320.312 Problem set 5 (chapter 9, 10)

9.4

Suppose that the quantity of plastic bottles produced (q) takes place in two locations. Capital inputs cannot change and changes in production use only labour as an input. The production function in location 1 is given by $q_1 = 10l_1^{0.5}$ and in the other location by $q_2 = 50l_2^{0.5}$.

a. If a single firm produces bottles in both locations, then it will obviously want to get as large an output as possible for a given labour input. How should the firm allocate labour between the two locations to do so? Explain precisely the relationship between l_1 and l_2.

b. Assuming that the firm operates in the efficient manner described in part (a), how does total output (q) depend on the total amount of labour employed (l)?

9.6

Suppose we are given the constant returns-to-scale *CES* production function

$$q = [k^{\rho} + l^{\rho}]^{1/\rho}.$$

a. Show that $MP_k = (q/k)^{1-\rho}$ and $MP_l = (q/l)^{1-\rho}$.

b. Show that $RTS = (k/l)^{1-\rho}$; use this to show that $\sigma = 1/(1-\rho)$.

c. Determine the output elasticities for k and l; and show that their sum equals 1.

d. Prove that

$$\frac{q}{l} = \left(\frac{\partial q}{\partial l}\right)^{\sigma}$$

and hence that

$$\ln\left(\frac{q}{l}\right) = \sigma \ln\left(\frac{\partial q}{\partial l}\right).$$

Note: The latter equality is useful in empirical work because we may approximate $\partial q / \partial l$ by the competitively determined wage rate. Hence s can be estimated from a regression of $\ln(q/l)$ on $\ln w$.

9.11

A firm producing polo sticks has a production function given by

$$q = 2\sqrt{kl}.$$

In the short run, the firm's amount of capital equipment is fixed at $k = 100$. The rental rate for k is $v = £1$ and the wage rate for l is $w = £4$.

a. Calculate the firm's short-run total cost curve. Calculate the short-run average cost curve.

b. What is the firm's short-run marginal cost function? What are the *SC*, *SAC* and *SMC* for the firm if it produces 25 polo sticks? Fifty polo sticks? One-hundred polo sticks? Two-hundred polo sticks?

c. Graph the *SAC* and the *SMC* curves for the firm. Indicate the points found in part (b).

d. Where does the *SMC* curve intersect the *SAC* curve? Explain why the *SMC* curve will always intersect the *SAC* curve at its lowest point.

10.1

Harry's Hardware is a small business that sells bags of cement in a market where it is a price-taker and $P = MR$ The prevailing market price of cement is €10 per 50 kg bag. Harry's total cost function is given by

$$TC = 0.05q^2 + 5q + 22$$

where q is the number of bags of cement Harry chooses to sell each week.

a. How many bags of cement must Harry sell to maximise profit?
b. Determine Harry's maximum profit on cement.
c. Graph these results and label Harry's supply curve.

10.5

Would a lump-sum profits tax affect the profit-maximising quantity of output? How about a proportional tax on profits? How about a tax assessed on each unit of output? How about a tax on labour input?