Scattering Phases in the $H \to WW^*$ channel

Master's Thesis Defense

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Introduction & Motivation

The Standard Model of Particle Physics

- Most successful theory up to now
- Describes 3 of 4 fundamental interactions and everyday matter as a Quantum Field Theory
  - Electromagnetic interaction
  - Weak interaction
  - Strong interaction
- Plus the Higgs-Sector providing masses to the particles
- Not the final answer: Dark matter, $g_\mu - 2$ anomaly, $\ldots$
  $\rightarrow$ Need for Physics beyond the SM
- **But** before studying BSM physics we need to fully understand the SM: perturbative and non-perturbative
- Brout-Englert-Higgs effect introduces some ambiguities

Electroweak-Higgs-Sector

- $\mathrm{SU}(2)_W \times \mathrm{U}(1)_\gamma$-Yang-Mills-Higgs-Theory
- Only Weak interaction $\rightarrow$ no fermions & photon, $m_W = m_Z$

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - \lambda \left(\phi^\dagger \phi - f^2 \right)^2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu \qquad D_\mu = \partial_\mu - igW_\mu$$

- Gauge symmetry $G(x) \in \mathrm{SU}(2)_W$:
  $W_{\mu\nu}(x) \rightarrow G(x) W_{\mu\nu}(x) G(x)^{-1}$ & $\phi(x) \rightarrow G(x)\phi(x)$
- Additional global $\mathrm{SU}(2)_c$ symmetry of the Higgs field

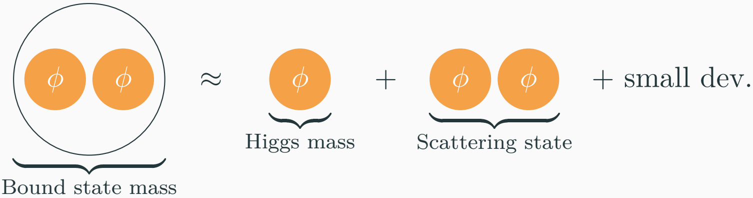
Gauge-Fixed Perturbation Theory

- PT around minimum of the potential $\rightarrow$ BEH-effect: Shifting the Higgs-field $\phi_i(x) = vn_i + h_i(x)$
- vn_i such that $|\langle\phi\rangle| = |v| = f$ $\rightarrow$ needs to fix the gauge
- In PT elementary fields are used to describe EW-phenomenology
- **But** they are gauge-dependent and thus unphysical (human choice)
- Though using a *wrong* premise PT agrees strikingly well with experiment
- Compare to intrinsically gauge-invariant objects (e.g. bound states) as the physical objects

FMS-Mechanism

[Fröhlich et al., PL **B97** (1980) and NP **B190** (1981)]

- Compatible with standard Perturbation Theory?
  $\Rightarrow$ Yes! Using FMS-Mechanism
- Maps Gauge SU(2) on Global (custodial) SU(2) $J^P \to J_c^P$
- e.g.: The mass spectrum of the Higgs-Boson (0_1^+)



- Lattice spectroscopy $\Rightarrow$ Mass spectrum stays the same

  [Maas, MPL **A28** (2013) / Maas and Mufti, **JHEP** (2014)]
- What about other quantities?

Research Goals

Study scattering properties of different physical Higgs-Bosons

1. SM Higgs $M_H \approx 125\,\text{GeV}$
   - Comparison of results with gauge-fixed PT
   - HL-LHC will be capable of measuring phase shift
     $\rightarrow$ direct comparison to measurement
2. Higgs with $M_H \gtrsim 2M_W \approx 160\,\text{GeV}$
   - Previous Lattice studies suggest no additional states
     $\rightarrow$ Lüscher analysis will clarify this issue
   - Possible implications for BSM Theories with additional Higgs-Bosons
     $\rightarrow$ could be hard to detect

Lattice Methods

Lattice Quantities vs. Infinite Volume Quantities

- Lattice quantities (e.g. mass) $\not\leftrightarrow$ Infinite volume quantities
- Two main sources
  1. Discretization artifacts $\xrightarrow{?}$ avoided by small lattice spacing
  2. Particle in a box $\rightarrow$ restricted interaction volume and wrap around effects
- For bound state energy and masses:

$$M_B(V) \approx m_{inf} + a_1 e^{-a_2 V}$$

- **But** particles in a box always interact $\rightarrow$ volume-dependent distortion of energy levels

Lattice energy spectrum $E_n(V) \leftrightarrow$ Continuum phase shift $\delta_l(E)$

Lüscher Analysis

[M. Lüscher, NP **B354** (1991) and **B364** (1991)]

- We consider 0_1^+ channel $\rightarrow$ only formula for spinless particle is needed
- Relation between energy levels E_n and phase shift δ_l

$$\tan\left(\delta_0(q_n)\right) = \frac{\pi^{\frac{3}{2}} q_n}{Z_{00}^{\vec{0}}\left(1, q_n^2\right)} \quad \text{with } E_n = 2\sqrt{m_{inf}^2 + \left(q_n \frac{2\pi}{N}\right)^2}$$

$$Z_{lm}^{\vec{d}}(r, q^2) = \sum_{\vec{x} \in P_{\vec{d}}} \frac{\left|\vec{x}\right|^l Y_{lm}(\vec{x})}{\left(\vec{x}^2 - q^2\right)^r} \quad \text{with } P_{\vec{d}} = \left\{\left(\vec{u} + \frac{\vec{d}}{2}\right) \,\middle|\, \vec{u} \in \mathbb{Z}^3\right\}$$

- Holds in threshold region: $2m_{inf}^{1_3^-} < E_n < \min\left(2m_{inf}^{0_1^+}, 4m_{inf}^{1_3^-}\right)$

Spectroscopy

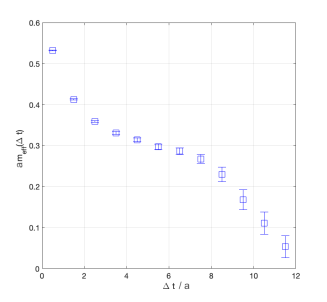
- Calculate correlators

$$C(\Delta t) = \left\langle \mathcal{O}^\dagger(\Delta t)\mathcal{O}(0) \right\rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n \Delta t}$$

- Sum over **all** states in the quantum number channel

$$E_{eff}\left(\Delta t + \frac{1}{2}\right) = \ln\left\{\frac{C(\Delta t)}{C(\Delta t + 1)}\right\}$$

- If dominated by E_n: $E_{eff} \approx E_n$
- Take periodicity in Δt and excited states into account $\rightarrow C(\Delta t) = \sum_n c_n \cosh\left(E_n \Delta t\right)$



Effective Mass

How to obtain information about further energy levels?

Variational Analysis

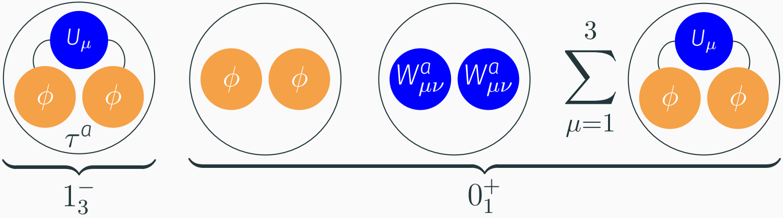
[M. Lüscher et al., NP **B339** (1990) / C. Michael, NP **B259** (1985)]

$$C_{ij}(\Delta t) = \left\langle \mathcal{O}_i^\dagger(\Delta t)\mathcal{O}_j(0) \right\rangle$$

Diagonalization disentangles physical states

$$\lambda_n(\Delta t) \propto e^{-E_n \Delta t}\left(1 + \mathcal{O}\left(e^{-\Delta E_n \Delta t}\right)\right)$$

Basis-Interpolators in the 1_3^- and 0_1^+ (physical Higgs) channel



$$C_{ij}^{1_3^-} = \left\langle \mathcal{O}_{i\,\mu}^{1_3^-\,a} \; \mathcal{O}_{j\,\mu}^{1_3^-\,a} \right\rangle \quad \text{for} \quad i = j = 1, \ldots, 10\,(2 \times 5 \text{ smear})$$

$$C_{ij}^{0_1^+} = \left\langle \begin{matrix} \mathcal{O}_i^{0_1^+} & \mathcal{O}_j^{0_1^+} \\ \mathcal{O}_i^{1_3^-}\mathcal{O}_i^{1_3^-} & \mathcal{O}_j^{1_3^-}\mathcal{O}_j^{1_3^-} \end{matrix} \right\rangle \quad \text{for} \quad |\vec{\mathbf{p}}|^2 = 0$$

$$\text{for} \quad i = j = 1, \ldots, 180\left((4 + 2) \times 3\,|\vec{\mathbf{p}}|^2 \times 5 \text{ smear} \times 2 \text{ squared}\right)$$

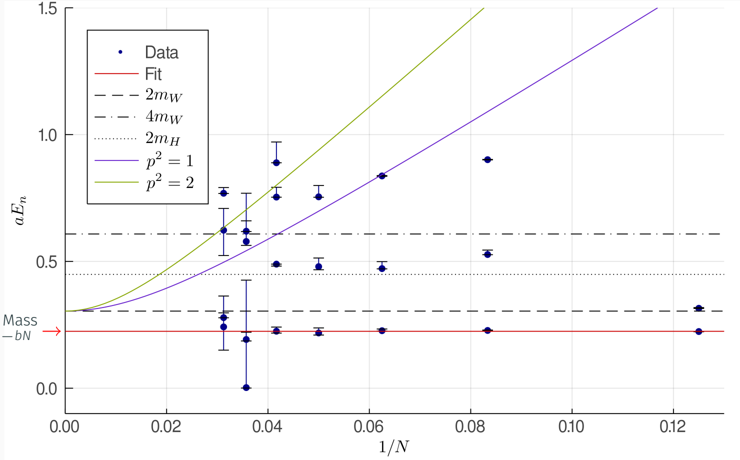
$$C_{ij}^{1_3^-} = \left\langle \mathcal{O}_{i\,\mu}^{1_3^-\,a} \; \mathcal{O}_{j\,\mu}^{1_3^-\,a} \right\rangle \quad \text{for} \quad i = j = 1, \ldots, 10\,(2 \times 5 \text{ smear})$$

$$\mathcal{O}_i^{0_1^+} \xleftarrow{-\vec{\mathbf{p}}} \bullet \xrightarrow{\vec{\mathbf{p}}} \mathcal{O}_i^{0_1^+}$$

$$C_{ij}^{0_1^+} = \left\langle \begin{matrix} \mathcal{O}_i^{0_1^+}\,\mathcal{O}_i^{0_1^+} & \mathcal{O}_j^{0_1^+}\,\mathcal{O}_j^{0_1^+} \\ \mathcal{O}_i^{1_3^-}\,\mathcal{O}_i^{1_3^-} & \mathcal{O}_j^{1_3^-}\,\mathcal{O}_j^{1_3^-} \end{matrix} \right\rangle \quad \text{for} \quad |\vec{\mathbf{p}}|^2 \neq 0$$

$$\text{for} \quad i = j = 1, \ldots, 180\left((4 + 2) \times 3\,|\vec{\mathbf{p}}|^2 \times 5 \text{ smear} \times 2 \text{ squared}\right)$$

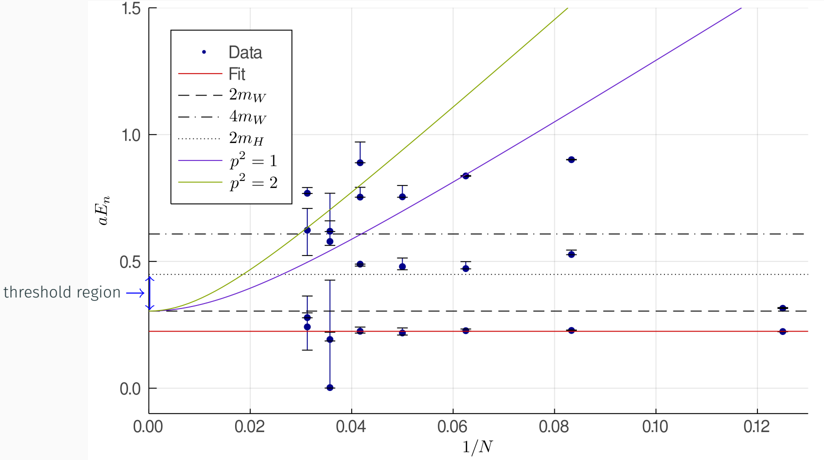
Lattice Energy Spectrum



cont. Higgs Mass
$\approx m + ae^{-bN}$

Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size N.

Lattice Energy Spectrum



Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size N.

Results

Parameter Sets

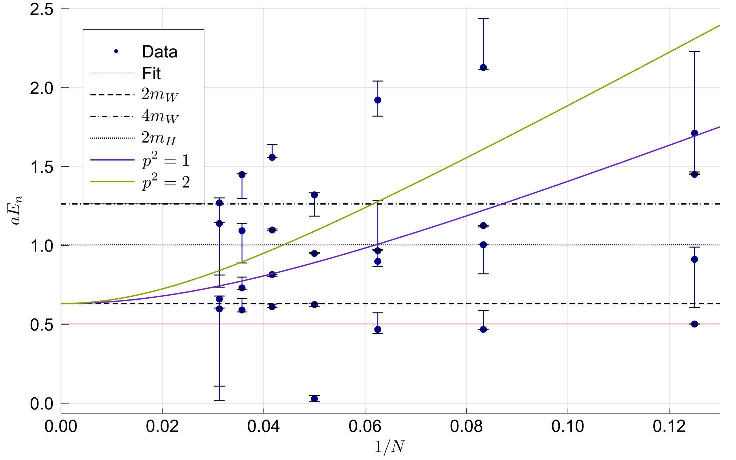
Set Name	$m_H \setminus$ GeV	$a^{-1} \setminus$ GeV	$\alpha_{200\text{ GeV}}$	N
Example	118 ± 1	528	0.119	8:4:32
Energy 1	141 ± 1	705	0.153	8:8:32
Energy 2	91 ± 32	532	0.664	8:8:32
Energy 3	107 ± 1	470	0.605	8:4:32
Energy 4	110 ± 1	454	0.128	8:8:32
Energy 5	116 ± 3	404	0.157	8:8:32
Energy 6	95 ± 7	329	0.492	8:8:32
Energy 7	95 ± 1	281	0.718	8:8:32
Lüscher 1	91 ± 4	328	0.492	8:8:32 +
Lüscher 2	103 ± 3	328	0.448	8:8:32 +
Lüscher 3	118 ± 2	317	0.219	8:4:32 +
Lüscher 4	130 ± 1	259	0.211	8:4:32 +

m_H is the mass of the lightest state in the 0_1^+ channel.

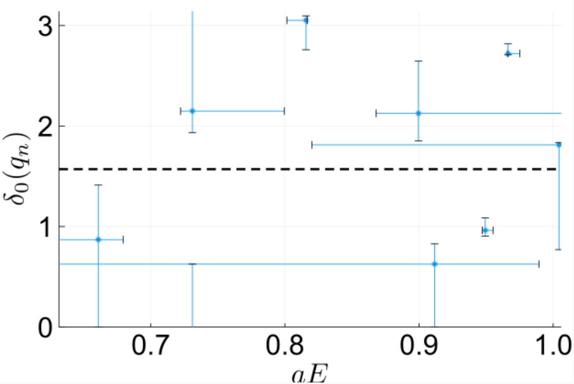
Scale a^{-1} is set by the mass of the lightest state in the 1_3^- channel: $m_W \stackrel{!}{=} 80.375$ GeV.

N are the used lattice sizes in the form $N_l : N_s : N_u = \left\{ N = N_l + nN_s \;\middle|\; n \in \mathbb{N}^0 \wedge N \leq N_u \right\}$.

Energy Spectrum



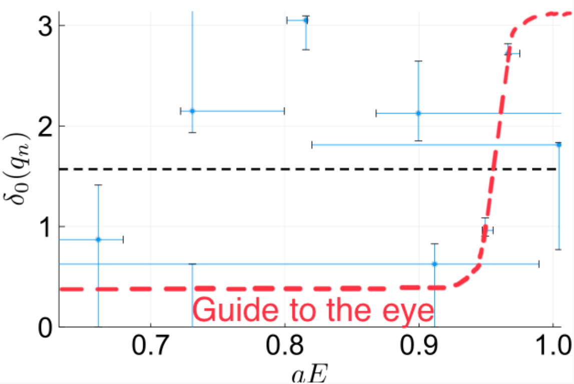
Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size N for *Lüscher4*.



(a) *Lüscher4*

Phase shifts in rad for the given parameter sets in the 0_1^+ channel over the energy in lattice units.

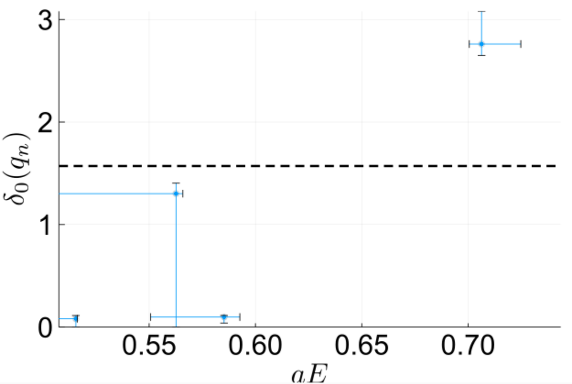
Phase Shifts



(a) *Lüscher4*

Phase shifts in rad for the given parameter sets in the 0_1^+ channel over the energy in lattice units.

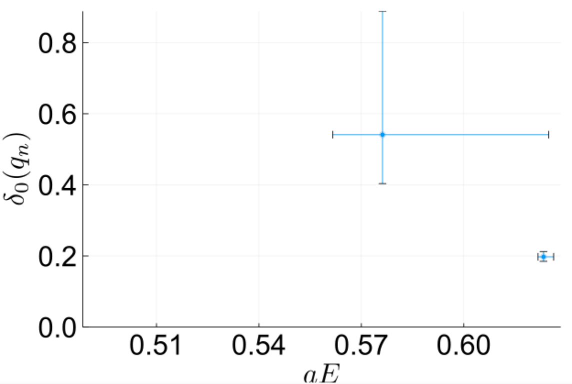
Phase Shifts



(b) *Lüscher3*

Phase shifts in rad for the given parameter sets in the 0_1^+ channel over the energy in lattice units.

Phase Shifts



(c) *Lüscher2*

Phase shifts in rad for the given parameter sets in the 0_1^+ channel over the energy in lattice units.

Conclusion & Outlook

1. Particle spectrum below threshold as expected from gauge-fixed PT
   $\rightarrow$ FMS mechanism still valid
   - Additional parameter sets
2. Resonance in threshold region seems possible
   $\rightarrow$ Additional observable particles?
   - Improve statistics, operator-basis and error estimation
   - Additional parameter sets along line of constant physics
   - Calculate decay-widths and couplings and compare to PT

+ Higgs mass always in region 90 GeV–150 GeV
  $\rightarrow$ Hierarchy problem?
  - Additional parameter sets

Thank you!

Gauge-invariant Perturbation Theory

[Fröhlich et al., PL **B97** (1980) and NP **B190** (1981) / A. Maas, PPNP **106** (2019), arXiv: 1712.04721]

1. Formulate gauge-invariant operator and correlator e.g.:

$$0^+ \text{ singlet:} \quad H(x) = \left(\phi_i^\dagger \phi_i\right)(x)$$

$$\langle H(x)H(y)\rangle = \left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle$$

2. Expand Higgs field in fixed gauge $\phi_i = vn_i + h_i$

$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = dv^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle +$$
$$+2v\left\{\left\langle \left(h_i^\dagger h_i\right)(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle + (x \leftrightarrow y)\right\} + \left\langle \left(h_i^\dagger h_i\right)(x)\left(h_j^\dagger h_j\right)(y)\right\rangle$$

3. Perform standard Perturbation Theory

$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = d'v^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}} +$$
$$+ \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}}^2 + \mathcal{O}(g^2, \lambda)$$

Lattice Action

SU(2)-Gauge-Higgs-Theory on the Lattice

[I. Montvay and G. Münster, Quantum Fields on a Lattice (1994)]

$$
\begin{aligned}
S = \sum_{x \in \Lambda} \Bigg[\beta \left(1 - \frac{1}{2} \sum_{\mu<\nu} \mathrm{Re}\{\mathrm{Tr}\,\{U_{\mu\nu}(x)\}\} \right) + \\
+ \gamma \left(\phi^\dagger(x)\phi(x) - 1 \right)^2 + \phi^\dagger(x)\phi(x) - \\
- \kappa \sum_{\pm\mu} \phi^\dagger(x) U_\mu(x) \phi(x + e_\mu) \Bigg]
\end{aligned}
$$

Lattice parameters: β, γ and κ

Continuum parameters: g, λ and f

Operator Basis

$$\mathcal{O}_\rho(x) = \phi^\dagger(x)\phi(x)$$

$$\mathcal{O}_W(x) = \mathrm{Tr}\left\{U_\mu(x)U_\nu(x+e_\mu)U_\mu^\dagger(x+e_\nu)U_\nu^\dagger(x)\right\}$$

$$\mathcal{O}_{0^+}(x) = \sum_{\mu=1}^{3}\mathrm{Tr}\left\{X^\dagger(x)U_\mu(x)X(x+e_\mu)\right\}$$

$$\mathcal{O}_{0_n^+}(x) = \sum_{\mu=1}^{3}\mathrm{Tr}\left\{\frac{X^\dagger(x)}{\sqrt{\det(X(x))}}U_\mu(x)\frac{X(x+e_\mu)}{\sqrt{\det(X(x+e_\mu))}}\right\}$$

$$\mathcal{O}^a_{1^-\mu}(x) = \mathrm{Tr}\left\{\tau^a X^\dagger(x)U_\mu(x)X(x+e_\mu)\right\}$$

$$\mathcal{O}^a_{1_n^-\mu}(x) = \mathrm{Tr}\left\{\tau^a\frac{X^\dagger(x)}{\sqrt{\det(X(x))}}U_\mu(x)\frac{X(x+e_\mu)}{\sqrt{\det(X(x+e_\mu))}}\right\}$$

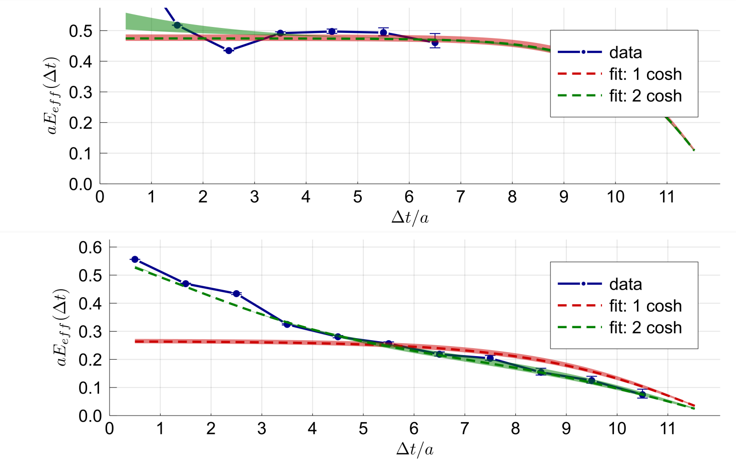
$$\mathcal{O}^{0_1^+}_{1-30} = \left\{\begin{array}{c} \mathcal{O}_W^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_\rho^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{0^+}^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{0_n^+}^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{1^-\mu}^{(0-4)a}(-\vec{\mathbf{p}})\,\mathcal{O}_{1^-\mu}^{(0-4)a}(\vec{\mathbf{p}}) \\ \mathcal{O}_{1_n^-\mu}^{(0-4)a}(-\vec{\mathbf{p}})\,\mathcal{O}_{1_n^-\mu}^{(0-4)a}(\vec{\mathbf{p}}) \end{array}\right\} \text{ such that } |\vec{\mathbf{p}}|^2 = 0$$

$$\mathcal{O}^{0_1^+}_{30-90} = \left\{\begin{array}{c} \mathcal{O}_W^{(0-4)}(-\vec{\mathbf{p}})\,\mathcal{O}_W^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_\rho^{(0-4)}(-\vec{\mathbf{p}})\,\mathcal{O}_\rho^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{0^+}^{(0-4)}(-\vec{\mathbf{p}})\,\mathcal{O}_{0^+}^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{0_n^+}^{(0-4)}(-\vec{\mathbf{p}})\,\mathcal{O}_{0_n^+}^{(0-4)}(\vec{\mathbf{p}}) \\ \mathcal{O}_{1^-\mu}^{(0-4)a}(-\vec{\mathbf{p}})\,\mathcal{O}_{1^-\mu}^{(0-4)a}(\vec{\mathbf{p}}) \\ \mathcal{O}_{1_n^-\mu}^{(0-4)a}(-\vec{\mathbf{p}})\,\mathcal{O}_{1_n^-\mu}^{(0-4)a}(\vec{\mathbf{p}}) \end{array}\right\} \text{ such that } |\vec{\mathbf{p}}|^2 = 1, 2$$

$$\mathcal{O}^{0_1^+}_{91-180} = \left(\mathcal{O}^{0_1^+}_{1-90}\right)^2$$

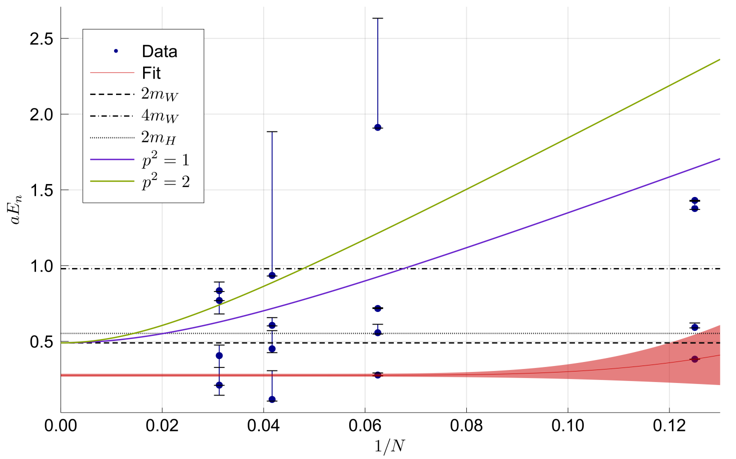
$$\mathcal{O}^{1_3^-\,a}_{1-10\mu} = \left\{\begin{array}{c} \mathcal{O}_{1^-\mu}^{(0-4)a}(\vec{\mathbf{0}}) \\ \mathcal{O}_{1_n^-\mu}^{(0-4)a}(\vec{\mathbf{0}}) \end{array}\right.$$

Example Fit



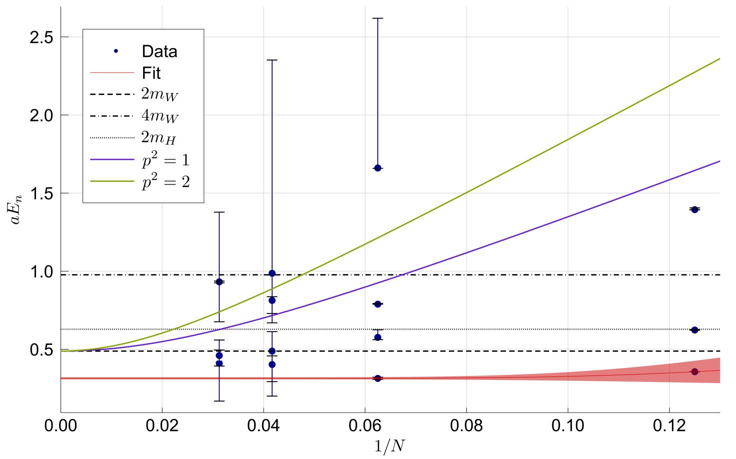
Excited (upper) and ground state (lower) fits for $N = 24$ in the 0_1^+ channel. Dashed lines are fits with error bounds of the fit indicated by ribbons. See text for details.

Energy Spectrum: *Lüscher1*



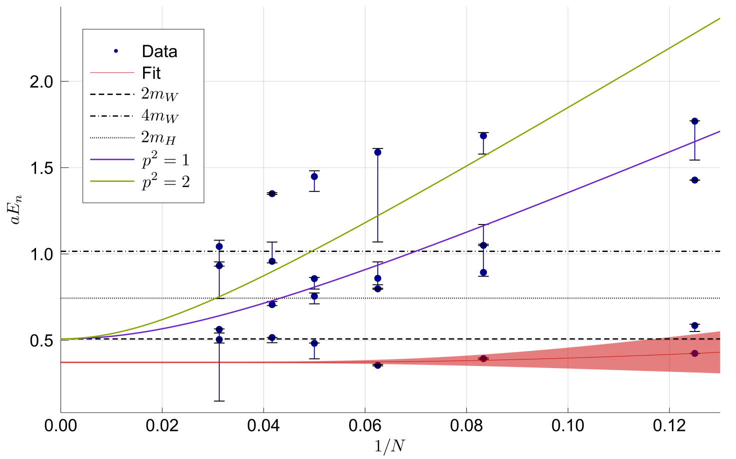
Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size *N* for *Lüscher1*.

Energy Spectrum: *Lüscher2*



Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size N for *Lüscher2*.

Energy Spectrum: *Lüscher3*



Energy spectrum E_n in the 0_1^+ channel (physical Higgs) as a function of the Lattice size N for *Lüscher3*.

Statistics

N	8	12	16	20	24	28	32
Example	95616	85558	81270	86760	172104	79638	12118
Energy 1	95616	-	81270	-	14325	-	11027
Energy 2	122160	-	119168	-	144053	-	17595
Energy 3	107488	121344	106848	91200	220837	87570	14425
Energy 4	123152	-	119840	-	43375	-	15798
Energy 5	92364	-	82390	-	131803	-	17339
Energy 6	81360	-	74140	-	98156	-	16745
Energy 7	162305	-	159400	-	93543	-	29331
Lüscher 1	137265	-	127990	-	121872	-	19665
Lüscher 2	208274	-	149740	-	107923	-	18343
Lüscher 3	291968	86460	259489	130640	199023	-	19772
Lüscher 4	464912	147122	238648	167890	232215	109667	29749

Number of used configurations for the Parameter Sets