VALIDATION OF A FLEXIBLE OPTIMAL CONTROL APPROACH FOR RF-PULSE-DESIGN INCLUDING RELAXATION EFFECTS AND SAR

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Abstract— *Radio frequency (RF) pulses are essential in MRI to excite and alter magnetization. We present and validate a flexible approach based on optimal control of the full time-dependent Bloch equation, including relaxation effects. A globally convergent trust-region Newton method with exact derivatives via adjoint calculus allows the efficient computation of optimal pulses. The results are validated on a 3T scanner and demonstrate the ability to generate optimized pulses for arbitrary flip angles and arbitrary slice profiles.*

Keywords— *MRI, RF-pulse-design, slice selective excitation, optimal control theory*

Introduction

For many applications in MRI there is still a demand for the optimization of slice selective RF pulses. Inhomogeneous RF fields and the restrictions of the Specific Absorption Rate (SAR) are a challenge for RF pulse design at high field strength. Short pulses in fast imaging typically suffer from a bad slice profile. Computing RF pulse shapes with a good slice profile, low SAR requirement and robust against RF in homogeneities is therefore still important and becomes critical for quantitative methods. Conventional design is based on simplifications of the Bloch equation, i.e. the small tip angle or the hard pulse approximation[1]. Applying iterative methods[1-3], including optimal control (OC) [4-7], are used to improve the excitation process. However, its computational effort and difficult implementation limits a general application. We present and validate a generalized approach based on optimal control of the full time-dependent Bloch equations that is able to handle arbitrary flip angles and target slice profiles. Furthermore, the flexibility of the formulation allows inclusion of relaxation effects and models the SAR by adding a regularization term in the cost function.

Methods

Theory The optimal control approach to pulse design consists in minimizing the functional

$$\mathrm{J}(\mathbf{M},\mathbf{u}) = \frac{1}{2}\int_{\mathrm{z_{min}}}^{\mathrm{z_{max}}} |\mathbf{M}(\mathrm{T},\mathrm{z}) - \mathbf{M_d}(\mathrm{z})|_2^2 \mathrm{dz} + \frac{\alpha}{2}\int_0^{\mathrm{T}} |\mathbf{u}(\mathrm{t})|_2^2 \mathrm{dt},$$

where the three dimensional magnetization vector $\mathbf{M}(\mathrm{T},\mathrm{z}) = (M_x(\mathrm{T},\mathrm{z}),\ M_y(\mathrm{T},\mathrm{z}),\ M_z(\mathrm{T},\mathrm{z}))^T$, is the solution of the time-dependent Bloch equation

$$\dot{\boldsymbol{M}}(t) = \gamma\mathbf{M}(\mathrm{t}) \times \mathbf{B}(\mathrm{t}) + \mathbf{R}(\mathbf{M}),$$

and describes the total precession of the nuclear magnetization in an external magnetic field **B** at the end of the excitation time $t = T$ for every point along the slice direction z from z_{max} to z_{min}. The relaxation term $\mathbf{R}(\mathbf{M}) = \left(-\frac{M_x}{T_2},\ -\frac{M_y}{T_2},\ -\frac{(M_z - M_0)}{T_1}\right)^T$ contains the initial magnetization M_0 and the longitudinal and transversal relaxation of the nuclear magnetization $\boldsymbol{M}$. $\mathbf{M_d}(\mathrm{z})$ is the desired slice profile and $\mathbf{B}(\mathrm{t},\mathrm{z}) = (\mathrm{u_x}(\mathrm{t})\mathrm{B_1}, \mathrm{u_y}(\mathrm{t})\mathrm{B_1}, \mathrm{G_s}(\mathrm{t},\mathrm{z}))$ includes the scaling $B_1 = 10^{-5}$ and the shape of the RF pulse $\mathbf{u}(\mathrm{t}) = (\mathrm{u_x}(t), \mathrm{u_y}(\mathrm{t}))^T$ to be optimized and the prescribed slice selective gradient $\mathrm{G_s}$. The last term in J incorporates the desire for minimal SAR of the optimized pulse and is weighted with α for a trait-off between the slice profile accuracy and the required pulse power.

The optimal control u minimizing J can be computed using a globally convergent trust-region Newton method[8] with a matrix-free iterative solution of the Newton step, where the gradient and application of the Hessian are calculated using the adjoint calculus, i.e., by solving in each step forward and backward Bloch equations. Since it is essential to have accurate derivative information, this is done via a numerical simulation of the full Bloch equation, where the time-stepping schemes are chosen such that the discretized derivatives coincide with the derivatives of the discretized functional (adjoint-consistency).

Implementation The described approach was implemented in MATLAB (The MathWorks, Inc., Natick, USA) and used to compute optimized pulses for a rectangular slice with a thickness of 20mm and a flip angle of 90° for a total excitation time of T=2.56ms starting from a zero initial guess. The slice selection gradients were taken from a gradient echo sequence on a 3T MR scanner (Magnetom Skyra, Siemens Healthcare, Erlangen, Germany).

Validation The results of the numerical optimization for the first profile were verified on the above mentioned scanner using a body coil and a cylinder phantom with a diameter of 140mm, a length of 400mm and relaxation times $T_1 \cong 102$ms, $T_2 \cong 81$ms and $T_2^* \cong 70$ms.

Results

Figure 1-3 show the results of the numerical optimization for the rectangular target profile. The optimized pulse $(B_{1,x}(t), B_{1,y}(t)) = (u_x(t)B_1, u_y(t)B_1)$ is given in Figure 1. It can be seen that $u_x(t)$ is similar, but not identical, to a standard sinc shape, and that $u_y(t)$ is nearly zero, which is expected due to the symmetry of the prescribed slice profile. Figure 2 contains the corresponding slice profile $M(T)$ obtained from a numerical solution of the Bloch equation and the experimental results (small crosses) which confirms the simulation. It shows an excitation with a steep transition between the in- and out-of-slice regions and a homogeneous flip angle distribution across the target slice. The phantom image is shown in Figure 3, where the slice profile in Figure 2 is indicated in red.

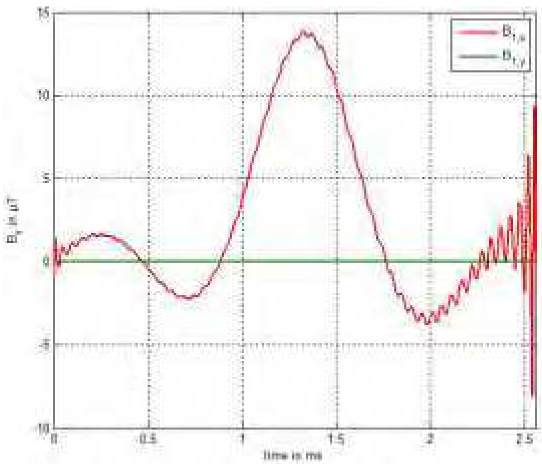

Figure 1: Numerical optimized RF-pulse ($B_{1,x}$ and $B_{1,y}$).

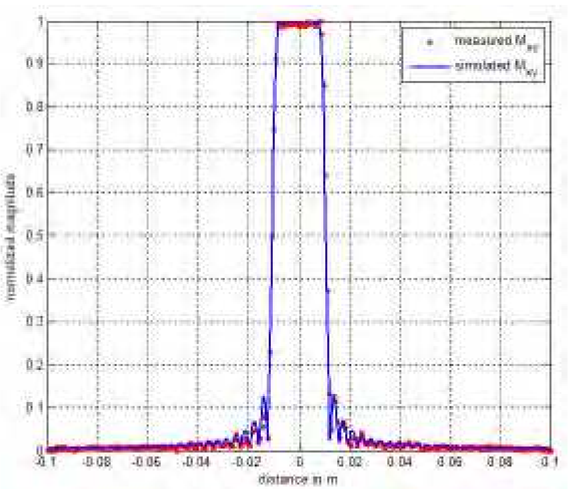

Figure 2: Comparison of the simulated magnetization pattern (solid line) with experimental data (crosses).

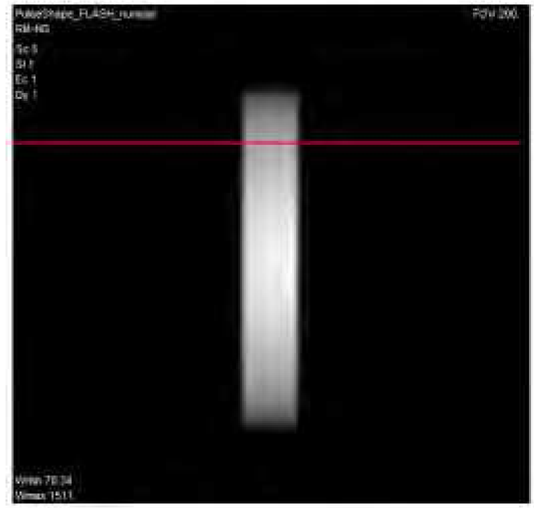

Figure 3: Reconstructed experimental phantom image using optimized pulse shown in Figure 1.

Discussion

The results indicate that the proposed approach allows a problem specific optimization for 1D-excitation. Due to the flexibility of the optimal control formulation, it is possible to include additional robustness (e.g., with respect to B_1 or B_0 inhomogeneities) as well as joint optimization of pulse and gradient shape including slew rate limitations.

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