An example of a Perfect Bayesian equilibrium in mixed strategy

A simplification of poker

Consider the following simplification of poker. There are 2 players: a professor and a student.

Before playing each player puts a dollar down. We do not consider this to be a choice.

- The professor draws a single card from a deck consisting of an equal number of kings and queens.
- After observing the card the professor either «bet» or «fold». If the professor folds the student wins the pot. If the professor bets, he places another dollar in the pot.

- The student then has the option between «fold» and «call».

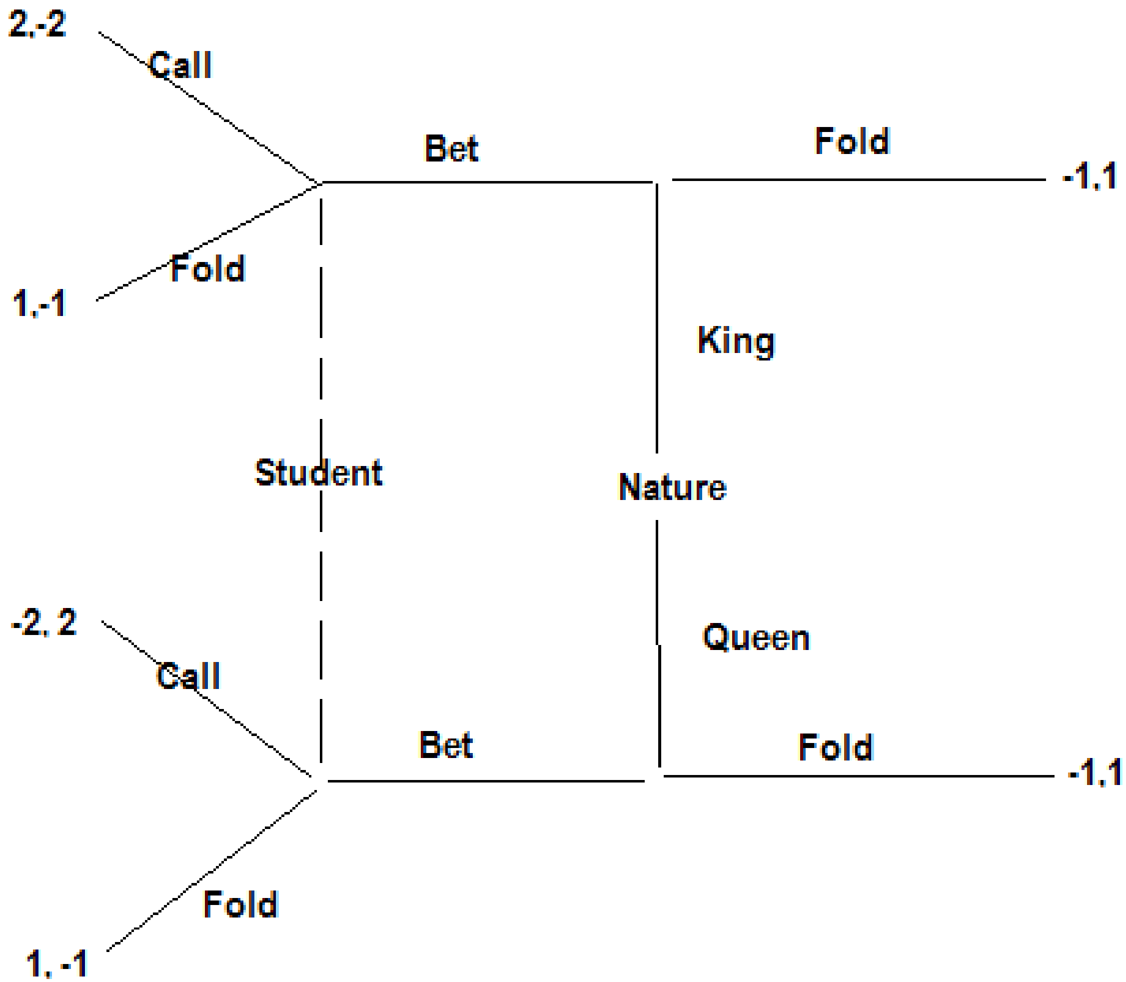
If the student folds, the professor takes the pot (net transfer of 1$ from the student to the professor).

If the student calls he adds another dollar to the pot.

Then the professor wins if he has a king (net transfer from student to professor of 2$)

and the student wins if the professor has a queen (transfer of 2$ from the professor to student)

The extensive form is the following:


2,-2
Call
Bet
Fold
-1,1
Fold
1,-1
King
Student
Nature
-2, 2
Call
Queen
Bet
Fold
-1,1
Fold
1, -1

This game has a unique equilibrium in mixed strategies. Let q be the probability that the professor bet when he has a queen. (when he has a king he always bet since he wins with pr. 1). Using Bayes' rule the probability the professor has a king when he bets is:

$$Pr(king|bet) = \frac{Pr(bet|king)\mathrm{Pr}(king)}{Pr(bet|king)\ \mathrm{Pr}(king)+\mathrm{Pr}(Queen|bet)\mathrm{Pr}(Queen)}$$

$$= \frac{1\times\frac{1}{2}}{1\times\frac{1}{2}+q\times\frac{1}{2}} = \frac{1}{1+q}$$

When the student calls its payoff is:

$$\frac{1}{1+q}(-2) + \left(1 - \frac{1}{1+q}\right) 2$$

Student's payoff when playing fold is -1.
He is better off calling than folding if $q > \frac{1}{3}$, he is indifferent when $q = \frac{1}{3}$, and obviously is better off folding when $q < \frac{1}{3}$.
Let consider the professor decision between betting or folding when he has a queen.
Let p be the probability that the student calls.

The professor is indifferent between folding and betting when:.

$$-1= (1-p)1-2p$$

Which gives p=$\frac{2}{3}$.

Then the equilibrium of the game is:

Professor: Bet with pr. 1 if king, bet with pr. q=$\frac{1}{3}$ if queen

Student: call with pr. p=$\frac{2}{3}$.

By successive elimination it can be shown that this is the unique PBE.