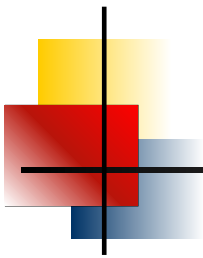


# *Information Economics*

Ronald Wendner

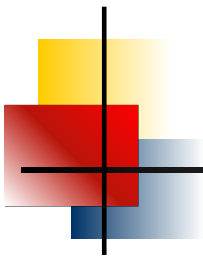
Department of Economics  
Graz University, Austria

Course: Information Economics (Adverse selection)



## Moral Hazard: Lessons

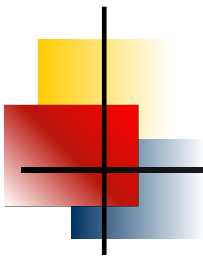
- Lesson 1* Moral hazard (hidden action): A's actions ( $e$ ) not observable (verifiable): contract cannot include effort level  $\rightarrow$  incentive problem
- Lesson 2* Franchise contract solves incentive problem, but at a cost: if A is RA franchise contract implies degree of risk sharing that is not Pareto efficient
- Lesson 3* Tradeoff between incentive problem and Pareto inefficiency problem
- Lesson 4* Incentive constraint: for any given wage schedule, P takes into account that A chooses u-max effort level (backward induction)
- Lesson 5* If  $e = e^{MIN}$  is optimal for P, the optimal contract equals symmetric information contract.
- Lesson 6* If  $e > e^{MIN}$  is optimal for P, IC implies a rising  $w(x_i)_{i=1}^n$ , in spite of A being RA



# Adverse Selection

- ▶ Problem of hidden information (player's type or payoff)
- ▶ Akerlof (1970), The market for lemons (QJE)

Adverse selection: only bad cars (lemons) survive  
bad quality drives good quality out of the market



# Adverse Selection

- ▶ Adverse Selection: base model w/ symmetric information
- ▶ One principal, two agents
- ▶ When principals compete for agents
  - benchmark: symmetric information
  - asymmetric information (separating, pooling equilibrium)
- ▶ Applications
  - Competition b/w insurance companies
  - Optimal licensing contracts
  - Regulation

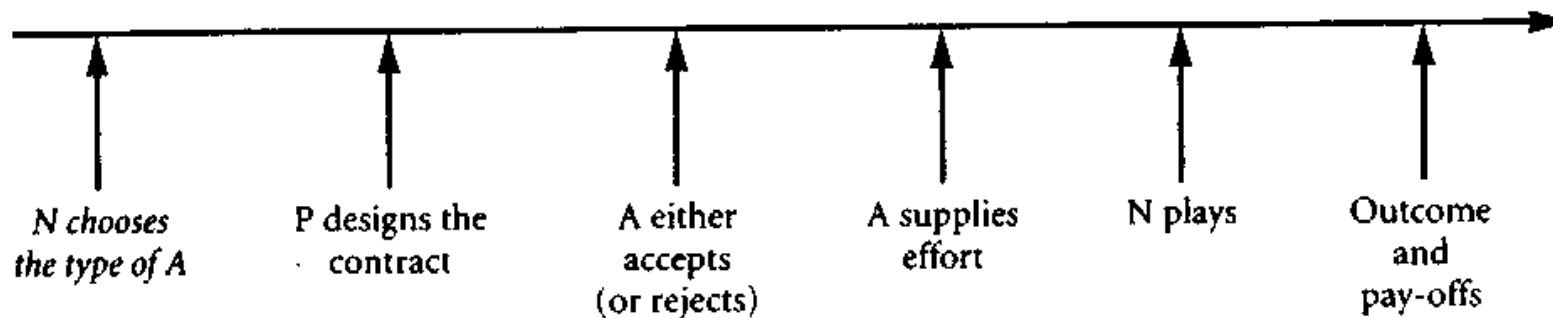
# Adverse Selection: benchmark – symmetric information

## ▶ Agents (RA)

$e$  verifiable

**type:** productivity; disutility  $v(e)$

## ▶ Timing



# Symmetric information

▶ A (RA)

- 2 types: G, B

$$U^G(w, e) = u(w) - v(e); U^B(w, e) = u(w) - k v(e); k > 1$$

$$u'(w) > 0, u''(w) < 0, v'(e) > 0, v''(e) > 0$$

▶ P (RN) 2 contracts:  $(w^G, e^G), (w^B, e^B),$

- $\max_{e,w} \Pi(e) - w$ , s.t.  $u(w) - v(e) \geq \underline{U}$  and  $u(w) - k v(e) \geq \underline{U}$

$$\Pi(e) \equiv \sum_{i=1}^n p_i(e) x_i, \Pi'(e) > 0, \Pi''(e) < 0$$

- P can identify type

- ▶ Participation constraints bind (why?)
- ▶ G-type

$$u(w^{G*}) - v(e^{G*}) = \underline{U}$$

efficiency condition:

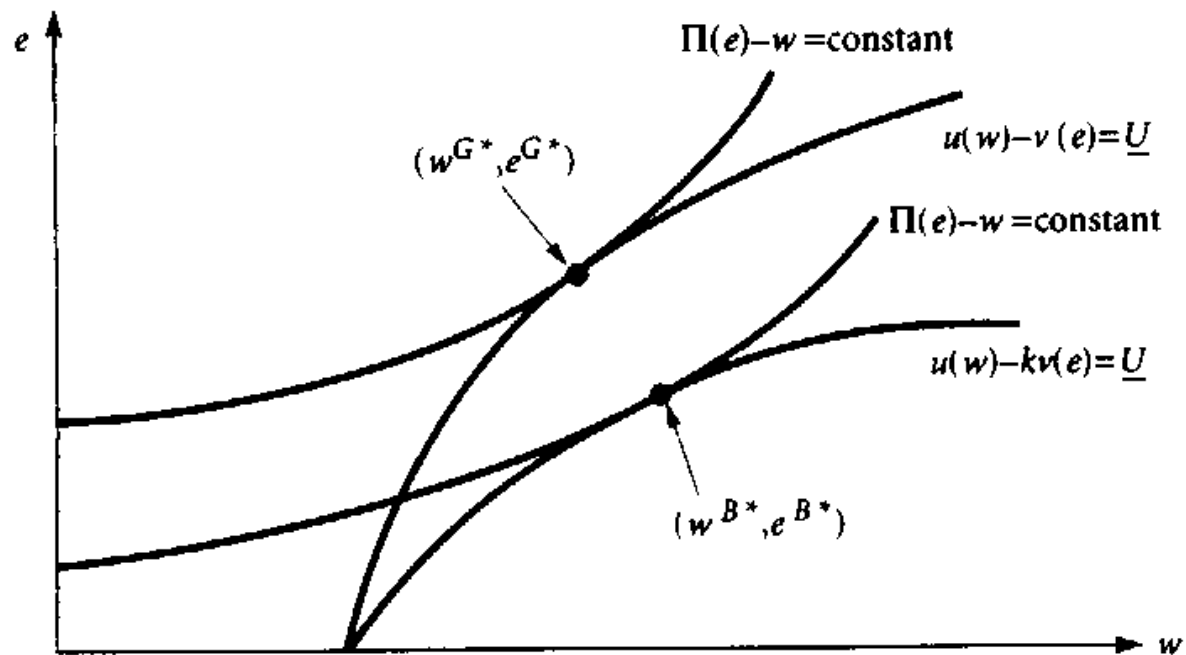
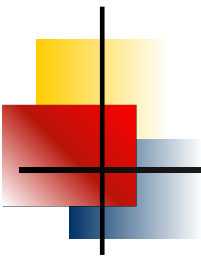
$$\Pi'(e^{G*}) = \frac{v'(e^{G*})}{u'(w^{G*})}$$

- ▶ B-type

$$u(w^{B*}) - k v(e^{B*}) = \underline{U}$$

efficiency condition:

$$\Pi'(e^{B*}) = \frac{k v'(e^{B*})}{u'(w^{B*})}$$



- ▶ G less costly:  $e^{G^*} > e^{B^*}$
- ▶  $w^{G^*} \gtrless w^{B^*}$ , 2 opposing effects
- ▶ G would prefer contract  $(w^{B^*}, e^{B^*})$  over  $(w^{G^*}, e^{G^*})$



# Asymmetric information (1 principal, 2 types of agents)

- ▶ P designs **self-selective (incentive compatible)** menu of contracts

$$\{(e^G, w^G), (e^B, w^B)\}$$

$q$  share of G-type (public knowledge)

$$\max_{\{(e^G, w^G), (e^B, w^B)\}} q [\Pi(e^G) - w^G] + (1 - q) [\Pi(e^B) - w^B]$$

$$\text{s.t. } u(w^G) - v(e^G) \geq \underline{U} \quad (1)$$

$$u(w^B) - k v(e^B) \geq \underline{U} \quad [\lambda] \quad (2)$$

$$\text{for } G \quad u(w^G) - v(e^G) \geq u(w^B) - v(e^B) \quad [\mu] \quad (3)$$

$$\text{for } B \quad u(w^B) - k v(e^B) \geq u(w^G) - k v(e^G) \quad [\delta] \quad (4)$$

- ▶ (3) + (2)  $\Rightarrow$  (1):

$$u(w^G) - v(e^G) \geq u(w^B) - v(e^B) \geq u(w^B) - k v(e^B) \geq \underline{U}$$

**only participation constraint of least-efficient type is binding**

- ▶ (3) + (4)  $\Rightarrow e^G \geq e^B$   
 $v(e^G) - v(e^B) \leq u(w^G) - u(w^B) \leq k [v(e^G) - v(e^B)]$   
 $[v(e^G) - v(e^B)] [1 - k] \leq 0$   
 as  $k > 1$ ,  $v(e^G) \geq v(e^B) \Leftrightarrow e^G \geq e^B$

- ▶ first order conditions

$$\mu - \delta = \frac{q}{u'(w^G)} \quad (5)$$

$$\lambda - \mu + \delta = \frac{1 - q}{u'(w^B)} \quad (6)$$

$$\mu - \delta k = \frac{q \Pi'(e^G)}{v'(e^G)} \quad (7)$$

$$\lambda k - \mu + \delta k = \frac{(1 - q) \Pi'(e^B)}{v'(e^B)} \quad (8)$$

## Characterization of optimal contract

- ▶ (5) + (6): **participation constraint of  $B$  binds:  $\lambda > 0$**

$$\lambda = q/u'(w^G) + (1 - q)/u'(w^B) > 0!$$

$B$  gets exactly  $\underline{U}$ ,  $G$  gets at least  $\underline{U}$

- ▶ **self-selection constraint of  $G$  binds:  $\mu > 0$**

from (5),  $\mu - \delta > 0$ , and  $\delta \geq 0$  by Kuhn-Tucker

- ▶  **$e^G > e^B$  (contracts differ)**

suppose  $e^G = e^B$

then  $w^G = w^B$  (from (3) &  $\mu > 0$ ),  $\lambda = 1/u'(w) = \Pi'(e)/[k v'(e)]$

then (5), (7):

$$\mu = q/u'(w) + \delta = q\lambda + \delta$$

$$\mu = q\Pi'(e)/v'(e) + k\delta = k(q\lambda + \delta)$$

but:  $q\lambda + \delta \neq k(q\lambda + \delta) \Rightarrow e^G \neq e^B$

- ▶  $e^G > e^B$  implies **not**:  $(\mu > 0)$  and  $(\delta > 0)$   
 otherwise, by (3)+(4):  $v(e^G) - v(e^B) = k[v(e^G) - v(e^B)]$ 
  - $\mu > 0 \Rightarrow \delta = 0$  self-selection constr. of B not binding

▶ Information rent (market power) of G

- from (3)+(2):

$$\begin{aligned}
 u(w^G) - v(e^G) &= u(w^B) - v(e^B) && \text{by (3)} \\
 &= u(w^B) - k v(e^B) + (k - 1)v(e^B) \\
 &= \underline{U} + \underbrace{(k - 1)v(e^B)}_{\text{information rent}} && \text{by (2)}
 \end{aligned}$$

# Adverse Selection: First Lessons

- ▶ Participation constraint binds only for agent with *highest* cost, other agent receives information rent  $(k - 1) v(e^B)$
- ▶ Incentive constraint binds only for agent with *lowest* cost
- ▶ Non-distortion at the top (agent G)

$$\Pi'(e^G) = \frac{v'(e^G)}{u'(w^G)}$$

- ▶ Distortion for agent B

$$\Pi'(e^B) = \frac{k v'(e^B)}{u'(w^B)} + \underbrace{\frac{q(k - 1)}{1 - q} \frac{v'(e^B)}{u'(w^G)}}_{\text{incomplete information}}$$

- $e^B \downarrow$  to make contract less attractive to G

$e^B \downarrow \Rightarrow v(e^B) \downarrow \Rightarrow$  information rent  $\downarrow$

**P minimizes information rent**

► Setup: several principals

- 2 types of agents: B, G; differing productivity; unique effort level
- results: success ( $x_S$ ), failure ( $x_F$ )
- probability of success  $p$ :  $p^G > p^B$
- payoffs:  $w_S, w_F$

- expected profit (of RN P):  $p(x_S - w_S) + (1 - p)(x_F - w_F)$

- expected utility (A):

$$U^G = p^G u(w_S) + (1 - p^G) u(w_F); U^B = p^B u(w_S) + (1 - p^B) u(w_F)$$

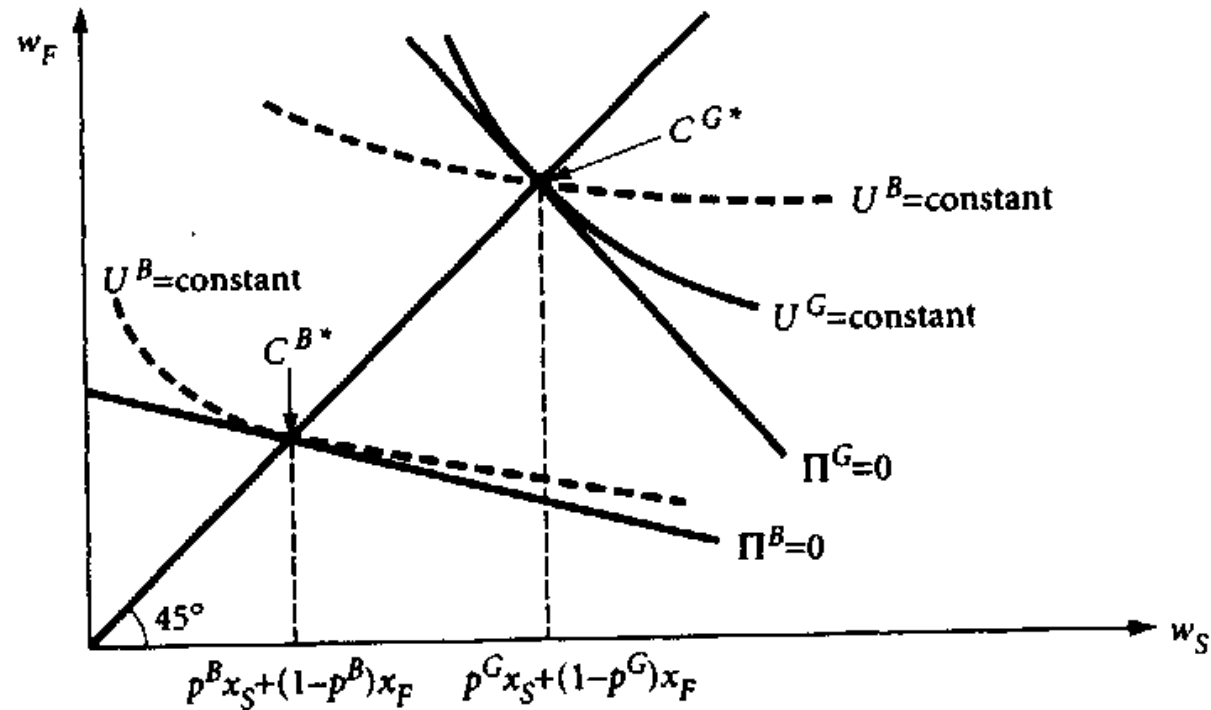
– disutility from *unique* effort level considered in  $\underline{U}$

- menu of contracts  $\{(w_S^G, w_F^G), (w_S^B, w_F^B)\}$

- ▶ Menu of contracts:  $C^T \equiv (w_S^T, w_F^T)$ ,  $T \in \{G, B\}$

$$\mathcal{L}^T = p^T(x_S - w_S^T) + (1 - p^T)(x_F - w_F^T) + \lambda [p^T u(w_S^T) + (1 - p^T) u(w_F^T) - \underline{U}]$$

- zero expected profits (competition among P)
  - $C^T$  are Pareto efficient
- ▶ Full insurance:  $w_S^T = w_F^T$ 
    - $\partial \mathcal{L} / \partial w_S^T = 0 = \partial \mathcal{L} / \partial w_F^T \Leftrightarrow \lambda u'(w_S^T) = 1 = u'(w_F^T) \lambda$
- ▶  $(w_S^G, w_F^G) \gg (w_S^B, w_F^B)$



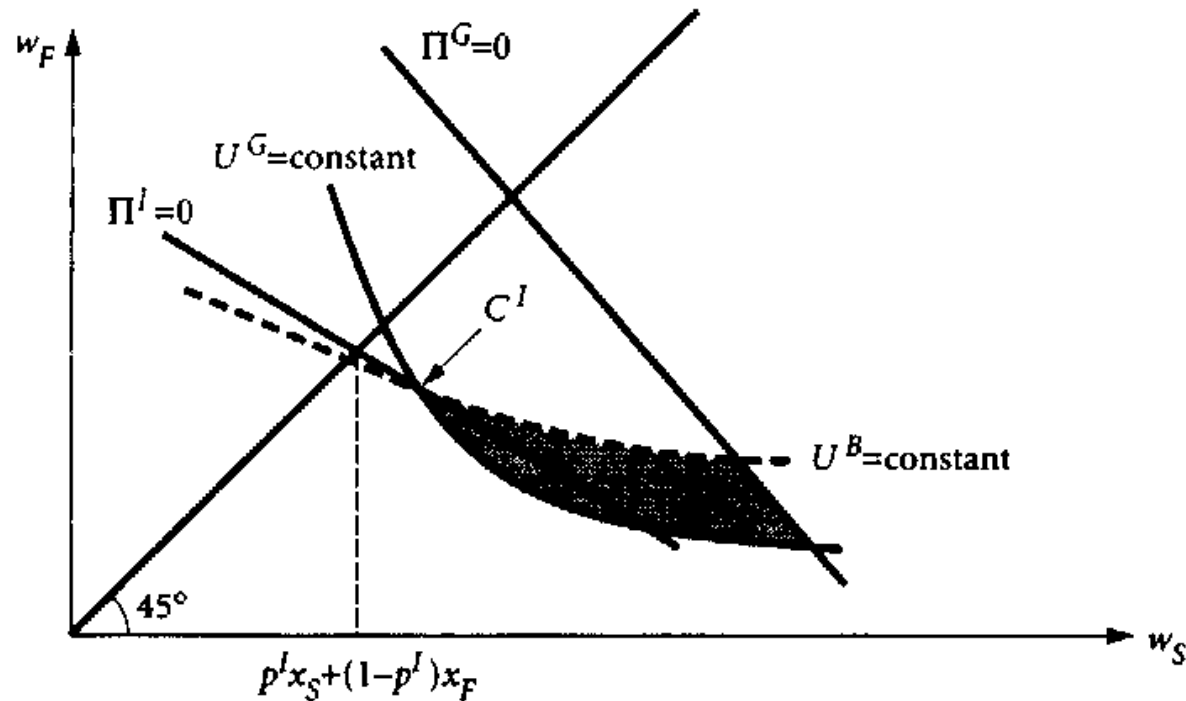
- ▶ Indifference curve is steeper for G than for B
- ▶ Isoprofit line is steeper for G than for B
- ▶ Self selection constraint **not** satisfied for **B**
  - B prefers  $C^{G*}$  over  $C^{B*}$



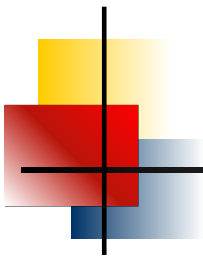
- ▶ Equilibrium contract  $\{C^G, C^B\} \equiv \{(w_S^G, w_F^G), (w_S^B, w_F^B)\}$ 
    - pooling (Bayesian Nash) equilibrium:  $C^G = C^B$
    - separating (Bayesian Nash) equilibrium:  $C^G \neq C^B$
  - ▶ Requirements: given  $\{C^G, C^B\}$ 
    - $\exists$  no other contract preferred to  $C^G$  only by  $G$ , with  $\Pi > 0$
    - $\exists$  no other contract preferred to  $C^B$  only by  $B$  with  $\Pi > 0$
    - $\exists$  no other contract preferred to  $C^T$  only by  $T$  with  $\Pi > 0$ ,  
 $T \in \{G, B\}$
- rules out B mimicking G
- zero expected profits (due to many P)

# No pooling equilibria

- ▶ Pooling zero profit line:
  - $p^I \equiv q p^G + (1 - q)p^B$
  - $\Pi^I \equiv p^I(x_S - w_S) + (1 - p^I)(x_F - w_F) = 0$

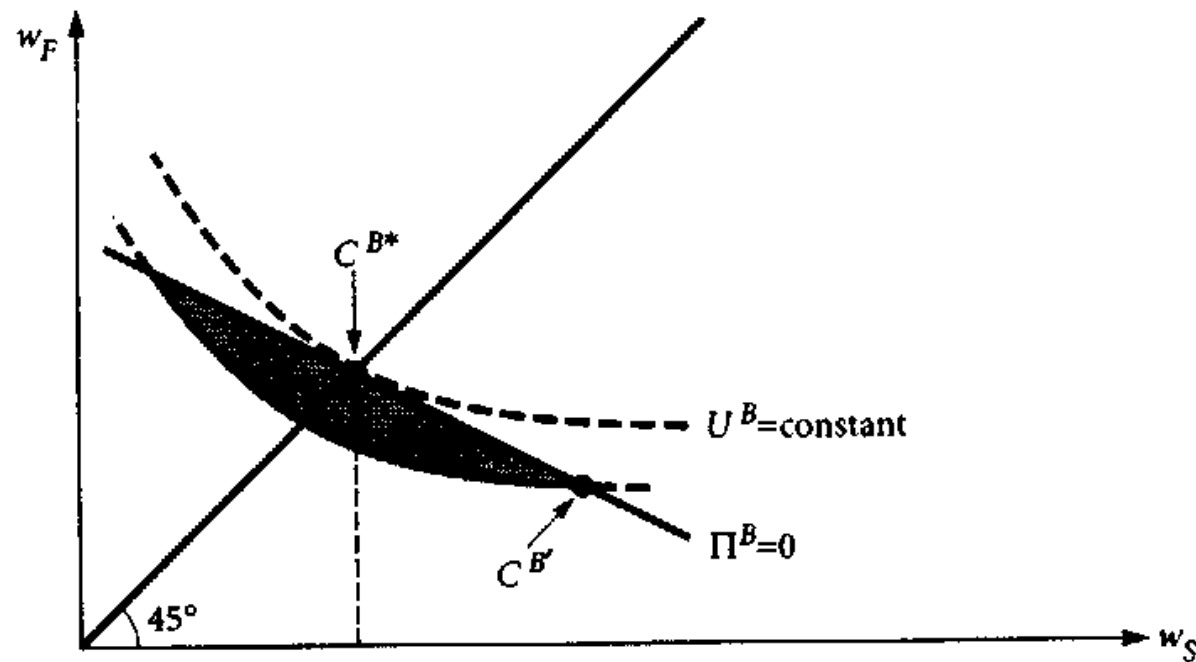


- ▶  $C^I$ : for every contract on pooling zero profit line,  $MRS_{S,F}^G > MRS_{S,F}^B$   
no pooling equ. b/c cream skimming



# Separating equilibria

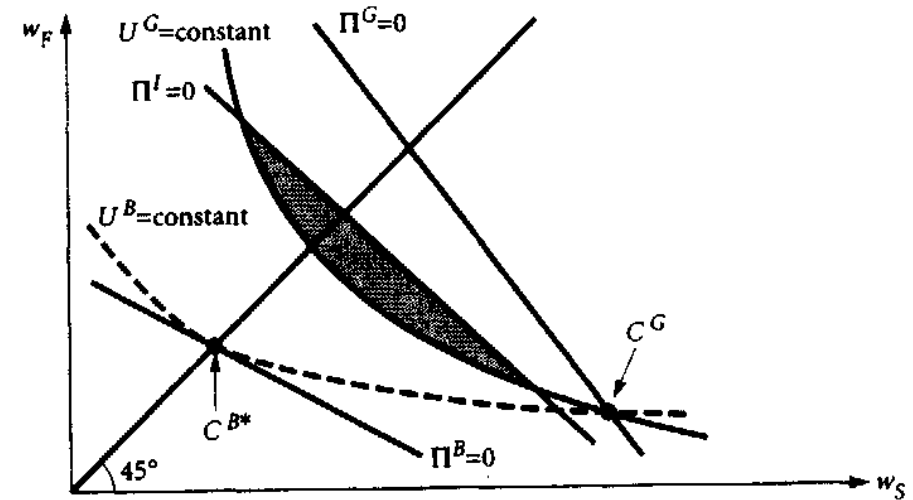
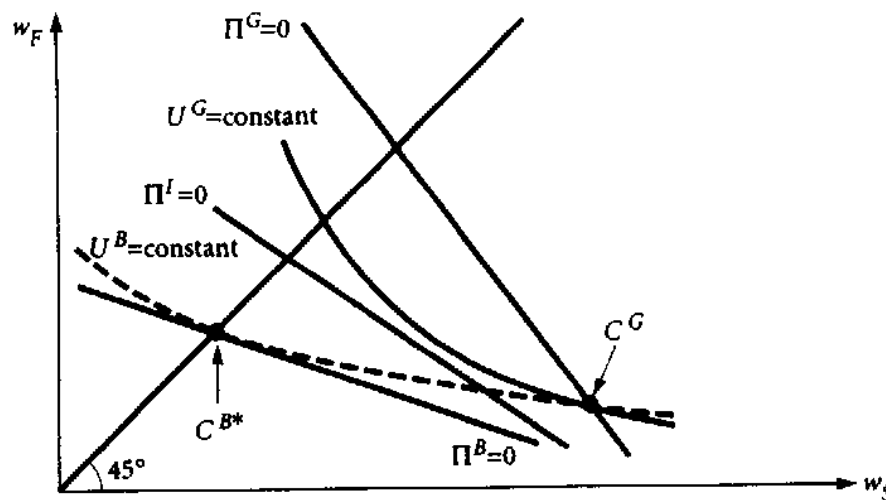
- ▶ B-type:  $C^B = C^{B*}$  (Nash equilibrium contract)



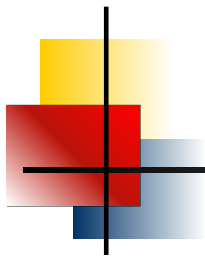
$C^{B'}$  violates equilibrium requirements



# Separating equilibrium exists for “low” $q$



- left: separating equilibrium exists
- right: equilibrium requirements violated; shaded area: both T prefer contract over their equilibrium contract (but positive expected profits)



*Lesson 1* Adverse selection (AS): hidden information about type (payoff) of A

*Lesson 2* Optimal contracts differ among type. AS may provoke

- good quality to drive out bad quality (lemons) = adverse selection  
or
- absence of equilibrium (market) in the extreme

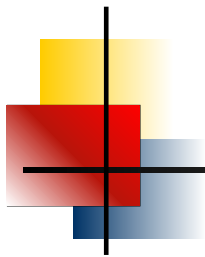
→ **inefficiency**: gains from trade go unexploited

*Lesson 3* AS: P designs menu of **self-selective** contracts

∴ Moral hazard: pooling equilibrium (same contract for all A)

*Lesson 4* Self-selection constraint rules out mimicking:

cost to P = information rent to A



## Lesson 5 1 P, 2 A:

PC is binding only for **least** productive A

SSC **not** binding for **least** productive A ( $G$  wants to mimic  $B$ )

non-distortion at the top ( $G$ )

P distorts  $C^B$  (lowers  $e^B$ ) in order to minimize information rent

Lesson 6 **Many P, 2 A:** Every equilibrium contract (if existing) is **separating** (due to cream skimming)

Lesson 7 Bad risks are fully insured (as w/ symmetric info)

Lesson 8 Good risk (while actuarially fair) get excess clause:  
less than full insurance due to signaling requirement.

## ▶ A. Single insurance company (P, RN)

- A: low (accident) risk  $\pi^G$ , high risk  $\pi^B > \pi^G$
- P proposes  $p$  per unit of benefit  $z$ , A choose coverage  $z$

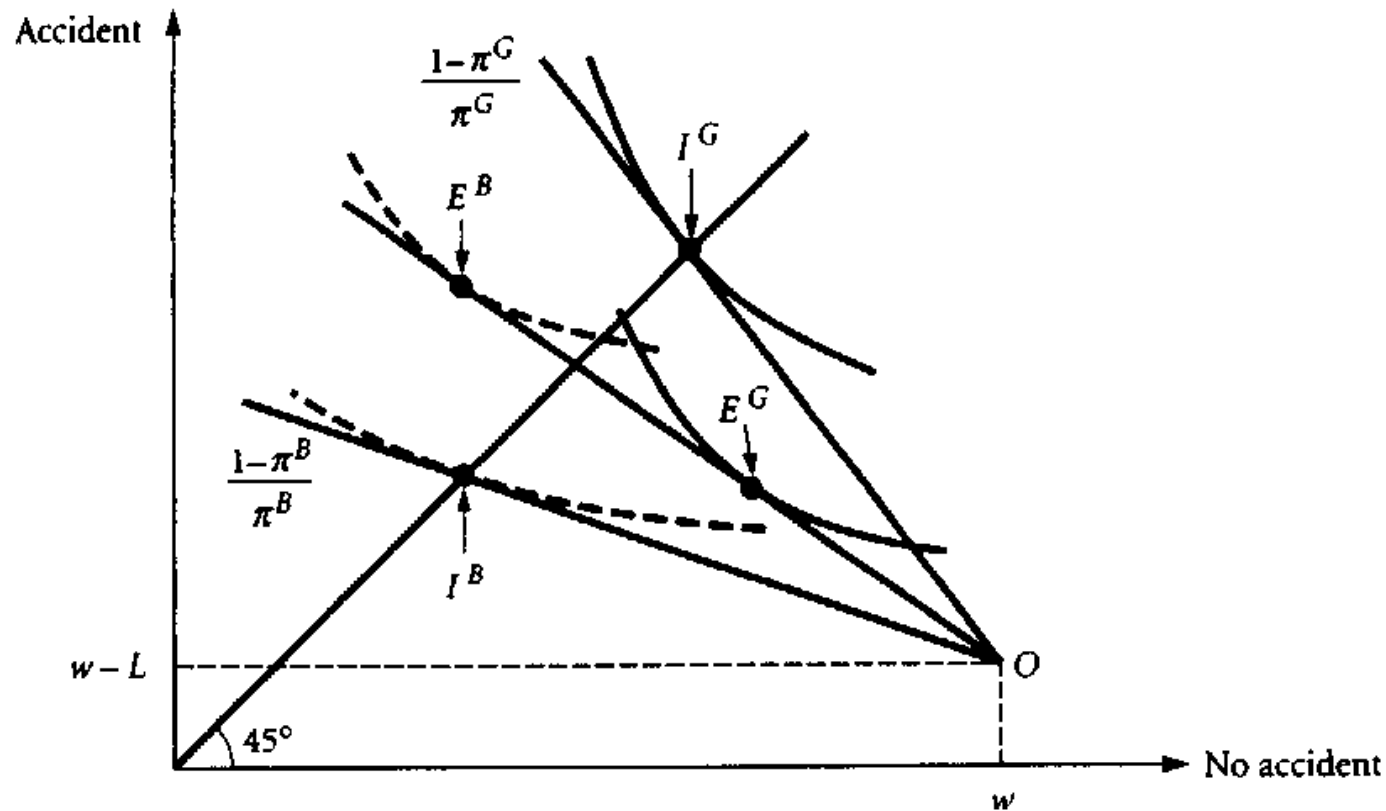
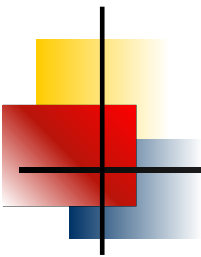
## ▶ Symmetric information (efficient sol)

- A  $\max_{\{z^T\}} \pi^T u(w - L - pz^T + z^T) + (1 - \pi^T) u(w - pz^T)$ ,  
 $T \in \{G, B\}$

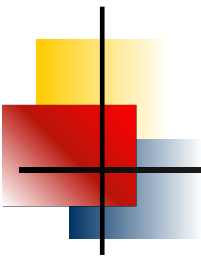
$$\frac{u'(w - L - pz^T + z^T)}{u'(w - pz^T)} = \frac{(1 - \pi^T)p}{\pi^T(1 - p)}$$

- for given  $p$ , as  $\pi^B > \pi^G$ :  $z^B > z^G$
  - actuarial fairness:  $p^T = \pi^T$
- $z^B = L = z^G$ ,  $p^B > p^G$





- asymmetric information  $\pi^B > p > \pi^G$
- $E^B$ : B overinsures,  $E^G$ : G underinsures (expected loss)
- $p \uparrow$  (flatter contract menu line) re-enforcing adverse selection!

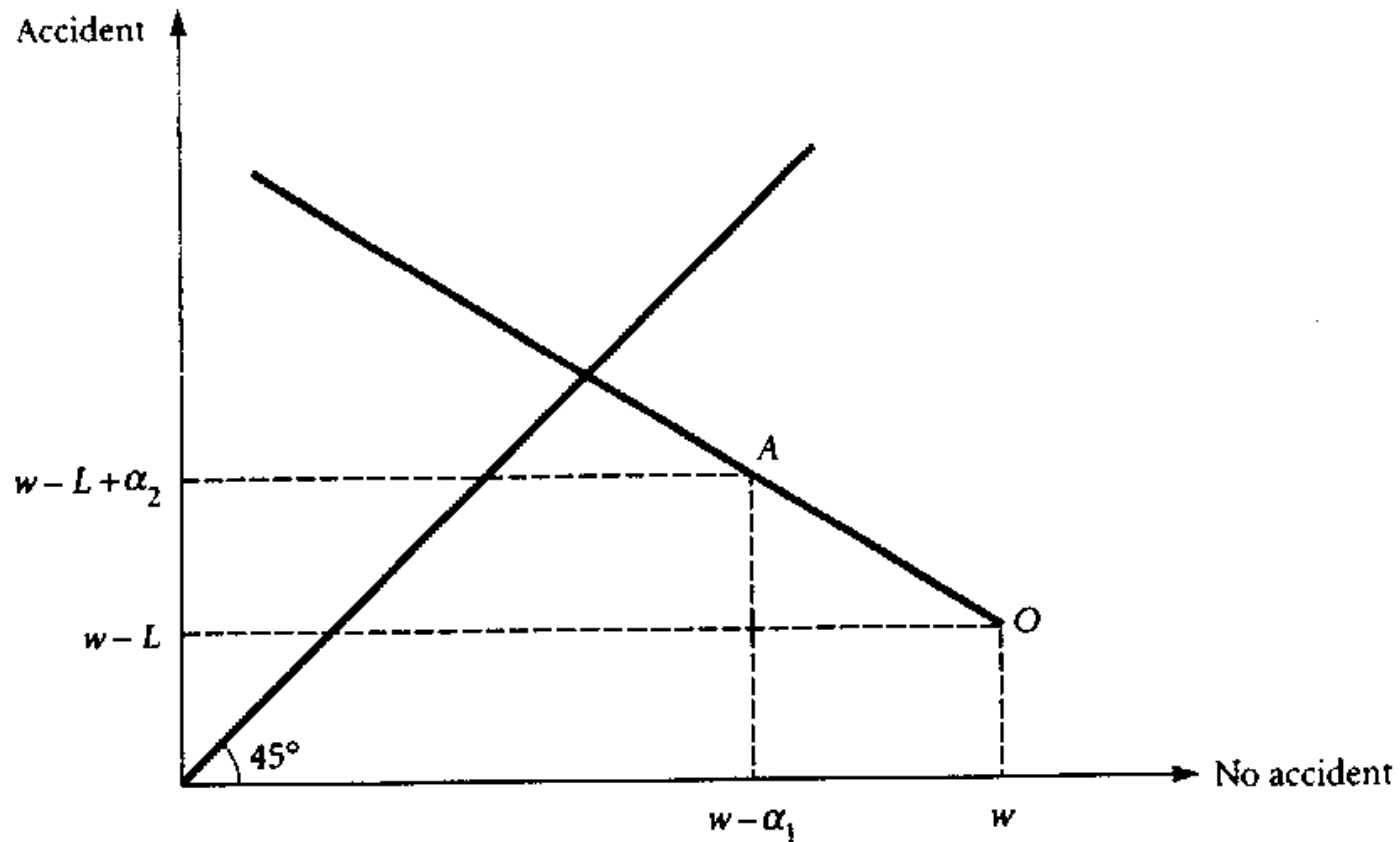


▶ Asymmetric information

- AS drives out “good” risk from the market
- inefficiently low insurance coverage for G
- possibly only equilibrium:  $I^B$  for B, 0 for G

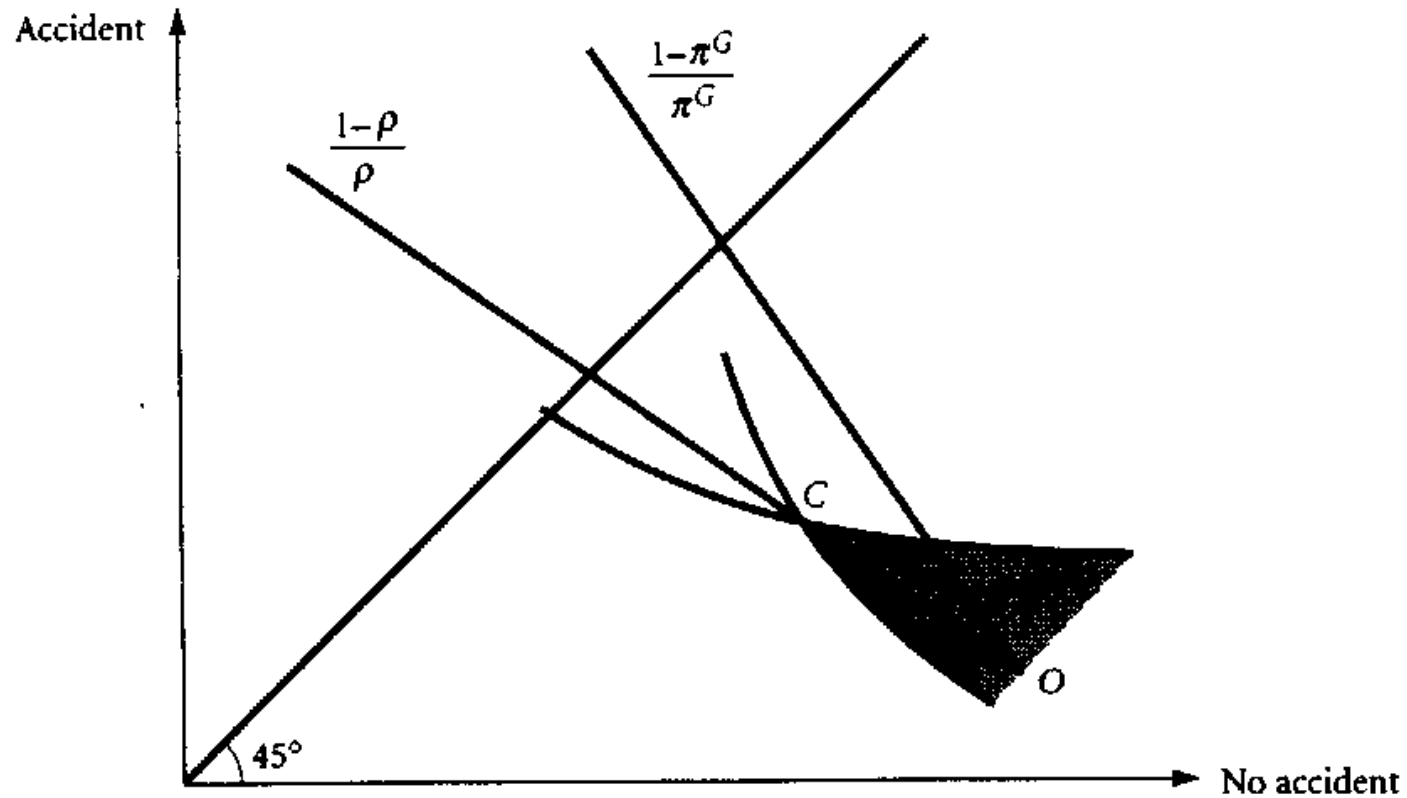
► B. Several insurance companies (P) offering  $(p, z)$  packages

- premium  $\alpha_1 \equiv pz$ ; net benefit  $\alpha_2 \equiv z - pz$
- contract  $(\alpha_1^T, \alpha_2^T)$ : pooling, separating



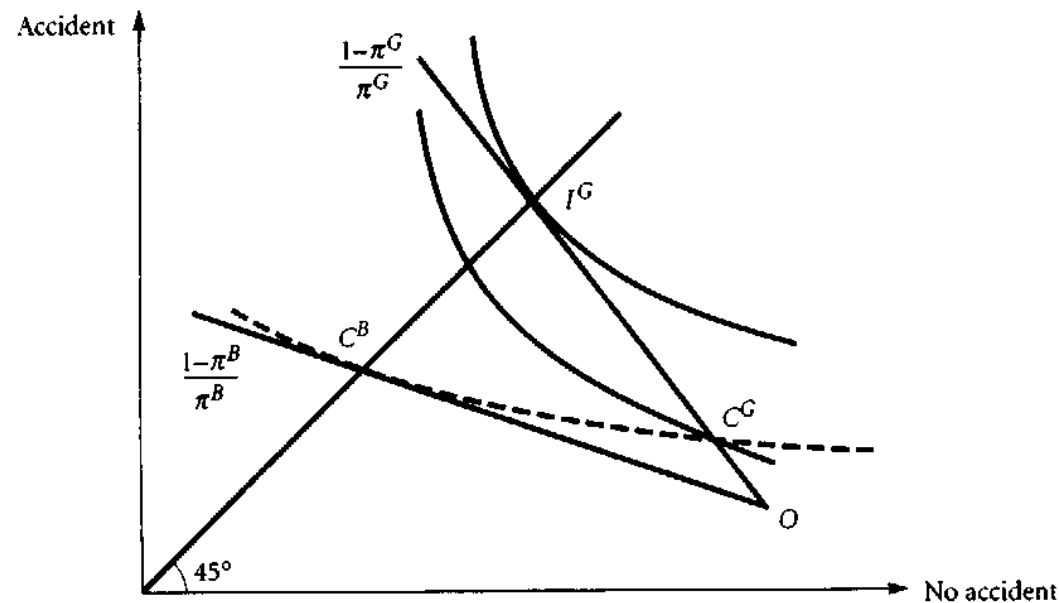
▶ Non-existence of pooling equilibrium contracts

- actuarially fair price  $\rho = q\pi^G + (1 - q)\pi^B$

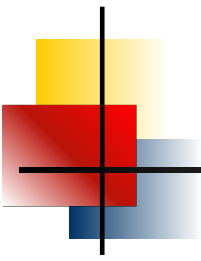


▶ MRS differ: cream skimming on zero-profit pooling line “below” C

▶ Separating contracts  $\{C^B, C^G\}$



- both contracts must be on respective zero profit lines
- $C^G$  must not be above B's indifference curve (expected loss)
- $C^G$  must not be below B's indifference curve (competition for G)
- $C^B$  must be on  $45^\circ$  line (most preferred one by B on zero profit line)



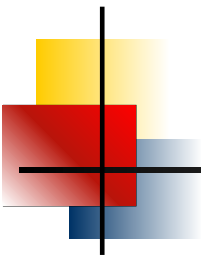
- ▶ Existence of separating contracts
  - no dominating pooling contract (holds if  $q$  “low”)
  - signalling  $\pi^G$  is costly (in terms of low coverage)
  - only pays if there is significant amount of bad risk in the market
  
- ▶ Compared to symmetric info equilibrium
  - B gets same contract
  - G gets partial coverage at lower price ( $p^G = \pi^G$ )  
... needs to signal  $G$ -characteristic by accepting lower coverage

## Application II: Optimal licensing contracts

- ▶ P research lab, selling license  $f$  / cost reducing technology
- ▶ A monopolist, AC  $c^0$ ; technology lowers  $c^0$  to  $c < c^0$
- ▶ Contract  $(F, \epsilon)$
- ▶ Symmetric information (backward induction)
- ▶ A

$$\Pi^m(c + \epsilon) = [p^m(c + \epsilon) - (c + \epsilon)] D^m(p^m(c + \epsilon))$$

$$p^m(c + \epsilon) \in \arg \max_p [p - (c + \epsilon)] D(p)$$



► P

$$\max_{F, \epsilon} F + \epsilon D^m(p^m(c + \epsilon))$$

$$\text{s.t. } \Pi^m(c + \epsilon) - F \geq \Pi^m(c^0) \quad [\lambda]$$

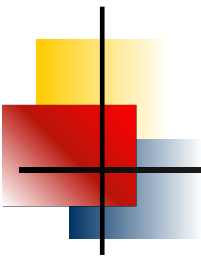
$$\epsilon \geq 0, \quad F \geq 0$$

► Optimal contract / symmetric information

- $\lambda > 0 \Rightarrow$  part. constraint binds
- $\epsilon^* = 0, F^* = \Pi^m(c) - \Pi^m(c^0)$



- ▶ Asymmetric information on cost:  $c^G < c^B < c^0$ 
  - $\epsilon^{G^*} = 0, F^{G^*} = \Pi^m(c^G) - \Pi^m(c^0)$   
 $\epsilon^{B^*} = 0, F^{B^*} = \Pi^m(c^B) - \Pi^m(c^0)$
  - sym. inf. contract not optimal, as  $G$  chooses  $(\epsilon^{B^*}, F^{B^*})$
  - $P$  needs to make  $(\epsilon^B, F^B)$  less attractive to  $G$ 
    - $\epsilon^B > 0$  (distortion)



$$\max_{F^G, \epsilon^G, F^B, \epsilon^B} q[F^G + \epsilon^G D^m(p^m(c^G + \epsilon^G))] + (1 - q)[F^B + \epsilon^B D^m(p^m(c^B + \epsilon^B))]$$

$$\Pi^m(c^G + \epsilon^G) - F^G \geq \Pi^m(c^G + \epsilon^B) - F^B \quad [\mu]$$

$$\Pi^m(c^B + \epsilon^B) - F^B \geq \Pi^m(c^B + \epsilon^G) - F^G \quad [\lambda]$$

$$\Pi^m(c^G + \epsilon^G) - F^G \geq \Pi^m(c^0) \quad [\rho]$$

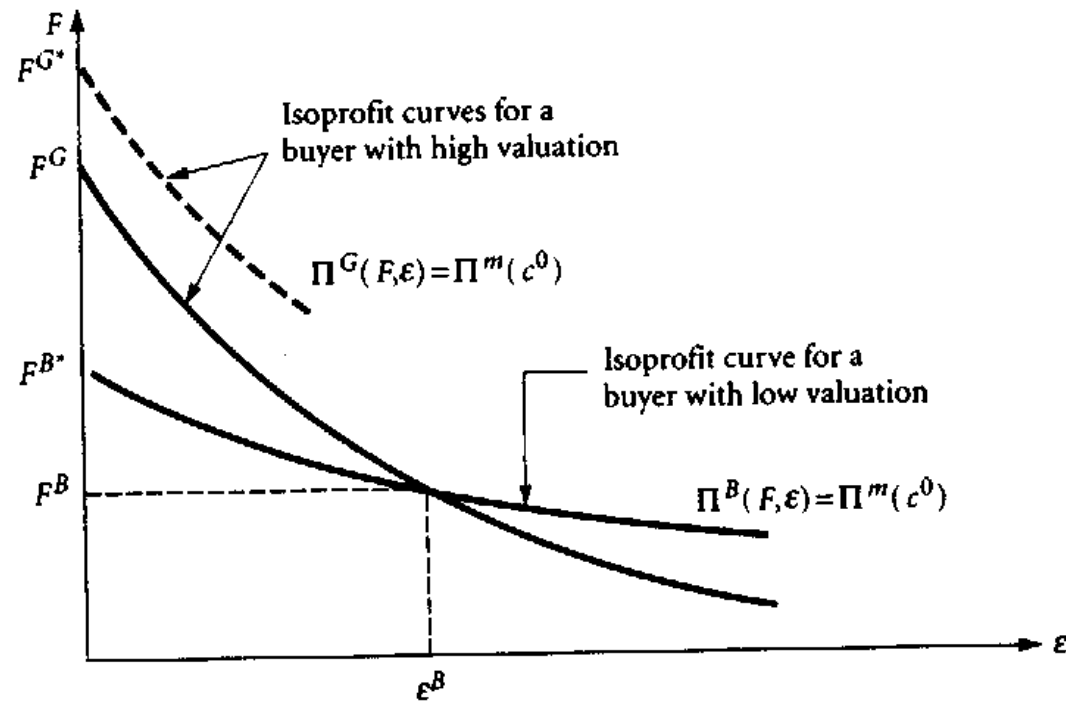
$$\Pi^m(c^B + \epsilon^B) - F^B \geq \Pi^m(c^0) \quad [\delta]$$

$$F^G \geq 0, F^B \geq 0, \epsilon^G \geq 0, \epsilon^B \geq 0$$

- × ○ PC of G not binding  $\rho = 0$ , info rent for G
- PC for B binding  $\delta > 0$
- SSC for G binding  $\mu > 0$ , and  $\lambda = 0$

► FOC imply separating equilibrium contracts

- $\epsilon^G = \epsilon^{G^*} = 0$  no distortion at the top
- $\epsilon^B > \epsilon^{B^*} = 0$  distortion to discourage mimicking
- $F^G < F^{G^*}$  information rent for G
- $F^B < F^G$



- ▶ Regulation of natural monopolist (A)
  - prices, quantities, subsidies set by public sector (P)
  - cost function of monopoly often private information
  - monopolist  $C(Q) = F + c Q$  (decreasing AC)
  - households  $U(Q)$ , paying  $T$
  - government
    - $S$ , with social cost  $(1 + g)S$ ,  $g > 0$
    - max consumer surplus + firm profits - social cost of subsidy

► Symmetric information

$$\max_{T, S, Q} [U(Q) - T] + [T + S - cQ - F] - [(1 + g)S]$$

$$\text{s.t. } T + S - cQ - F \geq 0 \quad [\lambda]$$

$$U(Q) - T \geq 0 \quad [\mu]$$

- both PC bind:  $\lambda = \mu = g > 0$
- optimal quantity decision:  $U'(Q) = c$  (MWP = MC)

► Asymmetric information about  $c$ :  $c^G < c^B$

- for finding Bayesian NE: P proposes  $\{(T^G, S^G, Q^G), (T^B, S^B, Q^B)\}$  given beliefs about types of monopolist  $q, (1 - q)$

$$\max_{(T^G, S^G, Q^G), (T^B, S^B, Q^B)} [q(U(Q^G) - T^G) + (1 - q)(U(Q^B) - T^B)]$$

$$+ q[T^G + S^G - c^G Q^G - F] + (1 - q)[T^B + S^B - c^B Q^B - F]$$

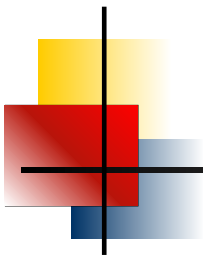
$$- (1 + g)[qS^G + (1 - q)S^B]$$

$$\text{s.t. } T^G + S^G - c^G Q^G - F \geq 0; \quad T^B + S^B - c^B Q^B - F \geq 0$$

$$U(Q^G) - T^G \geq 0; \quad U(Q^B) - T^B \geq 0$$

$$T^G + S^G - c^G Q^G - F \geq T^B + S^B - c^G Q^B - F;$$

$$T^B + S^B - c^B Q^B - F \geq T^G + S^G - c^B Q^G - F$$



- ▶ too complicated? **no!**
- ▶ eliminate some constraints and investigate foc
  - PC of G firm; SSC of B firm
  - G wants to mimic B, SSC of G binds
  - **no distortion at top:**  $U'(Q^G) = c^G$
  - **distortion for B**  $U'(Q^B) > c^B$  to make contract less attractive for G
  - **information rent for G**  
distortion for B lowers information rent
- ▶ optimal contract under asymmetric info
  - $Q^G = Q^{G*}$ ,  $Q^B < Q^{B*}$
  - G obtains an information rent
  - government distorts B contract to lower information rent