Competitive Equilibrium and the Core

Main objectives of today's session:

. price taking assumption
. bargaining solution vs. competitive equilibrium

- The Core

- Competitive Equilibrium and the Core

1 The Core of an Exchange Economy

- competition I: buyers and sellers meet at large marketplace – treat prices parametrically

  $\rightarrow$ price takers (market so large that individuals are powerless)

- competition II: every agent tries to do as well as possible by making the best available deals

  $\rightarrow$ competition as conflict

- comp I $\rightarrow$ WE, comp II $\rightarrow$ bargaining solution

$\rightarrow$ in large economies: WE $\Leftrightarrow$ bargaining solution

- exchange economy: $I > 1$, $L > 0$

  households: $\{X_i, \succsim_i\}_{i=1}^{I}$, $\{\omega_i\}_{i=1}^{I}$

  allocation: $x = (x_1, x_2, ..., x_I)$

  feasible allocation: $\sum_i x_{li} \leq \sum_i \omega_{li}$, $l = 1, ..., L$

  free disposal

  $\succsim_i$: weak mono, cont, strict conv

- coalition

  I hh, set of all hh: $H = \{1, 2, ..., I\}$

  2^I subsets of households: coalition

  $\rightarrow$ coalition: $S \subseteq H$

  endowment of coalition: $\sum_{i \in S} \omega_i$

  $\rightarrow$ coalition can realize any allocation: $\sum_{i \in S} x'_i \leq \sum_{i \in S} \omega_i$

- blocking of an allocation x

  x blocked if $\exists S \subseteq H$ and the coalition's S allocation x':

  (i) $\sum_{i \in S} x'_{li} \leq \sum_{i \in S} \omega_{li}$, $l = 1, ..., L$

  (ii) $x'_i \succsim_i x_i$ for all $i \in S$

  (iii) $x'_i \succ_i x_i$ for some $i \in S$.

Definition 1 *The core of an economy is the set of all allocations that are blocked by no coalition $S \subseteq H$.*

- if allocation $x = (x_1, x_2, ..., x_I)$ is a core allocation:

  $x_i \succsim_i \omega_i$ for all $i \in H$

  $\rightarrow$ give a proof

- if x' is any core allocation: x' is Pareto efficient

  $\rightarrow$ give a proof

2 Competitive Equilibrium and the Core

- WE allocation is a core allocation

  $\rightarrow$ core nonempty once WE exists

  - Walras law (monotonicity)
  - continuity of $z(p)$: cont, strict conv of $\succsim_i$

  $\rightarrow$ WE allocations $\subseteq$ core allocations

- large economy: core allocations are WE allocations

Definition 2 *Consider* $p \in \mathbb{R}^L_+$ *and* $x_i \in \mathbb{R}^L_+$, $i = 1, ..., I$. (p, x) *is a Walrasian equilibrium if:*

(i) $p \cdot x_i \leq p \cdot \omega_i$, $i \in H$
(ii) $x_i \succsim_i x'_i$ *for all* $x'_i \in \mathbb{R}^L_+$*:* $p \cdot x'_i \leq p \cdot \omega_i$
(iii) $\sum_i x_{li} \leq \sum_i \omega_{li}$, $l = 1, ..., L$
(with $p'_l = 0$ *for any* $l' = 1, ..., L$ *so that strict inequality holds.)*

Theorem 1 *Suppose* $\succsim_i$ *satisfy weak monotonicity and continuity. If* (p, x) *is a WE then* x *is a core allocation.*

Proof (contradiction).
Suppose not: $\exists\,(x', S)$ that blocks x:
$\sum_{i \in S} x'_{li} \leq \sum_{i \in S} \omega_{li}$, $l = 1, ..., L$
$x'_i \succsim_i x_i$ for all $i \in S$
$x'_i \succ_i x_i$ for some $i \in S$

x is a WE: for all $i \in S$
$x'_i \succsim_i x_i \Rightarrow p \cdot x'_i \geq p \cdot x_i$ for all $i \in S$
$x'_i \succ_i x_i \Rightarrow p \cdot x'_i > p \cdot x_i$ for some $i \in S$

thus: $\sum_{i \in S} p \cdot x'_i > \sum_{i \in S} p \cdot \omega_i$ $(*)$

coalitional feasibility: $\sum_{i \in S} x_i' \leq \sum_{i \in S} \omega_i$

thus, for $p \geq 0$:

$\sum_{i \in S} p \cdot x_i' \leq \sum_{i \in S} p \cdot \omega_i$ (**)

- $(*)$ and $(**)$ is a contradiction

- $\nexists \, (x', S)$ that blocks x

$\Rightarrow$ WE allocation is a core allocation.

Example. Consider $I = L = 2$. For both consumers: $u = (1 + x_1)^{1/2} (1 + x_2)^{1/2}$. Suppose $\omega_1 = (99, 0)$, and $\omega_2 = (0, 99)$. Define the core of the exchange economy.