Information Economics

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Economics, information, and incentives

Business- & economic models: multi-person decision problems

- full information or symmetric information
- asymmetric incomplete information

Multi-person decision problems

- often: bilateral relationship
  contractor: principal
  contractee: agent
    - firmowner (shareholders) – manager (effort)
    - employer – worker (effort, type)
    - insurance company – policy holder (effort)
    - bank – firm (project risk)

Differing or opposing objectives

- identify objectives in above examples

- Asymmetric information & differing objectives
  - hidden knowledge: adverse selection
  - hidden action: moral hazard
  - incentive problems $\leftarrow$ opposing objectives
    - inefficiencies: mutually beneficial trades go unexploited
- Remedies (incentives)
  - contract design
  - design of signalling and screening devices

Method

But how to analyze multi-person strategic decision problems with asymmetric information and/or opposing objectives?

- Game theory
  - multi-person strategic decision problems
  - strategic
    - behavior of one agent influences outcome of other agents
      example: duopoly
- Information economics $\leftarrow$ game theory

The grand plan

- Setting the stage
- Game theory
  - methodological basis
  - important problems involving strategic behavior
    - study game theory on its own right,
      not only to solve problems in information economics
- Information economics (adverse selection, moral hazard)
  insurance model framework
  - perfect competition as a benchmark case
  - application of game theory to derive solutions
    - signalling equilibria, screening equilibria
  - information economics: generalized framework

Today's plan: setting the stage

- Some important basics in probability theory
- VNM utility
- Games: a few conceptual issues

A few basics in probability theory

- definition and properties
  - "probability" is math language for dealing with (quantifying) uncertainty
  - sample space Ω; event A: $A \subseteq \Omega$
    - 2 types of drivers with acc. prob $\pi_i \in \{\underline{\pi}, \overline{\pi}\}$
    - continuum of types with $\pi_i \in [\underline{\pi}, \overline{\pi}]$

$$Pr : \Omega \to \mathbb{R}$$
$$Pr(A) \geq 0$$
$$Pr(\Omega) = 1$$
$$Pr(\varnothing) = 0$$
$$Pr(A^c) = 1 - Pr(A)$$

if $A \cap B = \varnothing$, then $Pr(A \cup B) = Pr(A) + Pr(B)$

if $A \cap B \neq \varnothing$, then $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A, B)$

A, B are independent $\Leftrightarrow Pr(A, B) = Pr(A)\, Pr(B)$

- **Independence.** Suppose $Pr(A) > 0$, $Pr(B) > 0$, and $A \cap B = \varnothing$. Then A, B are not independent. Proof ($\rightarrow$ class)

- Conditional probability

  - consider $A \subseteq \Omega$, $B \subseteq \Omega$
    restrict attention to $B \subseteq \Omega$, $Pr(B) > 0$

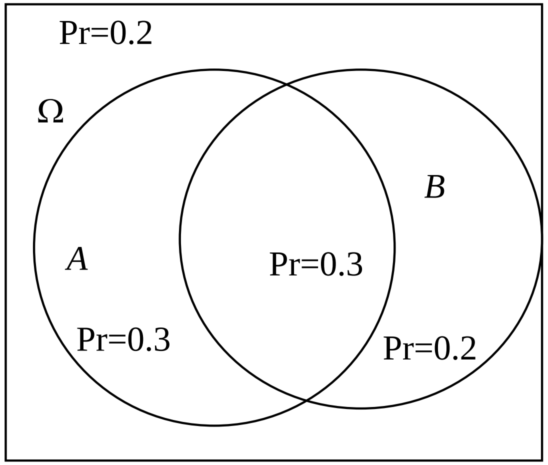
    $\rightarrow$ once we observe event B, what is the probability that event A has occurred?

    $$(A, B) \equiv A \cap B \equiv A \wedge B$$

    $$Pr(A|B) = Pr(A, B)/Pr(B)$$

  - If A independent of B, then $Pr(A, B) = Pr(A)\, Pr(B)$.
    Thus, $Pr(A|B) = Pr(A)$.

Example. Independence (Venn diagram)



$$Pr(A) = .6,\ Pr(B) = .5,\ Pr(A, B) = .3$$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A, B) = 0.8,$$
$$Pr(A \cup B)^c = 1 - Pr(A \cup B) = 0.2$$

$$Pr(A|B) = Pr(A, B)/Pr(B) = 0.3/0.5 = 0.6 = Pr(A)$$

- $Pr(A)$ indept. of whether or not B occurs

Pr of A in $\Omega = Pr(A, B)/Pr(B)$
(share of A in $\Omega =$ share of $A \cap B$ in B)

- Law of total probability

  let $A_1, \ldots, A_k$ be a partition of Ω:

  $Pr(B) = \sum_{i=1}^{k} Pr(B|A_i)\, Pr(A_i)$

- Bayes' theorem

  $Pr(A_i|B) = \frac{Pr(B,A_i)}{Pr(B)} = \frac{Pr(B|A_i)\, Pr(A_i)}{\sum_{j=1}^{k} Pr(B|A_j)\, Pr(A_j)}$

- Random variable: $X : \Omega \to \mathbb{R}$

$\to$ often values of X considered as random variable

  - example: 2 flips of a coin, X # of heads
  - $\Omega = \{(TT), (HT), (TH), (HH)\}$,
    $X(TT) = 0$, $X(TH) = X(HT) = 1$, $X(HH) = 2$
  - distribution: $Pr(X = 0) = .25$, $Pr(X = 1) = .5$, $Pr(X = 2) = .25$

- Discrete distributions: X countable

  Continuous distributions: X convex set

- Cumulated distribution function $F(x) : X \rightarrow [0,1]$
  - consider $a \in [\underline{\pi}, \overline{\pi}]$ then $F(a) = Pr(\pi \leq a)$
    - "share interpretation"
    - probability interpretation

- Probability density function $f(x) = \frac{\partial \, F(x)}{\partial \, x}$
  - consider $a, b \in [\underline{\pi}, \overline{\pi}]$, then

    $$\int_a^b f(\pi) d\pi = Pr(a \leq \pi \leq b) = F(b) - F(a)$$

- Properties

  $$Pr(\pi = a) = Pr(a \leq \pi \leq a) = F(a) - F(a) = 0$$

  $$Pr(\pi > a) = 1 - Pr(\pi \leq a) = 1 - F(a)$$

  $$F(a) = \int_{-\infty}^{a} f(\pi) d\pi, \quad \lim_{a \rightarrow \infty} F(a) = 1$$

- Expectation and variance

$$E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_{-\infty}^{\infty} x\, dF(x)$$

  - note: $dF(x)/dx = f(x) \Rightarrow f(x)dx = dF(x)$
  - example: $E[\pi] = \int_{\underline{\pi}}^{\overline{\pi}} \pi\, dF(\pi)$

$$V[x] = \int_{-\infty}^{\infty} [x - E[x]]^2\, f(x)\, dx = \int_{-\infty}^{\infty} [x - E[x]]^2\, dF(x)$$

- Example: cont. uniform distribution

  - $X \sim$ Uniform(a, b):
    $f(x) = 1/(b - a)$ for $x \in [a, b]$, otherwise $f(x) = 0$
    $F(x) = (x - a)/(b - a)$ for $x \in [a, b]$, otherwise $F(x) \in \{0, 1\}$

Von Neumann-Morgenstern utility

- set of uncertain outcomes $A = \{a_1, ..., a_n\}$ (consumption plans)
- gamble assigns probabilities to outcomes: $a_i \circ p_i$
  - set of gambles for given A:

    $$G = \{(a_1 \circ p_1, a_2 \circ p_2, ..., a_n \circ p_n) \,|\, p_i \geq 0, \sum_i^n p_i = 1\}$$

    a_i outcome, gamble, compund gamble
- 6 axioms of choice under uncertainty
  - (A.1) Completeness: $g, g' \in G$, either $g \succsim g'$, or $g \precsim g'$, or both
  - (A.2) Transitivity: $g, g', g'' \in G$, and $g \succsim g'$, $g' \succsim g''$ then $g \succsim g''$
    (A.1) + (A.2) allow for a complete ordering of A – order as follows:
    $a_1 \succsim a_2 \succsim .... \succsim a_n$
  - (A.3) Continuity:
    $\forall g \in G \;\; \exists \alpha \in [0,1] : g \sim (\alpha \circ a_1, (1-\alpha) \circ a_n)$
    example: $A = \{\$1000, \$10, \textit{death}\}$

Von Neumann-Morgenstern utility

- (A.4) Monotonicity (nonsatiation)
  $\alpha \geq \beta$: $(\alpha \circ a_1,\, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1,\, (1-\beta) \circ a_n)$

- (A.5) Substitution (linearity): $g, h \in G$,
  $g = (p_1 \circ g_1, ..., p_n \circ g_n),\; h = (p_1 \circ h_1, ..., p_n \circ h_n)$

  if $g_i \sim h_i$ for all i, then:

  $g \sim h \Rightarrow (\alpha \circ g, (1-\alpha) \circ h) \sim (\alpha \circ g, (1-\alpha) \circ g) = g$

$\rightarrow$ convex combinations are not *better*

VNM utility

- (A.6) Reduction to simple gambles

  compound gamble and effective probabilities

  let $g \in G$, and g_s simple gamble induced by effective probabilities: then $g \sim g_s$

- $\succsim$ observing (A.1) – (A.6) can be represented by a VNM utility function

  $u(g) = u(p_1 \circ a_1, ..., p_n \circ a_n) = \sum_{i=1}^{n} p_i \, u(a_i)$

  - $u(a_i) \equiv u(1 \circ a_i)$
  - VNM said to have "expected utility property" (linearity in effective probabilities)

$\rightarrow$ In situations with incomplete information:

- agents form beliefs using Bayes rule;
- agents' *payoff* (utility) functions are of VNM type.

Games: a few conceptual issues or ... A beautiful mind...

- Basic coordinates of (noncooperative) games
- John Nash (1928 –)

- Game theory: robust predictions in multi-person decision problems
- Nash equilibrium

Example: Prisoner's dilemma

- game consists of: players, actions, payoffs

		Bob fink	Bob mum
Adam	fink	-10 , -10	0 , -20
	mum	-20, 0	-1 , -1

- ○ players {Adam, Bob}
  ○ actions = {fink, mum}
  ○ payoffs (matrix)
- strategic decision problem
- prediction of game theory: Nash equilibrium: (fink, fink) $\rightarrow$ worst outcome!
  (idea behind leniency program)

Setting the stage: basic coordinates

- Game: every multi-person decision problem
  - game theory (noncooperative)
  - basic ingredients: players, actions, payoffs
- Types of games
  - time

    static (simultaneous-move) vs. dynamic (sequential-move)
  - information

    complete information vs. incomplete information (Bayesian games)

    perfect vs. imperfect information (dynamic games)

$\rightarrow$ incomplete information & information economics

- Solution concepts

	complete information	**incomplete information**
static games	iterated elimination of strictly dominated strategies; Nash equilibrium (NE)	Bayesian Nash equilibrium (BNE); Sequential equilibrium
dynamic games	backward induction outcome; subgame perfect Nash equilibrium (SPNE)	Perfect Bayesian equilibrium; Sequential equilibrium

- Representation of games
  - normal form = strategic form (matrix)
  - extensive form (tree)

To every extensive form game there corresponds a strategic form game. For a given strategic form game there may be different corresponding extensive form games.

- frequently: static games – normal form; dynamic games – extensive form

- Actions & strategies
  - set of actions
    - static vs. dynamic games
    - finitely vs. infinitely many actions
  - strategy: complete contingent plan that specifies a player's possible actions in every possible distinguishable circumstance.

		Player 2: L	Player 2: R
Player 1	T	-10 , -10	0 , -20
	D	-20, 0	-1 , -1

- Player 1 plays first, then player 2 moves
  - set of actions of 1: $\{T, D\}$; of 2: $\{L, R\}$
  - strategies of 1: $\{T, D\}$
  - strategies of 2:
    (L if 1 plays T, L if 1 plays D), (L if 1 plays T, R if 1 plays D), (R if 1 plays T, L if 1 plays D), (R if 1 plays T, R if 1 plays D)

- strategies: pure vs. mixed
  - sometimes it is in the best interest to mix pure strategies
    - pitcher-batter duel
  - pure strategy vs. mixed strategy equilibria
- hierarchy of solution concepts

  As games become progressively richer, equilibrium concepts need to be strengthened to rule out implausible equilibria.

  - sequential equilibria $\subseteq$ Bayesian Nash equilibria $\subseteq$ subgame perfect NE $\subseteq$ Nash equilibria
  - stronger equilibrium concept always survives weaker concept

Example: Prisoner's dilemma

		Bob F	Bob M
Adam	F	-10 , -10	0 , -20
	M	-20, 0	-1 , -1

2 players $\{1, 2\}$, actions=strategies $= \{F, M\}$

time: static (simultaneous move-, one shot-) game

information: complete and perfect

solution concept: (i) IESDS, (ii) NE

representation: normal form game

strategies: pure