



FULBRIGHT-NAWI GRAZ VISITING PROFESSORS 2024

Thursday, March 7th 2024

Heinrichstraße 36, HS 11.02

Program

15:10–15:15

Prof. Klemens Fellner, Dean of the Faculty of Natural Sciences

Welcome address

15:15–16:00

Prof. Felix Gotti (MIT, USA)

Atomicity and Factorizations in Monoid Algebras

16:00–16:30

Coffee Break

16:30–17:15

Prof. Dmytro Bilyk (University of Minnesota, USA)

Energy minimization problems on the sphere

The event will be also streamed online via <https://unimeet.uni-graz.at/b/mar-ppo-9tf-7i7>.

Abstracts

Felix Gotti: Atomicity and Factorizations in Monoid Algebras

Given an (additive) commutative monoid M and a commutative ring R , the monoid algebra of M over R , denoted by $R[M]$, is the commutative ring consisting of all polynomial expressions with coefficients in R and exponents in M , with addition and multiplication defined as for polynomial rings. In this talk, we will discuss some classical factorization properties in the setting of monoid algebras that are integral domains. We will give special consideration to the property of being atomic: an integral domain is atomic if every nonzero nonunit factors into irreducibles. In this direction, we will discuss recent progress on the ascent of atomicity from the pair (M, R) to the monoid algebra $R[M]$.

Dmytro Bilyk: Energy minimization problems on the sphere

Energy minimization problems arise naturally in various areas of mathematics: discrete and metric geometry, discrepancy theory, signal processing and frame theory, mathematical physics etc. In such problems, for a given kernel F one seeks to find finite point sets $\{z_1, \dots, z_N\}$ in a domain Ω which minimize the discrete energy

$$\sum_{i,j} F(z_i, z_j),$$

or Borel probability measures on Ω which minimize the energy integral

$$\int_{\Omega} \int_{\Omega} F(x, y) d\mu(x) d\mu(y).$$

This can be interpreted as finding the equilibrium position of N particles or a continuous unit charge under the interaction given by F . Typical choices of the interaction kernel F include powers of various distances (Riesz energies, distance sums/integrals), powers of the inner product (frame energies), as well as various other potentials, both repulsive and attractive-repulsive. A very interesting phenomenon that often occurs for some attractive-repulsive interactions is that minimizers tend to cluster, i.e. minimizing measures happen to be discrete. We shall discuss this and many other phenomena, in particular, relation to the discrepancy theory (Stolarsky identities), peculiar differences between minimizers of Riesz energies for geodesic and Euclidean/chordal distances (closely related to several conjectures by Fejes Tóth) and others. We shall mostly consider the case when the domain $\Omega = \mathbb{S}^{d-1}$ is a sphere. The talk will be accessible to a wide mathematical audience, including students.