

Daniel Reisinger

# Cascading tipping in real-world networks

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Institute of Biology, UniGraz

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Symposium, 2025



# What is tipping?

Tipping is the phenomenon of rapid reorganization of a system in response to small changes in environmental conditions.

Bastiaansen 2022



# Nature

## Tipping in shallow lakes

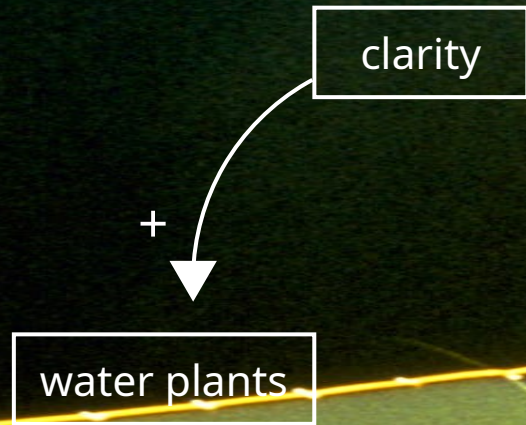


Photo credit: IISD Experimental Lakes Area (IISD-ELA)  
Icon credit: Surang (Flaticon)



# Nature

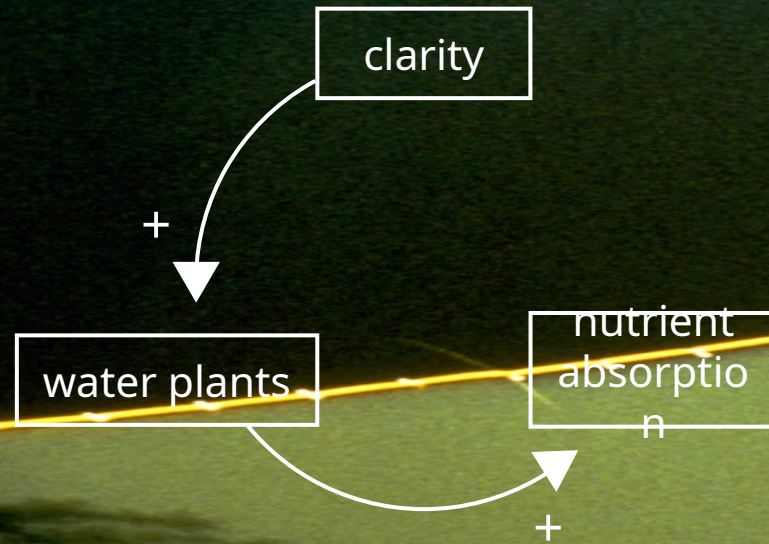
## Tipping in shallow lakes





# Nature

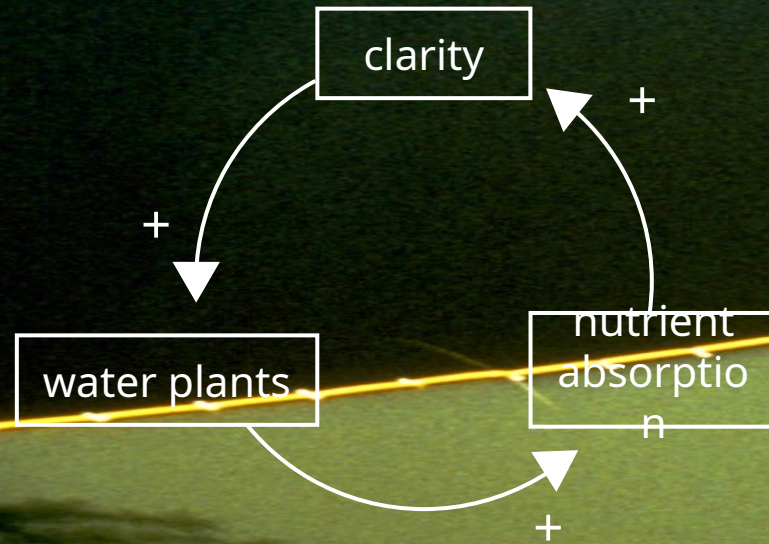
## Tipping in shallow lakes





# Nature

## Tipping in shallow lakes





# Nature

## Tipping in shallow lakes

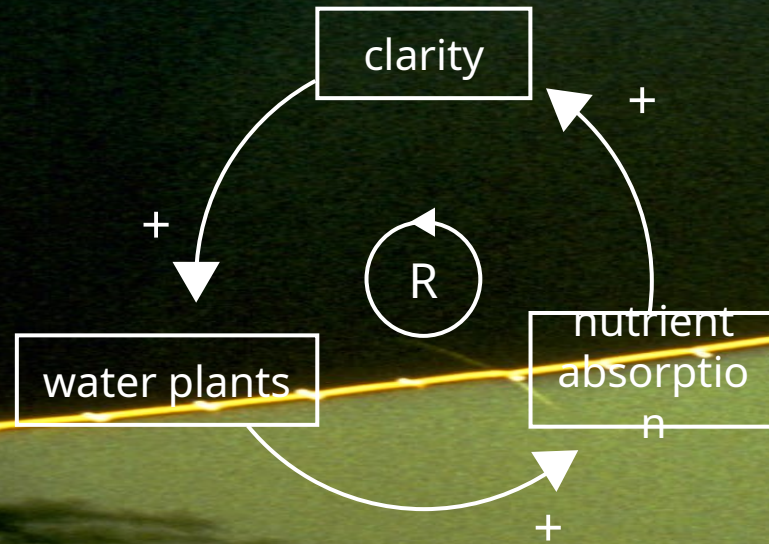


Photo credit: IISD Experimental Lakes Area (IISD-ELA)

Icon credit: Surang (Flaticon)



# Nature

## Tipping in shallow lakes

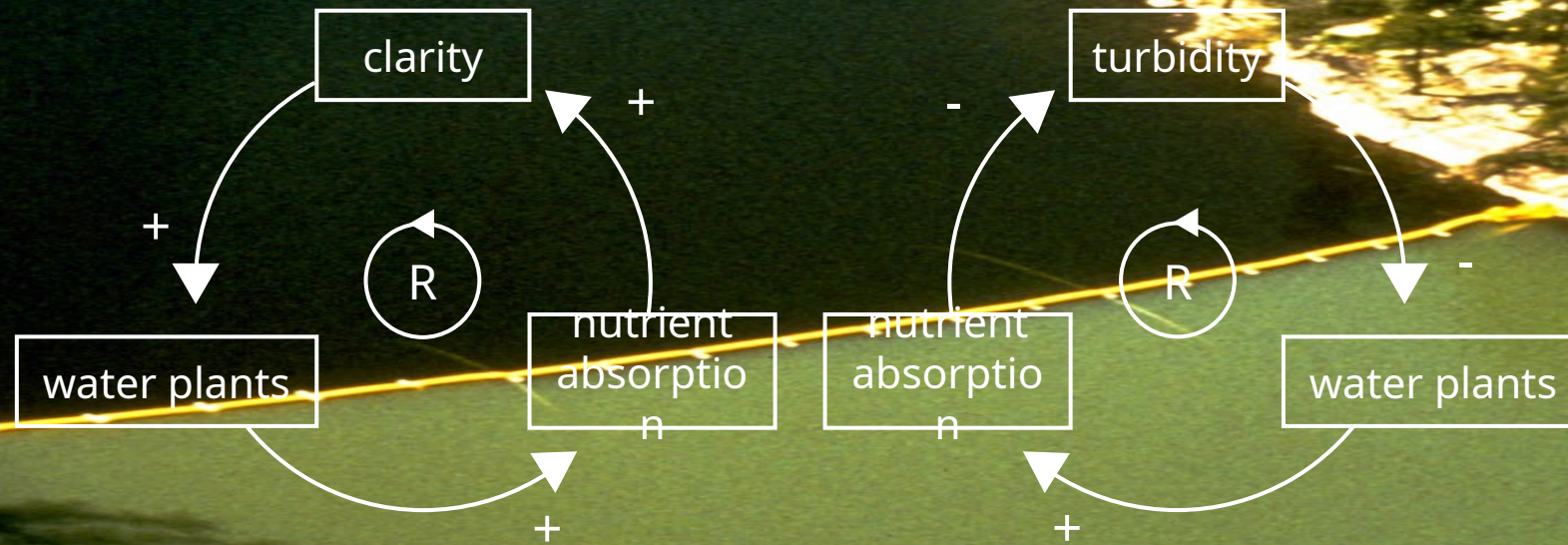


Photo credit: IISD Experimental Lakes Area (IISD-ELA)

Icon credit: Surang (Flaticon)



# Nature

## Tipping in shallow lakes

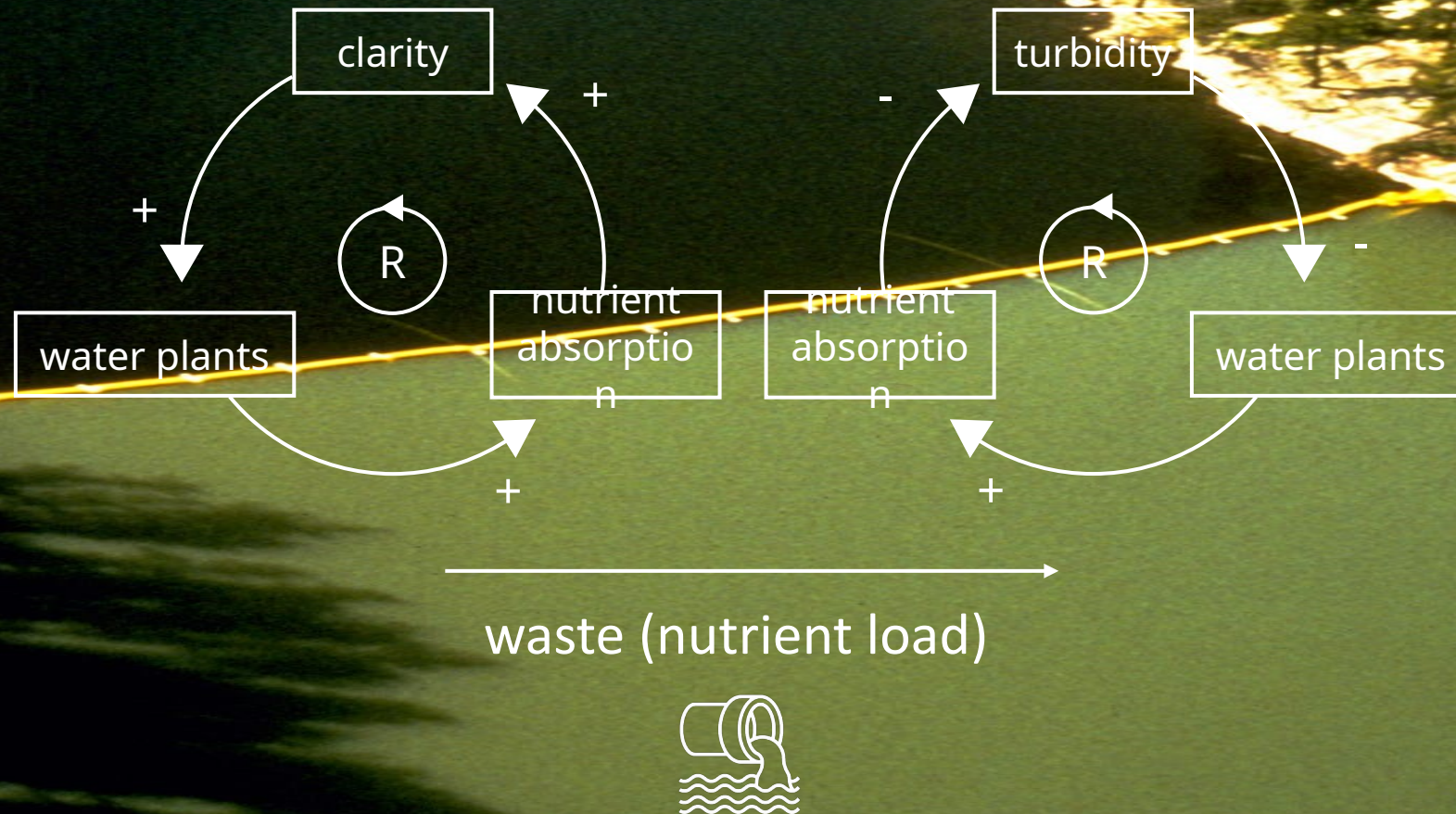


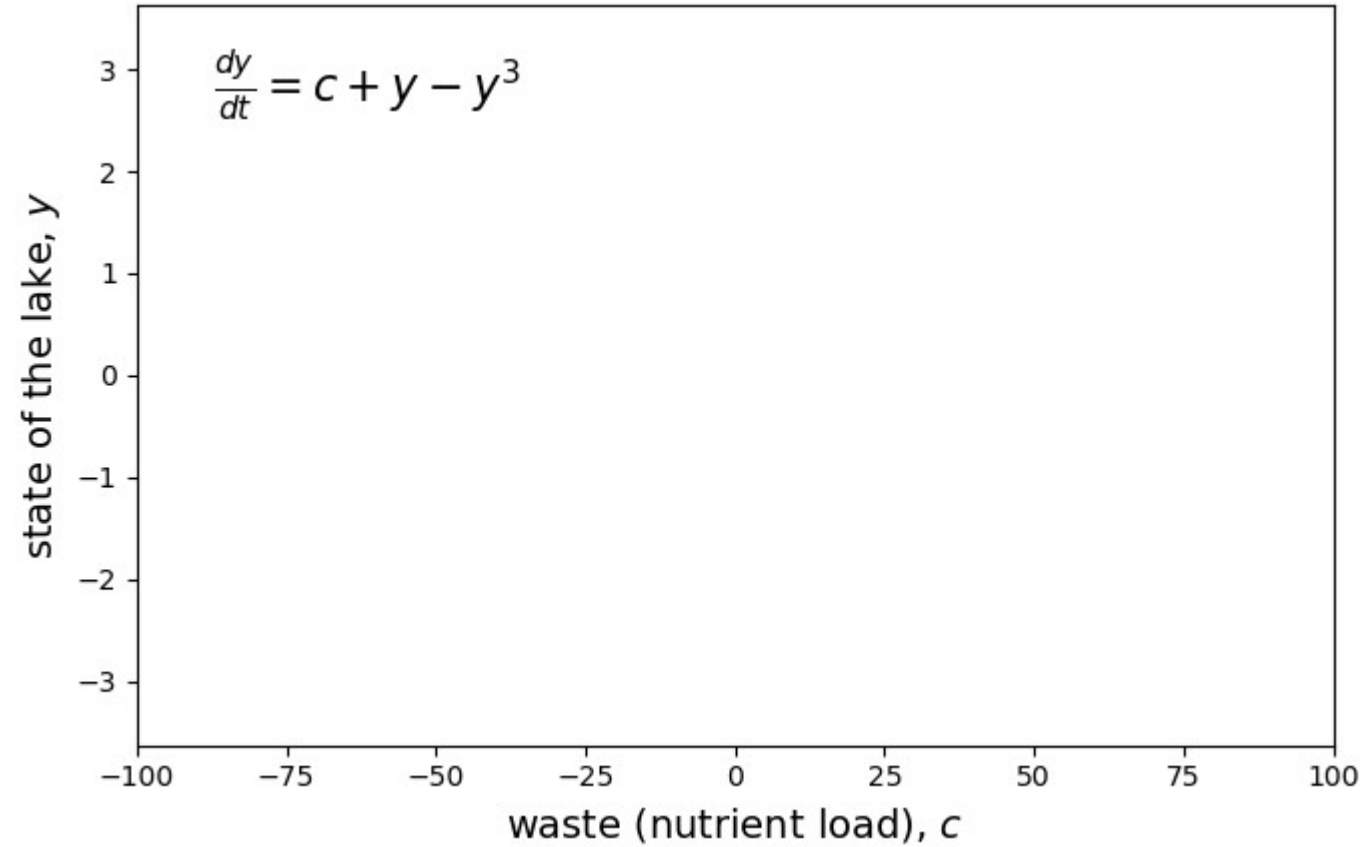
Photo credit: IISD Experimental Lakes Area (IISD-ELA)

Icon credit: Surang (Flaticon)



# Theory

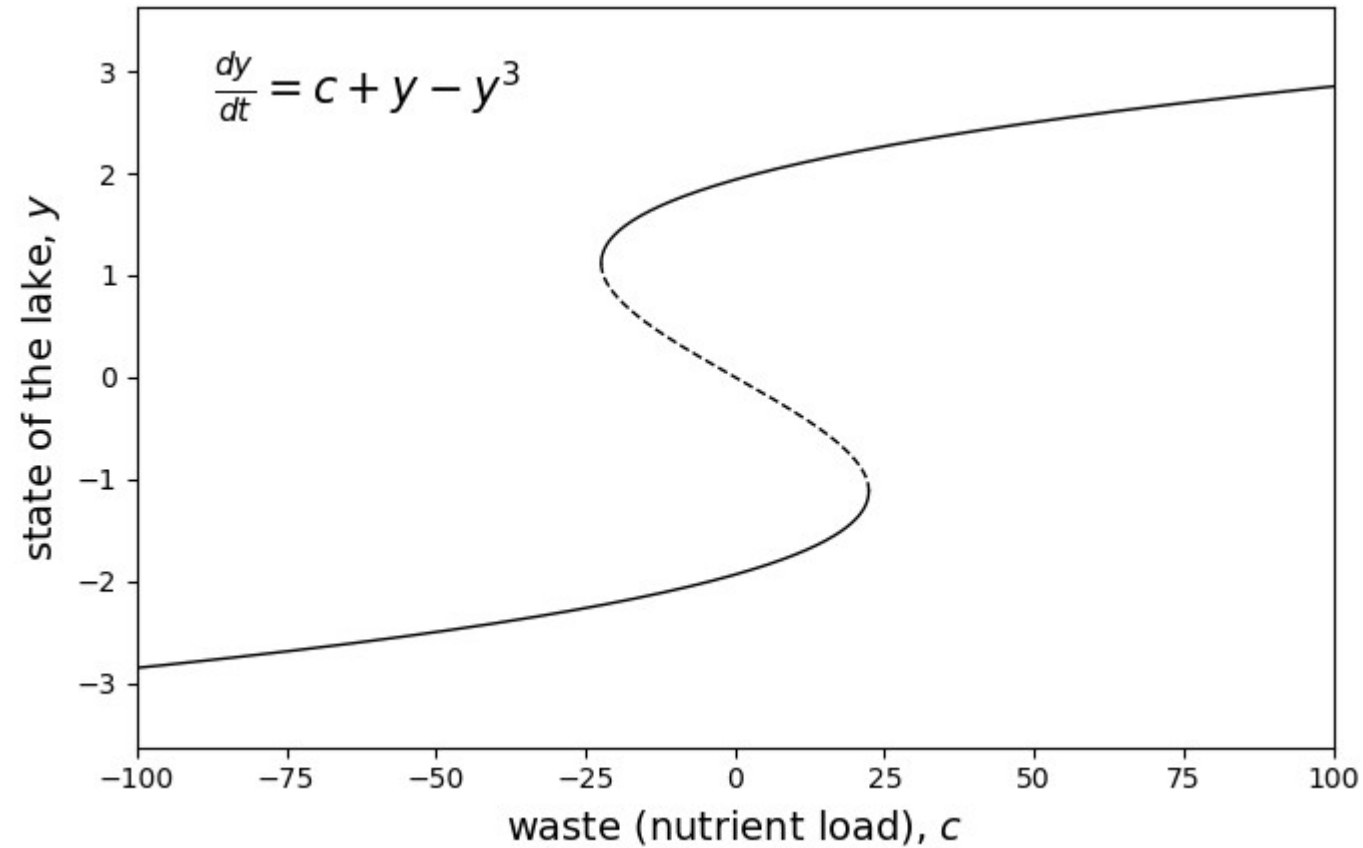
## Tipping in simulations of ODEs





# Theory

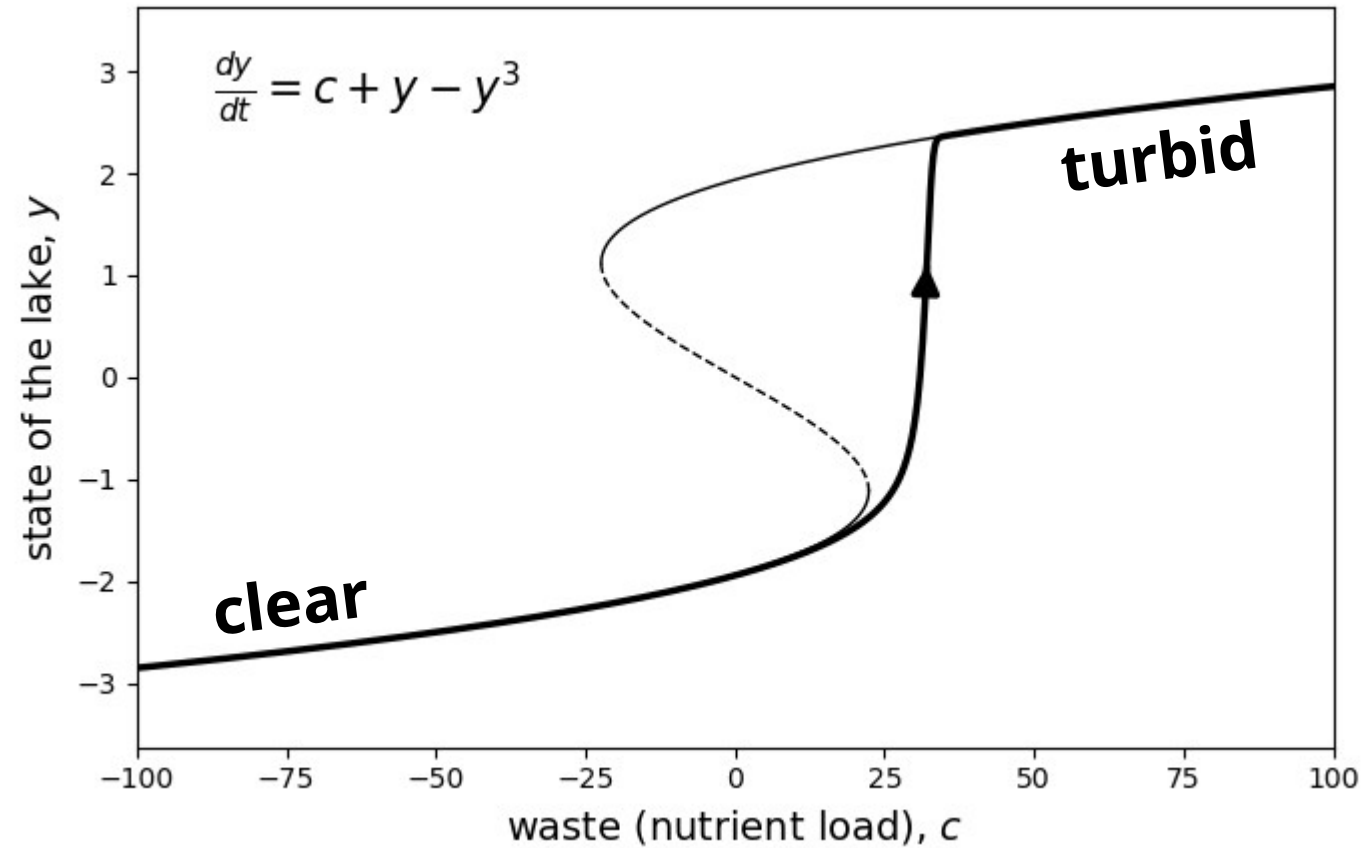
## Tipping in simulations of ODEs





# Theory

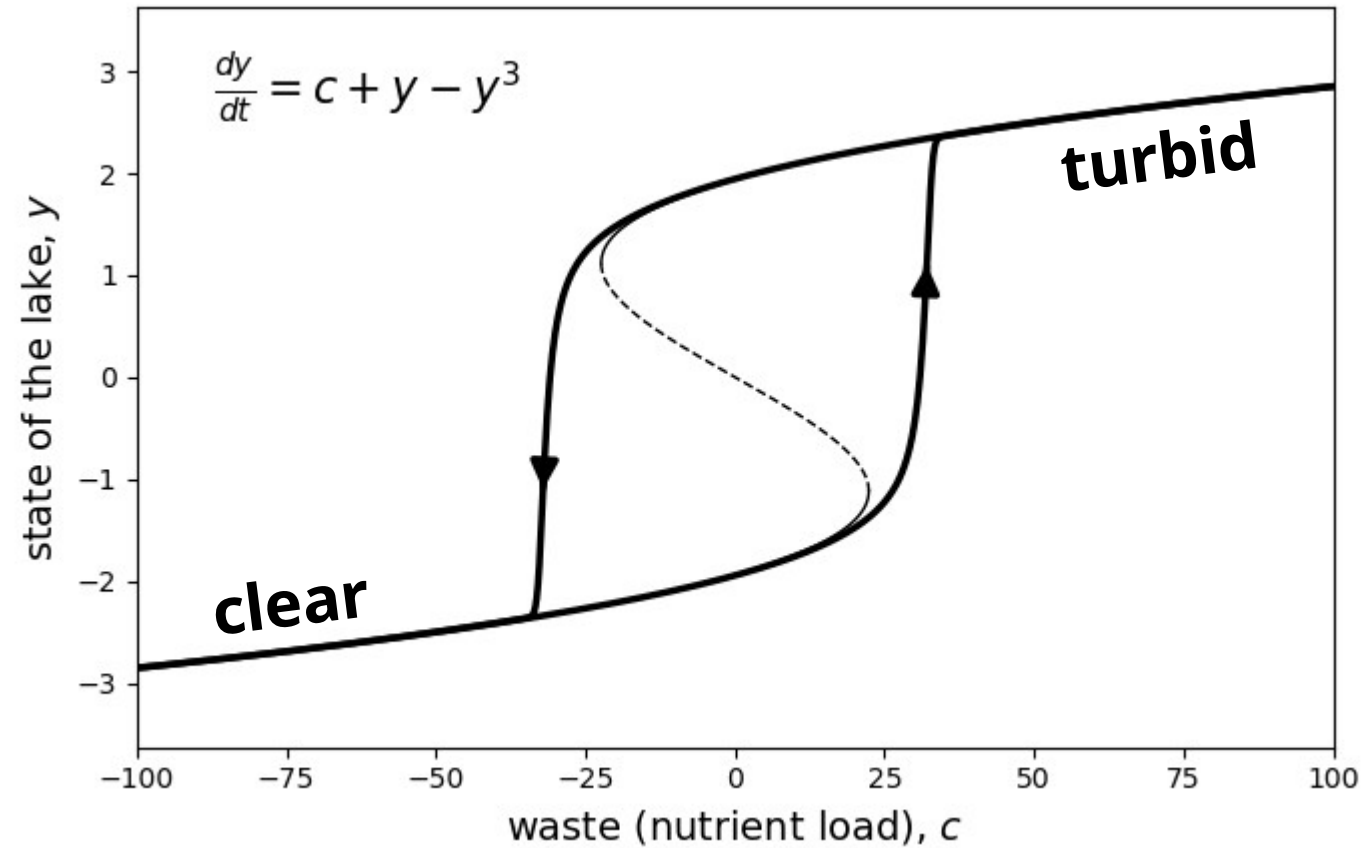
## Tipping in simulations of ODEs





# Theory

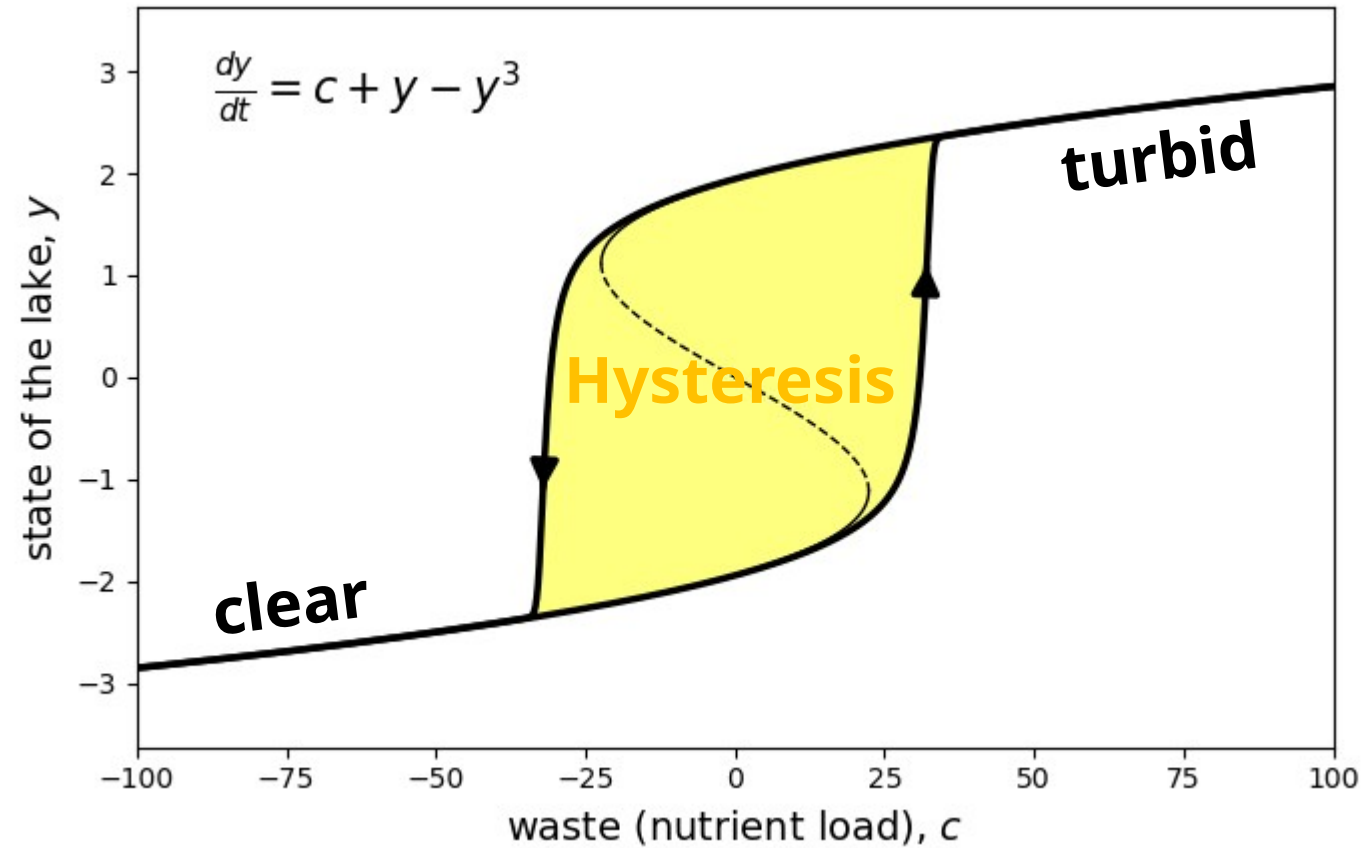
## Tipping in simulations of ODEs





# Theory

## Tipping in simulations of ODEs






# What is cascading tipping?

Cascading tipping is the phenomenon of rapid reorganization of individual sub-domains of the system with multiple points of return.

Bastiaansen 2022



A satellite image of Lake St. Clair, showing a large, irregularly shaped area of bright green water, indicating an algae bloom. The lake is surrounded by land, with a dense urban area on the left and a patchwork of agricultural fields on the right. The text "Nature" and "Algae bloom at Lake St. Claire" is overlaid on the image.

# Nature

## Algae bloom at Lake St. Claire

NASA Earth Observatory images by Joshua Stevens  
using Landsat data from the U.S. Geological SurveyCaption: Kathryn



A satellite image of Lake St. Clair, showing a massive green algae bloom that covers a large portion of the lake's surface. The bloom is a vibrant green color, contrasting sharply with the darker blue of the water. The surrounding land is a patchwork of green fields and brown urban areas. The text "Nature" is overlaid in white at the top center.

Nature

## Algae bloom at Lake St. Claire

“phosphorus from farm runoff  
combined with favorable weather  
and lake conditions to **produce a bloom**  
three times larger than previously  
observed”

- NASA 2015



A satellite image of Lake St. Clair, showing a large, irregularly shaped area of bright green water, indicating an algae bloom. The lake is surrounded by a dense urban area on the left and a patchwork of green and brown fields on the right. A white circle labeled 'z1' is drawn around a specific area of the bloom. The text 'Nature' and 'Algae bloom at Lake St. Claire' is overlaid in white at the top.

# Nature

## Algae bloom at Lake St. Claire

z1



A satellite image of Lake St. Clair, showing a large, irregularly shaped area of bright green water, indicating an algae bloom. The lake is surrounded by land, with a dense urban area on the left and a patchwork of agricultural fields on the right. Two white circles, labeled 'z1' and 'z2', are drawn on the green area of the lake. The text 'Nature' is in the top left, and 'Algae bloom at Lake St. Claire' is in the top center.

# Nature

## Algae bloom at Lake St. Claire

z1

z2



# Nature

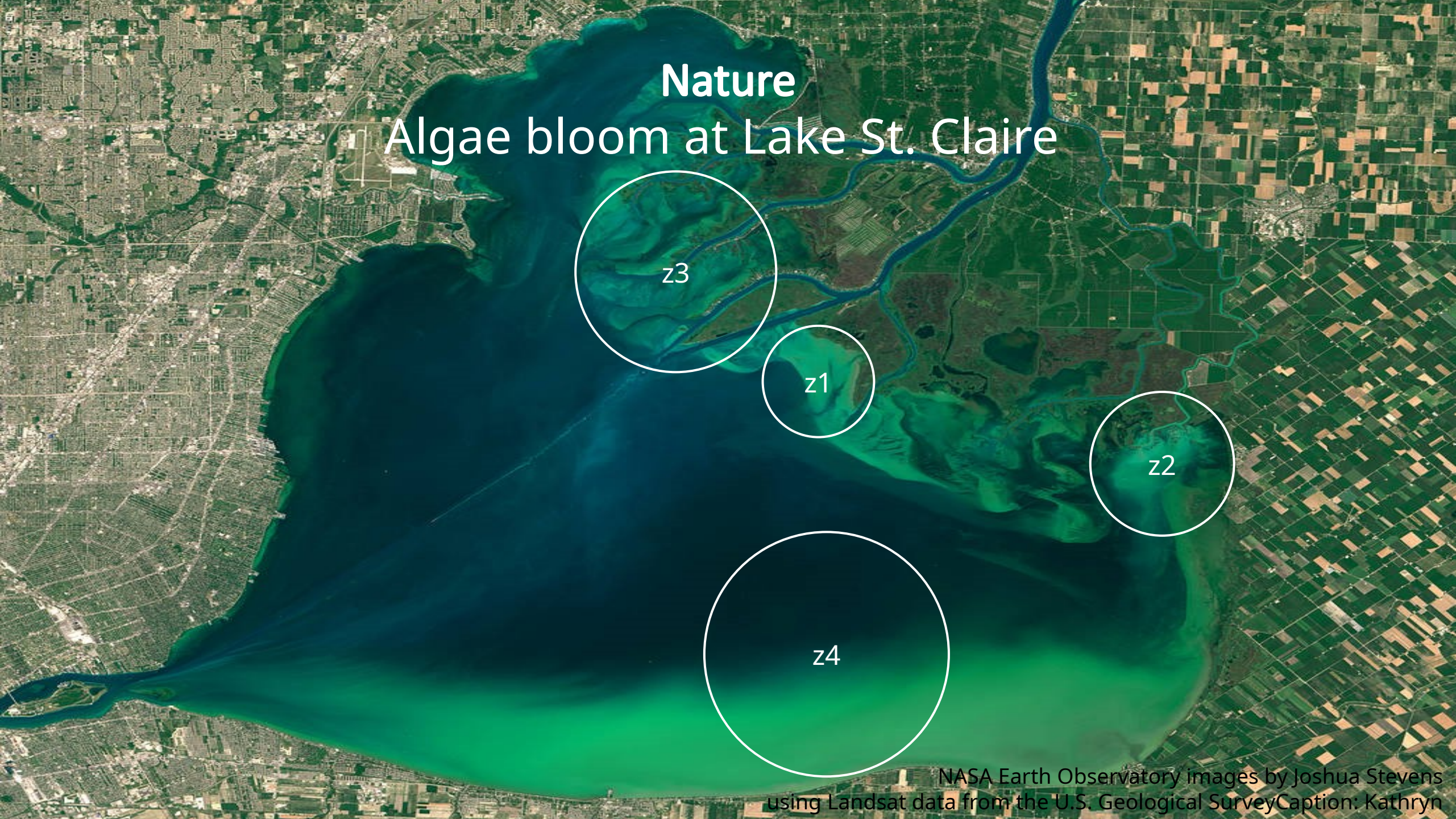
## Algae bloom at Lake St. Claire

z3

z1

z2



A satellite image of Lake St. Clair, showing a large, dark green area representing an algae bloom. The lake is surrounded by land, with a city visible on the left and agricultural fields on the right. Four white circles are overlaid on the lake, labeled z1, z2, z3, and z4, indicating specific sampling locations. The bloom is most concentrated in the upper right and lower right portions of the lake, with some lighter green areas in the center and lower left.

# Nature

## Algae bloom at Lake St. Clair

z3

z1

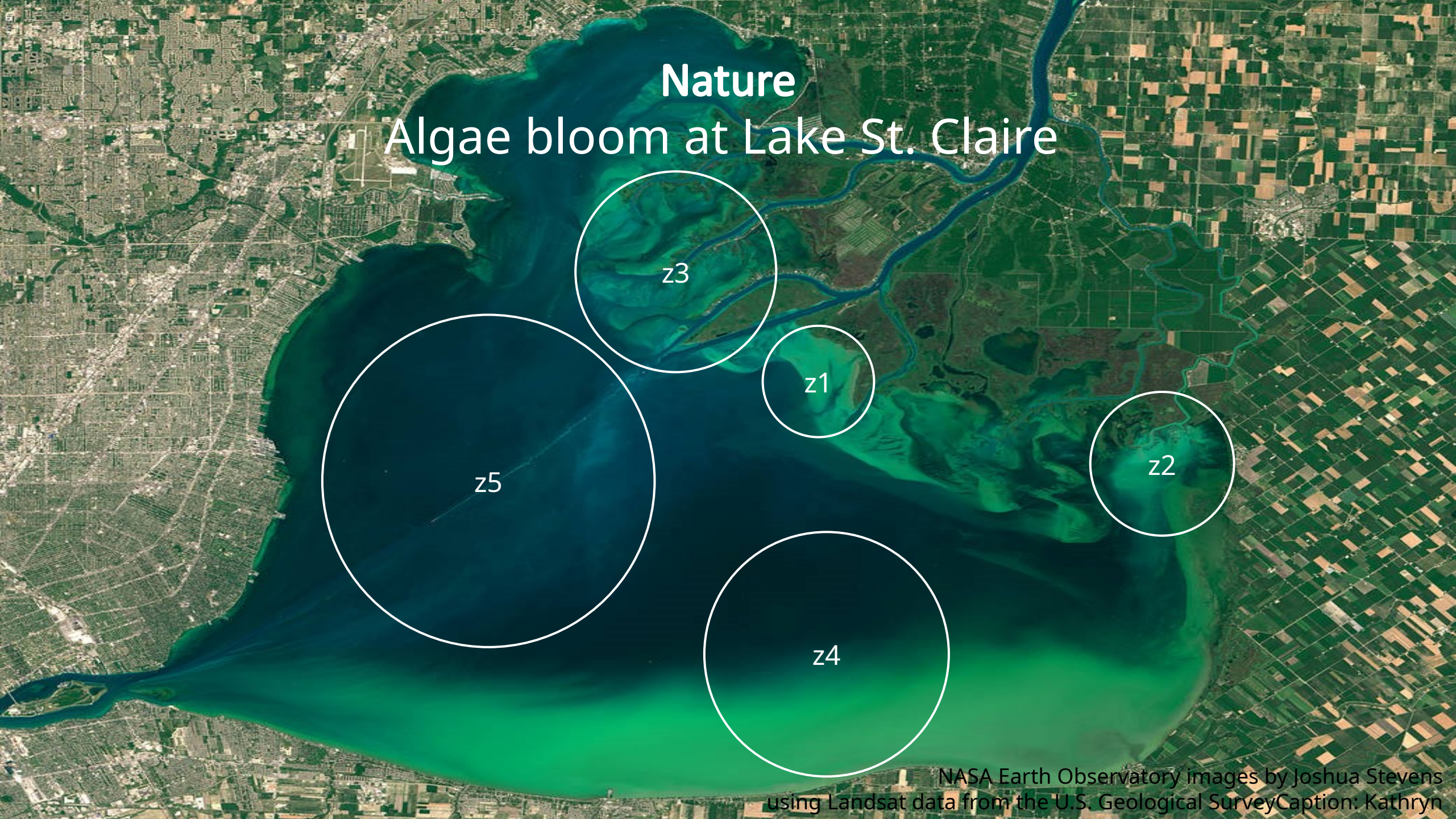
z2

z4



# Nature

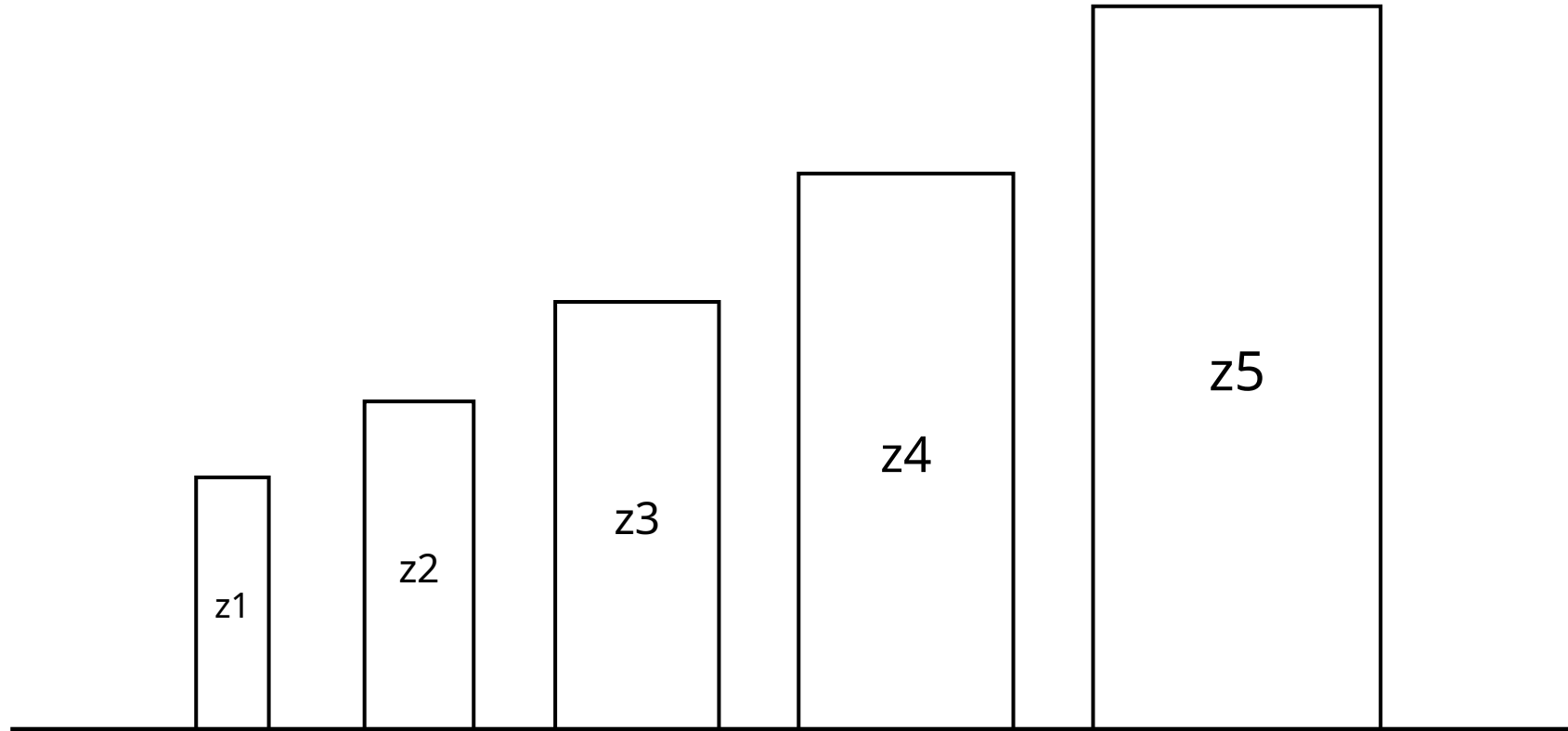
## Algae bloom at Lake St. Claire





# Dominos

## Cascading tipping example



**Subdomains of the system,  $z_1 \dots z_5$**

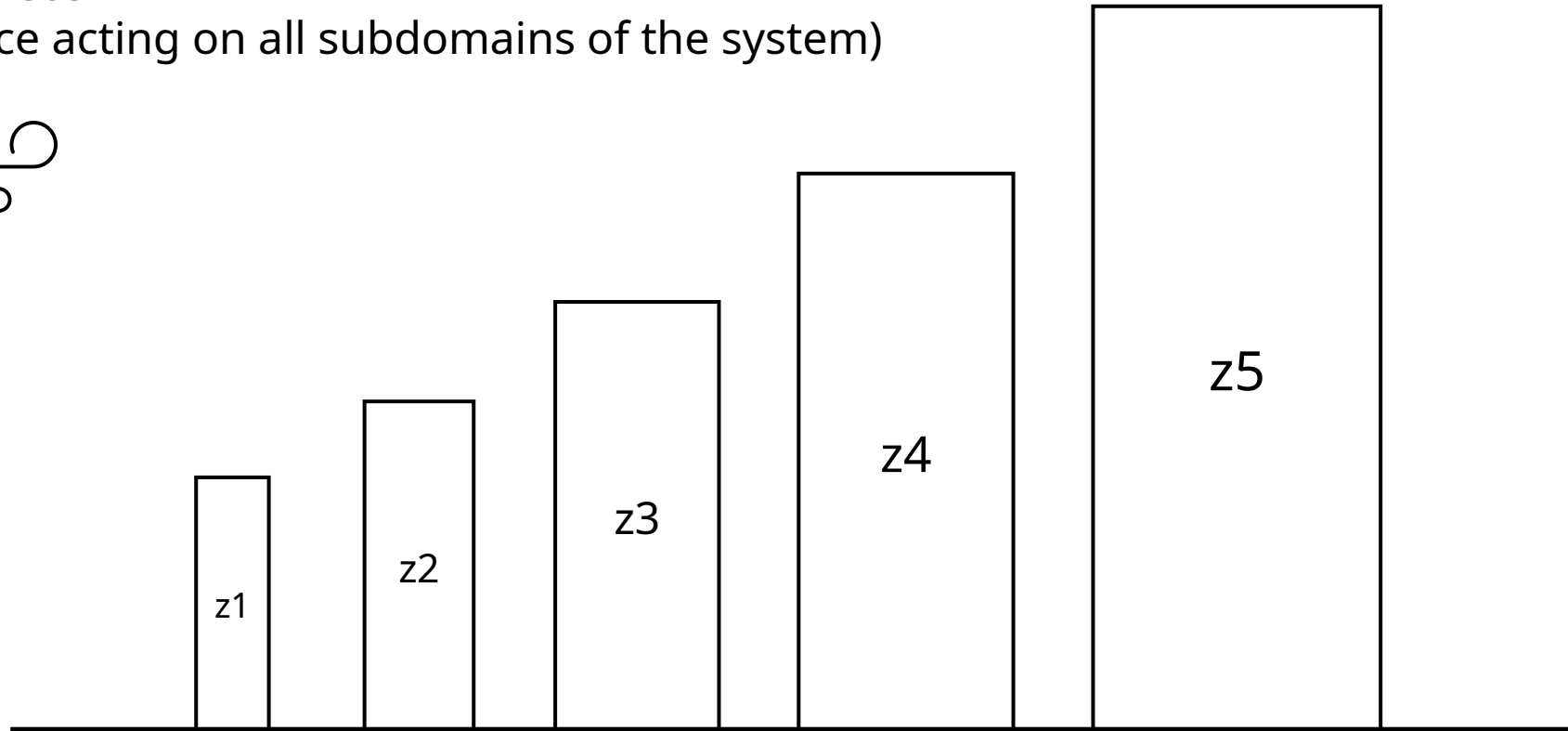
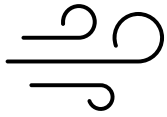


# Dominos

## Cascading tipping example

### Control parameter

(increasing force acting on all subdomains of the system)



Subdomains of the system,  $z_1 \dots z_5$

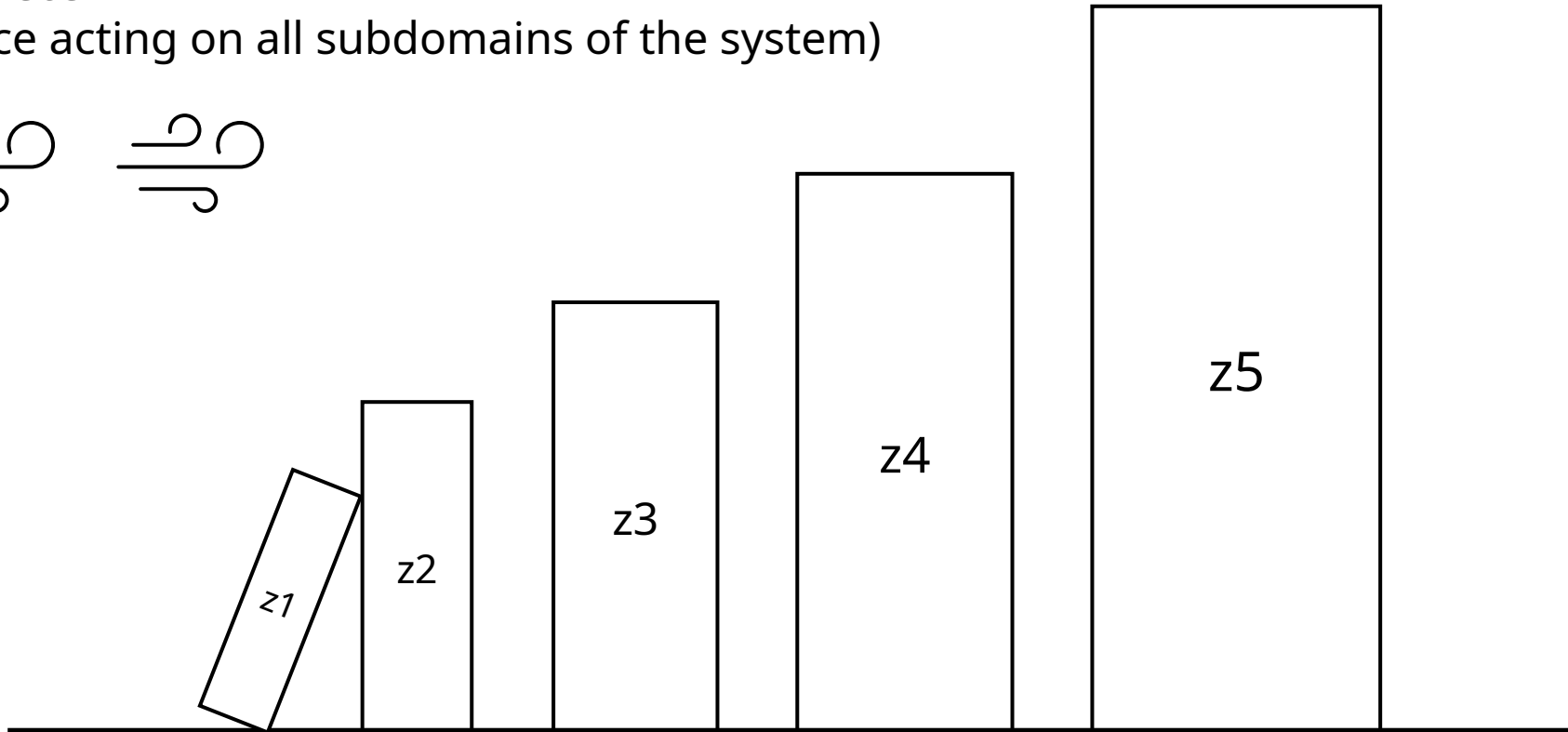
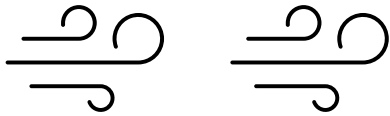


# Dominos

## Cascading tipping example

### Control parameter

(increasing force acting on all subdomains of the system)



Subdomains of the system,  $z1 \dots z5$

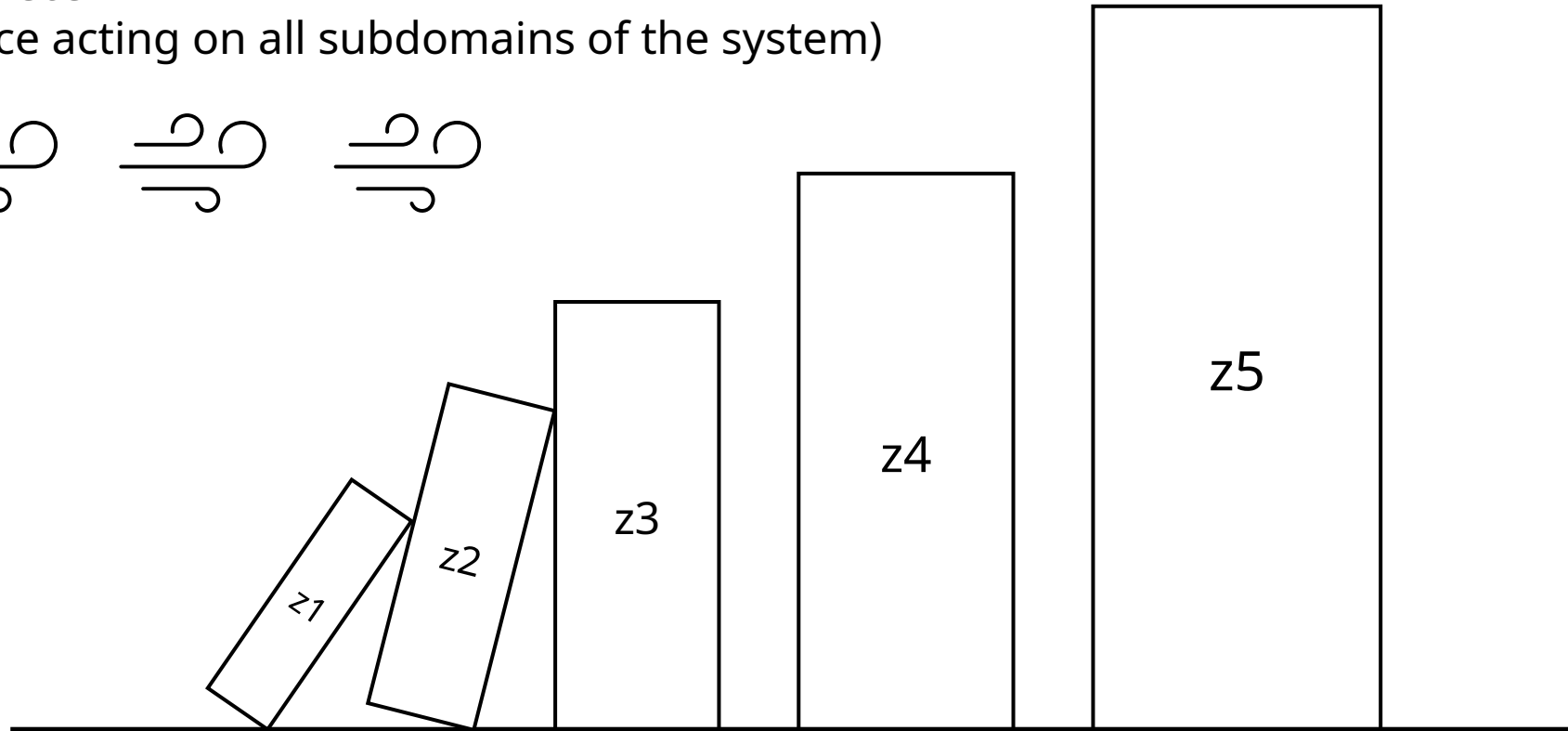
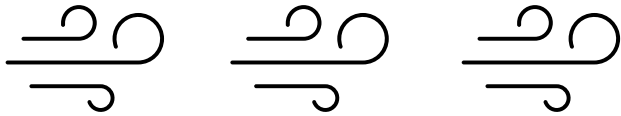


# Dominos

## Cascading tipping example

### Control parameter

(increasing force acting on all subdomains of the system)



Subdomains of the system,  $z_1 \dots z_5$

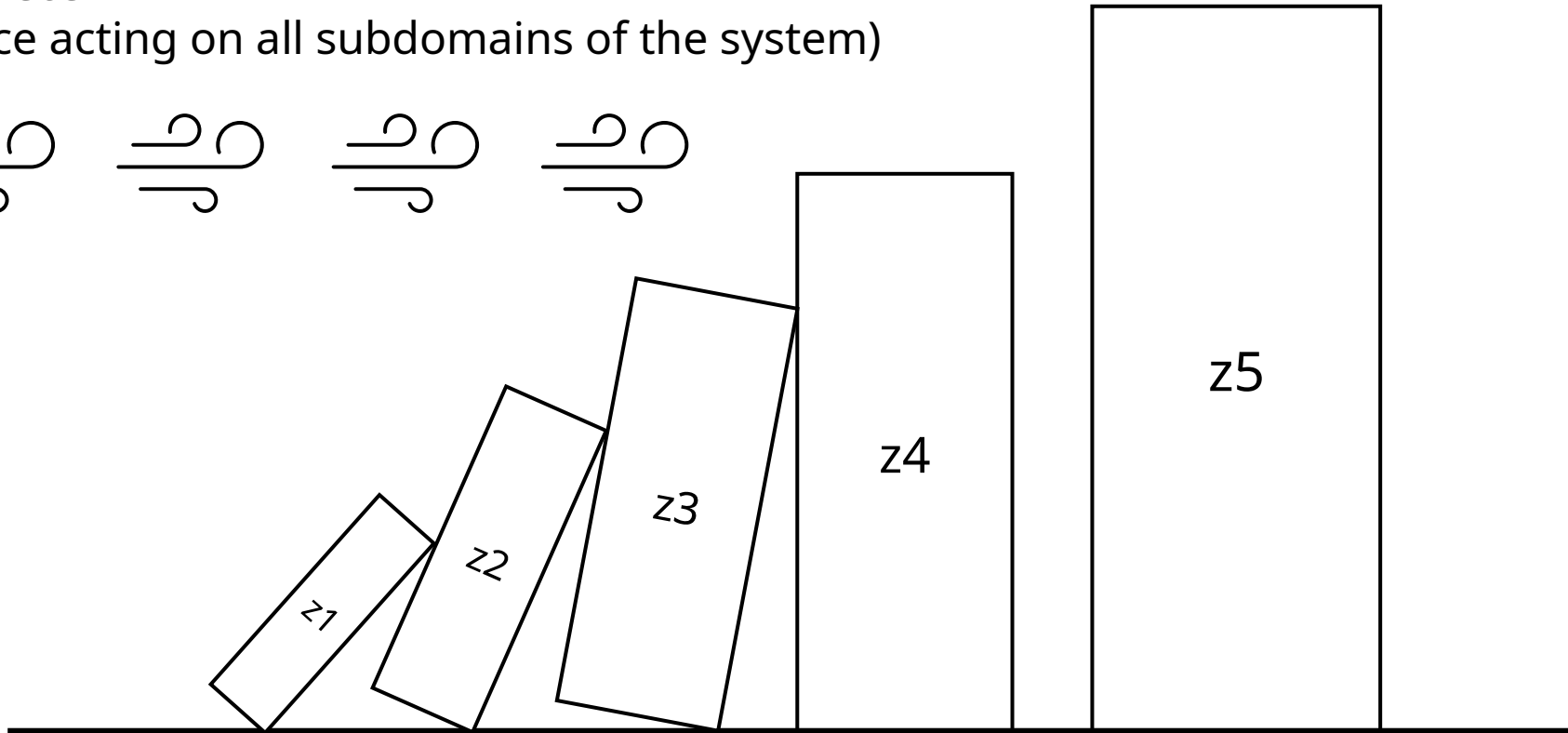
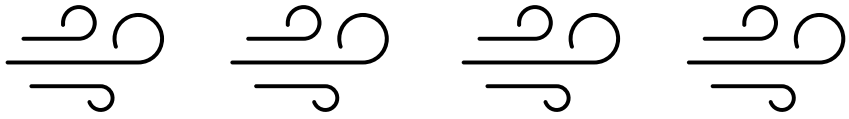


# Dominos

## Cascading tipping example

### Control parameter

(increasing force acting on all subdomains of the system)

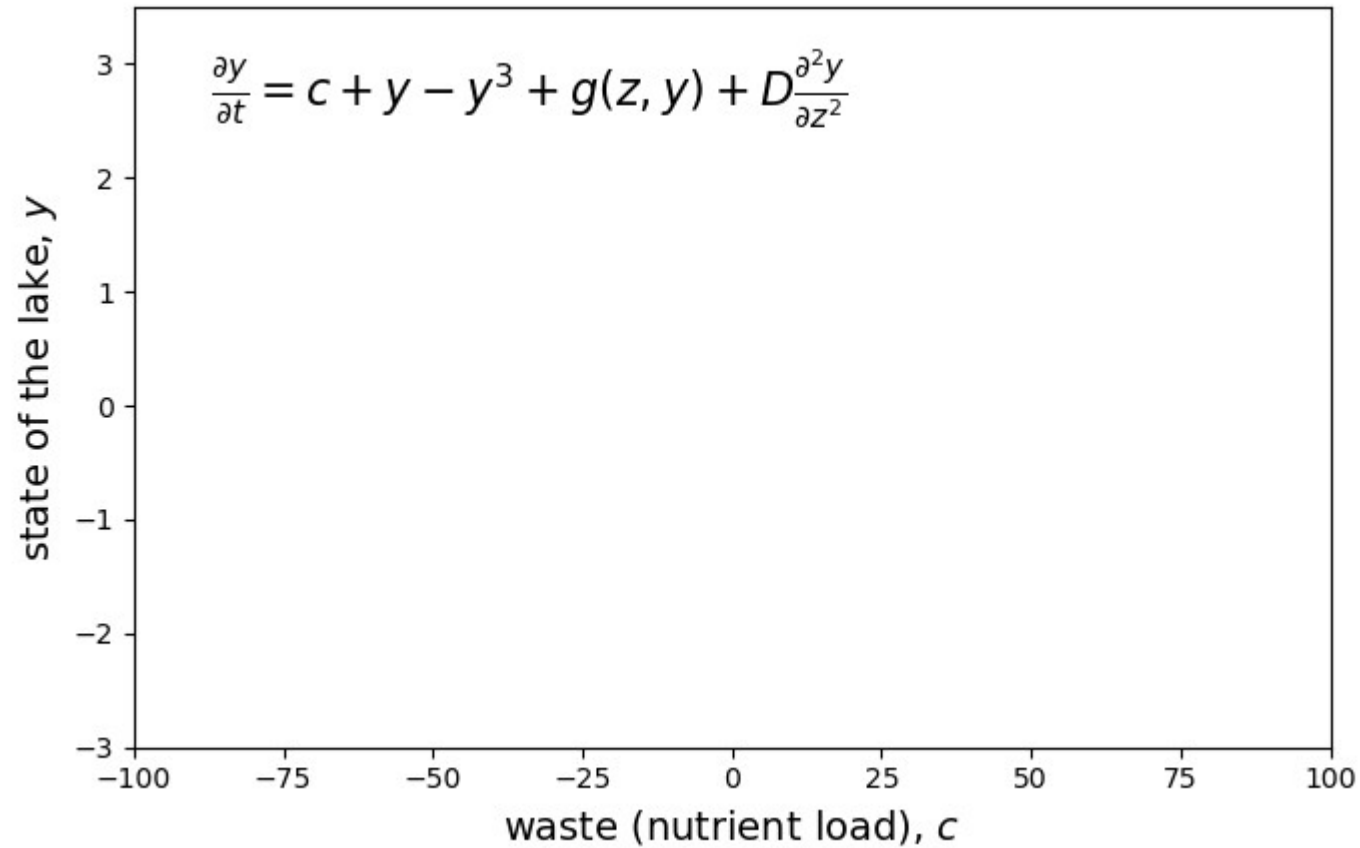


Subdomains of the system,  $z_1 \dots z_5$



# Theory

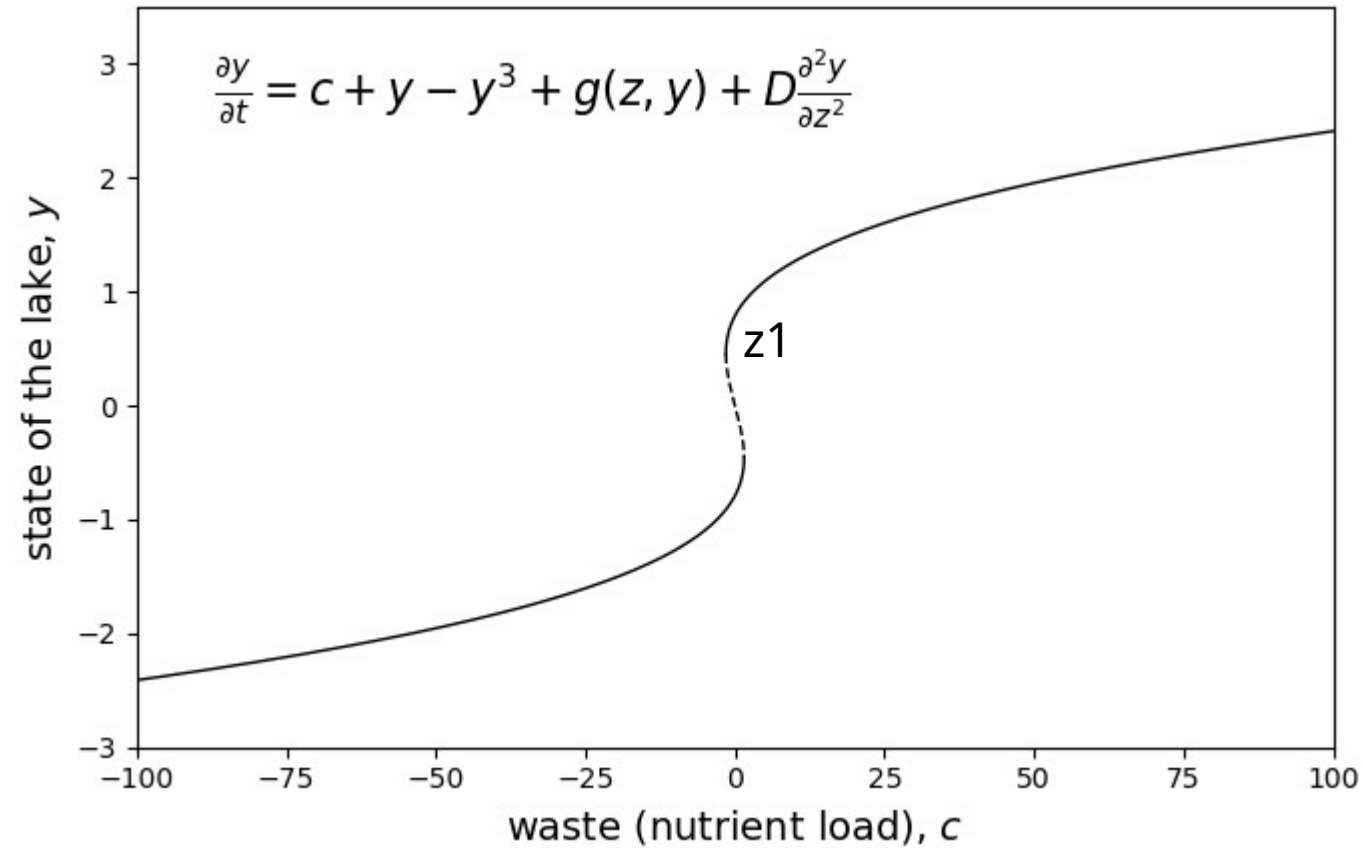
## Cascading tipping in simulations in PDEs





# Theory

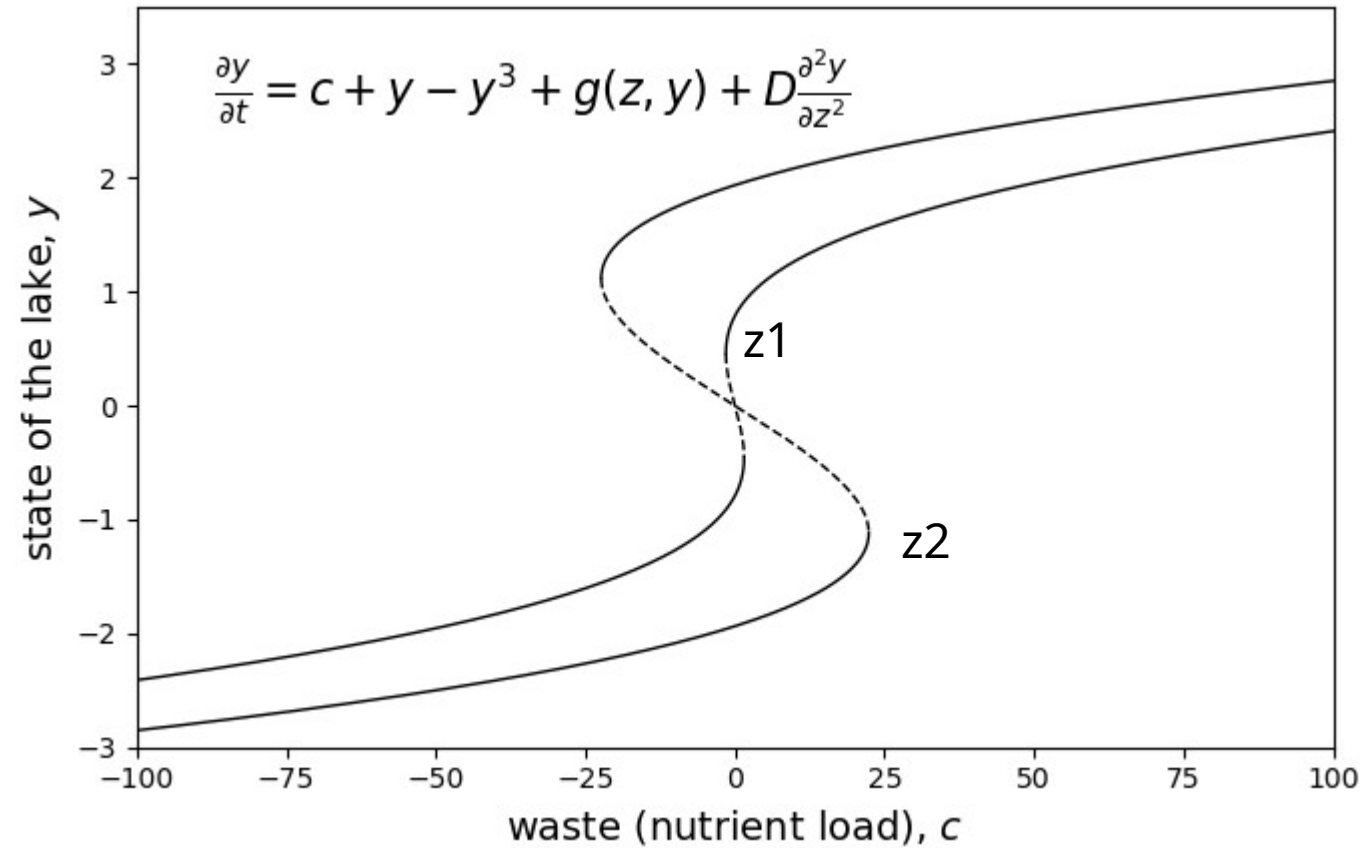
## Cascading tipping in simulations in PDEs





# Theory

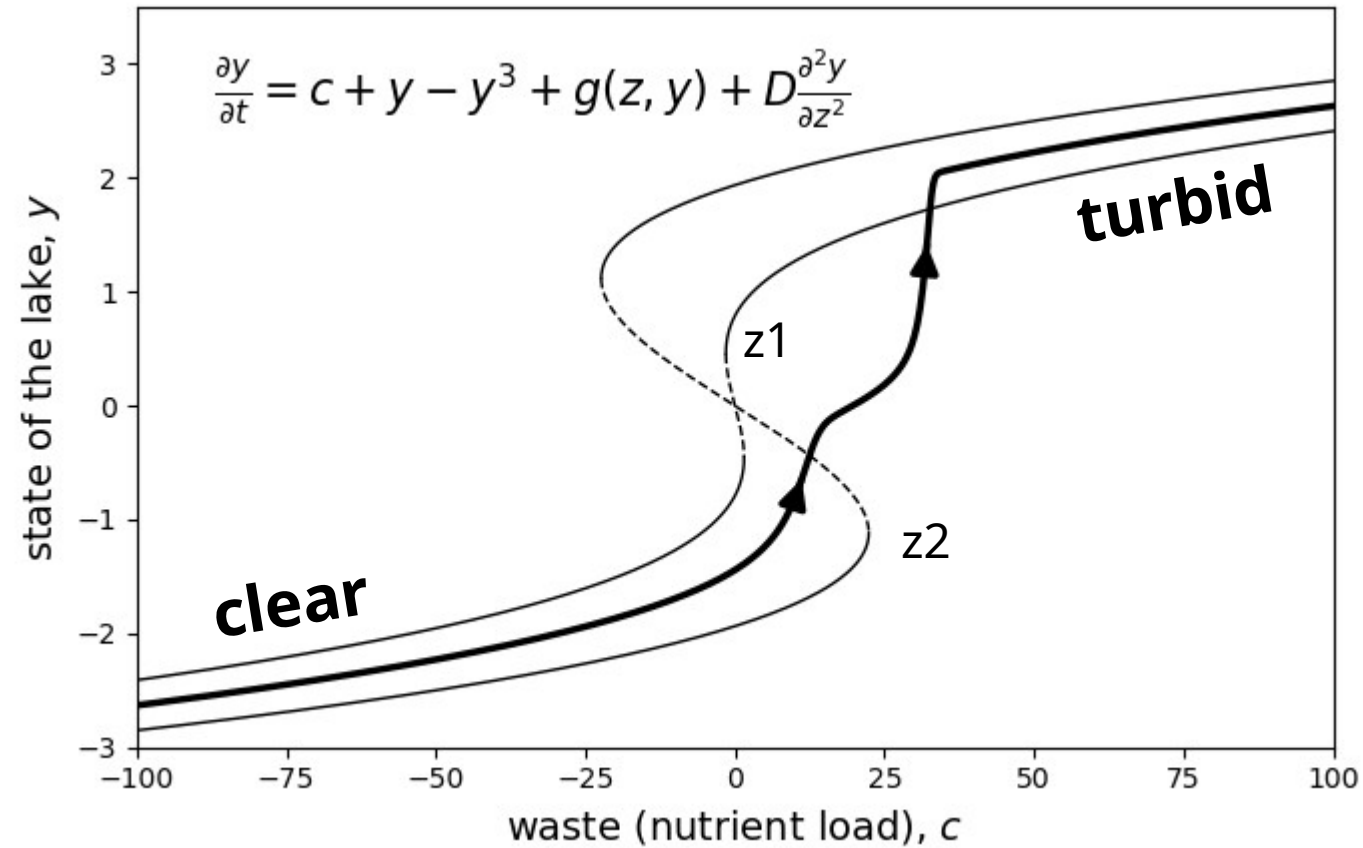
## Cascading tipping in simulations in PDEs





# Theory

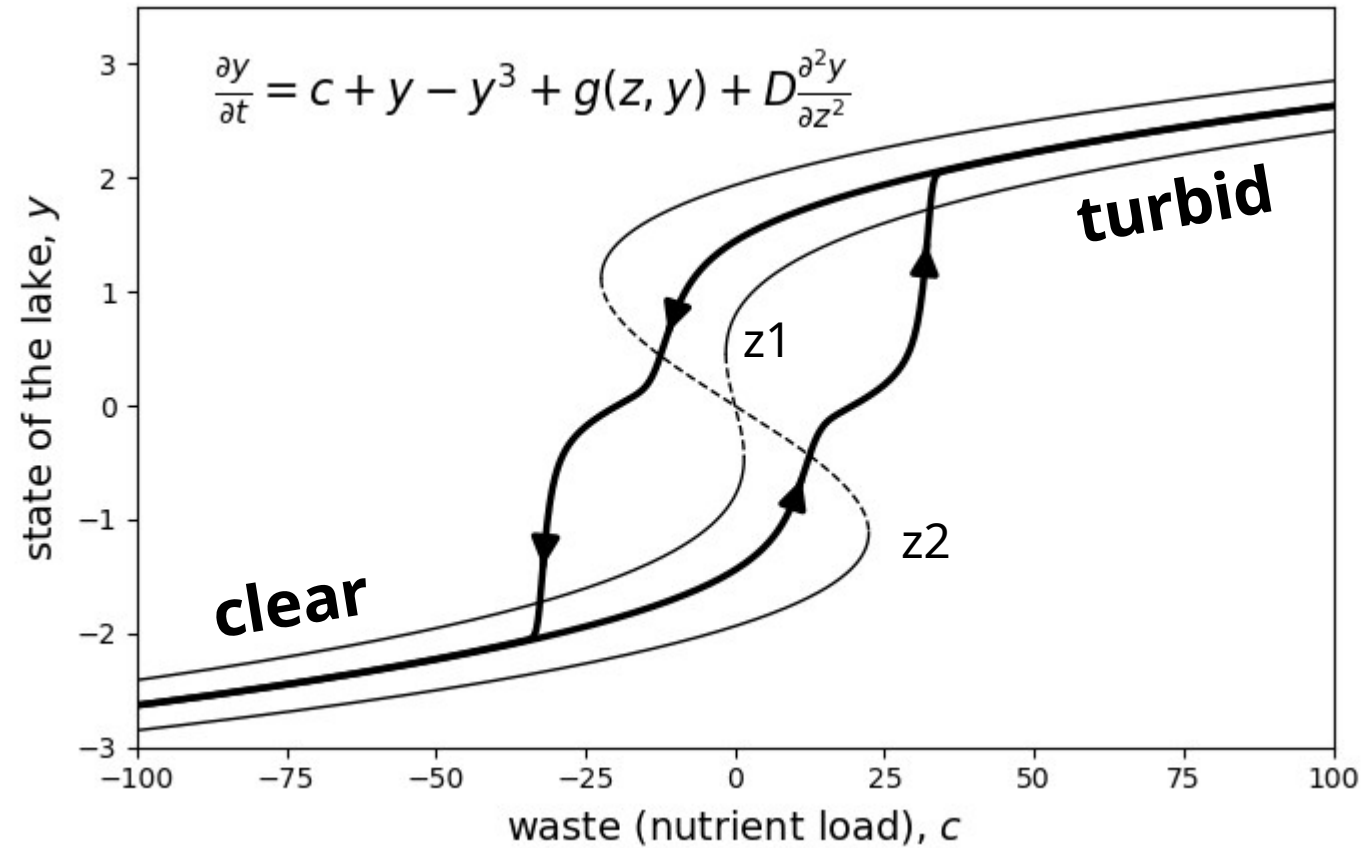
## Cascading tipping in simulations in PDEs





# Theory

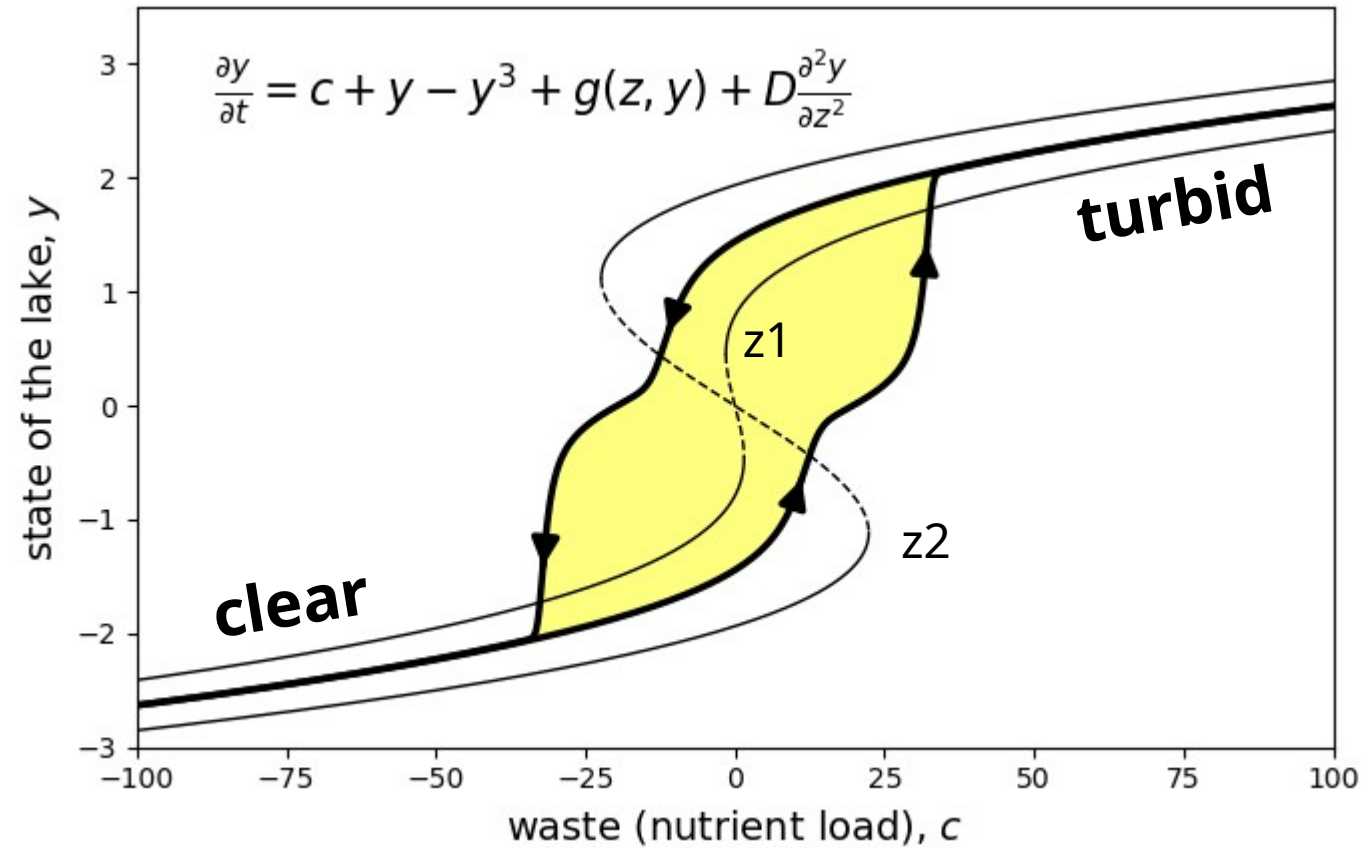
## Cascading tipping in simulations in PDEs





# Theory

## Cascading tipping in simulations in PDEs



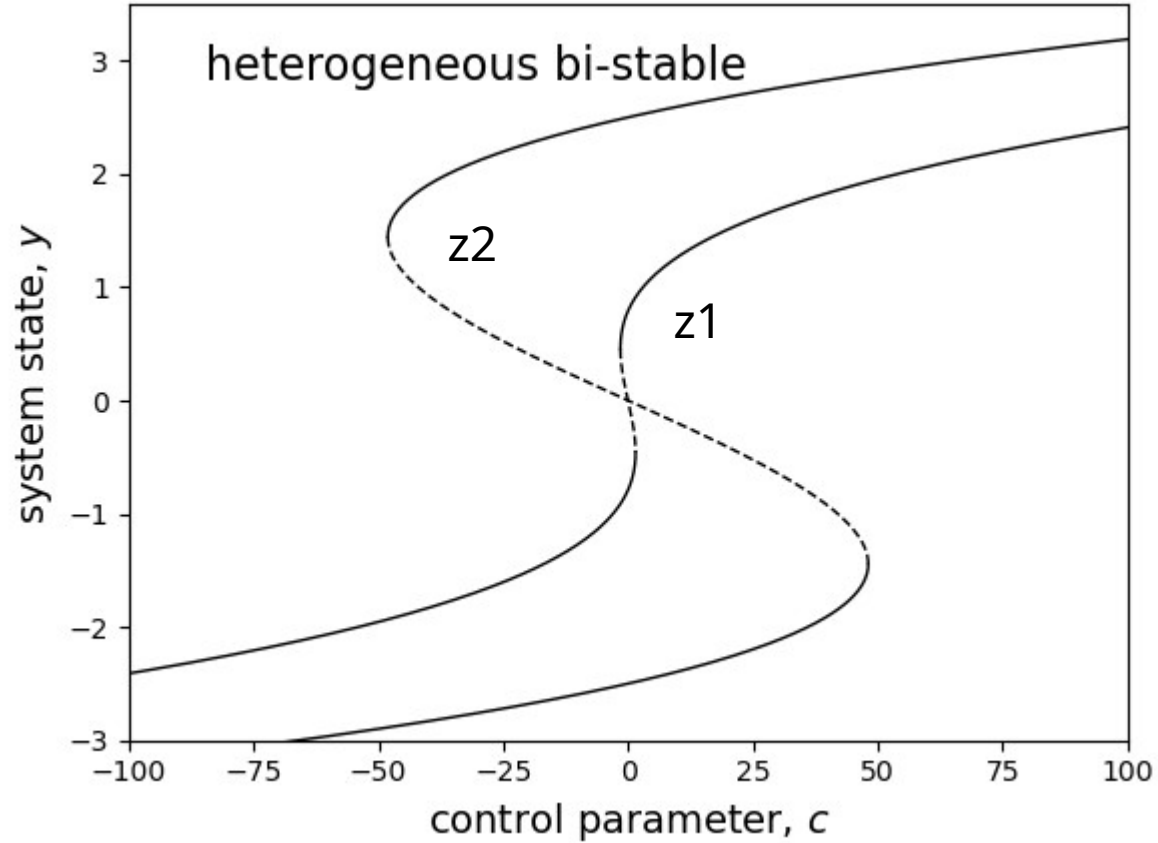


# Pitfall

Spatially heterogeneous bi-stability can look very similar to spatially homogenous multi-stability

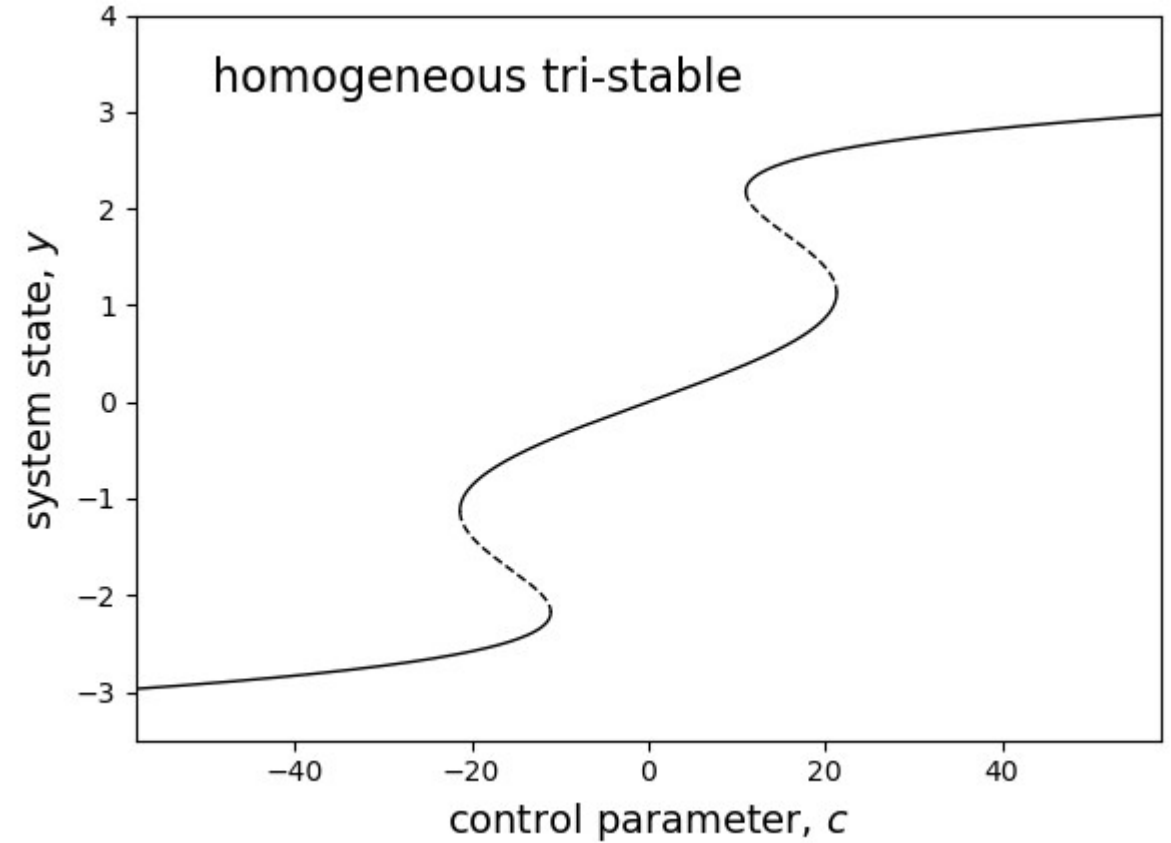


$$\frac{\partial y}{\partial t} = c + y - y^3 + g(z, y) + D \frac{\partial^2 y}{\partial z^2}$$



Bi-stable lake: **clear** vs **turbid**

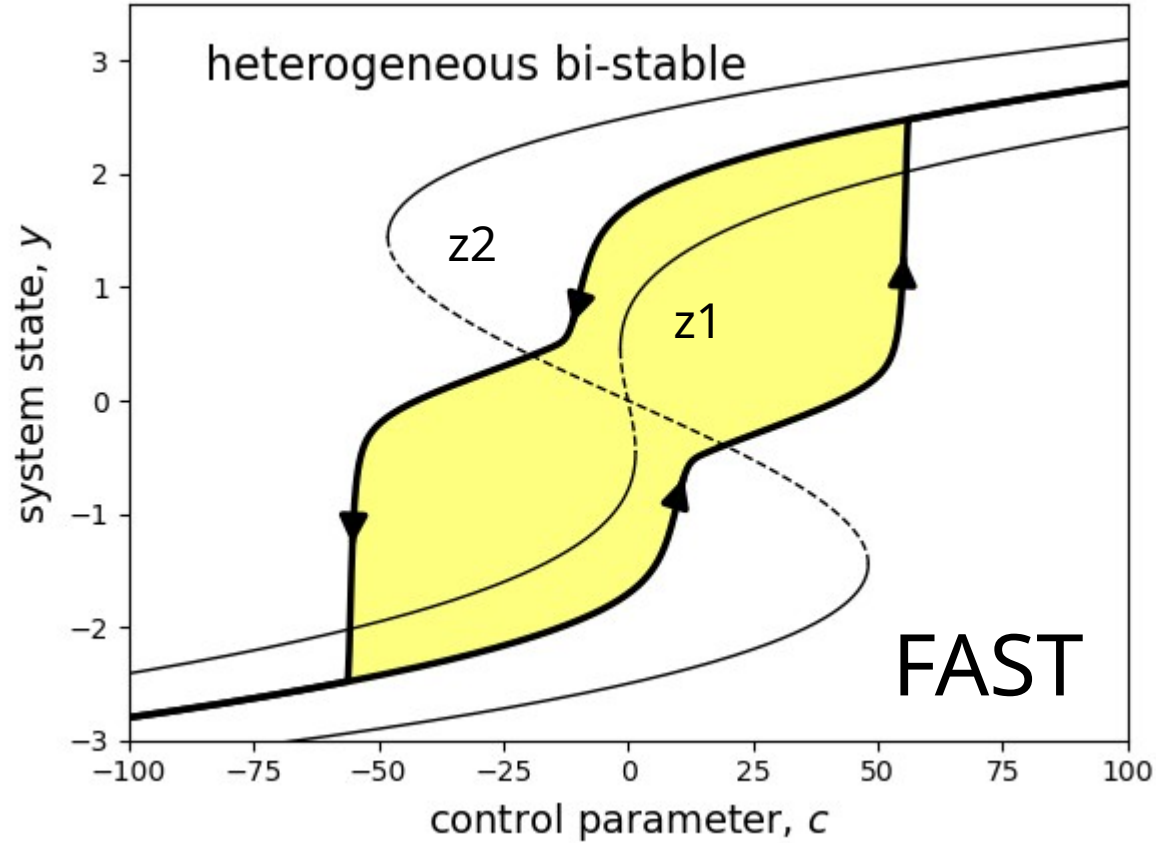
$$\frac{dy}{dt} = c - y + y^3 - y^5$$



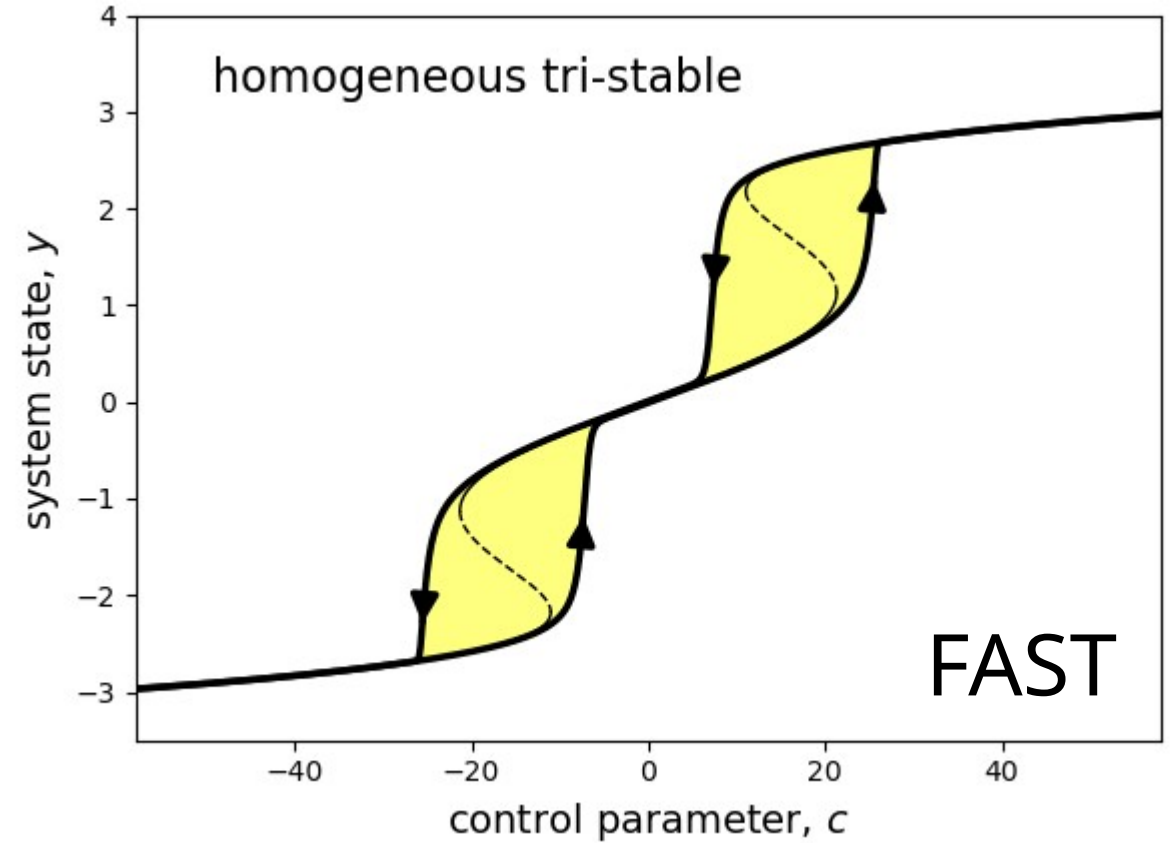
Tri-stable lake: **clear** vs **turbid** vs **sulfur**



$$\frac{\partial y}{\partial t} = c + y - y^3 + g(z, y) + D \frac{\partial^2 y}{\partial z^2}$$



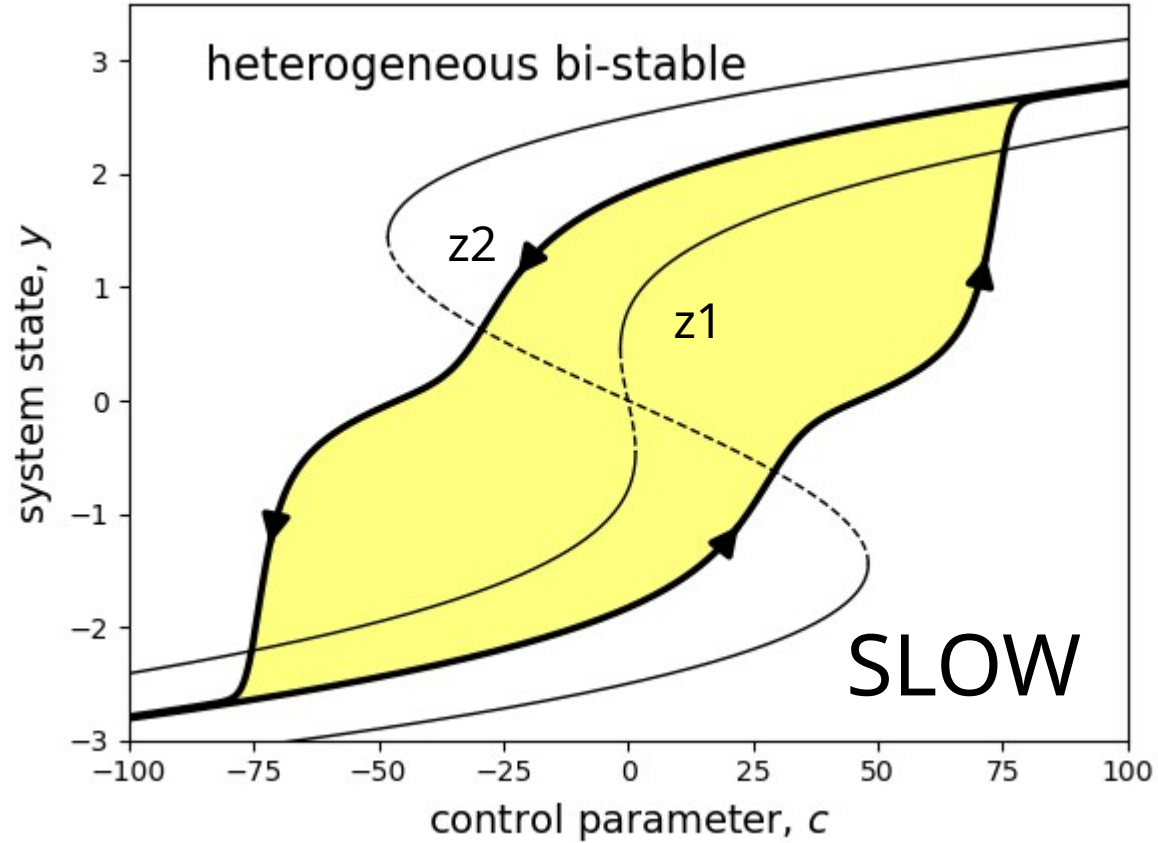
$$\frac{dy}{dt} = c - y + y^3 - y^5$$



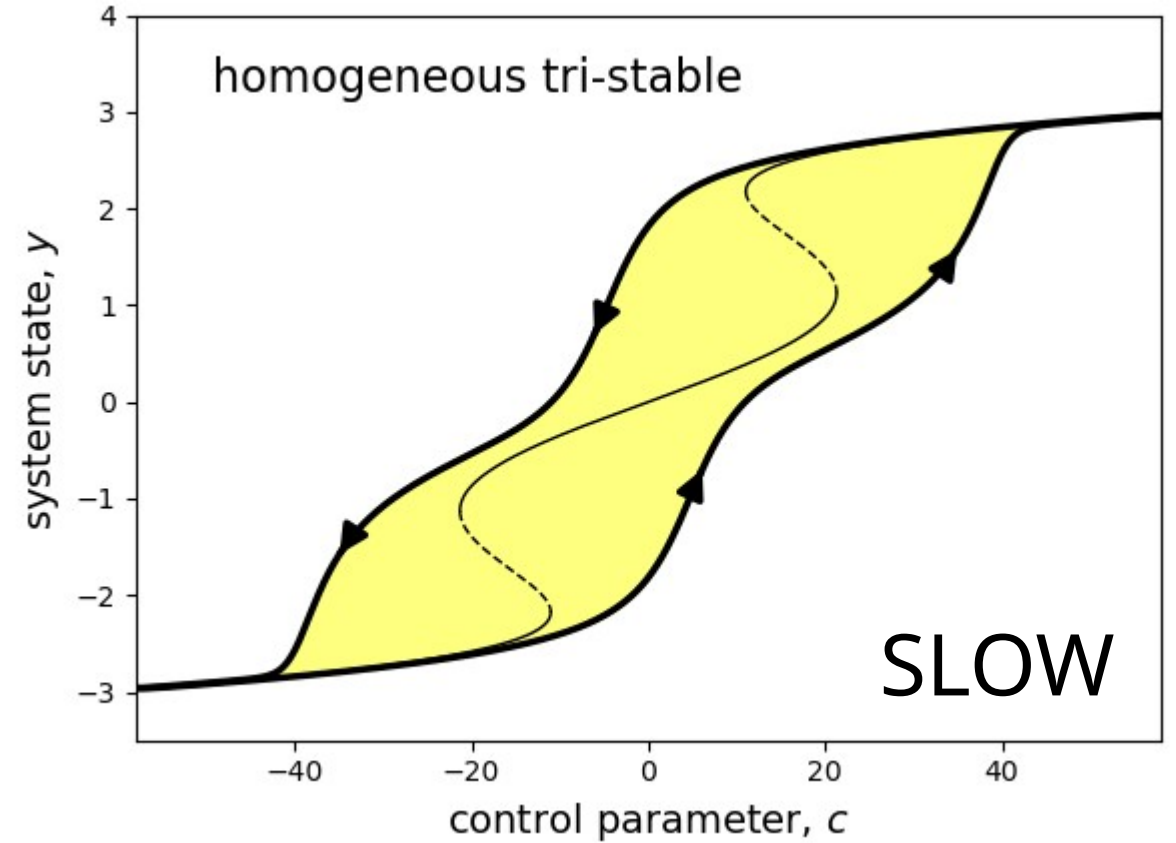
If the "lake" systems react **fast**



$$\frac{\partial y}{\partial t} = c + y - y^3 + g(z, y) + D \frac{\partial^2 y}{\partial z^2}$$



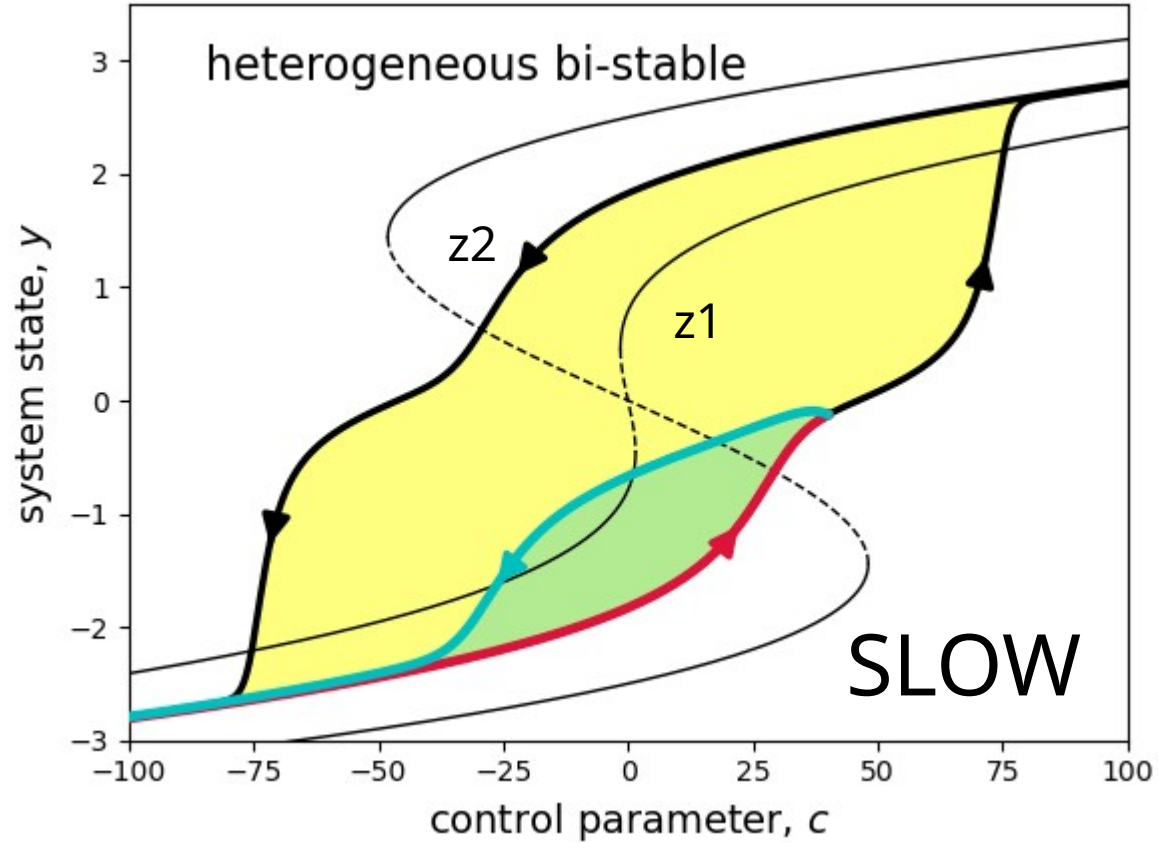
$$\frac{dy}{dt} = c - y + y^3 - y^5$$



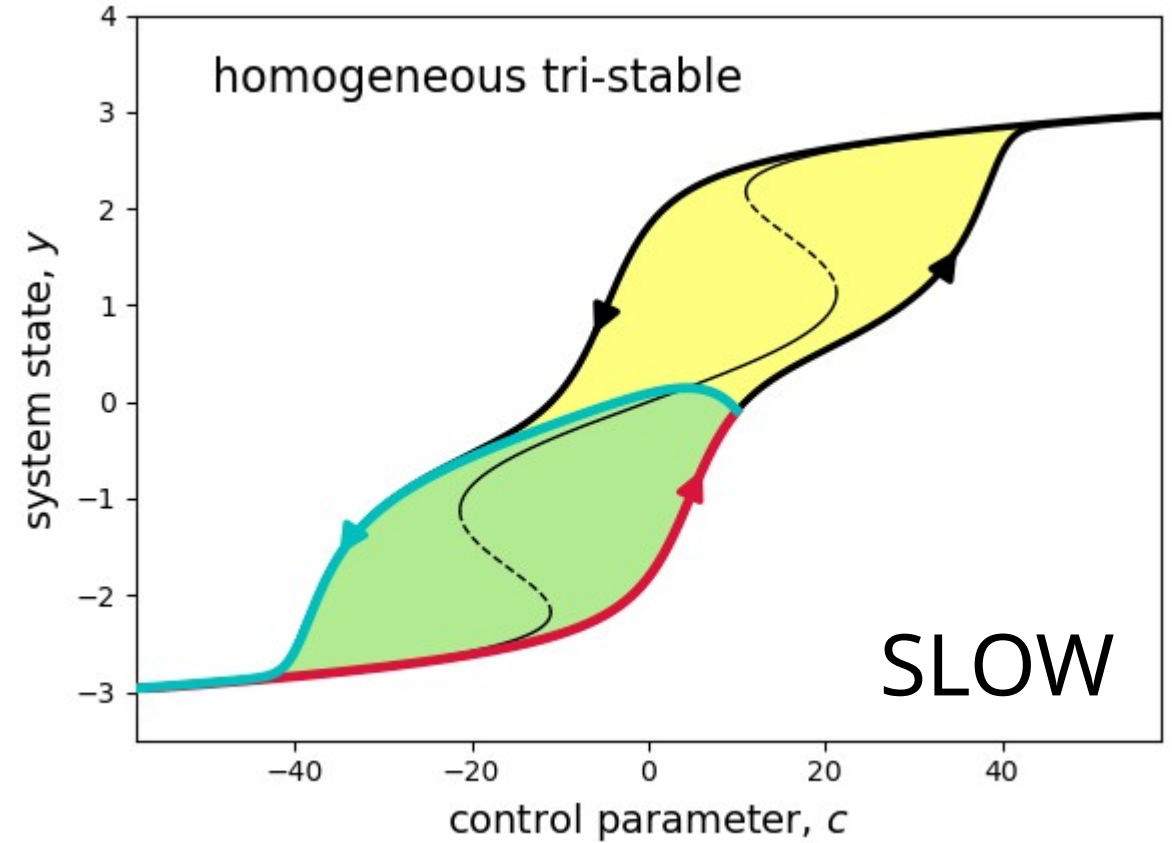
If the "lake" systems react **slow**



$$\frac{\partial y}{\partial t} = c + y - y^3 + g(z, y) + D \frac{\partial^2 y}{\partial z^2}$$



$$\frac{dy}{dt} = c - y + y^3 - y^5$$



How to tell them apart?

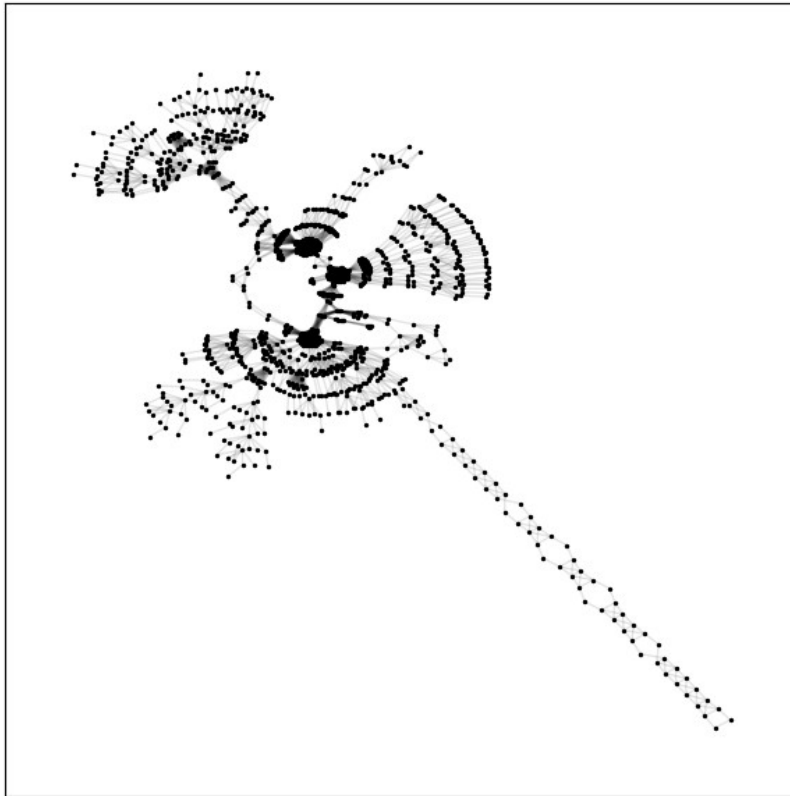


# Cascading tipping in networks

Using networks as a proxy for modular systems

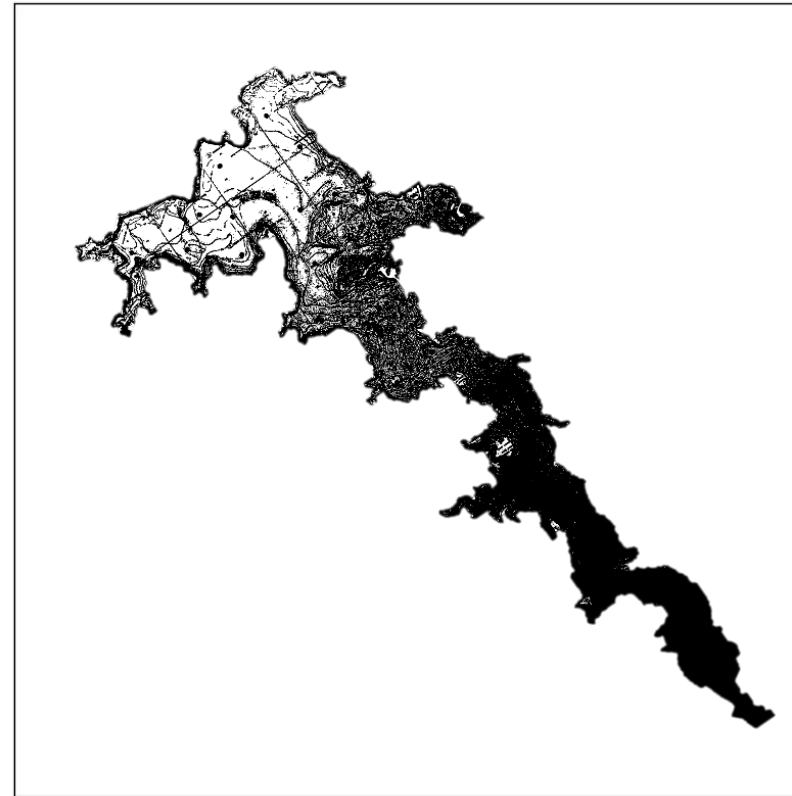


Eletrical network



Nodes: 1176  
Edges: 8688  
Avg. degree: 14  
Assortativity: 0.9  
Clustering: 0.4

Eagle mountain lake

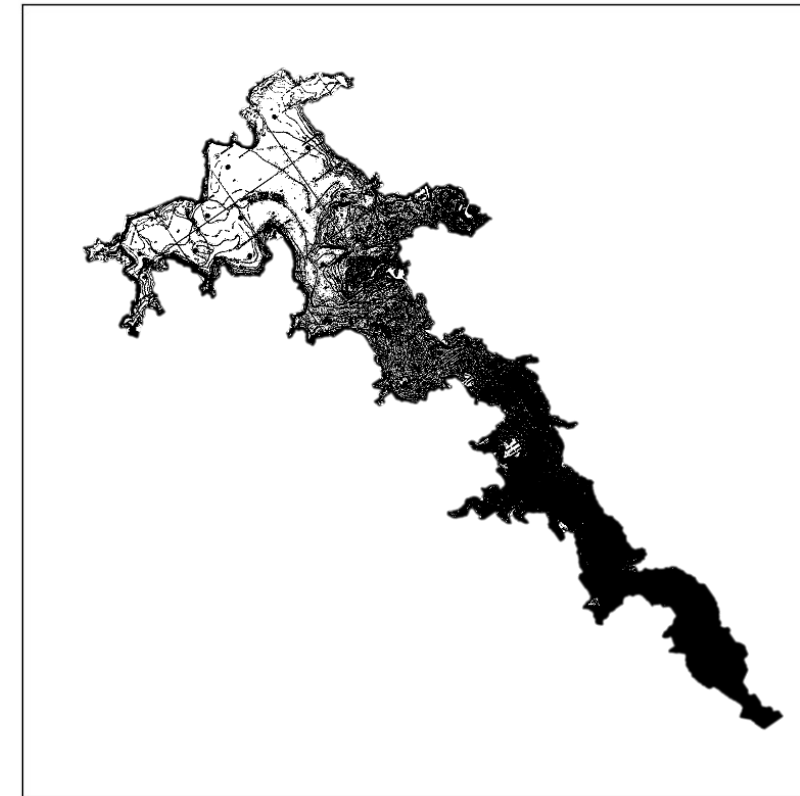
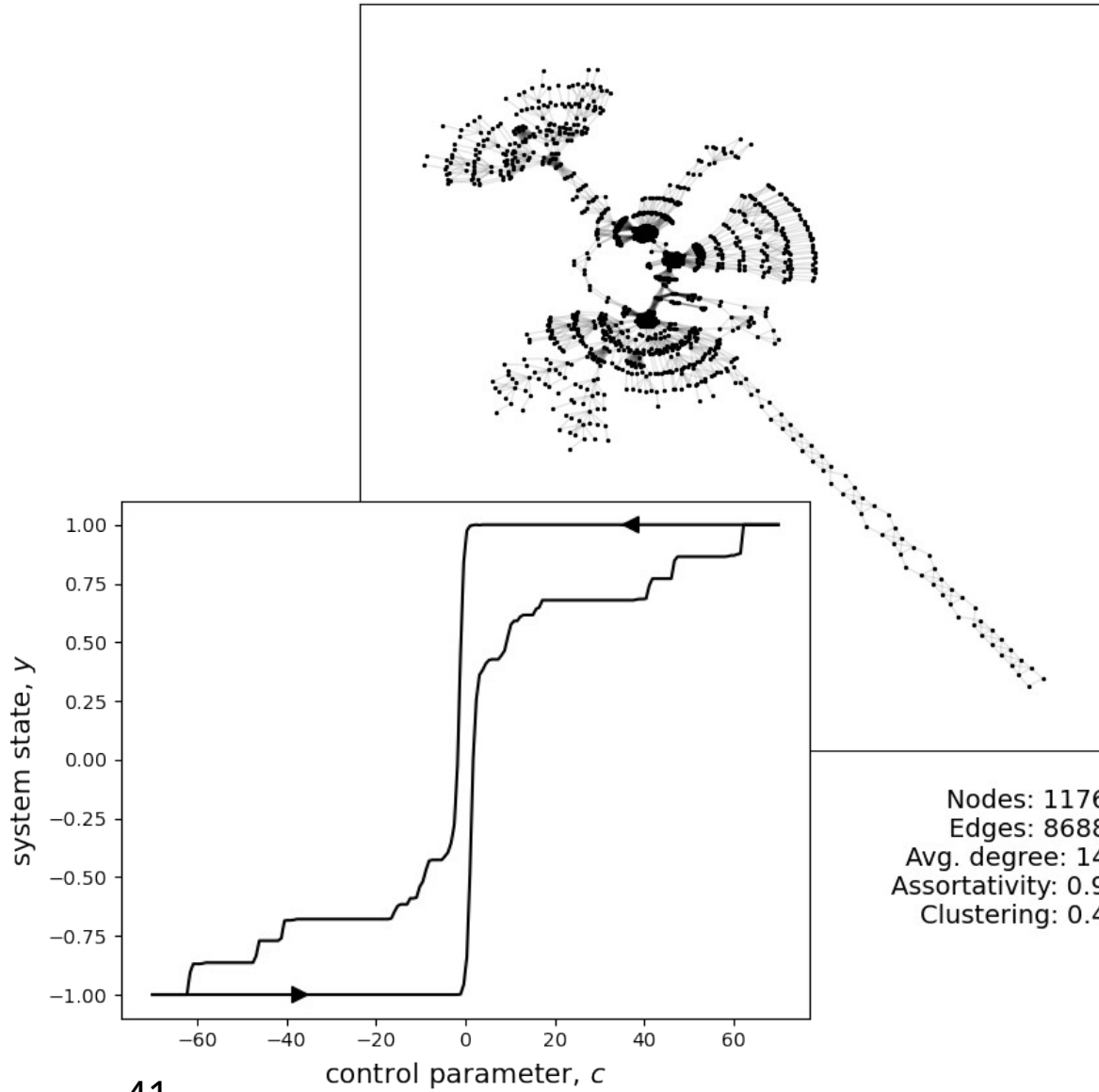


Location: North Texas  
Surface area: 35.18 km<sup>2</sup>  
Water volume: 221.88 hm<sup>3</sup>  
Surface elevation: 198m



Eletrical network

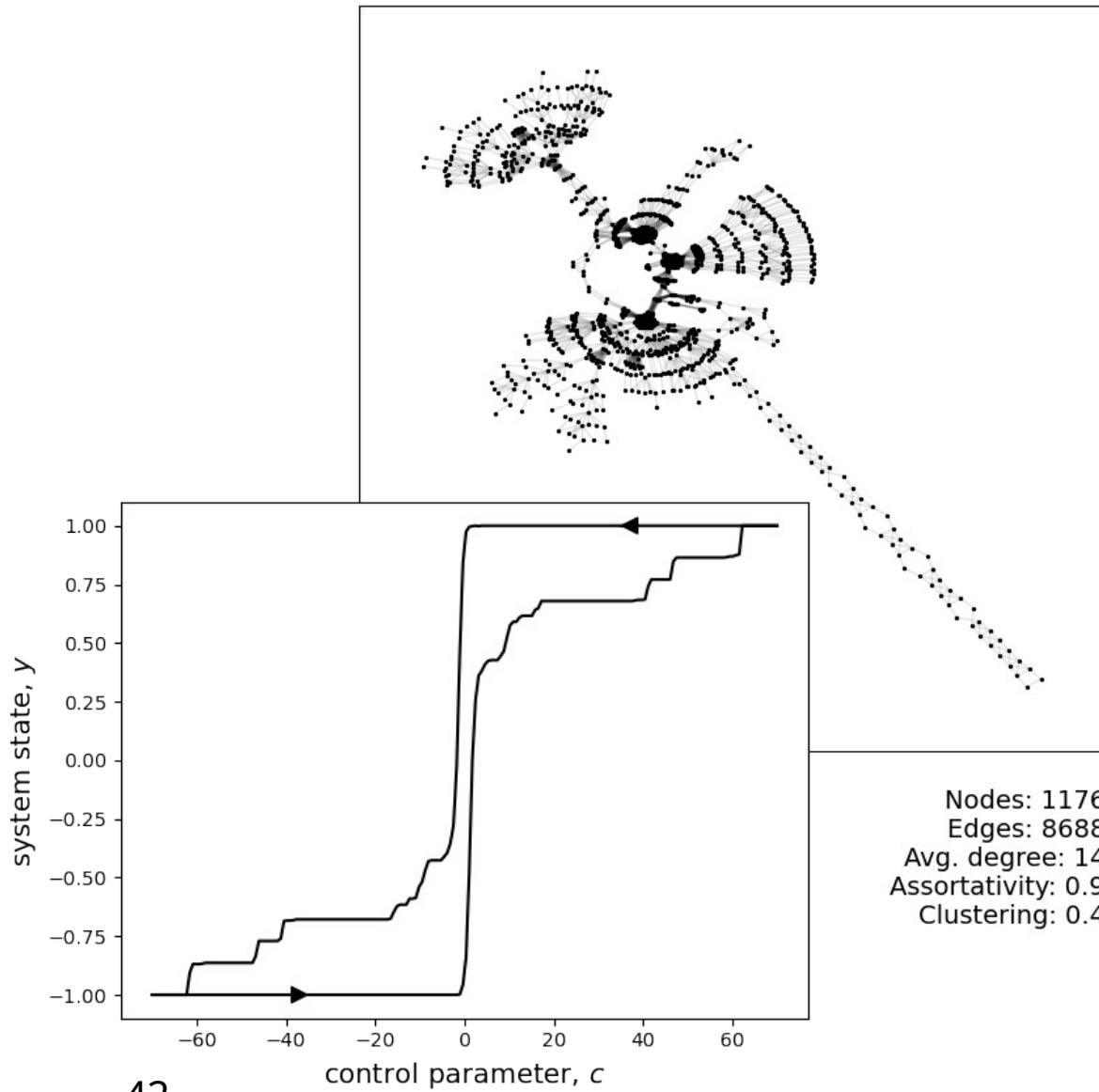
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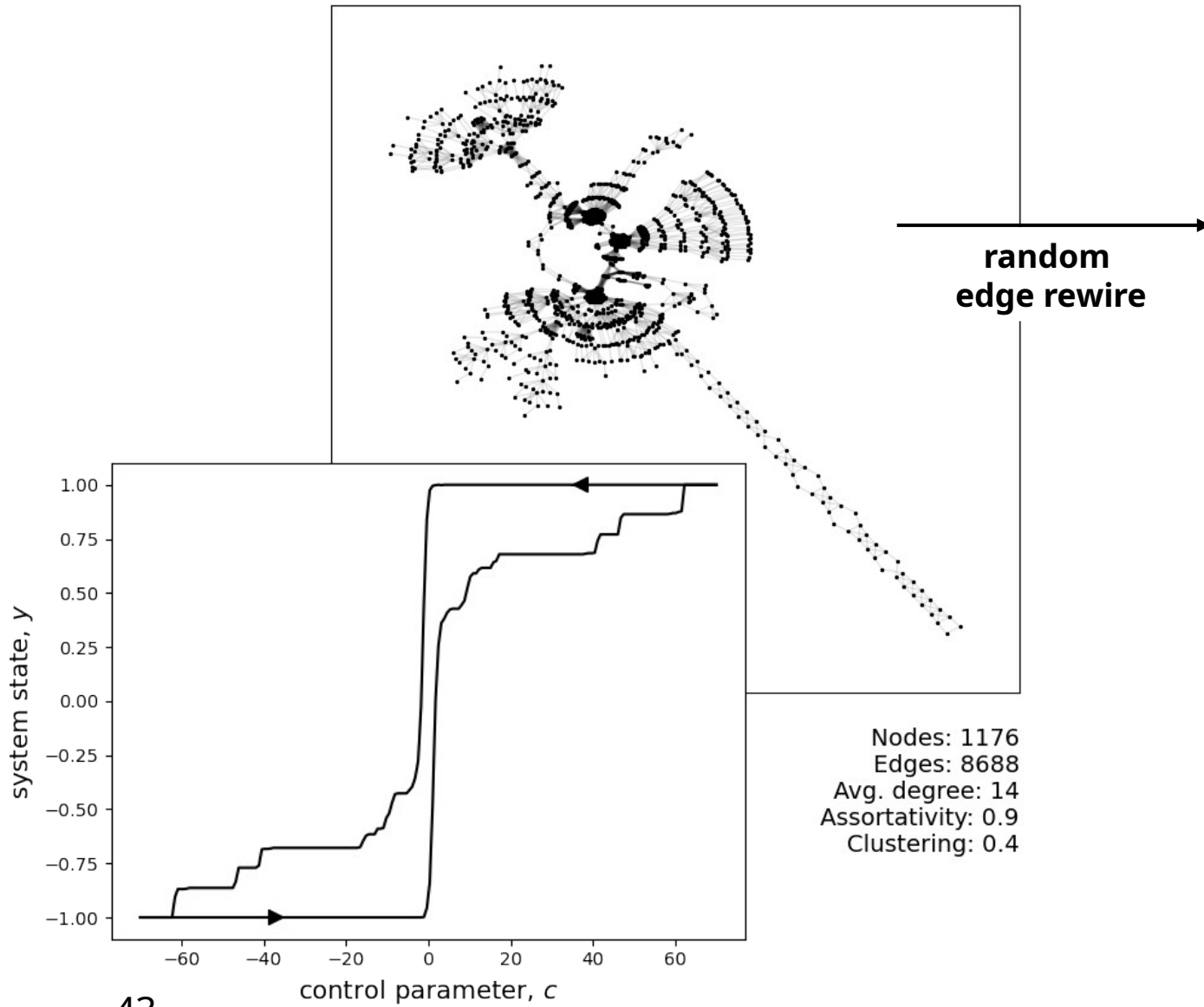


Eletrical network



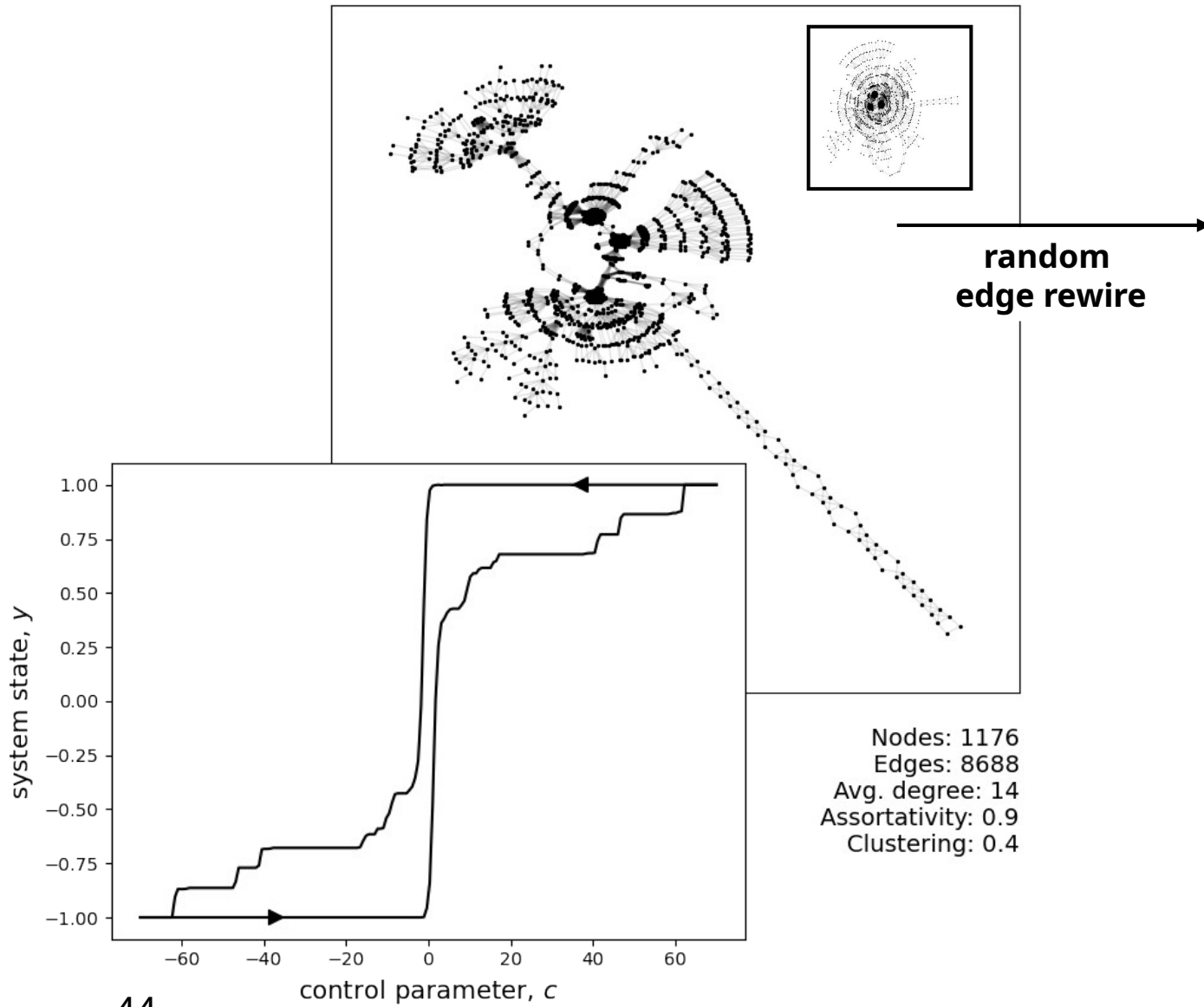


Eletrical network

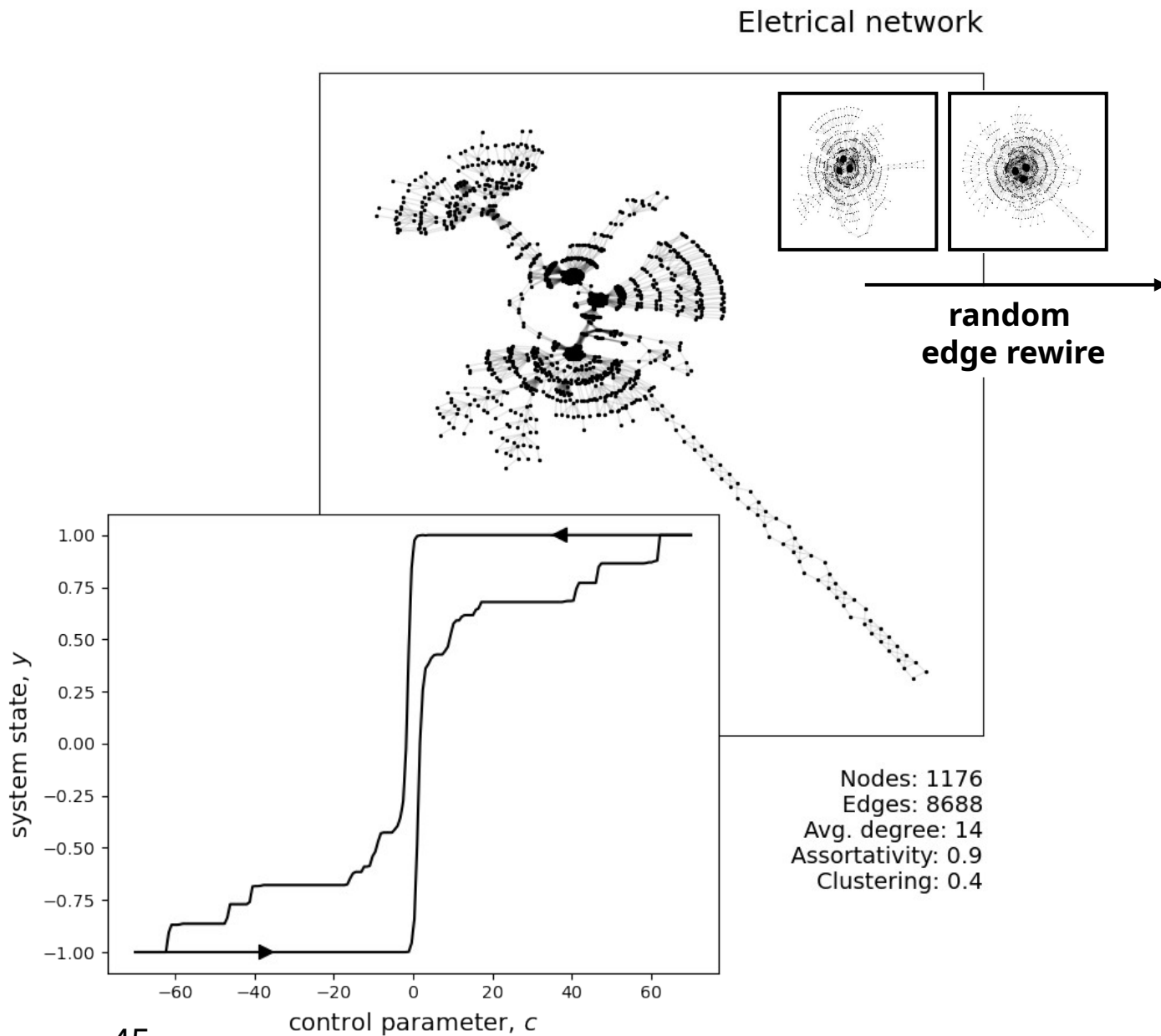




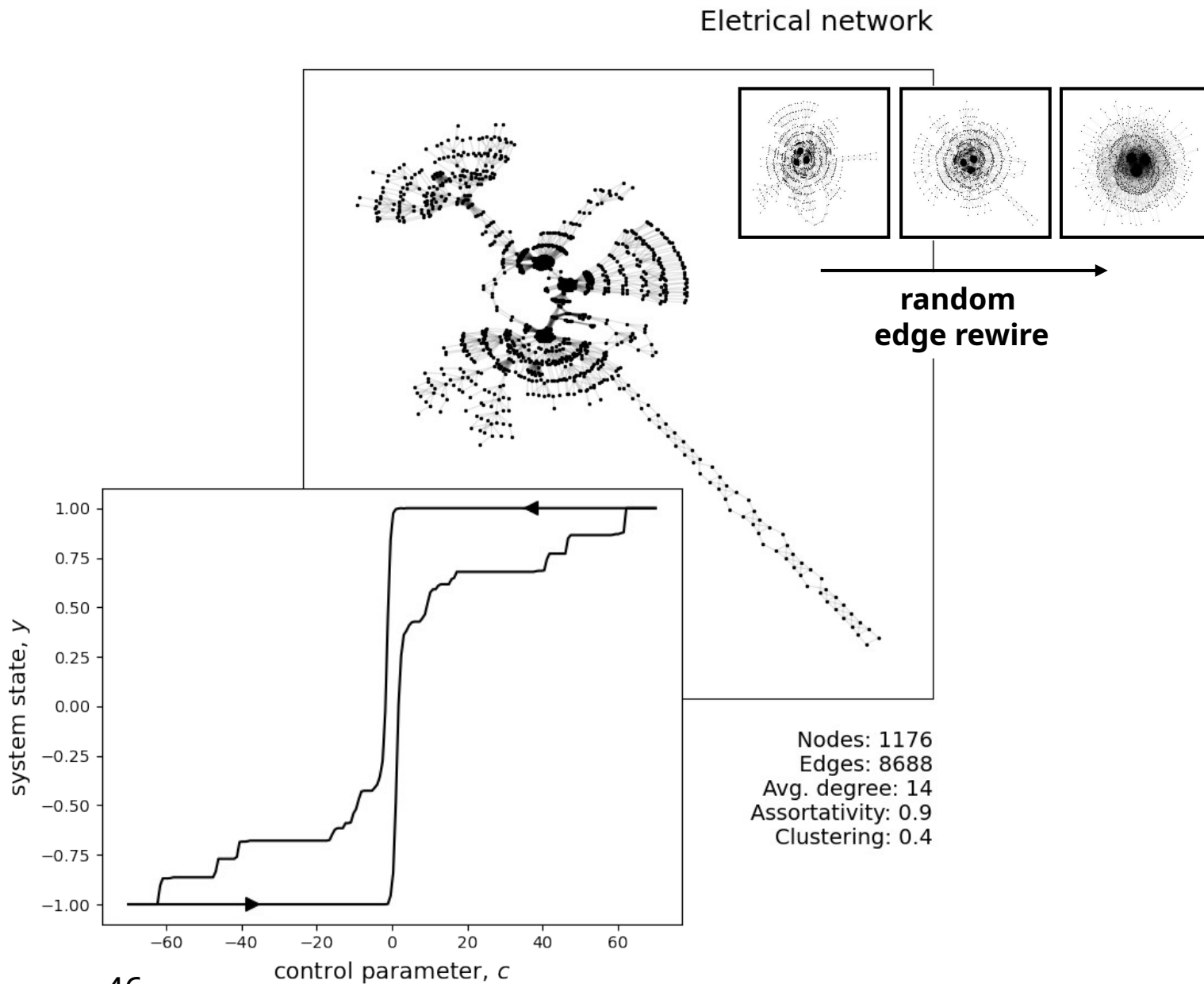
Eletrical network



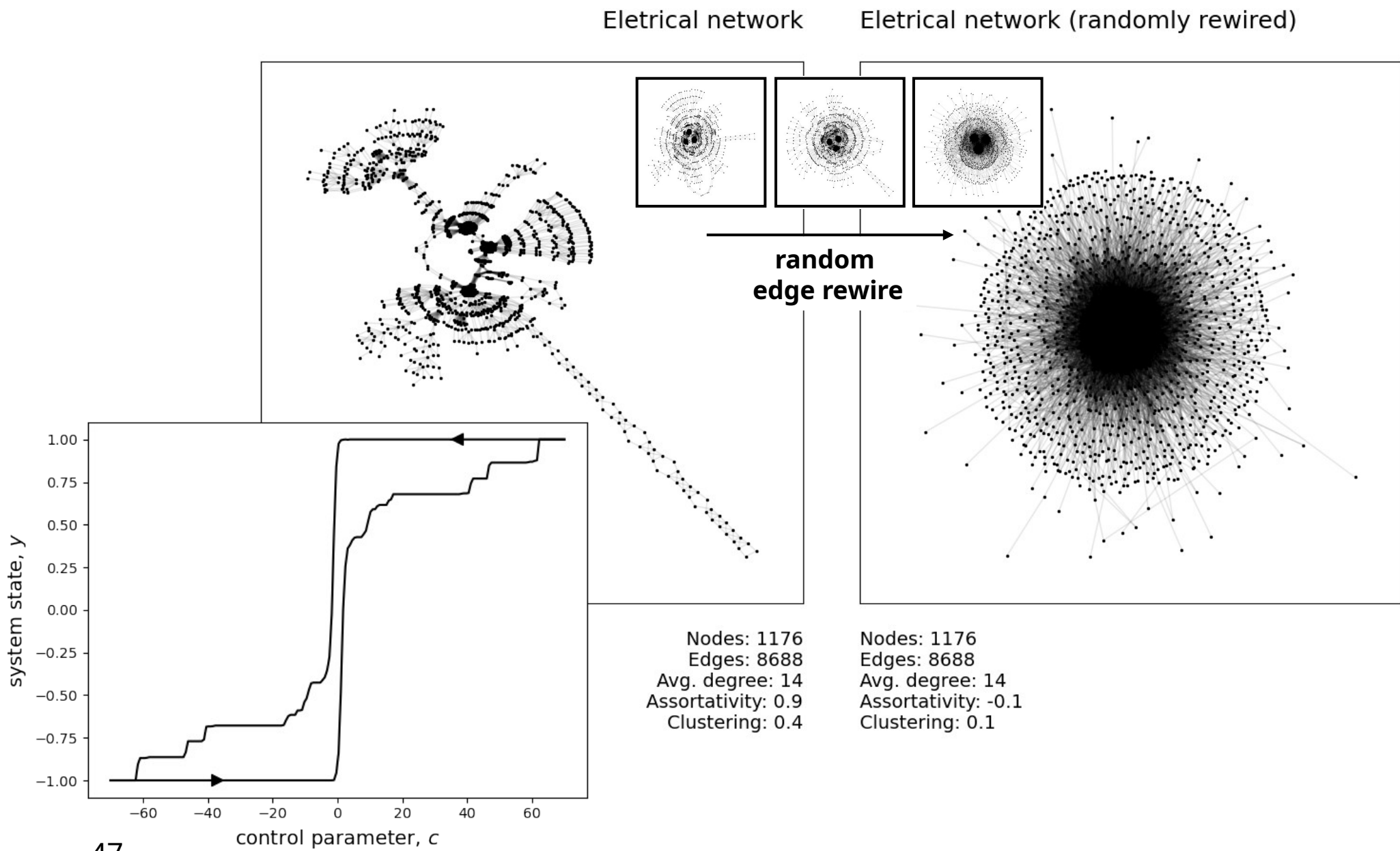




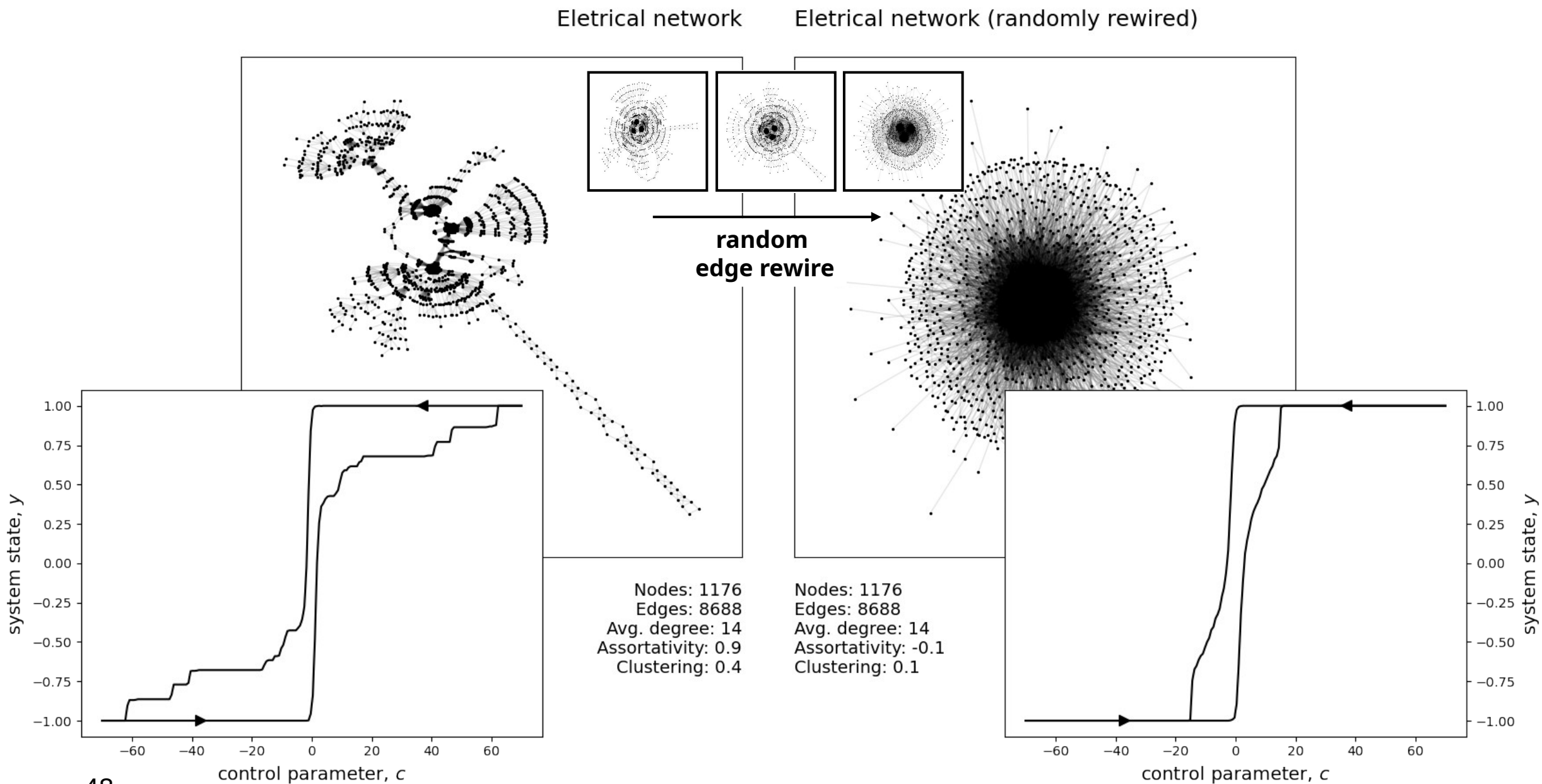












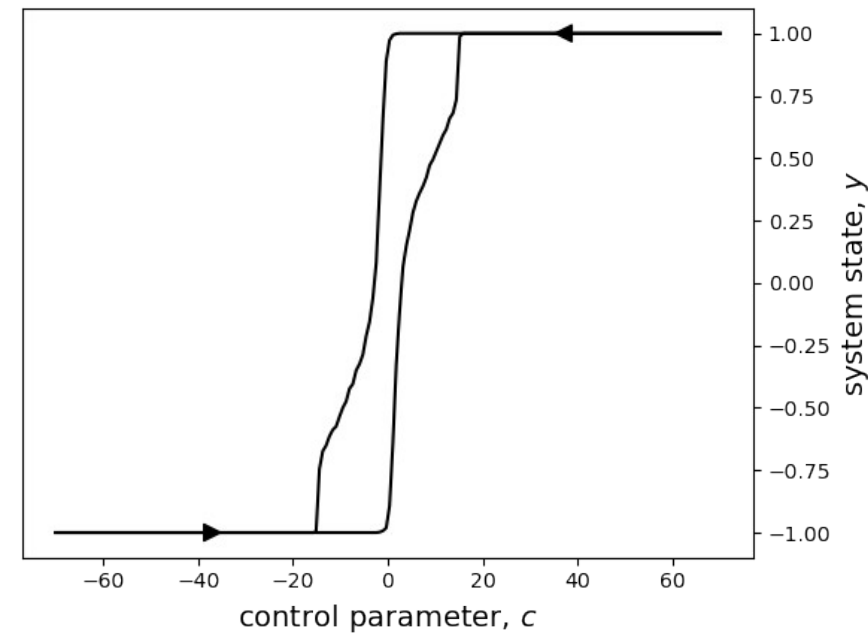
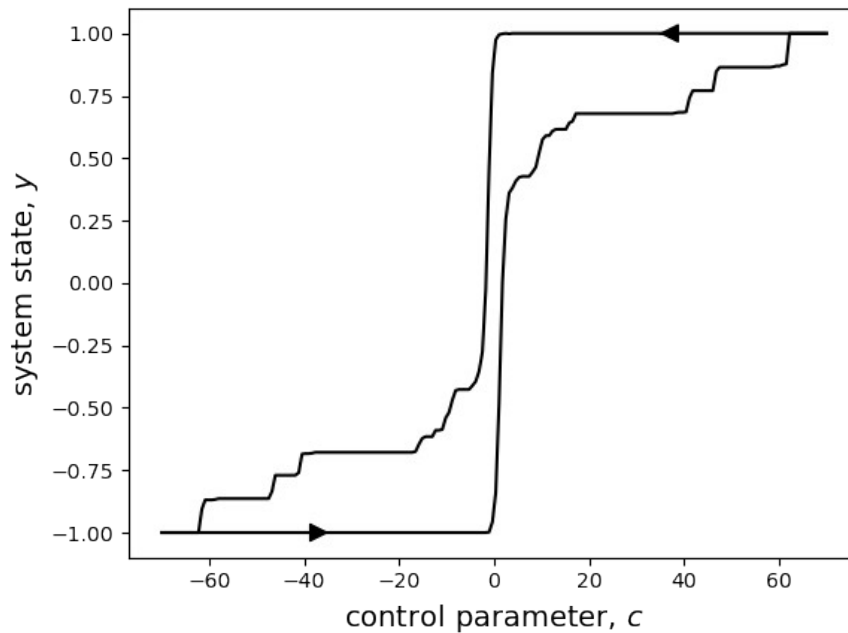


# Linear Regression Model

X ... Network properties

y ... Euclidean distance to original tipping shape

Assortativity Coefficient	Clustering Coefficient	Modularity	Transitivity	Betweenness Centrality	Eigenvector Centrality	Closeness Centrality
-0.570	0.027	-0.006	-0.269	-0.099	-0.400	-0.173



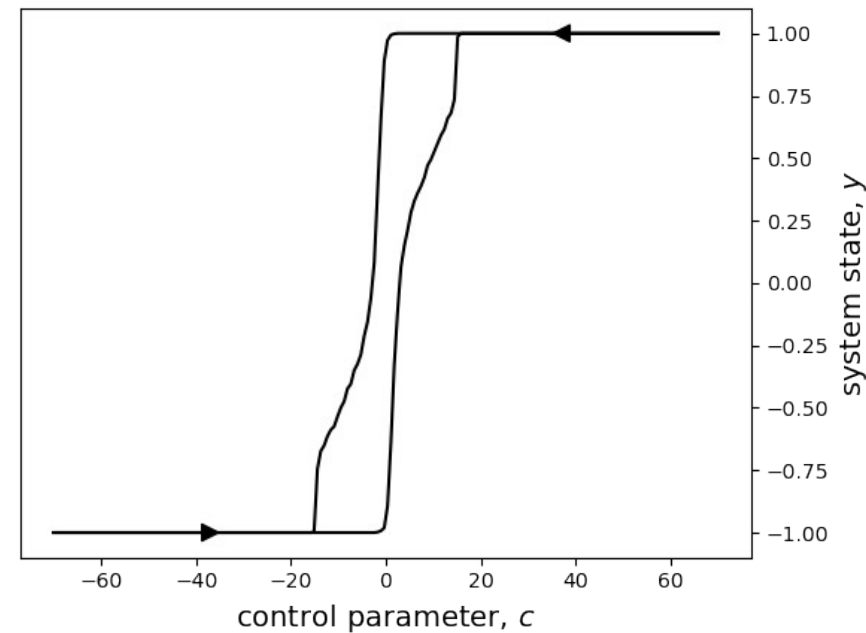
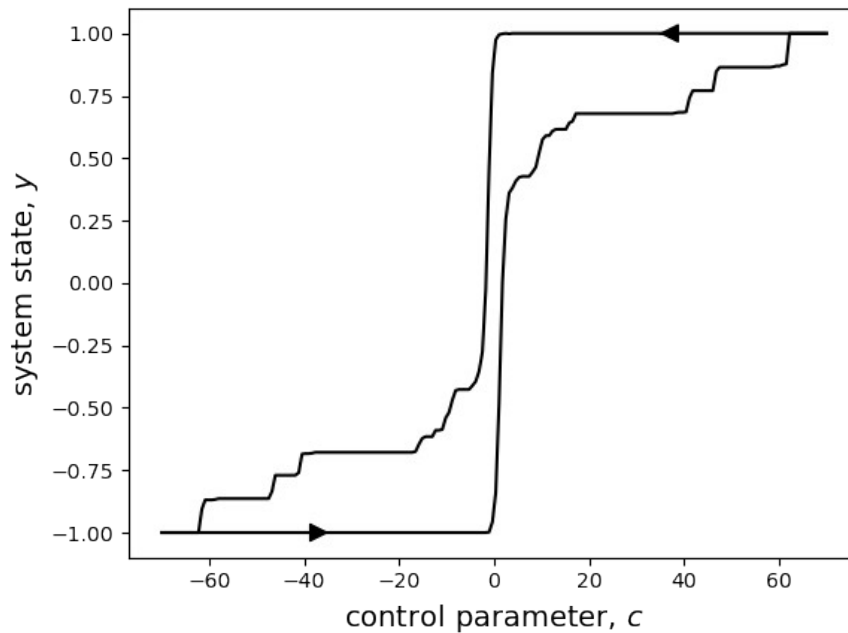


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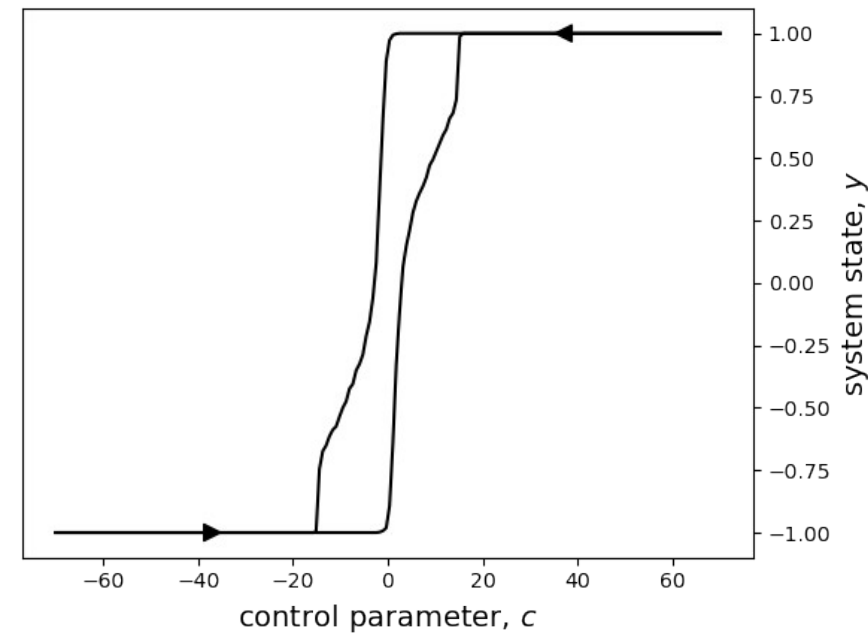
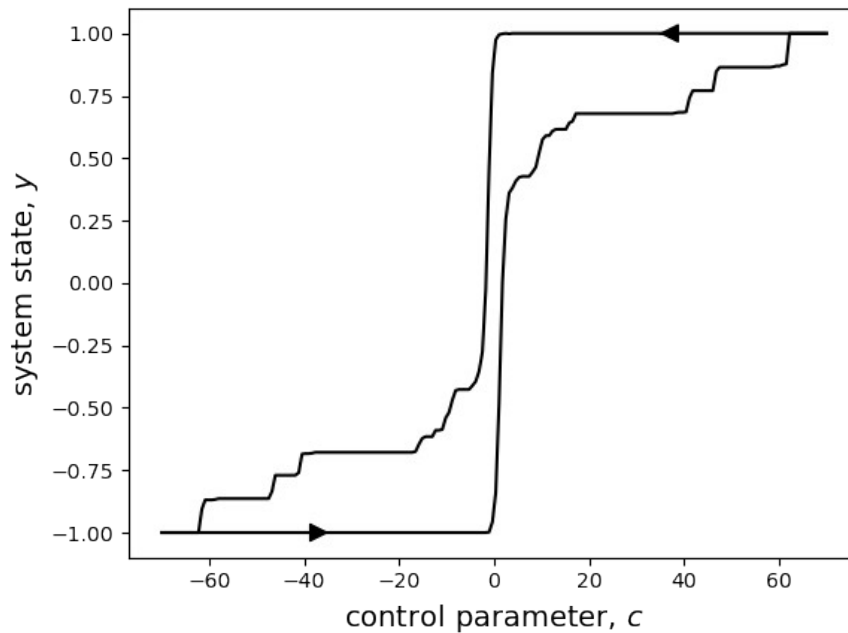


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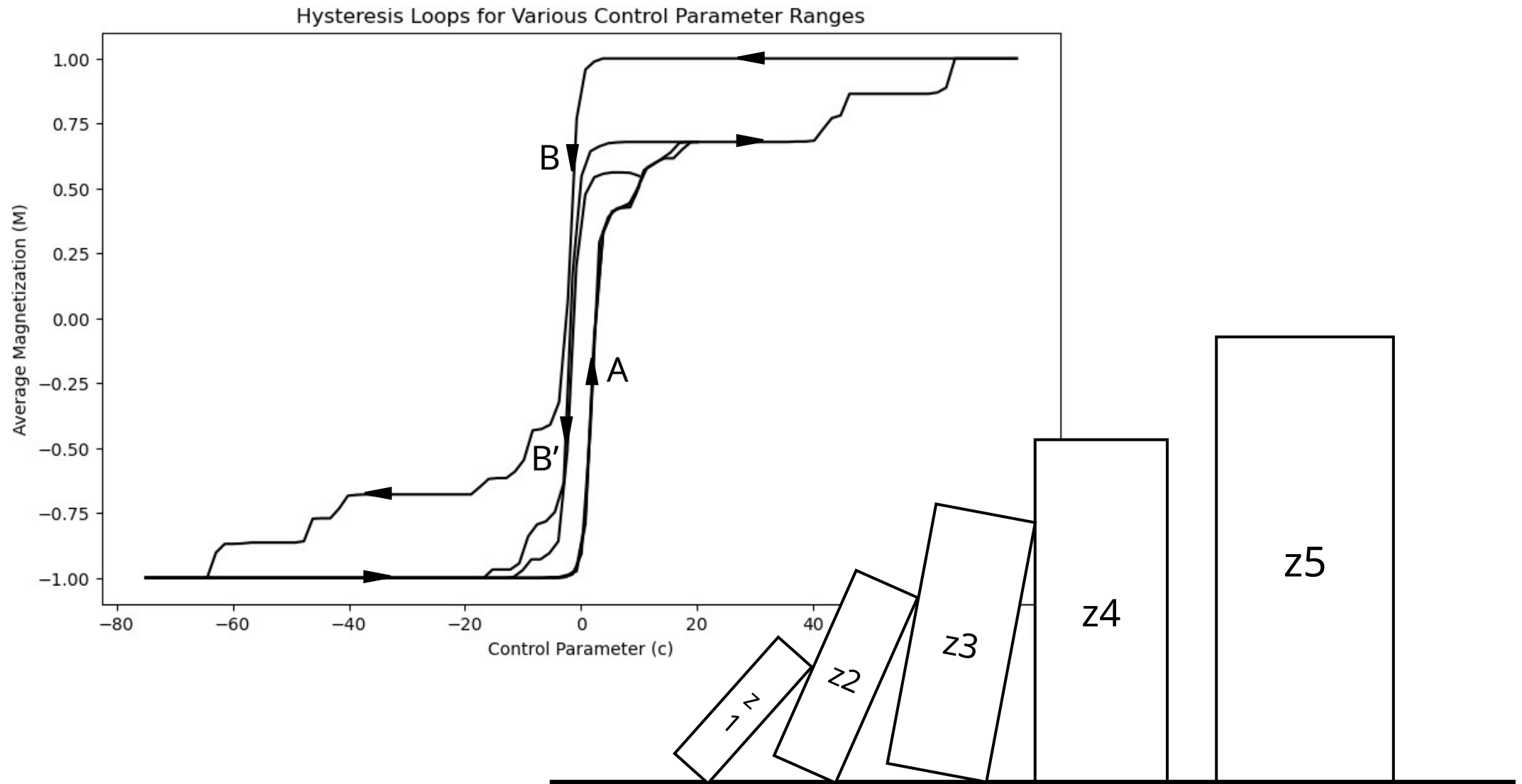




# Phenomenon of growing hysteresis

Just like we saw in the dominos example, the effect of hysteresis can cascade through the system.







# Outlook

- a. How to use this for early warning signal analysis?
- b. How to find appropriate diffusion equations?
- c. Better describe the phenomenon of growing hysteresis.