 0.085)$ and therefore uses less of the GPT as input (labor input is the same for processes 2 and 3 to keep matters simple). As depicted in Figure 2.8 for $t < 70$, the incumbent process gets superseded and an advantage for the third process against the second process prevails. Without further incidents, for $t > 70$ the market share of process 3 would increase and finally take over the market due to its cost advantage compared to process 2. This scenario occasionally changes if technical change in the GPT sector reduces its unit costs, possibly (not necessarily) leading to a switch of profitability in sector 1 as indicated in Figure 2.8 for $t > 70$: In sector 1 only the new processes 2 and 3 are depicted, and the new process in sector 2 is characterized by pure labor saving technical change with $l^2_{22} = 0.05$. The emergence of technical change in the GPT sector therefore changes the long-run behavior of sector 1, where now process 2 is on the way to dominate process 3.

Additionally, increasing labor productivity is indicated by a rising wage rate. This is a general property of the model: Whenever at least one commodity directly or indirectly is part of the numéraire basket $\mathbf{d}$, the wage rate increases in the course of the diffusion process; it actually never decreases.[13]

This is all the more the case for a GPT-sector, since a general purpose technology is inter alia characterized by its scope of improvement during its lifetime. After its arrival, the crude technology may take decades to mature and show its full potential. The relation between technical change in a GPT-producing sector and rising labor productivity in the application sectors is empirically studied by

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[13] A formal proof of this statement can be found in Chapter 1.

means of a structural decomposition analysis (SDA). Labor productivity growth is thereby measured as the relative change in the maximum wage rate as defined in equation (2.4). The SDA resembles growth accounting because the change in one macroeconomic variable – labor productivity growth– is broken down to its underlying sources (one of which is technical change). Subsequently, a cross-sectional analysis is applied and results are filtered for the ICT-sector.[14] We thus trace the development in aggregate labor productivity back to its driving sectors on the meso-economic level and show which role technical change[15] *within* the ICT producing sector played in this development. The findings are shown in Figure 2.9.

Technical change in the ICT-producing sector as measured from an input-output perspective is a developable indicator for improvements of the technology itself; especially since it does not consider capital goods which embody the bulk of technological change in ICT. Nevertheless, input-output data are capable of tracking process innovations on a meso-(economic) level. To underpin this analysis, the gray shades of the surface represent the degree of (local) innovation activity as given by the share of ICT patents in total patent applications.[16] The number of patents alone as a measure of technological change may be not satisfying either, first because the volume of patents just reflects the level of inventive activity, but does not say anything about how many of those inventions could be successfully introduced to the marketplace. Second, there have been important policy and institutional changes in the last decades that gave rise to the incentives for filing patents. On the other hand, a study by Kortum & Lerner (1998) shows that the increase in patent applications across the globe could be indeed attributed to technological change (Jovanovic & Rousseau, 2005b).

Turning back to Figure 2.9, technical change in the ICT-producing industries manifests itself in labor productivity growth not earlier than from the mid 1990s onwards. Dating the arrival of this GPT to the beginning of the 70ies, it thus took more than two decades for ICT to become a major source of productivity growth. Breaking down its effects on the sectoral level, ICT had its strongest impact on labor productivity growth in the following manufacturing industries: Machinery and equipment, Electrical, optical and medical instruments and Transport equipment.

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[14] A detailed description of the SDA and further results can be found in Strohmaier & Rainer (2013).

[15] By technical change, we refer to the change in the production process of the respective industry, as opposed to technological change embodied in a new product. With regard to ICT, *technological* change means the emergence of this new GPT and the consequences for the economic system via its diffusion. *Technical change* refers to the changes in the input composition of the ICT sector over time, which in turn affect all other industries tied to the ICT sector upstream (due to the change in demand for intermediate products) and downstream (due to the change in the supply of ICT products).

[16] ...filed by Danish applicants under PCT between 1977 and 2007. Data source: OECD.Stat.

It also significantly affected the construction sector. As regards the service sector, a high impact on Post and telecommunications, Real estate activities, Other business activities, Research & development and Public administration can be observed.

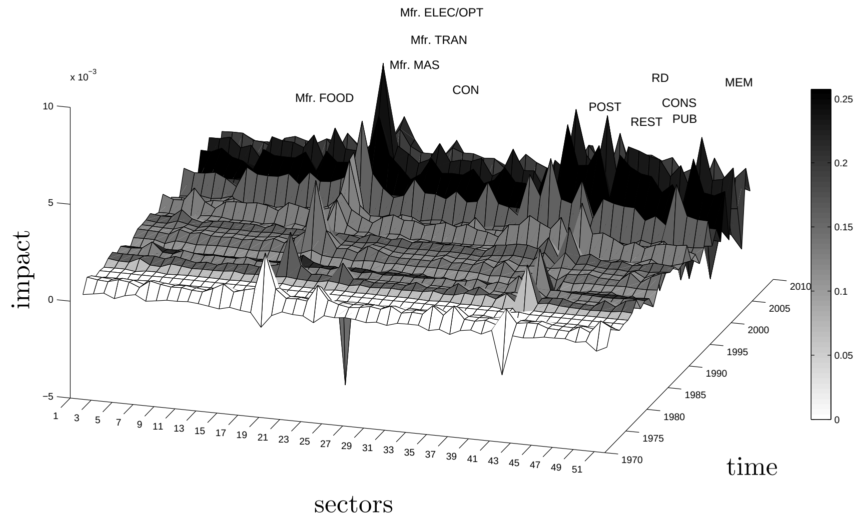

Figure 2.9: The contribution of technical change in the ICT manufacturing sector to sectoral labor productivity growth

Mfr.=Manufacturing of; FOOD=Food, beverages and tobacco; MAS=Machinery and equipment n.e.c.; OPT=Optical and medical equipment; TRAN=Transport equipment; CON=Construction; POST=Post and telecommunications; REST=Real estate activities; RD=Research and development; CONS=Consultancy etc.; PUB=Public administration; MEM=Activities of membership organizations n.e.c.

With regard to inventive activities, Figure 2.9 shows that the 1990ies were not only characterized by productivity gains due to improvements in ICT, but also by a surge in ICT patents; however, most of the important innovations, which aim at facilitating its wide-spread use, were already developed between 1975 and 1990, outside of Denmark. For example, the first microcomputers – commonly known as personal computers – were developed in 1975 by the Massachusetts Institute of Technology. In the same year Bill Gates and Paul Allen founded Microsoft. The market of PCs rose quickly when Apple introduced its first microcomputer in 1977 and even more, when IBM did so in 1981, equipped with DOS, the operating system by Microsoft. These developments in personal computers have been of particular importance for Denmark, since computers have been the most important import good for the economy, when it comes to ICT products. The second half of the 1980ies was characterized by the emergence of the Internet that went viral from the 1990ies onwards; In 1984, the domain name system was created, making the use of Internet way more customer-friendly. In 1987 followed the adaption of the TCP/IP standard protocol which gave a big boost to the number of users. All those innovations can be seen as an important step towards the era of e-commerce which started in 1995, when Amazon and Ebay went online. It is interesting to note that the *local* innovation activity[17] was highest between 1998 and 2003, at a late stage of the diffusion process. This indicates the long time span necessary for a GPT to reach maturity and for the economic system to adapt to the new technology, something which is resembled by the model results.

2.4 Conclusion

As concerns the theoretical part, starting point to simulate diffusion patterns of GPTs is a multi-sector model. Its dynamics is based on differentiated growth due to diverse profitability of production processes. The following stylized facts are reconstructed: (1) The emergence of a new GPT sector (by a product innovation) and technical change in a GPT sector (by process innovations) induce changing productivity in related sectors. This includes negative output growth after the emergence of a new GPT, if technical change is capital intensive. (2) By assuming a higher skill level necessary for processes which are related to the GPT sector, transitional wage inequality is demonstrated. (3) The S-shaped diffusion pattern, which prevails for successful innovations, is endogenized, and feedback effects between output growth and prices (respectively wages) are considered.

Furthermore, the theoretical analysis was tested against empirical evidence from data of the ICT sector in Denmark from 1966 to 2007. The main purpose of the empirical part was to show that ICT was not only a sectoral revolution; it transformed processes throughout the whole economy. Since it took several decades for this technology to pervade the production system, its impact could only be observed recently. As regards the consequences of ICT for the labor market, the diffusion of this technology can be associated with transitional wage dispersion in the ICT producing as well as ICT using industries.

The analysis of the role of ICT for labor productivity change in the rest of the economy also reveals industry clusters: The ICT sector had its strongest impact on technology-intensive manufacturing industries, such as Machinery and equipment or Transport equipment as well as on neighboring service sectors such as Post and telecommunications, Real estate and Other business activities. Likewise, the diffusion and intensity of utilization of ICT (depicted in Figure 2.4) supports the hypothesis by Antonelli (2003) that a new technology diffuses at a higher rate, the more similar are the factor endowments between the place of origination

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[17] as given by the number of patents from Danish enterprises

and the place of adoption: This is the case, for example, of Telecommunications, which represents a similar industry to the ICT services covered in our analysis (in fact, the new Danish industry classification from 2007 groups these two industries together[18]. Thus our results cautiously suggest that the composition of factors determines the speed and order of adoption, and eventually the composition of the economic system as a whole. However, a profound statement would require further examination which is beyond the scope of this paper. Another concept that was not discussed, but would be worthwhile applying empirically is industrial life cycle analysis (Saviotti & Pyka, 2008; Klepper, 2008). The entry and exit of enterprises in the ICT producing sector may foreshadow the maturity stage of the technology, while waves of entry/exit of enterprises in the application sectors reveal the inner-industrial diffusion process.

In general, findings promote the importance of the meso level as a unit of analysis, both from a theoretical and empirical perspective, as has been done, for example, by Dopfer & Potts (2008) and Saviotti & Pyka (2008); as it is the coordination among industries that determines the success or failure of a new technology regarding its impact on the economic system. A meaningful study of this sectoral interplay demands the differentiation of sectors according to their activities respective production processes. Such an analysis also represents a key tool for the design of effective policies fostering economic development via technological change.

Appendix: Industry classification

Table 2.1: Aggregation of Danish industries. Note: The numbers in the third column indicate the assignment of the respective sector to the Danish 130-industry-classification, the third column to ICT-producing, ICT-using and Non-ICT industries.

Code	Abbr.	Industry	Aggr.	ICT-class
1	AGR	Agriculture	1	Non-ICT
2	HORT	Horticulture, orchards etc.	2	Non-ICT
3	AGS	Agricultural services; landscape gardeners etc.	3	Non-ICT
4	FOR	Forestry	4	Non-ICT
5	FISH	Fishing	5	Non-ICT
6	MPET	Extr. of crude petroleum, natural gas etc.	6	Non-ICT
7	GCS	Extr. of gravel, clay, stone and salt etc.	7	Non-ICT
8	FOOD	Mfr. of food, beverages and tobacco	8-18	Non-ICT
9	TEXT	Mfr. of textiles, wearing apparel, leather	19-21	Non-ICT
10	WOOD	Mfr. of wood and wood products	22	Non-ICT
11	PAP	Mfr. of paper prod.; printing and publish.	23-26	Non-ICT

Continued on next page

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[18] Industry classifications subsume etablishments using similar raw material inputs, similar capital equipment, and similar labor in one class. In order to reflect the changing economy, classifications are regularly subject to revisions.

Table 2.1 – continued from previous page

Code	Abbr.	Industry	Aggr.	ICT-class
12	PET	Mfr. of refined petroleum products etc.	27	Non-ICT
13	CHEM	Mfr. of chemicals and man-made fibres etc.	28-35	Non-ICT
14	PLAST	Mfr. of rubber and plastic products	36-38	Non-ICT
15	NMET	Mfr. of other non-metallic mineral products	39-41	Non-ICT
16	BMET	Mfr. and processing of basic metals	42-47	Non-ICT
17	MAS	Mfr. of machinery and equipment n.e.c.	48-52	ICT-using
18	ICM	Mfr. of ICT equipment	53,55	ICT-producing
19	INSTR	Mfr. of electrical, optical and medical instruments	54,56	ICT-using
20	TRAN	Mfr. of transport equipment	57-59	ICT-using
21	FURN	Mfr. of furniture; manufacturing n.e.c.	60-62	Non-ICT
22	ELC	Electricity supply	63	Non-ICT
23	GWS	Gas and water supply	64-66	Non-ICT
24	CON	Construction	67-70	Non-ICT
25	REP	Sale and repair of motor vehicles etc.	71-73	ICT-using
26	WHO	Ws. and commis. trade, exc. of m. vehicles	74	ICT-using
27	RET	Retail trade of food etc.	75	ICT-using
28	DEP	Department stores	76	ICT-using
29	PHARM	Re. sale of phar. goods, cosmetic art. etc.	77	ICT-using
30	SALE	Re. sale of clothing, footwear etc.	78	ICT-using
31	OTH RET	Other retail sale, repair work	79	ICT-using
32	HOT	Hotels and restaurants	80-81	Non-ICT
33	LTRAN	Land transport; transport via pipelines	82-85	Non-ICT
34	WTRAN	Water transport	86	Non-ICT
35	ATRAN	Air transport	87	Non-ICT
36	OTRAN	Support. trans. activities; travel agencies	88-89	Non-ICT
37	POST	Post and telecommunications	90	ICT-using
38	FIM	Financial intermediation	91-92	ICT-using
39	INS	Insurance and pension funding	93-94	ICT-using
40	AUX	Activities auxiliary to finan. intermediat.	95	ICT-using
41	REST	Real estate activities	96-98	ICT-using
42	RENT	Renting of machinery and equipment etc.	99	ICT-using
43	ICS	Computer and related activities	100-101	ICT-producing
44	RD	Research and development	102-103	ICT-using
45	OBS	Other business activities	104-109	ICT-using
46	PUB	Public administration etc.	110-113	Non-ICT
47	EDC	Education	114-118	Non-ICT
48	HEALTH	Health care services	119-120	Non-ICT
49	INST	Social institutions	121-122	Non-ICT
50	DISP	Sewage and refuse disp. and similar act.	123-125	Non-ICT
51	MEM	Activities of membership organiza. n.e.c.	126	ICT-using
52	CULT	Recreational, cultural, sporting activities	127-128	Non-ICT
53	OSERV	Other service activities	129-130	ICT-using

Chapter 3

Population dynamics of competing production processes

This chapter was written in cooperation with Prof. Klemens Fellner, University of Graz, Institute for Mathematics and Scientific Computing.

This chapter provides an extension of the single-sector version of the multi-sector model introduced in Chapter 1. Fixed capital as input factor of production and a lagged price adjustment due to the finiteness of the production processes are introduced. The respective dynamic properties of the system are compared to the nondelayed system without fixed capital.

3.1 Introduction

Technical change comprises three aspects: the investigation of new products and processes, their introduction into the economic system and the subsequent diffusion through the system. Abstracting from the first two aspects, in this chapter the diffusion of existing processes as a result of cost advantages is modeled by dividing the population of (infinitesimally small) firms into different species of firms, each species being characterized by the respective process it uses to produce on homogeneous good. An exogenously given set of Leontief production processes constitutes the strategy set of firms.

The population dynamics, which is the time evolution of the market shares of the different species due to different speed of replication, is modeled as an epidemic process: At each time, some firm is drawn out of the continuum of infinitesimally small firms. This firm faces a binary choice between the recently used process (which is known to the firm) and some second process, which is also randomly chosen out of the set of existing processes; for this second process, which is found by monitoring competing firms, some belief concerning its productivity exists. Hence, the chance to take some specific process into consideration depends on the market share this process currently has. The belief about this alternative process – respectively the probability that is is implemented successfully – is normally distributed. This process of imitation alters the economic environment in that the real wage rate changes due to changing average costs of production (hence due to changing labor productivity). A feedback mechanism between firms' choices and costs of production therefore prevails.

The ensuing population dynamics is described by some replicator dynamics (Weibull, 1997). The respective analysis is concerned with the existence and stability of steady states as well as with the dynamics of the wage rate, which serves as *Ljapunov* function of the system. An in-depth analysis, including the analytical solution, is conducted for the case of two processes. There, an alternative representation of the system in terms of Lie symmetries and the usage of the wage rate as dependent variable provides additional insight into the transitional dynamics of the system. An extensions of the model towards fixed capital and price delay dynamics is also provided, leading to quasi lock-in effects of possibly inefficient processes.

This chapter is organized as follows. In Section 3.2 the general model setting is introduced with a restriction to labor and circulating capital as factors of production. The resulting replicator system is studied and some general properties concerning existence and stability of equilibria are derived; also a characterization of the dynamics of the process is provided by studying the time-evolution of the wage rate. In Section 3.3 fixed capital is added, leading to lagged diffusion due to the necessity of maintenance and building up of fixed capital. Finally, in Section 3.4 the increased diffusion velocity in case of lagged price adjustment is studied.

3.2 Nondelay population dynamics

3.2.1 Replicator dynamics

A homogeneous commodity is produced, used for final consumption and as an input factor of production. Constant returns to scale prevail and the production processes are of Leontief-type (input factors of production are therefore complementary). No entry and exit of firms is allowed for, and no costs of adoption exist if some firms changes its process strategy.

Let $\mathcal{I} = [1, I]$ denote the exogenously given set of I technologies. Each process $i \in \mathcal{I}$ is characterized by technical coefficients $a_i \in (0, 1)$ and $l_i > 0$ denoting the quantity of circulating capital and labor necessary to produce one unit of output. Taking the price of the produced commodity as numéraire, unit costs of production of process i are given by $a_i + wl_i$ for some given wage rate w. The rate of profits r_i of process i is then implicitly defined by

$$(1 + r_i)a_i + wl_i = 1. \tag{3.1}$$

The maximum rate of profits R_i can be earned by process i if $w = 0$, leading to

$$R_i = \frac{1 - a_i}{a_i}.$$

Vice versa, the maximum wage rate w_i which can be paid by firms using process i is defined by $r_i = 0$, leading to

$$w_i = \frac{1 - a_i}{l_i}. \tag{3.2}$$

From relation (3.1), rates of profits r_i and r_j of two different processes $i, j \in \mathcal{I}$ are related to each other linearly, independently of the prevailing wage rate w:

$$r_i = \frac{l_j(1 - a_i) - l_i(1 - a_j)}{a_i l_j} + \frac{l_i a_j}{a_i l_j} r_i$$

In a next step the wage rate w is determined endogenously as the maximum wage rate which can be paid by the average process with technical parameters

$$\bar{a} = \mathbf{a}^T \mathbf{q} \quad \text{and} \quad \bar{l} = \mathbf{l}^T \mathbf{q}.$$

In this definition, $\mathbf{a} = (a_1, \ldots, a_I)^T$ and $\mathbf{l} = (l_1, \ldots, l_I)^T$; $\mathbf{q} = (q_1, \ldots, q_I)^T$ denotes the vector of market shares q_i for all $i \in \mathcal{I}$. Due to equation (3.2), the wage rate w is then given by

$$w = \frac{1 - \bar{a}}{\bar{l}}. \tag{3.3}$$

This is equivalent to a commodity price being equal to average costs of production, since $\bar{a} + w\bar{l} = 1$.

The dynamics of the system accrues from firm decisions concerning the utilized process and hence due to changing market shares of processes. Changing market shares influence the rates of profits r_i via changing wages rates. Firms are characterized by strategic behavior. They operate some specific process and are similarly engaged in an ongoing search for cost saving processes which pay higher rates of profits. By chance at time t some firm using process i draws some other firm out of the continuum of firms. The drawn firm uses some process j which is not necessarily different to its own process i. This induces an epidemic process of spreading information in a complete network. The firm using process i will switch to process j if and only if it is convinced that its rate of profits can be increased by this action and that it can successfully implement the new process. Firms are myopic in that no memory exists and no expectations about future outcomes are included in the decision process.

Let $\mathbb{P}_{i\to j}$ denote the probability with which a firm believes that $r_j > r_i$, which is the probability that it switches to process j if it currently uses process i. Firm are homogeneous with respect to their beliefs. Since q_i denotes the probability of drawing a firm using process i, it holds that

$$\dot{q}_i = q_i \left[\sum_{j\in\mathcal{I}} q_j \mathbb{P}_{i\to j} - \sum_{j\in\mathcal{I}} q_j \mathbb{P}_{j\to i} \right] \quad \text{for all } i \in \mathcal{I}. \tag{3.4}$$

The first term denotes the inflow into process i, the second one denotes the outflow out of process i. Since $\mathbb{P}_{j\to i} = 1 - \mathbb{P}_{i\to j}$, equation (3.4) can be simplified to

$$\dot{q}_i = q_i \sum_{j\in\mathcal{I}} q_j \left(2\mathbb{P}_{i\to j} - 1\right) \quad \text{for all } i \in \mathcal{I}. \tag{3.5}$$

Taking $r_i - r_j$ to be normally distributed with variance σ, one gets

$$\mathbb{P}_{i\to j} = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{r_j - r_i}{\sqrt{2}\sigma}\right)\right].$$

With this specification, (3.5) becomes a system of differential equations describing the diffusion process of different technologies as the result of inter-personal communication in the presence of uncertainty:

$$\dot{q}_i = q_i \sum_{j\in\mathcal{I}} q_j \operatorname{erf}\left(\frac{r_j - r_i}{\sqrt{2}\sigma}\right) \quad \text{for all } i \in \mathcal{I}$$

For high uncertainty (large σ)and small differences of the rates of profits, the error function can be approximated by its first order Taylor expansion:

$$\operatorname{erf}\left(\frac{r_j - r_i}{\sqrt{2}\sigma}\right) \approx \sqrt{\frac{2}{\pi}} \frac{r_i - r_j}{\sigma}$$

Defining the average rate of profits $\bar{r} = \mathbf{q}^T\mathbf{r}$ with $\mathbf{r} = (r_1, \ldots, r_I)^T$ and rescaling time according to $t \mapsto t/\sqrt{2/(\pi\sigma^2)}$ this leads to the replicator system

$$\dot{q}_i = q_i(r_i - \bar{r}) = q_i[Q\mathbf{q}]_i \quad \text{for all } i \in \mathcal{I} \tag{3.6}$$

with coefficients $Q_{ij} = r_i - r_j$ of the matrix Q. With initial conditions $\mathbf{q}_0 > 0$ and $\|\mathbf{q}_0\|_1 = 1$, system (3.6) defines a non-linear, skew-symmetric generalized

Lotka-Volterra system of first order differential equations. It guides the evolution of market shares of competing technologies and is the subject matter of the subsequent analysis.

For the special case neutral technical change, characterized by unchanging proportions of input factors, it holds that $l_i/a_i = l_j/a_j$ for all $i, j \in \mathcal{I}$. In this case, $L = 0$ and $Q = A$ in (3.6). The system to be studied is then given by the following skew-symmetric Lotka-Volterra system with constant coefficients:

$$\dot{q}_i = q_i[A\mathbf{q}]_i = q_i \sum_{j\in\mathcal{I}} q_j A_{ij} \quad \text{with } A_{ij} = \frac{1}{a_i} - \frac{1}{a_j},$$

Otherwise, additional non-linearities increase the complexity of the analysis of the population dynamics.

3.2.2 Equilibria of the replicator system

As a first result, a set of equilibria of the replicator system (3.6) can be identified:

Proposition 5. *Each I-dimensional unit vector $\mathbf{e}_k$ with $k \in \mathcal{I}$ is an equilibirium of system (3.6) with $r_k(\mathbf{e}_k) = 0$.*

A mixed equilibrium with two processes $i \neq j$ exists, if $r_i(\mathbf{e}_j) = r_j(\mathbf{e}_i) = 0$. In this case, each convex combination $(1 - q)\mathbf{e}_i + q\mathbf{e}_j$ for $q \in [0, 1]$ is also an equilibrium, since

$$w_i = \frac{1 - a_i}{l_i} = \frac{1 - a_j}{l_j} = w_j \tag{3.7}$$

independent of q. w_i denotes the maximum wage rate of process i. This induces a partition of the set $\mathcal{I}$ into disjunct subsets

$$\mathcal{I}_d = \{i \in \mathcal{I}, d \in D \,|\, w_i = w_d\},$$

where D is the largest subset of $\mathcal{I}$ such that $w_i \neq w_j$ for all $i, j \in D$.

Acknowledging (3.1), $Q(\mathbf{q}) = A - w(\mathbf{q})L$ can be decomposed into constant, skew-symmetric matrices A and L, with coefficients

$$A_{ij} = \frac{1}{a_i} - \frac{1}{a_j} \quad \text{and} \quad L_{ij} = \frac{l_i}{a_i} - \frac{l_j}{a_j}.$$

The wage rate w is a scalar function depending on $\mathbf{q}$ given in equation (3.3).

Due to the Picard-Lindelöf existence theorem a unique, differentiable solution path $\{\mathbf{q}(t)\}_{t\geq 0}$, continuously depending on $\mathbf{q}_0$, exists, since A and L are bounded. Skew-symmetry of Q implies that $\|\mathbf{q}_0\|_1 = \|\mathbf{q}(t)\|_1 = 1$ for all $t > 0$. This ensures the market-share interpretation of $\mathbf{q}$ and can be seen by summing up (3.6) over the index i, leading to $\|\dot{\mathbf{q}}\|_1 = 0$. Due to this conservation law, the dimension of the system can be reduced by one through elimination of the k-th equation for any arbitrarily chosen $k \in \mathcal{I}$:

$$\begin{aligned}\frac{\dot{q}_i}{q_i} &= \sum_{j\in\mathcal{I}} Q_{ij}q_j = \sum_{j\in\mathcal{I}\setminus\{k\}} Q_{ij}q_j + \underbrace{\Big(1 - \sum_{j\in\mathcal{I}\setminus\{k\}} q_j\Big)}_{=q_k} Q_{ik} \\ &= Q_{ik} + \sum_{j\in\mathcal{I}\setminus\{k\}} [Q_{ij} - Q_{ik}]q_j = Q_{ik} + \sum_{j\in\mathcal{I}\setminus\{k\}} Q_{kj}q_j \qquad \text{for } i \in \mathcal{I}\setminus\{k\} \qquad (3.8)\end{aligned}$$

Stability of some equilibrium $\mathbf{e}_k$ can be obtained by calculation of the Jacobian matrix of the accordingly reduced system (3.8):

$$J_{ij}\big|_{\mathbf{e}_k} = \frac{\partial \dot{q}_i}{\partial q_j}\bigg|_{\mathbf{e}_k} = \begin{cases} r_i & \text{if } i = j \\ 0 & \text{otherwise.}\end{cases} \qquad \text{for } i,j \in \mathcal{I}\setminus\{k\} \qquad (3.9)$$

Hence, the Jacobian evaluated at some equilibrium $\mathbf{e}_k$ is diagonal with rates of profits of the not used processes being its eigenvalues. The introduction of some new process $i \neq k$ with some initial market share automatically leads to a reduction of the market share of the incumbent strategy. From expression (3.9) it follows that the system will return to the original equilibrium if $r_j(\mathbf{e}_k) < 0$, hence if the innovative technology incurs losses on the firm by which it is applied. As one consequence of this result, $\mathbf{e}_k$ is *evolutionary stable* if and only if $r_i(\mathbf{e}_k) < 0$ for all $i \neq k$. In case of mixed equilibria it holds that $r_i(\mathbf{e}_j) = 0$ for all $i, j \in \mathcal{I}_d$, and therefore the stability analysis of those equilibria defined by all convex combinations of $\{\mathbf{e}_i | i \in \mathcal{I}_d\}$ can be conducted by (3.9) for any $k \in \mathcal{I}_d$, since $J_{ij}|_{\mathbf{e}_k} = J_{ij}|_{\mathbf{e}_l}$ for all $k, l \in \mathcal{I}_d$ due to (3.7).

In addition to the local stability analysis, w provides a *Ljapunov* function along the trajectory $\mathbf{q}(t)$, showing global stability of equilibria $\mathbf{e}_k$ for which $r_i(\mathbf{e}_k) < 0$ for all $i \neq k$:

Proposition 6. *The wage rate w is increasing, rates of profits r_i are decreasing. Additionally, $\dot{\mathbf{q}} = 0 \Leftrightarrow \dot{w} = 0$.*

Proof. That $\dot{w}$ and $\dot{r}_i$ are inversely related is a consequence of (3.1):

$$r_i = \frac{1 - wl_i}{a_i} - 1 \quad \Rightarrow \quad \dot{r}_i = -\dot{w}\frac{l_i}{a_i} \qquad (3.10)$$

Differentiation of (3.3) leads to $\dot{\bar{a}} + \dot{w}\bar{l} + w\dot{\bar{l}} = 0$ and therefore to the proof of the proposition:

$$\begin{aligned} -\bar{l}\dot{w} &= \sum_{i\in\mathcal{I}} \dot{q}^i \left[a_i + w(t)l_i\right] \overset{(3.1)}{=} \sum_{i\in\mathcal{I}} \dot{q}_i \left[1 - r_i a_i\right] = 0 - \sum_{i\in\mathcal{I}} \dot{q}_i r_i a_i \\ &\overset{(3.6)}{=} -\sum_{i\in\mathcal{I}} q_i (r_i - \bar{r}) r_i a_i = -\sum_{i\in\mathcal{I}} q_i (r_i)^2 a_i \leq 0. \end{aligned}$$

The last equation follows from $\sum_{i\in\mathcal{I}} q_i a_i r_i = 0$, which can be seen by multiplying (3.1) with q_i, summing up and comparing the result with (3.3). □

3.2.3 Special cases of technical change

Pure capital saving technical change: For the special case of pure capital saving technical change, i.e. if labor input is the same for all processes ($l_i = l$ for all $i \in \mathcal{I}$), it can be shown that average rates of profits are always positive, since from equation (3.1) with $\bar{l} = l$ it follows that

$$\begin{aligned} \bar{r} = \sum_{i\in\mathcal{I}} q_i r_i &= \sum_{i\in\mathcal{I}} \frac{q_i}{a_i} \left[1 - wl - a_i\right] = \sum_{i\in\mathcal{I}} \frac{q_i}{a_i} \bar{a} - 1 \\ &= \Big(\sum_{i\in\mathcal{I}} q_i / a_i\Big)\Big(\sum_{i\in\mathcal{I}} q_i a_i\Big) - 1 \geq 0 \end{aligned} \tag{3.11}$$

Non-negativity prevails, since for $a_i \geq 0$ and $0 \leq q_i \leq 1$ with $\sum_{i\in\mathcal{I}} q_i = 1$ it holds that

$$\Big(\sum_{i\in\mathcal{I}} q_i / a_i\Big)\Big(\sum_{i\in\mathcal{I}} q_i a_i\Big) \geq 1.$$

An estimate for $\dot{w}$ and therefore for the pace of diffusion can now be derived:

Proposition 7. *In case of pure capital saving technical change, some constant C exists such that $0 \leq \dot{w} \leq C$.*

Proof. W.l.o.g., assume that $a_1 \geq a_2 \geq \cdots \geq a_I > 0$. $\bar{a} = \sum_{i\in\mathcal{I}} q_i a_i$ and $\overline{a^{-1}} = \sum_{i\in\mathcal{I}} q_i a_i^{-1}$ are convex combinations and therefore $a_I \leq \bar{a} \leq a_1$ and $a_1^{-1} \leq \overline{a^{-1}} \leq a_I^{-1}$. As a consequence, $\lim_{t\to\infty} w = w^* = (1 - a_I)/l$, implying $\lim_{t\to\infty} \bar{a} = a_I$ and $\lim_{t\to\infty} \mathbf{q} = \mathbf{e}_I$.

Moreover, with $l\dot{w} = \bar{r}\,\bar{a}$ we have with $\dot{\bar{a}} = -\bar{r}\,\bar{a} = -l\dot{w}$

$$\begin{aligned} l\ddot{w} = \dot{\bar{r}}\,\bar{a} + \bar{r}\,\dot{\bar{a}} &= \sum_{i\in\mathcal{I}} q_i (r_i - \bar{r}) r_i - \bar{r}\, l\dot{w} \\ &= \sum_{i\in\mathcal{I}} q^i r_i^2 - \Big(\sum_{i\in\mathcal{I}} q_i r_i\Big)^2 - \bar{r}\, l\dot{w} \end{aligned} \tag{3.12}$$

By Jensen's inequality, we have

$$\left[\sum_{i\in\mathcal{I}} q^i(r^i)^2 - \Big(\sum_{i\in\mathcal{I}} q^i r^i\Big)^2\right] \geq 0,$$

with equality if and only if either some i exists such that $q_i = 1$, or if $r_i = r_j$ for $i \neq j$ and $q_i, q_j > 0$. The former case exists only in equilibrium, and the latter is excluded by the assumption of different processes ($r_i = r_j$ is only possible for $a_i = a_j$). Thus, as long as there exist two indices $i \neq j$ such that $q_i, q_j \geq C_{ij} > 0$, we have

$$\sum_{i\in\mathcal{I}} q_i r_i^2 - \Big(\sum_{i\in\mathcal{I}} q_i r_i\Big)^2 \geq C(C_{ij}) > 0.$$

This implies that $\dot{w} > 0$ until the equilibrium is reached, since otherwise $\ddot{w} > 0$ as soon as $0 < \dot{w} < C(C_{ij})/\bar{r}$. Moreover, since $\ddot{w} \leq \sum_{i\in\mathcal{I}} q_i r_i^2 < C$, we have from (3.12) that $\dot{w} < C$ is uniformly bounded for all $t \geq 0$. □

Pure labor saving technical change: For pure labor saving technical change, hence if $a_i = a$ for all i, the rates of profits defined in equation (3.1) are given by

$$r_i = \frac{1 - wl_i}{a} - 1,$$

and the wage rate from expression (3.3) can be determined to

$$w = \frac{(1-a)}{\bar{l}}. \tag{3.13}$$

In this case, the dynamics of the wage rate depends on the variance

$$\mathrm{var}_{\mathbf{q}}(\mathbf{l}) = \sum_{i\in\mathcal{I}} q_i l_i^2 - \bar{l}^2 \geq 0$$

of labor input coefficients (which is non-negative due to Jensen's inequality). As for the case of pure capital saving technical change in Proposition 7, an upper bound $C_{\max}$ of $\dot{w}$ can be given:

Proposition 8. *In case of pure labor saving technical change,*

$$C_{\max} \geq \dot{w} = \frac{(1-a)^2}{a\,\bar{l}^3}\,\mathrm{var}_{\mathbf{q}}(\mathbf{l}) \geq 0.$$

Proof. Since

$$r_i - r_j = \frac{w}{a}\,(l_j - l_i) \overset{(3.13)}{=} \frac{1-a}{a\,\bar{l}}\,(l_j - l_i)\,,$$

the replicator system (3.6) can be simplified to

$$\frac{\dot{q}_i}{q_i} = \frac{1-a}{a\,\bar{l}} \sum_{j\in\mathcal{I}} (l_j - l_i) q_j = \frac{1-a}{a\,\bar{l}} (\bar{l} - l_i).$$

The dynamics of average labor input $\bar{l}$ then reads

$$\frac{1}{2}\frac{d}{dt}\left(\bar{l}^2\right) = \dot{\bar{l}}\,\bar{l} = \bar{l} \sum_{i\in\mathcal{I}} \dot{q}_i l_i = \frac{1-a}{a} \sum_{i\in\mathcal{I}} q_i (\bar{l} - l_i) l_i = -\frac{1-a}{a}\,\mathrm{var}(\mathbf{l}) \le 0, \qquad (3.14)$$

leading to the following time evolution of the wage rate:

$$\dot{w} \overset{(3.13)}{=} -\frac{1-a}{\bar{l}^2}\dot{\bar{l}} \overset{(3.14)}{=} \frac{(1-a)^2}{a\,\bar{l}^3}\,\mathrm{var}(\mathbf{l})$$

Finally, since $\bar{l} > 0$ and $\mathrm{var}(\mathbf{l}) < \infty$, some finite upper bound $C_{\max}$ exists. ☐

3.2.4 Two processes

General solution: For the case of two processes determined by technical coefficients a_i and l_i $(i = 1, 2)$, the replicator equation (3.6) reads

$$\dot{q} = q(1-q)[\alpha - w(q)\lambda] \qquad (3.15)$$

with $\alpha = 1/a_1 - 1/a_2$ and $\lambda = l_1/a_1 - l_2/a_2$. q denotes the market share of the second process.

The simplest case is Hicks-neutral technical change, for which the replicator system (3.15) reduces to the logistic equation

$$\dot{q} = \alpha q(1-q). \qquad (3.16)$$

By integration, equation (3.16) together with the initial condition $q(0) = q_0$ can be solved to

$$q(t) = \frac{q_0}{q_0 + (1-q_0)e^{-\alpha t}}.$$

As a first step towards solving the general replicator equation for the case of two processes, some canonical coordinate $z(q)$ of the logistic equation (3.15) can be looked for by studying infinitesimal similarity transformations. The latter can be represented by the differential operator $X = \eta(q)\partial_q$, which can be extended to the $(\dot{q}, q)$-space by means of the *prolongation*

$$\hat{X} = \eta(q)\partial_q + \eta_{,q}\dot{q}\partial_{\dot{q}}.$$

The respective *Lie symmetry* of the logistic equation (3.16) can be identified by solving

$$\hat{X}\Big|_{\dot{q}=\alpha q(1-q)} \left[\dot{q} - \alpha q(1-q)\right] = 0,$$

which explicitly reads

$$\frac{\eta_{,q}}{\eta} = \frac{1-2q}{q(1-q)}.$$

Integration yields $\eta(q) = q(1-q)$, leading to the canonical coordinate z by solving $Xz = 1$:

$$z(q) = \int \frac{dq}{q(1-q)} = \ln\frac{q}{1-q} \qquad (3.17)$$

The curvilinear coordinate $z = q/(1-q)$, motivated by expression (3.17), can be used to solve (3.15). From (3.3), the wage rate is determined by

$$w = \frac{1-\bar{a}}{\bar{l}} = \frac{1+z-(a_1+za_2)}{l_1+zl_2}. \qquad (3.18)$$

With auxiliary parameters $\mu_i = \alpha l_i - \lambda\,(1-a_i)$ and since $\dot{z}/z = \dot{q}/\big[q(1-q)\big]$, differential equation (3.15) reads

$$\frac{\dot{z}}{z} = \frac{\mu_1 + z\mu_2}{l_1 + zl_2}.$$

Given the initial condition $z_0 = z(0)$, integration yields the implicit solution

$$z\left(\mu_1+\mu_2 z\right)^D = Ce^{t\mu_1/l_1}, \qquad D = \frac{\mu_1 l_2}{\mu_2 l_1} - 1, \qquad C = z_0(\mu_1+\mu_2 z_0)^D. \qquad (3.19)$$

Expression (3.19) describes the diffusion of some new process, which step-by-step replaces an incumbent, less productive process. The exponent of the exponential function on the right-hand-side of expression (3.19) suggests the use of μ_1/l_1 as a measure of the diffusion velocity v_{diff} defined by

$$v_{\text{diff}} = \frac{l_2}{a_2}\left[\frac{1-a_1}{l_1} - \frac{1-a_2}{l_2}\right]. \qquad (3.20)$$

It characterized the pace at which some cost-saving innovation takes over the market by means of the technical coefficients of the two competing processes.

Wage dynamics: The two-process system (3.15) can be transformed from market shares $q(t)$ to a system describing the wage rate dynamics $w(t)$ directly:

Proposition 9. *In case of two processes ($I = 2$), the replicator dynamics (3.15) can equivalently be states in terms of the wage rate:*

$$\dot{w} = (\alpha - w\lambda)\frac{(1 - a_1 - wl_1)\,(1 - a_2 - wl_2)}{(1 - a_1)l_2 - (1 - a_2)l_1} \geq 0 \tag{3.21}$$

Proof. The coordinate transformation $q \mapsto w$ is given by equation (3.3), which for $I = 2$ reads

$$\begin{aligned} w &= \frac{1 - a_1 - q(a_2 - a_1)}{l_1 + q(l_2 - l_1)} \qquad \text{or} \\ q &= \frac{1 - a_1 - wl_1}{a_2 - a_1 + w(l_2 - l_1)} = \frac{l_1}{l_2 - l_1}\frac{\delta - w}{\gamma + w} \end{aligned}$$

with $\delta = (1 - a_1)/l_1$ and $\gamma = (a_2 - a_1)/(l_2 - l_1)$. Differentiation leads to

$$\dot{q} = \frac{l_1}{l_2 - l_1}\frac{-\dot{w}(\gamma + w) - \dot{w}(\delta - w)}{(\gamma + w)^2} = -\dot{w}\frac{l_1}{l_2 - l_1}\frac{\gamma + \delta}{(\gamma + w)^2}. \tag{3.22}$$

Using the expression

$$\begin{aligned} q(1 - q) &= \frac{l_1}{l_2 - l_1}\frac{\delta - w}{\gamma + w}\left[1 - \frac{l_1}{l_2 - l_1}\frac{\delta - w}{\gamma + w}\right] \\ &= \left(\frac{l_1}{l_2 - l_1}\right)^2 \frac{\delta - w}{\gamma + w}\left[\frac{l_2 - l_1}{l_1} - \frac{\delta - w}{\gamma + w}\right], \end{aligned}$$

one can insert (3.15) into (3.22) to get

$$\frac{l_1}{l_2 - l_1}(\delta - w)(\alpha - w\lambda)\left[\frac{l_2 - l_1}{l_1}(\gamma + w) - (\delta - w)\right] = -\dot{w}(\gamma + \delta).$$

This leads to

$$\dot{w} = \frac{1}{l_2 - l_1}\frac{\delta - w}{\gamma + \delta}(\alpha - w\lambda)\big[\underbrace{l_1(\delta - w) - (\gamma + w)(l_2 - l_1)}_{=1 - a_2 - wl_2}\big]$$

In a last step, one gets a first order differential equation (3.21) by re-inserting δ and γ. The sign is a consequence of Proposition 6. $\square$

The statement $\dot{w} = 0 \Leftrightarrow \dot{\mathbf{q}} = 0$ from Proposition 6 is for two processes a corollary of Proposition 9: The system is in equilibrium if $w = (1 - a_i)/l_i$ (if only process i is employed), or if $\alpha = w\lambda$ (if the two technologies jointly survive). In detail, steady states read

$$w_1^* = \frac{1 - a_1}{l_1} \qquad w_2^* = \frac{1 - a_2}{l_2} \qquad w_3^* = \frac{a_1 - a_2}{a_1 l_2 - a_2 l_1}$$

with w_i^* being the maximum wage rates which can be paid by firms using process i. The range of values of w is therefore given by $w \in [w_1^*, w_2^*]$ if $w_1^* < w_2^*$, which canbe assumed without loss of generality. Since $\dot{w} > 0$ if the system is not in equilibrium, hence if $w \in (w_1^*, w_2^*)$, $d\dot{w}/dw > 0$ for $w = w_1^*$ and $d\dot{w}/dw < 0$ for $w = w_2^*$, showing stability of w_2^* and instability of w_1^*.

In case of a degenerate interval with $w_1^* = w_2^*$, the denominator in (3.21) vanishes. The third steady state w_3^* resembles the case of two processes existing in equilibrium, which is the case for $(l_2 - l_1)/(a_1 l_2 - a_2 l_1) = 1$ as shown in Proposition 10.

3.2.5 Three processes

For the case of two technologies, diffusion is monotonic:

Proposition 10. *For two distinct processes ($I = 2$), the market share q of process $i = 2$ is a monotone function of time. It is constant over time if and only if either $q(0) \in \{0, 1\}$ or*

$$1 = \frac{l_1 - l_2}{l_1 a_2 - a_1 l_2}. \tag{3.23}$$

For $I > 2$, in contrast to the situation for $I = 2$, the market shares $q_i(t)$ possibly exhibit non-monotonic behavior over time. This can be seen in a general setting by projecting the system onto the 2-dimensional phase space with coordinates (q_1, q_2) by acknowledging the relation $\|\mathbf{q}\|_1 = 1$. The replicator system (3.6) then reads

$$\frac{\dot{q}_1}{q_1} = q_2 Q_{12} + \underbrace{(1 - q_1 - q_2)}_{=q_3} Q_{13} = q_2 \underbrace{(Q_{12} - Q_{13})}_{=-Q_{23}} + (1 - q_1) Q_{13} \tag{3.24}$$

$$\frac{\dot{q}_2}{q_2} = q_1 Q_{21} + \underbrace{(1 - q_1 - q_2)}_{=q_3} Q_{23} = q_1 \underbrace{(Q_{21} - Q_{23})}_{=-Q_{13}} + (1 - q_2) Q_{23}. \tag{3.25}$$

This leads to some differential equation in the phase space, given by

$$\frac{dq_1}{dq_2} = \frac{q_1}{q_2} \frac{Q_{13} - [q_1 Q_{13} + q_2 Q_{23}]}{Q_{23} - [q_1 Q_{13} + q_2 Q_{23}]}.$$

The isoclines can be calculated to

$$\begin{array}{rcl} \dot{q}_1 = 0: \ q_2 & = & (Q_{13} - Q_{13} q_1) / Q_{23} \\ \dot{q}_2 = 0: \ q_2 & = & (Q_{23} - Q_{13} q_1) / Q_{23}. \end{array} \tag{3.26}$$

For the special case of neutral technical change these isoclines are straight lines as depicted in Figure 3.1, since then the coefficients Q_{ij} are constants and $Q_{13} <$

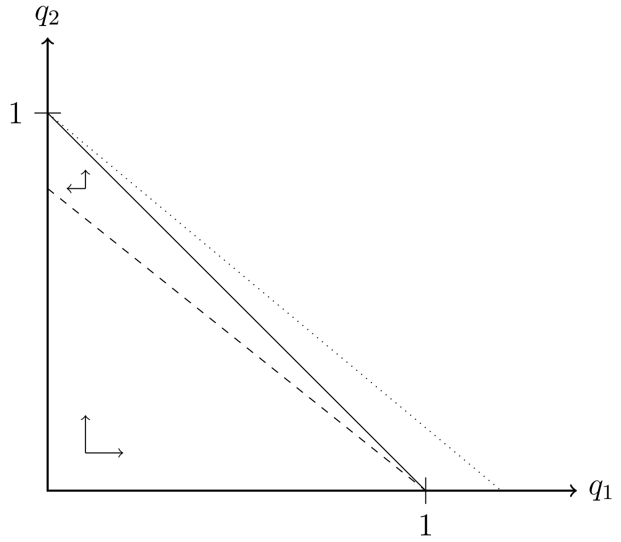

Figure 3.1: Phase space

Q_{23} can be assumed without loss of generality. The arrows in Figure 3.1, which are a consequence of expressions (3.26), indicate that the the system can show non-monotonic behavior. Below the dotted line both q_1 and q_2 are increasing, whereas above the dotted line only q_2 increases and finally converges towards the equilibrium $q_2 = 1$. The area above the solid line is of no relevance since $q_1 + q_2 \leq 1$.

3.3 Fixed capital

3.3.1 The dynamics of capacity utilization

In Section 3.2 only circulating capital which is used up by the production process was considered. For the set $\mathcal{I}$ of Leontief-processes, characterized by unit production input coefficients a_i and l_i of circulating capital and labor respectively, one can add *fixed capital* f_i, denoting the capacity of fixed capital necessary to produce one unit of output. This includes for example the use of machinery, of buildings, of land and natural resources as well as of labor skills. Fixed capital input f_i differs from circulating capital input a_i in that some positive amount of it remains valid after finishing the production process. Some fixed capital stock therefore exists, and depending on the level of production (the quantity of output) this stock of fixed capital has to be maintained or it even has to be extended.

Let total output x_i be produced by some stock $F_i \leq f_i x_i$ of fixed capital. Total output evolves according to $\dot{x}_i = r_i x_i$, which resembles the replicator equation

(3.6) if differentiated with respect to t. The *capacity utilization* $\kappa_i(t)$ of process i is defines as

$$\kappa_i(t) = \frac{f_i x_i(t)}{F_i(t)} \in (0,1]. \tag{3.27}$$

For a rate of depreciation of $\alpha_i \in (0,1)$ and investment $\nu_i F_i$ into fixed capital, the stock of fixed capital evolves according to

$$\dot{F}_i = -\alpha_i F_i + \nu_i F_i. \tag{3.28}$$

Hence, the time path of the capital utilization can be derived:

$$\frac{\dot{\kappa}_i}{\kappa_i} = r_i - \frac{\dot{F}_i}{F_i} = r_i + \alpha_i - \nu_i$$

To get maximum output growth, ν_i is chosen to be the smallest non-negative number such that $\kappa_i \leq 1$:

$$\nu_i = \begin{cases} 0 & \text{if } \kappa_i < 1, \\ \max\{r_i + \alpha_i, 0\} = (r_i + \alpha_i)_+ & \text{if } \kappa_i = 1. \end{cases} \tag{3.29}$$

Capacity utilization therefore evolves according to

$$\frac{\dot{\kappa}_i}{\kappa_i} = \begin{cases} r_i + \alpha_i & \text{if } \kappa_i < 1 \\ \min\{0, r_i + \alpha_i\} = (r_i + \alpha_i)_- & \text{if } \kappa_i = 1. \end{cases} \tag{3.30}$$

Analogue to (3.1), rates of profits r_i are then determined by the market clearing condition

$$(1 + r_i) a_i x_i + w l_i x_i + \nu_i F_i = x_i,$$

which is equivalent to

$$(1 + r_i) a_i + w l_i + \frac{\nu_i f_i}{\kappa_i} = 1.$$

Acknowledging (3.29), this implies rates of profits given by

$$(1 + r_i) a_i + w l_i + \delta_{\kappa_i,1}\ f_i\ (r_i + \alpha_i)_+ = 1, \tag{3.31}$$

where $\delta_{\kappa_i,1}$ denotes the *Kronecker*-delta, which is 1 if $\kappa_i = 1$ and 0 otherwise. This expression includes the necessity to invest into fixed capital if output growths in case of full capacity utilization. In this case, the rate of profit r_i is smaller than in the case of production with circulating capital only, which one can see by comparing equation (3.31) with (3.1). The replicator equation (3.6) together with system (3.30) yields a $2 \cdot I$-dimensional system of first order differential equations. For $f_i = 0$, hence for the case of processes which do not use any fixed capital, this system equals the initial replicator equation (3.6).

Full capacity utilization $\kappa_i = 1$ prevails in the long period position. From (3.31), for each process i the maximum wage rate is therefore given by

$$w_i = \frac{1 - a_i - f_i \alpha_i}{l_i}. \tag{3.32}$$

The claim for a positive wage rate w_i for all $i \in \mathcal{I}$ guarantees the viability of process i, constraining the technical coefficients a_i, f_i and α_i. Additionally, this hints at an altered wage setting rule given by

$$w = \frac{1 - \bar{a} - \sum_{i \in \mathcal{I}} q_i \alpha_i f_i}{\bar{l}}, \tag{3.33}$$

which replaces (3.3). Expression (3.33) takes into account that in the long period position fixed capital has to be replaced on a regular basis. The rate of extra profits r_i of process i can now be calculated from (3.31) by solving for $r_i + \alpha_i$ according to

$$(r_i + \alpha_i) a_i + (r_i + \alpha_i)_+ f_i = 1 - (1 - \alpha_i) a_i - w l_i \equiv \mu_i. \tag{3.34}$$

This indicated that $\operatorname{sgn}(r_i + \alpha_i) = \operatorname{sgn} \mu_i$, leading to

$$1 + r_i = \begin{cases} (1 - \alpha_i f_i - w l_i)/a_i \geq 1 & \text{if } \mu_i \geq 0, \kappa_i = 1 \\ (1 - w l_i)/a_i < 1 & \text{if } (\mu_i < 0, \kappa_i = 1) \text{ or if } \kappa_i < 1. \end{cases} \tag{3.35}$$

3.3.2 Effects of limited capacity on the diffusion path

Figure 3.2 shows that the need for building up fixed capital leads to a delay of the diffusion process: The solid line shows the diffusion of the innovation with unconstrained capacity. This is realized in the given example by assuming $\kappa_1 = \kappa_2 = 0.0001$; then it holds for all $t < 15$ (hence for the whole timespan looked at in this specific example) that $\kappa_i < 1$. The simulation shows that in this case κ_2 is monotonically increasing and that $\kappa_2(15) \approx 0.2$. κ_1 never exceeds 0.001. The diffusion process can therefore evolve similarly to the unconstrained diffusion process without fixed capital as studied in the preceding sections. This diffusion path without capacity constraint can be compared with some diffusion path constrained by full capacity utilization. Full capacity utilization slows down growth by the demand for additional fixed capital as indicated by equation (3.31). The respective example shows capital utilization of $\kappa_1(0) = \kappa_2(0) = 0.1$: The innovation, which shows output growth, works at full capacity utilization at time T_1 (see the dotted path depicting κ_2). At this point the dashed diffusion path with full capacity utilization lags behind the solid diffusion path with unconstrained capacity for the obvious reason as new fixed capital has to be build up. A similar

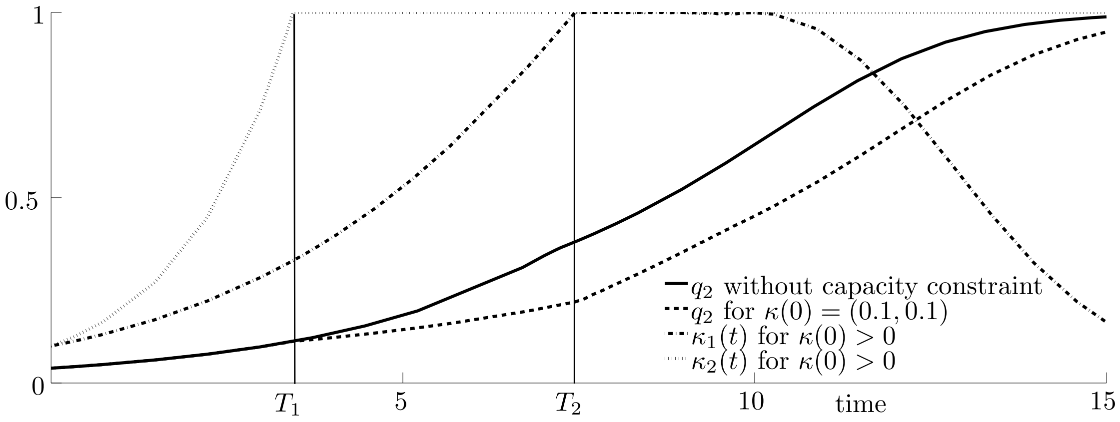

Figure 3.2: Diffusion path with and without capacity constraint: $(a_1, f_1, l_1, \alpha_1) = (0.2, 0.6, 0.5, 0.1)$ and $(a_2, f_2, l_2, \alpha_2) = (0.35, 0.6, 0.3, 0.1)$

effect, now for the incumbent process, can be observed at time T_2. Then for a limited period of time also the old process works at full capacity utilization ($\kappa_1(t)$ is the dash-dotted line), since up to T_3 it holds that $r_2(t) < 0$ but nevertheless $r_2(t) + \alpha_2 > 0$. The diffusion path at T_2 shows a kink, owed to the observation that now also firms using the incumbent process have to invest into fixed capital, since the output decline $r_2(t) < 0$ is not as pronounced as the depreciation rate α_2. In this case it holds that $r_2(t) + \alpha_2 > 0$. Only at time T_2 it holds that $r_2(t) + \alpha_2 = 0$, with $r_2(t) + \alpha_2 < 0$ for all $t > T_3$.

3.3.3 Lock-in due to limited capacity of fixed capital

From (3.32) one can see that for different kinds of innovations it is possible that more than just one process can survive in the long run, similar to (3.7) for the case without fixed capital. The difference between long- and short run behavior of the system now leads to the possibility of some *lock-in* of inefficient processes in the sense that $\lim_{t\to\infty} q_2(t) < 1$ despite the fact that $w_2 > w_1$ and therefore it would be optimal to have $\lim_{t\to\infty} q_2(t) = 1$. This possibility is shown in Figure 3.3, where for the easy of explanation pure capital saving technical change is assumed, with the innovation using fixed capital. The solid line shows the diffusion path of the innovation, which does not survive since $w_2 < w_1$, because from the beginning it works with full capacity utilization and always has to replace the depreciated fixed capital according to α_2. But for the case of unconstrained capacity, hence if $\kappa_2 = 0$, the upper branch of the dashed line shows that the innovation takes over and asymptotically replaces the incumbent process. For the case of initially

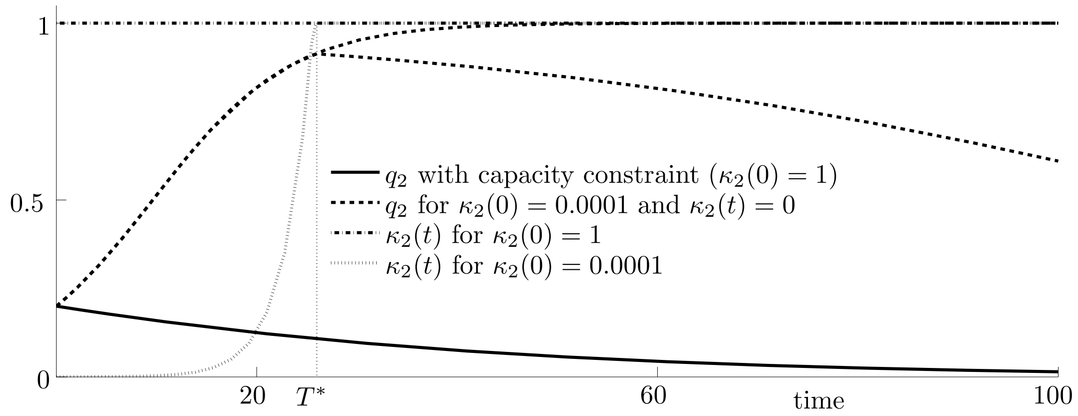

Figure 3.3: Lock-in for limited capacity: $(a_1, f_1, l_1, \alpha_1) = (0.4, 0, 1, 0)$ and $(a_2, f_2, l_2, \alpha_2) = (0.35, 0.3, 1, 0.2)$

unconstrained growth with $\kappa_2(0) = 0.0001$ at time T^* full capacity utilization is reached, and the innovation which up to this point looked as if it would win shows a kink in its path, since this is when in (3.29) suddenly investment into increasing capacity has to be undertaken and hence the rate of profits r_2 is negative for $t > T^*$.

Since the diffusion process only asymptotically converges towards some equilibrium, as soon as $\kappa_2(0) > 0$ – no matter how small this initial value of capital utilization in this specific example is chosen – some time T^* exists where $\kappa_2(t) < 1$ for $t < T^*$ and $\kappa_2(t) = 1$ for $t = T^*$. this leads to a reversal of the slope of the diffusion path of the innovation. Only for infinite capacity (hence for $\kappa_2 = 0$) one gets $\lim_{\kappa_2(0)\to 0} T^* = \infty$ and therefore the equilibrium with $q_2 = 1$ is attained.

That in this example $\kappa_2(t) = 1$ holds for all t if $\kappa_2(0) = 1$, respectively for $t > T^*$ if $\kappa_2 = 0.0001$, is owed to the choice of the technical coefficients, which lead to $r_2 + \alpha_2 > 0$ even if $r_2 < 0$. This observation in contrast to the example of Figure 3.2 suggests that there are limit cases defined by

$$\lim_{t\to\infty} r_i(t) + \alpha_i = 0, \tag{3.36}$$

indicating that even in case of negative rates of profits r_i the fixed capital stock is fully employed whenever

$$\lim_{t\to\infty} r_i(t) + \alpha_i > 0.$$

This can be seen from equation (3.30), since then enforced output decline due to a negative rate of profits is not as pronounced as the decline of the stock of fixed capital due to depreciation.

3.4 Price delay

3.4.1 Delay-differential replicator dynamics

In Section 3.2, within each time interval $[t, t+dt]$ an infinitesimal amount of output was produced instantaneously. Now the production period is 1, and therefore the purchase price $p(t-1)$of capital input at the beginning of production period t (only circulating capital is considered) differs from the selling price $p(t)$ at the end of the production period t. In contrast to (3.3), wages are now taken as numéraire. The price equation then reads

$$\bar{a}(t-1)p(t-1) + \bar{l}(t-1) = p(t). \tag{3.37}$$

For technology i, the rate of extra profits ρ_i is thus defined by

$$(1 + r_i(t))a_i p(t-1) + l_i = p(t), \tag{3.38}$$

which is analogue to the non-delay version (3.1). Explicitly, one gets for the rates of profits the expression

$$r_i(t) = -\frac{l_i}{a_i}\frac{1}{p(t-1)} + \frac{1}{a_i}\frac{p(t)}{p(t-1)} - 1. \tag{3.39}$$

The generalized Lotka-Volterra system (3.6) of differential equations hence turns into as system of delay-differential equations:

$$\dot{q}_i(t) = q_i(t)\sum_{j\in\mathcal{I}} q_j(t)\big(r_i(t) - r_j(t)\big)$$

Inserting extra profits from (3.39) then leads to

$$\dot{q}_i(t) = q_i(t)\sum_{j\in\mathcal{I}} q_j(t)Q_{ij}(t-1) \tag{3.40}$$

with coefficients

$$\begin{aligned} Q_{ij}(t-1) &= A_{ij}\; p(t)/p(t-1) - L_{ij}/p(t-1) \\ &\overset{(3.37)}{=} A_{ij}\; \bar{a}(t-1) + \big(A_{ij}\; \bar{l}(t-1) - L_{ij}\big)\,/p(t-1) \end{aligned}$$

which depend on $p(t-1)$ and on $p(t)$.

By multiplying (3.38) with $q_i(t-1)$, summation and comparison with (3.37), one gets

$$\sum_{i=1}^{n} q_i(t-1)r_i(t)a_i = 0, \tag{3.41}$$

which implies by differentiation that

$$\sum_{i\in\mathcal{I}} \dot{q}_i(t-1)r_i(t)a_i = -\sum_{i\in\mathcal{I}} q_i(t-1)\dot{r}_i(t)a_i.$$

Next, by using (3.41), one can calculate

$$\begin{aligned}\dot{\bar{a}}(t-1) &= \textstyle\sum_{i\in\mathcal{I}} \dot{q}_i(t-1)a_i = \sum_{i\in\mathcal{I}} q_i(t-1)[r_1(t-1)-\bar{r}(t-1)]a_i \\ &= -\textstyle\sum_{i\in\mathcal{I}} q_i(t-1)\bar{r}(t-1)a_i = -\bar{r}(t-1)\bar{a}(t-1),\end{aligned}$$

which equals the non-delay analog $\dot{\bar{a}} = -\bar{r}\bar{a}$.

Differentiating (3.37) and acknowledging (3.38) and (3.41), one can calculate

$$\begin{aligned}\dot{p}(t) &= \bar{a}(t-1)\dot{p}(t-1) + \sum_{i\in\mathcal{I}} \dot{q}_i(t-1)[a_i p(t-1) + l_i] \\ &= \bar{a}(t-1)\dot{p}(t-1) + \sum_{i\in\mathcal{I}} \dot{q}_i(t-1)[p(t) - r_i(t)a_i p(t-1)] \\ &= \bar{a}(t-1)\dot{p}(t-1) - p(t-1)\sum_{i\in\mathcal{I}} \dot{q}_i(t-1)r_i(t)a_i \\ &= \bar{a}(t-1)\dot{p}(t-1) - p(t-1)\sum_{i\in\mathcal{I}} q_i(t-1)r_i(t-1)r_i(t)a_i, \qquad (3.42)\end{aligned}$$

where the last equality follows from (3.40). In (3.42), the second term on the right hand side is non-positive except if $r_i(t)$ changes sign within $[t-1,t]$. Thus, if $\dot{p}(t-1) < 0$, then $\dot{p}(t) < 0$ should follow.

By calculating $r_i(t) - r_i(t-1)$ from (3.39), we obtain

$$r_i(t) - r_i(t-1) = -\frac{l_i}{a_i}\left[\frac{1}{p(t-1)} - \frac{1}{p(t-2)}\right] + \frac{1}{a_i}\left[\frac{p(t)}{p(t-1)} - \frac{p(t-1)}{p(t-2)}\right], \qquad (3.43)$$

where the first term is always negative if and only if $p(t-1) < p(t-2)$, and the second term is positive when – roughly speaking – $p(t-1)$ decays slower than $p(t)$. Thus, when the price level begins to decline, it is possible that $r_i(t) > r_i(t-1)$. This is in contrast to the non-delay model, where $\dot{r}_i(t) = -(l_i/a_i)\dot{w}(t) < 0$.

3.4.2 First simulations

For $t < 0$ only process 1 is in use, hence

$$p_0 \equiv p(t<0) = \frac{l_1}{1-a_1}.$$

Since for $t < 1$ input is determined solely by (a_1, l_1), also $p(t) = p_0$ and hence

$$p_0 \equiv p(t<1) = \frac{l_1}{1-a_1}.$$

At time $t = 0$ some other processes $i > 1$ enter the system, and hence the price dynamics is determined by

$$p(t) = l(t-1) + \bar{p}\prod_{k=1}^{[t]} a(t-k) + \sum_{\tau=1}^{[t]-1}\left[l(t-[t]+\tau-1)\prod_{k=1}^{[t]-\tau} a(t-k)\right]$$

due to (3.37). Expression (3.40) is therefore a system of delay-differential equations with $p(t) = p_0 = 1$ for $t \leq 0$. The delays are $\{1, 2, \ldots, [t]\}$ with $[t]$ denoting the largest natural number smaller than t. In contrast, without delay and with wages normalized to 1, the price is determined by $a(t)p(t) + l(t) = p(t)$, and hence $p(t) = 1/w(t)$ in the discussion of Section 3.2.

From Figure 3.4 it gets apparent that the diffusion process with time delay is faster because the price mechanism in contrast to the case without delay is more sluggish and hence the decline of the rate of profits of innovators is retarded as can be seen in Figure 3.5.

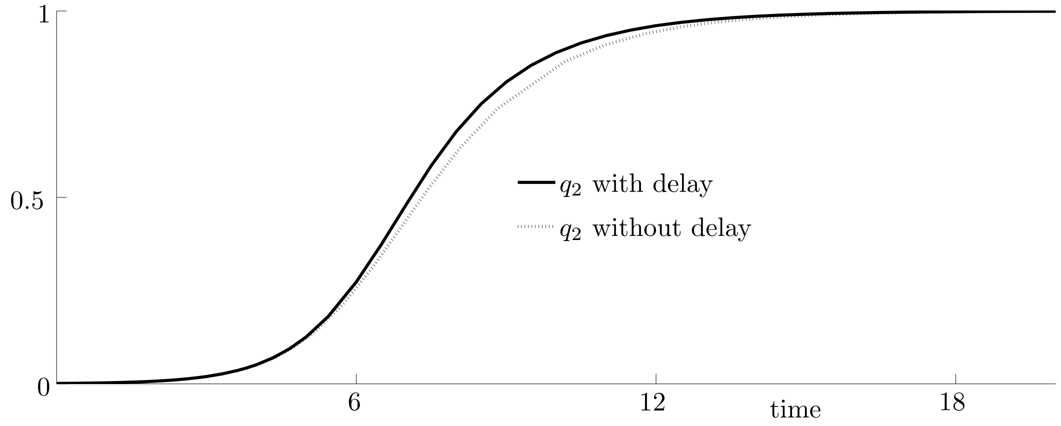

Figure 3.4: Diffusion with and without delay: $(a_1, l_1) = (0.4, 1)$ and $(a_2, l_2) = (0.2, 1)$

3.4.3 Rescaling of time to generate quasi lock-in effects

This lag in the price adjustment process leads to a richer dynamics in the case of more than two processes as indicated in Figure 3.6 for the case of three processes. The system with some general delay $\tau > 0$ is described by the system

$$\begin{cases} (1+r_i)a_i p(t-\tau) + l_i = p(t), \\ \dot{q}_i = q_i(r_i - \bar{r}). \end{cases}$$

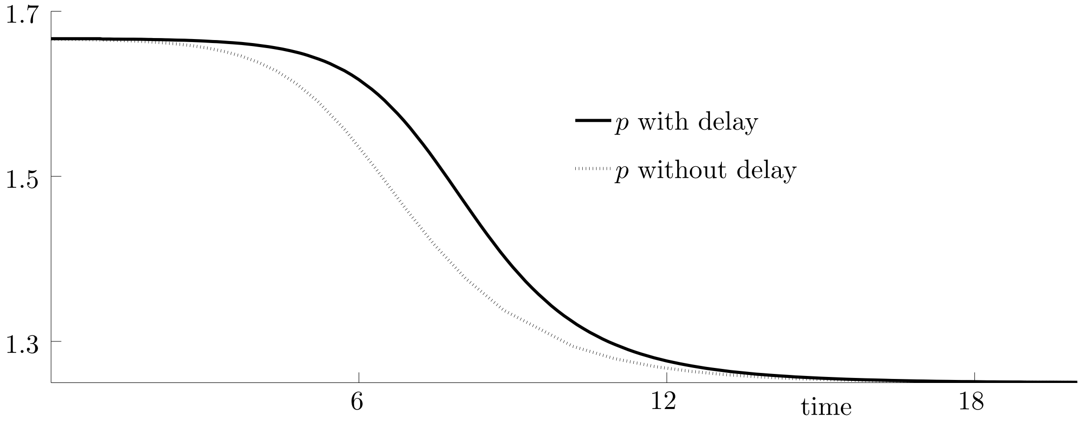

Figure 3.5: Price dynamics with and without delay: $(a_1, l_1) = (0.4, 1)$ and $(a_2, l_2) = (0.2, 1)$

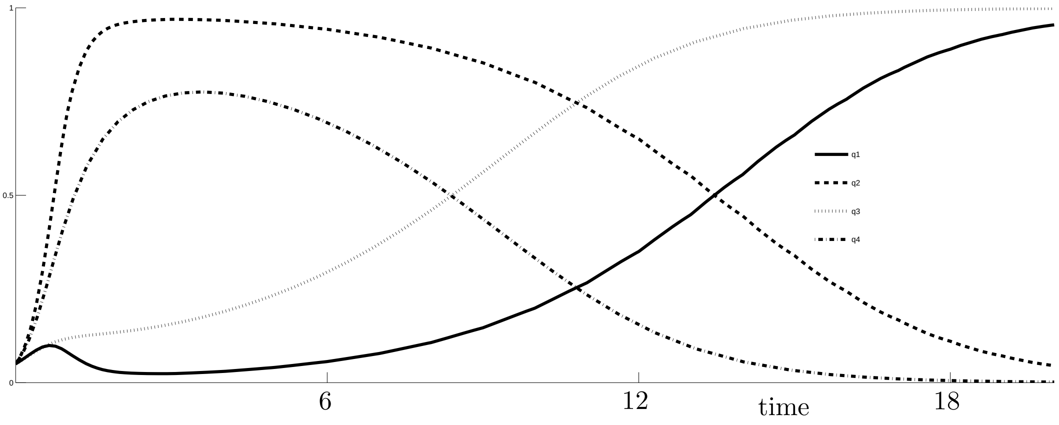

Figure 3.6: 3 Processes: $(a_1, l_1) = (0.6, 1)$, $(a_2, l_2) = (0.2, 1)$ and $(a_3, l_3) = (0.1, 1.2)$

Remember that the original nondelay replicator equation (3.6) was obtained by some re-scaling of time by the factor $1/\sqrt{2/(\pi\sigma)}$. This rescaling took notice of the uncertainty that some process can successfully be implemented by some firm. In a similar manner, on can now rescale the time axis according to $t \mapsto t/\tau$ to get a system with normalized delay:

$$\begin{cases}(1+r_i)a_i p(t-1) + l_i = p(t),\\ \dot{q}_i = q_i s\,(r_i - \bar{r}).\end{cases}$$

This re-introduces some constant $s = \tau$ into the replicator dynamics which has now two different interpretations. First, it can be understood as a parameter measuring uncertainty. The smaller $s = \sqrt{2/(\pi\sigma)}$, the slower the diffusion process takes place due to uncertainty of imitating firms. Secondly, in a delay-setting, small periods of production, hence small τ, lead to a re-scaling of the time axis slowing down the diffusion process.

We can now ask under which condition $r_i(t) > r_i(t-\tau)$ for a general delay $\tau > 0$. Necessarily,

$$\left[\frac{p(t)}{p(t-\tau)} - \frac{p(t-\tau)}{p(t-2\tau)}\right] > l_i \left[\frac{1}{p(t-\tau)} - \frac{1}{p(t-2\tau)}\right],$$

which yields

$$p(t)p(t-2\tau) - p^2(t-\tau) > l_i(p(t-2\tau) - p(t-\tau)).$$

Thus, with $p(t) = p(t-\tau) + \dot{p}(t-\tau)\tau$ and $p(t-2\tau) = p(t-\tau) - \dot{p}(t-\tau)\tau$, we have

$$0 > -l_i\tau\dot{p}(t-\tau) + O(\tau^2).$$

This yields a contradiction for sufficiently small τ, since $p(t)$ is monotone decreasing. Thus, $r_i(t) > r_i(t-1)$ is only possible for large enough delay. The appearance of $r_i(t) > r_i(t-1)$ is shown in Figures 3.7 and 3.8, the disappearance of $r_i(t) > r_i(t-1)$ for small delays is shown in Figure 3.9. This indicates that for large s and three processes $(a_1, l_1) = (0.6, 1)$, $(a_2, l_2) = (0.2, 1)$ and $(a_3, l_3) = (0.1, 1.2)$, some kind of temporary lock-in can be generated, with the process which dominates in the long run, in the short run almost vanishes and only after considerable time re-emerges.

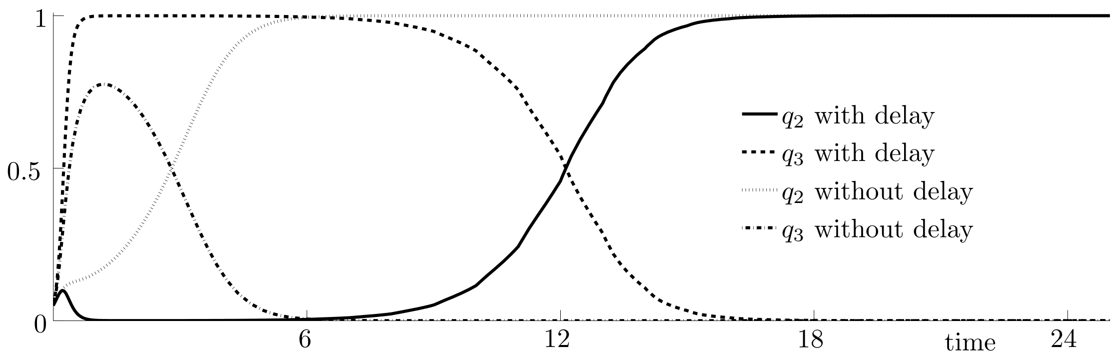

Figure 3.7: 3 Processes: $s = 3$

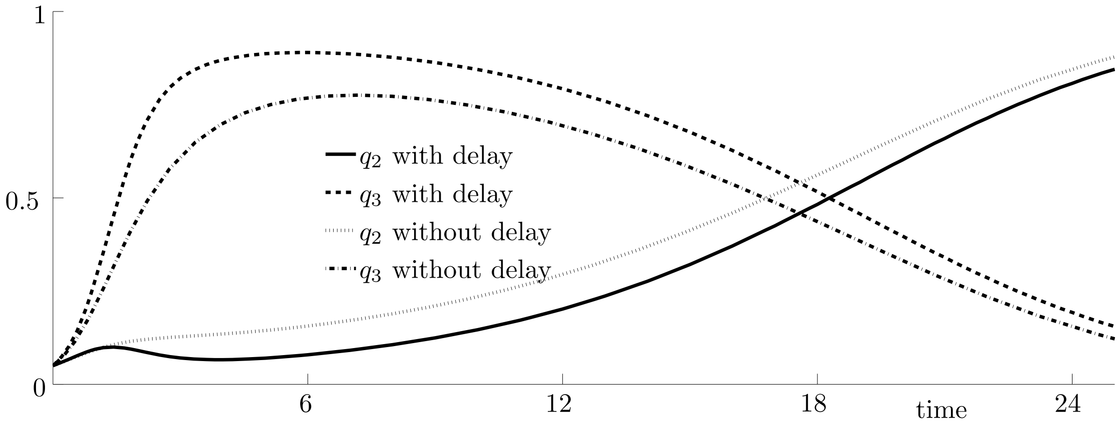

Figure 3.8: 3 Processes: $s = 0.5$

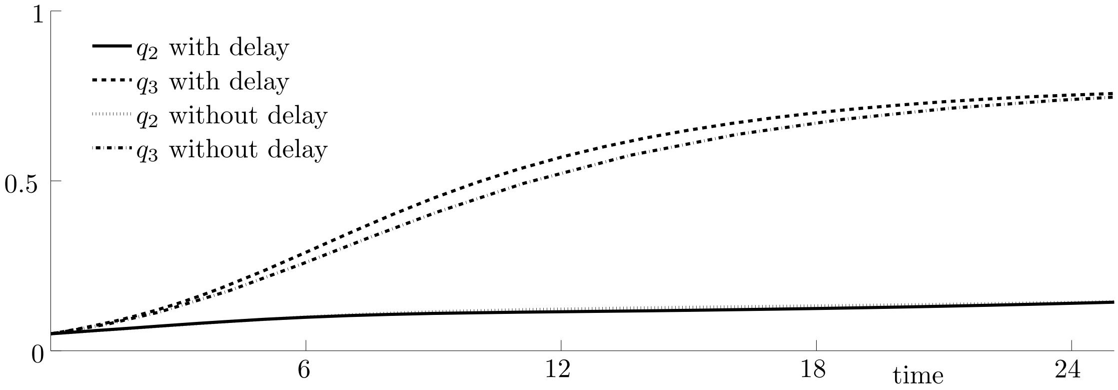

Figure 3.9: 3 Processes: $s = 0.1$

Chapter 4

Diffusion in a simple Classical model

This chapter contains an article, which I co-authored with David Haas. A preliminary version was presented at the *Graz Schumpeter Summer School* 2013 with a talk on *Diffusion Processes in Economic Systems: Population and Agent-based approaches.* The final version will be presented at the ISS Conference in Jena 2014 with a talk on *Diffusion in a simple Classical Model: Micro Decisions and Macro Outcomes.*

In this chapter we explore the interface between evolutionary and modern classical thinking by posing the question of how a long-period position is reached if multiple methods of production are simultaneously in use. Firm decisions on investment and on technology provide the basis of two possible mechanisms of convergence: differential growth and imitation. Both mechanisms rely on the concept of 'extra profits' and imply that during a period of disequilibrium economically superior methods of production gradually supersede inferior ones. The model reproduces the stylized fact of sigmoid diffusion curves and shows that diffusion leads to uneven growth with ambiguous long term effects, a change of income distribution and of the industry structure.

4.1 Introduction

Joseph A. Schumpeter defines economic evolution as "changes in the economic process brought about by innovation, together with all their effects, and the response to them by the economic system" (Schumpeter, 1939, p. 37). Based on Schumpeter's contribution, contemporary evolutionary economics explores "the sources of innovative novelties in economic practice, and the adaptation of the economic system to the potential contained in those novelties." (Metcalfe, 2008, p. 24) This strand of thinking views the economy as a disequilibrium process shaped by the interplay between variety generation and competitive selection.

In evolutionary economics, two basic models of evolution are in use. In the two-stage model, generation of variety and competitive selection do not act at the same time. Variety of behavior is taken to be given and fixed and the question of how innovations arise from within the system is set aside. The focus therefore is on the selection mechanism and its macro consequences. This view postulates convergence towards equilibrium: Since economic evolution depends on the presence of variety, and since selection destroys variety, evolution "consumes its own fuel. [...] Unless this variety is replenished, evolution will come to an end." (Foster & Metcalfe, 2001, p. 9) This 'selectionist view' on economic change does not account for endogenous variety creation. Including this aspect leads to the three-stage model of evolution in which variety generation and variety destruction act simultaneously and are mutually interdependent (Foster & Metcalfe, 2001).

Although the three-stage model is more sophisticated and comprehensive, viewing the generation of variety and selection as two distinct steps in the analysis of economic change allows one to study each of these two aspects of economic change in isolation. By this separation, economic change is grasped as a stylized sequence of three successive steps: Invention, Innovation and Diffusion. The third item of this 'Schumpeterian trilogy', diffusion of innovations, is connected to the evolutionary mechanism of selection (Stoneman, 2007). This scientific route also underlies Schumpeter's analysis. Schumpeter (2012 [1934]) admits that it is the interplay between the 'creative construction' of 'energetic' men and the passive 'hedonistic' mass that puts economic evolution at work and that the internal source of variation always will disturb any tendency to restore equilibrium. But in his analysis Schumpeter distinguishes between "*definite periods* in which the system embarks upon an excursion away from equilibrium and *equally definite periods* in which it draws toward equilibrium" (Schumpeter, 1939, p. 63; emphasis added). The first 'definite period' is shaped by invention, innovation and creative behavior; in the second 'definite period' adaptive behavior, which restores equilibrium, is seen as dominating. The system will, after having been disrupted by innovation, settle in a circular flow, in which its evolutionary potential is exhausted.

The circular flow is a state of the system in which no agent has an incentive to change his position. It is the bridge between evolutionary and modern classical thinking. Two papers explore the interface between the two schools of thought: Kurz (2008) discusses innovation within a classical two sector model and uses the classical long-period method to study the effects of new methods of production on prices and distribution. In his analysis, Kurz interprets a process innovation as a change in the data and evaluates the consequences for the economic system by comparing the long-period positions (hereinafter *LPP*) before the innovation enters the system and after the innovation has been fully absorbed. Steedman & Metcalfe (2011) argue that a full account of economic transformation also has to explain how new methods of production replace old ones. Within a one-sector framework they explore the process of adaptation taking place out of equilibrium and explain how competitive selection moves the system towards an LPP.

We add to this literature about how a long-period position is established after an innovative impulse has disrupted the circular flow by studying how the nature of the innovation effects aggregate growth and income distribution. What are the forces that lead the system towards a new position of rest? It is the aim of this paper to explore possible mechanisms of convergence and to study the disequilibrium dynamics for different kinds and intensities of technical change. The study of disequilibrium paths is relevant for two reasons: First, if disequilibrium prevails for a long time, understanding the dynamics outside equilibrium is crucial. Secondly, it adds to our understanding of how equilibria form and it illuminates phenomena characterizing disequilibrium, which do not appear in long-run equilibrium.

The paper proceeds in three steps. Section 4.2 presents basic concepts and assumptions. In Section 4.3 the core mechanisms of convergence, differential accumulation and imitation are formalized. Section 4.4 explores the macro regularities initiated by diffusion. Section 4.5 concludes.

4.2 The economy out of equilibrium

In a one-commodity world, a homogeneous good is produced by means of homogeneous labor and by the good itself. Production functions are of the Leontief-type and returns to scale are constant. There are no barriers to growth as labor is available in abundance. Take the economy to be in an LPP, “characterized by a uniform rate of profit and uniform rates of remuneration for each particular kind of ‘primary’ input in the production process” (Kurz & Salvadori, 1995, p. 1). This definition of an LPP implies that in terms of market shares only one method of production is in use, if one abstracts from the possibility that at the given level of wages and for a given normal rate of profit various methods of production just break even. An LPP is an equilibrium position, which is understood as the outcome of a disequilibrium process. As (Knell, 2008, p. 39) notes, “the uniform rate of profit describes the outcome of the competitive behavior of heterogeneous actors in the market, whereas profit-seeking entrepreneurs [...] minimize the cost of production because of the competitive process”. The mechanism which leads to such a state of uniformity relies on the assumption that markets are characterized by free competition. By definition, this implies the absence of substantial barriers of entry and exit and therefore allows both capital and labor to be fully mobile across sectors. Furthermore, free competition demands firms to have access to all known methods of production and that the availability of these methods is independent of firm size (Kurz & Salvadori, 1995, p. 17). Given these conditions, profit-seeking firms will look for the method of production which yields the highest rate of profit given current prices.

For a new method of production, which is superior in the sense that at the prevailing wage rate it has a cost advantage, it is reasonable to assume that it is not adopted instantaneously by all firms. Due to numerous constraints it is rather a gradual process by which new technological knowledge is absorbed. Several reasons can be found to explain this time lag of adoption: First, the pioneer might succeed in keeping the innovative method of production a secret for some time. Secondly, due to a lack of information about technical characteristics and experience, firms face uncertainty about the innovation's superiority. Thus the basis for a decision on technology is blurred and, given limited knowledge, an immediate adoption might be seen as involving high risk. Third, there will be limits to the ability to adopt novel business practices due to organizational and financial frictions. (Stoneman, 2002; Rogers, 2003; Baptista, 1999; Nelson *et al.*, 2004) Given that the speed at which an innovation spreads is not infinitely large, there is a period of disequilibrium in which multiple methods of production co-exist and rates of profit vary across firms.

Consider the case of two methods of production co-existing ($i = 1, 2$), the first being established and the second invading the system. Take the produced good as the numéraire and assume that on the output and on the labor market the law of one price holds. Wages are paid ex post. For some real wage rate w, the rate of profit $r_i(w)$ of production process i is then given by

$$(1 + r_i(w))a_i + wl_i = 1 \tag{4.1}$$

with a_i and l_i denoting the input of capital and labor respectively necessary to produce one unit of output. With x_i denoting the output produced by means of process i, let $q = x_2/x$ be the share of total output x produced by the innovative process 2, and $1 - q = x_1/x$ the market share of the incumbent process 1. Given the prevailing distribution of production methods across firms, the average amount of capital and the average amount of labor needed to produce one unit of output is computed as

$$\bar{a} = (1 - q)a_1 + qa_2 \quad \text{and} \quad \bar{l} = (1 - q)l_1 + ql_2.$$

The rate of profit $\bar{r}(w)$ of the average production process $(\bar{a}, \bar{l})$ – which in general does not coincide with the average rate of profit $(1 - q)r_1 + qr_2$ – is then given by

$$(1 + \bar{r})\bar{a} + w\bar{l} = 1. \tag{4.2}$$

The average production process is an abstract measure of the prevailing state of technical knowledge at a given moment of time. It reflects the normal conditions of production. By comparing r_i and $\bar{r}$, hence by measuring the distance of this method from the average method, the relative economic superiority of the method of production i is determined. To measure this relative economic superiority, the concept of *extra profits* is used. Defining the rate of extra profits by $\rho_i = r_i - \bar{r}$, equation (4.1) reads

$$(1 + \bar{r} + \rho_i)a_i + wl_i = 1. \tag{4.3}$$

Methods of production which yield positive (negative) extra profits have a cost advantage (disadvantage) compared with the average or normal conditions of production and are hence economically superior (inferior). Figure 4.1 illustrates the wage-profit relationship corresponding to equations (4.2) and (4.3).

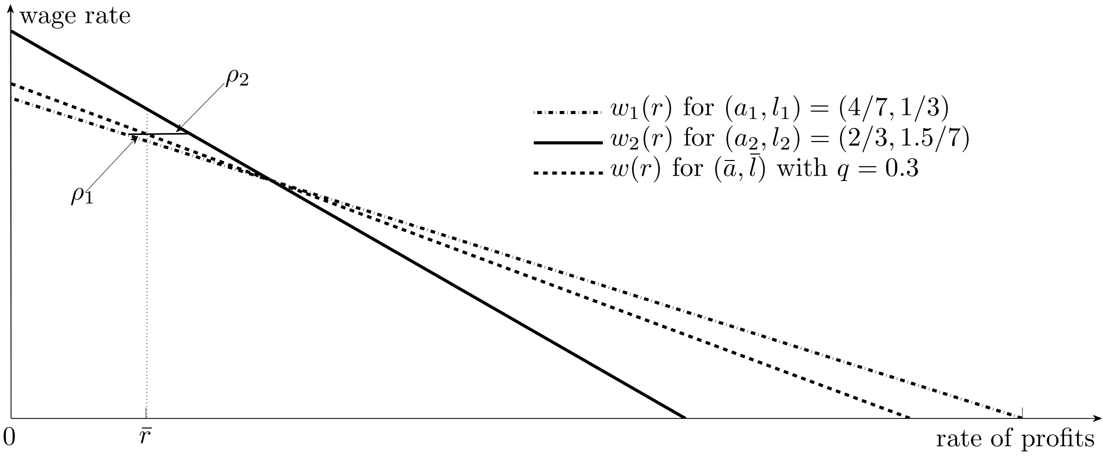

Figure 4.1: Wage-profit curves for different processes

Disequilibrium, a situation in which different methods of production co-exist and rates of extra profits are non-zero, is our starting point for the characterization of the economy's path towards an LPP. The system reaches a new LPP via two inter-related adjustments. First, an adjustment concerning quantities takes place. Its essence is that the relative significance of different methods of production changes over time: Superior methods gain ground, whereas inferior methods loose ground. The diffusion of an innovation is based on two mechanisms: *differential accumulation* and *imitation. Differential accumulation* relies on the idea that those firms which generate higher profits than others can expand at a higher rate. If there is a functional relation between past profits and firm growth, the relative significance of a cost-saving innovation will increase over time. *Imitation* is the adoption of a new method of production by a non-innovator and can be understood as the outcome of a choice-of-technique problem on the firm level. Both mechanisms are formalized in Section 4.3.

The second adjustment concerns income distribution. Since a superior method allows for a higher surplus of production, there is the question of how this surplus is distributed amongst capitalists and workers. Before the innovation enters the system with technology (a_1, l_1) and some exogenously given wage rate w_0, the (normal) rate of profit is given by $r(w_0)$. To account for the theoretical argument that profit due to innovation is transitory and acknowledging the stylized fact that the economy-wide profit rate has no long-term trend, we assume that in disequilibrium average conditions of production just generate the normal rate of profits and hence $\bar{r} = r$. This assumption implies that the wage rate adjusts according to average labor productivity. As a result of equation (4.2), the wage rate is then determined by

$$w = \frac{1 - (1 + r)\bar{a}}{\bar{l}}. \tag{4.4}$$

The wage rate is therefore endogenously determined by the normal conditions of production, influencing extra profits ρ_i by equation (4.3), which is now given by

$$(1 + r + \rho_i)a_i + wl_i = 1. \tag{4.5}$$

The adjustment of the wage rate due to a change in the relative significance of the methods of production in use thus feeds back on the diffusion process and implies a competitive pressure on firms: In order to generate the normal rate of profit a firm has to produce with the average conditions of production. Further it follows that notwithstanding profit-seeking behavior, the more widespread the use of a novel method the less profitable it becomes.

4.3 Mechanisms of diffusion

In this section we formalize the two mechanisms which account for the gradual pervasion of a process innovation. Diffusion by differential accumulation relies on assumptions on investment behavior and is studied in Subsection 4.3.1. In Subsection 4.3.2 the second mechanism, diffusion by imitation, is formalized. For both mechanisms extra profits are pivotal.

4.3.1 Firm growth

Firm growth as the driver of diffusion can be investigated by abstracting from the possibility that firms can change their current method of production. The possible strategy is therefore firm-specific investment, which is financed by past profits. According to equation (4.5), the output of a firm using i at time t is divided into wage payments, into capital investment to maintain the output-level and into profits. Output $x_{i,t}$ is produced by process i and hence divides into

$$x_{i,t} = w_t l_i x_{i,t} + a_i x_{i,t} + (r + \rho_{i,t}) a_i x_{i,t}.$$

To determine the next period's output $x_{i,t+1}$ produced by process i, the following variation of the classical investment hypothesis formulated at the level of firms is adopted: Let $s \in (0, 1]$ be the propensity to invest in case of a positive rate of profit $r+\rho_{i,t}$ and let $C_{i,t} = (1-s)\,(r + \rho_{i,t})\,a_i x_{i,t}$ denote consumption out of profit; there are no savings out of wages. Because the economy is out of equilibrium three cases have to be distinguished:

Case 1: $(r + \rho_{i,t}) > 0$. In this case the firm accumulates and hence total output produced by process i increases: $x_{i,t+1} \geq x_{i,t}$. The net-output $x_{i,t} - w l_i x_{i,t}$ which remains after paying wages is split up into investment $s(r+\rho_{i,t}) a_i x_{i,t}$ and capitalist consumption $C_{i,t} \geq 0$.

Case 2: $-1 < (r + \rho_{i,t}) \leq 0$. Using the same rule as in Case 1 would imply that firms using process i shrink at rate $s(r+\rho_{i,t})$ and that capitalist consumption $C_{i,t}$ would turn negative. Sticking to the assumption that no savings out of wages exist, the whole output $(1 + r + \rho_{i,t}) a_i x_{i,t}$ which is left over after paying wages is invested, implying $C_{i,t} = 0$. Total output produced by process i in this case declines, since $1 + r + \rho_{i,t} < 1$.

Case 3: $(r + \rho_{i,t}) \leq -1$. Since now $w l_i x_{i,t} \geq x_{i,t}$, firms using process i fail to be able to pay the total wage bill. This can happen because in period t the wage rate is determined after production has taken place. In this case it is assumed that the firm pays its laborers as far as it can and then leaves the market.

Summing up, output growth is given by

$$g_{i,t} = \frac{x_{i,t+1} - x_{i,t}}{x_{i,t}} = \begin{cases} s\,(r + \rho_{i,t}) & \text{in Case 1: } r + \rho_{i,t} > 0 \\ r + \rho_{i,t} & \text{in Case 2: } -1 < r + \rho_{i,t} \leq 0 \\ -1 & \text{in Case 3: } r + \rho_{i,t} \leq -1 \end{cases} \tag{4.6}$$

and illustrated in Figure 4.2. Comparing Cases 1 and 2 illustrates the asymmetry between firm growth and decline: The firm in the first case decides how much to grow, depending on its propensity s to invest; only if $s = 1$ it realizes its full growth potential. The firm in the second case has to de-accumulate if there is no capital injected from outside, i.e if $C_{i,t}$ is non-negative. From equation (4.6) it

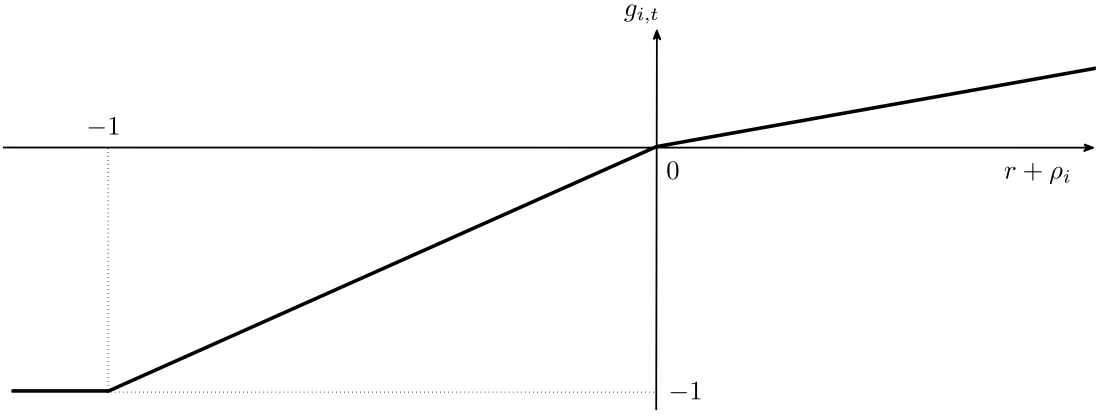

Figure 4.2: Kinked investment function for $s = 0.4$

follows that the market share $q_t = x_{2,t}/x_t$ of the innovation evolves according to

$$q_{t+1} = \begin{cases} \frac{1+s(r+\rho_{2,t})}{1+s(r+\bar{\rho}_t)} q_t & \text{in Case 1: } r+\rho_{1,t} > 0 \\ \frac{[1+s(r+\rho_{2,t})]q_t}{1+s(r+\bar{\rho}_t)+(1-s)(r+\rho_{1,t})(1-q_t)} & \text{in Case 2: } -1 < r+\rho_{1,t} \leq 0 \\ 1 & \text{in Case 3: } r+\rho_{1,t} \leq -1 \end{cases} \tag{4.7}$$

with $\bar{\rho}_t = (1-q_t)\rho_{1,t} + q_t\rho_{2,t}$ denoting average extra profits. In Case 2, the diffusion path takes place faster than for Case 1. This can be seen formally by acknowledging the negative term $(1-s)(r+\rho_{1,t})(1-q_t)$ in the denominator. An innovation meeting the condition of Case 3 leads to an extinction of the incumbent process as opposed to the asymptotic behavior of the diffusion curve for Cases 1 and 2. Two examples of diffusion paths of some specific innovative processes, which replace the incumbent process, are exemplified in Figure 4.3. They mimic the stylized fact of S-shaped curves of diffusion processes (see for example Stoneman, 2002).

The market share dynamics described by equation (4.7) is related to the model of Steedman & Metcalfe (2011), who assume that the propensity to invest is one ($s = 1$). Then the first two cases of (4.7) would coincide. By excluding Case 3, one gets

$$\frac{q_{t+1}-q_t}{q_t} = \frac{(1-q_t)(\rho_{2,t}-\rho_{1,t})}{1+r+\bar{\rho}_t}. \tag{4.8}$$

Extra profits ρ_i gained by process i therefore evolve over time according to the prevailing real wage rate w defined by equation (4.4). Steedman & Metcalfe (2011) determine prices by the marginal firm, hence by unit costs of production of the incumbent process. This implies that the original wage rate w_0, according to

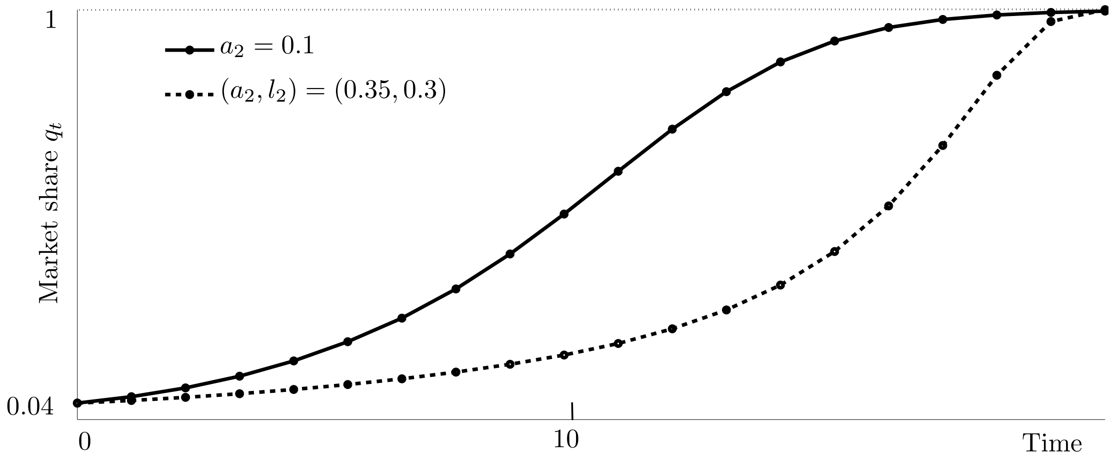

Figure 4.3: Examples of firm growth diffusion paths for $(a_1, l_1) = (0.2, 0.5)$, $r = 0.1$ and $s = 0.4$

equation (4.5), leads to constant extra rates of profit $\rho_1 = 0$ and $\rho_2 = (1 - w_0 l_2)/a_2$. Since in this case $\bar{\rho}_t = q_t \rho_2$, equation (4.8) is identical to the replicator equation

$$\frac{q_{t+1} - q_t}{q_t} = \frac{(1 - q_t)\rho_2}{1 + r + q_t \rho_2}$$

of Steedman & Metcalfe (2011). There, the feedback of changing wages is excluded from the analysis, leading to sustained positive extra profits of the innovative firms in case of unrestricted labor supply.[1] In contrast, in the present model with adapting real wages, guided by the wage setting mechanism (4.4), extra profits decrease and finally vanish.

4.3.2 Imitation

In the previous section firms were assumed to stick to their current method of production irrespective of its performance in terms of profits. Therefore diffusion takes place by differential growth alone. If, on the other extreme, firms are assumed not to grow, but to be concerned with choosing amongst available methods of production, diffusion is the outcome of imitative behavior of non-innovators. In

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[1] Steedman & Metcalfe (2011) propose a fixed supply of labor to bring the diffusion process to a halt as soon as all workers are headhunted by the innovative firm from the incumbent firms by infinitesimally larger wages (which hence do not influence extra profits). At the end of this diffusion process, a discontinuous jump of the wage rate towards the new level is assumed to restore the exogenously given original normal rate of profit.

contrast to the diffusion-by-growth mechanism, this approach involves interaction between firms to bring about knowledge transfer. To isolate the mechanism of imitation, take each firm to produce exactly one unit of output, not shrinking or growing in size regardless of the profits or losses it incurs.

At time t the state of firm k is given by $f_t(k) = i$, with $i \in \{1, 2\}$ depending on whether the firm is still using the incumbent process or if it has already switched to the new one. Let N be the fixed number of firms and n_t the number of firms using the innovative process. The market share of the innovation is then given by $q_t = n_t/N$ and, accordingly, the market share of the incumbent process is $1 - q_t = (N - n_t)/N$. Firms use the following behavioral rule: At each step in time, firms which use the incumbent process, and hence earn negative extra profits, decide on whether to imitate or not. Firms are myopic and have no a-priori knowledge about the innovation but only learn from some other firm, which is randomly drawn from the set of all firms. If the chosen firm also uses the incumbent process, nothing will change; if it already uses the innovation, the firm using the old process will switch to the new process with probability $\mathbb{P}_t$. This probability includes two aspects, *choice* and *capability*: It might be the case that one knows a superior process, but for whatever reason, for instance due to vested interests, the firm decides not to change its currently employed method; if a firm decides to adopt the innovation, obstacles such as a lack of financial resources, human capital (skills) or tacit knowledge may render the attempt to imitate unsuccessful.

The evolution of the expected number $\hat{n}_t$ of firms using the innovation is therefore given by

$$\hat{n}_{t+1} - \hat{n}_t = \mathbb{P}_t \cdot (N - \hat{n}_t)\hat{n}_t/N. \tag{4.9}$$

$N - \hat{n}_t$ is the number of firms using the old process and $\hat{n}_t/N$ is the probability that this firm choses an innovative firm with which to compare its process. The expected market share $\hat{q}_t$ of the innovation due to (4.9) is then determined by

$$\hat{q}_{t+1} - \hat{q}_t = (1 - \hat{q}_t)\hat{q}_t\mathbb{P}_t. \tag{4.10}$$

In a first approximation, one can take the probability that the innovation is adopted to be given by an exponential distribution, with the adoption probability negatively influenced by some parameter λ and positively influenced by the profit differential $\rho_{2,t} - \rho_{1,t} > 0$:

$$\mathbb{P}_t = 1 - e^{-\lambda(\rho_{2,t} - \rho_{1,t})}$$

For $\lambda \to \infty$ firms adopt the superior method whenever they get in contact with a firm already using it. Thus, equation (4.10) reduces to the logistic equation and diffusion becomes a pure epidemic process. In the other extreme, for $\lambda = 0$, no firm ever switches.

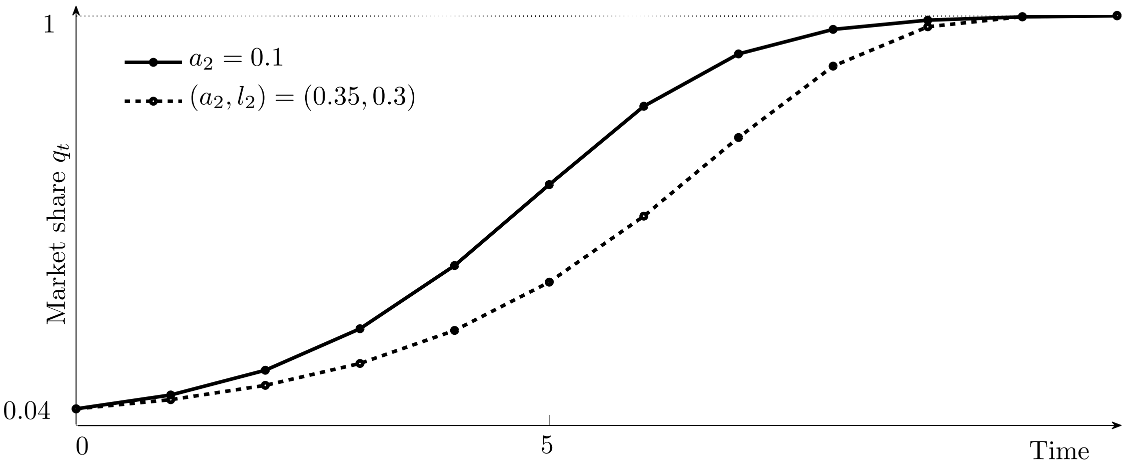

Figure 4.4: Examples of diffusion paths by imitation for $(a_1, l_1) = (0.2, 0.5)$, $r = 0.1$ and $\lambda = 2$

As shown in Figure 4.4, similar to the case of firm growth in Subsection 4.3.1, an S-shaped diffusion path emerges as indicated by the structure of equation (4.10). The latter resembles the *logistic equation* with some variable diffusion-factor $\mathbb{P}$, which serves as a measure for the *diffusion velocity.*

4.4 Disequilibrium dynamics

Based on the formalization of diffusion mechanisms, this section explores the aggregate dynamics of disequilibrium: What are the implications of the diffusion of a new method of production for the economy as a whole? As a preparatory step, in Subsection 4.4.1 we subdivide the factor space. In Subsections 4.4.2 and 4.4.3 we then analyze the dynamics of aggregate growth and the change of income distribution. Finally, in Subsection 4.4.4 we look at the industry structure.

4.4.1 The factor space

With reference to the incumbent method of production 1, any innovative process 2 is characterized in terms of the relative change of the capital and labor input coefficients

$$\Theta_a = \frac{a_2 - a_1}{a_1} \quad \text{and} \quad \Theta_l = \frac{l_2 - l_1}{l_1}.$$

Kind of technical change	**technical coefficients**	**in Figure 4.5**
capital saving and labor using	$\Theta_a < 0 \ , \ \Theta_l > 0$	ΔOEA
labor saving and capital using	$\Theta_a > 0 \ , \ \Theta_l < 0$	ΔOCD
pure capital saving	$\Theta_a < 0 \ , \ \Theta_l = 0$	$\overline{OA}$
pure labor saving	$\Theta_a = 0 \ , \ \Theta_l < 0$	$\overline{OC}$
combined factor saving	$\Theta_a < 0 \ , \ \Theta_l < 0$	$\Delta OABC$
neutral	$\Theta_a = \Theta_l < 0$	$\overline{OB}$
dominantly capital saving	$\Theta_a < \Theta_l < 0$	ΔOAB
dominantly labor saving	$0 > \Theta_a > \Theta_l$	ΔOBC

Table 4.1: Partition of the factor space I

For a given r and maximum rate of profit

$$R_1 = \frac{1 - a_1}{a_1}$$

of the incumbent process, the factor space is subdivided along two dimensions: First, according to different kinds of technical change, listed in Table 4.1; secondly, according to the degree of technical change, determining whether case 1, 2 or 3 of the investment function (4.6) applies, listed in Table 4.2.

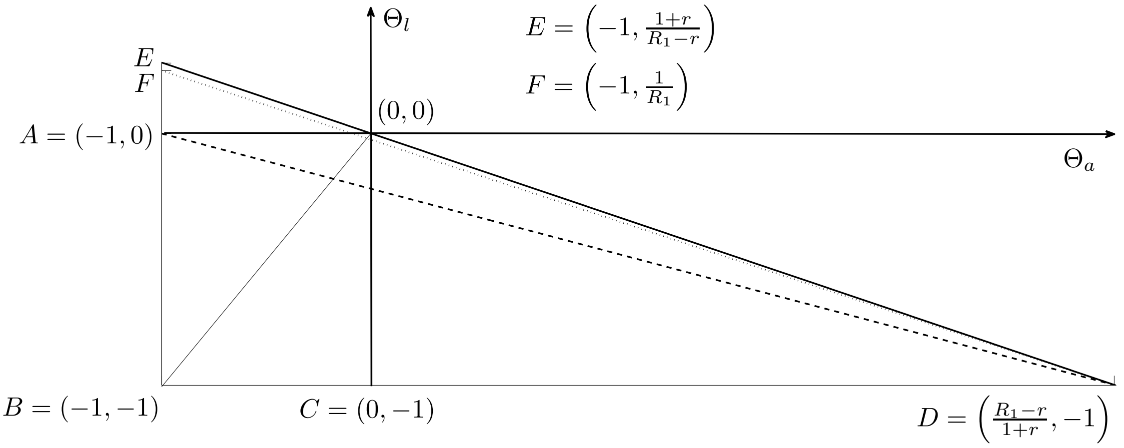

Figure 4.5: Partition of the factor space for an incumbent process for $R_1 = 4$ and $r = 0.1$

In Figure 4.5, the coefficients of an incumbent process are represented by the origin $(\Theta_a, \Theta_l) = (0, 0)$. Θ_l and Θ_a are bounded from below by -1. The line $B - C - D$ is characterized by $\Theta_l = -1$, i.e. by $l_2 = 0$. Similarly, $\Theta_a = -1$ holds for the line $B - A - F - E$, indicating that $a_2 = 0$.

Interval of total profits in the new LPP	**in Figure 4.5**
Case 1: $r \geq r + \rho_1\|_{q=1} \geq 0$	ΔDEF
Case 2: $0 \geq r + \rho_1\|_{q=1} \geq -1$	ΔDFA
Case 3: $-1 \geq r + \rho_1\|_{q=1}$	ΔDAB

Table 4.2: Partition of the factor space II

The downward sloping *iso-profit-rate line* $\overline{DE}$, given by

$$\overline{DE}: \quad \Theta_l = -\frac{1+r}{R_1 - r}\Theta_a,$$

defines the set of all methods of production (a_2, l_2) which have the same unit costs of production as the incumbent process. $\overline{DE}$ is plotted for some given positive normal rate of profit r in Figure 4.5. It divides superior methods – potential innovations – below the line from inferior ones lying above it. A method of production which lies above $\overline{DE}$ is not able to pervade the system since it exhibits higher unit costs of production. This line, which separates innovations from economically inferior methods of production, gets steeper for increasing r, with $(0, 0)$ as fixed point. If $r = R_1$, the iso-profit rate line is a vertical through the origin in Figure 4.5, implying that any method of production which is capital saving has a cost advantage irrespective of its labor coefficient.

Secondly, the line identified by separating innovations of cases 2 and 3 in Table 4.2 is defined by $r + \rho_1|_{q=1} = 0$: The line

$$\overline{DF}: \quad \Theta_l = -\frac{r}{R_1} - \frac{1+r}{R_1}\Theta_a$$

defines the set of all innovations for which the incumbent process generates exactly zero total profits in the *new* LPP. Note that for a new method of production within the triangle ΔDEF, in the new LPP the incumbent process yields a positive rate of profit. Nevertheless, the market share of the innovation asymptotically approaches 1. Despite positive absolute output growth, in relative terms the market share of the incumbent process vanishes.

Finally, the area ΔDFA contains innovations where the total rate of profit of the incumbent process lies within -1 and 0 in the new LPP. Any combination of a_2 and l_2 below

$$\overline{AD}: \quad \Theta_l = -\frac{1+r}{1+R_1}(1 + \Theta_a)$$

implies a rate of profit smaller than -1 in the new LPP. The three lines intersect at point D.

This discussion shows that for reasonable values of r the wedge of innovations which leave the inferior method with a positive rate of profit is very narrow. Thus the case in which profitability of inferior method turns negative is decisive in this model. In the following investigation we focus on this case and abstract from cases below $\overline{AD}$ in order to hold the number of firms in disequilibrium constant.

4.4.2 Aggregate growth

In this section we deal with the dynamics of the economy in disequilibrium. Both the nature of the invading method and the degree of technical change, together with the assumptions on investment behavior, play a role. Aggregate output is given by $x_t = x_{1,t} + x_{2,t}$. In the LPP, the aggregate growth rate is determined by $g_{LPP} = sr$. The aggregate growth rate g_t is given by

$$g_t = \frac{x_{t+1} - x_t}{x_t} = (1 - q_t)g_{1,t} + q_t g_{2,t}$$

with the growth rate $g_{i,t}$ of a firm using process i determined by equation (4.6). As long as $r + \rho_{1,t} > 0$ it follows that

$$g_t = (1 - q_t)s(r + \rho_{1,t}) + q_t s(r + \rho_{2,t}) = s\left(r + \bar{\rho}_t\right), \tag{4.11}$$

where $\bar{\rho}_t$ denotes the average rate of extra profit. Thus, the transient growth rate deviates from the long-run growth rate whenever $\bar{\rho}_t$ does not equal zero. Before we explore this effect, which we call *technology effect*, the second effect is introduced. The second effect, which renders aggregate growth uneven, emerges from the kink in the investment function. If the invading method lies within the area ΔDAF, equation (4.11) is replaced by

$$g_t = (1 - q_t)(r + \rho_{1,t}) + q_t s(r + \rho_{2,t}) \tag{4.12}$$

as soon as the profitability of the inferior method turns negative. This *investment effect* starts to work at some $q_t = q_0$, where $r + \rho_{1,t} = 0$, that is when the ruling wage rate equals the maximum wage rate $(1 - a_1)/l_1$ process 1 can pay. Given equation (4.4), q_0 is determined by

$$q_0 = \frac{r}{-(1 + r)\Theta_a - R_1\Theta_l}.$$

The technology effect: To rule out the investment effect, consider the case $s = 1$, in which the kink in the investment function and thus the asymmetry between firm growth and decline vanishes. The aggregate growth rate then is

$g_t = r + \bar{\rho}_t$. From equations (4.4) and (4.5) it follows that $(1-q_t)a_1\rho_{1,t} + q_t a_2 \rho_{2,t} = 0$, i.e. that

$$\bar{\rho}_t = -q_t \rho_{2,t} \Theta_a. \tag{4.13}$$

In disequilibrium it holds that q_t and $\rho_{2,t}$ are strictly positive. Therefore, the sign of $\bar{\rho}_t$ only depends on the sign of Θ_a. This implies that the sign of $\bar{\rho}_t$ remains constant. Three cases can be distinguished:

1. $\bar{\rho}_t < 0$ holds for labor saving and capital using technical change ($\Theta_a > 0$).
2. $\bar{\rho}_t = 0$ holds for pure labor saving technical change ($\Theta_a = 0$).
3. $\bar{\rho}_t > 0$ holds for capital saving technical change ($\Theta_a < 0$).

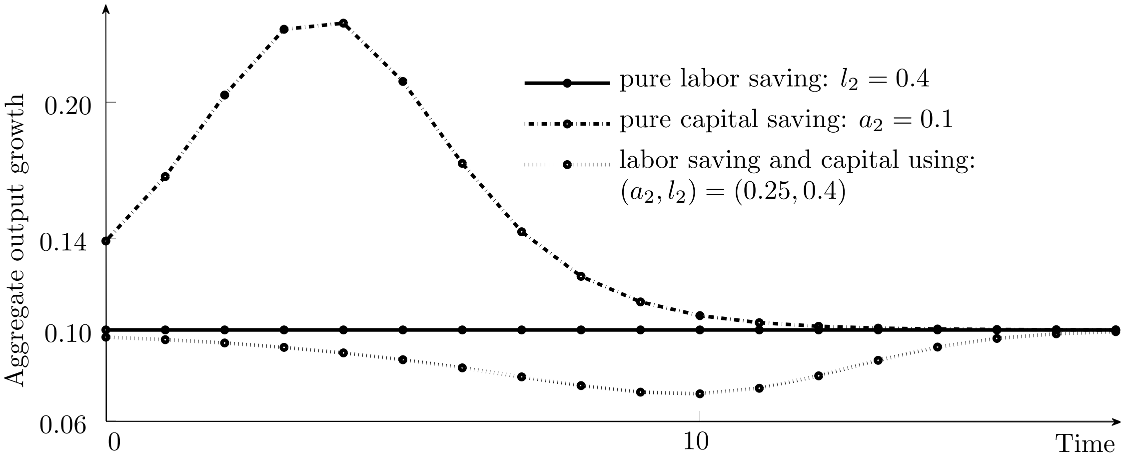

Figure 4.6: Technology effect for $(a_1, l_1) = (0.2, 0.5)$ and $r = 0.1$

Figure 4.6 illustrates the three possible patterns arising due to the technology effect. Whereas the diffusion of the pure capital saving method accelerates aggregate growth, labor saving and capital using technical change slows down economic growth; only the diffusion of the pure labor saving method shows no effect on aggregate growth.

The investment effect: The second important determinant of aggregate transient growth is the behavioral parameter s. For $s < 1$ and $q_t \in (q_0, 1)$, the investment effect is at work and g_t is determined by equation (4.12). The investment effect implies that during the diffusion the decline of the incumbent firm dominates growth, leading to a negative aggregate growth effect. The investment effect is illustrated in Figure 4.7 for different values of s. To isolate the investment effect, pure labor saving technical change is considered, because this combination of parameters does not harm steady growth for $s = 1$. The value of q_0 and the length of the period of disequilibrium are negatively correlated with the propensity s to invest. Also, the slow-down of aggregate growth is more pronounced for smaller values of s. For example, for $s = 0.25$ output declines while the superior method of production supersedes the inferior one.

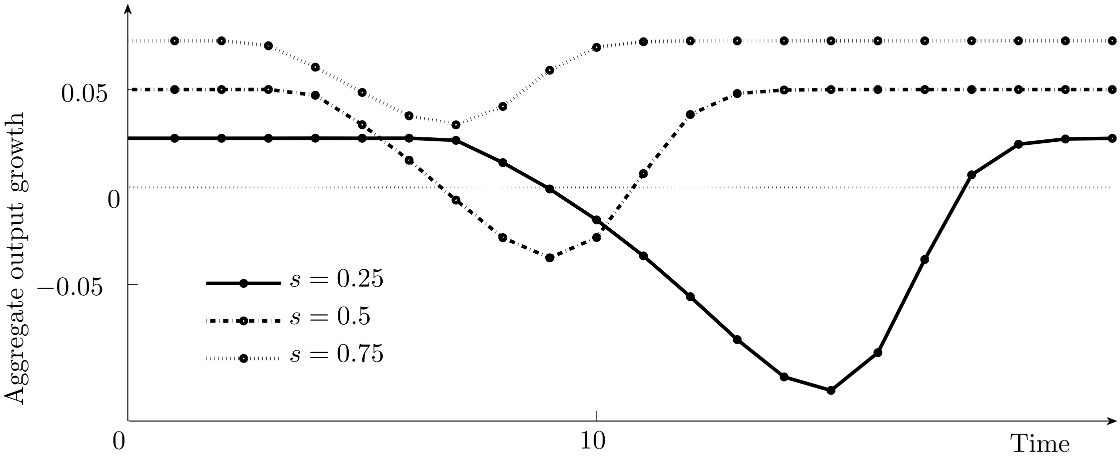

Figure 4.7: Investment effect for $(a_1, l_1) = (0.2, 0.5)$ and $(a_2, l_2) = (0.2, 0.4)$, with $r = 0.1$

Interference of the technology and the investment effect: The diffusion of a new method of production with $\Theta_a < 0$ small enough to turn the profit rate of the inferior method negative at some q_0 leads to a wave-like path of the aggregate growth rate for the following reasons: First, all firms experience a positive rate of profit and thus the technology effect accelerates growth. Yet, as soon as the profit rate of firms using the old method turns negative, aggregate growth is dampened due to the investment effect. Figure 4.8 provides an illustration.

Short term and long term effects: Transitional growth due to diffusion can be evaluated along two dimensions: the short term and the long term effect on output. The short term effect relates to the extent of the output slump after initially accelerated growth. A comparison of the first two examples given in Figure 4.8 with same capital input $a_2 = 0.1$ and different investment propensities s shows that a lower s implies a less pronounced upswing and a deeper downturn:

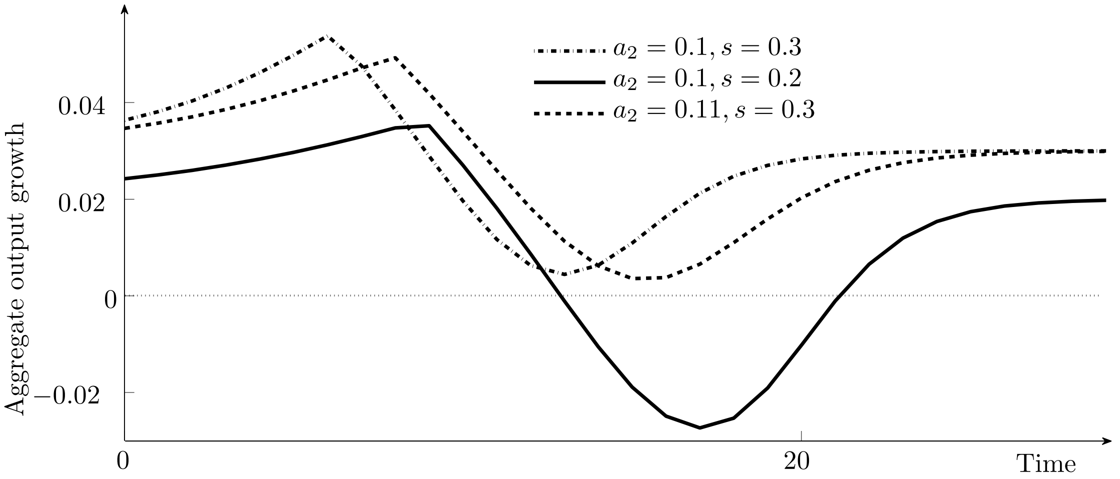

Figure 4.8: Interference of the technology and the investment effect for $(a_1, l_1) = (0.2, 0.5)$ and $r = 0.1$

The differences between the maximum growth rate and the minimum growth rate are 0.049 for $s = 0.3$ and 0.063 for $s = 0.2$ respectively.

The long term effect is the deviation of the output path from the *business-as-usual* (BAU) scenario, the hypothetical output path without diffusion taking place. This deviation is calculated as follows: For initial output x_0 and propensity to save s, BAU output at time T is given by $\tilde{x}_T = (1 + rs)^T x_0$. The relative deviation of the diffusion output from the BAU output in the long run is calculated as follows:

$$\Delta_s = \lim_{T\to\infty} \sqrt[T]{\prod_{t=1}^{T} \frac{1 + g_t}{1 + rs}} - 1 \tag{4.14}$$

This product series provides an assessment of the long term impact of uneven growth caused by diffusion. For the first two examples of Figure 4.8 with $a_2 = 0.2$, one gets $\Delta_{0.3} \approx -0,033$ and $\Delta_{0.2} \approx -0,232$.[2] Thus for $s = 0.3$ ($s = 0.2$) long-term output is 3.3% (23.2%) smaller than BAU output as shown in Figure 4.9. The analysis and numerical examples lead to the following observations: First, although technical change does not change the long run growth rate $g_{LPP} = sr$, short-term fluctuations due to diffusion in general have long-run implications on the level of output. Secondly, although an innovation may boost growth by accumulation via the technology effect, due to the investment effect the economy might end up with a lower output level compared to the business-as-usual scenario without diffusion.

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[2] The approximation arises from the truncation of power series in (4.14) at $T = 38$.

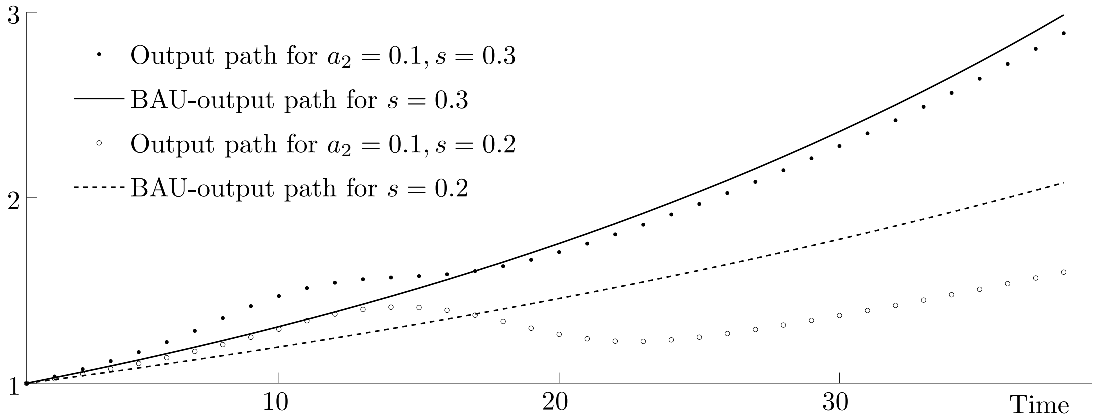

Figure 4.9: Long-term effects on the output level for $r = 0.1$

Summing up, the study of disequilibrium growth reveals that *creative destruction* (**?**, chapter VII), the replacement of old and inferior methods of production by new and superior ones and its consequences, manifests itself in different ways. In our model three intensities of creative destruction can be distinguished: (1) Asymptotic diffusion together with *relative decline* of firms using the inferior method yields a rather smooth growth path determined by technology only. Because the profit rate of the inferior methods remains positive, firms using it are not forced to exit but they co-exist with innovative firms even in the long run. (2) Asymptotic diffusion together with *absolute decline* of firms using the inferior method changes the growth regime as soon as profitability of the incumbent production process turns negative. For some kinds of technical change aggregate growth follows a wave-like path. As firms which use the inferior method gradually decline, in the long run only firms using the innovative process survive. (3) Diffusion in finite time due to *firms going bust*, which is the strongest evidence of creative destruction in out setting, is the third possible case.

4.4.3 Income distribution

In this section we explore the change of the income distribution due to the diffusion of a process innovation. To this end, the wage share ω is defined as $\omega = W/(W+P)$ with W denoting total wage payments and P total profits. In the LPP with method of production (a_i, l_i) being used, the wage share is given by

$$\omega_i = \frac{w_i l_i x}{w_i l_i x + r a_i x} = 1 - \frac{r}{R_i}. \tag{4.15}$$

with $w_i = [1-(1+r)a_i]/l_i$ denoting the wage rate process i can pay given the rate of normal profits r. $R_i = (1-a_i)/a_i$ is the maximum rate of profit of process i.

From equation (4.15) it follows that technical change influences income distribution, since the real wage rate rises due to the diffusion of a new method of production. A comparison of the wage share ω_1 before the innovative process enters with the wage share ω_2 after diffusion is complete shows the following:

1. If the innovation is capital using ($\Theta_a > 0$), $R_2 < R_1$ and the wage share falls: $\omega_2 < \omega_1$.

2. If the innovation is pure labor saving ($\Theta_a = 0$), $R_2 = R_1$ and the wage share does not change: $\omega_2 = \omega_1$.

3. If the innovation is capital saving ($\Theta_a < 0$), $R_2 > R_1$ and the wage share increases: $\omega_2 > \omega_1$.

Because the difference between the two maximum rates of profit is given by $R_2 - R_1 = -\Theta_a/a_2$, there is a symmetry between the technology effect on growth and the change in income distribution: Pure labor saving technical change neither affects aggregate growth nor income distribution, whereas capital using technical change dampens aggregate growth and reduces the wage share. All other forms of technical change increase both aggregate growth and the wage share. But, whereas the technology effect is related to the average rate of extra profits, the effect on income distribution arises from the change in the maximum rate of profit alone. Even in disequilibrium, income distribution is not influenced by the dynamics of extra profits but evolves according to

$$\omega_t = 1 - \frac{r}{R_t}$$

with $R_t = (1-\bar{a}_t)/\bar{a}_t$. The income distribution in disequilibrium therefore only depends on the exogenously given normal rate of profit and on the change in the normal conditions of production due to the diffusion. Even if the average *rate* of extra profit is non-zero, equation (4.13), which is equivalent to

$$\rho_{1,t} a_1 x_{1,t} + \rho_{2,t} a_2 x_{2,t} = 0,$$

implies that total extra profits always sum up to zero. It follows that extra profits only redistribute income within the group of capitalists but that they do not have any direct effect on the wage share. Indirectly, extra profits act on ω_t via its impact on q_t. This result is a consequence of the wage setting rule given by equation (4.4).

4.4.4 Industry structure

In this section we explore a combination of differential accumulation and imitation to evaluate the change of the industry structure as a consequence of the diffusion process. In the diffusion-by-growth model of Section 4.3.1, firms using different methods of production experience different growth histories. More precisely, two growth paths exist, one for the group of innovators and one for the group of non-innovators. Non-imitating firms gradually go out of business, and only innovating firms survive. Abstracting from the entry of new firms, the market structure in the new LPP depends on how many firms have innovated at the beginning. In the pure imitation model there is no growth. Firm size is taken to remain constant in order to isolate the effect of imitation on the diffusion process (see Section 4.3.2).

If diffusion is the outcome of both investment and adoption decisions, each firm k at time t is in a state $f_t(k) \in \{1, 2\}$ producing output x_t^k by means of process $i \in 1, 2$. To calculate x_{t+1}^k according to the respective imitation and investment behavior, the case of changing capital demand for unit production has to be taken into account. For some firm k, equation (4.6) is replaced by

$$\frac{x_{t+1}^k - x_t^k}{x_t^k} = \frac{a_{f_t(k)}}{a_{f_{t+1}(k)}} \cdot \begin{cases} s\,(r + \rho_{f_t(k),t}) & \text{in Case 1: } r + \rho_{f_t(k),t} > 0 \\ r + \rho_{f_t(k),t} & \text{in Case 2: } -1 < r + \rho_{f_t(k),t} \leq 0 \\ -1 & \text{in Case 3: } r + \rho_{f_t(k),t} \leq -1. \end{cases}$$

Hence, firm output changes from period t to $t + 1$ due to accumulation and is rescaled due to a change of the method of production if $\Theta_a \neq 0$. Irrespective of this rescaling effect, adding imitative behavior to the growth model speeds up the diffusion process. But, if firms both grow and imitate, the output path of the single firm and its long-run market share depend on its timing of imitation and on how much of the growth potential is left, which in turn depends on the investment and adoption decisions of all firms. A hint on the micro growth dynamics and the resulting industry structure is given for the following specification illustrated in Figure 4.10. Starting with 100 firms of equal size, in the new LPP the four initial innovators control about 40 % of the market. As a result some dimension of heterogeneity among firms persists in the long run.

4.5 Conclusions

Building on Kurz (2008) and Steedman & Metcalfe (2011) the paper explores the question of how a long-period position is established when multiple methods of production are simultaneously in use within a simple classical model. Whereas the LPP is characterized by uniformity of technology and profitability, in disequilibrium different methods of production co-exist and profit rates of firms differ.

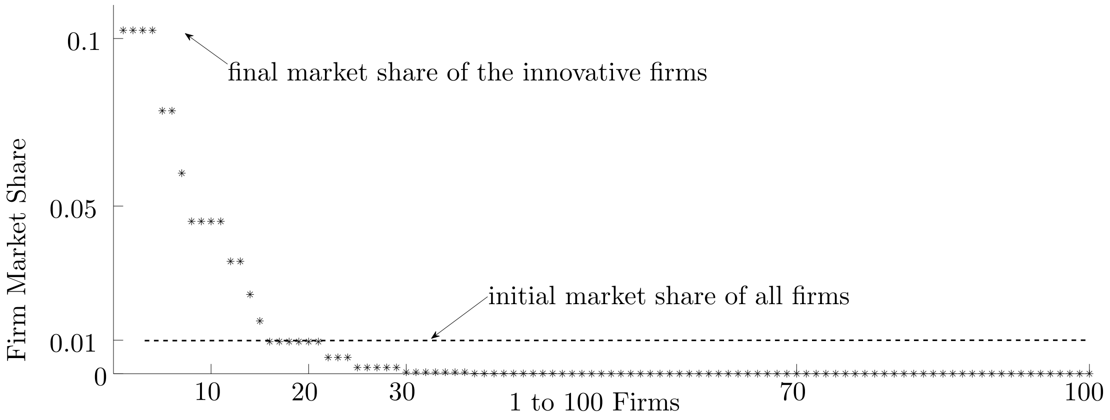

Figure 4.10: Firm size distribution in the new LPP for $s = 0.4$, $\lambda = 0.5$, $(a_1, l_1) = (0.2, 0.5)$ and $(a_2, l_2) = (0.2, 0.4)$. The initial firm market share equals 0.01

Our model is based on the concept of extra profits. This measure relates activated methods of production to the average method of production, with the latter reflecting the normal condition of production.

The dynamics of the model depends on the change in the prevailing significance of actual methods in use. Firm decisions on investment and on technology provide the basis for two mechanisms driving the diffusion process. On the one hand, differential growth relies on autonomous investment decisions of firms. Because the profitability of inferior methods of production might turn negative, an asymmetry between firm growth and firm decline arises. Whereas growth is the outcome of a purposeful firm decision, decline is a consequence of producing below the normal conditions of production. Thus, one can speak of a potential to grow, but not of a potential to shrink. Imitation on the other hand relies on direct interaction between firms. To explore the relation between differential growth and imitation, a simple imitation mechanism has been proposed relying on epidemics and on a comparison of profitability determining the probability to adopt. Both mechanisms are based on the concept of 'extra profits' and account for the stylized fact of sigmoid diffusion curves. Because of the assumed wage adjustment mechanism, for both mechanisms it holds that the more widespread the innovation the less profitable it is. This non-intended effect of individual profit-seeking behavior is at the heart of the convergence argument.

We examine uneven growth patterns, which arise due to the interference of technology and investment effects. The former relates to the kind of technical change, the latter is due to the degree of technical change, which has an effect on firm investment. The path towards the new LPP is studied for different kinds of technical change. Furthermore, the degree of technical change influences whether the innovation takes over the market asymptotically or in finite time, and it determines whether firms using the incumbent process stay in the market, implying different intensities of Schumpeterian *creative destruction*. Finally, uneven growth results in ambiguous results in long-term changes of the output level, revealing the permanent effect of technical change on economic performance.

Part II

Methodological considerations

Introduction: Is it science or is it art?

Contemporary economics to a great extent relies on mathematically formulated theories (Blaug, 2003), and the framework discussed in Part I of my doctoral thesis is part of this approach to describe economic systems. It provides an instance in which way mathematics provides *concepts* (respectively *conceptual metaphors*) such as population, species, diffusion processes and others, to understand certain aspects of economic systems. Acknowledging the importance of metaphors as figures of speech it makes sense to take a closer look at mathematics as a specific kind of language used for communication within the community of economists.

Expressing economic statements in mathematical terms is capable of reducing the complexity of economic problems and it facilitates the proof of coherence as well as the derivation of new propositions:

> As to mathematical language, why should we persist in using everyday language to explain things in the most cumbrous and incorrect way, [...] when these same things can be stated far more succinctly, precisely and clearly in the language of mathematics? (Walras, 1977 [1874], p. 72)
>
> Above all, the things to be considered are so complex that they can hardly—and never entirely accurate—be expressed by words. In addition, the basics are of exceptional clarity , but the difficulty stems from the outright and correct deduction from relatively simple data. This task now is just one of that kind for which the mathematical analysis can be utilized, and here it achieved successes. For this purpose it is essential, and this is where the mathematical method provides some results which cannot be derived without its help. (*Das Wesen*, p. 454–455)

The source concepts provided by mathematics do not exactly coincide with the target concepts of the observed economic system. This is no specific characteristic of mathematics but of any language, necessarily containing some kind of predetermination of the structure imposed on the subject matter. This situation is captured by the term *symmetry*: a symmetrization of reality is carried out to organize and to structure observations not only in an intuitive manner but in a communicable way. In this sense there is a similarity between the explanation of what is a theoretical concept and the idea of symmetries. They are two side of the same coin:

> Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection. (Weyl, 1952, p. 5)

This aspect is further elaborated in Chapter 7 by discussing the influence of beauty and aesthetics of an economic theory on its subsequent success within the economic community.

For the special case of mathematical theories, formalization therefore induces some structure on our perception of reality. Each economic theory can therefore be regarded as a piece of art, a metaphorical expression of how economic systems can be conceived. This view is closely related to what Deirdre N. McCloskey means when writing that "*Economists Are Poets*" in her book *The Rhetoric of Economics* (1998, p. 12):

> The project here is to overturn the monopolistic authority of Science in economics by questioning the usefulness of the demarcation of science from art. (McCloskey, 1998, p. 22)

This similarity between science and art is made explicit by the following example:

> Jede Sprache entwirft uns ein etwas anderes Bild unserer Welt, ähnlich wie z.B. drei Maler von ein und derselben Landschaft drei verschiedene Bilder malen werden. Allerdings können diese Verschiedenheiten nicht beliebig sein: Es muss schließlich noch ein Bild der gleichen Welt bleiben. (Wulff, 2006, p. 187)

In this vein in Part I of my doctoral thesis an attempt was made to get clear about methodological issues of economic theory as utilized .

In Chapter 5 mathematical economics is scrutinized by evaluating Schumpeter's thoughts on this topic, including the related problems of methodological pluralism and the problem of economic policy advice. The attitude of Schumpeter for using mathematics is controversially discussed in Chapter 6, where Veblen's reluctance towards the use of mathematics is contrasted with Schumpter's affection towards pure economics as point of departure of dynamic economic theories. In Chapter 7 finally the problem of economic policy advice is addressed by comparing economics with physics from a methodological point of view.

Chapter 5

Schumpeter on *Pure Economics*

This chapter is a summary of the talk on *Mathematical Economics and Methodological Pluralism: A Critical Appraisal of Schumpeter's "Pure Economics"*, which I gave at the *II International Schumpeter's Forum of Economics. Joseph Alois Schumpeter's Scientific Heritage and Today: A View from the Past into the Future* in Chernivtsi 2013.

Schumpeter's reflections on essence and scope of pure economics in his early writings (1906–1914) are discussed. This discussion is embedded in an explanation of how language and concepts influence economic reasoning. Special emphasis is put on Schumpeter's arguments in favor of the use of mathematics in the field of pure economics and on his plea for methodological pluralism in economics. For both aspects the complexity of economic systems is decisive. In this context, also Schumpeter's cautionary guidelines concerning the applicability of pure economics to problems of economic policy are outlined.

5.1 Introduction

In his first book *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie* (1970 [1908]; *Das Wesen* hereafter), Joseph A. Schumpeter (1883–1950) wanted "to scrutinize the foundations, methods and main results of *pure economics* as accurately as possible." (*Das Wesen*, p. 20; emphasis added) In his discussion he referred to Léon Walras (1834–1910), William Stanley Jevons (1835–1882) and Carl Menger (1840–1921) "who had the claim to put exact economics on a sound basis." (*Das Wesen*, p. 15) Not only the exposition of static economics, which Schumpeter at his time considered to be the only well elaborated part of economics; also a critical discussion of the methodological basics of the then dominant state of the art of pure economics was included: "How contemporary pure economics can be evaluated, what its nature, its methods and results are like, where and how to proceed: this is what we want to work out. Its limitations and weak points shall be highlighted [...].” (*Das Wesen*, p. XVIII) Schumpeter considered it necessary to provide “a final assessment on the epistemological value of our discipline” (*Das Wesen*, p. 524):

> Indeed in no other discipline such an appendix to one’s own exposition would be necessary. Objectives and methods, significance of the results – these things are much too clear to be discussed in front of a broader audience: There only some are in doubt and many do trust. Only in our case it’s the other way round. (*Das Wesen*, pp. 524–525)

This methodological assessment included a discussion of the role of mathematics as well as a plea for methodological pluralism, two aspects which are related to the complexity of economic systems.

The literature of the second half of the 20th century on the mathematization of economics (Weintraub, 2002; Blaug, 2003), on its critical discussion (Samuelson, 1952; Debreu, 1991), and on methodological pluralism (Salanti & Screpanti, 1997) refer to Schumpeter. His legacy to a considerable extent revolves around his 1912 published book *Theorie der wirtschaftlichen Entwicklung* (1952 [1926]; *Theorie* hereafter) and its English version *Theory of Economic Development* (2012 [1934]), containing his ideas on economic development (Swedberg, 1991, p. 22). Also his follow-up books *Business Cycles* (1939) and *Capitalism, Sozialism and Democracy* (2010 [1942]), dealing with the dynamics of social and economic systems, gained prominence.

Less is known about Schumpeter’s writings about the role of mathematics in the field of pure economics and, more generally, about his discussion of economic methodology. Primary sources for these issues are, besides *Das Wesen* and parts of *Theorie*, his articles *Über die mathematische Methode der theoretischen Ökonomie* (1906; *On the Mathematical Method of Theoretical Economics*) and *Die “positive” Methode in der Nationalökonomie* (1914; *The “positive” Method of Economics*). Schumpeter’s esteem for mathematics is discussed in surveys about his life and thinking for instance by Andersen (2009, Chapter 3.1) and by Shionoya (1997, pp. 43–45, 129–130). In these books some attention is also dedicated to Schumpeter’s ideas on methodological pluralism, which he discussed especially with respect to the *Methodenstreit* of the late 19th century (Andersen, 2009, Chapter 3.4 and Shionoya, 1997, Chapters 5, 8).

This chapter provides a unifying discussion of Schumpeter’s attitude towards mathematization and methodological pluralism in view of the complexity of economic systems. The relevance of mathematics in the context of economic theorizing is discussed in Section 5.2. In Subsection 5.2.1 arguments for the use of mathematics in the realm of pure economics are outlined. These are critically discussed and embedded in a more general philosophical framework of *concepts* in Subsection 5.2.2. Subsection 5.2.3 illustrated the discussion by outlining Schumpeter's thoughts on the use of abstract mathematical concepts. Then, in Section 5.3 the complexity of economic systems is given as a further argument for the mathematical toolbox. Schumpeter's claim for methodological pluralism especially due to the complexity of economic systems is outlined in Subsection 5.3.1. This leads over to the practical problems of economic policy advice discussed in Subsection 5.3.2, where Schumpeter's remarks concerning the limited applicability of pure economics to practical economic problems is outlined. Finally, Section 5.4 concludes.

5.2 Formalizing economic theories

5.2.1 Mathematics as a language

Schumpeter's arguments concerning the use of the mathematics were systematically expounded in his article *Über die mathematische Methode der theoretischen Ökonomie* (1906), and they were referred to in *Das Wesen* when appropriate. Mathematics was seen as a tool to avoid logical fallacies in the course of deductive reasoning, also facilitating the derivations of new propositions:

> [M]athematics provides a) a far more perfect representation, which enables a far better development of given concepts and a more precise, scientific reasoning. b) But in addition it is more than a method of representing; it is an independent method of research. (Schumpeter, 1906, p. 45)

It is a necessary precondition for some specific science to apply mathematics that it "is concerned with quantitative, mathematically expressible concepts, to which numbers can be assigned. And this condition definitely applies to our discipline." (Schumpeter, 1906, p. 33) Schumpeter perceives this observation as a chance to use mathematics in pure economics as the only sub-discipline of the social sciences "– and this advantage ought to be taken!" (Schumpeter, 1906, p. 36) Its usefulness is testified by the observation that "mathematics anyway cannot be avoided in our field of research; it is applied even by its harshest critics. This is because propositions about quantities are mathematical by nature, be they expressed by symbols or by words." (Schumpeter, 1906, p. 36) Schumpeter in this context echoes an argument put forward by Jevons, who wrote that "[o]ur science must be mathematical, simply because it deals with quantities. Wherever the things treated are capable of being greater or less, there the laws and relations must be mathematical in nature." (Jevons, 1965 [1871], p. 3)

This *quantity-argument* must not be mixed up with the use of numbers per se, which is part of economic statistics. To clarify this point, Schumpeter in his posthumously published *History of Economic Analysis* divided economics into

> history, statistics, and 'theory.' The three together make up what we shall call Economic Analysis. [...It is] useful [...] to introduce a fourth fundamental field to complement the three others [...]: the field that we shall call Economic Sociology (Wirtschaftssoziologie). [... E]conomic analysis deals with the questions how people behave at any time and what the economic effects are they produce by so behaving; economic sociology deals with the question how they came to behave as they do. (Schumpeter, 2006 [1954], pp. 10, 19)

Schumpeter had economic 'theory' or *pure economics* in mind when discussing mathematical economics. And pure economics is concerned

> with quantitative but hence not with numerical relations. It is most important, first, to know about the existence of functional relationships and, secondly, to know as many properties as possible from this function. (Schumpeter, 1906, p. 37)

This is the *structure-argument*, advocating mathematics as an important part of pure economics as a specific field of research concerned with the revelation of deductively derived general laws. Mathematics is therefore used as a symbolic language, facilitating the study of economic systems, since "[t]he symbols of mathematical books are not different in nature from language, adapted to the notions and relations which we need to express." (Jevons, 1965 [1871], p. 5)

It was the intention of Schumpeter and Jevons to clarify that the use of mathematics as a kind of shorthand notation of statements in terms of mathematical symbols does not harm or alter the meaning of economic theory. The utilization of mathematical symbols "does not change the line of reasoning, which determines nature and results of the respective method." (Schumpeter, 1906, p. 36) This quotation of Schumpeter resembles Jevons' statement that "[mathematical symbols] do not constitute the mode of reasoning they embody; they merely facilitate its exhibition and comprehension." (Jevons, 1965 [1871], p. 5) Mathematics can therefore be used to express an economic theory in some formal language, facilitating the proof of coherence. Once formalized, logical fallacies of verbally expressed propositions and the defective application of mathematics can be unveiled: "This reveals one of its advantages: errors can be discovered more easily." (Schumpeter, 1906, p. 32) However, the use of mathematics does not say anything about the quality of the underlying economic theory: "The wrong remains wrong, but not the mathematical symbols bear the blame. Mathematics does not protect against error." (Schumpeter, 1906, p. 31)

5.2.2 Concepts and mathematical structures

Similar to Schumpeter and Jevons, Paul Samuelson formulated the connection between mathematics and some economic theory with recourse to the property of mathematics as a language. That mathematics is a language to formulate economic theories and that it is the means by which economics can be labeled as science was the standpoint taken by Samuelson, who stated that mathematics is not only some kind of language, but that

> [m]athematics is language[, since, i]n principle, mathematics cannot be worse than prose in economic theory; in principle, it certainly cannot be better than prose. For in deepest logic–and leaving out all tactical and pedagogical questions–the two media are strictly identical. (Samuelson, 1952, p. 56)

This understanding of mathematics as language per se is more narrow than an understanding of language as suggested by philosophers of language, which differentiate between "natural languages like English, and invented languages like those of logic and mathematics." (Soames, 2010, p. 1) Thus, mathematics is not language, but it is a special kind of language. Consequently, using the formalism of mathematics does not provide a one-to-one translation of verbally formulated theories, but

> [m]athematics provides [the economist] with a language and a method that permit an effective study of economic systems of forbidding complexity; but it is a demanding master. It ceaselessly asks for weaker assumptions, for stronger conclusions, for greater generality. In taking a mathematical form, economic theory is driven to submit to those demands. [...] Mathematics also dictates the imperative of simplicity. It relentlessly searches for short transparent proofs and for the theoretical frameworks in which they will be inserted. Participating in that pursuit, economic theory was sometimes drawn by drives toward greater generality and toward greater simplicity in the same direction, rather than in opposite directions. (Debreu, 1991, p. 4)

The use of any specific kind of language therefore pre-determines how economic systems are perceived. Explaining economic systems by means of models hence implies the use of certain *metaphors*:

> Economics uses mathematical models, statistical tests and market arguments, which look alien to the literary eye. But looked at closely they are not so alien. They may be seen as figures of speech–metaphors, analogies, and appeals to authority. (McCloskey, 1998, p. xix)

The use of metaphors as well the the embedding of the used language into the conceptual framework of the audience is therefore inevitable. It is language use and the selection of chosen *source concepts*, incorporated in our communication and in our thoughts, which shape our understanding of the *target concept*, the economic system. Each individual is acquainted with certain source concepts, which are used to explain not so well understood target concepts. These concepts are linked together by *conceptual metaphors*:

> A conceptual metaphor consists of two conceptual domains, in which one domain is understood in terms of another. A conceptual domain is any coherent organization of experience. (Kovecses, 2002, p. 4)

In formal economic theories specific mathematical concepts are utilized to understand economic systems by imposing some specific structure onto them and vice versa economic considerations induce mathematical concepts. Mathematics is the target domain, and the economic system is the source domain in case of mathematically formulated economic theories. The propositions of the theory are then *metaphorical linguistic expressions*, namely "words or other linguistic expressions that come from the language or terminology of the more concrete conceptual domain" (Kovecses, 2002, p. 4) It is the aim of economic theory to extend the set of known concepts to describe economic phenomena. This especially includes the use of mathematical formalism, since especially mathematics is a proper tool to broaden the set of available concepts: "The systematic construction of such new patterns is the business of mathematics." (Hayek, 1964, p. 55) This view on economic theorizing provides a strong argument for methodological pluralism as discussed in Section 5.3.1, with each approach providing some metaphor highlighting different aspects of the target domain 'economic system'. This is no argument for the arbitrariness of economic models:

> [T]he constraint that limits the excessive production of metaphor is that there must be a similarity between the two entities compared. If the two entities are not similar in some respect, we cannot metaphorically use one or talk about the other. (Kovecses, 2002, p. 77)

5.2.3 Mathematics as a distinct method of research

As discussed by Schumpeter in *Das Wesen*, *differentiability* and *equilibrium* are two specific examples how the existence of mathematical concepts influences economic theorizing . Differential calculus was one of the better known parts of mathematics in the late 19th century within the community of economists. As Weintraub (2002, Ch. 1) showed, this was closely related to the university education of that time.

Following Schumpeter's understanding of pure economics, there is one defining property which serves as the starting point of each purely economic investigations: "Each economic agent at each time owns specific types and quantities of commodities." (*Das Wesen*, p. 120) It is the aim of pure economics to answer the questions "Which types and quantities of which commodities do individuals own? How can this particular distribution and the corresponding behavior of individuals be explained?" (*Das Wesen*, pp. 120–121) It is the variety of commodities and the number of members of the economic system which complicates things, since "almost all goods are interrelated. In a complete economy, between each two commodities at each time a fixed exchange ratio exists; in other words, each commodity can be bought or sold for some given price." (*Das Wesen*, p. 49) Pursuing the development of the system over time, one observes that quantities and prices are prone to change:

> Indeed, commodities change over time; but only slowly and gradually, and if short time periods are considered, one can observe that the overwhelming majority of goods keeps coming up. Only rarely it happens that some commodity disappears or that some new one emerges. (*Das Wesen*, pp. 127–128)

As a first approximation, unchanging conditions can be scrutinized by using some static theory. Two restrictions are imposed on the system by Schumpeter to identify it as being static: "First, that in successive time periods, by and large, the same commodities are produced and consumed; and secondly, that also quantities stay constant." (*Das Wesen*, p. 127–128) Since these conditions are best met within short periods of time, his notion of a static equilibrium is considered as prevailing in the short-run. It is a "snapshot of the economy in time. It shows all processes in a certain stage and apparently resting." (*Das Wesen*, p. 142) Schumpeter's argument on behalf of this approach invokes inertia of economic systems: For theoretical investigations an exact equilibrium is introduced by assuming that "kinds and quantities as well as the mode of use of goods do not change at all, and that there is no tendency to change for the commonest goods." (*Das Wesen*, p. 128) This approach suggests the notion of some equilibrium in the sense of time-independent variables, namely prices and quantities:

> The situation where any exchange stops is of fundamental significance for us: it characterizes the interrelation between quantities of goods purchased by individual economic agents, and it is this system of boundary points of goods purchase containing the formal identities to be studied by us without considering specific commodities and specific individuals. (*Das Wesen*, p. 129)

Pure economic theory therefore provides "a belt of equations as the boundary condition for the sphere of economic influence of individuals." (*Das Wesen*, p. 129) As a consequence, "not economic individuals or specific goods, but certain processes and relations are the content of our discussion." (*Das Wesen*, p. 131) Changes are allowed to occur only gradually; only infinitesimal changes are considered, since otherwise too many unknown changes within the system would occur:

> The value function to remain the same, quantities may not change too much. Would one get much more or much less of any good as in actuality, the whole economy would proceed differently. New modes of use would emerge such that the value function would change. (*Das Wesen*, pp. 456–457)

This restriction to small changes is the entry of the mathematical concept of *differentiability*: "To attain scientific accuracy, we must decrease the increments infinitely" (Jevons, 1965 [1871], p. 216) – which coincided with Jevon's perception of the mathematics he had in mind: "A mathematical law is in theory always continuous, so that the doses considered are indefinitely small and indefinitely numerous." (Jevons, 1965 [1871], p. 214) In this instance the available mathematical toolbox influences the kind of mathematics applied to economic theory (Weintraub, 2002). And secondly the kind of economic theorizing demands some specific way of doing mathematics, implementing the mathematical formalism as a specific kind of language, serving as a scientific tool in its own right, using its own concepts. The differential calculus introduces additional properties like continuity and (continuous) differentiability, which serve as an extension of the original idea of marginalism in economics:

> Mathematics has a correcting function. To be precise, it is not right to identify marginal value by the value of the last unit of the respective good; because also within this unit, marginal values are not the same. There is rather an infinitesimally small part to be considered. (Schumpeter, 1906, p. 42)

The discussions concerning the applicability of the infinitesimal calculus accrue from the assumed infinite divisibility of commodities: "Assumptions as for instance continuity of functions are abstractions or fictions, which are never realized in reality. The same happens in other disciplines, and also in pure non-mathematical economics it does not pose any conceptual difficulties." (Schumpeter, 1906, p. 38) The defense of applying the differential calculus is concerned with the objection that commodities are not infinitely divisible. But it is only the "tendency to acquire an infinitesimally small part of such an indivisible good." (Schumpeter, 1906, pp. 38–39) This implies the application of the differential calculus:

> [B]elieving that the quantities with which we deal must be subject to continuous variation, I do not hesitate to use the appropriate branch of mathematical science, involving though it does the fearless consideration of infinitesimally small quantities. The theory consists in applying the differential calculus [...]. As the complete theory of almost every other science involves the use of that calculus, so we cannot have a true theory of Economics without its aid. (Jevons, 1965 [1871], p. 3)

5.3 Complexity and methodological pluralism

5.3.1 Methodological pluralism in economics

Summing up the preceding discussion, economists describe economic systems by some economic theory. Defining *economic systems* thus implies a delineation of the science of economics against related sciences. The definition of economic systems as the subject matter of economics, the research programs which are initiated, as well as the tools employed to advance them show historical path dependence:

> [S]cience as a whole has never attained a logically consistent architecture; it is a tropical forest, not a building erected according to blueprint. [...] One of the consequences of this is that the frontiers of the individual sciences or of most of them are incessantly shifting and that there is no point in trying to define them *either by subject or by method.* (*History*, p. 9)

This insight is condensed in Jacob Viner's saying that "[e]conomics is what economists do" (cited by Boulding, 1941, p. 3). It is one of the many consequences of the observation that economic systems are complex systems, with the term *complexity* facing a similar problem of definition: "[T]here is no agreed-upon definition of such a complex term as 'complexity.'" (Rosser, 1999, p. 170) Our understanding of the respective system is therefore shaped by the way it is described, hence by the language we use, hence by the concepts at our disposal. Adding to the discussion of Section 5.2.2, talking and writing about complex systems is accomplished by means of languages, aiming at capturing at least "three interrelated features of complex systems–their creation, their structure, and the stability of their structure." (Kovecses, 2002, p. 137)

The complexity of economic systems motivates the *complexity-argument* (besides the *quantity* and the *structure* argument in Section 5.2.1) as an argument for the usefulness of mathematics in the field of economics:

> As to mathematical language, why should we persist in using everyday language to explain things in the most cumbrous and incorrect way, [...]

> when these same things can be stated far more succinctly, precisely and clearly in the language of mathematics? (Walras, 1977 [1874], p. 72)

This suggests that the use of mathematics not only simplifies the communication about complex systems, but also facilitates the deduction of non-trivial propositions:

> Above all, the things to be considered are so complex that they can hardly–and never entirely accurate–be expressed by words. In addition, the basics are of exceptional clarity, but the difficulty stems from the outright and correct deduction from relatively simple data. This task now is just one of that kind for which the mathematical analysis can be utilised, and here it achieved successes. For this purpose it is essential, and this is where the mathematical method provides some results which cannot be derived without its help. (*Das Wesen*, pp. 454–455)

This statement was put forth by Schumpeter for pure, static economics. For the case of dynamic economics he was not able to practice this claim accordingly: In *Theorie* as well as in his *Business Cycles* no mathematics can be found, since "a living piece of reality lies behind all of my ideas, and it is this that makes ... my theories so refractory to mathematical formulations." (Schumpeter, quoted by Swedberg, 1991, p. 118)

Hence it is the complexity of economic systems which indicates not only the use of mathematics but also an appreciation of methodological pluralism: Historical (statistical) versus pure (theoretical) economics, first principles based on individualistic versus group behavior, and static versus dynamic economics, are three instances. For the first case, Schumpeter stated that

> both 'methods' [(positive and pure economics)]–and also many more–are equally indispensable for us to find our way through the chaos of the social world. [The former is able] to solve problems inaccessible for pure economics. (Schumpeter, 1914, p. 2107)

Schumpeter has already been able to find this insight into the complementarity of theory and empirical work in Jevon's writings. Jevon defined the *complete method* as a combination of these two branches:

> In the absence of complete statistics, the science will not be less mathematical, though it will be immensely less useful than if it were, comparatively speaking, exact. [The] deductive science of Economics must be verified and rendered useful by the purely empirical science of Statistics (Jevons, 1965 [1871], p. 12, 22)

The second issue is outlined in the chapter *Der methodologische Individualismus* (methodological individualism; *Das Wesen*, Part I, Chapter VI). Whether economic theories are built on individual behavior or on social groups are according to Schumpeter complementary approaches, each one with a certain range of validity. The respective limitations depend on the questions to be answered. That Schumpeter's choice for further investigations is methodological individualism is based on the anticipation of results: "The individualistic approach in a great measure leads to useful results; within pure economics the social perspective does not provide any substantial advantage and is therefore superfluous." (*Das Wesen*, p. 95) This does not imply a general rejection of a social perspective, since

> the members of an economy are extensively interlinked, and hence effects and correlations cannot readily be seen by studying individuals. Methodological individualism and social results are anything but incompatible, but for pure theory the individual method is indispensable. (*Das Wesen*, pp. 93, 96)

Also the difference between static and dynamics economic theorizing is pivotal to the thinking of Schumpeter. The conscious distinction of these two fields dates back to John S. Mill (1806–1873), who evaluated the content of the first three books of his Principles of Political Economy as follows:

> All this, however, has only put us in possession of the economical laws of a stationary and unchanging society. We have still to consider the economical condition of mankind as liable to change. . . thereby adding a theory of motion to our theory of equilibrium–the Dynamics of political economy to the Statics. (Mill, 2009 [1885], Book IV, Chapter I; quoted by Schumpeter, 2006 [1912], p. 85)

In a similar manner, Schumpeter's second source for his perception of the dichotomy between static and dynamic economics , John B. Clark (1847–1938), advocated the view that

> static laws are [...] real laws. [...] We study them separately, in order that we may understand one part of what goes on in dynamic society. [... I]t is necessary to study the forces of progress. To influences that would act if society were in a stationary state, we must add those which act only as society is thrown into a condition of movement and disturbance. This will give us a science of Social Economic Dynamics. (Clark, 1923 [1899], pp. 30–31)

For Mill and Clark "the 'dynamic' moment is a disturbance of the static equilibrium and an investigation of the effects of this disruption and the then newly emerging equilibrium.” Schumpeter augments this notion of dynamic economics by adding a “theory of disturbances” (Schumpeter, 2006 [1912], p. 92, Footnote): “By ‘development’, only those changes of the circular flow of the economy are intended, which the system generates from within.” (Schumpeter, 2006 [1912], p. 95) This definition explains his assessment that a new methodology for dynamic economics has to be developed: “Static and dynamic are totally different fields of research, involving different methods.” (*Das Wesen*, p. 182)

Complexity therefore encourages diverse approaches:

> Almost each ‘direction’ and each single author is in the right with his statements: The way they are meant, and acknowledging their ends which they are constructed for, most propositions are true, and only rarely we are not able to extract any sense out of some statement. (*Das Wesen*, p. VI)

Additionally, Schumpeter denounced harmful discussions between different schools of economic thought, where “bordering on vandalism each glimmer of knew insight is enforced, regardless that neither in science nor in real life almost alike nothing utterly true and nothing utterly wrong is said.” (*Das Wesen*, pp. 428–429) Hence, different research questions imply different methodologies, and allowing for a multiplicity of approaches improves the comprehension of economic systems. This does not contradict his notion of the incommensurability of different theories within pure economics (*Das Wesen*, p. 325), since each theory employs counterfactuals, based on different concepts.

5.3.2 Limitations of *pure economics*

The issue of using mathematical concepts is an ontological problem of economics. It raises the problem of the limitations of pure economics both for general insights into the functioning of economic systems as well as for economic policy advice. In the first three chapters of Part V in *Das Wesen* Schumpeter discussed the gains and limitations of pure economics: “[E]conomic theory is indispensable, but just as any abstract science it provides only a modest share to our understanding of reality.” (Schumpeter, 1914, pp. 2102–2103) Despite this dismal prospect Schumpeter aimed at understanding “interrelations or functional relations” of economic quantities to “understand these quantities and their dynamics.” (*Das Wesen*, p. 33) “Correlating different states of the economy” (*Das Wesen*, p. 37) is the task of pure economics, which in his definition is “an abstract image of specific economic facts, a scheme that is used to describe them” (*Das Wesen*, p. 527). He claimed (and it was his intention) to add “nothing but the description of economic facts to their understanding.” (*Das Wesen*, p. 37)

Insights by means of pure economics were assessed as modest, and Schumpeter conceded “that pure economics is not capable of dealing with issues of social policy with its tools. These problems are a distinct field of research, exhibiting a different character and requiring alternative methods.” (*Das Wesen*, p. 51) Schumpeter concluded “that from pure economics [one] can learn nothing concerning practical problems: Different matters are important for the practitioners and for the theorists.” (*Das Wesen*, p. XI) This outlook Schumpeter kept throughout his scientific life, in his *History of Economic Analysis* complaining that pure economics is inappropriately applied to solve practical problems: “The habit of applying results of this character to the solution of practical problems we shall call the Ricardian Vice.” (Schumpeter, 2006 [1954], p. 448)

That pure economics cannot be used directly to tackle practical problems explains Schumpeter’s affection towards methodological pluralism. It is therefore no incident that he was one of the founding members of the Econometric Society, being its president 1940–1941:

> Much of what we want to know about economic phenomena can be discovered and stated without any technical, let alone mathematical, refinements upon ordinary modes of thought, and without elaborate treatment of statistical figures. Nothing is farther from our minds than any acrimonious belief in the exclusive excellence of mathematical methods, or any wish to belittle the work of historians, ethnologists, sociologists, and so on. We do not want to fight anyone, or, beyond dilettantism, anything. We want to serve as best as we can. [...] We should not indulge in high hopes of producing rapidly results of immediate use to economic policy or business practice (Schumpeter, 1933, pp. 5, 12)

5.4 Conclusions

In the late 19th century, the Methodenstreit between the German Historical School and the Austrian School was relevant for economists who thought about the foundations of this science. Knowing this controversy, Schumpeter took pains to clarify the fundamental principles and the scope of pure economics. Two goals were at least approximately met by his efforts. First, he outlined an idea of what pure economics is and what it can accomplish. Secondly, he argued in favor of methodological pluralism in the field of economic research.

Pivotal in discussing pure economics was the utilization of mathematics. Three arguments were put forth to motivate the introduction of mathematical symbolism in economics: the ‘quantity’, the ‘structure’, and the ‘complexity’ argument. The first is related to the observation that economic activities are concerned with quantitative concepts (prices and quantities of commodities), whereas the second is concerned with the structure of economic systems. Thirdly, the complexity argument is discussed by Schumpeter in favor of the utilization of mathematics. But he also advocated a kind of understanding of how science functions as a symbiotic system of different theories and approaches by advocating methodological pluralism in economics.

Finally, calling attention to the limitations of pure economic theory for solving practical problems of economic policy is of fundamental importance to Schumpeter. It is a consequence of both the discussions concerning mathematical methods as well as methodological pluralism. This to a great extent follows from the static property of pure economics. Schumpeter's statements that 'static and dynamic economics are something completely different' and that 'pure economics cannot readily be applied to practical economic problems' are interconnected by acknowledging that the real economy is not in equilibrium and one simply cannot know exactly what will happen next.

Chapter 6

Beyond pure economics

This chapter is a re-print of the article *The dichotomy between Statics and Dynamics in J. A. Schumpeter, T. B. Veblen and beyond* (Rainer & Schütz, 2014) which was submitted to the *European Journal for the History of Economic Thought* and is currently under review.

The discussion on the dichotomy between statics and dynamics is currently resolved by concentrating on its mathematical meaning. Yet, a simple formalization masks the underlying methodological discussion. Overcoming this limitation, in this chapter Schumpeter's and Veblen's viewpoints on dynamic systems generating change from within are discussed. Their perspectives on dynamic economic theorizing and evolutionary economics are compared, and it is shown how the two economists differed in their theorizing with respect to their intellectual and methodological origins. A focus is set on historical aspects of the distinction between statics and dynamics, which might be useful for future evolutionary theorizing.

6.1 Introduction

The field of *Political Economy* in the late 19th century was a multifaceted and lively body of thought. Several antagonisms appeared as a result of controversial viewpoints, interests and intellectual backgrounds of the respective authors. Methodological discussions, argued out with differing intensity were part of this. *The Scope and Method of Political Economy* by John Neville Keynes (1963 [1890]), one of the standard books on economic methodology of those days, reflects the discussed topics. Positive versus normative economics (Chapters II and III) were on the agenda just like the relation of economics to adjacent fields of research such as sociology, economic history and statistics (Chapters IV, IX and X). A great deal of methodological discussions of that time included the importance of deductive versus inductive reasoning (Chapters VI and VII). In this context, the definition and limitations of pure economics were discussed (Chapter V and Notes to Chapter IX). One dichotomy which Keynes only touched upon (Note B of Chapter IV) is that between static and dynamic economics. Hence, also this antagonistic aspect was recognized, and Joseph Alois Schumpeter in *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie* (1970 [1908]; henceforth *Das Wesen*) took pains to highlight its importance:

> My exposition is based on the fundamental distinction between economic 'statics' and 'dynamics', an aspect of utmost importance. For the moment, methods of pure economics suffice only for the former, for which its most important results are obtained. 'Dynamics' differs in every respect from 'statics', methodologically as well as in terms of content. (*Das Wesen*: XIX)

He mentioned that this division was better understood by American economists, especially by John Bates Clark: "For his theory the discrimination of statics and dynamics is crucial." (Schumpeter, 1910, p. 957) Apart from Clark's effort to promote a dynamic view, it was another American economist – Thorstein Bunde Veblen – who dedicated some of his work, clarifying the dichotomy between statics and dynamics and putting economics on an evolutionary basis. This long-lasting discussion on the classification of economics into 'static' and 'dynamic' theories was critically reflected about 50 years later by Fritz Machlup (1959), who listed 43 different definitions on the separation of these two terms, brought forth by 37 economists at his time and before him. He therefore revealed a fundamental problem concerning the respective discussions, namely that not only a rigorous definition was lacking, but also that economists used these discussions – consciously or unconsciously – to single out their own work as opposed to their predecessors and contemporaries:

> For more than twenty years I have been telling my students that one of the widespread uses of 'Statics' and 'Dynamics' was to distinguish a writer's own work from that of his opponents against whom he tried to argue. Typically, 'Statics' was what those benighted opponents have been writing; 'Dynamics' was one's own, vastly superior theory. (Machlup, 1959, p. 100)

Machlup suggested to avoid the use of the terms 'statics' and 'dynamics' whenever possible and to replace the latter by descriptions like Growth Theory, Theory of the Evolution of Economic Institutions, Time-Series Analysis, Trend Analysis, Sequence Analysis, Period Analysis, and so on, such that "more often than not we should be able to do without the terms Statics and Dynamics." (Machlup, 1959, p. 110)

In the more recent history of economics this fundamental discussion was resolved by sticking to the mathematical meaning of statics and dynamics. Theories are defined as either static, if they are formalized by means of time-independent variables, or as dynamic, if they contain inter-temporal dependencies of time-dependent variables. Hence, the terminology was adapted to formal methods, admittedly more precise and therefore less exposed to ambiguity. The clarification of the two terms from a mathematical viewpoint can be judged as a step towards a better foundation of economic dynamics. Yet, looking back in history, the discussion of the dichotomy between dynamics and statics from the standpoint of Schumpeter and Veblen more than 100 years ago shows that the discussion of these two terms in a formal framework would mask the underlying methodological discussion. Both Schumpeter and Veblen developed economic theories, which were and still are labeled as 'dynamic' or 'evolutionary', without having had recourse to mathematical formalization.

In this chapter we clarify what Schumpeter and Veblen meant when talking about dynamic economic systems. Both Schumpeter and Veblen are considered as pioneers of modern evolutionary economics. The paper hence adds to the literature both on Schumpeter (Andersen, 2006, 2009; Shionoya, 1997; Swedberg, 1995, 2013) and on Veblen (Hodgson, 2008; Latsis, 2010; Rutherford, 1996, 2011). In this literature hardly any effort to combine their perspectives on economic methodology can be found – a topic covered by the current paper. Their notion of evolutionary economics and also their understanding of dynamic economic theorizing will be scrutinized. A comparison is drawn in which way Schumpeter and Veblen understood economic systems as dynamic systems. It is shown how the two economists differed in their theorizing, reflecting also their different intellectual and methodological origins. The article therefore provides insights into the general discussion of static versus dynamic economic theorizing as well as into the different lines of argument put forth by Veblen and Schumpeter. The focus is on historical aspects of the distinction between statics and dynamics, creating a basis for evolutionary theorizing. The paper proceeds as follows: In Section 6.2, Schumpeter's and Veblen's evaluation of static economic theory are outlined. Their arguments in support of economics becoming an evolutionary science are put forth. In Section 6.3 their paths towards evolutionary economics are discussed. Section 6.4 concludes.

6.2 The relevance of static economics

Static economics is concerned with the functional relation between quantities, prices and other variables, setting aside a stream of endogenous causal relations which would be under study if allowing for inter-temporal dependencies. In *Das Wesen*, Schumpeter summarized the state-of-the-art of pure, static economics:

"Certain relationships of dependence, or functional relations, are [...] the subject matter." (*Das Wesen*, p. 33) In case of a static theory, the information is gained by looking at mutual constraints of economic quantities: "A belt of equations constrains the sphere of economic influence of the individual." (*Das Wesen*, p. 132) A static perspective comprises a "snapshot of the economy in time. It shows all processes in a certain stage and apparently resting." (*Das Wesen*, p. 142) The assumption of static behavior of economic agents, who act according to their individual circumstances, was assumed to answer the following questions: "Which kind and quantity of commodities do the individual economic agents own? How can this specific distribution of goods and the behavior of economic agents be understood?" (*Das Wesen*, pp. 120–121)

In a further step, comparative static analysis – which was then called method of variations – included the investigation of different constellations between economic variables. Within this method, as part of static theorizing, Schumpeter departed from static situations, allowing for small changes (*Das Wesen*, Part IV): "The variations are a reaction against the perturbation of equilibrium, leading to a new state of equilibrium, which – like the former – is also uniquely determined." (*Das Wesen*, pp. 451–452) Statics and comparative statics were two partial aspects of pure economics, and they were the most elaborated ones in the perception of Schumpeter (*Das Wesen*). Yet, if the system is 'too far' from equilibrium, static theory is no longer appropriate to explain the properties of the respective system. So, what to do then? An extension towards dynamics was provided by Schumpeter in *Theorie der wirtschaftlichen Entwicklung* (Schumpeter, 2006 [1912], 1952 [1926], 2012 [1934]; henceforth *Theorie*), where he introduced "a theory of the transition of the economy from a respectively given center of gravitation to another ('dynamics'), as opposed to the theory of the circular flow itself [...] ('statics')." (Schumpeter 1952 [1926]: 99) This approach was conducted within the realm of pure economics. Advancing towards a dynamic economic theory beyond pure economics, by including economic sociology, Schumpeter argued in his 'lost 7th chapter' of the first edition of *Theorie* that "no dynamic equilibrium exists. Economic development by its nature is a disturbance of the existing static equilibrium without any tendency to strive towards this or towards any other equilibrium." (Schumpeter, 2006 [1912], p. 489)

This early attempt to understand economic dynamics was clarified in *Capitalism, Socialism and Democracy* (2010 [1942]; henceforth *Capitalism*), where he wrote that "[a]s a matter of fact, capitalist economy is not and cannot be stationary. Nor is it merely expanding in a steady manner. It is incessantly being revolutionized from within by new enterprise" (*Capitalismd*, p. 28). Already ten years before Schumpeter, Veblen in his article *Why is economics not an evolutionary science?* (1898) advocated evolutionary theorizing in economics. He opted for a rigorous change of economic methodology and asserted that "the science stands in need of rehabilitation. [... It] is helplessly behind the times, and unable to handle its subject-matter in a way to entitle it to standing as a modern science" (Veblen, 1898, p. 373)), drawing thereby a parallel to the history of anthropology. Schumpeter as preparatory work for a dynamic economic theory considered the investigation of functional relations, hence the discussion of static economic theories, as pivotal to "understand these quantities and their dynamics" (*Das Wesen*, p. 33). Veblen on the contrary was concerned with the revelation of causal relations. He immediately looked for dynamic economic theories in which the past state of affairs influences the present one, and this occurs as a continuing sequence. Such "an evolutionary economics must be the theory of a process of cultural growth [...] a theory of a cumulative sequence of economic institutions stated in terms of the process itself." (Veblen, 1898, p. 393)

Further, Veblen not only drew attention to the necessity to understand economic systems as evolutionary systems, stimulated from within, but also denounced those economists who did not follow his advice. As concerns economics and its earlier development, for Veblen it had failed to keep up with time and still belonged to the pre-evolutionary sciences. In particular, this failure arose from its taxonomic character, combined with an ideal to explain every phenomenon in terms of some "natural law" (Veblen, 1898, p. 378). With such a pre-evolutionary scientific method, there was lack of a dynamic perspective or too much of a "constraining normality" (Veblen, 1898, p. 379). Economics thus did not become an evolutionary science, since "it is this facile recourse to inscrutable figures of speech as the ultimate terms of the theory" (Veblen, 1898, p. 383). This insight is similar to how Schumpeter judged pure static economics: "Dynamic phenomena in our field of research play a greater role than in the other exact disciplines. This substantially limits the epistemic value of [static economics]." (*Das Wesen*, p. 573)

Against an extension of pure static economics towards comparative statics and towards the investigation of equilibrium paths, Veblen asserted that observed changes are only interpreted as a deviation from the normal state (i.e. the static equilibrium) or as disturbing factors. Instead, for economic analysis to become dynamic the so-called disturbing factors were required to take center stage. For Veblen, within an evolutionary economic perception there would be no room for controlling principles such as equilibrium or similar concepts, since these restrict a dynamic and evolutionary perspective: "At its worst, it is a body of maxims for the conduct of business and a polemical discussion of disputed points of policy." (Veblen, 1898, p. 384)

Apart from this critique, both Veblen and Schumpeter also observed efforts for change aimed at pushing economics more towards a dynamic, evolutionary theory. Schumpeter found them in John Stuart Mill's *Principles of Political Economy*:

> All this, however, has only put us in possession of the economical laws of a stationary and unchanging society. We have still to consider the economical condition of mankind as liable to change...thereby adding a theory of motion to our theory of equilibrium–the Dynamics of political economy to the Statics. (Mill 2009 [1885], Book IV, Chapter I; quoted in Schumpeter, 1952 [1926], p. 85)

Also Veblen honored Mill's *Principles* as being focused on the explanation of dynamic processes:

> J. S. Mill's doctrines of production, distribution, and exchange, are a theory of certain economic processes and [...] he deals in a consistent and effective fashion with the sequences of fact that make up his subject matter. (Veblen, 1898, p. xx)

Yet, as already noted, Veblen saw the failure of the earlier history of political economy to develop an evolutionary science rooted in its devotion to explain every economic phenomenon in terms of some standard case or definitive formulation. Within contemporary works, he retrieved the same failure. Veblen (Veblen, 1908, 1909) referred to J.B. Clark's distinction between statics and dynamics as put forth in *The Distribution of Wealth* (1923 [1899]). There, the view was advocated that

> static laws are [. . .] real laws. [. . .] We study them separately, in order that we may understand one part of what goes on in dynamic society. [. . . I]t is necessary to study the forces of progress. To influences that would act if society were in a stationary state, we must add those which act only as society is thrown into a condition of movement and disturbance. This will give us a science of Social Economic Dynamics. (Clark, 1923 [1899], pp. 30–31)

Veblen rejected this complementarity of static and dynamic theorizing, since according to him this blurs what dynamics is really about. Hence, Clark's avenue towards a dynamic theory was considered as a dead end by Veblen. He therefore strongly criticized Clark:

> For all their use of the term 'dynamic', neither Mr. Clark nor any of his associates in this line of research have yet contributed anything at all appreciable to a theory of genesis, growth, sequence, change, process, or the like, in economic life. They have had something to say as to the bearing which given economic changes, accepted as premises, may have on valuation, and so on distribution; but as to the causes of change or the unfolding sequence of the phenomena of economic life

> they have had nothing to say hitherto; nor can they since their theory is not drawn in causal terms [...]. (Veblen, 1909, pp. 620–621; italics added)

In a similar vein, Schumpeter remarked on Clark's approach (Schumpeter, 1910). He credited Clark with having dealt seriously with the problem of economic dynamics: "An attempt to provide a theory of economic development and hence to extend the limitations of theoretical economics towards the frontiers of economic life, was carried out only by Clark." (Schumpeter, 1910, p. 961) However, Schumpeter was not completely satisfied with Clark's approach. After revising Clark's theory he concluded that it is "doubtful whether this is a satisfying theory of economic development." (Schumpeter, 1910, p. 962)

Further evidence for change in the economic method towards abandoning its non-evolutionary character and opening economics to a more evolutionary perspective were found as well in the later history of political economy. These efforts – according to Veblen – were pursued by the Austrian School and by Marginalism. However, similar to Clark's approach, these efforts were not considered as successful:

> [T]he Austrians have on the whole showed themselves unable to break with the classical tradition that economics is a taxonomic science. The reason for the Austrian failure seems to lie in a faulty conception of human nature [...]. [T]he human material with which the inquiry is concerned is conceived in hedonistic terms; [...] The hedonistic conception of man is that of a lightning calculator of pleasures and pains, who oscillates like a homogenous globule of desire of happiness under the impulse of stimuli that shift him about the area, but leave him intact. He has neither antecedent nor consequent. He is an isolated, definitive human datum, in stable equilibrium except for the buffets of the impinging forces that displace him one direction or another. (Veblen, 1898, p. 389)

Thus, also within the later history of political economy, efforts made to transform economic theorizing into an evolutionary science did fail. According to Veblen, this was simply the case because too little attention was paid to human action. Behavioral traits were just wrapped under a certain stereotype (the rational utility or profit maximizing agent), which did not allow to study changing human behavior and its consequences on economic development and, vice versa, the changing patterns in human action to changing external circumstances:

> The economists have accepted the hedonistic preconceptions concerning human nature and human action, and the conception of the eco-

> nomic interest which a hedonistic psychology gives does not afford material for a theory of the development of human nature. Under hedonism the economic interest is not conceived in terms of action. It is therefore not readily apprehended or appreciated in terms of a cumulative growth of habits of thought, and does not provoke, even if it did lend itself to, treatment by the evolutionary method. (Veblen, 1898, p. 394)

Schumpeter in a similar manner understood hedonistic behavior as a characteristic feature of economic agents in the static circular flow: "Past economic periods govern the activity of the individual [...]. All the preceding periods have [...] entangled him in a net of social and economic connections which he cannot easily shake off." (Schumpeter, 2012 [1934], p. 6) However, for a dynamic and evolutionary theorizing he regarded this stereotype as insufficient. Hence, Schumpeter and Veblen were in accordance concerning their assessment of the evolutionary character of economic systems. Nonetheless, they took different paths to formulate theories covering this dynamism. Veblen relied on the methodology of evolutionary sciences like biology and anthropology in developing an evolutionary economics. What inter alia accounted for an evolutionary science is that it is "placed in antithesis to the taxonomic and methods and ideals of the pre-evolutionary days" (Veblen, 1899, p. 123). Schumpeter, in contrast, took the static economy as a point of departure to explain economic evolution. Even in his most evolutionary book Capitalism he did not abandon it completely: "Capitalist reality is first and last a process of change. [...But] the question [...of] a perfectly equilibrated stationary condition of the economic process is [...] almost, though not quite, irrelevant." (Schumpeter, 1910, p. 394) Veblen, on the contrary, mainly criticized static economic considerations and the respective pure economic theories.

6.3 Towards an evolutionary perspective in economics

Schumpeter and Veblen aimed at developing some distinct dynamic economic theory. The former stated that dynamic economics needed some fundamentally different methodology, something which apparently was shared by the latter, who demanded a new kind of economics. Due to their different attitudes concerning the usefulness of static economics, they took different paths to accomplish their similar goal. They got their inspiration from different sources: Veblen included research fields like psychology, anthropology and biology into his considerations, which he labeled "evolutionary sciences" (Veblen, 1898, p. 374). Similarly to Veblen, Schumpeter was keen to push economics towards 'dynamics', "which is part of economics, but outside of [the static] system" (*Das Wesen*, p. 614). This research project was common to Veblen and Schumpeter. Consequently, Veblen's inclination to absorb the insights of other evolutionary sciences required economics to be, like "[a]ny evolutionary science[, ...] a close-knit body of theory. It is a theory of a process, of an unfolding sequence." (Veblen, 1898, p. 375) Hence, from a methodological viewpoint, the principle of cause and effect – in the sense of an explanation in terms of inter-temporally dependent and successive events – was urged to take center stage. What counted for a modern science is that "the modern scientist is unwilling to depart from the test of causal relation or quantitative sequence. When he asks the question, Why? he insists on an answer in terms of cause and effect." (Veblen, 1898, p. 377) This allowed the modern scientist to analyze dynamic processes as cumulative causation. According to Veblen, putting the principle of cause and effect into the foreground implies that "[t]here is no ultimate term, and no definitive solution except in terms of further action." (Veblen, 1899, p. 124) This claim for causality was in contrast to what pure static economics might afford, which first and foremost aimed at providing an understanding of "functional relations" (*Das Wesen*, p. 33), guaranteeing logical coherence.

New methodological approaches were therefore envisaged by Veblen and Schumpeter. The former was convinced not only that anthropology, psychology, and biology provide a methodological benchmark for economics, but also that these sciences contribute important insights into economic phenomena. However, different sciences exist, to be distinguished by their subject matter as well as by their methodology. And there are cross-fertilizations between different fields of research by various spill-over effects. Schumpeter in *Das Wesen* took pains, first, to delineate pure economics from other sciences, and, secondly, to scrutinize these crossover effects: "Even if comparing two sciences calls forth many concerns, nevertheless it cannot be denied that it adds to the understanding of their nature." (*Das Wesen*, p. 536) This was exemplified with respect to biology, which was capable "[of] scrutiniz[ing] the essence e.g. of economic activity, and it figures out human motivations, something economists cannot do." (*Das Wesen*, p. 538) This argument he still supported about thirty years later, when he wrote that Biology provides input for economists, for example "such a thing as social and economic Darwinism." (Schumpeter, 2006 [1954], p. 25) Such a contribution to economics was denied with regard to the field of psychology in his early writings: "[N]o methodological or tangible relation between economics and psychology exists to which we have to refer in order to arrive at our results." (*Das Wesen*, p. 544) In his later writings, this extreme viewpoint was qualified: "[I]t is necessary to glance occasionally at the developments in the field of professional psychology" (Schumpeter, 2006 [1954], p. 25).

Compared to this incidental orientation towards psychology, Schumpeter had a more differentiated opinion concerning the importance of economic history and of anthropology for economics. It was the subfield of static economics, for which he neglected any value of this and related disciplines, "[which] are of little use – if they are of any use." (*Das Wesen*, p. 553) Nevertheless he knew of the importance of these fields for dynamic aspects of economic theory, since "[f]actual reports and theories about actual events [...] are of fundamental importance for the theoretical economists as soon as he leaves the narrow limits of his exact system" (*Das Wesen*, p. 552) In contrast to Veblen, who promoted an interdisciplinary perspective in order to understand dynamic economic systems, Schumpeter regarded an almost completely 'intra-disciplinary' perspective of four different fields of research as important for economics:

> What distinguishes the 'scientific' economist from all the other people who think, talk, and write about economic topics is a command of techniques that we class under three heads: history, statistics, and 'theory.' The three together make up what we shall call Economic Analysis. [...It is] useful [...] to introduce a fourth fundamental field to complement the three others [...]: the field that we shall call Economic Sociology (Wirtschaftssoziologie). [... E]conomic analysis deals with the questions how people behave at any time and what the economic effects are they produce by so behaving; economic sociology deals with the question how they came to behave as they do. (Schumpeter, 2006 [1954], pp. 10, 19; italics added)

It was therefore considered important both to take pure economics ('theory') seriously and at the same time to incorporate related fields into economic considerations. Particularly the mutual dependence of economic and sociological research was highlighted by Schumpeter on various occasions, especially in *Capitalism*.

In a similar vein, Veblen was keen to implement an evolutionary view on economics (Veblen, 1898), which includes sociological aspects and accounts for the connection between social conditions and economic behavior: "There is the economic life process still in great measure awaiting theoretical formulation." (Veblen, 1898, p. 387) Thus, economics should have turned its back on hedonism and focus instead on individual behavioral traits – "it is the human agent that changes" (Veblen, 1898, p. 387). This would also allow the application of the principle of cause and effect in a more stringent way and would have permitted the perception of economic processes as being characterized by cumulative change – the second main focus, considered by Veblen: "All economic change is a change in the economic community,– a change in the community's methods of turning material things to account. The change is always in the last resort a change in habits of thought." (Veblen, 1898, p. 391) In *The Limitations of Marginal Utility* (1909), a similar idea concerning the methodology of economics was put forth. There, the claim for economics to deal more intensely with behavioral traits was reinforced:

> In so far as modern science inquires into the phenomena of life, whether inanimate, brute or human, it is occupied about questions of genesis and cumulative change, and it converges upon a theoretical formulation in the shape of life-history drawn in causal terms. In so far as it is a science in the current sense of the term, any science, such as economics, which has to do with human conduct, becomes a genetic inquiry into the human scheme of life; and where, as in economics, the subject of inquiry is the conduct of man in his dealings with the material means of life, the science is necessarily an inquiry into the life-history of material civilization, on a more or less extended or restricted plan. (Veblen, 1909, pp. 627–628)

A more concrete implementation of these evolutionary ideas and early claims are found in his book *The instinct of workmanship and the state of the industrial arts* (1918 [1914]; henceforth *Instinct*). There, Veblen embedded the description of evolutionary processes into a detailed treatise on human behavior. The understanding of individual behavioral traits on the micro-level was considered as a key to gain knowledge of evolutionary phenomena on a more aggregate level. Borrowing from the terminology of evolutionary sciences, he described human behavior as follows:

> [T]he life of the species is conditioned by the complement of instinctive proclivities and tropismatic aptitudes with which the species is typically endowed. Not only is the continued life of the race dependent on the adequacy of its instinctive proclivities in this way, but the routine and details of its life are also, in the last resort, determined by these instincts. These are the prime movers in human behaviour, as in the behaviour of all those animals that show self-direction or discretion. Human activity [...] can never exceed the scope of these instinctive dispositions, by initiative of which man takes action. [...] These native proclivities alone make anything worth wile, and out of their working emerge not only the purpose and efficiency of life but its substantial pleasures and pains as well. (*Instinct*, p. 1)

In *Instinct*, Veblen considered human behavior as being to some degree "tropismatic" and guided by instincts, but in the end "the ways and means of accomplishing those things which the instinctive proclivities so make worthwhile are a matter of intelligence." (*Instinct*, pp. 5–6) Thus, on the one hand human behavior was considered as being instinctive but on the other hand human action was purposeful and intentionally motivated. Further, Veblen distinguished between stable and changing patterns of human behavior, which allowed him to leave behind the hedonistic stereotypes studied in pre-evolutionary sciences. He referred to the adaptive and flexible components of human behavior in the following way:

> All instinctive behaviour is subject to development and hence to modification by habit [...] In man the instincts appoint less of a determinate sequence of action, and so leave a more open field for adaption of behaviour to the circumstances of the case [...] habits take on more a cumulative character, in that the habitual acquirements of the race are handed on from one generation to the next, by tradition, training, education, or whatever general term may best designate that discipline of habituation by which the young acquire what the old have learned. (*Instinct*, pp. 38–39)

This behavioral approach to describe economic dynamics in one aspect fits Schumpeter's notion of Methodological Individualism, which "only means that one ought to start with the actions of individuals to describe economic processes." (*Das Wesen*, p. 90) This statement was pursued both in his static theory in *Das Wesen* and in his dynamic theory within the realm of pure economics in *Theorie*. In the former case, the equilibrium conditions were derived on the basis of individual exchange relations of hedonistic agents, and in the latter case a second type of idealized agent – the entrepreneur – with a different kind of behavior was introduced:

> In contrast to the 'economic man,' who carefully calculates marginal costs and revenues of alternative courses of action on the basis of known data, the entrepreneur must be a man of 'vision,' of daring, willing to take chances, to strike out, largely on the basis of intuition, on courses of action in direct opposition to the circular flow. (Schumpeter, 2012 [1934], p. xxi)

By leaving the field of pure economics towards an evolutionary theory with this focus on behavioral aspects, Schumpeter additionally considered the feedback of social circumstances onto individual behavior. Already at the beginning of his career he stated: "Indeed we concede that social influences determine individual actions" (*Das Wesen*, p. 93). This idea was carried over to his later writings as can be seen for example in *Capitalism*:

> The economic interpretation of history does not mean that men are [. . .] wholly [. . .] actuated by economic motives. On the contrary, the explanation of the role and mechanism of non-economic motives and the analysis of the way in which social reality mirrors itself in the

> individual psyches is an essential element of [Marx's] theory and one of its most significant contributions. (*Capitalism*, p. 10)

A similar idea was also put forth by Veblen (1918 [1914]), for whom cultural, institutional and organizational changes on the aggregate level feed back on the individual level. Hence, individual human behavior was considered as being socially determined and guided by socio-economic circumstances. Despite socio-economic conditions established specific stable habits of thought, routines and norms in a society, these were not seen as being of a completely persistent character but these were expected to undergo change, which required an adaption of human behavior and led to "new habits of work and of thought in the community, and so [these changes] continually instill new principles of conduct" (Veblen, 1918 [1914], p. 17). Thus, on the one hand, consecutive change in the socio-economic environment was recognized by Veblen and Schumpeter. For both of them, this appeared on the individual as well as on the aggregate level, showing in institutional and organizational innovations or in "unremitting changes and adaptions that go forward in the scheme of institutions, legal and customary" (*Instinct*, p. 17). On the other hand, dynamic change was considered to take place in the material environment, reflecting what Schumpeter in Theorie described as process and product innovations. This perception of dynamic economic systems Schumpeter stressed when introducing his concept of creative destruction:

> The opening up of new markets, foreign or domestic, and the organizational development from the craft shop and factory to such concerns as U. S. Steel illustrate the same process of industrial mutation–if I may use that biological term–that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism. (*Capitalism*, p. 73)

It is therefore innovative behavior, which according to Schumpeter initiated economic evolution: "The changes in the economic process brought about by innovation, together with all their effects, and the response to them by the economic system, we shall designate by the term Economic Evolution." (Schumpeter, 1939, p. 83) Veblen put forth a similar idea: With the turn of societies to modern, civilized ones,

> [t]he complex of technological ways and means grows by increments that come into the scheme by way of improvements, innovations, expedients designed to facilitate, abridge or enhance the work to be done. Any such innovation that fits workably into the technological scheme, and that in any appreciable degree accelerates the pace of that scheme

> at any point, will presently make its way into general and imperative use, regardless of whether its net ulterior effect is an increase or a diminution of material comfort or industrial efficiency. (Veblen, 1918 [1914], p. 314)

Given the evolution of technologies, institutions and organizations as well as their interdependency to individual behavior, these forms of evolutionary change are not independent of each other, neither for Schumpeter, nor for Veblen. Both forms of evolutionary change, according to Veblen's understanding, appear as cumulative causation. Hence, "[w]hen a step in development has been taken, this step itself constitutes a change of situation which requires a new adaption; it becomes the point of departure for a new step in the adjustment" (Veblen, 1899, p. 191). For Veblen this process of continuous changes, both in the socio-economic environment and in the material environment, is the basis for the long-term evolution of societies: from predatory to industrial ones. Malcolm Rutherford made the twofold dimension of evolutionary change within Veblen's approach explicit: "Technological change in Veblen's work occurs at a pace and in a direction affected by the existing institutional framework as manifested in the habitual ways of thinking" (Rutherford, 1996, p. 38). These evolutionary patterns of economic and social history, the mutual interdependence of technical circumstances and social institutions (respectively individual habits and routines) are prominent in Schumpeter's thinking as well. In this respect, he was influenced by the ideas of Karl Marx:

> What [Marx's] theory really says may be put into two propositions: (1) The forms or conditions of production are the fundamental determinant of social structures which in turn breed attitudes, actions and civilizations. [...] (2) The forms of production themselves have a logic of their own; that is to say, they change according to necessities inherent in them so as to produce their successors merely by their own working. (*Capitalism*, p. 10–11)

Schumpeter hence takes the ideas of Marx to express his own evaluation of this twofold dimension of evolutionary change: "Both propositions undoubtedly contain a large amount of truth and are [...] invaluable working hypotheses" (*Capitalism*, p. 11). Thus, also Schumpeter's appraisal of Marx stresses that "[t]echnology has institutional consequences by altering material circumstances and the methods, patterns, and habits of life and thought of individuals." (Rutherford, 1996, p. 39)

6.4 Conclusion

In its contemporary meaning, a theory is called 'dynamic' as soon as it is represented by time dependent variables. Schumpeter as well as Veblen offered a view on economic dynamics, which goes beyond what contemporary economists regard as dynamic. Both held sophisticated views on the interpretation of 'dynamics' and its suitability for uncovering dynamic phenomena. Using a metaphor, one can say that both climbed the same hill of evolutionary economic theorizing from different base camps and with diverse equipment. In their dynamic economic theorizing, Veblen relied on anthropology, psychology and biology, whereas Schumpeter was closer to sociology and history. Another demarcation line between these two economists can be found with respect to the appraisal of pure economics. This is well reflected in their stance on their contemporaries' works – such as J. B. Clark – on the dichotomy between statics and dynamics and its relation to pure economic theorizing. One can say that for Schumpeter pure static economic theorizing represents the basis on which some evolutionary theory can build on, whereas Veblen regarded it as a cage, from which economics ought to break out. Despite these differences, their basic understandings of dynamic economic systems show striking similarities, especially concerning their notion of change arising from within the system.

First steps towards a combination and towards unifying single elements of their approaches can be found in some of John Maurice Clark's works (Clark, 1918, 1961). On the one hand J.M. Clark advocated his father's – J.B. Clark's – ambitions as to the development of a dynamic theory. On the other hand, J.M. Clark is credited with having stroke the balance between Veblen's repudiation and Schumpeter's appraisal of pure economics and its relation to evolutionary theorizing. Both in a Veblerian and Schumpeterian tradition, J.M. Clark (Clark, 1918, p. 196) considered one of the main problems in economic theorizing at his time "how to proceed from static to dynamic economics." Further, "[t]he significant field for present work lies in the development of more realistic economics, which may be defined, in contradistinction to statics, as dynamics." (Clark, 1918, p. 19p) In his dynamic theorizing, J.M. Clark tried to develop instruments, which capture the dynamics of economic systems. Similar to both Veblen and Schumpeter, he held the view that dynamic change emerges from within the economic system. The inclusion of social conditions and economic behavior are considered as essential for a dynamic theorizing. He also put forth interdisciplinary ideas, and hedonic behavior was regarded as insufficient for a dynamic economic analysis. Besides, in an evolving dynamic system cumulative change in both the material and the socio-economic environment is generated by innovations of different types. Competition is recognized as a dynamic and evolutionary process.

J.M. Clark thus elaborated on elements of both Schumpeter's and Veblen's approach. Nonetheless, he held the view that "[u]nlike statics, dynamics is in its infancy, and very possibly is destined always to remain in that stage" (Clark, 1918, p. 199). Keeping his diagnosis on the then prevailing state of the discussion on this topic in mind – truthfully, the dichotomy between 'statics' and 'dynamics' was and is a dynamic field of research.

Chapter 7

Comparing physics and economics

This chapter is a summary of talks given at the *Graz Schumpeter Summer School* 2011 titled *A Background-Story about Neoclassical Hegemony*, at the EAEPE conference 2011 in Vienna titled *The science of economics: Internal and external feedback effects*, at the junior fellow conference *Die Dynamik normativer Ordnungen - Beharrung, Bewegung, Bruch* titled *Lock-In im ökonomischen Wissenschaftsbetrieb: Analysen und Auswege* (Goẗhe University Franfurt am Main) and at the INEM and ESHET conferences 2012 in St. Petersburg titled *Economic Methodology Revisited.*

Two interrelated topics are discussed, namely (1) the epistemic foundations of economics and (2) sociological forces within the economic community. Both topics are important to understand the dilemma of economic policy advice. (1) From a methodological point of view, physics sometimes is regarded as benchmark-science, to be copied by economists. Common features and fundamental differences in physics and economics are discussed. (2) Economic theory by means of policy advice also influences the economic system. This influence is intensified by the emergence of dominant economic theories. The respective mechanisms are of importance, since the methodological dilemma gets intensified in case of a strong economic mainstream.

7.1 Introduction

An economic crisis implies problems for economists, who aim at predicting the outcome of economic policy action. In sound economic times, contemporary macroeconomic research concerning the forecast of the future time-path of important economic variables, such as unemployment and output, is increasingly successful (Christoffel *et al.*, 2010). The situation changes, once the economic system is in turmoil. Then the economic systems reveals its complexity and its evolutionary character, with governing laws of motion differing from those in times of smooth economic development.

Economic theories explaining out-of-equilibrium dynamics therefore ought to include dynamic and evolutionary elements as argued by Thorstein Veblen (1857–1929) in his article *Why is Economics Not an Evolutionary Science?* (Veblen, 1898; see Chapter 6). As argued in Chapter 6 with recourse to Joseph A. Schumpeter (1883–1950), it is fruitful to abstract from dynamic properties despite the evolutionary character of economic systems in order to get grasp of the basic correlations between economic objects. This was carried out by generations of economists like Adam Smith (1723–1790), David Ricardo (1772–1823), Alfred Marshall (1842–1924), Paul Samuelson (1915–2009), and many others. Building on these insights, dynamics aspects can be taken into account. Evolutionary economics (Schumpeter, 2006 [1912]; Nelson & Winter, 1982; Dopfer, 2005) as well as contemporary Dynamic (Stochastic) General Equilibrium models (Fernández-Villaverde, 2010) take steps in this direction.

This succession of research programs – from static to dynamics – can also be found in the field of physics. There the theory of Galileo Galilei (1564–1642) on the same speed of falling bodies of different weight (see Subsection 7.2.3) was covered by the more general theory of Isaac Newton's (1643–1727) classical mechanics, which itself was incorporated into the formulation of classical thermodynamics by Ludwig Boltzmann (1844–1906). Hence, the study of simple systems advanced towards increasingly complex theories. But in physics, the situation described by some idealizing model can be evaluated by means of laboratory experiments and field observations, something which is not possible with similar rigor in economics. Economists therefore commence their scientific work with theories, which cannot be found or reproduced in reality.[1] This asks for an investigation both of the basic assumptions and of the predictions of an economic model, and logical coherence gets an important feature of economic theories (see Section 7.2.3).

It is the aim of this chapter to discuss the problems of economic theory concerning tangible policy advice by means of philosophical and sociological reasoning. The former relates to the literature on the philosophical foundations of economics, as surveyed in Hausman (2008), and the latter is in the tradition of the sociology of science as outlined in Kuhn (1970) and recently by Ravetz (2012). Philosophical foundations of economics are discussed in Section 7.2, with a special focus on differences between economics and physics. Section 7.3 discusses consequences of sociological forces within the community of economists, which influence the outcome of policy advice and thus explain one channel of influence of economics on the

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[1] Even if, for instance, the Solow growth model approximately reproduces the respective data (Acemoglu, 2008, Chapter 3), this is more or less an exercise of curve fitting, giving evidence for some regularities. But these data-sets are generated by economies which not even approximately match the assumptions of the model economies.

economic system. In Section 7.4 conclusions are drawn and the consequences of the preceding analysis for economic policy decision makers and for the evaluation of their actions are discussed.

7.2 What economics can accomplish

7.2.1 The purpose of scientific research

Gaining insight into the principles of nature is of interest for two fundamentally different reasons. (1) It provides orientation in a complex environment, which at first glance seems chaotic and unstructured. This goal is met for instance by religion, by myths, by philosophical discussions as well as by common sense reasoning and, eventually, by science. (2) Knowing how nature functions facilitates the active design of the environment by means of technical devices. This is true for natural sciences, with cars, computers and many more things of every day's life, witnessing its success; also economics launched the utilization of tools such as money and markets.

Understanding and shaping nature are interrelated aspects, as new technical devices may open up new questions by revealing new phenomena, and theoretical insights on the other hand may lead the way to new applications. Besides technical inventions such as for instance cars, which are an advancement of the motorized coaches of the late 19th century also economic concepts, which shape our economic environment, were developed. Contemporary modes of payment, like bank transfer and credit cards, are examples of advancements of paying cash. *Gaining insight* can have different meanings. (1) The individual categorization of complex phenomena is related to the term *Verstehen* of Max Weber (1864–1920; see Weber, 1922, §1.I.5). (2) The pragmatic maxim of Charles S. Peirce (1839–1914; see Peirce, 1902) applies, with a focus on practical applicability of theories.

Both approaches are complemented by scientific knowledge. In the institutionalized setting of academic research, a dispute about what ought to be accepted as scientific knowledge is conducted by the philosophy of science. One of its main issues is the demarcation problem, which was prominently discussed by Karl R. Popper (1902–1994) in his early writing *Logik der Forschung* (1935; *The Logic of Scientific Discovery*). The demarcation line between scientific and non-scientific standards is not a sharp frontier but a gradual passage from what is accepted as scientifically valuable and what not. It can therefore be regarded as some informal norm (North, 1990) or paradigm, which gets formalized, evaluated and adapted time and again (Kuhn, 1970). It depends on historical and cultural circumstances, as well as on technological possibilities. Hence, accepted scientific methodology is exposed to similar forces as norms and institutions in social systems, since a scientific community is a social system as investigated by the sociology of science (see for example Ravetz, 2012).

Two counteracting tendencies drive the evolution of modern scientific theories, on which policy decision makers rely. On the one hand, the subject matters and respective methodologies get increasingly diversified. The scientific community as a social evolutionary system evolves from simple to increasingly complex structures; but also a tendency towards a unification of methodology exists within some science and between sciences. Causes of the intra-disciplinary unification of scientific methodology are outlined in Section 7.3. The inter-disciplinary methodological unification is based on the abstracting nature of science, leading to the observation that different fields of research can be described by similar conceptual frameworks. Inter-disciplinary stimulations are the result, as for example in the 19th century Newtonian mechanics influenced classical and neoclassic economics (Mirowski, 1991); and Charles Darwin's (1809–1882) theory of evolution (Darwin, 1859) had an impact on evolutionary economics and institutionalism (Veblen, 1899).

One specific example of modern economic theorizing is the adaption of the concept of the Brownian motion of gas particles (Einstein, 1905), as a model for the stochastic process of option pricing, described by Black & Scholes (1973). More generally, theories are influenced by the language in which they are formulated (see Chapter 5). Objects of study and their interrelations are described by some language, which co-determines the perception of the world and the limits of some theory. This also holds for natural sciences (Wulff, 2006). Especially the language of mathematics is of importance in contemporary economic theorizing (as well as in most natural sciences, including physics and biology). Inter alia, at least three important ingredients exist: Firstly, logical reasoning as introduced by Aristotle (384–322 BC) in his *Organon* into western philosophy; it is the formal framework, which ensures internal coherence of some theory. Secondly, set theoretical foundations as introduced by Cantor (1874) are part of the basis of modern mathematics, and at the latest since Arrow & Debreu (1954) formulated the existence theorem of competitive equilibria, set-theory and convex analysis are integral parts of modern economic theory. And, finally, it was David Hilbert (1862–1943), who advocated the axiomatization of mathematics, an approach which was originally introduced by Euklid (360–280 BC) in his *Elements*. The use of axiomatically founded theories can be observed throughout modern science, introducing structuralism into the philosophy of science.[2]

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[2] The focus of this analysis is put on axiomatically founded theories. But this is no necessity. To indicate one alternative approach, take the field of artificial intelligence and the utilization of neural networks (e.g. Lek & Guégan, 2000). Axioms are replaced by some network, which can learn by being exposed to real world data. This is a fundamentally different approach of how knowledge can be gained and how complex dynamic systems can be studied.

7.2.2 Characterizing economic theory

Theory-building in economics ranges from verbal reasoning (appreciative theorizing) to mathematical modeling approaches:

> Although [*formal* and *appreciative* theorizing] are quite different, both kinds of theorizing are necessary for economic understanding to progress satisfactorily, and there are strong if subtle connections between them. (Nelson & Winter, 1982, p. 46)

Thus, verbal and formal models are developed by economists, with the difference being one of language, capable of focusing on different relationships and phenomena. In case of mathematically formulated theories, originally unintended characteristics are added, or vice versa the mathematical toolbox at hand suggests specific assumptions. This is important to acknowledge, as throughout the last century, mathematical reasoning within economics took off (Blaug, 2003; see Chapter 5). For instance, take the explanation of price formation as brought forth by marginalism (Jevons, 1965 [1871], and proximate literature) and promoted by constraint optimization approaches (Samuelson, 1947, and proximate literature). The Lagrange formalism used there does not work without differentiability of the utility function. Thus, available mathematics added a special feature to the theory (differentiability; see Chapter 5). The demand for logically consistent and mathematically tractable theories therefore implies the use of mathematical gimmicks. This language-dependent feature of some theory does not only apply to the differences between verbal and formal arguments, but to the difference between languages and cultures. Chinese and Latin-Romanic tongues lead to different perceptions of reality, implying different cultural heritages and therefore generate different theoretical scientific frameworks (Wulff, 2006, p. 187). Each language is specific in describing economic phenomena. This implies a strong argument for methodological pluralism (see Chapter 5).

The characteristics of economic models can be worked out by comparing it with physics as a science dealing with a different subject matter. Economic models are based on specific assumptions, and they aim at explaining specific economic phenomena. This strategy is also utilized by physics. The success of physical sciences with the advent of Newton's classical mechanics led economists to the belief that economics can be practiced by applying the same methodology. Milton Friedman (1912–2006) in his influential article *The Methodology of Positive Economics* (Friedman, 1953) prominently explicated this view. Friedman claimed predictability of economic theories to be the yardstick for their evaluation, something which worked properly in case of physics and was promoted by him as the predominant strategy of economic scientific research. As indicated in the Introduction to this chapter, forecasting is to some extent possible in economically sound times, if some near-equilibrium situation prevails. Things get problematic in times of change, which indicate a state of the system off equilibrium. In this case, economics and physics differ from each other fundamentally from a methodological point of view.

Newton based his theory of gravitation on a set of basic assumptions (axioms), which can be formalized (translated into mathematical language). This leads to abstract objects such as a point mass (classical mechanics). Succeeding theories introduced concepts such as space-time (general theory of relativity), fields (electromagnetism) and quantum fields (elementary particle physics). From an ontological point of view, none of these objects exist ‘in reality’, but the respective theories appropriately describe reality (respectively our perception of reality) within certain boundaries. This was the scheme adopted by Friedman (1953), where the standpoint was defended that also in economic theory the underlying assumptions (and objects) do not matter: only the predictive force of the theory counts. And economic objects like the representative household, the profit maximizing firm, the aggregate production function and other concepts are comparable with respect to their ontological meaning to those just mentioned concepts invented by physicists. Hence, there are similarities in the structural framework, but additionally there are fundamental differences, which emerge as soon as observations of the subject matter come into play.

7.2.3 Economic theory and data

Two kinds of observation exist: either by means of laboratory experiments or by observing the system without artificially set boundary conditions (which is the typical mode of data-collecting not only in economics, but also in astronomy and meteorology, for example). One reason of the success of physics is the possibility to construct appropriate experiments to compare the theory’s predictions with observed data. As an example, take Galilei’s assertion that two bodies regardless of their weight fall at the same speed, if simultaneously unhanded from the same height. This statement was shown to be true on theoretical ground in his book *Discorsi e Dimostrazioni Matematiche* (1638, in the chapter on *natural acceleration*) by some simple thought experiment, based on the concept of *weight* of some object. Now go to the laboratory, take a vacuum pump and a long pipe, and organize a race between two balls of different weight. Even if your gauge is sufficiently precise, the theoretical prediction will always be rejected because inevitable measuring errors occur. But after a sufficiently large test series, statistical tests of significance will affirm the Null-Hypothesis for an appropriate level of significance. Two goals are met by this kind of research. First, a theoretical model is set up to gain some idea of how the world can be explained. Some natural law is invented (not detected, to be clear), which, secondly, can be utilized to predict the behavior of a well-defined closed system. These two issues are intertwined (as is the practice of doing research by theory and experiment) in a complementary manner. The latter is capable of leading the way to technical applications, whereas the former serves the needs of mankind for guidance in a complex world.

Physics was quite successful in combining theory and experiment, and human kind exploits this success by developing new products and new production processes as a result of research and development efforts. In this respect, Newtonian mechanics can be regarded as being as true as quantum mechanics, as long as the field of application of the respective theory is chosen properly. The difference between these two physical theories is the theoretical framework, since different concepts are introduced. In the case of Newtonian mechanics it is the phase space of some system of mass points, spanned by space and momentum; in the case of quantum theory, it is the function space of wave functions of the elementary particles of the system. This view deviates from the view that theories are nested, one (quantum mechanics) including the other (Newtonian mechanics). Indeed on formal ground Newtonian mechanics can be shown to be a limit case of quantum mechanics (as well as of relativity theory; see any textbook on quantum mechanics and relativity theory).

Physical and economic modeling facilitate the understanding of how physical respectively economic systems behave in the presence of well specified circumstances. But in economics there is a lack of possibilities of falsifying some theory, therefore the evaluation of economic theories has to differ from physics; hence, it also has to deviate from the methodology of the natural sciences. Since the basic assumptions of economic models are quite restrictive, relying on the ceteris paribus assumption to fix boundary condition, real-world observations do not lead to an appropriate evaluation of an economic theory; and laboratory experiments of social systems are not possible due to the reflexive autopoietic characteristic of economic systems (see for example Luhmann, 1986). This gap of experiments in economics cannot be filled by experimental economics (for a survey of this topic see for example Plott & Smith, 2008). In experimental economics the microeconomic behavior of agents is tested. These findings can be incorporated into the assumptions concerning the behavior of economic agents influencing macroeconomic outcomes. But the behavior of economic agents in every day's life cannot be simulated under laboratory conditions: Complex biological systems, as well as systems investigated by astronomy and meteorology, are autopoietic. This characterizes the capability of these systems to develop macro-patterns, which cannot be predicted properly by studying the micro-behavior of the respective particles the system consists of. To make things worse for economists, the reflexivity-property in social systems (hence also in economic systems) which is added to the autopietic characteristic includes a change of behavior on the individual level as a response to macro-patterns. An isolated study of the elements of the system (hence of economic agents) is of a different quality as in the natural sciences.

One further comment can be made concerning the evaluation of economic theories. Friedman (1953) claimed economic theories to be evaluated by their predictive force. This is possible only with restrictions as outlined in the Introduction: in economically sound times; and this only because the extrapolation of some curve fitting process cannot be far from the truth for short time predictions. The alternative is a rigorous inspection of the assumption (of the axioms) of the theory. But as a consequence of the discussion above, this tells us only little about the practicability of some theory. This dilemma is sharpened by acknowledging that economic theories do not only explain the world outside, but that they are themselves a crucial part of economic life. The existence of economic systems depends on the existence of economic theory, and both interact with each other. As historical and cultural circumstanced shape economic life, the respective theories accrue out of this social environment. This aspect is surveyed for example by Canterbery (2011), where the history of economic thought is embedded into the history of mankind. Vice versa, economic theories directly shape economic system as MacKenzie (2006) shows for the financial markets, coining the term *performativity* (MacKenzie *et al.*, 2007). George Soros' *reflexivity* (Soros, 2008) also covers this phenomenon. Hence, economic theory influences policy decision making and therefore also economic systems themselves.

7.3 Internal feedback effects

7.3.1 How economic theories succeed

Scientific methodology tends to be multifaceted and at the same time has the tendency to get standardized as scientific communities are social systems, exposed to evolutionary forces. Following the argumentation of Herbert Spencer (1820–1903) in his *First Principles* (Spencer, 1864), evolutionary (social) systems develop an increasingly complex structures as time goes by. This tendency is counteracted by the formation of social norms, which facilitate social life by creating continuance (North, 1990); in case of science, they guide education, collaboration and evaluation. The situation in economics underpins this analysis, since a wide range of schools of thought exists, demonstrating the evolutionary forces towards increasing multiplicity of methods; nevertheless a strong branch of mainstream economics existed throughout certain ages. Two examples are the dominance of Keynesian economics in the aftermath of World War II and the nearly monopolistic position of neoclassical economics at the end of the 20th century.[3]

It is the aim of this section to explain some of the underlying forces driving economic sciences towards a strong mainstream.[4] From an evolutionary point of view (surviving-of-the-fittest), one can argue that those theories prevail, which offer the best explanation of real world phenomena. This implies the surviving of the theory with the best fit concerning the available data in the sense of Popperian falsificationism. Falsificationism means that some theory is scientific if and only if it can (at least in principle) be falsified. But in Section 7.2.3 it was argued that for economics this is not an appropriate strategy of evaluation of some economic theory. From a sociological and psychological point of view, a heuristic line of argument can be provided, explaining certain characteristics of some theory, which enhance its success within the economic community. Three of them are: aesthetics, simplicity and authority. They are present in the case of neoclassical economics, which is the prevailing research paradigm at the beginning of the 21st century. Firstly, apart from technical difficulties, there is hardly anything to say against the simplicity and aesthetics of constrained optimization. Writing down a complex problem in just two elegant lines is pretty appealing. And authorities are also easy to find in the history of neoclassical economics. Alfred Marshall and his textbook *Principles of Economics* (1920 [1890]) and Paul Samuelson with his textbook *Foundations of Economic Analysis* (1947) are only two examples. As one theory gains some prominence, internal feedback effects within the community of economics can be identified, which boost a further diffusion of the respective theory within the community. This mechanism, which is the fourth cause of monopolization, is explained in the following subsection.

7.3.2 Internal feedback effects within the economic community

Science aims at producing scientific knowledge, and the community of economists can be regarded as a facility to produce economic knowledge in terms of scientific papers. Evaluation of scientific knowledge in this context is not only a matter of philosophy, which was discussed in Section 7.2, but it is also sociologically determined. Within the scientific community, quality is measured by means of citations. What can be counted – and this is state of the art in contemporary evaluation of scientific success – is the number of articles published in certain journals; the

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[3] See Palley (2004) on the handover of power from Keynesian to neoclassical economics, shifting economic policy towards neoliberalism and thus demonstrating the strong relation between economic theory and policy.

[4] See Dequech (2007) for a discussion and definition of what can be regarded as *mainstream economics*.

number of citations of published articles, weighted by the importance and ranking of the respective journals (Bornmann *et al.*, 2008; Lee *et al.*, 2010) provides information about the quality of certain scientific contributions. This generates an incentive for scientists to publish as many papers as possible in journals as reputable as possible.[5]

To exemplify the situation, assume the existence of two methods, A and B, which are able to deal with some specific economic issues. Let one of these two methods be the dominant one in the sense that more economists apply it and more journals publish articles using this method. As a second assumption, two different types of economists exist, each preferring one of the methods at hand. Each scientist is assumed to have one chance to decide, which method to apply throughout his scientific career. The more dominant method A is, the easier it is for a young scientist to get a foothold in the economic community by also applying A. Thus, there is a positive feedback concerning the use of one specific method. If the dominance of A gets overpowering, even for type-B economists it becomes more profitable to apply method A. The presented model is inspired by Arthur (1989) and is a variation of the choice-of-technique problem for producing firms. An economist in this context can be interpreted as a firm, producing scientific papers, while increasing returns external to the firm and internal to the industry (the economic community) prevail. The result is a possible lock-in of some method, which further on constitutes some informal norm (the mainstream) within the economic community. A second positive feedback loop on the level of the journals exists: The more prestigious a journal is, the more often its articles are read; hence, they are cited more often, leading to a higher impact factor of the journal, thus increasing its standing. The just described relation between economists and journals is indicated by arrow e in Figure 7.1. Even more, a network of editors exists, with one editor sitting in the board of several journals, such that even the journals themselves are closely tied together (see Hodgson & Rothman, 1999). This implies network externalities in the publishing industry.

At least two counteracting forces in both feedback loops exist in addition to the evolutionary argument Spencer, 1864. Firstly, filling a niche by applying an alternative method, contradicting the prevailing mainstream, might be honored and thus reduces the monopolistic forces identified above. Indeed, throughout the past decade a diversification of economic methodology can be observed (Colander *et al.*, 2004). Secondly, high-level journals attract the best economists (however ‘best’ is defined). Thus, they can publish the most promising papers and eventually are actually of better quality than journals with a lower impact factor. This would

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[5] Almost inevitably, this incentive structure leads to the occurrence of dubious scientific practices, as documented by Bennett & Taylor (2003) for the case of medical research. Guest authorship, paper slicing and pressure authorship are just some examples.

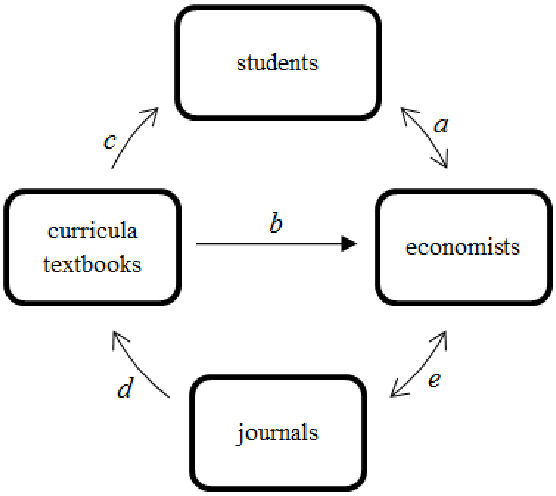

Figure 7.1: Feedback-loops within the economic community

imply that network externalities de facto lead to better quality and therefore lead to an efficient outcome. Continuing the preceding discussion, the most prestigious journals are most likely to supply the content of economic textbooks (even if this occurs with some time-lag), which is indicated by arrow d in Figure 7.1. This implies a bias of undergraduate and graduate students towards mainstream economics due to the literature they have at their disposal (arrow *c* in Figure 7.1). Johansson (2004) for instance investigates the bias of contemporary graduate textbooks towards non-evolutionary economics. Since the teaching of senior economists is also guided by textbooks (arrow *b* in Figure 7.1) as well as by the lecturers personal opinion and scientific background (arrow *a* from economists to students in Figure 7.1), these tendencies shape the undergraduates attitudes towards certain methodological approaches within the economic community. This was notably demonstrated by Colander & Klamer (1987) and Colander (2005). It is depicted in Figure 7.1 by arrow *a* from students to economists, as an undergraduate gets an economist and hence transfers his convictions from the class of students to the class of economists.

In the face of the existing monopolizing tendencies within the economic community, at a glance depicted in Figure 7.1, a discussion on the broadness and depth of graduate and undergraduate economic curricula evolved (Colander & McGoldrick, 2010). Acknowledging the hitherto alleged reasoning, the focus ought to be put on a balanced curriculum, including different approaches. This implies a supporting education in sociology, philosophy and in the history of economic thought (Kurz, 2011). This yields knowledge of the evolution of economic theories and therefore provides hints how to develop and implement new ideas. Furthermore, past theories are not always out-dated, but just not part of mainstream economics. For the case of economic policy advice, this pluralistic approach to economic methodology prevents narrow-minded argumentation, which is of importance especially in time of economic turmoil.

7.4 Conclusions

In this chapter I argued that economics is not capable of properly dealing with data in the sense that reliable forecasts are possible. For policy decision makers and analysts, this has at least two implications. Firstly, extrapolations of economic data-sets for the sake of forecasting the consequences of some specific policy action cannot be conducted with the necessary accuracy; hence, there is only limited capability to predict the net outcome of some policy action, since fundamental uncertainty in the sense of Frank H. Knight (1885–1972) prevails (Knight, 1921). Secondly, economic theory influences economic policy, and indirectly therefore influences the dynamics of the system. The results may be intended or not, they may be valued positively or not, but the policy action per se cannot be evaluated, as the outcomes of possible alternative choices or of some business-as-usual policy (the default-option) are not known.

These two dilemmas for politicians, namely a lack of a-priori certainty about the outcome of their decisions and the limited a-posteriori possibility to evaluate the respective decisions, are not easy to resolve. Problems get even worse, if some dominant economic theory governs policy action, as can be witnessed by the parallel advent of neoclassical dominance in the field of economics and neoliberal politics in the 1980s and 1990s (Plehwe *et al.*, 2007). Several reasons for the existence of some predominant theory were given, suggesting that one way to overcome the just stated policy dilemmas is the broad support for methodological pluralism, which is a direct consequence of the discussion of unifying tendencies within the economic community.

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