Signaling Games

Gibbons Ch.4

Signaling Games.

Perfect Bayesian Equilibrium in signaling games

A signaling game is a dynamic game of incomplete information with 2 players: a Sender (S) and a receiver (R). The timing is:

1. Nature draws a type $t_i \in T=\{t_1 \; .... \; t_I\}$ for the Sender according to the probability distribution $p(t_i)$, with $p(t_i) > 0$ and

$$p(t_1)+.....+p(t_I)=1.$$

2. Sender observes t_i and chooses a message m_j from a set $M=\{m_1, ..., m_J\}$

3. Receiver observes m_j (but not the type) and then chooses an action $a_k \in A=\{a_1,\ldots.,a_k\}$
4. Payoff are given by $U_S(t_i, m_j, a_k)$ and $U_R(t_i, m_j, a_k)$.

Note that the first move is by nature.

We can now translate the informal statements of requirement 1 through 3 for Perfect Bayesian Equilibrium into a formal definition of PBE in Signaling games.

<u>Signaling Requirement 1</u>: After observing any message m_j the receiver must have a belief about which types could have sent m_j . Denote this belief by the probability distribution

$$\mu(t_j \mid m_j)$$

with $\mu(t_j \mid m_j) > 0$ and $\sum \mu(t_j \mid m_j) = 1$.

<u>Signaling Requirement 2R:</u> For each m_j, the Receiver action must maximize his/her expected utility given the belief:

$$\underset{a_k}{Max} \quad \sum_{t_i}^{T} \mu\left(t_i \middle| m_j\right) U_R(t_i, m, a_k,)$$

The same requirement applies to the Sender who has complete information so that the requirement semplifies to:

<u>Signaling Requirement 2S:</u> For each t_j, the Sender's message $m^*(t_j)$ must maximize his/her expected utility given the Receiver strategy $a^*(m_j)$:

$$\underset{m \in M}{Max}\, U_s(t_i, m_j, a^*(m_j))$$

Finally, for messages on the equilibrium path we have to apply requirement 3 to the Receiver belief:

Signaling Requirement 3: For each m_j in M, if there exists t_j in T such that $m^*(t_j) = m_j$, then the Receiver's belief at the information set corresponding to m_j must follow from Bayes' rule and the Sender' strategy:

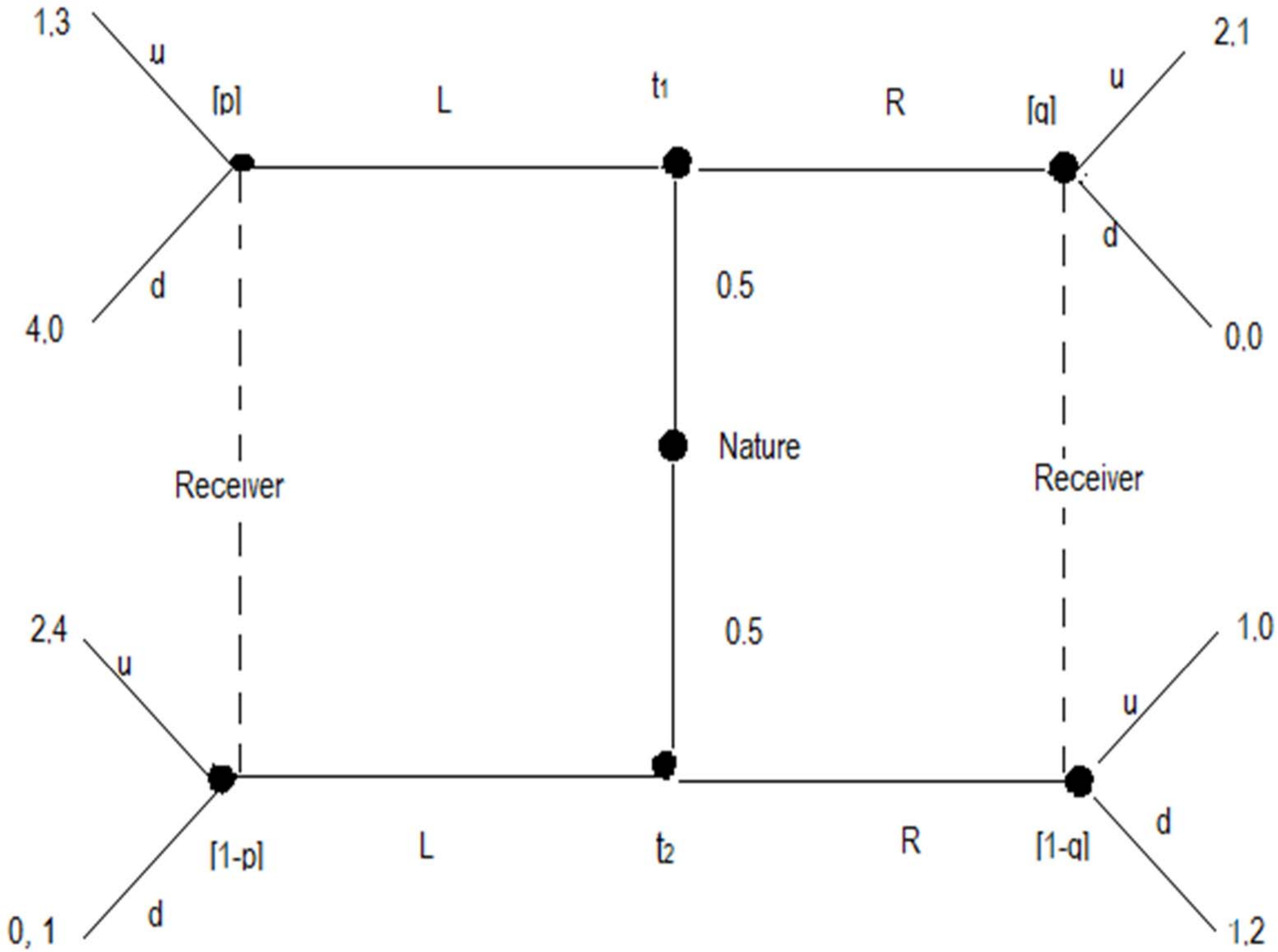
$$\mu(t_i|m_j) = \frac{p(t_i)}{\sum\limits_{t_i \in T_i} p(t_i)}$$

Definition: A pure-strategy Perfect Bayesian Equilibrium in a signaling game is a pair of strategies $m^*(t_j)$ and $a^*(m_j)$ and a belief $\mu(t_j \mid m_j)$ that satisfy Requirements (1), (2R), (2S), and (3).

If the sender's strategy is pooling (all types send the same message) we call the equilibrium pooling, if different types send different messages we call the equilibrium separating.

Consider the following signaling game with 2 types, 2 messages. The game starts with a move by Nature that chooses the type of the Sender, who then selects a message for the receiver.

In this game the message consists of the choice between Left and Right.


1,3
u
[p]
L
t1
R
[q]
u
2,1
4,0
d
0.5
d
0,0
Nature
Receiver
Receiver
2,4
u
0.5
1,0
u
d
0, 1
d
[1-p]
L
t2
R
[1-q]
1,2

There 4 possible pure-strategy perfect Bayesian equilibria in this game:

1. Pooling on L,
2. Pooling on R
3. Separating with t_1 playing L and t_2 playing R
4. Separating with t_1 playing R and t_2 playing L.

<u>1.Pooling on L</u>. If both types play L, there is no updating :

$p=(1-p)=1/2$.

Receiver plays u with expected payoff $3p+4(1-p) >$ Expected payoff from d if L is observed $(=1-p)$.

t_1 gets 1, t_2 gets 2.

How would Receiver answer if R were observed? Is crucial to determine whether pooling on L is optimal.

If Receiver plays u, t_1 deviates and plays R because in this way gets 2 rather than 1.

t_2 does not deviate (2 > 1). To support pooling on L we need the Receiver to play d.

This is optimal only if :

$$q1 < 0q+2(1-q) \Rightarrow \quad q < 2/3.$$

Thus the equilibrium is defined by:

$$[(L, L), (u, d), p=1/2, q \leq 2/3]$$

<u>2. Pooling on R</u>. => q=(1-q)=1/2. In this case we know Receiver plays d.

If Receiver plays d, t_1 does not want to play R, (he would deviate and play L: 1 > 0).

=> No pooling equilibrium on R .

3. Separating with t_1 playing L => p = 1, q = 0. Receiver plays u if L and d if R. But if receiver plays u when he observes L , t_2 finds more profitable playing L which gives him a payoff of 2 instead of 1 , => t_2 deviates. There is no equilibrium.

We have to check the last possible equilibrium:

4. Separating with t_1 playing R => p = 0 and q = 1. The Receiver best response is (u,u). t_1 has no incentive to deviate , nor has it $t_{2.}$
Then [(R, L), (u,u) , p = 0, q = 1] is a separating perfect Bayesian equilibrium.

<u>Summarizing, there are 2 equilibria: one pooling and one separating</u>.