A Veblenian Externality, Economic Growth, and Corrective Taxation\footnote{We thank the participants of research seminars at Umea University, the University of Udine, the University of St. Gallen, and Brunel University for helpful comments. We are also grateful to Thomas Aronsson, Evangelos Dioikitopoulos, and Xavier Raurich for insightful discussions on a former version of this paper. We retain sole responsibility for any remaining errors.}

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\textbf{Abstract.} This paper addresses, in an endogenous growth (via public investment) context, the effects of conspicuous (relative) consumption on economic growth and on optimal taxation. In contrast to the prior literature, this paper accounts for the “Veblenian characteristic” – the consumption-leisure complementarity – of relative consumption. To be visible, relative consumption requires both individual leisure- and social interaction time. Conspicuous consumption – via a rise in labor supply – makes public investment more productive and, thereby, raises endogenous growth. If the externality associated with social interaction time becomes important enough for displaying conspicuous consumption, optimal (corrective) consumption- or wage tax rates may become negative. If the government chooses its investment share optimally, the optimal capital income tax is zero in spite of the consumption externality.

\textbf{Keywords and Phrases:} Consumption externality, Veblen, consumption-leisure complementarity, public capital, endogenous growth, optimal taxation.

\textbf{JEL Classification Numbers:} D62, D91, E21, H21, O41

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“Conspicuous consumption of valuable goods is a means of reputability to the gentleman of leisure.”

– Veblen (1899, p. 64)

1. Introduction *** motivation: overwhelmed

In his *Theory of the Leisure Class*, Veblen (1899), emphasized not only the quest for status – via conspicuous consumption – as an important component of the pursuit of self-interest, but also that leisure is important when signaling social status. The present paper addresses, in an endogenous growth context, the effects of conspicuous consumption (or, more generally, a consumption externality) when an explicit role of leisure is considered, on economic growth as well as on optimal taxation. Thus, the perspective is both positive, in the sense that the model aims to explain possible implications of such social comparisons, and normative in the sense that it discusses what the government ought to do in such a world.

In contrast to the prior literature, this paper accounts for what is here denoted the “Veblenian characteristic” – the consumption-leisure complementarity – of relative consumption. Veblen (1899) emphasizes that displaying conspicuous consumption takes leisure time. In order to display a new luxury car, it takes leisure time to drive around the corner. And in order to show that one can afford a dinner at a luxury restaurant, it takes leisure time to eat and enjoy the sophisticated foods served in such a restaurant. More generally, it makes sense to consider consumption and leisure as complements when it comes to displaying status. Moreover, to be visible it is not sufficient that the individual him- or herself has a certain amount of leisure, also other people need to have some leisure to observe the conspicuous behavior. Indeed, an individual cannot display wealth effectively when being alone in a fancy restaurant, and when being alone on the street in a luxury car. We will refer to average leisure time as *social interaction time*, to distinguish it from the individual’s own leisure time. Average leisure – that is, social interaction time – allows the society to receive and evaluate the signals from the individual, and correspondingly allows the individual to observe the signals from others. We

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2 Veblen (1899) was not the first philosopher (economist) to articulate this observation. Adam Smith (1759) devoted a section to conspicuous consumption in his *Theory of Moral Sentiments*. Even in the ancient world, Plato, in *The Republic* (book II) wrote: “Since ...appearance tyrannizes over truth and is lord of happiness, to appearance I must devote myself.”

3 Different terms have been used in the literature. They include conspicuous consumption, relative consumption, status consumption, keeping up with the Joneses (KUJ), or simply consumption externality. Here, we use these terms synonymously.
call a consumption externality, augmented by this Veblenian characteristic, a *Veblenian consumption externality*.

A Veblenian consumption externality introduces two distinct externalities. First, consumption of an individual household has a direct (negative) impact on utility of all other households via their consumption reference levels. Second, the leisure choice of an individual household also has an effect on utility of all other households via the social interaction component. By devoting more time to leisure, an individual is able to better display her consumption (status). The social interaction component implies that an individual can observe others’ consumption better if they increase their leisure, and hence that others can observe that individual’s consumption better if she or he raises leisure. Clearly, this externality is not considered by that individual. Whether or not the externality from increased leisure dominates the externality from increased consumption, has stark consequences for the design of an optimal policy scheme.

This paper is related to two different literatures, one on consumption externalities and growth and the other on consumption externalities and optimal taxation.

A sizeable body of literature has discussed the impact of consumption externalities on economic growth. Alvarez-Cuadrado *et al.* (2004) show that the desire to catch up with the Joneses raises (lowers) the consumption growth rate if consumption exceeds (is lower than) the consumption reference stock. In the presence of capital externalities, a consumption externality results in an increase in the balanced growth rate (Brekke and Howarth 2002, Carroll *et al.* 1997, Corneo and Jeanne 1997, Liu and Turnovsky 2005).

None of these papers, however, takes into account the Veblenian characteristic of a consumption externality. In the present paper, the engine of economic growth is public infrastructure investment, which causes a positive production externality to an individual firm. The higher the (elastic) labor supply, the more productive a given amount of public investment is. In this setting, the Veblenian characteristic has a stark impact on endogenous growth.

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4 In a model where people’s utility depends negatively on others’ current and past consumption.
There is also a sizeable body of literature discussing optimal taxation in the presence of consumption externalities (for some examples, cf. Boskin and Sheshinski 1978, Layard 1980, Ljungqvist and Uhlig 2000, Aronsson and Johansson-Stenman 2008 and 2010, Micheletto 2008, and Eckerstorfer and Wendner 2013). From this literature it is well established that an externality due to conspicuous consumption concerns calls for some corrective element in the tax system. For example, Aronsson and Johansson-Stenman (2008) show that the presence of such a consumption externality typically implies substantially higher marginal income tax rates. Eckerstorfer and Wendner (2013) demonstrate that the specific design of optimal consumption taxes depends strongly on the consumption reference levels (specific reference groups) of individual households.

Aronsson and Johansson-Stenman (2013a) is the only paper with a specification that includes something similar as a Veblenian externality. Yet, their setting is quite different. For example, in their model people care about relative consumption with respect to a reference consumption level, which in turn is given by a leisure-influenced average of the consumption level, such that those with a high leisure level will have a higher weight. Here, in contrast, the degree to which relative consumption matters, i.e. the degree of positionality, is a function of both individual leisure and social interaction time (i.e. others’ leisure). Moreover, their model is static, while the present model is dynamic and allows for a growing economy.

This paper brings about two sets of results. The first set relates to the impact of the Veblenian consumption externality on equilibrium labor supply and growth in the framework of a competitive economy. The second set of results relates to optimal taxation in the presence of public investment- and Veblenian consumption externalities.

In a competitive equilibrium, the Veblenian consumption externality raises the marginal rate of substitution of consumption for leisure. As a consequence, the consumption externality always raises equilibrium labor supply, thereby the endogenous growth rate of per capita output and consumption. This growth rate, however, is affected by the specific tax regime employed to finance a given level of public investment. Specifically, in case the consumption externality is strong enough, a consumption tax is the least distortionary means of financing public investment and gives rise to the highest endogenous growth rate.

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5 Aronsson and Johansson-Stenman (2013b) and Arrow and Dasgupta (2009) present models where utility also depends directly on relative leisure, i.e. own leisure in relation to others’ leisure.
In a social optimum, the optimal consumption- and wage taxes are both time-dependent along a transition path. The main reason consists in the Veblenian characteristic that attaches a leisure weight to the relative consumption term. As leisure changes in time along a transition path so does the leisure weight. As a consequence, the corrective consumption- and wage taxes are also required to change over time, along a transition path. Only in the absence of the Veblenian characteristic are optimal consumption- and wage taxes constant along a transitional path.

Along a balanced growth path (BGP), the sign and magnitudes of optimal tax rates depend on the Veblenian consumption externality term in the social optimum relative to that in a decentralized economy. We let \( v \) denote this ratio. If only individual leisure time matters but social interaction time did not matter, \( v \) would always be lower than unity. In this case, the optimal consumption- or wage tax rates are strictly positive and proportional to \( v \) in order to internalize the Veblenian consumption externality. The Veblenian characteristic lowers the optimal tax rates, as it lowers the households’ willingness to give up leisure for additional (conspicuous) consumption. Households aim at complementing conspicuous consumption with leisure time. If, however, social interaction time matters strongly enough, \( v \) exceeds unity. In this case, the externality associated with social interaction time takes over, and optimal consumption- and wage taxes are required to encourage a lowering of leisure time, so the tax rates become negative. This possibility, however, is entirely due to the Veblenian characteristic, that is, with the externality associated with social interaction time.

The sign and magnitude of the optimal capital income tax rate depends on the public investment share relative to the optimal one. If the government chooses the public investment share optimally, the optimal capital income tax rate is zero. This Chamley (1986) – Judd (1985)- like result holds in spite of the Veblenian consumption externality.

The proceeding Section 2 presents the endogenous growth model with elastic labor supply. The Veblenian consumption externality is discussed in detail. Section 3 analyzes the effects of the Veblenian consumption externality on equilibrium labor supply and equilibrium endogenous growth. Section 4 sets up the socially optimal transitional- and balanced growth paths. It analyzes the optimal consumption-, wage- and capital income taxes. In Section 5, numerical simulations shed light on the possible quantitative effects of the Veblenian externality on labor supply, endogenous growth, the propensity to consume in equilibrium, as
well as on optimal corrective tax rates for various parameter choices regarding the Veblenian externality. Section 6 concludes the paper.

2. The competitive economy

In this section we outline firm behavior, the endogenous growth engine, preferences, budget and resource constraints, and equilibrium conditions.

Firm- and government behavior

Firm- and government behavior follows commonly made assumptions, see, e.g., Turnovsky (2000, p. 468 ff). The economy is populated by homogeneous agents whose number is normalized to unity and fixed in a continuous time framework. At every date \( t \), the representative agent is endowed with one unit of time that can be allocated either to leisure, \( l_t \), or to work, \( 1-l_t \). Output of the representative firm, \( y_t \), is determined by the Cobb-Douglas production function

\[
y_t = A k_t^\alpha (1-l_t) H_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0,
\]

where \( k_t \) denotes private capital, \( H_t \) is the flow of government spending for public investment, \( A \) represents a productivity parameter, and parameter \( \alpha \) denotes the elasticity of output with respect to private capital.\(^6\) Government spending is considered a pure public good. It interacts with labor supply to yield \( (1-l_t) H_t \) efficiency units of labor. From the point of view of an individual firm, \( H_t \) is exogenous. Thus, from the firm perspective production is characterized by constant returns to scale, while there are increasing returns to scale in production from a social perspective (due to \( H_t \)).

The competitive firm hires factors up to the point where the respective marginal product corresponds to its cost (with \( H_t \) considered exogenous to the firm):

\(^6\) A notation alert is in order here. Although we consider time to be continuous, we use the subscript notation throughout. Consider any variable \( x_t \). Then, \( x_t \equiv x(t) \).
\[ w_t = (1 - \alpha) \frac{y_t}{1 - l_t}, \quad r_t = \alpha \frac{y_t}{k_t}, \]  

(2)

where \( w_t \) and \( r_t \) respectively denote the wage- and interest rate.

The government spends a fixed proportion of output, \( h \), on public investment:

\[ H_t = h y_t. \]  

(3)

We assume that the government runs a balanced budget at each point in time, i.e. total tax revenue equals public expenditures. In addition to the investment share \( h \), the government’s instruments include four taxes: taxes on capital- and wage income, \( \tau_{k_t} \) and \( \tau_{w_t} \), a consumption tax, \( \tau_{c_t} \), and a non-distortionary poll tax, \( t_t \), i.e. a tax that the individual faces regardless of consumer choices in terms of labor supply and savings. We express the poll tax by a share \( t_t \) in output, such that \( t_t = t_t y_t \) (recall that \( y \) is exogenous to the individual). The (binding) government budget constraint at each point in time is then given by:

\[ \tau_{w_t} w_t (1 - l_t) + \tau_{k_t} r_t k_t + \tau_{c_t} c_t + t_t y_t = H_t = h y_t. \]  

(4)

Given (3), the economy’s resource constraint becomes\(^7\)

\[ \dot{k}_t = (1 - h) y_t - c_t. \]  

(5)

Considering (1) and (3), aggregate output as viewed from the social perspective becomes

\[ y_t = \left( A h^{1 - \alpha} \right)^{\frac{1}{\alpha}} (1 - l_t)^{(1 - \alpha) / \alpha} k_t. \]  

(6)

**Household behavior**

The population size is normalized to unity. The representative individual faces given market prices, is equipped with perfect foresight and chooses a plan \( (c_t, l_t), t \geq 0 \) \( \{1\} \) so as to

\[
\max_{\{c_t, l_t\}} U_0 = \int_{t=0}^{\infty} u(c_t, c_t - c'_t, l_t, \bar{t}) e^{-\rho t} dt, \quad \rho > 0,
\]  

(7)

where \( u(\cdot) \) denotes the instantaneous utility function, \( \rho \) is the constant pure rate of time preference, \( c'_t \) is a reference consumption stock, and \( \bar{t} \) denotes the population average of leisure time, here interpreted as social interaction time. Individual households take \( c'_t \) and \( \bar{t} \) as exogenously given (both variables are further discussed below). The individual, at each date, faces the private household’s flow budget constraint

\(^7\) For the sake of simplicity, we thus assume that the rate of depreciation is zero. This assumption does not affect the insights of the paper.
\[ \dot{k}_t = (1 - \tau_{wt})w(1 - l_t) + (1 - \tau_{kt})r_y k_t - (1 + \tau_{ct})c_t - t_{yt}, \]  

(8)

and the boundary (transversality) conditions:

\[ \begin{align*}
  k_0 & \text{ given,} \\
  \lim_{t \to \infty} \mu_t k_t e^{\rho t} & = 0,
\end{align*} \]

(9)

where \( \mu_t \) represents the shadow price of a unit of capital.

In order to be able to derive algebraic solutions, we consider instantaneous utility functions that are further specified as follows:

\[
\begin{align*}
  u(c_t, c_t' - c_{t'}') & = \left[ \frac{(1 - \eta(l_t, \bar{l}_t))c_t + \eta(l_t, \bar{l}_t)(c_t - c_t')l_{t'}^\phi}{1 - \theta} \right]^{1 - \theta} \\
  & = \frac{[c_t - \eta(l_t, \bar{l}_t)c_t']^{1 - \theta}}{1 - \theta} l_{t'}^{1 - \phi}, \quad \phi > 0, \ \theta \geq 1.
\end{align*}
\]

(10)

Several comments are in order. First, for simplicity, consider the conventional case without positional concerns, such that \( \eta(l_t, \bar{l}_t) = 0 \). In this case, \( \theta \) is the absolute elasticity of marginal utility, and parameter \( \phi \) determines the elasticity of utility with respect to leisure, \( \phi(1 - \theta) \). In the simulations below, we choose \( \theta \geq 1 \), as is overwhelmingly suggested by the literature.\(^8\)

This restriction, though, is not required for the theoretical analysis.

Consider next the more interesting case in which \( 0 < \eta(l_t, \bar{l}_t) < 1 \), such that \textit{relative consumption}, \( c_t - c_t' \), does matter for instantaneous utility. It is then easy to show from (10) that \( \eta(l_t, \bar{l}_t) \) reflects the degree of positionality, following the definition in Johansson-Stenman et al. (2002) and Aronsson and Johansson-Stenman (2010), as follows:

\[
\begin{align*}
  \frac{u_{c,c'}(c_t, c_t - c_{t'}', l_t, \bar{l}_t)}{u_{c,c}(c_t, c_t - c_{t'}', l_t, \bar{l}_t) + u_{c,c'}(c_t, c_t, l_t, \bar{l}_t)} & = \eta(l_t, \bar{l}_t).
\end{align*}
\]

(11)

Assume a small consumption increase ceteris paribus at time \( t \), which will cause an increased instantaneous utility for two reasons, namely that both absolute and relative consumption increases. The degree of positionality \( \eta(l_t, \bar{l}_t) \) will then measure the fraction of the instantaneous utility increase that is due to increased relative consumption. As such, when

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\(^8\) Hall (1988, p. 350) favors a value of (at least) \( \theta = 5 \).
\( \eta(l, \overline{l}) \) approaches 1, absolute consumption does not matter at all, and all that matters is relative consumption, \( c_i - c'_i \). If, say, \( \eta(l, \overline{l}) = 0.3 \) then 30% of the marginal instantaneous utility of consumption stems from a rise in relative consumption term, while the remaining 70% directly arise from a rise in absolute consumption.

Second, the reference consumption stock \( c'_i \), which is taken as exogenously by individual households, is given by
\[
\begin{align*}
  c'_i &= \beta \int_{-\infty}^t e^{-\beta(t-\tau)} \overline{c}_i \, d\tau, \\
  &\quad \beta > 0. 
\end{align*}
\]  
(12)
The reference consumption stock is an exponentially declining sum of the economy-wide average consumption. Differentiating (12) with respect to time yields the following rate of adjustment for the reference stock
\[
  \dot{c'}_i = \beta(\overline{c}_i - c'_i). 
\]  
(13)
The parameter \( \beta \) reflects the relative importance of recent consumption in determining the current reference stock. As \( \beta \to \infty \), \( c'_i \to \overline{c}_i \), that is, the consumption reference level approaches the present economy-wide average consumption level. With \( c'_i \) determined by (13), the economy-wide consumption imposes an externality on the individual household. This externality is frequently referred to as catching-up with the Joneses (CUJ) externality, whereas the limit case where \( \beta \to \infty \), such that the reference consumption is given by the present average consumption level, results in what is typically referred to as keeping-up with the Joneses externalities. Our model is thus general enough to encompass the latter form as a special case. CUJ models are also often referred to as models with external habit formation.

Third, the introduction of the consumption externality also impacts on the elasticity of marginal utility of consumption with respect to consumption. It turns out to be useful to define the ratio of reference consumption to the present average consumption as
\[
  \gamma_i = \frac{c'_i}{\overline{c}_i}. 
\]  
(14)
Thus, for a growing economy (and a finite \( \beta \)) it follows that \( \gamma_i < 1 \). The absolute elasticity of marginal utility of consumption with respect to consumption is given by

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\(^9\) Recall that as households are homogeneous and the population size is normalized to unity, average consumption \( \overline{c}_i = c_i \).
\[
\frac{\theta}{1 - \gamma_t \eta(l_t, \bar{l}_t)} \geq \theta.
\]

Ceteris paribus, the elasticity of marginal utility rises in the reference function \( \eta(l_t, \bar{T}) \), specifically it increases upon a rise in \( l_t \).

Fourth, in contrast to earlier work on consumption externalities that have utilized specific functional forms, we let the degree of positionality depend on both own and others’ leisure time. This *Veblenian characteristic* captures two ideas. On the one hand, more own leisure allows a household to display conspicuous consumption more effectively. That is, leisure complements relative consumption, and the marginal degree of positionality increases in individual leisure. On the other hand, it is social interaction time, as captured by \( \bar{T} \), that makes it possible for others to observe and evaluate this displayed conspicuous consumption. We summarize this Veblenian characteristic of the consumption externality as follows:

\[
0 \leq \eta(l_t, \bar{T}_t) < 1, \quad \eta_t(l_t, \bar{T}_t) > 0, \quad \eta_T(l_t, \bar{T}_t) > 0.
\]

Thus, the degree of consumption positionality increases with both own and others’ leisure time. One specific example is \( \eta(l_t, \bar{T}_t) = \lambda l_t^a \bar{T}_t^b \), with parameters \( \lambda, a, b > 0 \) determining the impact of leisure on the degree of positionality. Clearly, the standard case of a consumption externality without the Veblenian characteristic is recovered by \( \eta(l_t, \bar{T}_t) = \eta_t(l_t, \bar{T}_t) = \eta_T(l_t, \bar{T}_t) = 0 \).

Moreover, for analytical simplicity we assume the reference function to be homogeneous of degree \( d \) in individual leisure.

**Assumption.** \( l_t \eta_t(l_t, \bar{T}_t) = d \eta(l_t, \bar{T}_t), \quad (d + \phi) \gamma_t \eta(l_t, \bar{T}_t) < \phi, \quad d \geq 0 \).

The parameter \( d \) represents the elasticity of the degree of positionality with respect to a rise in leisure. While the iso-elasticity of \( \eta(l_t, \bar{T}) \) is not a necessary assumption, it allows for a more straightforward interpretation of the optimal taxation results below. As \( d \geq 0 \), a rise in leisure increases the marginal degree of positionality. The restriction \( (d + \phi) \gamma_t \eta(l_t, \bar{T}_t) < \phi \) is necessary and sufficient for a positive individual marginal utility of leisure.
The dynamic system

Equipped with the above description of the economy, we are now able to characterize the dynamic system. Let us first, in order to simplify notation, define the following two variables:

\[ x_t \equiv \frac{c_t}{y_t} \] (the average propensity to consume);

\[ z_t \equiv \frac{y_t}{k_t} \] (the capital productivity).

Also, let \( g_{vt} \equiv \frac{v_t}{v_t} \) be the growth rate of some variable \( v \). We can now define the individual current value Hamiltonian as

\[
H(c_t, l_t, k_t, \mu_t) = \frac{c_t - \eta(l_t, l_t) c_t'}{1 - \theta} - \mu_t \left[ (1 - \tau_{wt}) w_t (1 - l_t) + (1 - \tau_{ct}) r_t k_t - (1 + \tau_{ct}) c_t - t_w v_t \right],
\]

where \( \mu_t \) is the costate variable.

Differentiating (17) with respect to the control variables \((c_t, l_t)\) and evaluating at \( l_t = l_t \) and \( c_t = c_t \) yields the intratemporal optimality condition

\[
x_t(l_t) \equiv \frac{c_t}{y_t} = \frac{l_t}{1 - l_t} \frac{(1 - \alpha)(1 - \tau_{wt})}{\phi(1 + \tau_{ct})} V(l_t, \gamma_t),
\]

\[
V(l_t, \gamma_t) \equiv \frac{\phi}{\phi - \eta(l_t, l_t) \gamma_t d + \phi} > 0.
\]

The function \( V(l_t, \gamma_t) \) captures the direct impact of the Veblenian consumption externality. Assumption (A.1) ensures positivity of \( V(l_t, \gamma_t) \). In the absence of a consumption externality, \( V(\cdot) = 1 \).

**Lemma 1.** Assume (A.1). Then, ceteris paribus, the propensity to consume, \( x_t \), is an increasing function of leisure, \( l_t \).

*Proof.* The lemma follows immediately from the fact that \( V(l_t, \gamma_t) \) increases in leisure due to (A.1).

Lemma 1 states an important property of \( x_t \) that is exploited in the propositions below. It is important to note that (18) is only an optimality condition, no market-clearing/equilibrium condition. A further implication of (18) is that, ceteris paribus (for a given leisure demand),
an exogenous rise in the degree of positionality raises the propensity to consume out of income, via an increase in $V(l_t, \gamma_t)$. Both properties will be heavily employed below.

From (6), we derive the capital productivity

$$z_c(l_t) \equiv \frac{V}{k_t} = A^{1/\alpha} h^{\frac{1}{\alpha}} (1-l_t)^{\frac{1}{\alpha}-1},$$

(19)

and we note that

$$g_k \equiv \frac{\dot{k_t}}{k_t} = (1-h-x_t)z_t = \left(1-x_t(1+\tau_{ct})-(1-\alpha)\tau_{ct} - \alpha \tau_{ct} - t_{ct}\right)z_t,$$

(20)

where $g_k$ is the rate of growth of capital. Next, we derive the Euler equation. Differentiation of $\partial H / \partial c_t = 0$ with respect to time, and considering (12) afterwards, yields:

$$\theta \frac{\dot{c}_t}{c_t} = -\frac{\dot{\mu}_t}{\mu_t} - \frac{\dot{t}_{ct}}{1+\tau_{ct}} - \frac{i}{l_t} \left(\phi(\theta - 1) - \theta \frac{\dot{\eta}(l_t)\gamma_t}{1-\eta(l_t)\gamma_t}\right) + \theta \frac{\dot{\eta}(l_t)l_t\gamma_t}{1-\eta(l_t)\gamma_t} \gamma_t,$$

(21)

where $\dot{\eta}(l_t) \equiv \eta(l_t, l_t)$.

Furthermore, $-\partial H / \partial k_t = \dot{\mu}_t - \rho \mu_t$ gives

$$-\frac{\dot{\mu}_t}{\mu_t} = \alpha z_t (1-\tau_{ct}) - \rho.$$

(22)

First-order conditions (21) and (22) imply:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha(1-\tau_{ct})z_t - \rho}{\theta} - \frac{i}{l_t} \left[\frac{\phi(\theta - 1)}{\theta} - \frac{\dot{\eta}(l_t)\gamma_t}{1-\eta(l_t)\gamma_t}\right] - \frac{\dot{t}_{ct}}{\theta(1+\tau_{ct})} + \frac{\dot{\eta}(l_t)l_t\gamma_t}{1-\eta(l_t)\gamma_t} \gamma_t.$$

(23)

Finally, we differentiate (18) with respect to time, and consider both (6) and (20) to arrive at:

$$\frac{\dot{c}_t}{c_t} = \frac{i}{l_t} \left[\frac{1}{l_t} - \frac{1-\alpha}{1-\alpha} l_t \frac{V(l_t, \gamma_t)\gamma_t}{V(l_t, \gamma_t)}\right] + \left[1-h-x(l_t, \gamma_t)\right]z(l_t) + \frac{V(l_t, \gamma_t)\gamma_t}{V(l_t, \gamma_t)} \gamma_t,$$

(24)

\[ - \frac{\dot{t}_{ct}}{1-\tau_{ct}} - \frac{\dot{t}_{ct}}{1+\tau_{ct}}. \]

The two-dimensional dynamic system of the decentralized economy consists of two differential equations in the variables $(l_t, \gamma_t)$. Considering both (23) and (24) yields the differential equation of $l_t$. The differential equation of $\gamma_t$ is derived from (13) and (14).
\[ \frac{i_t}{l_t} = \Omega(l_t, \gamma_t)^{-1} \left[ \frac{\alpha_z (1-\tau_w)}{\theta} - (1-h - x(l_t, \gamma_t)) z(l_t) + \hat{T}_t + \left[ \frac{\hat{\eta}(l_t)\gamma_t}{1-\hat{\eta}(l_t)} - \frac{V(l_t, \gamma_t)\gamma_t}{V(l_t, \gamma_t)} \right] \frac{\dot{\gamma}_t}{\gamma_t} \right], \]

\[ \Omega(l_t, \gamma_t) = \frac{1}{1-l_t} - \frac{\alpha}{\alpha} \frac{l_t}{1-l_t} + \frac{(\theta-1)\phi}{\theta} + \frac{V(l_t, \gamma_t)\gamma_t}{V(l_t, \gamma_t)} - \frac{\hat{\eta}(l_t)\gamma_t}{1-\hat{\eta}(l_t)} \frac{\dot{\gamma}_t}{\gamma_t}, \]

\[ \hat{T}_t \equiv \frac{\hat{\tau}_w}{\theta} + \frac{(\theta-1)\hat{\tau}_a}{\theta(1+\tau_a)}, \]

\[ \frac{\dot{\gamma}_t}{\gamma_t} = \beta \left( \frac{1}{\gamma_t} - 1 \right) - \frac{c_t}{c_t}, \]

(25)

where \((x(l_t, \gamma_t), z(l_t))\) are given by (18) and (19), and \(\frac{c_t}{c_t}\) is given by either (23) or (24).

### 2.3 Steady State and balanced growth

The economy will, in a steady state, follow a balanced growth path (BGP), defined as a path along which \(c_t, y_t,\) and \(k_t\) grow at constant growth rates. In steady state, we have \(i_t = \dot{y}_t = 0\) and all tax rates are constant. Hence, \(x(l, \gamma)\) and \(z(l)\) are constant, and by definition \(c_t, y_t,\) and \(k_t\) grow at the same constant endogenous growth rate.

Labor supply can only be expressed implicitly for the BGP:

\[ \frac{\alpha z(l)(1-\tau_k) - \rho}{\theta} = (1-h - x(l, \gamma)) z(l) = g. \]

(26)

This condition follows from both intratemporal as well as intertemporal optimization. It represents the well-known Keynes-Ramsey rule. The left hand side represents the difference between the rate of interest and the pure rate of time preference, divided by the absolute elasticity of marginal utility. The right hand side represents the optimal consumption growth rate in a steady state. Variable \(g\) denotes the endogenous balanced growth rate of \(c_t, y_t,\) and \(k_t\). We employ (26) for deriving (and proving) Proposition 1.

Before moving on, we briefly need to address the differential equation of \(\gamma_t\). In a steady state, \(\dot{\gamma}_t = 0\). From (25), it follows that \(g = \beta (\gamma^{-1} - 1)\). Thus, we note that for a strictly positive endogenous growth rate, \(\gamma < 1 \iff c > c^r\).\(^{10}\) Moreover, the left hand side of (26) implies a

\(^{10}\) Strictly speaking, this holds only for \(\beta < \infty\). If \(\beta \to \infty\), then \(\gamma = 1\), while the endogenous growth rate is still strictly positive.
negative relationship between the endogenous growth rate and leisure: $g(l)$, which allows us to express

$$\gamma(l) = \frac{\beta}{\beta + g(l)} \quad (27)$$

for a BGP. That is, $\gamma$ is rising in $l$. Considering (27), we define:

$$\hat{V}(l) \equiv V(l, \gamma(l)),$$
$$\hat{x}(l) \equiv x(l, \gamma(l)),$$

where the signs follow directly from (18). By investigating the left- and right-hand sides of (26), we are able to graphically analyze both the value of $l$ on the BGP and the impact of the consumption externality on $l$.

$$\hat{x}(l) = l - \alpha(1 - \tau) z(l) \quad \text{FOC for control variables} (c,l)$$

$$z(\hat{x}(l)) = \frac{\rho}{\alpha(1 - \tau) - \theta(1 - h - \hat{x}(l))} \quad \text{Keynes-Ramsey rule}$$

$$z(l) = (Ah^{1-a})^{1/\alpha} (1 - l)^{(1-\alpha)/\alpha} \quad \text{Production technology}$$

$$\gamma(l) = \frac{\beta \theta}{\beta \theta + [\alpha z(l)(1 - \tau) - \rho]}$$

Ceteris paribus, a rise in leisure demand raises $x(l)$ (cf. Lemma 1) and lowers $z(l)$. Graphically, the BGP can be analyzed by the joint consideration of $z(l) = (Ah^{1-a})^{1/\alpha} (1 - l)^{(1-\alpha)/\alpha}$ and $\hat{z}(l) \equiv z(\hat{x}(l))$ in $(l,z)$ plane.
Figure 1. Equilibrium leisure demand in the steady state.

Note. The figure is based on $\eta(l, T) = \lambda l^a T^b; \hat{\eta}(l) = \lambda l^{0.5}$ as well as on the following parameter values: $\alpha = 1/3, \beta = 0.3, \rho = 0.02, \phi = 0.6, \theta = 3, \lambda = 0.2, h = 0.2, A = 2, d = 0.5$.\(^{11}\)

The dashed line represents the production technology. It directly follows from the ex-post production function \((6)\) that the higher is leisure, the lower is employment and thereby output-to-capital. The Veblenian consumption externality does not impact on this line. The solid line represents the Keynes-Ramsey rule (solved for $\hat{z}(l)$). Ceteris paribus, a higher $l$ raises $\hat{x}(l)$ by Lemma 1, and lowers the balanced growth rate, as seen in \((26)\). For the lower balanced growth rate to be optimal, the rate of interest – that is $\hat{z}(l)$ – needs to decline. Therefore, the solid line is declining in $l$. The Veblenian consumption externality impacts only on the $\hat{z}(l)$-function via its impact on $\hat{x}(l)$.

We are now prepared to prove existence of a steady state, using two existence conditions:

\[
h \leq 1 - \alpha(1 - \tau_k) / \theta , \quad \text{(E.1)}
\]

\[
\frac{(1 - \alpha)(1 - \tau_w)}{(1-h)(1+\tau_k)\phi + (1-\alpha)(1-\tau_w)} > \left( \frac{\rho}{\alpha(Ah^{-\alpha})^{1/\alpha}(1-\tau_k)} \right) \quad \text{.} \quad \text{(E.2)}
\]

\(^{11}\) The shape of the figure is robust with respect to the specification of $\hat{\eta}(l)$, as long as Assumption (A.1) is satisfied.
Proposition 1. (Existence)
Assume (A.1), (E.1) and (E.2). Then a unique steady state \((l^*, \gamma^*)\) with strictly positive endogenous growth exists and satisfies the transversality condition (9). The steady state is associated with a BGP for which \(g_c = g_y = g_k = g > 0\).

Proof. See Appendix.

Existence condition (E.1) ensures that in a steady state, if it exists, \(l^* > 0\). Intuitively, (E.1) ensures that the asymptote, as depicted in Figure 1, occurs where \(l > 0\). As \(l^*\) is located to the right of the asymptote, \(l^* > 0\). Existence condition (E.2) is a sufficient condition for a steady state to exist. It ensures that \(\hat{z}(l)\) “cuts” \(z(l)\) from above for some \(l < 1\) (to the right of the asymptote). An equivalent interpretation of (E.2) is given in the Appendix.

The unique steady state \((l^*, \gamma^*)\), associated with a strictly positive endogenous growth rate, is a saddle point and is saddle-point stable (see Appendix).

The endogenous growth rate of the economy (of per capita consumption) is given by:

\[
g(l) = (1 - h - \hat{x}(l))z(l).
\]  

A rise in equilibrium labor supply (a decline in \(l\)) unambiguously raises the economy’s growth rate, as it makes public investment, \(h\), more productive. A remaining question, that we will deal with in the next section, is how a Veblenian consumption externality affects the equilibrium choice of labor supply (and hence leisure demand).

3. A Veblenian consumption externality and growth

Equations (28) as well as Figure 1 are well suited to investigate the impact of the consumption externality on labor supply and growth along the BGP. First, we note that \(\hat{\eta}(l)\) does not affect the \(z(l)\)-curve. Next, the consumption externality affects the \(\hat{z}(l)\)-curve only via its’ impact on \(\hat{x}(l)\). In the following propositions, we consider an increase in the degree of positionality (not caused by a rise in \(l\)), that is, a rise in \(\hat{\eta}(l)\) for any given \(l\).

---

12 By the fact that \(\lim_{l \to 1} x(l) = \infty\), \(l^* < 1\), if a steady state exists (see Appendix).

13 For example, if \(\hat{\eta}(l) = \lambda + \beta l^\gamma\), we will consider the effects of a change in \(\lambda\).
Proposition 2. (Impact of the Veblenian Consumption Externality)

If Assumptions (A.1) and \( \theta \geq 1 \) hold, then:

\[
\frac{\partial l^*}{\partial \hat{\eta}(l)} < 0, \quad \frac{\partial \gamma^*}{\partial \hat{\eta}(l)} < 0, \quad \frac{\partial g^*}{\partial \hat{\eta}(l)} > 0.
\]

The Veblenian characteristic, as given by \( \hat{\eta}_l(l) > 0 \), lowers the impact of a rise in \( \hat{\eta}(l) \) on \( l^* \) and \( g^* \).

Proof. From (28), it follows that \( \partial \hat{x}(l) / \partial \hat{\eta}(l) > 0 \), as \( \hat{\eta}_l > 0 \). That is, for a given \( l \), a rise in \( \hat{\eta}(l) \) unambiguously raises \( \hat{x}(l) \). Therefore, the \( \hat{z}(l) \) - curve shifts downwards. Considering Figure 1, as a consequence, \( l^* \) essentially declines in equilibrium. Thus, from (29) it follows that \( g(l^*) \) increases. As a consequence of the rise in the endogenous growth rate, (27) implies a decline in \( \gamma^* \).

As a secondary effect, in the presence of a Veblenian externality, the degree of positionality \( \hat{\eta}(l) \) declines due to the reduction of leisure in equilibrium. Hence, this decline in \( \hat{\eta}(l) \) lowers the primary effect shown in part 1 of the proposition. ||

Thus, an increase in the degree of positionality gives in a steady state rise to increased labor supply (decreased leisure) and, as a consequence through the effects of public investments, an increased consumption growth rate. The latter implies that the ratio between the reference consumption level (which consists of a weighted average of past consumption levels) and the present consumption level will decrease.

Intuitively, as a consequence of the rise in \( \hat{\eta}(l) \), the Veblenian term \( \hat{V}(l) \) rises, and so does the propensity to consume \( x(l) \) ceteris paribus (cf. (28)). Graphically, the \( \hat{z}(l) \) - curve shifts downwards in Figure 1. That is, for given \( l \), \( \hat{z}(l) < z(l) \). While \( \hat{z}(l) \) determines the optimal consumption growth rate \( \hat{z}(x(l))(1-h-x(l)) \), \( z(l) \) determines the rate of interest \( \alpha z(l)(1-\tau_i) \). It follows that – for given \( l \) – the optimal consumption growth rate falls short of the desired consumption growth rate:

\[
\frac{\hat{z}(x(l))(1-h-x(l))}{\alpha z(l)(1-\tau_i)} < \frac{\alpha z(l)(1-\tau_i) - \rho}{\theta}.
\]

In this situation, households wish to raise their consumption growth rate. The only channel available to the households is an increase in labor supply (reduction in \( l \)). The lowering in
leisure demand raises the optimal consumption growth rate to the point at which equality in (30) is established, that is at which Keynes-Ramsey rule (28) holds.

Proposition 2 delivers several results. A rise in the strength of the consumption externality, \( \hat{\eta}(l) \), raises the endogenous growth rate of the economy – via an increase in equilibrium labor supply. The growth engine in this model is the externality generated by public investment. This investment is the more productive the higher is labor supply – cf. (6). Households respond to the increase in \( \hat{\eta}(l) \) by raising labor supply, as the marginal rate of substitution of consumption for leisure increases. Thereby, the labor share rises, which increases the endogenous growth rate.

As a consequence of the lowering in \( l \), \( \hat{\eta}(l) \) declines. The more elastic \( \hat{\eta}(l) \) responds to the decline in \( l \), the stronger is the weakening impact of the Veblenian characteristic on \( dl^*, d\gamma^* \) and \( dg^* \). In terms of the elasticities – see Assumptions (A.1) and (A.2) below – , the weakening effects of the Veblenian characteristic are the stronger, the higher are the elasticities of the degree of positionality with respect to leisure and social interaction time. This makes intuitive sense, as \( \hat{\eta}(l) \), increases in leisure. In this case, status is displayed not simply by consuming a commodity, but by complementing the act of consumption with an appropriate amount of leisure- and social interaction time. This leisure-consumption complementarity lowers the extent by which a household is prepared to give up leisure for an additional unit of consumption.

The consumption externality also impacts on the propensity to consume. For a given labor supply, \( x(l) \) rises in the strength of the consumption externality. This corresponds to conventional wisdom. When people become more positional, they tend to increase their propensity to consume out of income. However, there is a second effect at work. A rise in \( \hat{\eta}(l) \) also raises equilibrium labor supply. A rise in labor (lowering in \( l \)), however, lowers \( x(l) \), according to Lemma 1.

To study the impact of an increase in \( \hat{\eta}(l) \) for a given \( l \), we parameterize the degree of positionality as follows: \( \hat{\eta}(l; \lambda) \) with \( \frac{\partial \hat{\eta}(l; \lambda)}{\partial \lambda} > 0 \). An increase in the parameter \( \lambda \) raises
\( \hat{\eta}(l; \lambda) \) for given \( l \). Furthermore, we define the elasticity of some variable \( x \) with respect to \( y \) by \( \varepsilon_{xy} \equiv (\partial x / \partial y)(y / x) \). Moreover, let \( \tilde{d} \equiv \hat{\eta}(l_i) / \hat{\eta}(l_i) \). We can now state

**Proposition 3.** The impact of a rise in the degree of positionality on the propensity to consume is ambiguous. Specifically,

\[
\frac{\partial x(l^*)}{\partial \hat{\eta}(l)} \geq 0 \Leftrightarrow \frac{\partial \hat{\eta}(l; \lambda)}{\partial \lambda} \leq -\tilde{d} \frac{\varepsilon_{\hat{\eta}}(1 + \varepsilon_{vl})}{\varepsilon_{vl}}. \tag{31}
\]

(i) In the absence of the Veblenian characteristic, \( \tilde{d} = 0 \), a rise in the degree of positionality always raises \( x(l^*) \).

(ii) In the presence of the Veblenian externality, \( \tilde{d} > 0 \), \( \partial x(l^*) / \partial \hat{\eta}(l) > 0 \) is the more likely, the lower is \( |\varepsilon_{\hat{\eta}}| \).

**Proof.** The left hand side of (31) is strictly positive by construction. If \( \tilde{d} = 0 \), the right hand side equals zero, and (i) follows. If \( \tilde{d} > 0 \), the right hand side is strictly positive too. From Proposition 2 it follows that \( \varepsilon_{\hat{\eta}} < 0 \); (18) implies that \( \varepsilon_{vl} > 0 \). Inequality (31) is derived in the Appendix. ||

According to Proposition 3 the propensity to consume increases, due to a rise in the degree of positionality, as long as the primary effect – the rise of \( x(l) \) due to the increase in \( \hat{\eta}(l) \) (for a given \( l \)) – dominates the secondary effect – the lowering of \( x(l^*) \) via the decline in \( l^* \) due to the increase in \( \hat{\eta}(l) \). This is typically the case, as is indicated below, by numerical simulations for various values of the Veblenian externality parameters. Only for the case that the primary effect is dominated by the secondary effect, that is \( |\varepsilon_{\hat{\eta}}| \) is “large”, \( x(l^*) \) declines as of a rise in the degree of positionality.

4. A Veblenian consumption externality and optimal fiscal policy

So far we have dealt with positive analysis of individual behavior and equilibrium outcomes. In this section we will instead deal with normative issues and consider how the government
should optimally respond to the positional externalities and the Veblenian characteristics. Contrary to the individual, the social planner takes all externalities – that is, \( \dot{c}' = \beta(c_t - c'_t) \), \( T_t = \bar{I}_t \) and \( H_t = hy_t \) – into account when deciding on \((c_t, I_t, h)\), \( t \geq 0 \). A tilde indicates optimal values, according to the social planner’s (welfarist) optimality criterion. The social planner chooses \((c_t, I_t, h)\), \( t \geq 0 \) so as to

\[
\max_{\{c_t, I_t, h\}} U_0 = \int_{t=0}^{\infty} \left[ \left( c_t - \tilde{\eta}(l) c'_t \right)^\theta \right]^{1 - \theta} e^{-\rho t} dt
\]

subject to the economy’s resource constraint as given by (5) and (6) with \( k_0 \) given, subject to the reference consumption constraint \( \dot{c}' = \beta(c_t - c'_t) \) with \( c'_0 \) given, and subject to the transversality conditions

\[
\lim_{t \to \infty} e^{-\rho t} \mu c_i k_i = 0, \quad \lim_{t \to \infty} e^{-\rho t} \mu c_i c'_i = 0.
\]

In what follows, we assume that \( \tilde{\eta}(l) \) is homogeneous of degree \( \tilde{\alpha} \).

**Assumption.** \( \tilde{\eta}(l) I_i = \tilde{\alpha} \tilde{\eta}(l) \), \( (\tilde{\alpha} + \phi)\gamma \tilde{\eta}(l) < \phi, \quad \tilde{\alpha} \geq \alpha \)

(A.2)

Assumption (A.2) parallels (A.1). The inequalities ensure that the marginal utility of leisure is positive and declining in \( l \). Observe that \( \tilde{\alpha} \geq \alpha \).\(^{14}\)

### 4.1 Intratemporal optimality conditions

The Hamiltonian of the social planner’s economy is given by:

\[
H(c_t, I_t, h, k_t, c'_t, \mu, c_i) = (1 - \theta)^{-1} \left[ c_t - \tilde{\eta}(l) c'_t \right]^{1 - \theta} l_t^{\theta(1-\theta)} + \mu \beta(c_t - c'_t) + \mu \beta k_t \left[ A^{1/\alpha} h^{1-\alpha} (1 - l) \right]^{1-\alpha} (1 - h) - c_i.
\]

Analogously to the prior section, we define the Veblenian consumption externality term by

\[
\tilde{V}(l_t, \gamma, q_i) = \frac{1}{1 - \beta q_i} \frac{\phi}{\phi - \tilde{\eta}(l) \gamma (\tilde{\alpha} + \phi)} \geq 1,
\]

where \( q_i \equiv \frac{\mu c_i}{\mu k_t} \),

that is, \( q_i \) denotes the shadow price of the reference consumption stock relative to the shadow price of physical capital. As the reference consumption stock impacts

\[
^{14} \tilde{\alpha} = d + \eta_x(\bar{I}_t, \bar{I}_t) / \eta(\bar{I}_t, \bar{I}_t) \geq d \text{ by Assumption (A.1)}.\]
negatively on intertemporal utility, \( q_c < 0 \) (when \( \hat{\eta}(l_i) > 0 \)).

The social planner takes the impact of an individual’s leisure decision on social interaction time into account, while an individual household doesn’t. The necessary first-order conditions for \((c_i,l_i)\) yield the optimal propensity to consume out of income

\[
\tilde{x}(l_i,\gamma_i,q_i) \equiv \frac{\tilde{c}_i}{\bar{y}_i} = \frac{l_i}{1-l_i} \frac{1-\alpha}{1-h} \phi \frac{\tilde{V}(l_i,\gamma_i,q_i)}{\alpha}.
\]

(36)

In what follows, we compare (36) with the market economy’s propensity to consume as given by (28). In order to compare \(\bar{x}(l_i)\) with \(\tilde{x}(l_i)\), it is useful to investigate the properties of

\[
v(l_i,\gamma_i,q_i) \equiv \tilde{V}(l_i,\gamma_i,q_i) / V(l_i,\gamma_i).
\]

**Lemma 2.** Let \(v(l_i,\gamma_i,q_i) \equiv \tilde{V}(l_i,\gamma_i,q_i) / V(l_i,\gamma_i)\).

(i) If \(\hat{\eta}(l_i) = 0\), \(\tilde{V}(l_i,\gamma_i,q_i) = V(l_i,\gamma_i) = 1\), thus, \(v(l_i,\gamma_i,q_i) = 1\).

(ii) If \(\hat{\eta}(l_i) > 0\),

\[
\tilde{V}(l_i,\gamma_i,q_i) \geq V(l_i,\gamma_i) \iff v(l_i,\gamma_i,q_i) \geq 1 \iff \frac{1}{1-\beta q_i} \leq \frac{\phi-(d+\phi)\hat{\eta}(l_i)\gamma_i}{\phi-(d+\phi)\hat{\eta}(l_i)\gamma_i}. \tag{37}
\]

**Proof.** The Lemma follows directly from the definition of the Veblenian externality terms.

The degree of positionality, \(\hat{\eta}(l_i)\), affects the decentralized allocation differently from the centralized allocation, as individual households – in contrast to a social planner – consider \((c_i',T_i)\) as exogenous. Specifically, the social planner takes into account that a rise in \(l_i\) not only raises individual leisure time but also social interaction time. That is, the social planner takes into account that \(\hat{\eta}(l_i) \geq \eta_i(l_i,T_i)\). In our framework, this reduces to \(\tilde{d} \geq d\). There is a tradeoff between the consumption externality at the one hand and the externality stemming from social interaction time at the other hand. Two cases emerge.

First, if social interaction time is not a significant component of the degree of positionality, then \((\tilde{d} - d)\) is small. In this case, the right hand side of (37) is just slightly smaller then one,
while the left hand side is substantially smaller than one.\textsuperscript{15} In this case, the consumption externality requires $\tilde{V}(l_t, y_t, q_t) < V(l_t, y_t) \Leftrightarrow v(l_t, y_t, q_t) < 1$. As the propensity to consume out of income increases in the Veblenian externality term, $v(l_t, y_t, q_t) < 1$ lowers $\bar{x}(l_t, y_t, q_t)$ relative to $x(l_t, y_t)$. This reflects the standard case according to which, for a given $l_t$, positional consumers overconsume, and the centralized propensity to consume is lower than the decentralized one.

Second, suppose the social interaction time is a significant component of the degree of positionality. Then, if $(\tilde{d} - d)$ is strictly positive and, according to (37), large enough, $\tilde{V}(l_t, y_t, q_t) > V(l_t, y_t) \Leftrightarrow v(l_t, y_t, q_t) > 1$. In this case, the externality from social interaction is dominant: $v(l_t, y_t, q_t) > 1$ raises $\bar{x}(l_t, y_t, q_t)$ relative to $x(l_t, y_t)$. This reflects a non-standard case that is entirely due to the Veblenian consumption externality. If the Veblenian characteristic is important enough, then households gain by displaying status via additional leisure rather than via additional consumption.

Statement (ii) of Lemma 2 identifies the condition for the former (for the latter) case to hold. Below, it turns out that this condition strongly affects the nature of optimal tax policy.

We next turn to the optimal level of public investment. To obtain the socially optimal level, $\tilde{h}$, we differentiate the current-value Hamiltonian associated with (32) with respect to $h$:

$$\tilde{h} = 1 - \alpha .$$

The optimal public investment share equals the elasticity of public investment $H_t$ in production (1). This result is well known from Barro (1990) in an endogenous growth context without consumption externalities. First-order conditions (36) and (38) follow from optimally choosing the control variables $(c_t, l_t, h)$. We refer to these conditions as intratemporal optimality conditions.

In order to calculate the optimal consumption- and wage tax rates, we consider $\bar{x}(l_t, y_t, q_t) = x(l_t, y_t)$:

\textsuperscript{15} In the absence of the social interaction effect – that is, $\eta_l(l, \tilde{T}) = 0$, or equivalently $\tilde{d} = d = 0$, as a special subcase, the right hand side of (37) equals one, while the left hand side is strictly less than one.
\[
\frac{1 - \bar{\tau}_{wt}}{1 + \bar{\tau}_{ct}} = \left(1 - \frac{h - \tilde{h}}{\alpha}\right)v(l_t, \gamma_t, q_t),
\]
\[
\tau_{ct} = 0 \Rightarrow \bar{\tau}_{ct} = 1 - \left(1 - \frac{h - \tilde{h}}{\alpha}\right)v(l_t, \gamma_t, q_t),
\]
\[
\tau_{wt} = 0 \Rightarrow \bar{\tau}_{ct} = \left(1 - \frac{h - \tilde{h}}{\alpha}\right)v(l_t, \gamma_t, q_t)^{-1} - 1.
\]

The optimal tax rates depend on both the deviation of the public investment share from the optimal one, \(h - \tilde{h}\), and the social Veblenian consumption externality term relative to the decentralized one, \(v(l_t, \gamma_t, q_t)\). Several remarks are in order. First, in the absence of the consumption externality (\(v(\cdot) = 1\)), the sign of \((h - \tilde{h})\) alone determines the signs of the optimal consumption- and wage tax rates. Specifically, \(\text{sgn} \bar{\tau}_{ct} = \text{sgn}(h - \tilde{h}) = \text{sgn} \bar{\tau}_{wt}\).

Moreover, if \(h = \tilde{h}\), both optimal tax rates are nil. In this case, public investment is best financed by the lump sum tax only. Intuitively, if \(h < \tilde{h}\), the rate of interest is below its optimal level. Consequently, households save less than is optimal. In this situation, one option for raising savings to the optimal level consists in subsidizing labor supply (\(\bar{\tau}_{wt} < 0\)). In response, households raise labor supply, thereby income and savings.

Second, in the presence of a catching-up with the Joneses externality, the consumption- or wage tax needs to differ from zero in order to internalize the consumption externality, even if \(h = \tilde{h}\). Specifically, if \(\tilde{\eta}(l) > 0\),
\[
\tau_{ct} = 0 \Rightarrow \bar{\tau}_{wt} = 1 - v(l_t, \gamma_t, q_t),
\]
\[
\tau_{wt} = 0 \Rightarrow \bar{\tau}_{ct} = \frac{1}{v(l_t, \gamma_t, q_t)} - 1.
\]

Both the consumption and the wage tax can be used to target the consumption-leisure externality. In the presence of the Veblenian characteristic, the optimal tax rates are either positive or negative. If \(v(l_t, \gamma_t, q_t) < 1\), the optimal tax rates are positive. This corresponds to the case in which the consumption externality dominates the externality due to social interaction time. In other words, \((\tilde{d} - d)\) is small in the sense defined by (37). If, however, \(v(l_t, \gamma_t, q_t) > 1\), the externality due to social interaction time dominates the consumption externality. This case becomes more likely the large is the difference \((\tilde{d} - d)\). In this case, households gain by displaying status via additional leisure rather than via additional
consumption. It is then optimal to subsidize consumption (labor supply) in order to indirectly tax leisure. These results are summarized as follows:

**Proposition 4. (Intratemporal optimality).**

*Assume (A.1), , and (A.2).*

(a) The optimal public investment share is given by \( \tilde{h} = 1 - \alpha \).

(b) The optimal wage- and consumption tax rates are given by

\[
\frac{1 - \tilde{\tau}_{w} + \alpha}{1 + \tilde{\tau}_{c}} = \left( 1 - \frac{h - \tilde{h}}{\alpha} \right) v(l, \gamma, q).
\]

(c) Suppose \( h = \tilde{h} \). If \( \hat{\eta}(l) = 0 \) , \( \tilde{\tau}_{w} = \tilde{\tau}_{c} = 0 \). If \( \hat{\eta}(l) > 0 \), and according to (37) the difference \( (\hat{d} - d) \) is small (is large), then \( \tilde{\tau}_{w}, \tilde{\tau}_{c} > 0 \) (then \( \tilde{\tau}_{w}, \tilde{\tau}_{c} < 0 \)).

(d) The Veblenian characteristic is necessary for the corrective tax rates to be negative. Specifically, the degree of positionality needs to be sufficiently elastic with respect to social interaction time (\( \tilde{d} \) must be large enough).

*Proof.* The results follow directly from Lemma 2 and the above discussion. ||

In the absence of the Veblenian characteristic of the consumption externality \( (d = \tilde{d} = 0) \), \( v(\cdot) = v(q) = 1/(1 - \beta q) < 1 \). That is, \( \tilde{\tau}_{w} > 0 \) or \( \tilde{\tau}_{c} > 0 \). As soon as \( \tilde{d} > d \geq 0 \), the Veblenian characteristic raises \( v(l, \gamma, q) \) ceteris paribus, thereby lowering the optimal consumption- and wage taxes. Intuitively, with \( d, \tilde{d} > 0 \), the consumption externality raises the marginal rate of substitution of consumption for leisure by less as compared with \( d = \tilde{d} = 0 \). Thus, the externality is less distortionary, and the optimal tax rates are lower.

The possibility of a negative optimal consumption- or wage tax cannot occur in the absence of the Veblenian characteristic (of the social interaction externality). If \( \tilde{d} > d \) by a large enough amount, leisure itself becomes a means of displaying status (and more so than consumption). In this case, optimal consumption- and wage taxes are required to discourage leisure demand, thus they need to be negative.

**4.2 Intertemporal optimality**
In order to derive the dynamic system of the social planner’s economy, we essentially follow the steps applied for the market economy above. There is one difference though. As there are two state variables, we have deal with two co-state variables, $\mu_{st}$ and $\mu_{ct}$. As already indicated in (35), we introduce $q_t = \mu_{ct} / \mu_{st}$ and derive also a differential equation in $q_t$.

The necessary first order conditions of the optimal control problem give rise to

$$
\begin{align*}
-\frac{\dot{\mu}_{st}}{\mu_{st}} &= \left((1-h)\dot{z}_t - \rho\right), \\
\frac{\dot{c}_t}{c_t} &= \left((1-h)\dot{z}_t - \rho + \frac{\dot{\eta}(\tilde{l}_t)}{\theta} \frac{\dot{\tilde{y}}_t}{\tilde{y}_t} - \frac{\dot{\phi}(\theta-1)}{\theta} \frac{\dot{\tilde{a}}\tilde{a}(\tilde{l}_t)}{\theta} \frac{\tilde{y}_t}{\tilde{y}_t} + \frac{\beta q_t}{\theta(1-\beta q_t)} q_t \right). \\
\end{align*}
$$

(41)

Log-differentiating (36) with respect to time and considering (41) yields a differential equation for $\tilde{l}_t$. The three-dimensional dynamic system of the decentralized economy consists of three differential equations in the variables $(\tilde{l}_t, \tilde{y}_t, q_t)$.

$$
\begin{align*}
\dot{\tilde{l}}_t &= \Omega(\tilde{l}_t, \tilde{y}_t, q_t)^{-1} \left\{ \frac{\dot{z}_t(1-h) - \rho}{\theta} \left(1-h - \tilde{x}_t(\tilde{l}_t, \tilde{y}_t, q_t)\right) \dot{z}_t(\tilde{l}_t) \\
&+ \left[ \frac{\dot{\eta}(\tilde{l}_t)}{1-\dot{\eta}(\tilde{l}_t)} \frac{\dot{\tilde{y}}_t}{\tilde{y}_t} - \frac{\dot{\phi}(\theta-1)}{\theta} \frac{\dot{\tilde{a}}\tilde{a}(\tilde{l}_t)}{\theta} \frac{\tilde{y}_t}{\tilde{y}_t} \right] \dot{\tilde{y}}_t \right(\theta - 1) \beta q_t \right\}, \\
\Omega(\tilde{l}_t, \tilde{y}_t, q_t) &= \frac{1}{\tilde{l}_t} \left(1 - \alpha \right) + \frac{\theta(\theta-1)}{\theta} + \frac{\tilde{v}_t(\tilde{l}_t, \tilde{y}_t, q_t) \tilde{l}_t}{\tilde{v}(\tilde{l}_t, \tilde{y}_t, q_t)} - \frac{\dot{\phi}(\theta-1)}{\theta} \frac{\dot{\tilde{a}}\tilde{a}(\tilde{l}_t)}{\theta} \frac{\tilde{y}_t}{\tilde{y}_t} \left(\theta - 1\right) \beta q_t \right\}, \\
\dot{\tilde{y}}_t &= \beta \left(\frac{1}{\tilde{l}_t} - 1\right) \frac{\dot{c}_t}{c_t}, \\
q_t &= \frac{\dot{z}_t(\tilde{l}_t)(1-h) + \beta + \dot{\eta}(\tilde{l}_t)}{1-\theta} \frac{1-\beta q_t}{q_t},
\end{align*}
$$

(42)

where $(\tilde{x}_t, \tilde{z}_t)$ are given by (36) and (19), and $\dot{c}_t / c_t$ is given by (41).

Based on the dynamical system (42), by comparing (41) with (22), it becomes evident that the optimal capital income tax is given by

$$
\bar{\tau}_{st} = \frac{h - \bar{h}}{\alpha}.
$$

(43)

For $h < \bar{h} = (1 - \alpha)$ it is optimal to subsidize capital in order to raise the decentralized rate of interest to the optimal level. It is striking, though, that the optimal capital income tax rate is independent of the CUJ consumption externality, once the consumption- or wage tax rates are set optimally. The Veblenian characteristic, however, requires optimal consumption- and wage taxes to change along the transition path. The main reason is that a change in labor...
supply, along a transition path, impacts on the degree of positionality, $\hat{\eta}(l)$. Obviously, $v(l_t,\gamma_t,q_t)$ changes over time and so do the optimal consumption- or wage tax rates.

4.3 Steady State and balanced growth

The economy will, in a steady state corresponding to the social optimum, follow a BGP, defined as a path along which $c_t$, $y_t$, and $k_t$ grow at the same constant growth rate. In steady state, we have $\dot{l} = \dot{\gamma} = \dot{q} = 0$. Corresponding to (28), we have:

$$\dot{x}(\tilde{l},\tilde{\gamma},q) = \frac{\tilde{l}}{1-\tilde{l}} \frac{1-\alpha}{\phi} \frac{1-h}{\alpha} \tilde{v}(\tilde{l},\tilde{\gamma},q),$$

$$\dot{z}(\tilde{x}) = \frac{\rho}{1-h-\theta(1-h-\tilde{x}(\tilde{l},\tilde{\gamma},q))},$$

$$\ddot{z}(\tilde{l}) = \left( Ah^{1-a} \right)^{1/\alpha} (1-\tilde{l})^{(1-a)/\alpha}$$

with $\tilde{\gamma} = \frac{\beta}{\beta + (1-h-\tilde{x}(\tilde{l},\tilde{\gamma},q))\tilde{z}(\tilde{l})}$, $q = -\frac{\ddot{\eta}(\tilde{l})}{(1-h)\tilde{z}(\tilde{l}) + \beta(1-\tilde{\eta}(\tilde{l}))}$, and $\tilde{l}$ is implicitly given by $\tilde{z}(\tilde{x}(\tilde{l},\tilde{\gamma},q)) = \tilde{z}(\tilde{l})$.

Comparing (44) with (28), the optimal corrective taxes along a BGP are given as follows:

**Proposition 5.** Assume (A.1), and (A.2). The optimal (corrective) tax program along a BGP is given by:

$$\frac{1-\bar{\tau}_w}{1+\bar{\tau}_c} = \frac{1}{1-\frac{h-\tilde{h}}{\alpha}} v(\tilde{l},\tilde{\gamma},q),$$

$$\bar{\tau}_k = \frac{h-\tilde{h}}{\alpha},$$

$$\tilde{l}_c = (1-\alpha)(1-\bar{\tau}_c) - \tilde{x} \bar{\tau}_c.$$  

The optimal choice of public investment is given by $\tilde{h} = 1-\alpha$. If $h = \tilde{h}$, then $\bar{\tau}_k = 0$, regardless of the (Veblenian) consumption externality.

**Proof.** Follows directly from above discussion.

Proposition 5 allows for several insights. First, the consumption externality, be it of a Veblenian nature or not, only affects the optimal consumption- and wage tax rates. Second, if $d > a \geq 0$, the Veblenian nature lowers the optimal consumption- and wage tax rates.
Moreover, only in the presence of the Veblenian characteristic, the optimal consumption- and wage tax rates can become negative.

Third, if \( h = \bar{h} \), the optimal capital income tax rate is nil. In this case, the Chamley (1986) – Judd (1985) result holds in spite of the Veblenian consumption externality. Finally, in the absence of a consumption externality, the optimal lump sum tax share, \( \bar{t}_s \), equals \((1 - \alpha) = \bar{h}, \) paralleling Barro (1990). In the presence of a consumption externality, as long as \( \nu(l) < 1 \), we have \( \bar{t}_w, \bar{t}_c > 0 \), and \( \bar{t}_y < \bar{h} \). A part of the public expenses is financed by corrective consumption- or wage taxation. However, once people start displaying via leisure rather than consumption, that is, \( \nu(l) > 1 \), optimal consumption and wage tax rates become negative. In this case, \( \bar{t}_y > \bar{h} \), as not only the public investment but also the consumption- or wage subsidies need to be financed by the lump sum tax revenue.

5. Veblenian consumption externality: results from numerical simulations

Propositions 2 and 3 imply the qualitative result that a rise in the degree of positionality lowers leisure and raises the endogenous growth rate in the steady state equilibrium. Considering reasonable calibrations, two main quantitative questions suggest themselves. First, how does an increase in the degree of positionality quantitatively affect \((\bar{l}^*, \bar{g}^*)\) for different values of the parameters that define the Veblenian externality? Second, how do different values of these parameters affect optimal corrective tax rates? Numerical simulations, addressing these questions, are presented in the following.

For the numerical simulations, we specify

\[
\eta(l, T) = \lambda l^d T^{\bar{d} - d} \Rightarrow \hat{\eta}(l) = \lambda l^{\bar{d}}, \quad \bar{d} > d \geq 0, \quad \lambda > 0, \tag{46}
\]

where thus the parameters \( \lambda, \beta, d \) and \( \bar{d} \) define the Veblenian externality. What we call baseline values of the background parameters are listed in Table 1. Tables below are based on these parameter values, which, except for the parameters related to the consumption externality, may be considered standard and noncontroversial.
Several remarks are in order. The baseline values of the parameters\(^{16}\) give rise to a base case for which the endogenous variables assume the following values.

### Table 2. Base case

<table>
<thead>
<tr>
<th>Steady state</th>
<th>(l^* = 0.43)</th>
<th>(\gamma^* = 0.85)</th>
<th>(g^* = 1.73%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of positionality</td>
<td>(\hat{\eta}(l^*) = 0.22)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, production</td>
<td>(x^* = 0.75)</td>
<td>(z^* = 0.44)</td>
<td></td>
</tr>
</tbody>
</table>

In the base case, with \(l^* = 0.43\), individuals roughly devote half of their time (after sleeping) to work and half of it to leisure.\(^{17}\) The value for \(\gamma^*\) is a mere reflection of parameter \(\beta\). The larger \(\beta\) the closer is \(\gamma^*\) to one. The endogenous growth rate amounts to 1.73\%, which is close to the stylized fact for industrial countries, according to which the average yearly growth rate of income per capita amounts to about 1.8\% (Jones and Romer, 2010). The (endogenous) degree of positionality \(\hat{\eta}(l^*) = 0.22\) value is slightly above the lower bound of empirical estimates. Compiling several empirical studies, Wendner and Goulder (2008) find that the degree of positionality falls into the range of \(\hat{\eta}(l^*) \in [0.2,0.4]\). Other studies find empirical evidence for even larger values of \(\hat{\eta}(l^*)\) (cf. Johansson-Stenman et al. 2002, Solnik and Hemenway 1998, 2005). Newer empirical studies corroborate this evidence (Alvarez-Cuadrado et al. 2012, Dynan and Ravina 2007). The propensity to consume equals \(x^* = 0.75\), which implies a saving rate of \(s^* = 0.25\).

---

\(^{16}\) For the numerical simulations, the rate of depreciation is considered strictly positive to avoid unrealistically high endogenous growth rates.

\(^{17}\) In the literature, \((1 - l^*)\) is sometimes interpreted as labor force participation rate. According to Table 2, \((1 - l^*) = 0.57\), a value that falls well into the range of observed labor force participation rates for many countries.
The parameters of primary interest are those associated with the Veblenian consumption externality: \((\beta, \lambda, d, \tilde{d})\). With the exception of the degree of positionality, the empirical literature does not provide firm conclusions as to the likely magnitudes of these parameters. To clarify the potential quantitative role of these parameters, we vary them in the following numerical simulations. Table 3A displays the effects of varying parameters \((\beta, \lambda, d, \tilde{d})\) for the key variable \(\hat{\eta}(l^*)\). The subsequent Table 3B shows the effects of varying these parameters for the main endogenous variables. The shaded cells in Table 3B correspond to the base case.

Table 3A shows two main effects. First, a rise in \(\lambda\) – holding constant \((d, \tilde{d})\) –, raises \(\hat{\eta}(l^*)\). A doubling of \(\lambda\), though, leads to less than a doubling in \(\hat{\eta}(l^*)\) due to the associated decrease in equilibrium labor supply (cf. Proposition 2). A rise in \(\beta\) does not significantly impact on this observation. Second, a rise in \(\tilde{d}\) – holding constant \(\lambda\) –, lowers \(\hat{\eta}(l^*)\). This is due to specification (46) according to which the term \(l^\tilde{d}\) declines in \(\tilde{d}\).

Table 3A. Steady state effects of Veblenian externality parameters on \(\hat{\eta}(l^*)\)

<table>
<thead>
<tr>
<th>(\hat{\eta}(l^*))</th>
<th>(\beta = 0.1)</th>
<th>(\beta = 0.3)</th>
<th>(\beta = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. (\lambda = 0)</strong></td>
<td>(d, \tilde{d}) arbitrary</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Panel B. (\lambda = 0.8)</strong></td>
<td>(d = 0.3, \tilde{d} = 0.3)</td>
<td>0.56</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>(d = 0.3, \tilde{d} = 0.9)</td>
<td>0.34</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(d = 0.3, \tilde{d} = 1.5)</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>Panel C. (\lambda = 1.6)</strong></td>
<td>(d = 0.3, \tilde{d} = 0.3)</td>
<td>0.89</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(d = 0.3, \tilde{d} = 0.9)</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(d = 0.3, \tilde{d} = 1.5)</td>
<td>0.36</td>
<td>0.35</td>
</tr>
</tbody>
</table>

In order to correctly interpret Table 3B, one needs to distinguish (i) a change in \((d, \tilde{d})\) for a given \(\lambda\) from (ii) a change in \(\lambda\) for given \((d, \tilde{d})\). In case (i), we compare values within a panel, whereas in case (ii) we compare values between Panels (but with the same values of \((d, \tilde{d})\).
Panel A of Table 3B presents steady state values in the absence of the Veblenian consumption externality. A rise in $\beta$, by raising the weight of the reference consumption level, only increases $\gamma^*$ without affecting the other variables. In Panels B and C of Table 3B, a rise in $\beta$ has only minor effects – the higher the $\beta$ the (slightly) more responsive are endogenous variables to changes in $(\lambda, d, \tilde{d})$.

For a given $\lambda$, a rise in the social interaction time parameter $\tilde{d}$ lowers $\hat{\eta}(l^*)$. Consequently, $l^*$ rises and $g^*$ declines. Likewise, for given values $(d, \tilde{d})$, a rise in $\lambda$ raises $\hat{\eta}(l^*)$ and lowers $l^*$. This was shown in Proposition 2 already. It is worth noting, though, that the impact of the Veblenian externality parameters on $(l^*, g^*)$ is substantial. For the numerical simulations documented in Table 3B, $l^* \in [0.08, 0.43]$. Corresponding to these results, $g^*(\%) \in [1.11, 5.38]$.

### Table 3B. Steady state effects of Veblenian externality parameters on $(l^*, \gamma^*, g^*, x^*)$

<table>
<thead>
<tr>
<th></th>
<th>$\beta = 0.1$</th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l^*$</td>
<td>$\gamma^*$</td>
<td>$g^*$</td>
<td>$x^*$</td>
</tr>
<tr>
<td>Panel A. $\lambda = 0$</td>
<td>$\lambda = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d, \tilde{d}$ arbitrary</td>
<td>0.49 0.90 1.11 0.73</td>
<td>0.49 0.96 1.11 0.73</td>
<td>0.49 0.98 1.11 0.73</td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 0.3$</td>
<td>0.30 0.77 2.98 0.77</td>
<td>0.26 0.90 3.36 0.77</td>
<td>0.25 0.94 3.47 0.77</td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 0.9$</td>
<td>0.38 0.82 2.12 0.76</td>
<td>0.37 0.93 2.25 0.76</td>
<td>0.37 0.96 2.28 0.76</td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 1.5$</td>
<td>0.43 0.85 1.73 0.75</td>
<td>0.42 0.94 1.79 0.75</td>
<td>0.42 0.97 1.81 0.75</td>
</tr>
<tr>
<td>Panel B. $\lambda = 0.8$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 0.3$</td>
<td>0.14 0.68 4.62 0.78</td>
<td>0.09 0.85 5.24 0.78</td>
<td>0.08 0.90 5.38 0.78</td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 0.9$</td>
<td>0.30 0.77 2.93 0.77</td>
<td>0.28 0.90 3.18 0.77</td>
<td>0.27 0.94 3.24 0.77</td>
</tr>
<tr>
<td>$d = 0.3, \tilde{d} = 1.5$</td>
<td>0.37 0.82 2.22 0.76</td>
<td>0.36 0.93 2.34 0.76</td>
<td>0.36 0.96 2.37 0.76</td>
</tr>
</tbody>
</table>

Notes. $g^*$ is shown in percentage points.

At the same time, the impact on the propensity to consume is minor due to Proposition 3. The change in the Veblenian externality parameters goes hand in hand with an offsetting decline in the equilibrium leisure supply. These findings are summarized in
Result 1. (Quantitative impact of a change of \((\beta, \lambda, d, \dd)\) on \((l^*, g^*, x^*)\)

(i) Small variations in the Veblenian externality parameters \((\beta, \lambda, d, \dd)\) imply large changes in the endogenous steady state variables leisure and growth rate \((l^*, g^*)\).

(ii) Variations in the Veblenian externality parameters \((\beta, \lambda, d, \dd)\) imply only minor changes in the propensity to consume.

Finally, we consider the impact of the Veblenian consumption externality parameters on the optimal tax rates – given that the government investment share is chosen optimally. The latter implies \(\tau_k = 0\), as shown by Proposition 5.\(^{18}\)

***********

Table 4. Steady state effects of Veblenian externality parameters on optimal fiscal policy

<table>
<thead>
<tr>
<th></th>
<th>(\tau_c = 0)</th>
<th>(\tau_w = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\bar{\tau}_w) (in %)</td>
<td>(\bar{l}_y)</td>
</tr>
<tr>
<td><strong>Panel A. (\lambda = 0)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d, \dd) arbitrary</td>
<td>0.00</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>Panel B. (\lambda = 0.8)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d = 0.3, \dd = 0.3)</td>
<td>1.48</td>
<td>0.59</td>
</tr>
<tr>
<td>(d = 0.3, \dd = 0.9)</td>
<td>-1.86</td>
<td>0.61</td>
</tr>
<tr>
<td>(d = 0.3, \dd = 1.5)</td>
<td>-2.09</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>Panel C. (\lambda = 1.6)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d = 0.3, \dd = 0.3)</td>
<td>2.81</td>
<td>0.58</td>
</tr>
<tr>
<td>(d = 0.3, \dd = 0.9)</td>
<td>-3.79</td>
<td>0.62</td>
</tr>
<tr>
<td>(d = 0.3, \dd = 1.5)</td>
<td>-4.13</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Notes. All results are based on the baseline values of the background parameters. As the government chooses \(h\) optimally, \(\bar{\tau}_k = 0\) in all cases. \(\bar{\tau}_i\) is the lump-sum revenue – net of revenue (or expenses) arising from corrective taxation – required to finance the optimal level of public investment. The grayed row corresponds to the baseline parameterization regarding the Veblenian positionality parameters.

Result 2. (Quantitative impact of a change of \((\beta, \lambda, d, \dd)\) on \((\bar{\tau}_c, \bar{\tau}_w)\))

Assume the public investment share is set optimally. Then, the Veblenian externality, as measured by the difference \((\dd - d)\), strongly impacts on the corrective consumption- and

\(^{18}\) This also holds when \(\delta > 0\), in which case \(\bar{\tau}_k = \left(z \left(1 - (1 - \alpha)\right)\right) / (az - \delta)\).
wage tax rates. In particular, even for moderate levels of the leisure- (interaction time-) externality, the corrective tax rates become negative.

As shown in the table, the Veblenian characteristic has a strong impact on the corrective tax rates. If social interaction time becomes important enough, the corrective tax rates become negative. Considering the magnitude of the corrective tax elements, though, the rates are rather small for the chosen baseline parameterization. [some more interpretation is needed here]

6. Conclusions

This paper shows that the Veblenian characteristic of a consumption externality – the consumption-leisure complementarity of relative consumption – has significant implications for economic behavior and optimal fiscal policy. Informally, Veblen (1899) already brought forth this argument. We consider the effects of an exogenous increase in the degree of positionality. By exogenous, we mean a rise in the degree of positionality for a given level of leisure/labor – possibly from zero to some positive level. We show that the Veblenian characteristic has two major implications.

*******
First, it lowers the positive impact of conspicuous consumption on economic growth. That is, an exogenous increase in the degree of positionality lowers leisure in equilibrium, which lowers individuals’ degrees of positionality. It is interesting to note that in spite of the fact that households understand that they need more leisure to display conspicuous consumption, they actually engage in more labor supply due to positional preferences. This behavior accords with the fact that ... l declining book...etc

Second ... lowers optimal tax rates + going negative if one externality dominates the other one but this comes with 2 cases regarding optimal taxation
Numerical simulations (not reported in this paper, but available upon request) show...

There are close to no empirical studies considering the quantification of our findings. 
Empirical studies shall rank highly on the research agenda for the future discussion on positional consumption.

7. Appendix

Proof of Proposition 1. Where no confusion can arise, we omit the time-indexes below. 
(i) Function \( \hat{x}(l) \) is continuously increasing in \( l \), as shown by Lemma 1. We note that 
\( x(0) = 0 \) and \( x(1) \to +\infty \). The existence proof is based on growth rates as derived from (28):

\[
\hat{g}(l) = \left( 1 - h - \frac{l}{1-l} \frac{(1-\alpha)(1-\tau)}{\phi(1+\tau)} \hat{V}(l) \right) \left( Ah^{1-\alpha} \right)^{1/\alpha} (1-l)^{(1-\alpha)/\alpha}, \\
g(l) = \frac{\alpha \left( Ah^{1-\alpha} \right)^{1/\alpha} (1-l)^{(1-\alpha)/\alpha} (1-\tau_k) - \rho}{\theta},
\]

where \( \hat{g}(l) \) follows directly from capital accumulation (20), and \( g(l) \) represents optimal consumption growth according to the Keynes-Ramsey rule. It can be easily verified that \( g'(l) < 0 \), and \( \hat{g}'(l) < 0 \) for \( l \) not too large (\( \hat{g}'(l) > 0 \) for \( l \) close to 1). Figure A.1 depicts the two curves.

Figure A.1 Existence of a BGP with \( g(l^*) > 0 \)
Step (i). The first existence condition ensures \( \hat{g}(0) > g(0) \). Considering (47), this condition amounts to 
\[
-\left( Ah^{1-\alpha} \right)^{1/\alpha} \left[ \alpha(1 - \tau_k) - \theta(1 - h) \right] + \rho > 0.
\]
A sufficient condition for this inequality to hold is:
\[
\left[ \alpha(1 - \tau_k) - \theta(1 - h) \right] \leq 0 \iff h \leq 1 - \alpha(1 - \tau_k) / \theta.
\] (E.1)

Considering Figure 1, existence condition (E.1) ensures that the specific \( l \) defining the asymptote is strictly positive.\(^{19}\) As this \( l \) is strictly smaller than \( l' \), (E.1) ensures \( l' > 0 \) (if it exists). As \( g(l) \) is strictly monotonously declining and \( g(1) = -\rho / \theta < 0, \ l' < 1 \) (if it exists).

Step (ii). The second existence condition ensures existence of \( l' \). We show that \( l_i > l_0 \). As a consequence, there exists \( l' < l_0 \) so that \( \hat{g}(l') = g(l') > 0 \). Specifically, we define:
\[
l' \equiv \min\{ l \in (0,1) : \hat{g}(l) = g(l) \}, \quad l_0 \equiv \min\{ l \in [0,1] : \hat{g}(l) = 0 \}, \quad \text{and} \quad l_i \text{ is the unique solution to } g(l) = 0.
\]
Given these definitions, we (implicitly) derive \( (l_0, l_i) \) from (47):
\[
l_0 = \frac{(1-h)(1+\tau_{\phi})}{(1-h)(1+\tau_{\phi}) + (1-\alpha)(1-\tau_{\phi}) \hat{V}(l_0)},
\]
\[
l_i = 1 - \left( \frac{\rho}{\alpha \left( Ah^{1-\alpha} \right)^{1/\alpha} (1-\tau_{\phi})} \right)^{a/(1-\alpha)}.
\] (48)

The sufficient existence condition
\[
\frac{(1-\alpha)(1-\tau_{\phi})}{(1-h)(1+\tau_{\phi}) + (1-\alpha)(1-\tau_{\phi})} > \left( \frac{\rho}{\alpha \left( Ah^{1-\alpha} \right)^{1/\alpha} (1-\tau_{\phi})} \right)^{a/(1-\alpha)}
\] (E.2)

ensures \( l_i > l_0 \).

Stability. Define the Jacobian of the dynamic system in the neighborhood of the steady state by:
\[
J = \begin{bmatrix}
\dot{\gamma}_x & \dot{\gamma}_y \\
\dot{i}_x & \dot{i}_y \\
\end{bmatrix}
\]
(49)

Claim. Assume \( \dot{\gamma}_x < 0 \). Then the determinant of \( J \) is negative.

\(^{19}\) Considering (28), the asymptote is defined by \( l : \alpha(1 - \tau_k) = \theta \left( 1 - h - x(l) \right) \).
If the claim is true, the Jacobian matrix exhibits one positive- and one negative eigenvalue.

With \( l \) being a jump variable and \( \gamma \) being a predetermined variable, the steady state \((l^*,\gamma^*)\) is saddle-point stable.

\[ \dot{\gamma}_i = -\gamma(\frac{d}{c})_i. \]

As is depicted in Figure A.1, \( \dot{g}_i(l) = (\frac{d}{c})_i < 0 \) at the steady state. Thus, \( \dot{\gamma}_i > 0 \).

\[
\begin{align*}
\dot{\gamma}_i \bigg|_{\gamma^*} &= \left( \Omega(l,\gamma) \right)^{-1} \frac{\partial}{\partial l} \left[ \frac{\dot{\gamma}_i}{\gamma} \left( \frac{\dot{\gamma}_i}{\gamma} - V(l,\gamma) + 1 \right) \right] \\
&= \left( \Omega(l,\gamma) \right)^{-1} \frac{\dot{\gamma}_i}{\gamma} \left( \frac{\dot{\gamma}_i}{\gamma} - V(l,\gamma) + 1 \right) + \frac{\dot{\gamma}_i}{\gamma} \left( \frac{\dot{\gamma}_i}{\gamma} - V(l,\gamma) + 1 \right) > 0
\end{align*}
\]

The last inequality follows from (A.1) and the facts that \( \eta < 1 \) and \( \gamma < 1 \) in a steady state with a strictly positive growth rate. Thus, \( \dot{\gamma}_i > 0 \).

Finally, \( \dot{l}_i > 0 \), as was shown in the literature for this model without positional concerns (see Turnovský 2000, p.475). The presence of positional concerns (with \( 0 < \eta < 1 \)) does not change the sign of this derivate.\(^{20}\)

The signs indicated in (49) follow.\(^{21}\)

**Proof of Proposition 3.** In the formulae below, we consider the definition of \( V(l,\gamma) \) as given by (18). The impact of a rise in \( \dot{\eta}(l) \) is given by

\[
\frac{\partial x(i')}{\partial \dot{\eta}(l)} = \frac{i(1-\alpha)\gamma(1-\tau w)(d+\phi)(\dot{\eta}_l(l) + l_\eta^* \dot{\eta}(l))}{(1-\tau w)(\phi - \gamma)(d+\phi)\dot{\eta}(l)} + i_\eta \left( \frac{(1-\alpha)(1-\tau w)}{(1-\tau w)(\phi - \gamma)(d+\phi)\dot{\eta}(l)} \right)
\]

\[
\begin{align*}
&= \dot{\eta}_l(l) + l_\eta \dot{\eta}(l) + l_\eta \left( \frac{\phi - \gamma(d+\phi)\dot{\eta}(l)}{(1-\tau w)(\phi - \gamma)(d+\phi)\dot{\eta}(l)} \right) \\
&= \dot{\eta}_l(l) - \frac{l_\eta \dot{\eta}(l)}{l} \left( \frac{\dot{\eta}_l(l)}{\dot{\eta}(l)} + \frac{\phi - \gamma(d+\phi)\dot{\eta}(l)}{(1-\tau w)(\phi - \gamma)(d+\phi)\dot{\eta}(l)} \right) \\
&= \dot{\eta}_l(l) - \frac{\phi - \gamma(d+\phi)\dot{\eta}(l)}{\dot{\eta}(l)} = \dot{\eta}_l(l) - \frac{\phi - \gamma(d+\phi)\dot{\eta}(l)}{\dot{\eta}(l))}.
\end{align*}
\]

\(^{20}\) In the model without a catching-up with the Joneses externality, the dynamic system is one-dimensional in \( l \). Thus, the steady state is unstable, and the model does not exhibit any transitional dynamics.

\(^{21}\) The authors pursued numerical simulations for a wide range of parameter values. In all numerical simulations, without exception, we found \( \dot{\gamma}_i < 0 \).
It follows that

$$\frac{\partial x(l)}{\partial \hat{\eta}(l)} \geq 0 \iff \hat{\eta}(l) \geq \hat{d} \frac{\varepsilon_\eta(1 + \varepsilon_{Vl})}{\varepsilon_{Vl}}.$$  

References


