An example of a Perfect Bayesian equilibrium in mixed strategy
A simplification of poker

Consider the following simplification of poker. There are 2 players: a professor and a student. Before playing each player puts a dollar down. We do not consider this to be a choice.

- The professor draws a single card from a deck consisting of an equal number of kings and queens.
- After observing the card the professor either «bet» or «fold». If the professor folds the student wins the pot. If the professor bets, he places another dollar in the pot.
• The student then has the option between «fold» and «call».

If the student folds, the professor takes the pot (net transfer of 1$ from the student to the professor). If the student calls he adds another dollar to the pot.

Then the professor wins if he has a king (net transfer from student to professor of 2$) and the student wins if the professor has a queen (transfer of 2$ from the professor to student)

The extensive form is the following:
This game has a unique equilibrium in mixed strategies. Let q be the probability that the professor bet when he has a queen. (when he has a king he always bet since he wins with pr. 1).

Using Bayes’ rule the probability the professor has a king when he bets is:

\[
Pr(\text{king}|\text{bet}) = \frac{Pr(\text{bet}|\text{king}) Pr(\text{king})}{Pr(\text{bet}|\text{king}) Pr(\text{king}) + Pr(\text{Queen}|\text{bet}) Pr(\text{Queen})}
\]

\[
= \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + q \times \frac{1}{2}} = \frac{1}{1+q}
\]
When the student calls its payoff is:

\[
\frac{1}{1+q}(-2) + (1- \frac{1}{1+q})2
\]

Student’s payoff when playing fold is -1.
He is better off calling than folding if \( q > \frac{1}{3} \), he is indifferent when \( q = \frac{1}{3} \), and obviously is better off folding when \( q < \frac{1}{3} \).

Let consider the professor decision between betting or folding when he has a queen.
Let \( p \) be the probability that the student calls.
The professor is indifferent between folding and betting when:

\[-1 = (1-p)1-2p\]

Which gives \( p = \frac{2}{3} \).

Then the equilibrium of the game is:
Professor: Bet with pr. 1 if king, bet with pr. \( q = \frac{1}{3} \) if queen
Student: call with pr. \( p = \frac{2}{3} \).

By successive elimination it can be shown that this is the unique PBE.