Mathematics for Microeconomics

Sample Questions

In the following, you find a number of sample questions. Some of those questions are old exam questions. I try to roughly assign the questions to one topic each (but in many cases, one question might be associated with several of the listed topics).

Topics in Differentiation

1. Employ the chain rule to find the first derivative of $y(x) = k\, e^{x^2} + f$ with respect to x.

2. Consider $y(t) = a(t)^{4\,x}\, b(t)^{2\,y}$. Employ log-differentiation to calculate the growth rate of $y(t)$.

3. Derive the (first order) partial derivatives of $x(t)$ with respect to variable t. (Remember that $\dot{y}(t) \equiv \partial\, y(t)/\partial\, t$.)

(a) $x(t) = \dot{y}(t)/y(t)$
(b) $x(t) = y^{1+c\,\ln t}$.

Implicit Function Theorem

1. Consider demand for vitamin pills by $d(p, a)$, depending on the price p, and advertising expenses, a. Furthermore, consider its supply function $s(p, b)$, where b captures some cost, external to the firm. Suppose, the price mechanism works and the market for good x is in equilibrium.

(a) How does the market clearing price change upon a rise in advertising expenses? Find the expression for $d\,p/(d\,a)$, and provide all sufficient conditions for the existence of this derivative.

(b) What is the sign of expression $d\,p/(d\,a)$? Under which (reasonable) conditions will the sign of $d\,p/(d\,a)$ be positive?

Optimization Theory, Classical Programming

1. A Silicon Valley firm produces computer chips by employing labor, l, and capital k according to the following production function: $f(l,k) = A\,l\,\sqrt{k}$, where parameter A measures technological development.

(a) By investigating the Hessian matrix, argue whether the production function is concave or convex.

(b) Suppose both factor prices are equal: $w = r$. Calculate the *cost minimizing* factor input combination k/l.

2. Consider an opportunity set $X \subset \mathbb{R}^N$, with $x \in X$, i.e., $x = (x_1, x_2, ..., x_N)$. Furthermore, consider an objective function $f(x) : X \to [0,1]$. Provide sufficient conditions for the existence of an $x^\star \in X$ such that $f(x^\star) \geq f(x)$ for all $x \in X$.

3. Generally, second order conditions (SOC) of constrained optimization problems depend on both the objective function, $f(x_1,\, x_2,\, ..., x_n)$, and the constraint, $g(x_1,\, x_2,\, ..., x_n) = 0$. Consider, now, a constrained optimization problem with two choice variables and a single *linear* constraint. For this problem, the SOC is:

$$f_{11}\, f_2^2 - 2\, f_{12}\, f_1\, f_2 + f_{22}\, f_1^2 < 0\,.$$

For this problem, the SOC is independent of the constraint — how come? Explain why neither $g(x_1,\, x_2,\, ..., x_n)$ nor any derivative of $g(x_1,\, x_2,\, ..., x_n)$ enters the SOC.

Nonlinear Programming

1. Consider the following utility function: $u(x,y) = b\, \ln(x) + y$. The prices of (x,y) are: (p_x, p_y). The maximum amount of income to be spent is: $I = 50$.

(a) Suppose $50 < b\, p_y$. Calculate the utility maximizing quantities x^*, y^*.

$x^* =$ .... $y^* =$ ....

(b) Suppose $50 > b\, p_y$. Calculate the utility maximizing quantities x^*, y^*.

$x^* =$ .... $y^* =$ ....

Matrix Algebra

1. Consider the following system of two equations:

$$4x + 7y = a\,,$$
$$x - y = b\,.$$

Employ Cramer's rule to calculate x and y.

2. Consider a utility function $u(x_1, x_2) = \alpha \ln x_1 + \beta \ln x_2$.
(a) Find the eigenvalues (e_1, e_2) of the Hessian of $u(x_1, x_2)$.

$e_1 =$... $e_2 =$...

(b)Determine the parameter restrictions on α and β that ensure strict concavity of $u(x_1, x_2)$.

Parameter restriction on α: ....

Parameter restriction on β: ....

Functions and Sets

1. Consider the following Cobb-Douglas function:

$$f(x_1,\, x_2) = a\, x_1^{\alpha}\, x_2^{\beta} + b\,.$$

(a) Derive a condition for concavity of $f(x_1,\, x_2)$.
(b) Derive a condition for convexity of $f(x_1,\, x_2)$.
(Hint: Consider the leading principal minors of the Hessian matrix.)

2. Consider function $g(x_1, x_2)$.
(a) Impose sufficient conditions on $g(x_1, x_2)$ for strict quasiconcavity.
(b) Suppose, $g(x_1, x_2) = x_1\, x_2^{\alpha}$. Which condition is required for quasiconcavity?

Propositional Logic, Proof Strategies

1. Establish the truth of the following statement φ:

φ: $[x \leq 0] \Rightarrow [x^2 \leq (x-1)^2]$

Use the method of proof by the contrapositive.

2. Establish the truth of the following statement: "If x and y are integers, and $(x\,y)$ is an odd number, then x and y are both odd." (Use the method of proof by the contrapositive.)