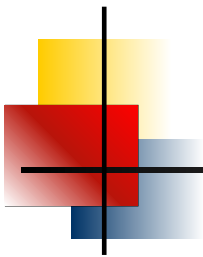


Information Economics

Ronald Wendner

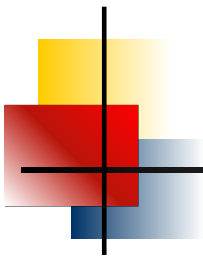
Department of Economics
Graz University, Austria

Course # 320.412 (part 4)



Games of incomplete information

- ▶ Games of incomplete information
 - incomplete information
 - type space, beliefs
- ▶ Associated games that can be solved
- ▶ Cournot competition with incomplete information
- ▶ Incomplete information and mixed strategies (BoS)
- ▶ Further examples



Games of incomplete information

- ▶ Information: **incomplete** versus imperfect
 - *imperfect*: not all **information sets** are singletons; types of other agents are known
 - *incomplete*: **types** (payoff functions) of other agents are not known
 - examples: Cournot, insurance market, market for used cars, BoS

- ▶ Additional ingredients
 - type spaces
 - beliefs about others' types
 - *expected* payoff functions

Incomplete information

▶ type space

- T_i set of possible types of player i
- $T \equiv \times_{i=1}^N T_i$, $t \in T$ joint type vector (space)

▶ beliefs

- $p_i(t_{-i}|t_i)$, for all i , for all t_i
- $p_i(t_{-i}|t_i) \in [0, 1]$, and $\sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) = 1$
- prior beliefs identical \rightarrow nature chooses type vector
 \rightarrow posterior (conditional) beliefs formed via Bayes rule

▶ $u_i : S \times T \rightarrow \mathbb{R}$ expected payoff

Example. Prior vs. posterior beliefs

- ▶ Common joint distribution: $p(t) > 0, \sum_{t \in T} p(t) = 1$

Bayes rule:
$$p_i(t_{-i}|t_i) = \frac{p(t_i, t_{-i})}{\sum_{t_{-i} \in T_{-i}} p(t_i, t_{-i})}$$

		t_2			
		L	M	R	
t_1	T	0.4	0.1	0.1	0.6
	D	0.3	0.1	0.0	0.4
$p(t_2)$		0.7	0.2	0.1	1.0

$$T_i = \{T, D\}, T_j = \{L, M, R\}$$

$$T = \times_{i=1}^2 T_i = \{(T, L), (T, M), (T, R), (D, L), (D, M), (D, R)\}$$

$t \in T$, e.g., $t = (T, L)$, $p(t) = p(t_1, t_2) = 0.4$ (prior believe)

► Posterior beliefs

- verify $\sum_{t \in T} p(t) = 1$

- $p(t_i) = \sum_{t_{-i} \in T_{-i}} p(t_{-i}, t_i)$

$$p(T) = p(L, T) + p(M, T) + p(R, T) = 0.6$$

- $p_i(t_{-i}|t_i)$: e.g., $p_1(t_2|t_1) = p_1(M|T) = ?$

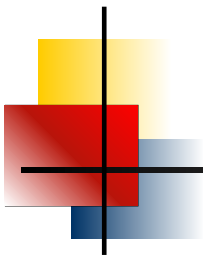
$$\text{Bayes' rule: } p_1(M|T) = \frac{p(M, T)}{p(T)} = \frac{0.1}{0.6} = 0.17$$

► Independence: $p(T, L) = p(T)p(L)$ but: $0.4 \neq 0.6 * 0.7 = 0.42$

t_i not independent of $t_j \Leftrightarrow$ prior belief \neq posterior belief

- prior belief $p(T, M) = 0.1$

- posterior belief $p_1(M|T) = 0.17$



Bayesian Game

$$G = (p, T_i, S_i, u_i)_{i=1}^N$$

- $u_i : S \times T \rightarrow \mathbf{R}$
- p prob. distribution over T

▶ Cournot duopoly w/ incomplete information

- both players: constant marginal cost ; player 2: c_H, c_L .
type space

	Player 2		
	c_L	c_H	$p(t_1)$
Player 1	$1 - \theta$	θ	1.0
$p(t_2)$	$1 - \theta$	θ	1.0

$$T_1 = \{1\}, T_2 = \{c_L, c_H\}, T = T_1 \times T_2 = \{(1, c_L), (1, c_H)\}$$

independence: $(1 - \theta) = p(1, c_L) = p(1) p(c_L) = 1 (1 - \theta)$.

→ $p(c_L) = p_1(c_L|1)$ (prior = posterior belief)

- ▶ Associate equivalent game G^* : each type is separate player
 - each **type** of player chooses her strategy
 - nature, using p , randomly chooses type vector
those players actually play the game
 - joint pure strategy $s^* = (s_1(t_1), \dots, s_N(t_N))_{t \in T}$ **where $*$ \rightarrow types**
 s^* is a joint pure strategy for every $t \in T$
 - Example. $N = 2$, player 1 has 1 type, player 2 has 2 types
2 type vectors: $t, t' \in T$
one joint strategy for t , $s(t)$; and one joint strategy for t' , $s'(t')$
 $\rightarrow s^* = (s(t), s(t'))$
 - expected payoffs (VNM!): **note – payoff of type, not of player**
$$v_{t_i}(s^*) = \sum_{t_{-i} \in T_{-i}} p_i(t_{-i} | t_i) u_i(s^*, t_1, \dots, t_N)$$

▶ **Theorem.** A Bayesian NE of G is a NE of the associated G^* .

Example. Cournot duopoly w/ incomplete information

	Player 2		
	c_L	c_H	$p(t_1)$
Player 1	$1 - \theta$	θ	1.0
$p(t_2)$	$1 - \theta$	θ	1.0

$$s(t) \equiv (q_1, q_{cL}), s'(t') \equiv (q_1, q_{cH})$$

expected payoffs (* = type dependent)

$$v_1(s^*) = (1 - \theta) u_1(s, (1, c_L)) + \theta u_1(s', (1, c_H))$$

$$v_{2,cL}(s^*) = u_2(s, (1, c_L))$$

$$v_{2,cH}(s^*) = u_2(s', (1, c_H))$$

► Bayesian Nash equilibrium requires:

$$v_1(\hat{s}^*) \geq v_1(s_1, \hat{s}_{-1}^*),$$

$$v_{2_L}(\hat{s}^*) \geq v_{2_L}(\hat{s}_1^*, s_{2_L}), v_{2_H}(\hat{s}^*) \geq v_{2_H}(\hat{s}_1^*, s_{2_H})$$

every player chooses q_i so as to maximize expected payoff

- ▶ $P(Q) = a - Q$, $Q = q_1 + q_2$, $Q < a$, $C_1(q_1) = c q_1$, $0 < c < a$,
 $C_2(q_2) = c_2 q_2$, $c_2 \in \{c_L, c_H\}$

$S_i = [0, \infty)$, as for $Q \geq a$, $P(Q) = 0$, no firm produces $q_i \geq a$

firm 2, type c_H : $\max_{q_2 \in S_2} q_2 [a - q_2 - \hat{q}_1 - c_H]$

firm 2, type c_L : $\max_{q_2 \in S_2} q_2 [a - q_2 - \hat{q}_1 - c_L]$

firm 1

$\max_{q_1 \in S_1} \theta [a - q_1 - \hat{q}_2(c_H) - c] q_1 + (1 - \theta) [a - q_1 - \hat{q}_2(c_L) - c] q_1$

→ $\hat{q}_2(j) = (a - \hat{q}_1 - c_j)/2$, $j \in \{H, L\}$

→ $\hat{q}_1 = [\theta [a - \hat{q}_2(c_H) - c] + (1 - \theta) [a - \hat{q}_2(c_L) - c]] / 2$

▶ Result

- $\hat{q}_2(c_H) = \frac{a-2c_H+c}{3} + \frac{1-\theta}{6}(c_H - c_L)$
- $\hat{q}_2(c_L) = \frac{a-2c_L+c}{3} - \frac{\theta}{6}(c_H - c_L)$
- $\hat{q}_1 = \frac{a-2c+\theta c_H+(1-\theta)c_L}{3}$

▶ Interpretation

- incomplete information affects Bayesian *Nash equilibrium*

- $\hat{q}_2(c_H) > \hat{q}_2 = \frac{a-2c_2+c_1}{3}$

- $\hat{q}_2(c_L) < \frac{a-2c_2+c_1}{3}$

- $\hat{q}_1 = \frac{a-2c+\theta c_H+(1-\theta)c_L}{3}$

- information rent

► Information rent

- suppose 2 is of type c_H

– $\hat{q}_2(c_H) > \hat{q}_2$ b/c $\hat{q}_1(\theta < 1) < \hat{q}_1(\theta = 1)$

- suppose 2 is of type c_L

– $\hat{q}_2(c_L) < \hat{q}_2$ b/c $\hat{q}_1(\theta > 0) > \hat{q}_1(\theta = 0)$

Incomplete information and mixed strategies (BoS)

- ▶ Incomplete information: payoff (rather than type)

		Pat	
		Opera	Fight
Chris	Opera	$2+t_c, 1$	$0, 0$
	Fight	$0, 0$	$1, 2+t_p$

private information: t_c, t_p

- $t_c \sim \text{Uniform}[0, x], t_p \sim \text{Uniform}[0, x]$
- type spaces: $T_c = [0, x], T_p = [0, x]$

beliefs about x

- density for $x \in [0, x] = 1/(x - 0) = 1/x$

- ▶ Chris plays Opera if $t_c > c, c = \text{critical value}$

Pat plays Fight if $t_p > p, p = \text{critical value}$

Chris: $Pr(t_c > c) = 1 - Pr(tc \leq c) = 1 - F(c) = 1 - (c - 0)/(x - 0) = 1 - c/x$

Pat: $Pr(t_p > p) = 1 - Pr(tp \leq p) = 1 - F(p) = 1 - (p - 0)/(x - 0) = 1 - p/x$

		Pat	
		Opera (p/x)	Fight ($1 - p/x$)
Chris	Opera ($1 - c/x$)	$2+t_c, 1$	$0, 0$
	Fight (c/x)	$0, 0$	$1, 2+t_p$

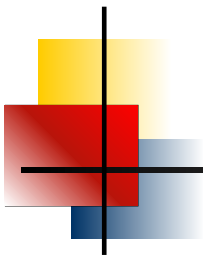
▶ Chris

- expected payoff of Opera: $(2 + t_c)p/x + 0(1 - p/x) = (2 + t_c)p/x$
- expected payoff of Fight: $0 p/x + 1 (1 - p/x) = 1 - p/x$
- play Opera if $(2 + t_c)p/x \geq 1 - p/x \Leftrightarrow t_c \geq x/p - 3 \equiv c (*)$

▶ Pat

- expected payoff of Opera: $1(1 - c/x) + 0 c/x = 1 - c/x$
- expected payoff of Fight: $0 (1 - c/x) + (2 + t_p)c/x = (2 + t_p)c/x$
- play Fight if $(2 + t_p)c/x \geq 1 - c/x \Leftrightarrow t_p \geq x/c - 3 \equiv p (**)$

▶ $(*)+(**)$ $\Rightarrow c = p$ and...



▶ Pure strategy Bayesian NE

○ Chris: play Opera if $t_c \geq c = [-3 + \sqrt{9 + 4x}]/2$

Pat: play Fight if $t_p \geq p = [-3 + \sqrt{9 + 4x}]/2$

▶ Probabilities

$$\frac{x - p}{x} = \frac{x - c}{x} = 1 - \frac{-3 + \sqrt{9 + 4x}}{2x}$$

○ $\lim_{x \rightarrow 0} \frac{x - p}{x} = \lim_{x \rightarrow 0} \frac{x - c}{x} = \frac{2}{3}$

○ In limit, incomplete information BoS approaches behavior in mixed strategy NE in BoS with complete information.

○ **Interpretation of mixed strategies** (Harsanyi, 1973): A mixed-strategy NE in a game with complete information can be interpreted as a pure strategy Bayesian NE in a closely related game with a bit of incomplete information.