320.312 Problem set 3 (chapter 5)

5.5

Suppose the utility function for goods x and y is given by utility $= U(x, y) = xy + y$.

a. Calculate the uncompensated (Marshallian) demand functions for x and y, and describe how the demand curves for x and y are shifted by changes in I or the price of the other good.

b. Calculate the expenditure function for x and y.

c. Use the expenditure function calculated in part (b) to compute the compensated demand functions for goods x and y. Describe how the compensated demand curves for x and y are shifted by changes in income or by changes in the price of the other good.

5.7

Show that the share of income spent on a good x is $s_x = \dfrac{d\ln E}{d\ln p_x}$, where E is total expenditure.

5.11 Quasi-linear utility (revisited)

Consider a simple quasi-linear utility function of the form $U(x, y) = x + \ln y$.

a. Calculate the income effect for each good. Also calculate the income elasticity of demand for each good.
b. Calculate the substitution effect for each good. Also calculate the compensated own-price elasticity of demand for each good.
c. Show that the Slutsky equation applies to this function.
d. Show that the elasticity form of the Slutsky equation also applies to this function. Describe any special features you observe.

5.13 Price indifference curves

Price indifference curves are iso-utility curves with the prices of two goods on the X- and Y-axes, respectively. Thus, they have the following general form: $(p_1, p_2) | v(p_1, p_2, I) = v_0$.

a. Derive the formula for the price indifference curves for the Cobb–Douglas case with $\alpha = \beta = 0.5$. Sketch one of them.
b. What does the slope of the curve show?
c. What is the direction of increasing utility in your graph?