

# On Gauge Bosons, observable Particles and the Brout-Englert-Higgs mechanism

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Bernd Riederer, Axel Maas

PAPAP 2019 Rome/L'Aquila, 05 November 2019

University of Graz

 @b\_riederer



**NAWI Graz**  
Natural Sciences



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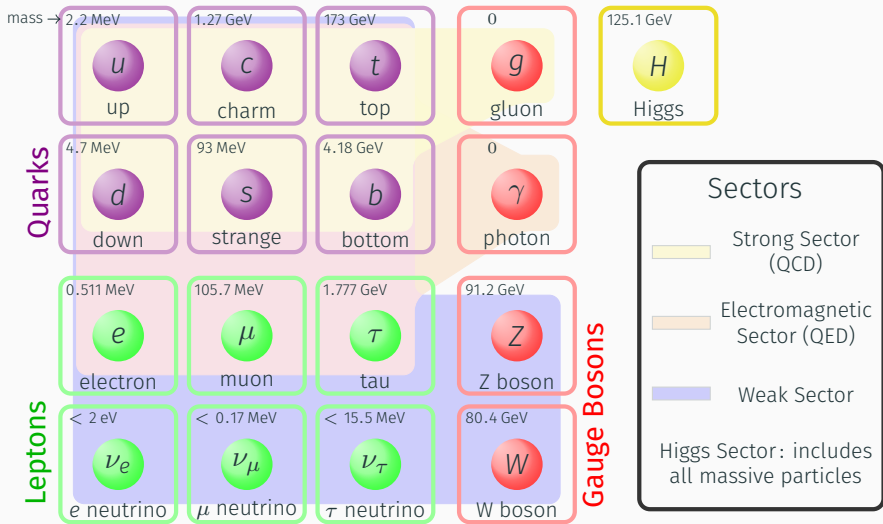
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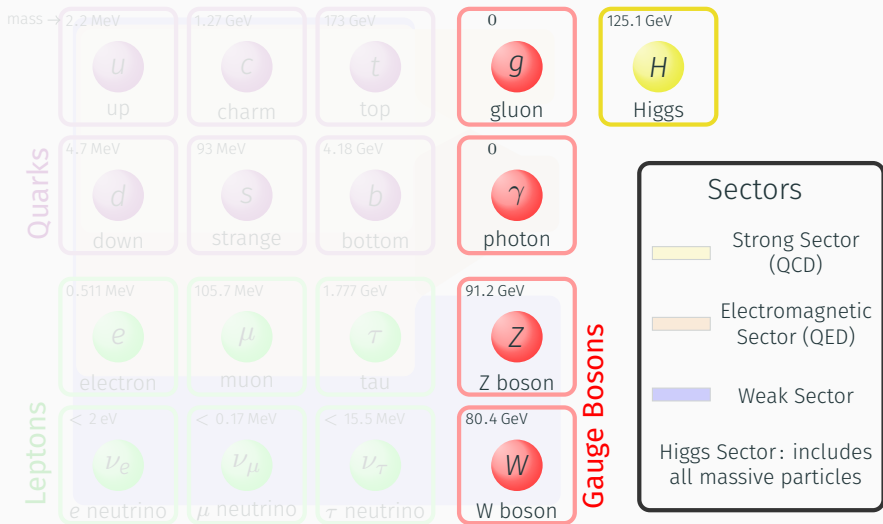
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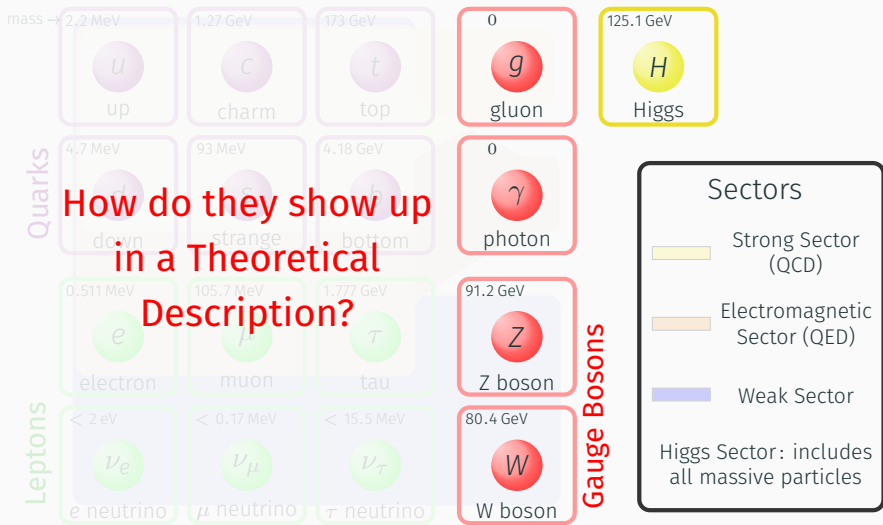
# Setting the Scene



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# Local Gauge Symmetry

## Electrodynamics

- Formulation using  $E$ - and  $B$ -Fields not suitable for a QFT
- Vector Potential  $A_\mu = (\varphi, A) \rightarrow$  covariant description

$$B = \nabla \times A$$

$$E = -\nabla\varphi - \partial_t A$$

- Mathematical Tool with redundant degrees of freedom

$$\underbrace{3}_{\# \text{ E-field comps.}} + \underbrace{3}_{\# \text{ B-field comps.}} - \underbrace{4}_{\# \text{ Maxwell eqs.}} = 2 \neq \underbrace{4}_{\# \text{ A-field comps.}}$$

- Invariant under Gauge Transformation ( $\Lambda(t, r) =$  arbitrary)

$$A \rightarrow A + \nabla\Lambda$$

$$\varphi \rightarrow \varphi - \partial_t\Lambda$$

- Quantization: Vector Potential  $A_\mu$  becomes Photon field
- Photons are the excitation quanta of the Photon field



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# Local Gauge Symmetry

- Mechanism is invertible:  
**Gauge Symmetry**  $\leftrightarrow$  **Equations of Motion**
- QCD and Electroweak interaction:  
different details, same principle
- A Gauge Symmetry allows to choose an  
internal coordinate system for the  
symmetry at every space-time point
- Observables need to be independent of  
this **human choice**
- All Gauge Bosons are massless<sup>1</sup>



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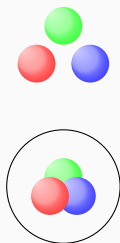


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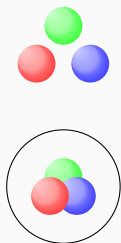


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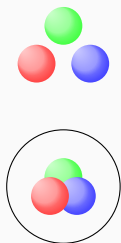


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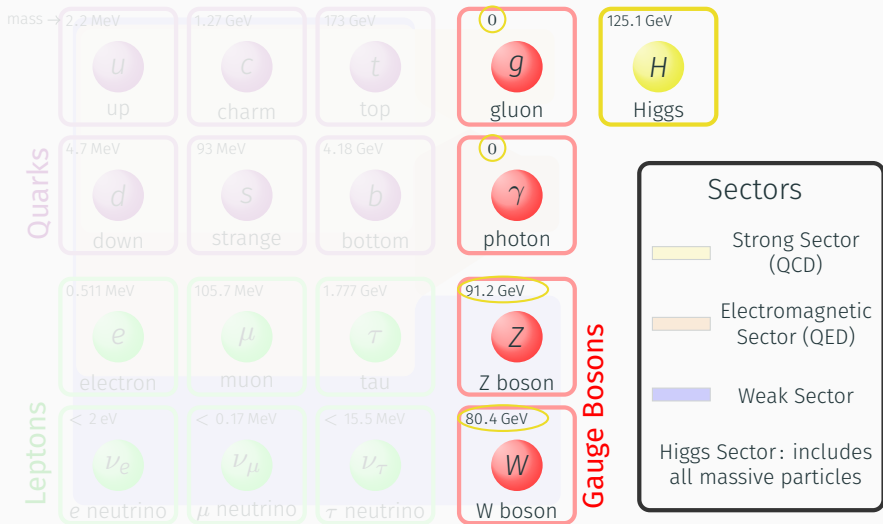
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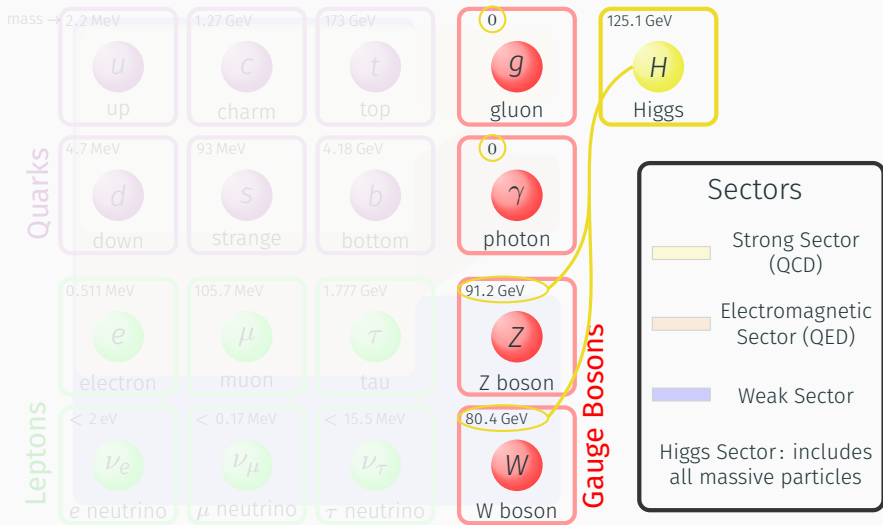
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# Massive Gauge Bosons





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- Need for Brout-Englert-Higgs-Mechanism
- Heuristic Explanation: Condensed Higgs Field slows down the Gauge Bosons  $\rightarrow$  gain mass
- Through Interaction  $\Rightarrow m_W \propto v^2$
- Theoretical Approach:
  - Higgs field  $\phi$  is split<sup>1</sup>:  $\phi(x) = v + h(x)$
  - Vacuum expectation value  $v$ : fixed in a specific gauge
  - Fluctuation Field  $h(x)$ : such that  $\langle h(x) \rangle = 0$

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- Quite subtle issue known since the 1970's
- $v$  actually carries a 'gauge index'
- Even worse: there exist gauges with  $v = 0 \Rightarrow$  Gauge Bosons (and Higgs itself) stay massless
- W/Z-Bosons are also gauge dependent  $\rightarrow$  **Cannot be Observable!**



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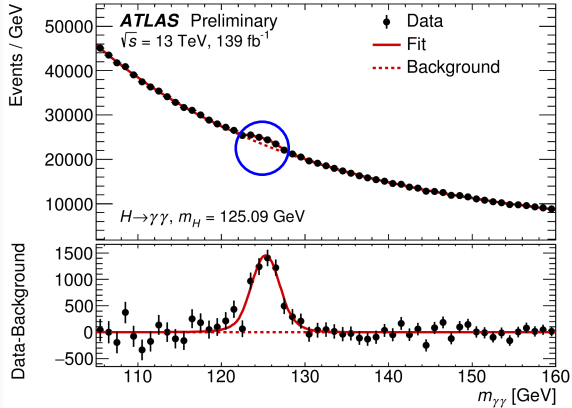
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Theory tells us that Gauge Bosons should **not** be observable  
Particles!

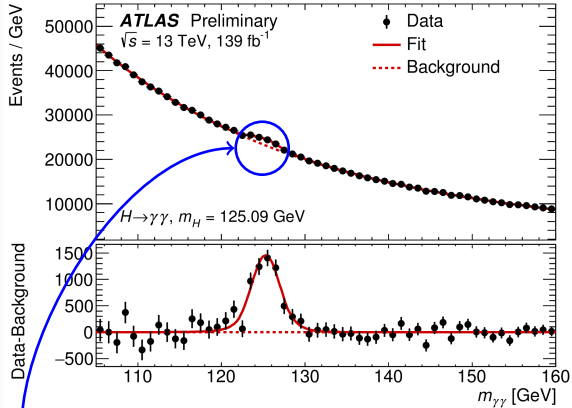
# Observation of Gauge Bosons

[From Report: ATLAS-CONF-2019-029]



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What did we actually observe?



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- Theory tells us: We need invariant states
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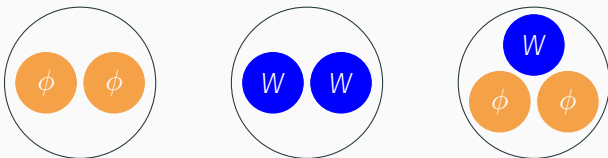
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  - Elementary Fields ( $\phi, W, \dots$ ) ✗
  - Gauge choice needs to be canceled  $\rightarrow$  more than one particle
  - Composite Particles ( $H, V, \dots$ ) ✓
- Observable particles: Higgs-Higgs, W-Ball, Higgs-Higgs-W etc.

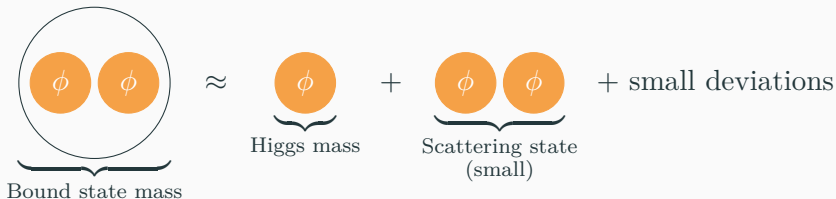


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- e.g.: The Mass spectrum of the Higgs-Boson

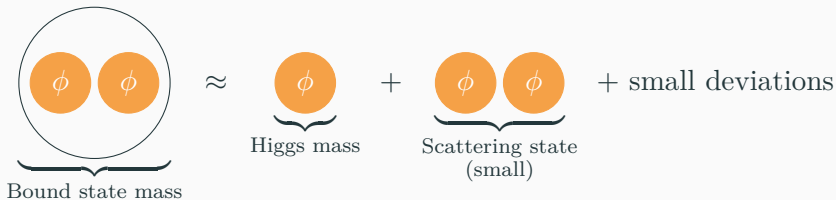


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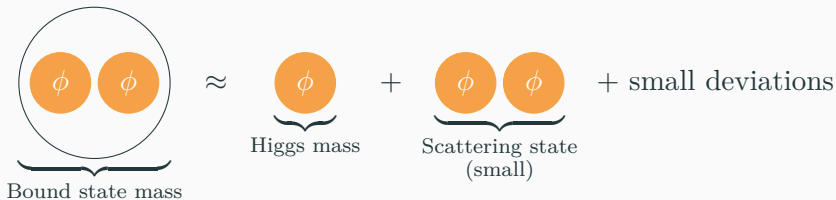
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- **Confirmed** on the Lattice



# Testing the theory

- Investigation of the Electroweak-Higgs Sector of the SM<sup>1</sup> using Gauge-invariant observables
- Lattice Simulations
  - Idea: Take Euclidean-Pathintegral ( $t \rightarrow i\tau$ )  
 $\int \mathcal{D}\Phi e^{-\int d^D x \mathcal{L}_E[\Phi(x)]}$  and make it discrete:  $\int \rightarrow \sum$
  - Calculate Observables as a sum over configurations created by MC-Simulation with weight  $e^{-\int d^D x \mathcal{L}_E[\Phi(x)]}$
  - Includes non-perturbative effects  $\rightarrow$  test for PT
- $H \rightarrow WW$ -decay
  - Phaseshift can be calculated from PT and will be measurable at HL-LHC  $\rightarrow$  direct test of PT and FMS
  - Scattering of Bound-States could mimic "new Physics"

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<sup>1</sup>Also full SM and BSM theories (including Quantum Gravity [A. Maas, arXiv: 1908.02140]) are investigated by colleagues

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  - Calculate Observables as a sum over configurations created by MC-Simulation with weight  $e^{-\int d^D x \mathcal{L}_E[\Phi(x)]}$
  - Includes non-perturbative effects  $\rightarrow$  test for PT
- $H \rightarrow WW$ -decay
  - Phaseshift can be calculated from PT and will be measurable at HL-LHC  $\rightarrow$  direct test of PT and FMS
  - Scattering of Bound-States could mimic "new Physics"

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<sup>1</sup>Also full SM and BSM theories (including Quantum Gravity [A. Maas, arXiv: 1908.02140]) are investigated by colleagues

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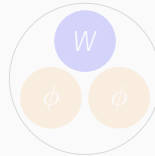
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Higgs



W-Ball

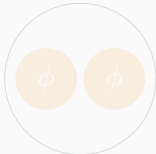


W/Z-Boson

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(Also for QCD and QED sector)



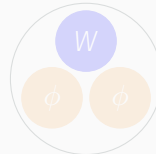
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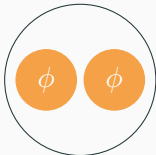


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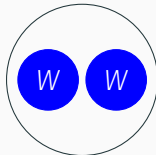
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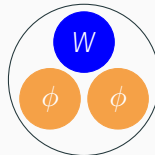
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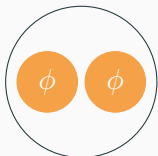


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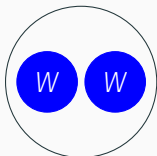
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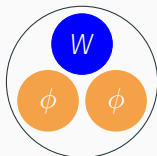
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# Questions?

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 @b\_riederer



**NAWI Graz**  
Natural Sciences





1. Formulate gauge-invariant operator and Correlator e.g.:

$$0^+ \text{ singlet: } H(x) = \left( \phi_i^\dagger \phi_i \right) (x)$$

$$\langle H(x)H(y) \rangle = \left\langle \left( \phi_i^\dagger \phi_i \right) (x) \left( \phi_j^\dagger \phi_j \right) (y) \right\rangle$$

2. Expand Higgs field in fixed Gauge  $\phi_i = vn_i + h_i$

$$\begin{aligned} \left\langle \left( \phi_i^\dagger \phi_i \right) (x) \left( \phi_j^\dagger \phi_j \right) (y) \right\rangle &= dv^4 + 4v^2 \left\langle \Re \left[ n_i^\dagger h_i \right]^\dagger (x) \Re \left[ n_j^\dagger h_j \right] (y) \right\rangle + \\ &+ 2v \left\{ \left\langle \left( h_i^\dagger h_i \right) (x) \Re \left[ n_j^\dagger h_j \right] (y) \right\rangle + (x \leftrightarrow y) \right\} + \left\langle \left( h_i^\dagger h_i \right) (x) \left( h_j^\dagger h_j \right) (y) \right\rangle \end{aligned}$$

3. Perform standard Perturbation theory

$$\begin{aligned} \left\langle \left( \phi_i^\dagger \phi_i \right) (x) \left( \phi_j^\dagger \phi_j \right) (y) \right\rangle &= d'v^4 + 4v^2 \left\langle \Re \left[ n_i^\dagger h_i \right]^\dagger (x) \Re \left[ n_j^\dagger h_j \right] (y) \right\rangle_{\text{tl}} + \\ &+ \left\langle \Re \left[ n_i^\dagger h_i \right]^\dagger (x) \Re \left[ n_j^\dagger h_j \right] (y) \right\rangle_{\text{tl}}^2 + \mathcal{O}(g^2, \lambda) \end{aligned}$$



# Lattice Formulation

## 1. Discretize the action on Euclidean $N^d$ -hypercubic

[I. Montvay and G. Münster, Quantum Fields on a Lattice (1994)]

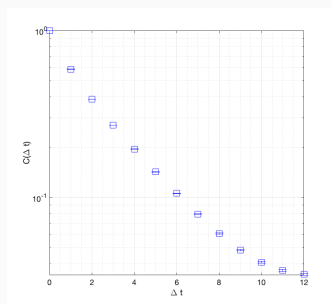
$$S_{cont.} = \int d^4x \left[ -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi - \lambda (\phi^\dagger \phi - f^2)^2 \right]$$

$$S_{latt.} = \sum_x \left[ \frac{\beta}{d_F} \sum_{\mu < \nu} \text{Re} \{ \text{Tr} (1 - U_{x,\mu\nu}) \} - \kappa \sum_{\pm\mu} \phi_x^\dagger U_{x,\mu}^R \phi_{x+\mu} + \phi_x^\dagger \phi_x + \gamma (\phi_x^\dagger \phi_x - 1)^2 \right]$$

## 2. Do Monte Carlo for Gauge-Links (Heatbath) and Higgs-Field (Metropolis)

# Spectroscopy

- Calculate Correlators  
$$C(\Delta t) = \langle \mathcal{O}(\Delta t) \mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0 | \mathcal{O} | n \rangle|^2 e^{-E_n \Delta t}$$
- Correlator contains information about **all** Operators with same quantum numbers
- For large times  $\Delta t \gg E_{n>0}^{-1} \Rightarrow C(\Delta t) \propto e^{-E_0 \Delta t}$



Typical Correlator

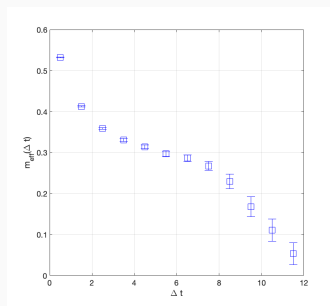
# Effective Energy Curves

- Calculate an effective Energy from Correlator

$$C(\Delta t) = \sum_n c_n e^{-E_n \Delta t}$$

$$E_{eff} \left( \Delta t + \frac{1}{2} \right) = \ln \left\{ \frac{C(\Delta t)}{C(\Delta t + 1)} \right\}$$

- If dominated by  $E_n$ :  $E_{eff} \approx E_n$
- Better: Take periodicity in  $\Delta t$  and excited states into account  
→ fit  $C(\Delta t)$  to  $\cosh$ -behaviour



Effective Mass

# Variational Analysis

- Calculate Correlator-Matrix

$$C_{ij}(\Delta t) = \langle \mathcal{O}_i(\Delta t) \mathcal{O}_j^\dagger(0) \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j^\dagger | 0 \rangle e^{-E_n \Delta t}$$

- Diagonalization disentangles physical states

$$\lambda_n(\Delta t) \propto e^{-E_n \Delta t} (1 + \mathcal{O}(e^{-\Delta E_n \Delta t}))$$

[C. Michael, NP B259 (1985) / M. Lüscher et al., NP B339 (1990)]

- Interpolators should have large overlap with Eigenstates  $|n\rangle$
- Eigenvectors give overlap of interpolators with the Eigenstates

- Infinite Volume Spectrum can be calculated from Spectrum of different finite Volumes

[M. Lüscher, CMP 105 (1986)]

- Needs to consider Bound-State nature
- Parametrization of Volume dependence ( $a$  and  $b$ : fit parameters)

$$m(V) = m_{inf} + ae^{-bV} \quad (1)$$

- Finite momentum states get additional polynomial contributions

# Lüscher Analysis II

- Lüscher method: Particles in a box always interact  $\rightarrow$  distortion of Energy levels

[M. Lüscher, NP B354 (1991) and NP B364 (1991)]

- Relation between Energy levels  $E_n$  and Phaseshift  $\delta_l$  for spinless particles in elastic region ( $2M < E_n < 4M$ )

$$\tan(\delta_l(q_n)) = \frac{\pi^{\frac{3}{2}} q_n}{Z_{00}^{\vec{0}}(1, q_n^2)} \quad \text{with } E_n = 2\sqrt{M^2 + \left(q_n \frac{2\pi}{L}\right)^2}$$

$$Z_{lm}^{\vec{d}}(r, q^2) = \sum_{\vec{x} \in P_{\vec{d}}} \frac{|\vec{x}|^l Y_{lm}(\vec{x})}{(\vec{x}^2 - q^2)^r} \quad \text{with } P_{\vec{d}} = \left\{ \left( \vec{u} + \frac{\vec{d}}{2} \right) \mid \vec{u} \in \mathbb{Z}^3 \right\}$$

- Different Volumes or moving Scattering states needed for reliable data  $\rightarrow$  further things need to be considered