320.312 Problem set 1 (chapter 3)

3.1

Graph a typical indifference curve for the following utility functions, and determine whether they have convex indifference curves (i.e., whether the *MRS* declines as x increases).

a. $U(x, y) = 3x + y$.

b. $U(x, y) = \sqrt{x \cdot y}$.

c. $U(x, y) = \sqrt{x} + y$.

d. $U(x, y) = \sqrt{x^2 - y^2}$.

e. $U(x, y) = \dfrac{xy}{x + y}$.

3.10 Cobb–Douglas utility

Example 3.3 shows that the *MRS* for the Cobb–Douglas function

$$U(x, y) = x^{\alpha} y^{\beta}$$

is given by

$$MRS = \frac{\alpha}{\beta}\left(\frac{y}{x}\right).$$

a. Does this result depend on whether $\alpha + \beta = 1$? Does this sum have any relevance to the theory of choice?

b. For commodity bundles for which $y = x$, how does the *MRS* depend on the values of α and β? Develop an intuitive explanation of why, if $\alpha > \beta$, $MRS > 1$. Illustrate your argument with a graph.

c. Suppose an individual obtains utility only from amounts of x and y that exceed minimal subsistence levels given by x_0, y_0. In this case,

$$U(x, y) = (x - x_0)^{\alpha}(y - y_0)^{\beta}$$

is this function homothetic?

3.12 CES utility

a. Show that the CES function

$$\alpha \frac{x^\delta}{\delta} + \beta \frac{y^\delta}{\delta}$$

is homothetic. How does the *MRS* depend on the ratio y/x?

b. Show that your results from part (a) agree with our discussion of the cases $\delta = 1$ (perfect substitutes) and $\delta = 0$ (Cobb–Douglas).

c. Show that the *MRS* is strictly diminishing for all values of $\delta < 1$.

d. Show that if $x = y$, the *MRS* for this function depends only on the relative sizes of α and β.

e. Calculate the *MRS* for this function when $y/x = 0.9$ and $y/x = 1.1$ for the two cases $\delta = 0.5$ and $\delta = -1$. What do you conclude about the extent to which the *MRS* changes in the vicinity of $x = y$? How would you interpret this geometrically?

3.13 The quasi-linear function

Consider the function $U(x, y) = x + \ln y$. This is a function that is used relatively frequently in economic modelling as it has some useful properties.

a. Find the *MRS* of the function. Now, interpret the result.
b. Confirm that the function is quasi-concave.
c. Find the equation for an indifference curve for this function.
d. Compare the marginal utility of x and y. How do you interpret these functions? How might consumers choose between x and y as they try to increase their utility by, for example, consuming more when their income increases? (We will look at this 'income effect' in detail in the Chapter 5 problems.)
e. Considering how the utility changes as the quantities of the two goods increase, describe some situations where this function might be useful.