320.312 Problem set 2 (chapter 4)

4.9 Suppose that we have a utility function involving two goods that is linear of the form $U(x, y) = ax + by$. Calculate the expenditure function for this utility function. Hint: The expenditure function will have kinks at various price ratios.

4.10 Stone–Geary utility

Suppose individuals require a certain level of food (x) to remain alive. Let this amount be given by x_0. Once x_0 is purchased, individuals obtain utility from food and other goods (y) of the form

$$U(x, y) = (x - x_0)^{\alpha} y^{\beta},$$

where $\alpha + \beta = 1$.

a. Show that if $I > p_x x_0$ then the individual will maximise utility by spending $\alpha (I - p_x x_0) + p_x x_0$ on good x and $\beta(I - p_x x_0)$ on good y. Interpret this result.

b. How do the ratios $p_x x/I$ and p_y/I change as income increases in this problem?

4.11 CES indirect utility and expenditure functions

In this problem, we will use a more standard form of the CES utility function to derive indirect utility and expenditure functions. Suppose utility is given by

$$U(x, y) = (x^{\delta} + y^{\delta})^{1/\delta}$$

[in this function the elasticity of substitution $\sigma = 1/(1 - \delta)$].

a. Show that the indirect utility function for the utility function just given is

$$V = I(p_x^r + p_y^r)^{-1/r},$$

where $r = \delta/(\delta - 1) = 1 - \sigma$.

b. Show that the function derived in part (a) is homogeneous of degree zero in prices and income.

c. Show that this function is strictly increasing in income.

d. Show that this function is strictly decreasing in any price.

e. Show that the expenditure function for this case of CES utility is given by

$$E = V(p_x^r + p_y^r)^{1/r}.$$

f. Show that the function derived in part (e) is homogeneous of degree one in the goods' prices.

g. Show that this expenditure function is increasing in each of the prices.

h. Show that the function is concave in each price.