Problem Set 2

For answering the following questions, please use your copy of Mas-Colell/ Whinston/Green.

Problem 2.1 Show that the Cournot- and Engel aggregations, as given by

$$\sum_{l=1}^{L} p_l \frac{\partial x_l(p,w)}{\partial p_k} + x_k(p,w) = 0\,, \quad k = 1, ..., L\,,$$
$$\sum_{l=1}^{L} p_l \frac{\partial x_l(p,w)}{\partial w} = 1$$

lead to the following elasticity formulas:

$$\sum_{l=1}^{L} b_l(p,w)\varepsilon_{lk}(p,w) + b_k(p,w) = 0\,,$$
$$\sum_{l=1}^{L} b_l(p,w)\varepsilon_{lw}(p,w) = 1\,,$$

where $b_l(p,w) = p_l x_l(p,w)/w$ represents the budget share spent on commodity l.

Problem 2.2 Define monotonicity and local nonsatiation. Then, prove the following claim. "If $\succsim$ satisfy monotonicity then $\succsim$ satisfy local nonsatiation."

Problem 2.3 Define strong monotonicity and monotonicity. Then, prove the following claim. "If $\succsim$ satisfy strong monotonicity then $\succsim$ satisfy monotonicity."

Problem 2.4 Consider the case with $L = 2$, $\nabla(x) \gg 0$, $p \gg 0$, and $w > 0$. The acronym "UMP" stands for *utility maximization problem.* Show that the following claim is true (make use of the method of proof by the contrapositive). “If x solves the UMP then $p \cdot x = w$.

Problem 2.5 Consider the following situation where $L = 2$, $X = \mathbf{R}^2_+$, and some consumption bundle $y \in X$. The upper level set of y, $X^+(y)$, is defined as $X^+(y) = \{x \in X \,|\, x \succsim y\}$. Prove the following claim (by proof by the contrapositive). “If $\succsim$ are continuous then the set X^+ is closed.” (Hint: You need to negate an implication when using the definition of continuity of $\succsim$. So, make sure you carefully state what the negation of an implication is.)

Problem 2.6 Suppose, $p \gg 0$, $w > 0$, $x^* \in X$, where X is convex, $\succsim$ is continuous, locally nonsatiated and strictly convex, and there exists an $x \in X$ such that $p \cdot x < w$. If x^* is expenditure minimizing, then x^* is utility maximizing.

Give a proof by the contrapositive.