Modelling radiative transfer of young single and binary star-systems

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Contents:

Preface

This work focuses on models of stars surrounded by matter calculated with RADMC-3D, henceforth Radmc3d. Its goal is firstly to provide a guide for newcomers to this program, since the manual itself is rather detailed and easily confusing for someone who is not used to use either "Linux" or fortran90, secondly, in order to find realistic parameters one needs to understand the basics of star evolution and radiative transfer.

The first chapter summarizes the evolution of stars in order to understand what scenarios are realistic for Radmc3d. Even though the program gives us the option to set up very unlikely constellations, e.g. very large yet very cold stars, it is useful to understand why most stars are on the main sequence or on the giant branches and not on other locations in the HRD. Section 1.3 also explains how the temperature and size of a star can be calculated.

The second chapter is to provide a very short overview of the most important inputs for Radmc3d, with focus on the syntax for placing two or more stars within a model. While the program can work with velocity files of dust and gas moving in certain manners, they are not used in the models provided here. The chapter also covers the theory of radiative transfer very briefly. Since the computation is quite complex, only the very basics are explained.

The third chapter is the main part of this work, containing different models and scenarios. Chapter 3.1, where single stars are modelled, mainly shows the opening of the spherical envelope towards a disk. Please note that jets have not been included due to their rather complex nature, e.g. the velocity of the gas towards the observer. Chapter 3.2 shows binary stars with shared and single envelopes, showing how two stars interact with the gas.

Since the physics of stars involves almost all aspects of physics, not only mechanics (e.g. structure of stars) but also quantum mechanics (e.g. core stability of dwarfs, fusion) and electrodynamics (refraction and scattering of radiation) too, it is not possible to cover all these aspects in detail.

1. Introduction into the physics of stars and envelopes

Radmc3d gives the option to set up models, independent of physical relevance. To figure out if our model is useful, it is necessary to understand what parameters are realistic for stars. While the program may give us the option to set up a star with a radius 10 times the radius of the sun and a temperature of 1 K, it is not a realistic setup for a star and therefore we need to know the parameters that stars can have.

This chapter will explain what stars are, how they are created and how different observables are related to each other.

In astrophysics it is common to refer to properties of the sun when describing a star, hence the following units besides the metric system are used:
[M_s] Mass of the Sun ($1{,}9884 \cdot 10^{30}$ kg)
[R_s] Radius of the sun ($6{,}96342 \cdot 10^{8}$m)
Henceforth the index "S" always refers to the sun.

1.1 Star formation and Main sequence

Stars are hot, self-gravitating balls of gas, whose central temperatures are high enough to sustain fusion reactions. [1] The size, surface temperature and mass of a main sequence star can vary over a broad spectrum e.g. Gliese 581 with ~ 3450 K, 0.03 M_s and 0.4 R_s and ζ Ophiuchi with ~33 000 K, 20 M_s and 8.5 R_s. [2]
Even though their size and temperature can vary over a broad spectrum, parameters of stars are not random, they follow a certain relation only changing in the early and late stages of their evolution.

1.1.1 Formation of stars

Stars are usually formed in areas with a, relative to the interstellar medium, higher density. The gas usually consists of ~70% H_2 by mass (therefore molecular clouds) with the rest being mainly CO and He. About 1% are heavier elements in form of dust particles. Interstellar clouds have typical densities of 100 particles/cm^3, diameters of ~$9{,}5 \cdot 10^{14}$km, masses up to $6 \cdot 10^{6}$ M_s and an average temperature of 10 K. [3]

While H_2 has no dipole moment and cannot be observed at 10-20K. It is therefore not suited for observation. CO, the second most abundant gas in interstellar clouds, emits in the radio when rotational transitions occur, which are traceable even at low densities. These transitions are so called quadrupole transitions. Assuming that the ratio of H_2 to CO molecules is constant, measuring the amount of CO also leads to the amount of H_2 of the cloud.

Other methods to determine the amount of H_2 are the observation through satellites capable of measuring the far-infrared. Since the atmosphere of the earth is blocking most of it, only satellites outside the atmosphere can detect it. [4]
A very special method is the observation of the wavelength-dependent extinction. High frequencies are scattered more than low frequencies and therefore stars that shine through dust clouds seem reddened if there is a higher density of dust particles. The mass of the cloud can be determined, assuming that the ratio of H_2 to dust is constant.

The main forces in interstellar clouds are the gravitational pulls towards the high-density cores within the cloud and the kinetic energy of the gas pressure. Once the cloud reaches a certain mass, the so-called Bonnor-Ebert mass, M_{BE}, the gravitational pull will be stronger than the gas pressure and the cloud will start to collapse.

$$M_{BE} = \frac{C_{BE} v_t^4}{\sqrt{P_0 G^3}} \quad [5]$$

where G is the gravitational constant, $v_t = \sqrt{\frac{kT}{\mu}}$ the isothermal sound speed, with μ being the molecular mass and C_{BE} a dimensionless constant.

Since the low-density areas consist mainly of H_2 which does not, as previously stated, absorb infrared emission that is created by the transformation of gravitational potential energy, most of the energy is radiated away. As density increases, the opacity of the gas rises and leads to absorption of the emitted radiation by the gas. The increase of the temperature stops the collapse in the core, but the outer gas keeps moving towards the centre. This leads to a second shock that, again, increases the temperature. Eventually, a stat called *First Hydrostatic Core* is formed. [6]

The heating of the core leads to the dissipation of H_2 to H, which allows the gravitational collapse to continue. This state is now called *protostar.* While the star is still gaining mass through the gas that falls towards the centre, raising the density and temperature again, the dust closest to the core evaporates, leading to a so called “opacity gap”.
Once the density and temperature in the core are high enough to start the ionisation of H, the emission will slow down the material falling into the core.

Since the total angular momentum of the collapsing cloud is unequal 0, the incoming gas cannot move in a straight line towards the centre. With the gravitational force pulling towards the core and the force resulting from the rotation, the gas slowly forms a disk.

Please note that, if the cloud has a magnetic momentum, the creation of a star becomes much more complex. Due to the size of the topic it is advised, if further information is needed, to read advanced literature on this topic, e.g. Maurizio Salaris and Santi Cassisis *“Evolution of Stars and Stellar Populations”*.

The star then will continue to follow the so called *Main Sequence*.

1.1.2 Main Sequence

All information we get from stars comes in form of electromagnetic radiation. The two most basic properties are the luminosity, the total energy emitted per unit time, and the temperature at the photosphere. Since stars are assumed to be almost blackbodies, their temperature is mostly given as *effective temperature* T_{eff}. While the temperature always has to be determined by the spectrum of a star, it is possible to measure the luminosity of a star at a given distance. Since the determination of the effective temperature and the luminosity is time-consuming, two alternative quantities are commonly used to describe a star. The *absolute magnitude* M_v in the visual band, hence V band, can be used to describe the intrinsic *Flux* F_V which is the energy in the visual in a unit wavelength per unit area per time. [7] With increasing distance to the object, the flux decreases. For determining the absolute brightness of a star, a distance of 10 pc is assumed.

$$M_V = -2.5 \log(F_v(10\,pc)/F_0(10pc))$$

This formula can be generalized to all distances and all wavelengths.

Given that the flux decreases with r^{-2} the *apparent magnitude*, the magnitude an object has when observed from earth can be written as [8]:

$$m_\lambda = M_\lambda + 5\log(\frac{r}{10pc})$$

Stars with higher masses tend to be more luminous. The relationship between the luminosity and the mass of a star is given by:

$$\left(\frac{L}{L_s}\right) = \left(\frac{M}{M_s}\right)^a$$

Where L is the star's luminosity and M is its mass. The exponent *a* is a value between 1 and 6. For stars on the main sequence it is 3.5 if their mass is between 2 and 20 M_s. [9] Note that very large stars often surpass the 20 M_s limit for this formula and have to be computed differently.

The quantity that can be used instead of T_{eff} is the colour of the star. Since the star is assumed to be modelled reasonably well as a blackbody, its colour is representative of its temperature. The ratio of the F_V and the F_B flux, also called (B-V), i.e. the difference between the apparent magnitude in the blue B-band and the V-band is often utilized. This assumes that between the observer and the star there is nothing, e.g. dust or gas. It is common to use *spectral types*, which are defined by relative strengths of lines in their spectrum. The spectral types correlate with T_{eff} and are, from highest T_{eff} to lowest, O, B, A, F, G, K, M, with only historical reasoning of their letters and order. Though the spectral types are actually defined by certain absorption and emission lines within certain frequencies (e.g. Class O is the ratio of N IV λ 4058 to N III λ 4634) and not by the T_{eff} itself they strongly correlate with their real temperatures.

The spectral types and the brightness are combined in the so-called Hertzsprung-Russell diagram, hence HRD.
As seen in Figure 1 [10], most stars lie in a certain region. This region is the main sequence introduced above.

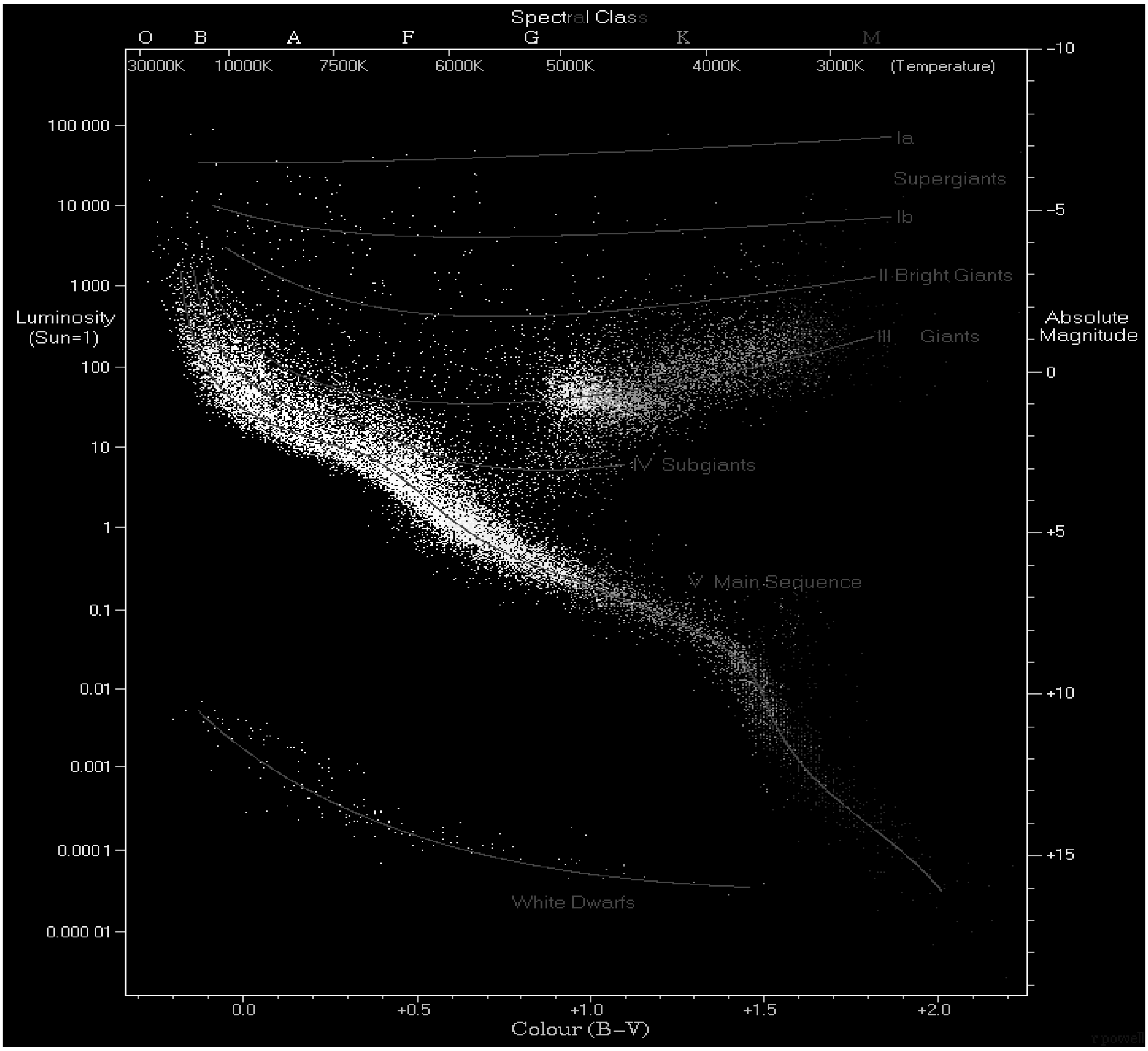


Figure 1 The plot of several thousand stars in the HRD, showing the position of stars, as predicted by the HDR, close to the Main sequence.

During the formation of stars, the proto-star follows a certain path in the HRD, its so-called pre-main sequence evolutionary track. While infant stars with masses lower than 3 M_s contract while keeping up their temperature and basically "fall down" in the HRD (this line is the so-called *Hayashi-track*) [11], stars with higher masses develop a fully radiative interior, which leads from the *Hayashi* to a pronounced *Henyey-track* [12], a bent track as seen in Figure 2.

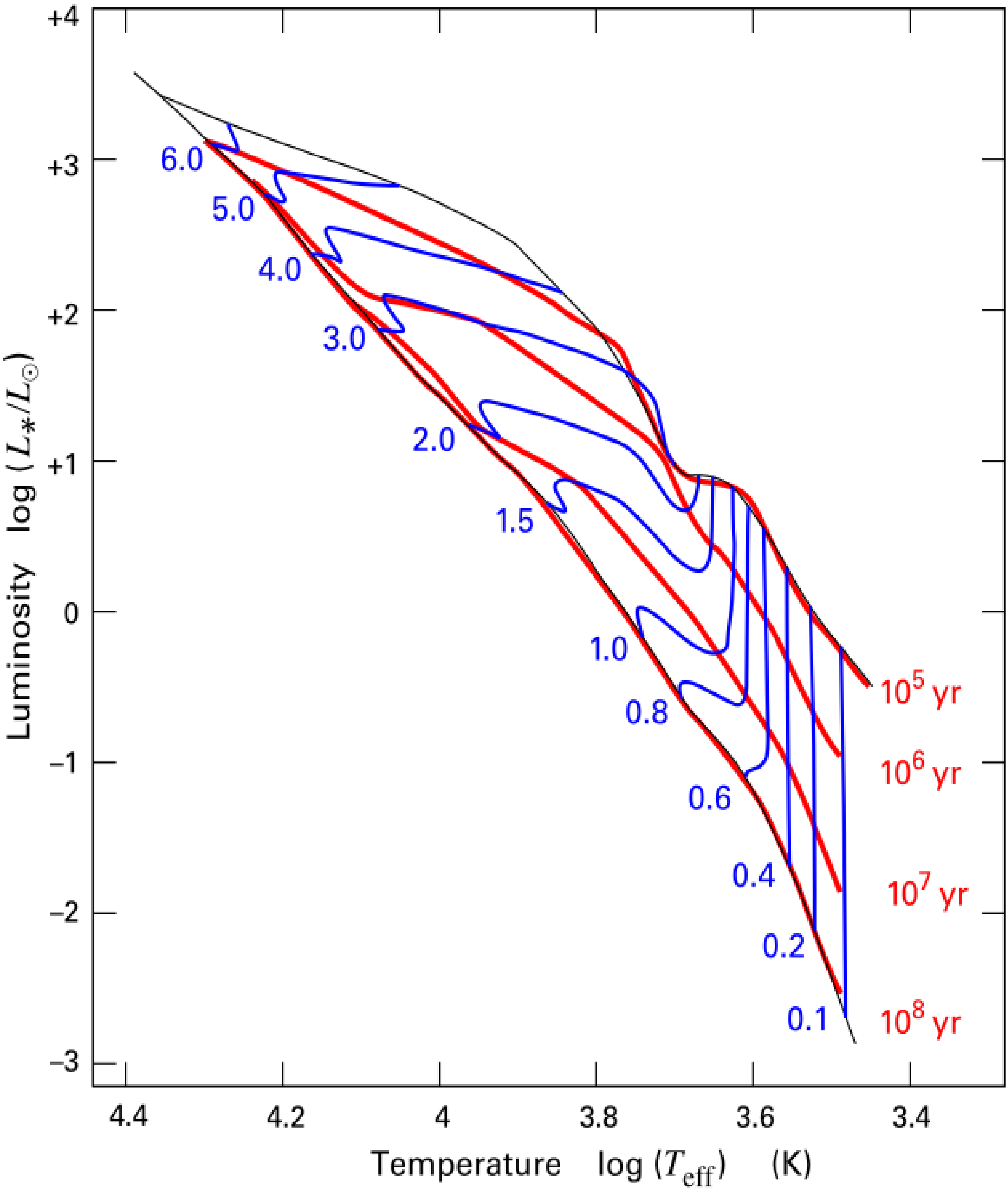


Figure 2 Stellar evolution tracks (blue lines) for the pre-main-sequence. At the right lower end are the Hayashi tracks which, are then bent and form the Henyey tracks. Note that larger stars are born directly at the Henyey track. The red lines are isochrones at certain ages. The blue numbers on the left show the masses of the stars in M_s [13]

One explanation of the gathering of objects on the Main Sequence is that stars, in order to be stable, have to be in hydrostatic equilibrium for a long period of time. This means that stars with a certain mass and radius must have a certain temperature, that ensures an equilibrium between gravity and gas-, (for massive stars also radiation-) pressure.

The stellar radius and temperature are connected through:

$$L_{star} = 4\pi(R_{star})^2\sigma_B\left(T_{eff}\right)^4$$

Where σ_B is the Stefan-Boltzmann constant and L is the luminosity. [14] The main sequence is therefore the relation of $L(T_{eff})$ over different stellar masses. The reason why so little stars are showing off the giant- or main branch is, that the states in between are very unstable and therefore have very short durations or are physically not possible.

For stars on the main sequence there is a clear relation between the mass of a star and the speed it burns hydrogen, leading to shorter lifespans of high-mass stars and very long lives for low-mass stars, e.g. stars with mass of 60 M_s and spectral type O5 live about $3.4\cdot10^6$years while stars with 1 M_s and spectral type G2V live for about $1\cdot10^{10}$ years. [15]

During their lifetime, stars move up in the Hertzsprung-Russell-Diagram, because their core becomes more and more filled with yet non-burnable helium, causing the star to contract and increase the density and temperature once again which will then let outer hydrogen spheres start fusion and thus leading to an increase in radius and brightness over time.

While the cores of stars merge hydrogen to helium for the longest time, they will eventually run low on material and thus leading to the end of the main sequence.

For further information to this rather complex topic please see Steven W. Stahler and Francesco Pallas "*The Formation of Stars.*"

1.1.3 End of the Main Sequence

As the star burned hydrogen to helium, the helium rich centre is surrounded by a sphere of hydrogen-burning material which will continue to add helium to the inner core that will eventually contract from its own gravity and the pressure from the layers above. The shell burning expands the envelope, which leads to an increase of brightness but decreases its temperature and thus moves the star rapidly to the right upper part of the HRD, the so-called *giant branch*. Eventually the star will reach temperatures in its core to fuse helium, increasing its temperature and moving him to the left in the HRD, to the so-called *horizontal branch*. [16]

Because of the very broad range of masses of stars and their very different endings of their lives, we must differentiate between the 3 most important regimes for this stage.

Stars with masses between the theoretical lower limit of star masses, about 75 masses of Jupiter (M_J) [17] up to ~ 0.4 M_S are not heavy enough to start the helium burning and eventually form a *white dwarf*, a high-density object which is stabilized by the degeneracy pressure of electrons rather than gas pressure.

Stars with ~0.4 up to 2.3 M_S can burn helium at the end of their lifetime and thus enter the giant branch resulting in becoming a *red giant*, a star with a very much higher radius than the main sequence star had. Ultimately they are not able to fuse heavier elements than helium they will end in a *white dwarf*. The stars lose shells that form the so-called *planetary nebulas*.

With masses >2.3 M_S stars are heavy enough not only to reach the helium burning stage but also to burn carbon and heavier elements. They will form *supergiants* as seen in the HRD, and eventually lose their outer shells leaving a *dwarf* or end in a supernova. The shells form planetary nebulas or supernova remnants.

If the remaining mass is high enough, the star will end in a supernova and leave either a neutron star, an object so dense that not even the degeneracy pressure of electrons, but only the degeneracy pressure of neutrons stops the collapse. If the remaining mass of the dying star is greater than ~3 solar masses [18] a black hole will form. Black holes are so heavy that not even light can escape and thus can only be observed through gravitational effects or the radiation from the accretion disks around them.

Remains of the dead stars finally contribute to interstellar matter from which the next generation of stars form. Note that all elements above iron are created by supernovae.

Dwarf stars and giants can, to a certain degree, be modelled with Radmc3d. It should be considered that the program was not created to model black holes or neutron stars.

1.2 Envelopes and Dust disks

Since the focus of the given models in this work lies on stars within envelopes and dust disks, it is necessary to explain the formation of these in detail.

After the giant molecular cloud collapsed and the proto-star is formed the star is surrounded by a circumstellar envelope. Due to the conservation of angular momentum the star increases its rotation speed as mass flows from the outer spheres into the centre. While the gas close to the plane of rotation does not fall directly into the star due to centripetal forces, the material that is closer to the axis of rotation falls faster into the star. Parts of the material closer to the rotation axis is pulled by the denser material in the equatorial plane and reduces the effects of the angular momentum conservation through friction. This creates at first a cone-like opening of the circumstellar envelope ultimately leading to *circumstellar discs*.

Along the rotation axis a bipolar outflow is created, containing hot gas being pushed away into the space. For further information on bipolar outflows read Adam Franks *“Bipolar outflows and the evolution of stars”*. [19]

While the gas in the interstellar matter only absorbs certain wavelengths, the dust-particles can absorb many wavelengths and bands and are therefore essentially for the cooling of new born stars. [20]

While most of the gas in the disc will eventually be pushed away due to the radiation pressure, parts are accreted. The dust will be less affected by the radiation pressure, leaving mainly particles created through collision in the envelope, which will eventually form protoplanets or fall into the star. This is called a *debris disk*. During this process the ratio of dust to gas changes drastically depending on the gas and radiation pressure of the star.

While the bipolar outflows, so called jets, do not last for very long, the circumstellar discs do last up to ~10 Myr [21]. Note that hotter, more massive stars do not have disks for that long due to the higher radiation pressure.

It is assumed that envelopes and disks are connected to young stellar objects. But there are also older stars found with circumstellar envelopes, e.g. Mira variable, making it necessary to consider such setups for models of evolved stars too.

1.3 Temperatures and spectrum of stars

While it was mentioned what properties stars on the main sequence have, it is still necessary to consider temperatures of stars that are no longer on the main sequence. White dwarfs, can reach low surface temperatures, e.g. down to ~3000 K, while giant stars can reach very high surface temperatures up to ~210 000 K, [22] since their high radiation pressure pushed off the layers that fuse hydrogen, exposing the layers beneath which burn at higher temperatures.

Radmc3d can plot the spectra, making it possible to match models to data. Therefore it is necessary to explain how the spectra of stars are composed.
Since stars can be modelled reasonably well as black bodies, we can derive the effective temperatures from the luminosity as stated in 1.1.2 and can compare the corresponding black body

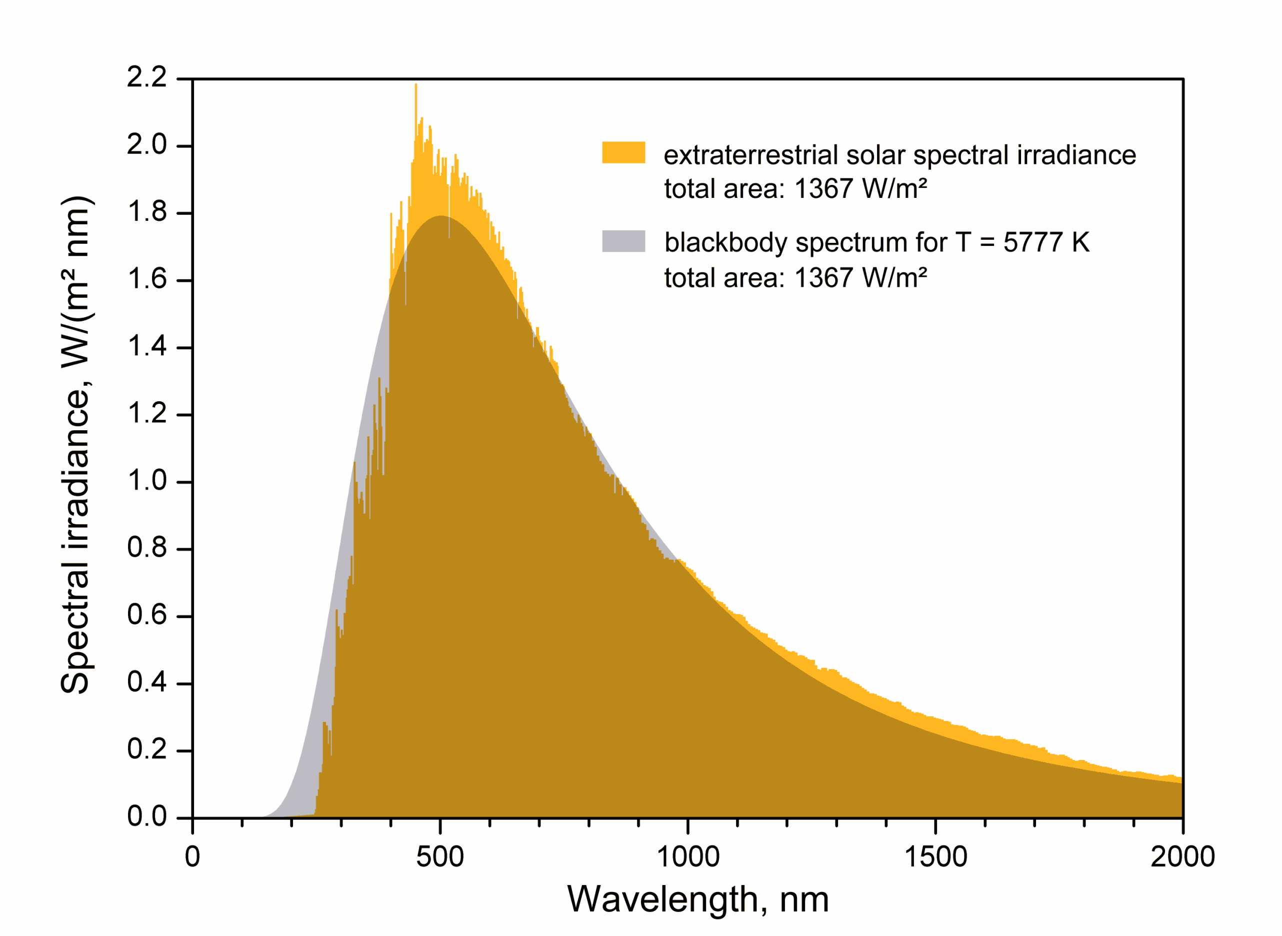


Figure 3 Compare of Sun's spectrum and an ideal black body spectrum with the same luminosity

with the measured spectrum.

As seen in Figure 3 [23] stars are not perfect blackbodies but quite well represented by them.

Stars show absorption and sometimes also emission lines in their spectra, depending on their metallicity and their surrounding material which leads to the spectral classes as stated before.
It is common for proto-stars and very young stellar objects to have a second peak, due to the emission of the surrounding gas at lower temperatures. This excess emission can be measured at infrared wavelengths.
Note that the gas can have other energy sources too, e.g. if it is very fast rotating or due to magnetic flux, altering the spectrum. In certain cases, e.g. if the star formed in a planetary nebular with a high metallicity, the spectrum can be misleading, letting stars seem older than they actually are due to a higher metallicity than a star of a certain age should have (younger stars have lesser metallicity due to the fact that they either are not yet hot enough to fuse elements above helium or did not have enough time to produce the metallicity found in the outer shells).

2 Radiative transfer

2.1 Introduction into radiative transfer

Radiative transfer describes the change of intensity and energy of radiation through absorption, scattering and emission. When modelling stars that are located in clouds of gas and dust it is of utmost importance to consider the influence of the matter on radiation.

“The analysis of radiation field often requires us to consider the amount of radiant Energy, dE_ν , in a specified frequency interval (ν , $\nu+d\nu$) which is transported across an element of area $d\sigma$ and in directions confined to an element of solid angle $d\omega$, during a time dt. This energy, dE_ν is expressed in terms of the *specific intensity* (hence intensity), I_ν, by

$$dE_\nu = I_\nu * \cos\vartheta \, d\nu \, d\sigma \, d\omega \, dt$$

where ϑ is the angle which the direction considered makes with the outward normal to $d\sigma$.” [24]

Since small spatial changes in the absorption and emission coefficients already require massive calculations computers have to be used. The basic idea of most of the programs used for this send out photons, in general, according to the spectrum of the central source from the light sources (e.g. stars, hot dust clouds) and then calculate the intensity an observer in a certain distance would measure after refraction, emission and absorption. Since this is a statistical approach, the number of photons sent through the material influence the resolution of the output.
The program used for this work was Radmc3d which was created to compute continuum transfer.

2.2 Radmc3d

Radmc3d was written by C.P. Dullemond with substantial contributions from: A. Pohl, M. Min, A. Juhasz, R. Shetty, F. Sereshti, T. Peters, B. Commercon and has been developed between 2007 and 2010 at the Max Planck Institute for Astronomy in Heidelberg, and is continued to be developed at the Institute for Theoretical Astrophysics of the University of Heidelberg.

It is based on Fortran90 and IDL, but was used in this thesis with GDL (Gnu-Data-Language). Due to the lack of compatibility with *Windows* it was run on a *Windows 10* OS with an *Ubuntu* distribution emulated.
While it may be possible to make it run on *Windows* it is strongly advised not to do so, as the manual is written for *Linux* based OS and most commands, like make and bash are not yet implemented into the *Windows* OS.

Radmc3d is a software package for astrophysical radiative transfer calculations in arbitrary 1-D, 2-D or 3-D geometries. It is mainly written for continuum radiative transfer in dusty media, but also includes modules for gas line transfer and gas continuum transfer. [25]
It calculates the radiative transfer after getting input from either ascii or binary files and is able to generate output in the same formats or as spectra or images within its own GUI.

While Radmc3d is capable of computing very complex model setups, e.g. moving gases or Doppler-effects within these gases and so on, the models presented here serve rather an educational than a scientific purpose and are therefore computed to introduce the basic concept.

For modelling stars surrounded by gas and dust, it is assumed that the photons come from a star or protostar within the cloud and finally a spectrum can be derived that the observer would see. Radmc3d gives the option to choose if the stars should be treated as spheres, it can also be chosen how many photons are emitted. Due to long processing times, it is strongly advised to run tests with lower photon numbers (e.g. $N=10^3$-10^6) depending on the density of the gas. Since Radmc3d computes the scattering and absorption due to the gas and dust, an increase of material leads to an increase of these effects. The processing time for each model is also dependent on the distribution of the material, e.g. circumstellar disks include less material than spherical envelopes if the density of the gas and dust is the same, therefore the processing time for higher densities is in general lower for circumstellar disks than for spherical envelopes.

To specify the density or temperature structure (or any other spatial variable) as a function of spatial location we must have a grid. The grid basically defines the way you split up the space, allowing you to observe more important parts (e.g. very small objects within the dense dust) very close while spending not too much processing time for the rather unimportant parts.

There are two types of grids given in the manual:
" **Structured grids**
In Cartesian coordinates a grid defines the size of the cells that your space contains.
In Spherical coordinates, it also defines the size of the cells, but they are now curved and dependent on r, θ and φ.
Please note that spherical coordinates cannot be used to simulate more than one star!
While the Cartesian grid should be in 3-D, it is possible to make a 1-D model in spherical coordinates, which would mean that the grid is only dependent on r.

Unstructured grids
For some applications, it may be more convenient to specify spatial variables not on a structured grid, but on a semi-random set of points in 3-D space." [26]

The main part of the programming was done in the problem_setup file, in which the parameters like number, temperature and radius of stars, mass of the gas and its distribution are set. These parameters are then created within the input files. The most important parameters will be listed here and their purpose and structure will be explained. For further details and not mentioned parameters see the manual of Prof. Dullemond.

2.2.1 Basic Structure and Setup

In this section, it will be specified which inputs and what structure was used. For details on the program itself please refer to the manual or contact Prof. Dullemond. Note that variables from the code have been marked with bold characters.

The main input file is the problem_setup file. As mentioned before it allows to specify the stars, the composition and density distribution of the dust and gas.

While not necessary, it is advised to implement natural constants, or place the natural constants file with the problem_setup file. In general, this contains a list with the solar mass (MS), the solar temperature (TS) the solar radius (RS) and the astronomical unit (AU) as well as the model parameters (e.g. the inner and outer radius of a circumstellar disk), which will be needed for the dust distribution e.g. the densities of the gases. After these the grid parameters are defined, giving the amount and the size of the cells.

```
; Grid Parameters
;
nx = 50L ny = 50L nz = 50L
sizex = 10*AU sizey = 10*AU sizez = 10*AU
;
xi= -sizex + 2*sizex*dindgen(nx+1)/(1.d0*nx)
yi= -sizey + 2*sizey*dindgen(ny+1)/(1.d0*ny)
zi= -sizez + 2*sizez*dindgen(nz+1)/(1.d0*nz)
xc = 0.5d0 * ( xi[0:nx-1] + xi[1:nx] )
yc = 0.5d0 * ( yi[0:ny-1] + yi[1:ny] )
zc = 0.5d0 * ( zi[0:nz-1] + zi[1:nz] )
```

Figure 4 Grid parameters
Example of a structured Cartesian grid

- **Nx**, **Ny**, **Nz** give the number of grid cells for each axis
- **sizex**, **sizey** and **sizez** give the size of each cell.
- **Xi**, **Yi** and **Yi** give the edges of the cells and their shape. In this example the cells are cubic.
- **Xc**, **Yc** and **Zc** specify the centre of the individual cells and are later used for the dust density.

  The next parameters to be defined are the stars and the model itself.

```
; star parameters
Mstar1 = MS
Rstar1 = RS
Tstar1 = TS
Pstar1 = [0.,0.,0.]
Mstar1 = 2* MS
Rstar1 = 2*RS
Tstar1 = 2*TS
Pstar1 = [-1*AU,0.,0.]
```

Figure 5Parameters for a binary star system

In this example one star has the mass, radius and the effective temperature of the sun while the other has 2 times higher values. **Pstar1** gives the location of the first star, in this particular case the star is placed in the origin. The second star is placed 1.49*10^{13}cm to the negative x axis. While this sets the parameters for the stars, please note that this is not yet the star.inp file which will actually write the input file.

Very important for modelling radiate transfer are the wavelengths that are observed. It is important to state which wavelengths should be used for the radiative transfer and therefore it is necessary to write the input for the wavelength_micron.inp file. This file sets the wavelengths that are observed and what ranges are covered. Below you see an example for a wavelength_micron.inp in a problem_setup.pro:

```
; Write the wavelength_micron.inp
;
Lambda1 =0.1d0
Lambda2 = 7.0.d0
Lambda3 = 25.d0
Lambda4 = 1.0d4
n12 = 20
n23 = 100
n34 = 30
lam12 = lambda1 * (lambda2//lambda1)^(dingen(n12)/(1.d0*n12))
lam23 = lambda2 * (lambda3/lambda2)^(dingen(n23)/(1.d0*n23))
lam34 = lambda3 * (lambda4/lambda3)^(dingen(n34)/(1.d0*n34))
lambda = [lam12, lam23, lam34]
nlam = n_elements(lambda)
```

Figure 6 Example of writing the setup for the wavelength

Lambda1,**2** and **3** set up the wavelengths, **n12**, **n23** and **n34** specify how many steps are in a certain range, so between 0.1 and 7·10^{-6}m, there are 20 steps, between 7 and 25 are 100 and so on. It should be mentioned that the wavelength with the highest intensity in the scenario must be covered within the wavelength grid.

```
; write the wavelength file
;
Openw, 1, ´wavelength_micron.inp´
Printf,1,nlam
For ilam=0, nlam-1 do printf,1,lambda[ilam]
Close,1
```

Figure 7 Example of a wavelength file

This file written here will contain all wavelengths. The exact wavelengths depend on how many steps in each interval were set up in the previous part. While the order of the wavelengths does not have to be always decreasing, it always has to be monotonically. [27]
It is possible to add certain wavelengths if necessary, e.g. if there is an additional radio source.

While we still set up the star parameters before, it is necessary to write a star.inp file.

```
; write the stars.inp file
;
Openw, 1, ´stars.inp´
Printf,1,2
Printf,1,2,nlam
Printf,1, ´´
Printf,1, rstar1, mstar1, pstar1[0], pstar1[1], pstar1[2]
Printf,1, rstar2, mstar2, pstar2[0], pstar2[1], pstar2[2]
Printf,1, ´´
For ilam=0, nlam-1 do printf,1,lambda[ilam]
Printf,1, ´´
Printf,1, -tstar1
Printf,1, -tstar2
Close,1
```

Figure 8 Example of stars.inp file with 2 stars

Please note, if the first printf,1,2 line is changed to printf,1,1 that the wavelength_micron.inp file will be read in Hertz and the structure of the .inp file changes. Therefore, it is strongly advised not to change this if not necessary. The second printf,1,2, nlam sets the number of stars to 2. As mentioned above, it is not possible to put 2 stars within a spherical coordinate system due to the structure of the program.

The following inputs assume that the first printf line has not been changed to 1.

Nlam is the number of frequency points. This needs to be equal to the number of points in the wavelength.inp file, which has been set to 150 in all of the presented models. The numbers behind the parameters indicate which star has which properties e.g. **Pstar1[0,1,2]** prints the position of the first star and **tstar2** gives the negative blackbody temperature of the second star.

A crucial input for the modelling of stars is the dust. It is necessary to specify the dust, its opacity and its density distribution. While the program itself has little restriction on what kind of densities and distributions one uses, even small changes in this section can lead to massive increases of computation time. It is therefore advised to run the models with lower density or with less photons at first.

At first it is needed to set up the distribution of the densities.

```
; Dust density model
xx    = rebin(xc,nx,ny,nz)
yy    = transpose(rebin(yc,ny,nx,nz),[1,0,2])
zz     = transpose(rebin(zc,nz,ny nx),[2,1,0])
rr      = sqrt(xx^2+yy^2+zz^2)
rr1    = sqrt((xx+1*AU)^2+(yy)^2+(zz)^2)
rhod  = rho0 * exp(-(rr^2/radius^2)/2.d0)
rhod1= rho1 * exp(-(rr^2/radius^2)/2.d0)
```

Figure 9 Example of a setup for a density model with 2 different densities

Xx, **yy** and **zz** rearrange the coordinates of the grid cells in a format that is then used to compute the array of radii **rr** and **rr1**. **Rhod** and **Rhod1** set up the distribution of the gas, in this case a Gaussian distribution. Note that these are two different densities with the same distribution simulating that there are two gases present.

Again, even though it is possible to use other distributions, it is strongly advised to keep the distribution Gaussian, linear or functions that have a declining density towards the edges of the model since even little changes in the distribution of gas can lead to a massive increase of time needed to compute the radiative transfer.

The next file to be created is the density file.

```
; write the dust_density.inp file
;
Openw, 1, ´dust_density.inp´
Printf,1,1                   ;file format
Printf,1,nx*ny*nz      ; number of cells
Printf,1, 1                 ; Nr. of species
For iz=0, nz-1 do begin
        For iy=0, ny-1 do begin
                For ix=0,nx-1 do begin
                        Printf,1,rhod[ix,iy,z]
                Endfor
        Endfor
Endfor
Close,1
```

Figure 10 Example of a dust_density.inp file.

The first **printf**,1,1 is used to set up the format this file is written in. The second is for setting up the number of cells and the third gives the amount of different dust species. Please note that if two or more dust species are used, they have to be placed one after another.

The last non-optional input in order to specify the dust needed, is the dust_opacity.inp file. It contains the amount and the species of the dust, e.g. silicate or dusts with a certain amount of carbon. The input files are created by other programs which are not integrated in Radmc3d, but the program provides a variety of different dust types in the example files.
Obviously, the amount of dust species has to be the same for dust_density.inp and dust_opacity.inp.

```
; Dust opacity control file
;
Openw, 1, ´dustopac.inp´
Printf,1,'2              format number of this file '
Printf,1,'1              Nr. Of dust species'
Printf,1,'============================================'
Printf,1, '1             way in which this dust species is read'
Printf,1, '0             0=Thermal grain'
Printf,1, 'silicate      extension of the name of dustkappa_***.inp file'
Printf,1,'-----------------------------------------------------------------------------------'
Close,1
```

Figure 11 Example of a Dustopac.inp file with a silicate dust species.

If a second type of dust is added, it is necessary to, again, specify the way in which the dust species is read, whether it is treated as a blackbody and the extension of the dustkappa_***.inp file. Also, the extra line beneath has to be included.
For the program, itself it is not necessary to modify the dust type. It is possible to copy the line block as often as needed. This usually occurs if two or more density distributions are implemented, e.g. a binary star system with an envelope around each of the stars and around both. This scenario would need 3 densities and therefore 3 types of dust species, yet all species could be the same.

The very last file that has to be created is the AMR grid file. It specifies what coordinate system is used and what size the cells have. The shape of the cells already has been defined with the grid parameters.

```
; AMR Grid File
;
Openw, 1, ´amr_grid.inp´
Printf,1,1              ;format number of this file '
Printf,1,0              ;grid style 0 is regular grid without amr
Printf,1,0              ; Coordinate system
Printf,1,0              ;gridinfo
Printf,1,1,1,1          ; includes the coordinates
Printf,1, nx,ny,nz      ; size of the grid
For i=0, nx do printf,1,xi[i]*0.2
For i=0, ny do printf,1,yi[i]*0.2
For i=0, nz do printf,1,zi[i]*0.2
Close,1
```

Figure 12 AMR grid file

3 Modelling stars

All the models used in this section have been created with Radmc3d and are structured as explained in 2.2. While there are some setups for single stars in spherical coordinates, most of the single star models and all of the binary models are written in Cartesian coordinates. While the setups may seem random, the used parameters have been selected to show the effects clearly, especially for the binary systems, where effects are hard to see if the difference in temperature is not high enough.
If further details are necessary, the code for each of the setups can be found on the same webpage from which this thesis was downloaded.

All stars will be shown in at least 3 different wavelengths that will vary depending on the setup itself and the according spectrum. The spectral energy distribution (SED) of the models are placed at the end of every model. The resolution of the figures was limited to 800 pixels by the program. While the GUI in Radmc3d allows to change the units for the SED-axes, it is not possible change the scaling of the axis within the GUI.

The densities given for the models are the densities at the centre of the distribution, which is in most cases the centre of the star. Even though this is not realistic, it is necessary for the computation within Radmc3d. For some models term “radius” for the envelope is used. This refers to the mathematical maximum of the used Gaussian distribution and is not to be understood as a sharp ending of the distribution.

While the size of the models differ, depending on the setup, the models within one setup stay the same size if not stated differently.

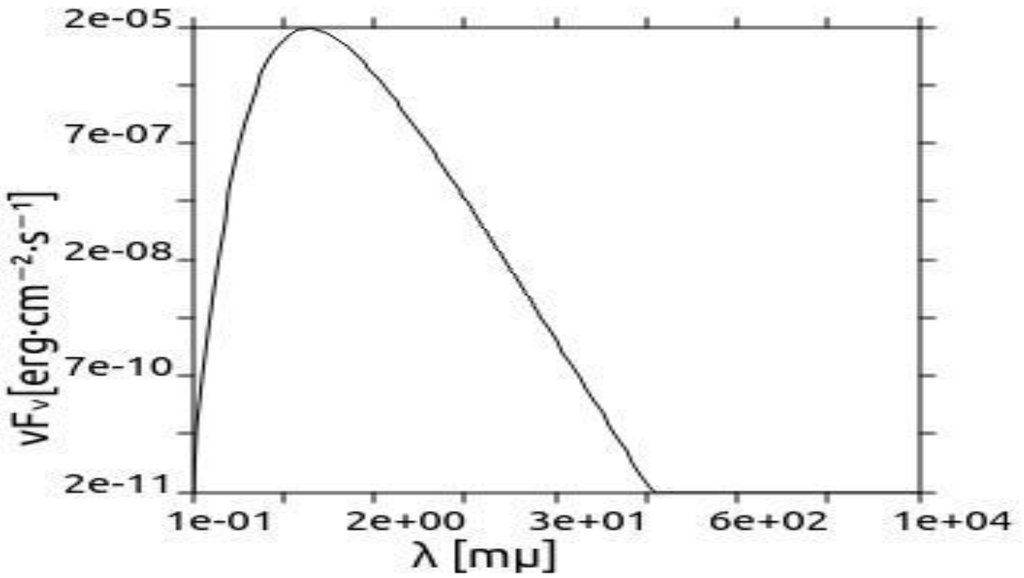
Note that for some models no SED could be derived due to unknown errors within the program.

3.1 Models of single stars

3.1.1. Single Star without envelope

As stated in 2.2, some stars, especially young stellar objects, are surrounded by dust and gas. Please note that the models include different stars, e.g. a variation of temperature and radius. For details read the description of the figures and see the attached source codes. All stars have been coloured for demonstration purpose.

The first model simulates a sun-like star (a star with T_s, M_s and R_s) without any dust and gas surrounding it. It is simply for showing the quality of the SED.



SED 1 SED of a star with the properties of the sun

Note the axes are not in the same units as in Figure 3 and therefore seem a bit compressed. The maximum and the form of this SED are the same as they are supposed to be, with a peak at $0.63 \cdot 10^{-1}$µm compared to the maximum power per percentage bandwidth output of the sun measured at $0.635 \cdot 10^{-1}$µm.

Since the first plot serves only the purpose to demonstrate the accuracy of the program and only a single bright pixel would be visible, no figure has been included.

Note that due to the simplified structure, the SEDs of the models can differ from real systems. For increased accuracy, more input is needed, e.g. Doppler-effects or induction heating.

3.1.2. Single star with envelope

The second model is a sun-like star surrounded by gas and silicate dust with a density of 10^{-14} g/cm^3 in the centre.
Note that the spherical envelope has yet no cone, simulating a very young star.

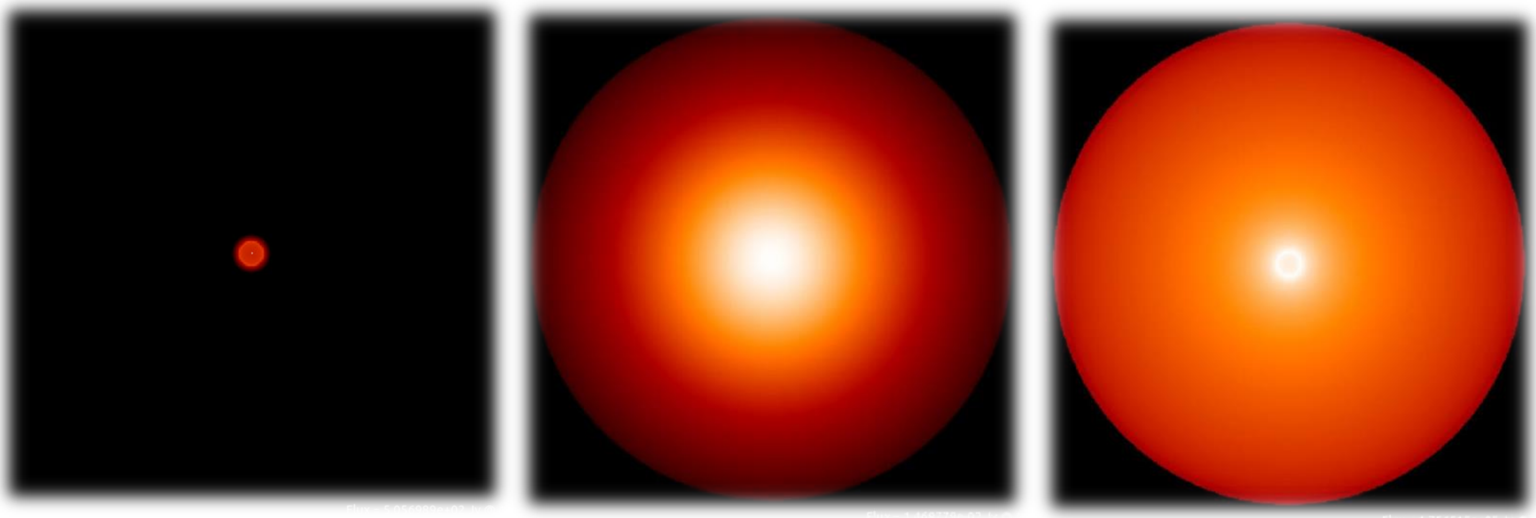
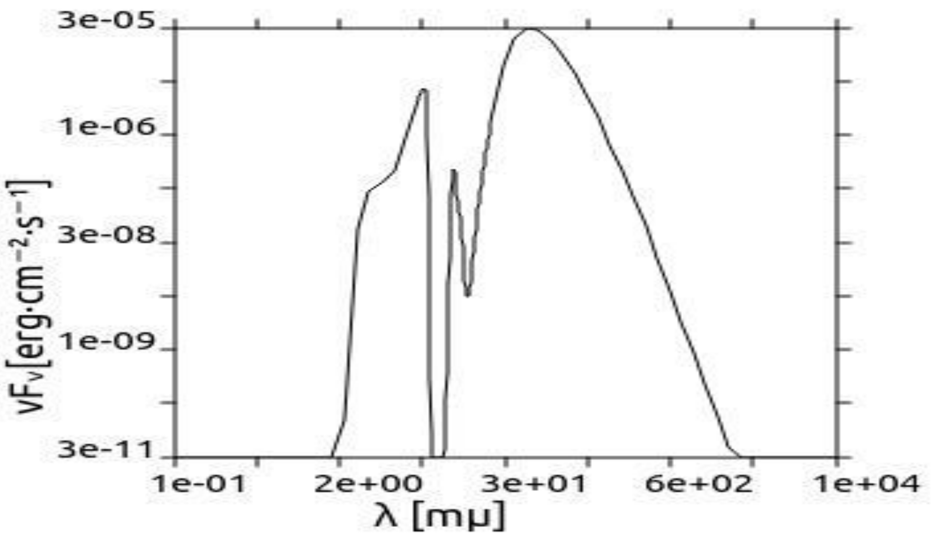


Figure 13: Sun-like star in a spherical envelope at (from left to right) 4µm, 10µm and 160 µm, the total size of the model is ~ 200 · AU

The gas emits little energy at 4 µm but with longer wavelengths the dust and gas in the envelope start to contribute more and more.



SED 2 Modelled spectrum of a star in spherical envelope.

The peaks in SED 2 come from the used gas-dust type. Overall the spectrum looks very much like a cooler sun-like star because the gas resembles a large, but cool black body. Imprinted is the silicate absorption around 10 micron.

3.1.3. Single star with cone

The third model shows the same star with his cone opened at the bottom and the top side with a 45° opening, slowly forming a disk as described in the star evolution process above. The density of the silicate dust is 10^{-14} g/cm^3.

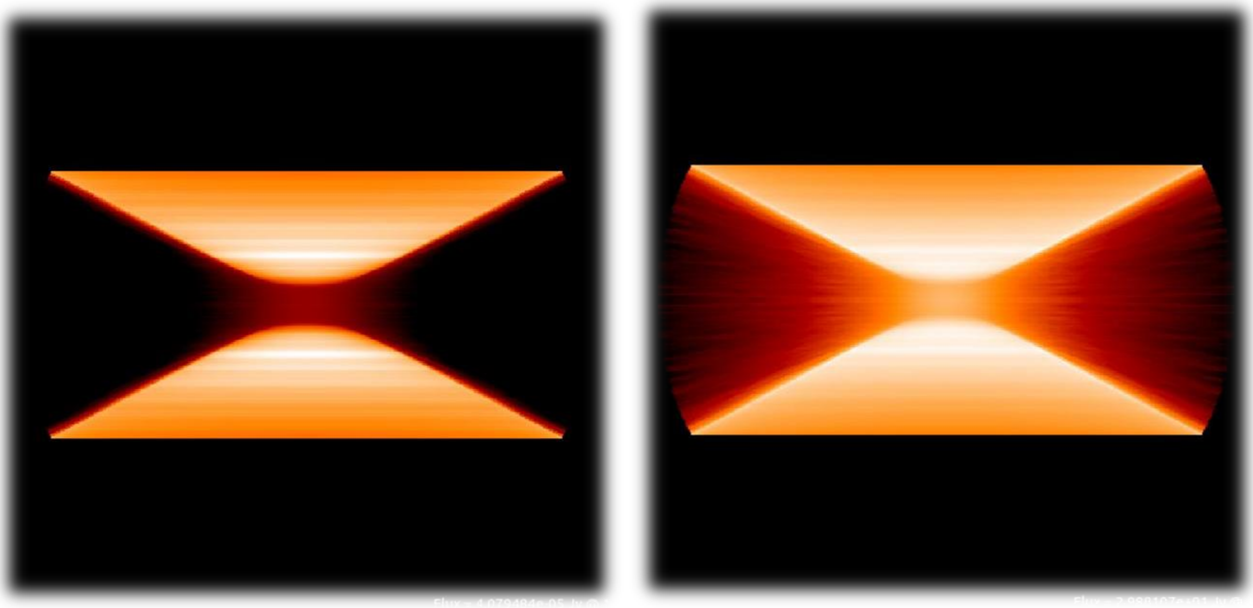


Figure 14: Sun-like star with cone opening at 10µm and 20 µm, the total size of the model is ~ 200 · AU

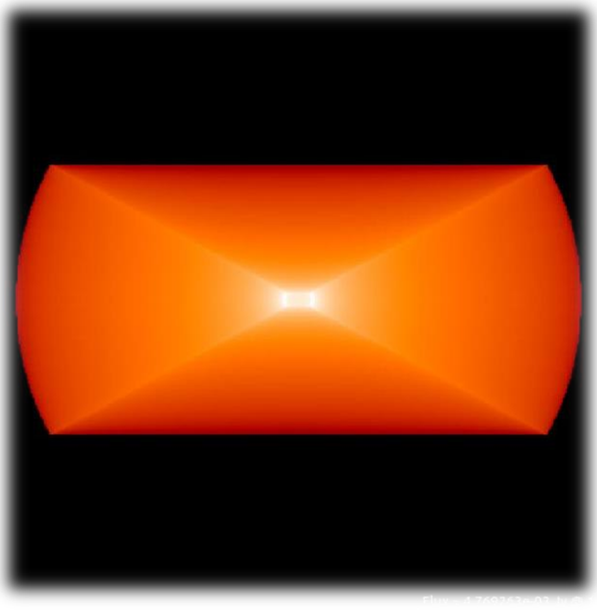
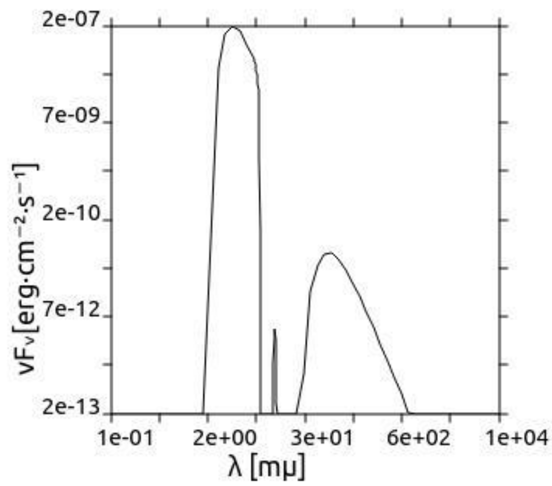


Figure 15: Sun-like star at 160 µm



SED 3 SED of a sun-like star with cone

Note that the first two figures show that the star illuminates the cone. The last figure shows the star at 160 µm and shows, that at some point the gas itself is emitting radiation, as explained in the theoretical part above.

The SED is an overlay of 2 black bodies, once the star and once the material surrounding the star with its silicate feature (e.g. the peak at ~ 10 micron).

3.1.4. Single star in circumstellar disk

The fourth model is an almost finished disk, filled with silicate dust with a density of $3 \cdot 10^{-18}$ g/cm^3, around a sun-like star. Here both seen pole-on (90° inclination) and seen edge-on (0°). The size of the inner radius is ~ 0.3· AU, the outer radius of the disk is ~ 1· AU.

Pole-on:

Figure 16: Sun-like star in a circumstellar disk at 10 and 160 µm, the total size of the model is ~2 · AU

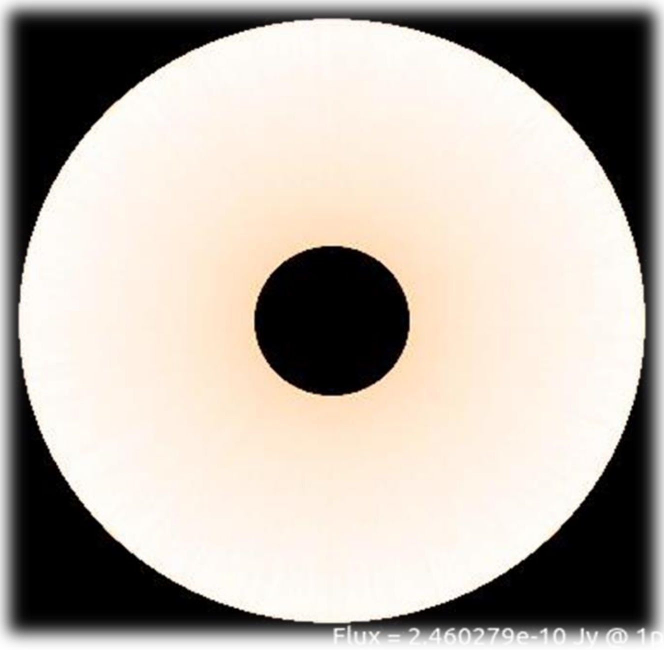


Figure 17: Circumstellar disk around a sun-like star at 10000 µm

Due to the low density of the disk and hence its little contribution towards the emission, Radmc3d could not plot the disk at 160 microns. Note that this happens at almost all frequencies due to the very weak emission. Only at very high and very low frequencies the emission of the disk is high enough, compared to the star, to be plotted again. In Figure 17 the luminosity of the star is too faint for the program to be shown.

Edge-on:

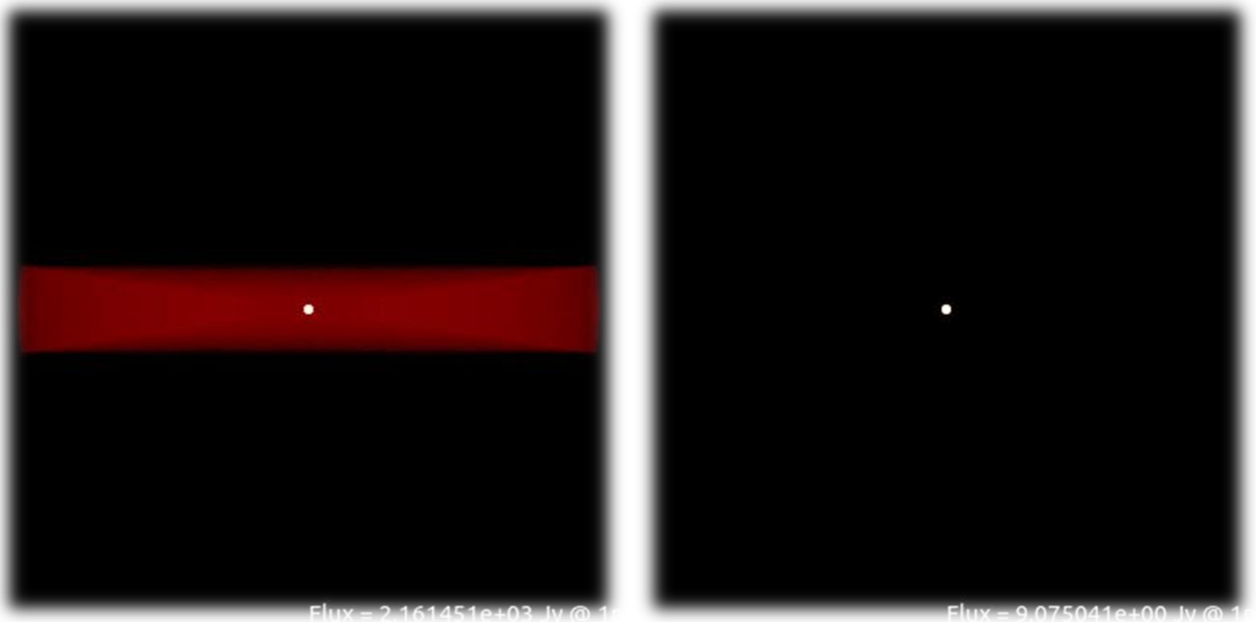


Figure 18: Sun-like star in a circumstellar disk at 10µ and 160 µ, the total size of the model is ~2 $\cdot AU$

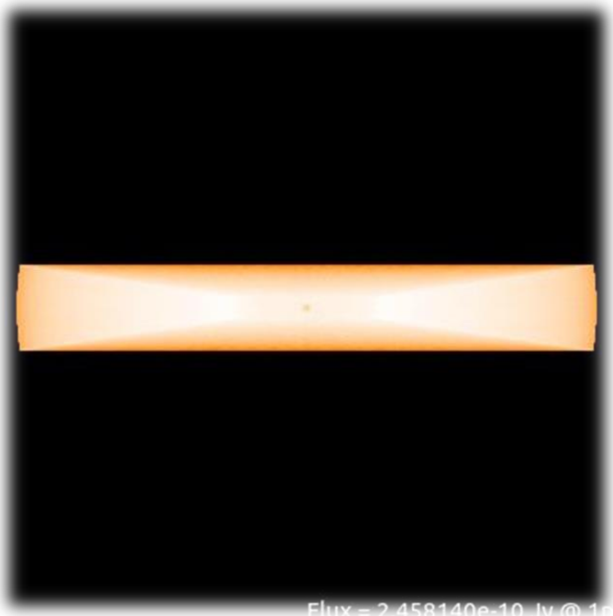


Figure 19: Circumstellar disk at 10000 µm

Figure 19 shows similarities to Figure 15 with the cone being wider, illustrating the evolution from a spherical envelope to a circumstellar disk.

While in Figure 18 (right) the circumstellar disk is too faint, compared to the central star, to be shown it is opposite in Figure 19, where the star is too faint compared to the circumstellar disk, to be shown.

Due to problems within Radmc3d no SED could be derived for this model.

3.1.5. Single star in multi-layer envelope

The last single star model is a simulation of 3 layers of different densities and gas-dust types (the inner and outer envelope are carbonate dust particles with silicate in the middle envelope) around a star with M_s, $2 \cdot R_s$ and $3 \cdot T_s$. The luminosity can be derived as stated in 1.1.2 and is, in this setup 324 times the luminosity of the sun. The densities are 10^{-15} g/cm^3 in the centre but their envelopes have different radii. The inner envelope has a radius of ~15· AU, the second of ~30· AU and the third has a radius of ~55·AU. Stars can be covered in different layers of material during their evolution. This model is to show the potential SED of a star that lies within these layers.

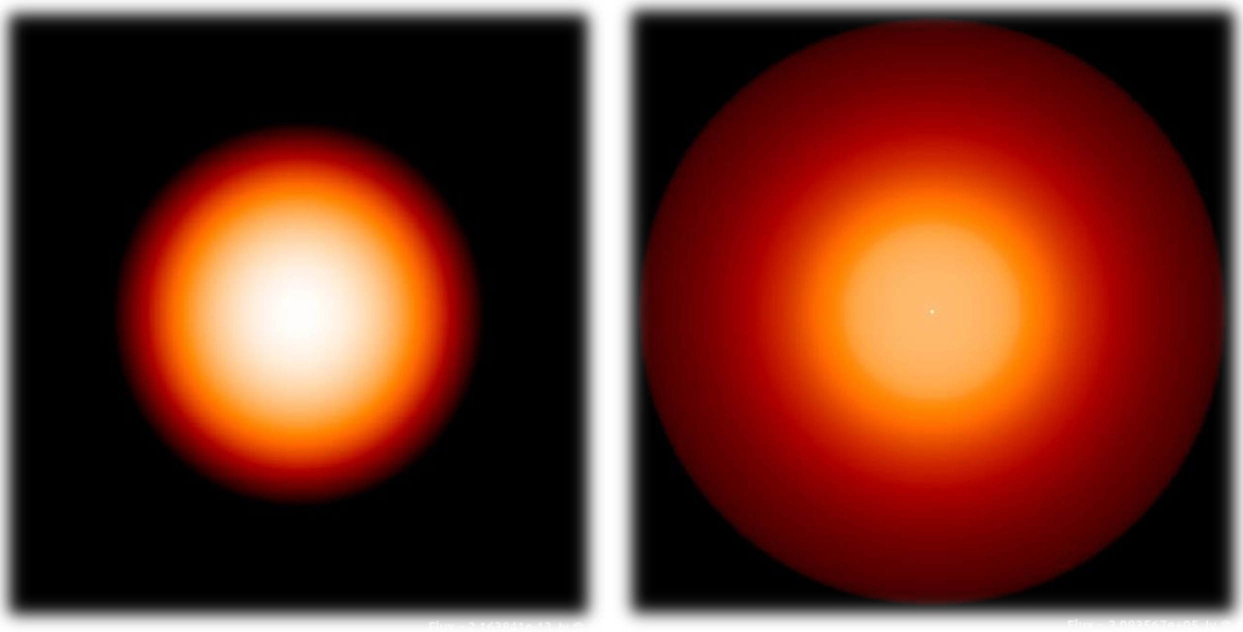


Figure 20: Star in multi-layer envelope at 1µm and 3µm, the shown frame is ~ 120 $\cdot AU$

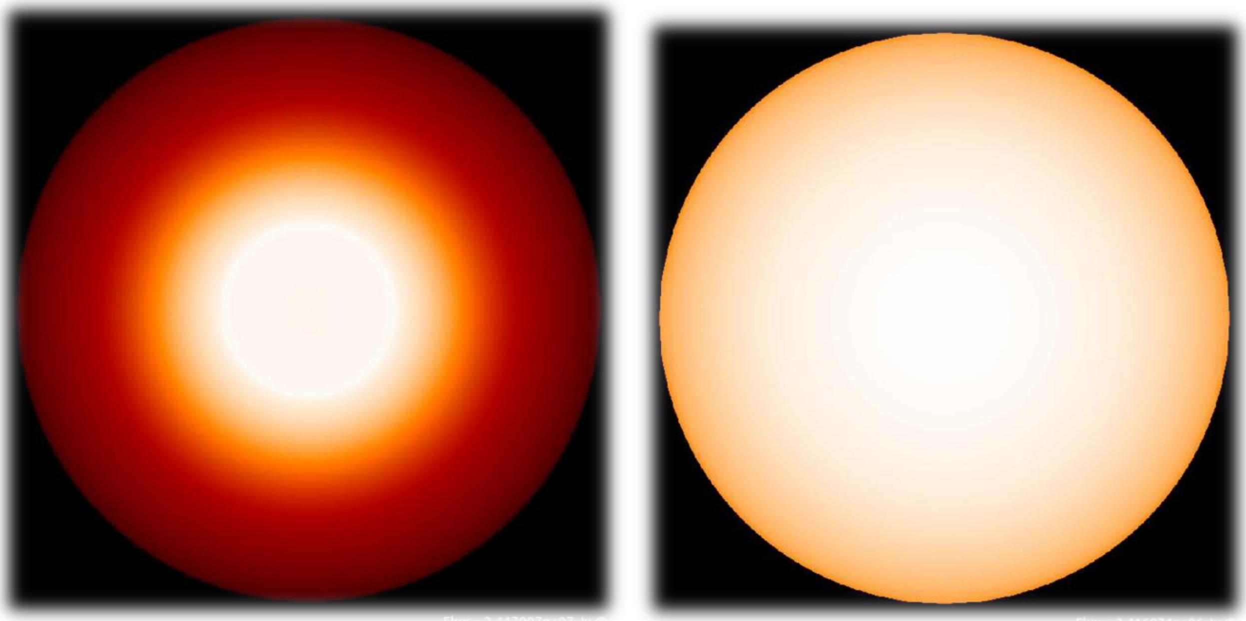


Figure 21: Star at 7 µm and 10 µm,

Figure 20 on the right shows, that the dust surrounding the star emits more at 1 µm than at 3 µm, therefore it seems more faint at 3 µm but due to the bigger surface, the absolute emission is higher at 3 µm than at 1 µm.

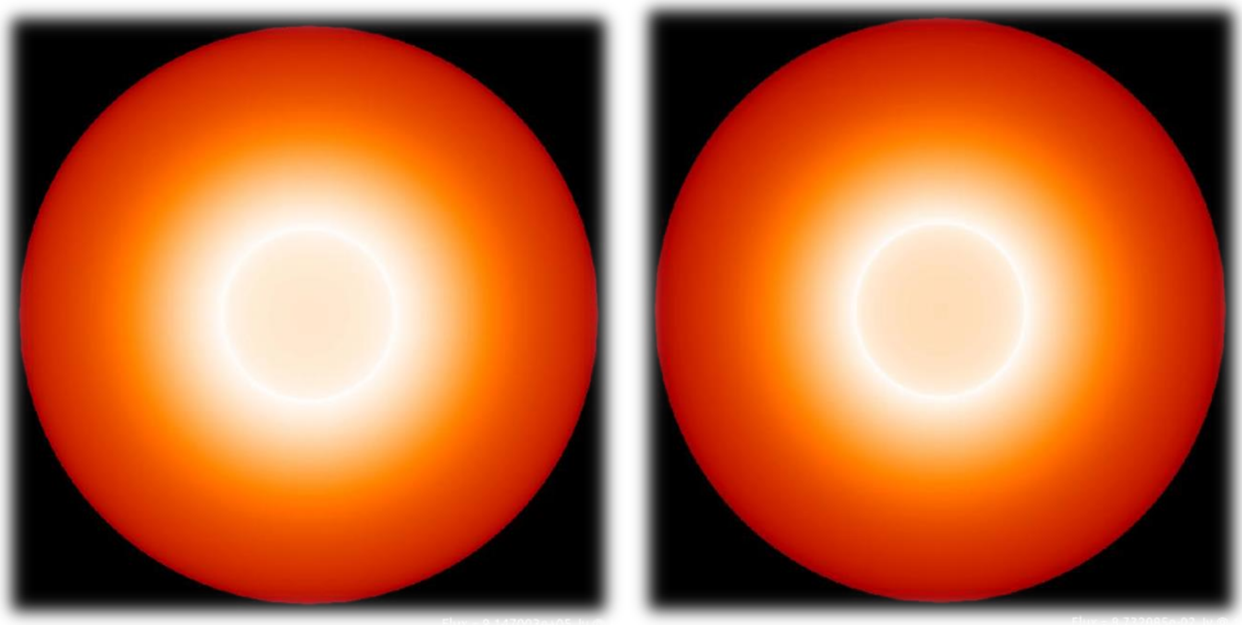
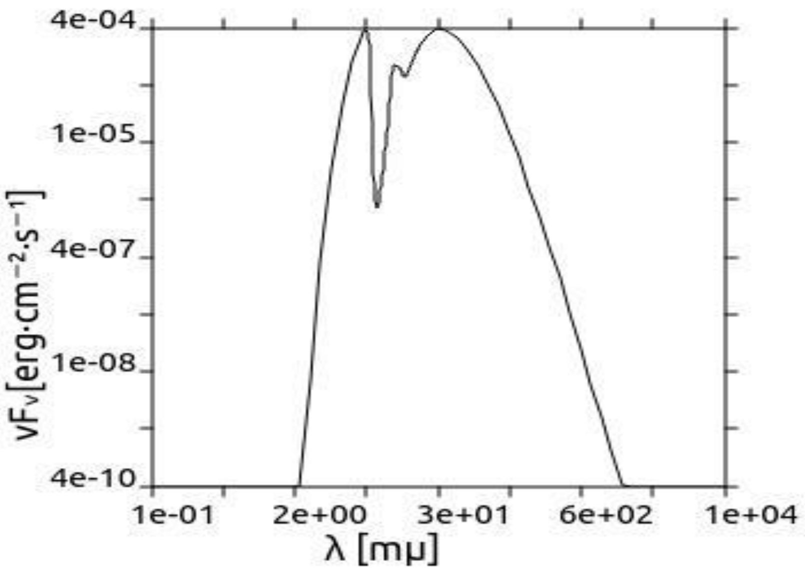


Figure 22: Star at 160 µm and 10000 µm

The envelope appears very small at shorter wavelengths, because the dust surrounding the star absorbs all the energy. At longer wavelengths the outer shells emit the lower frequencies, leading to Figure 22 showing that the system even shows layers at very long wavelengths.

The SED shows that silicates and carbonate are in the envelope, due to the very specific peaks from silicate and the extra emission of carbon.



SED 4 SED of a star with 3 different density layers.

Models of binary systems

3.2.1. Binary system with circumstellar disk

In this subsection several models of binary systems are shown. At first, a binary system, where a sun-like star has a circumstellar disk filled with silicate dust and the second star, with $10 \cdot T_s$ and $1 \cdot R_s$, is located at a distance of $0.7 \cdot$AU in y and $0.5 \cdot$AU in z axis. The disk has an inner radius of 0.6 AU and an outer radius of 1 AU. The density of the silicate disk $3 \cdot 10^{-14}$ g/cm^3.

Edge-on:

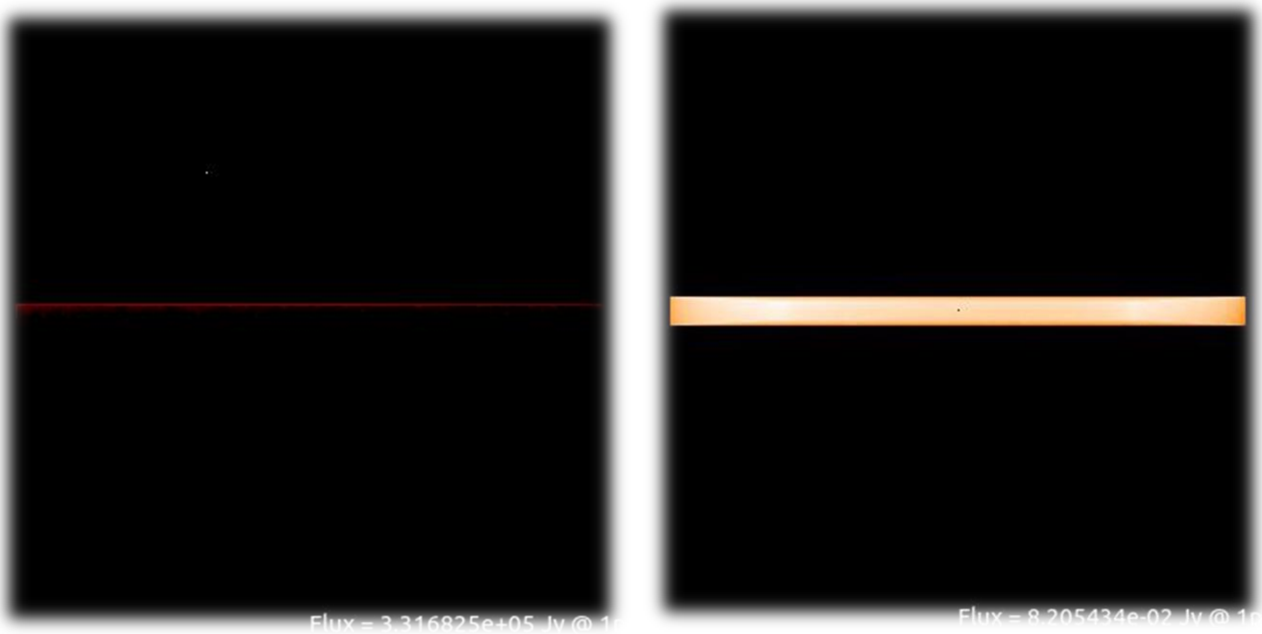


Figure 23 Binary circumstellar disk with a $10 \cdot T_s$ star placed outside the centre: left 0.83 µm and right 1000 µm, the total size of the model is ~2 · AU

Pole-on:

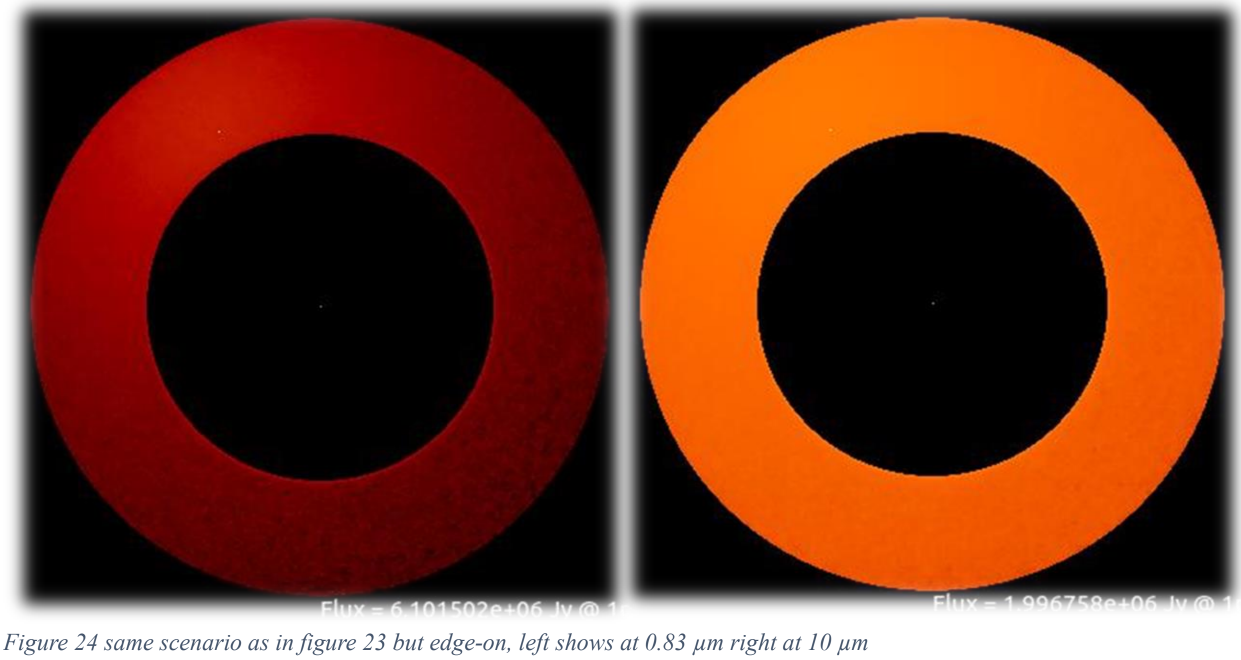


Figure 24 same scenario as in figure 23 but edge-on, left shows at 0.83 µm right at 10 µm

At short wavelengths, the hotter star is the dominant photon source and illuminates the disk, Figure 23 (left) and 24 (left), only the surface of the disk gets illuminated. Due to the placement of the star and the symmetry of the disk, the material closer to the star above the disk is illuminated more than the material farther away. This explains why the left part of the disk in Figure 23 (left) seems to be a little bit brighter than the right part.

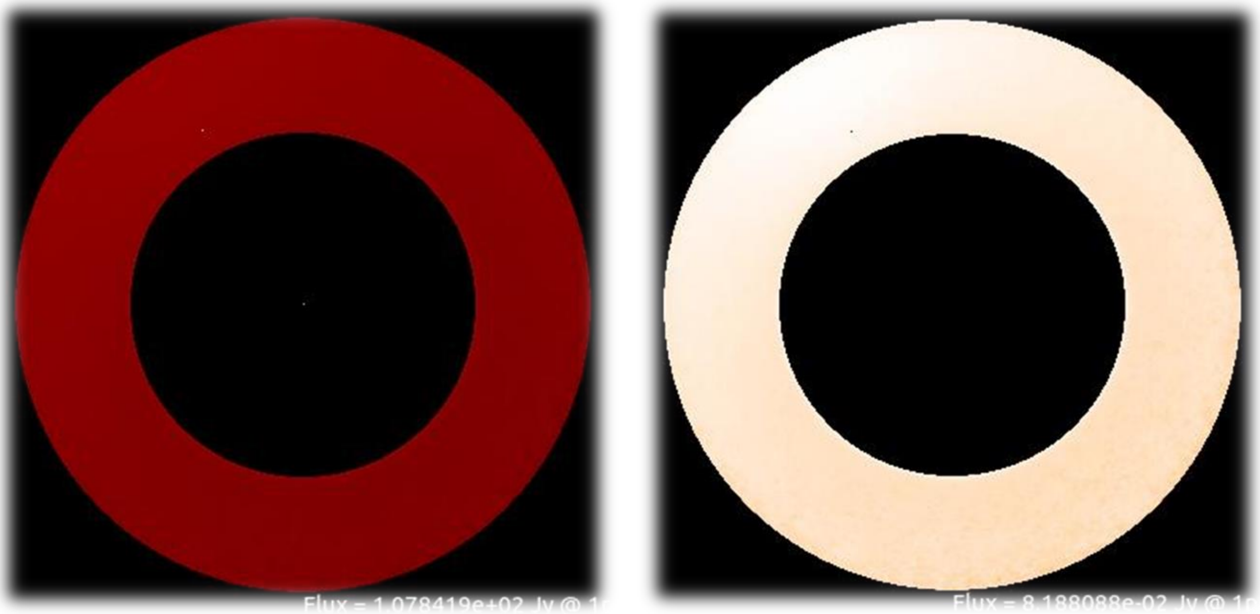


Figure 25 same scenario as in figure 23 and 24, at 140 µm and 1000 µm

At lower frequencies, the cooler star starts to contribute and illuminates the disk from the centre as seen in Figure 23 (right) and 25 (right).
Please note that the white pixel represents the star. Due to the size of the disk the star seems very small. At 1000 µm the central star is too weak to be shown in the figure.

As mentioned before, it was not possible to calculate the SED of circumstellar disk due to a yet unknown problem.

3.2.2. Binary systems with one envelope

The next models show a binary system, where a sun-like star is embedded either within a single- (left) or within a multi-layer envelope (right). The single layer has only one layer of silicate dust (with 10^{-16} g/cm^3 density) which has a Gaussian distribution. The multi-layer model has 2 different gas types in its envelope (once silicate with 10^{-16} g/cm^3 and once a carbonate dust with 10^{-15} g/cm^3) which are also Gaussian distributed.
Both models have a star with $10 \cdot T_s$, $10 \cdot M_s$ and $50 \cdot R_s$ in $10 \cdot$AU distance illuminating them. The radius had to be set this big in order to make the star visible in the first few figures.
Shown on the left is the single layer and on the right side the multi-layer scenario. Note that the wavelengths are not necessarily the same, but are rather selected to show the effects in the best possible way. Please note, that, while similar in their construction, the models have important differences. The single-layer setup has its outer radius at $5 \cdot$AU, while the multi-layer has the first outer radius at 4 and the second at $7 \cdot$AU.

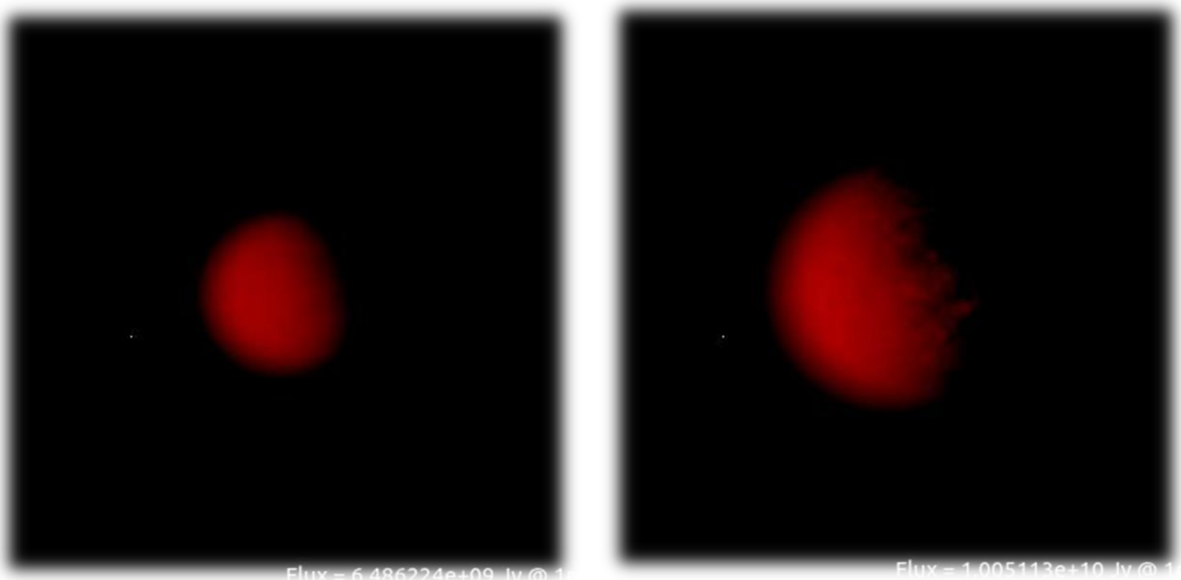


Figure 26 on the left the single-layer model at 0.36 µm on the right side the multi-layer model at 0.31 µm, the total size of the model is ~ 40· AU

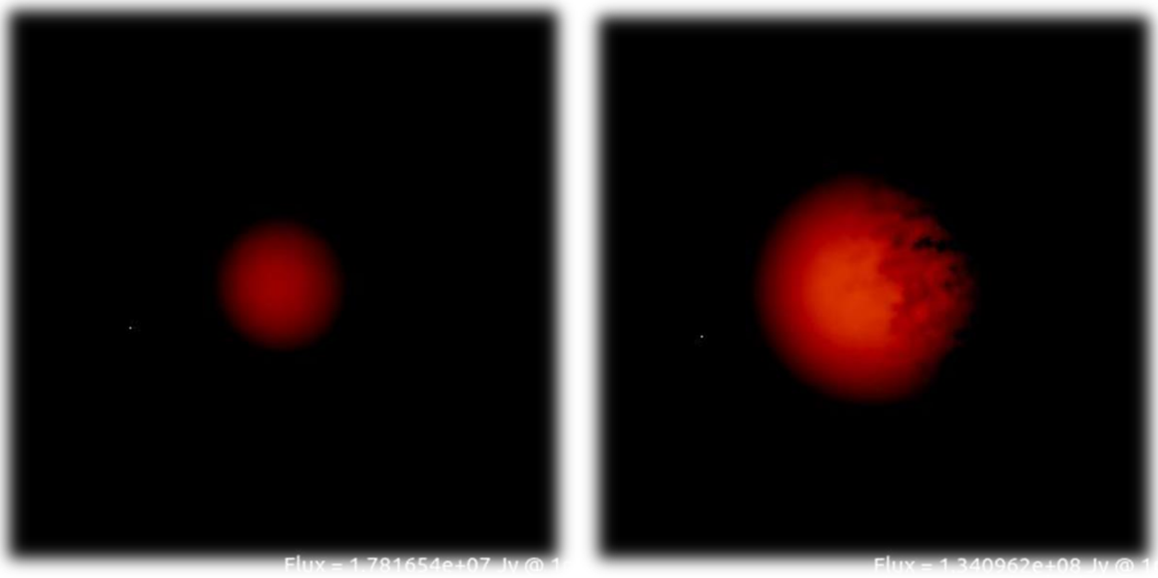


Figure 27 left at 7 µm, right at 5 µm

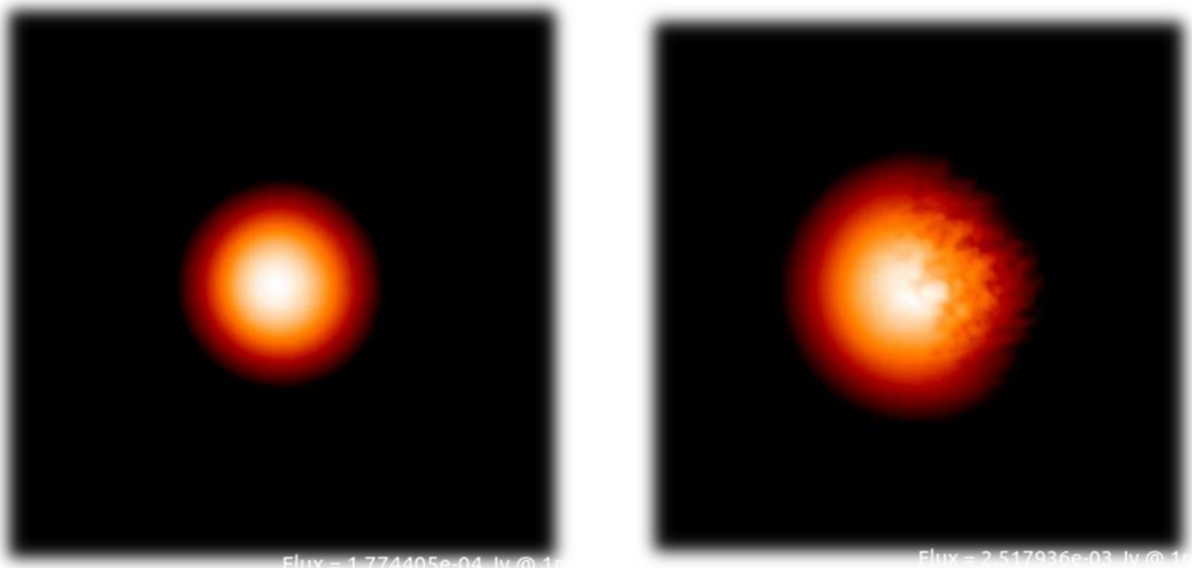
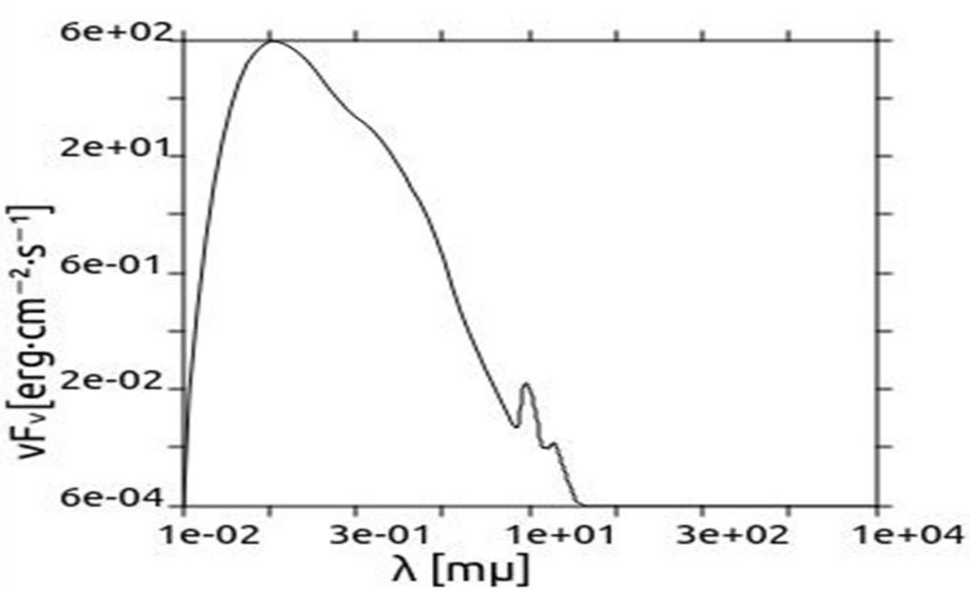


Figure 28 both scenarios at 10000 µm

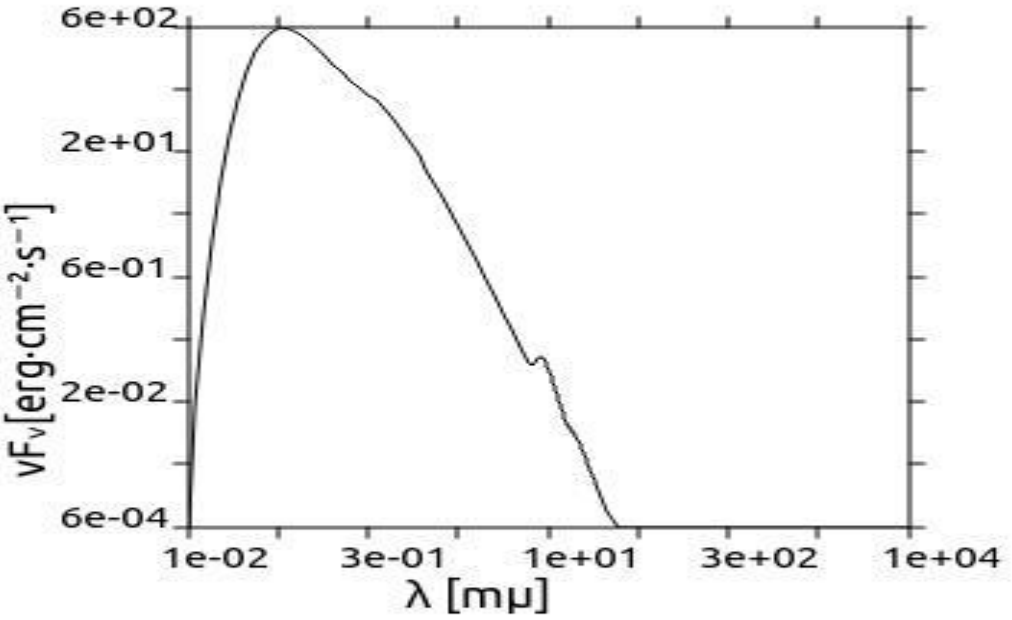
It is visible, that the envelope on the right hand consists of more than one gas, showing a non-linear temperature distribution, while the single layer shows an almost homogeneous temperature distribution.



SED 5 SED of the single-envelope scenario

The SED of this scenario is dominated by the radiation emitted at short wavelengths due to the fact that the hotter star is not surrounded by any gas and therefore contributes massively to this SED.

SED 5 is an overlay of the SED of a black body (the star without the envelope) and the emission of the optically thin envelope layer that is illuminated from the outside.
The second star is behind such a dense envelope that he barely contributes at all.



SED 6 SED of the multi-layer scenario

The multi-layer embedded star heats up the high-density envelope and therefore shines stronger in infrared due to the size of the envelope. The silicate peak at 10 microns reappears in SED 6, which is a complex overlay of the SED of the black body (the star without the envelope) and the emission and absorption of layers of the envelope that are illuminated by the star. Note that the silicate emits at 10 microns due to the fact that it is an optically thin layer in this setup.

Also, note, that in the multi-layer scenario, the central star is not hot enough to illuminate the envelope completely, leading, with the radiation of the outer star, to a not perfect spherical envelope radiation. In the single envelope scenario, the density is low enough for the cooler central star to illuminate it completely. Thus, seeming almost perfect spherical at very long wavelengths. In Figure 26, due to the very short wavelengths, only the hotter star contributes a significant amount of radiation and shapes the envelope not perfectly spherical.

3.2.3. Binary system with two envelopes and constant background density

The last setup consists of 2 sun-like stars in their own envelopes with a constant background. While the star in the left is surrounded by less dust and gas, the star on the right side is covered in a 10 times denser envelope. The whole volume is filled with gas and dust of constant density. While the first star is positioned in the centre, the second star is positioned in a distance of $\sqrt{3}$ AU (-1 AU from the centre on every axis). The dust particles consist of silicate and carbonate gas with densities of 10^{-18} for the background density and 10^{-16} (for the star in the left side) and 10^{-15} g/cm^3 (for the star on the right side). Note that due to the placement of the off-centre star, it seems that the stars are not of equal size.

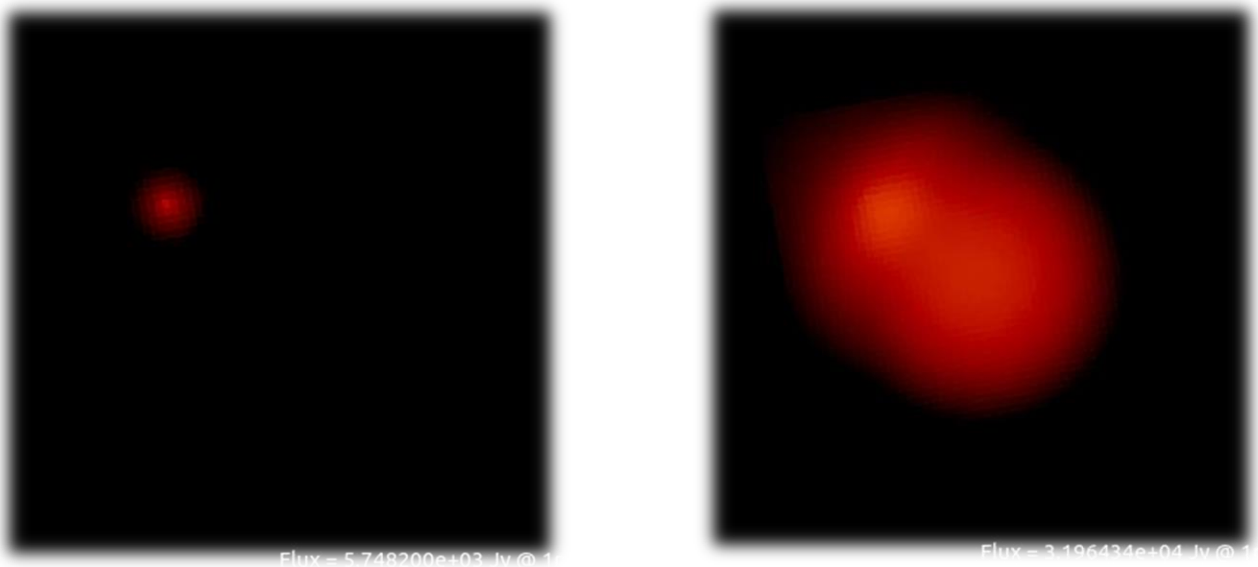


Figure 29 left at 1 µm right at 5 µm, the total size of the model is ~5·AU

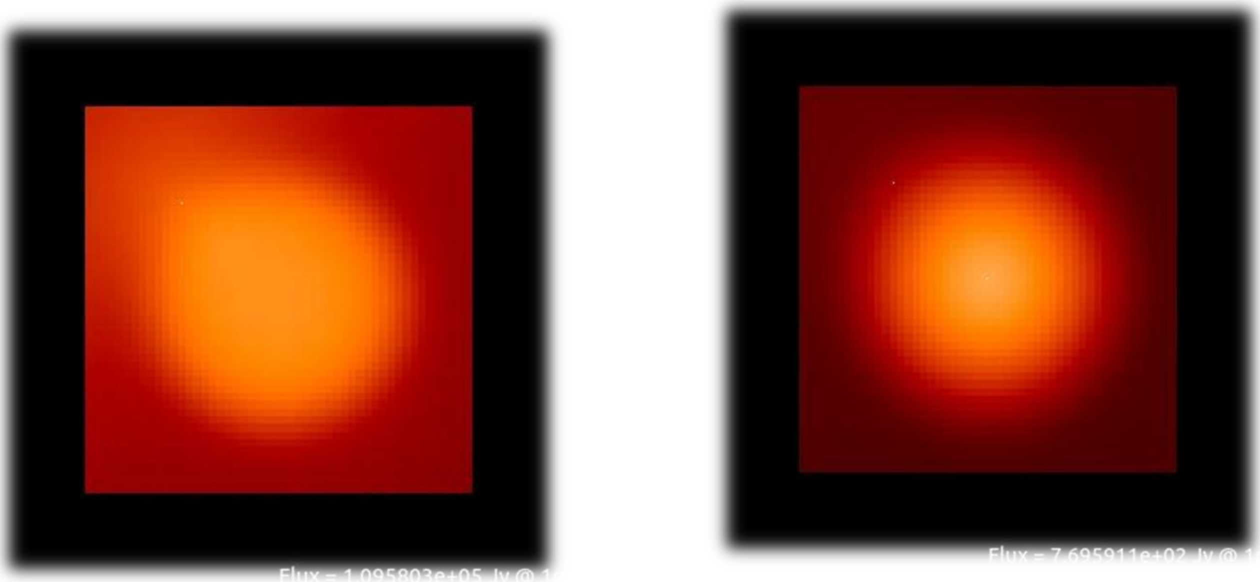


Figure 30 left at 17 µm, right at 160 µm

At first, the envelope with less gas and dust is visible, but with increasing wavelength the other envelope contributes. Figure 30 (left) shows, that at medium wavelengths it is still visible that these are two stars.

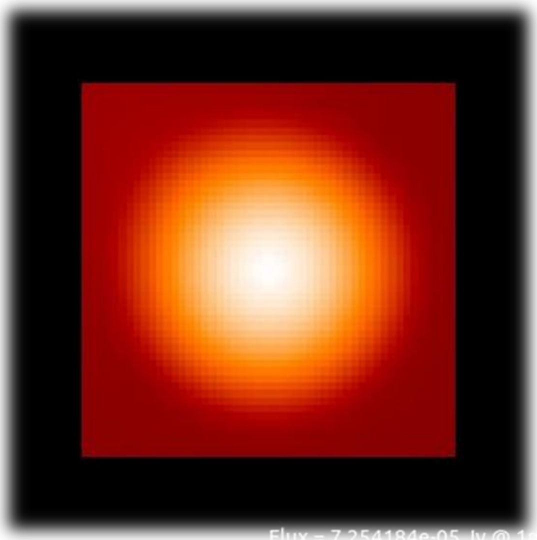
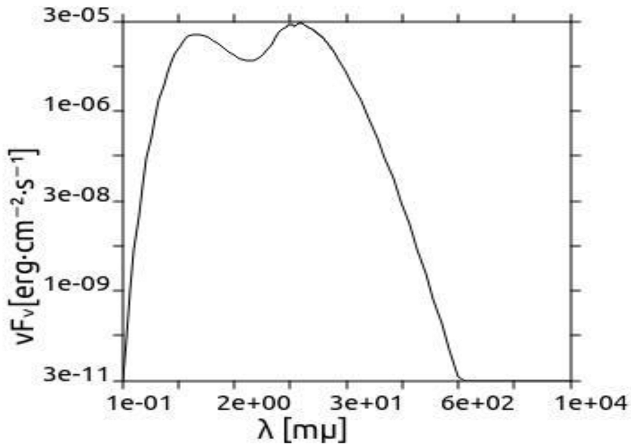


Figure 31 at 10000 µm



SED 7 SED of the binary-system with linear background density

The right side and Figure 31 let the stars seem like one gigantic star with a radius of ~ 2.5·AU and medium temperature. The background dust is clearly visible at Figure 30 and 31. SED 7 shows the overlay of the SED of the two stars with their envelopes and the SED of the constant background.

While it is possible to differentiate both stars at very short wavelengths, they look like one gigantic star at lower frequencies where the envelope contributes massively to the SED. Note that this effect is supported by the fact that the second star is placed almost behind the first one, which causes the light of the off-centre star, to interact not only with the off-centre star's envelope but also with the central star's envelope. This leads to a weaker contribution to the SED from the off-centre star.

While these models present possible scenarios, they are not scientific simulations yet. For using these scenarios, one would need to compare optical thickness and other data with real measurements.
What is left to do is to make models with different opening angles and orientations, e.g. two stars with their own envelopes and their cones being opened into different directions or star systems where every star has got an own circumstellar disk with different inclinations.

4 Appendix

4.1 Installation Guide

Due to *Windows* being the most common Operating system (hence OS) and the fact that Radmc3d can only be run within a *Linux* based OS it seems necessary to mention how the program was installed. As explained in the Radmc3d section, it might be possible to install the program directly within Windows, however, it has not been possible for me to do so.

While the manual explains how to install the program within *Linux*, this section will describe how to *emulate* a totally free OS within *Windows* with all the programs necessary, so one can then follow the manual. Note, that the emulation of another OS does not change or delete your files within windows but only uses up space on the hard drive and resources from the computer itself, e.g. memory and processors, which can, if simultaneously used, slow down the original OS a bit.

At first an Ubuntu distribution has to be downloaded, e.g. from https://www.ubuntu.com/download/desktop and saved. Do not install it yet! Then we need to download a virtual machine, in my case *Oracle VM VirtualBox* and install it. Note that you should run it in administrator mode. During the installation, you will be asked if you want USB support, it is advised to select the "Entire feature will be unavailable" feature, making it easier to uninstall the virtual box later.

After the installation, select the NEW button within *VirtualBox* and select the OS you have downloaded, e.g. Ubuntu 14 x64. During the process, you will get to allocate RAM and if you want to create a virtual hard drive if not already existent. The most important section is, if you want to allocate dynamically storage space or fixed space. Since Radmc3d does not use a lot of space, it is advised to run the dynamically allocation, as it uses only the amount of storage which is really used, for example if you use the fixed allocation, e.g. 8 GB, there will be always 8 GB used on your hard drive even if you have only used up 20 MB. With dynamically allocation the virtual machine would use 20 MB.

Now select the folder to create files and set the upper limit of the storage space. After this process, you can change the created OS, e.g. click on the change button and then the storage panel to select which file you want to use for the OS. Now run the OS.
For a more detailed guide please visit the German site:
http://de.wikihow.com/Ubuntu-in-VirtualBox-installieren

Then download Radmc3d from http://www.ita.uni-heidelberg.de/~dullemond/software/radmc-3d/
From this point on, the installation follows the manual. Instead of IDL you can download GDL. The easiest way to start the problem_setups is to go into the folder of the problem, e.g. run_simple_1_layers, right klick and select the command line and then type:

make cleanall (in case you changed your problem_setup)

gdl (if you got idl, you type idl)

.r problem_setup.pro (if the problem_setup file was renamed, you enter filename.pro)

Exit

Radmc3d mctherm
Viewimage
After the last command, there should be a Graphic User Interface, hence GUI, where your setup is visualized.

4.2 References

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The used models can be found on the webpage where this PDF has been provided.