Talk

Perturbation Theory in Continuum

Φ^4-Theory

Held at the *Lattice Seminar WS 2018/19*

by Bernd Riederer, BSc

at 31.10.2018

Contents

1 Introduction

In this talk I will give a short reminder of (QFT) perturbation theory in the continuum. Therefore I will use as a toy model the so called ϕ^4-theory to illustrate the most important steps. The whole talk follows closely [1].

To start with we need the Langrangian of the theory.

1.1 Lagrangian and Hamiltonian of the system

The Lagrangian $\mathcal{L}$ of the ϕ^4 theory is given by the one of a real scalar field plus a potential term proportional to ϕ^4.

$$\mathcal{L}(\partial_\mu\phi, \phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

Sometimes the factor $1/4!$ is also absorbed into the coupling constant λ. The equations of motion can be derived from the Euler-Lagrange equations and are given by:

$$\left(\partial_\mu\partial^\mu + m^2\right)\phi = -\frac{\lambda}{3!}\phi^3 \tag{2}$$

For $\lambda = 0$ the solution is given by the free Klein-Gordon field. However for $\lambda \neq 0$ this equation can not be solved by a Fourier analysis anymore.

Therefore we now want to divide the Lagrangian into a free part $\mathcal{L}_0$, which can be solved analytically in closed form, and an interaction part $\mathcal{L}_{int}$ which depends on the coupling constant.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} = \mathcal{L}_0 - \frac{\lambda}{4!}\phi^4 \tag{3}$$

Since $\mathcal{L}_{int}$ does not depend on ∂_μ the equal time commutation relations for the canonical conjugate field ϕ and Π are unaffected and the (canonical) quantization process keeps the same. [1] From this also follows that the Hamiltonian H can be split into a free part H_0 and an interaction part H_{int} which will be usefull later on.

$$H = \int d^3x\mathcal{H} = \int d^3x\mathcal{H}_0 + \int d^3x\mathcal{H}_{int} \tag{4a}$$

$$H_{int} = \int d^3x\mathcal{H}_{int} = -\int d^3x\mathcal{L}_{int} = \frac{\lambda}{4!}\int d^3x\phi^4 \tag{4b}$$

[1] For the whole derivation of the following procedure I will assume canonical qunatization. The pathintegral quantization will be used for the perturbation theory on the lattice in the next talk. However restricting to canonical quantization has no influence on the results (as it should be).

Now we have finished to define the (already quantized) theory (see canonical quantization of the Klein-Gordon field) and started already to split it into an interacting part and free part for which we already know the solution i.e. the Klein-Gordon filed equations. So the idea is to assume that λ is small and therefore can be treated as a small perturbation to the free system.

2 Perturbation expansion

The aim of this section is to develop a formalism, that allows us to express physical quantities (i.e. Observables) of the full field in terms of the free field where we assume that the interaction is given by small perturbation defined by the coupling constant λ. For now we will stick to calculation of so called *Two point Correlator functions.*

2.1 2-point Correlator

A two point correlator is given by:

$$\langle \Omega \,|\, T\left\{\phi(x)\phi(y)\right\} \,|\, \Omega\rangle \tag{5}$$

where T is the time-ordering symbol (only inserted for later convenience) and $|\Omega\rangle$ denotes the ground state of full theory. Note however that $|\Omega\rangle \neq |0\rangle$ the ground state of the free theory. This quantity can be physically interpreted as the amplitude for a propagation of a particle from x to y. In the free theory this propagation is described by the Feynman propagator:

$$\langle 0 \,|\, T\left\{\phi(x)\phi(y)\right\} \,|\, 0\rangle_{free} = D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon} \tag{6}$$

The first step in finding the Correlator of the interacting theory will be to express $\phi(x)$ of the full theory in terms of $\phi(x)$ in the free theory. Therefore we need a short excursion to the different pictures in quantum mechanics.

2.2 Pictures in QFT

In quantum mechanics there a three (equivalent) pictures for describing systems: the *Heisenberg*-picture, the *Schrödinger*-picture and the *Interaction*-picture.

The difference between the Heisenberg and the Schrödinger picture differs by a change of basis with respect to time-dependency. The standard operator type in

QFT is a Heisenberg operator. Therefore Heisenberg operators won't get an index here.

Heisenberg	Time dependent operators $\phi(x)$
Schrödinger	Time independent operators $\phi_S(\vec{x}) := \phi_S(t_0, \vec{x})$

There is a connection between Heisenberg and Schrödinger picture via the time evolution operator:

$$\phi(x) = e^{iH(t-t_0)}\phi_S(\vec{x})e^{-iH(t-t_0)} \tag{7}$$

Now we define a new picture, the *Interaction* picture. It is a mixture of both and defined as follows:

$$\phi(x)\Big|_{\lambda=0} = e^{iH(t-t_0)}\phi_S(\vec{x})e^{-iH(t-t_0)}\Big|_{\lambda=0} = e^{iH_0(t-t_0)}\phi_S(\vec{x})e^{-iH_0(t-t_0)} =: \phi_I(x) \tag{8}$$

The operator in the interaction picture is now given by the time evolution of the free (Schrödinger operator) field with respect to the free Hamiltonian. However we still have to express the interaction picture in terms of the free field in the Heisenberg picture since this are our solutions from the discussion of the Klein-Gordon field. So the final result of the Operator for the interacting field in terms of interaction picture operators (operator of the free field) is given as:

$$\phi(x) = e^{iH(t-t_0)}\underbrace{e^{-iH_0(t-t_0)}\phi_I(x)e^{iH_0(t-t_0)}}_{\phi_S(\vec{x})}e^{-iH(t-t_0)} =: U^\dagger(t,t_0)\phi_I(x)U(t,t_0) \tag{9}$$

with $U(t,t_0) = e^{iH_0(t-t_0)}e^{-iH(t-t_0)}$ the time evolution operator of the interaction picture.

2.3 Time-evolution operator

We now want to evaluate this expression further. Since H and H_0 are operators which **do not** (neccessarily) commute (i.e. $[H, H_0] \neq 0$) the Baker-Campbell-Hausdorff formula to express U in terms of H_{int} would lead to a very complicated expression. Therefore we need to find another solution. However it can be seen that $U(t,t_0)$ is the unique solution to the time-dependent Schrödinger equation with initial condition $U(t_0, t_0) = 1$.

$$\begin{aligned} i\frac{\partial}{\partial t}U(t,t_0) &= i\frac{\partial}{\partial t}e^{iH_0(t-t_0)}e^{-iH(t-t_0)} = \\ &= ie^{iH_0(t-t_0)}[iH_0 - iH]e^{-iH(t-t_0)} = \\ &= e^{iH_0(t-t_0)}\underbrace{[H - H_0]}_{H_{int}}e^{-iH(t-t_0)} = \\ &= \underbrace{e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)}}_{H_I}e^{iH_0(t-t_0)}e^{-iH(t-t_0)} = \\ &= H_I U(t,t_0) \end{aligned} \tag{10}$$

with $H_I = e^{iH_0(t-t_0)}H_{int}e^{-iH_0(t-t_0)} = \frac{\lambda}{4!}\int d^3x\phi_I(x)^4$. So $U(t,t_0)$ is the unique solution of a Schrödinger equation, which only depends on the free fields and therefore $U(t,t_0)$ has to be of the form $U(t,t_0) \sim exp(-iH_It)$. I claim now that the solution of $U(t,t_0)$ is given by a power series in λ which looks like:

$$\begin{aligned} U(t,t_0) = 1 + (-i)\int_{t_0}^{t} dt_1 H_I(t_1) + (-i)^2\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1)H_I(t_2) + \\ + (-i)^3\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 H_I(t_1)H_I(t_2)H_I(t_3) + \ldots \end{aligned} \tag{11}$$

This can easily be shown by differentiating with respect to t. Each term in the sum gives the previous one times $-iH_I(t)$ and also the initial condition is obviously fullfilled. Since the terms of $H_I(t)$ are in time order for all integrals it can be easily shown that the following identity holds (graphical proof):

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1)H_I(t_2) = \frac{1}{2}\int_{t_0}^{t} dt_1 dt_2 T\left\{H_I(t_1)H_I(t_2)\right\} \tag{12}$$

and for terms of order n the factor in front becomes $1/n!$. Therefore the power series can be rewritten.

$$\begin{aligned} U(t,t_0) &= 1 + (-i)\int_{t_0}^{t} dt_1 T\left\{H_I(t_1)\right\} + \frac{(-i)^2}{2!}\int_{t_0}^{t} dt_1 dt_2 T\left\{H_I(t_1)H_I(t_2)\right\} + \\ &+ \frac{(-i)^3}{3!}\int_{t_0}^{t} dt_1 dt_2 dt_3 T\left\{H_I(t_1)H_I(t_2)H_I(t_3)\right\} + \cdots = \\ &:= T\left\{exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right\} = \\ &= T\left\{exp\left(\frac{-i\lambda}{4!}\int_{t_0}^{t} dt' \int d^3x \phi_I(t', \vec{x})^4\right)\right\} \end{aligned} \tag{13}$$

where the time-order of a function has to be understood as the time ordering of each term of the taylor series for the function. Now we have arrived at the point where we can rewrite the interacting fields completely in terms of the free fields $\phi_I(x)$. Also note that this expression by now is an **exact** reformulation of the problem. If one could calculate all (infinite) terms of this power series the result should be correct for any theory. However in perturbation theory we assume that the coupling is small and therefore higher order terms can be neglected. Although it has to be mentioned that it is **not** assured that this series has to converge at all. The next thing we need for the correlator is the ground state of the full theory given by $|\Omega\rangle$.

2.4 Ground state of the full theory $|\Omega\rangle$

To find a representation of the ground state we start with the time evolution of the ground state for the free theory.

$$e^{-iHt}|0\rangle = \sum_n e^{-iE_n t}|n\rangle\langle n\,|\,0\rangle \tag{14}$$

where $H|n\rangle = E_n|n\rangle$. Now we have to assume that $\langle\Omega\,|\,0\rangle \neq 0$ which has to hold because otherwise H_I would not be a small perturbation at all. We now can extract the groundstate of the full theory from the sum above where we define $H|\Omega\rangle = E_0|\Omega\rangle$ (this just defines $|\Omega\rangle$ as the ground state of H) with $E_0 < E_n \forall n$.

$$e^{-iHt}|0\rangle = e^{-iE_0 t}|\Omega\rangle\langle\Omega\,|\,0\rangle + \sum_{n\neq 0} e^{-iE_n t}|n\rangle\langle n\,|\,0\rangle \tag{15}$$

to get rid off the higher excitations $n > 0$ we can take the limit by sending t to ∞ in a slightly imaginary direction: $t \to \infty(1 - i\varepsilon) := \infty'$. Then the ground energy term dies slowest. Now we got an expression for the ground state. Also I will replace t by $t + t_0$ which in the limit has no effect. The full expression for the ground state now reads as:

$$\begin{aligned}|\Omega\rangle &= \lim_{t\to\infty'}\left(e^{-iE_0(t+t_0)}\langle\Omega\,|\,0\rangle\right)^{-1} e^{-iH(t+t_0)}\underbrace{e^{+iH_0(t+t_0)}}_{H_0|0\rangle=0\to 1}|0\rangle = \\ &= \lim_{t\to\infty'}\left(e^{-iE_0(t_0-(-t))}\langle\Omega\,|\,0\rangle\right)^{-1} U(t_0,-t)\,|0\rangle\end{aligned} \tag{16}$$

This implies that we can get the ground state of the full theory by evolving the free field with the operator U from $-t$ to t_0. For the Bra-state a similar relation can be obtained:

$$\langle\Omega| = \lim_{t\to\infty'} \langle 0|\, U(t,t_0)\left(e^{-iE_0(t-t_0)}\,\langle 0\,|\,\Omega\rangle\right)^{-1} \tag{17}$$

Now we have got all the pieces together for the full correlator.

2.5 Full Correlator

For simplicity I will assume that $x^0 > y^0 > t_0$ (neglect time ordering operator). Putting everything together yields:

$$\begin{aligned}
\langle\Omega\,|\,\phi(x)\phi(y)\,|\,\Omega\rangle = &\lim_{t\to\infty'} \langle 0|\, U(t,t_0)\left(e^{-iE_0(t-t_0)}\,\langle 0\,|\,\Omega\rangle\right)^{-1}\times\\
&\times U^\dagger(x^0,t_0)\phi_I(x)U(x^0,t_0)\times\\
&\times U^\dagger(y^0,t_0)\phi_I(y)U(y^0,t_0)\times\\
&\times\left(e^{-iE_0(t_0-(-t))}\,\langle\Omega\,|\,0\rangle\right)^{-1}U(t_0,-t)\,|0\rangle =\\
&=\lim_{t\to\infty'}\left(e^{-iE_02(t)}|\langle 0\,|\,\Omega\rangle|^2\right)^{-1}\times\\
&\times\left\langle 0\,\middle|\,U(t,x^0)\phi_I(x)U(x^0,y^0)\phi_I(y)U(y^0,-t)\,\middle|\,0\right\rangle
\end{aligned} \tag{18}$$

To simplify this even more we can divide by 1 in the form of $\langle\Omega\,|\,\Omega\rangle$ to get rid off the prefactor. Since everything on both sides of this expression is in time order we can now use again the time ordering operator and get a very simple expression for the full correlator valid for arbitrary x^0 and y^0:

$$\langle\Omega\,|\,T\left\{\phi(x)\phi(y)\right\}|\,\Omega\rangle = \lim_{t\to\infty'}\frac{\left\langle 0\middle|T\left\{\phi_I(x)\phi_I(y)exp\left[-i\int_{-t}^{t}dt'H_I(t')\right]\right\}\middle|0\right\rangle}{\left\langle 0\middle|T\left\{exp\left[-i\int_{-t}^{t}dt'H_I(t')\right]\right\}\middle|0\right\rangle} \tag{19}$$

Our final result for the full theory is:

> The **2-point correlator** function of the interacting ϕ^4-theory can be written as a power series in λ, where the expansion coefficients are **n-point correlators** of the free theory. Also, again this expression is **exact**.

Further this can be extended to n-point correlators of the full theory by adding the same number of ϕ_I in the numerator.

3 Calculating n-Point Correlators (Wick's Theorem)

We have now reduced our problem of calculating n-point correlators of the interacting theory to calculating n-point correlators of the free theory. For $n = 2$ the correlator is given by the Feynman propagator as stated in the beginning (see eq. 6). For higher n one could evaluate the expressions by brute force plugging in the ϕ_I in terms of creation and annihilation operators. However this is already very tidious for $n = 3$ and therefore we are now going to simplify these calculations. The idea is to replace the time-order by normal order, so all normal ordered products will vanish when calculating the vacuum expectation value. To start with we will have a look at $\langle 0 \,|\, T\left\{\phi_I(x)\phi_I(y)\right\} \,|\, 0\rangle$. First split $\phi_I(x)$ into positive- and negative frequency parts:

$$\phi_I(x) = \phi_I^+(x) + \phi_I^-(y) \tag{20}$$

where

$$\phi_I^+(x) = \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}a_{\vec{p}}e^{-ip\cdot x} \qquad \phi_I^-(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}a_{\vec{p}}^\dagger e^{+ip\cdot x}$$

$$\phi_I^+(x)\,|0\rangle = 0 \qquad \langle 0|\,\phi_I^-(x) = 0$$

Now we will rewrite the time ordered product in terms of normal ordered ones. We start with $x^0 > y^0$:

$$\begin{aligned}
T\left\{\phi_I(x)\phi_I(y)\right\} &= \phi_I^+(x)\phi_I^+(y) + \phi_I^+(x)\phi_I^-(y) + \\
&+ \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y) = \\
&= \phi_I^+(x)\phi_I^+(y) + \phi_I^+(x)\phi_I^-(y) + \\
&+ \underbrace{\phi_I^-(y)\phi_I^+(x) - \phi_I^-(y)\phi_I^+(x)}_{=0} + \\
&+ \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y) = \\
&= \phi_I^+(x)\phi_I^+(y) + \phi_I^-(y)\phi_I^+(x) + \\
&+ \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y) + \\
&+ \left[\phi_I^+(x), \phi_I^-(y)\right] = \\
&= :\phi_I(x)\phi_I(y): + \left[\phi_I^+(x), \phi_I^-(y)\right]
\end{aligned} \tag{21}$$

where $: a_1 a_2^\dagger a_3 := a_2^\dagger a_1 a_3$ is the normal order operator which brings all operators into such an order that all creation operators are left to all annihilation operators without adding commutators. For $y^0 > x^0$ the result would look the same except for the commutator where y and x would be exchanged. Therefore we define a new quantity, the *contraction* as follows:

$$\overbrace{\phi_I(x)\phi_I}(y) := \begin{cases} \left[\phi_I^+(x), \phi_I^-(y)\right] & \text{for } x^0 > y^0 \\ \left[\phi_I^+(y), \phi_I^-(x)\right] & \text{for } y^0 > x^0 \end{cases} \tag{22}$$

Now we have found a relation between time-ordering and normal-ordering for two fields:

$$T\left\{\phi_I(x)\phi_I(y)\right\} = :\phi_I(x)\phi_I(y) + \overbrace{\phi_I(x)\phi_I}(y): \tag{23}$$

Also when sandwhiching this relation between vacuum stated we find that:

$$\overbrace{\phi_I(x)\phi_I}(y) = D_F(x-y) \tag{24}$$

which is nothing else than the Feynman propagator. To extend this now to an arbitrary number of fields one finds that:

$$T\left\{\phi_I(x_1)\phi_I(x_2)\ldots\phi_I(x_n)\right\} = :\phi_I(x_1)\phi_I(x_2)\ldots\phi_I(x_n) + \text{all possible contractions}: \tag{25}$$

This identity is known as *Wick's Theorem*. The proof will be skipped here but can easily done by e.g. induction. The real benefit of this theorem is now that in the vacuum expectation value all terms which are not fully contracted vanish and fully contracted terms can easily be replaced by the corresponding Feynman propagators. Also it can be shown that only even n-point correlators survive, since for an odd number there is no possibilty of full contraction. So Wick's theorem can be written in an easy form:

$$\langle 0\,|\,T\left\{\phi_I(x_1)\phi_I(x_2)\ldots\phi_I(x_n)\right\}|\,0\rangle = \begin{cases} 0 \text{ for } n = 2k+1, k \in \mathbb{N}_0 \\ \sum\limits_{\text{pairs}} D_F(x_{p_1} - x_{p_2})\ldots D_F(x_{p_{2k-1}} - x_{p_{2k}}) \end{cases} \tag{26}$$

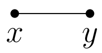
Summary of our achievement:

> We achieved to reduce the n-point correlators of the free theory to a sum over products of simple Feynman propagators of the free theory.

4 Feynman Rules

Now we come to the point where we can put everything together and find a diagramatic representation for our formulas. The first thing is to introduce that the Feynman propagator (in position space) is denoted as follows:

$$D_F(x-y) = \qquad (27)$$



Now we go back to our 2-point correlator of the full theory $\langle \Omega \,|\, T\{\phi(x)\phi(y)\} \,|\, \Omega \rangle$ (see eq. 19). We will have a closer look at the numerator and now expand the power series. Also remember that $\int dt H_I(t) = \int d^4z (\lambda/4!)\phi_I(z)^4$. So the numerator reads as:

$$\begin{aligned} &\left\langle 0 \middle| T\left\{ \phi_I(x)\phi_I(y) exp\left[\frac{-i\lambda}{4!} \int d^4z \phi_I(z)^4 \right] \right\} \middle| 0 \right\rangle = \\ &\left\langle 0 \middle| T\left\{ \phi_I(x)\phi_I(y) \left[1 + \left(\frac{-i\lambda}{4!} \right) \int d^4z \phi_I(z)^4 + \right.\right.\right. \\ &\left.\left.\left. + \frac{1}{2!} \left(\frac{-i\lambda}{4!} \right)^2 \int d^4z d^4z' \phi_I(z)^4 \phi_I(z')^4 + \dots \right] \right\} \middle| 0 \right\rangle \end{aligned} \qquad (28)$$

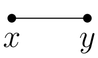
The first term in the power series is the Feynman propagator in eq. 27. However the second term is already very interesting. We will now have a closer look at this using Wick's theorem. I will restrict myself to the fully contracted terms:

$$\begin{aligned} &\phi_I(x)\phi_I(y)\phi_I(z)\phi_I(z)\phi_I(z)\phi_I(z) + \phi_I(x)\phi_I(y)\phi_I(z)\phi_I(z)\phi_I(z)\phi_I(z) \quad + \\ &+\phi_I(x)\phi_I(y)\phi_I(z)\phi_I(z)\phi_I(z)\phi_I(z) + \dots \quad = \\ &3D_F(x-y)D_F(z-z)D_F(z-z) + 12D_F(x-z)D_F(z-y)D_F(z-z) \end{aligned} \qquad (29)$$

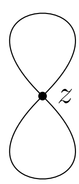
Now we can use the diagramatic representation for the Feynman propagator to visualize this equation.

$$\left\langle 0 \middle| T\left\{ \phi_I(x)\phi_I(y) \left(\frac{-i\lambda}{4!} \right) \int d^4z \phi_I(z)^4 \right\} \middle| 0 \right\rangle =$$

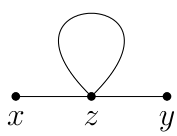
$= 3 \times$



$\times$



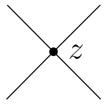
$+ 12 \times$



(30)

Here it has to be said that a 4-point vertex gets an additional factor.

$$\frac{-i\lambda}{4!}\int d^4z = \qquad (31)$$



Further it has to be mentioned that the disconnected terms (those diagrams seperated by a multiplication which do not contain any of the fieldparameters in the full correlator on the left side) are the so-called *vacuum bubbles* and have no physical meaning. This can easily be seen, because if one also calculates the denominator only those parts are reproduced. Therefore the disconnected parts come from the Hamiltonian part alone. It is possible to seperate the full correlator into a part of all connected diagrams multiplied by the exponential of the disconnected diagrams and therefore they are canceled out by the denominator. From all this we can now define the Feynman rules in position space for the ϕ^4 theory.

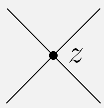
4.1 Position Space

1. For each propagator:

$$D_F(x-y) =$$

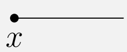
2. For each 4-point vertex (the only possible vertex):

$$-i\lambda\int d^4z =$$



3. For each external point:

$$1 =$$



4. Divide by symmetry factor:

$$\frac{1}{s} = \frac{1}{n!}\left(\frac{1}{4!}\right)^n E$$

with E the degeneracy of the diagram under interchange of points

The real benefit of this Feynman rules now is that the whole perturbation series for a n-point function of the full theory can be obtained by drawing all possible diagrams with n external points. The order of the perturbation series is thereby determined by the number of included vertices. Further for the ϕ^4-theory only even n-point functions exist since the 4-point vertex is the only possible vertex and for n and odd number we would need at least a 3-point vertex.

4.2 Momentum Space

In physics is it however often useful to calculate things in momentum representations, therefore the Feynman rules have to be translated into a momentum representation. The starting point therefore is again the Feynman propagator of the theory. As stated in eq. 6 the Feynman propagator is given by:

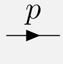
$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon} = \int \frac{d^4p}{(2\pi)^4} D(p) e^{-ip\cdot(x-y)} \tag{32}$$

When now 4 lines meet at a vertex momentum conservation is automatically imposed this can easily be seen by calculating the vertex itself using the position space rules and then insert the propagator in its momentum representation. The z-dependent parts (for a vertex at point z) then yields:

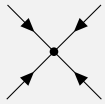
$$\int d^4z e^{-ip_1 z} e^{-ip_2 z} e^{-ip_3 z} e^{-ip_4 z} = (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 + p_4) \tag{33}$$

From this the Feynman rules in momentum representation can be obtained.

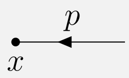
1. For each propagator:

$D(p) =$ 

2. For each 4-point vertex (the only possible vertex):

$-i\lambda =$ 

3. For each external point:

$$e^{-ipx} =$$ 

4. Impose (four-)momentum conservation at each vertex:

$$(2\pi)^4 \delta^{(4)} \left(\sum_i p_i \right)$$

5. Integrate over each undetermined momentum (not fixed in loops):

$$\int \frac{d^4 p}{(2\pi)^4}$$

6. Divide by symmetry factor.

5 Conclusion

Now let me summarize what we have achieved. We started out with an interacting theory where the equations of motion can not be solved in closed form simply by fourier analysis. Therefore we developed a procedure to treat the interacting part as a small perturbation to the free system using the definition of the quantum mechanical pictures. Further we introduced a new picture, the Interaction picture, which allowed us to write the fields and the Groud state of the full theory in terms of the free fields. Then we started to think about calculating n-point functions of the full theory to understand the dynamics of this theory. We found that it is possible to rewrite the n-Point correlators as a power series in the coupling parameter of the interacting part and therefore express the e.g. 2-point correlator of the full theory as a sum of n-point correlators of the free theory.

Under application of Wick's theorem, which allows us to replace time-ordered products by normal ordered ones plus so called contractions, we were able to simplify the calculations a lot. In the end we arrived at the result, that it is possible to represent the terms of the power series in a diagrammatic way and therefore (theoretically) calculate contributions to the n-point correlator simply by drawing all possible diagrams and using the assigned rules to get back to the formulas.

Literature

References

[1] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to Quantum Filed Theory*. Boulder, CO: Westview, 1995. Chap. 4, pp. 77–126.