320.312 Problem set 6 (chapter 12)

12.1

Suppose the production possibility frontier for guns (x) and butter (y) is given by

$$x^2 + 2y^2 = 900.$$

a. Graph this frontier.

b. If individuals always prefer consumption bundles in which $y = 2x$, how much x and y will be produced?

c. At the point described in part (b), what will be the *RPT* and hence what price ratio will cause production to take place at that point? (This slope should be approximated by considering small changes in x and y around the optimal point.)

d. Show your solution on the figure from part (a).

12.2

Suppose two individuals *A* and *B* each have 10 hours of labour to devote to producing either ice cream (x) or chicken soup (y). A's utility function is given by

$$U_S = x^{0.3} y^{0.7},$$

whereas *B*'s utility function is given by

$$U_J = x^{0.5} y^{0.5}.$$

The individuals do not care whether they produce x or y, and the production function for each good is given by

$$x = 2l \text{ and } y = 3l,$$

where l is the total labour devoted to production of each good.

a. What must the price ratio, p_x/p_y, be?
b. Given this price ratio, how much x and y will *A* and *B* demand?
c. How should labour be allocated between x and y to satisfy the demand calculated in part (b)?

12.10 An example of Walras' law

Suppose there are only three goods (x_1, x_2, x_3) in an economy and that the excess demand functions for x_2 and x_3 are given by

$$ED_2 = -\frac{3p_2}{p_1} + \frac{2p_3}{p_1} - 1,$$
$$ED_3 = -\frac{4p_2}{p_1} - \frac{2p_3}{p_1} - 2.$$

a. Show that these functions are homogeneous of degree 0 in p_1, p_2 and p_3.
b. Use Walras' law to show that, if $ED_2 = ED_3 = 0$, then ED_1 must also be 0. Can you also use Walras' law to calculate ED_1?
c. Solve this system of equations for the equilibrium relative prices p_2/p_1 and p_3/p_1. What is the equilibrium value for p_3/p_2?

12.12 Initial endowments, equilibrium prices and the first theorem of welfare economics

In Example 12.3 we computed an efficient allocation of the available goods and then found the price ratio consistent with this allocation. That then allowed us to find initial endowments that would support this equilibrium. In that way the example demonstrates the second theorem of welfare economics. We can use the same approach to illustrate the first theorem. Assume again that the utility functions for persons A and B are those given in the example.

a. For each individual, show how his or her demand for x and y depends on the relative prices of these two goods and on the initial endowment that each person has. To simplify the notation here, set $py = 1$ and let p represent the price of x (relative to that of y). Hence the value of, say, A's initial endowment can be written as $p\bar{x}_A + \bar{y}_A$.

b. Use the equilibrium conditions that total quantity demanded of goods x and y must equal the total quantities of these two goods available (assumed to be 1000 units each) to solve for the equilibrium price ratio as a function of the initial endowments of the goods held by each person (remember that initial endowments must also total 1000 for each good).

c. For the case $\bar{x}_A = \bar{y}_A = 500$, compute the resulting market equilibrium and show that it is Pareto efficient.

d. Describe in general terms how changes in the initial endowments would affect the resulting equilibrium prices in this model. Illustrate your conclusions with a few numerical examples.