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BACHELOR'S THESIS

# The Grand Tack

# An Overview Of The Next Big Thing In Planetary Evolution

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## Abstract

*The Grand Tack Model is a recent hypothesis concerning the evolution of the solar system's planets. It uses the in- and then outwards migration of giant planets Jupiter and Saturn to explain among other things the small mass of Mars as well as the composition of the Asteroid Belt. Here we will take a closer look at certain important features of the Grand Tack. We will discuss the classical theory of planetary formation and then the different types of planetary migration and how they can be implemented in the Grand Tack, as well as the constraints that the model has to satisfy to lead to the state of the Nice model which describes processes that happen after the Grand Tack and links it to our current planet states. We then will go into the consequences that arise from the Grand Tack and how these can help to test and improve the model. Then we will look at different alternative explanations for the problems for which the Grand Tack was proposed (namely the small Mars problem and Asteroid Belt structure), and will compare these other models with the Grand Tack regarding their problems.*

## I. Introduction

Our theory of planetary formation is in its core a simple one. Gas and dust form a disk around a newly formed star and this disk segregates into planetesimals which in turn collide with each other to form planets. But this is the most simplistic form of this theory and it has many problems which have to be accounted for.

In the last decade, telescopes discovered hundreds of exoplanets around other stars, and many of them were gas giants like Jupiter and Saturn which revolve around their stars at extremely close distances. Of course, the means of detection used for finding exoplanets favor such 'Hot Jupiters'; but they still make up a great percentage of exoplanets even when we take this bias into account. Planetary formation can not explain how gas giants are able to form inside of the Ice Line, where their primitive atmosphere would have been blown away by solar wind and radiation pressure and where they would not have had enough material to form in the first place.

Thus, planetary migration was introduced. Two (later three) different mechanisms were identified that terrestrial and giant planets can use to change the size of their semi-major axis.

Another problem stems from computer simulations that tried to reconstruct how our own solar system formed. Most of these models failed to resemble its current state. Planetesimals accreted in the Asteroid Belt, which had a few magnitudes more mass, and Mars usually was much bigger than it really is. Varying the simulation's parameters helped: For example, a disk which is truncated to a small ring between 0.7 AU and 1.0 AU was able to recreate small Mercury and Mars outside the ring while keeping Venus and Earth inside it heavy. This raised another question: If this is the correct explanation, what truncated the disk?

The Grand Tack Model delivers an explanation to this and a few other questions. The term 'Tack' comes from sailing, where a boat is going at an acute angle against the wind and changes its direction in fast maneuvers repeatedly from left to right and back. This sudden turn is called a tack, and it is exactly what is happening in the Grand Tack with our big gas planets Jupiter and Saturn.

Beyond the Ice Line there is a lot more material for planets to accrete and so Jupiter and Saturn are already more or less finished when the terrestrial planets just begin their formation. Jupiter wanders inwards via Type II migration and by doing so concentrates the solid objects in the gas disk. Saturn begins its migration later via Type III which is a lot faster than Type II and so manages to catch up with Jupiter, which at this moment has compressed the planetesimal disk down to a distance of 1.0 astronomical units from the Sun. When Saturn's and Jupiter's orbital periods reach a resonance (most variants of the Grand Tack use 2:3), their inwards movement stops and reverts - the famous Tack happens. At this point the two planets have truncated the disk to make our small Mars possible and additionally depleted the Asteroid Belt and scattered different types of asteroids through the solar systems, explaining today's Asteroid Belt's structure of two distinct groups of asteroids as well as how water-rich comets made it into the inner solar system and therefore to Earth.

# II. PLANETARY FORMATION

## I. Star Formation

Let us begin right at the beginning - the birth of a star.
Stars begin their existence in Giant Molecular Clouds (GMCs) which are in astronomical terms very dense and cold. They contain hundreds of molecules (mostly $H_2$) per cubic centimeter and have a temperature of about 30 Kelvin. Higher temperatures prevent gas atoms to form molecules in hotter clouds. If a GMC is big enough, it can collapse under its own gravity. The Jeans formula can give us a minimum mass for the cloud's collapse depending on its density and temperature (other effects, such as magnetic fields and turbulences, increase this minimum mass).

$$M \propto \sqrt{\frac{1}{\rho}\left(\frac{kT}{Gm}\right)^3} \tag{1}$$

$\rho$ is the density of the gas cloud, $k$ is Boltzmann's constant, $T$ the cloud's temperature, $G$ the gravitational constant, and $m$ the molecular mass. $\rho$ and $T$ are assumed to be constant everywhere in the molecular cloud. This is only an approximation, as star formation areas usually aren't spherical and boring but filamented and complex, with ionized particles causing magnetic fields which restrict other charged particle's movements and additionally increase the critical mass. When the GMC collapses its density increases, which in turn decreases the Jeans mass. As individual parts of the cloud now can collapse independently to the rest, the cloud segregates. It splits up into a number of cores which all compress on their own because their density is at least a little bit greater than in the overall cloud; many of these cores will become a star or a system of multiple stars. Because such clouds have (despite their small rotational energy) due to their size a substantial amount of angular momentum, at some point the cloud's cores are too small to allow further collapse. Gas is unable to simply fall down onto the recently formed protostar but instead forms a differentially rotating disk around it. Within the disk the different orbiting velocities allow the transport of angular momentum outwards so that gas can slowly but steadily wander inwards and be accreted by the protostar. This disk therefore is called the accretion disk or, because this is also the disk from which the planets draw their material, the protoplanetary disk. At this stage we have a protostar in the middle of our system. It is still increasing its mass, albeit at a slower rate. Its temperature is usually about 5000 K because the contraction releases the gas' potential energy as heat. Revolving around it is a disk of gas and dust particles [Bennett, 2010].

## II. The Protoplanetary Disk

We assume that this protoplanetary disk consists of the same composition of elements as the (proto)star. The biggest part of it is Hydrogen and Helium gas, which make up approximately 99 % of the disk's mass. The rest are other gases, ices, and dust. Ices are made up mostly of $H_2O$ and $CO_2$, with an ensemble of different molecules added such as $CH_4$ and $NH_3$. The disk's vertical height is defined by its scale height. As [Armitage, 2017] was able to show, disks are usually lightly flared - or as Armitage put it, the disk possesses a bowl-like shape. The temperature is assumed to be the same at every height in the disk from its surface to the midplane although it of course decreases with increasing distance to the star.

The gas in the protoplanetary disk rotates somewhat slower than it would due to Kepler's laws of motion. Because the gas pressure gradient points outwards we have a force acting against the star's gravity, lessening the force that the gas experiences. The condensed particles in the disk (which are usually about 1 $\mu m$ in diameter), namely the condensed dust and further out also the ice, do not feel this pressure and therefore orbit the star a little bit faster. They behave like all other objects on Keplerian orbits and have a velocity

$$v_K = \sqrt{\frac{GM}{r}} \,. \tag{2}$$

The difference in speed of gas and particles is about one percent of $v_K$. This difference results in a drag exerted by the gas on the dust particles, which a) makes them lose kinetic energy and therefore lets them move inward, and b) works against their inclination, effectively concentrating them in the disk's midplane [Weidenschilling, 1980].

Because the radiation and particles that get sent out from the protostar blow away most of the disk, one can only speculate how massive the Sun's protoplanetary disk was. By estimating the amount of non-hydrogen material in our planets and assuming the composition of the disk to be the same as the Sun's we can calculate how much gas there had to be in the disk *minimally*. This is called the minimum mass solar nebula (MMSN) and amounts to a few percent of the solar system's mass [Chambers, 2004]. Of course this assumes that all dust has been incorporated into planet formation, so we can expect the real disk mass to be larger than the MMSN. The most common formula for calculating the MMSN was presented by [Hayashi, 1981]:

$$\Sigma = \Sigma_0 \left(\frac{r}{1AU}\right)^{-1.5} , \quad \Sigma_0 = 1700 g cm^{-2} \,, \tag{3}$$

where $\Sigma$ is the surface density of the disk at distance $r$ and $\Sigma_0$ is the surface density at 1 AU. Integration over $0.35 \leq r \leq 36$ AU gives us a total mass of 0.013 solar masses [Hayashi, 1981].

One can measure the mass of disks (and if not more so at least as important, the mass distribution) around other stars, usually via two different methods. One can measure the intensity of light the disk emits, coming either from its gas or its solid particles. Since $H_2$ and Helium do not emit radiation at the energy levels corresponding to the expected temperatures in disks we have to rely on other molecules - usually carbon monoxide. Then when we assume that the amount of CO in these disks is the same as in comparable warm molecular clouds [Lacy et al., 1994], we can calculate the mass of the disk. Another possibility is to use Hydrogendeuteride. HD has the advantage that we know the ratio of Deuterium to Hydrogen very well. CO on the other side probably is much rarer in protoplanetary disks than it is in molecular clouds, probably because of photodissociation. The disadvantage of HD is that the used spectral line (J = 1 - 0) only emits at $T > 20K$, therefore colder gas farther out in the disk can not be detected [Powell et al., 2017].

The second method is measuring the thermal emission of dust particles. The gas disk is highly transparent for the infrared spectrum, so we can assume that no light got lost on its way. By measuring the intensity of the received light (which is proportional to the dust mass and its temperature) we can calculate the disk mass by assuming a dust-to-gas ratio. Here we are facing the same problem as before: we do not know how much dust there is, and how it is distributed. As mentioned, gas drag sends bigger particles into the Sun. Combined with the gas loss due to the disk's dissipation, we have a dust-to-gas-ratio that varies over time. Also, the dust's emissivity depends on the grain size, which again changes over time. Still, the dust method is our best way to derive the disk mass. It and the measurement of HD lines produce similar results while the CO method can yield up to ten times less mass.

[Powell et al., 2017] were able to demonstrate a new method by which disk masses could be determined. The last fact, the dependence of grain size, can actually be used to determine a dust distribution. The maximum particle size decreases further out as bigger particles need more time to form and move inwards during this time [Birnstiel and Andrews, 2014]. By this the age of the disk can be connected to the maximum particle size at a certain radius, which can be measured by observing different wavelengths. [Powell et al., 2017] applied their method for the disk of the near T Tauri star TW Hydrae, for which the disk mass actually was a little higher than the MMSN.

There is one remaining question we want to answer here: Is the disk mass correlated to its host star mass or is it roughly the same for every star independent of its size? Analysis of protoplanetary disks around other stars from [Andrews et al., 2013] suggests that those two masses are correlated, as one's intuition would tell. They determined the ratio of disk mass to star mass to about 0.2 to 0.6 percent. Please note that this is much less than the MMSN, because the density decreases over time and this is an average over disks of different age. The initial surface density can be assumed to be ten times higher with a few percent of the star mass [Robinson et al., 2006]. But more important than the value itself is that this ratio is nearly constant over star masses. This has interesting implications regarding different formation theories, which we will discuss later.

The dust particles have small relative velocities $v_rel$ to each other, so they tend to stick together when they collide. This produces planetesimals, growing steadily in size. There is a critical point in this accretion when the chunks of matter are about 1 meter in diameter, where the effects of gas drag are so big that theoretically the gas should prevent planetesimals from growing. This part is the least understood in the planetesimal theory of planetary formation.

## III. Use Of Turbulences

There are two big theories to explain how planetesimals form, each of them carrying its own unsolved problem. We'll go through them in a short way and then introduce turbulences as a possible explanation and a compromise between them both. This part of planetary formation theory is highly debated and we can not be sure how planetesimals really develop; the important part here is that these theories all end up in the same place, one which allows us to proceed in creating planets.

The first scenario is formation through accretion. Small particles, as mentioned, stick together due to cohesive effects. Later when they reach high masses, they can also attract other planetesimals and also gas if it is still present via their gravitational force. But, as we said, the gas stops this development. At a certain size those planetesimals tend to not only stop growing but also to lose angular momentum and energy and then fall into their star. This is a very fast process, the drift can make up to 1 AU in 500 years [Cuzzi et al., 1993]. According to our understanding of a disk's lifetime this means that when the gas has dissipated all planetesimals should have disappeared in the star. Thus, the core accretion model predicts that planetesimals form very quick, which is contrary to the observed structure of primitive meteorites. [Cuzzi et al., 2008].

The second scenario is the gravitational instability model. Compare the beginning of this chapter, where a gas nebula collapses under its own gravity to form a star. Something similar happens in the protoplanetary disk. Locally, gas can be assumed to move with the same velocity and to have the same density everywhere, but this is not the case on a larger scale. As simulations of [Boss, 2011] show, disks tend to fragment into spiral arms and clumps which can collapse under their own self-gravity. This process is important for planets at large separations to their stars with semi-major axes > 20 AU, since in the core accretion model migration prevents the formation of planets at such separations [Chambers, 2006]. Giant planets are believed to have cores of solid matter making up a fraction of their mass. In the core accretion model these cores exist from the beginning and then accrete gas; in the gravitational instability model, we do not have this constraint. Gas could collapse together with solids and form clumps early on; then the solid parts can take their time to form a core while the rest of the clump collapses very similar to the star formation process. Up to this point, gravitational instability looks like the way to go here. But we have to face a big problem: Given that the composition of the disk is the same as that of the star, the ratio of solids to gas is not big enough for a collapse to happen [Chambers, 2006].
Turbulences in the nebula might solve this problem. As dust settles in the midplane and rotates faster than the gas, turbulence is produced by this shear velocity which stirs up the dust plane and broadens it. Therefore turbulence in a disk can be estimated by the vertical distribution of particles. If a disk is too turbulent the collapse or accretion of dust is prevented, but most disks only show signs of weak turbulence. [Cuzzi et al., 2008] presented a model in which turbulences lead to regions with local particle density maxima. The particle mass in there can be about 100 times higher than the disk average. These regions, called clumps, still fail to collapse like they would in the gravitational instability model. But their own gravity prevents these clumps from being destroyed and homogenized by gas drag and allows its density to slowly increase, using that turbulence.

Now, what can the distribution of exoplanets tell us about which theory is the right one? Since the ratio of disk and star mass is quite constant this implies that the number of gas exoplanets which formed through core accretion is bigger around more massive stars and remains constant when we look at gravitational instability [Andrews et al., 2013]. Observations indicate that core accretion is the dominant process of planetesimal formation, but other processes might still apply. As we now discussed how planetesimals might form, we now go to the next question: Why do giant planets form so much faster than terrestrial ones?

## IV. The Ice Line

Solar radiation becomes increasingly weaker the more distant we are to the Sun. Intensity decreases with $1/r^2$. This means that our protoplanetary disk gets colder the farther we go outside. The disk consists of three categories of components: gas ($H_2$ and He), ice, and dust. Under the conditions in the disk the gas is never able to condense into a solid form (or liquid, although this distinction is irrelevant and will not be made here). Dust already condenses under high temperatures and therefore the terrestrial planets, which lie close to the Sun, are composed mostly of these materials. What we are calling ice here means a lot of different molecules, made mostly of non-metal elements: water, methane, ammonia, carbohydrates. These molecules all condense at lower temperatures than rock and metal. The distance from the Sun where this temperature is reached is called the Ice Line and lay approximately at 3.5 AU at the time when giant planets formed, but moves during the evolution of the disk. Beyond this point there is a lot more solid material than inside of it. This, paired with the less dense gas, makes the accretion of solids into planetesimals much easier and therefore faster [Bennett, 2010].

Protoplanetary disks have an observed lifetime of $10^6$ to $10^7 years$, so giant planets have much less time to form than terrestrial ones. Because after the disk dissipates there is no gas left for giant planets to form (as mentioned earlier, $H_2$ and He never condense, so they are lost when the disk is gone). For this reason gas planets cannot form inside the Ice Line: when their solid cores would eventually be big enough to accrete gas from the disk, the gas is long gone.

## V. Phases of Formation

Once we have planetesimals with sizes of about 1 km (either through accretion or collapse), planetary formation can be divided into different phases. The first one is the 'runaway growth': we start with these planetesimals, which are already big enough to interact gravitationally. Due to their mass, gas drag is now neglectable.

Bigger objects perturb smaller ones which become faster and have more eccentric orbits. These small ones on the other hand tend to circularize the bigger one's orbits due to their homogenous distribution and great numbers. *Dynamical friction* is the keyword here - it means that bigger bodies move slower relative to each other while smaller ones move faster. A low relative velocity means that planetesimals collide more gentle and are more likely to stick together instead of crashing into and bouncing off of each other, probably splitting into smaller pieces. So, bigger bodies grow faster than smaller ones and grow even faster the more mass they gain [Chambers, 2009].

[Nagasawa et al., 2007] describe this process with the gravitational focusing factor

$$f_G = 1 + \frac{v_{esc}^2}{v_{rel}^2}, \tag{4}$$

where $v_{rel}$ is a particle's velocity relative to its velocity on an unperturbed Keplerian orbit and $v_{esc}$ is its escape velocity. $f_G$ modifies a planetesimal's gravitational cross section and thus its ability to capture other bodies. Small planetesimals have high relative velocities and low escape velocities, so $f_G \approx 1$. Big planetesimals have low $v_{rel}$ and high $v_{esc}$ and therefore are more able to collect other (especially small) planetesimals. This leads to the runaway growth, which means that some planetesimals grow very big while the remaining small ones stay small.

Let us look beyond the Ice Line again for a moment: Due to $f_G$ big planetesimals have a gravitational cross-section (the area where gas and solids get caught because of gravitational pull) much bigger than their geometrical cross-section (where gas could collide with the planetesimal)

[Pollack et al., 1995]. This enables them to accrete substantial amounts of gas in the short time given for this process. Giant planets form much faster than terrestrial ones. Almost all models of terrestrial planet formation start with already finished gas giants [Walsh, 2012].

On both sides of the Ice Line runaway growth stops sooner or later: Namely when the disk is depleted from medium-mass planetesimals and the biggest have masses about 100 times higher than the mean [Chambers, 2004]. Thus begins the phase called 'oligarchic growth'. The big planetesimals (now called planetary embryos, or protoplanets) slow their growth because they perturb the smaller ones and increase their relative velocities. Now, $v_{rel}$ depends more on the masses of these embryos than on the mass distribution which enables dynamical friction. This effect is stronger the more massive the protoplanet is and enables smaller ones to catch up, which leads to similarly massive bodies in each region. These planetary embryos sweep up the remaining planetesimals and each other until only one big planetary embryo dominates his area of influence [Kokubo and Ida, 1998]. This area in which an object can accrete matter is called its feeding zone and depends on its mass. All protoplanets now have their own feeding zone where they grow, while not perturbing each other very much [Chambers, 2004]. The smaller planetesimals in each feeding zone cannot grow anymore since their orbits become more eccentric and collisions no longer lead to merging but to fragmentation. When eventually all matter in this zone is accreted, the embryo would have reached its isolation mass $M_{iso}$ which is about 0.1 to 0.2 Earth masses. But oligarchic growth ends when its mass is about $M = M_{iso}/2$. Then, the embryos are no longer kept quiet by the dampening of small planetesimal's gravitation and start gravitationally perturbing each other, making their orbits become more eccentric and inclined [Chambers, 2009].

This is the point where the Grand Tack model strikes: Usually, simulations would just look at these embryos perturbing each other, colliding, very slowly accreting or deflecting the last planetesimals, and slowly turning into planets. As [Nagasawa et al., 2007] mentioned, many calculations have been made, with the result that the outcome of this process depends a lot on chance: Do two embryos collide or do they just pass each other, throwing them into different orbits and maybe out of the system, or into other embryos? For an example, let us look at [Chambers, 2004]: Four numerical simulations were performed, each with a few dozens of embryos, each of them with a mass between one and ten percent of Earth's mass. They interacted with each other until planets with stable orbits remained. The four outcomes were different to each other, but one of them was able to reproduce small Mercury and Mars and bigger Venus and Earth. But, as J.E. Chambers admits in [Chambers, 2004], these simulations still fail to explain why Mars' mass is as small as observed, and why the Asteroid Belt is now so empty when it had to be much more massive in earlier times [Wetherill, 1992].

The Grand Tack provides us with a means to explain this. Around the time when oligarchic growth stops, Jupiter starts its movement inwards, causing havoc in the disk. But before we discuss that central point of this thesis let us first take a look at why and how Jupiter (and Saturn, for that matter) would be able to migrate through the disk.

# III. PLANETARY MIGRATION

THERE IS a famous quote out of Iain M. Banks' novel *Excession*: "*An Outside Context Problem was the sort of thing most civilisations encountered just once, and which they tended to encounter rather in the same way a sentence encountered a full stop.*"
Hot Jupiters were an Outside Context Problem to the theory of planetary formation. With its roots reaching back as far as Kant, and cast into a solid theory by Safronov in 1969, this theory described our own solar system well. But the first observations of exoplanets revealed a surprising result: Many of them had semi-major axes much smaller than those found in our own solar system. Additionally these planets had masses comparable to those of our own giant planets, Jupiter and Saturn. Although there are planets with masses between our terrestrial planets and the ice giants Neptune and Uranus whose composition is unclear - they may be very big terrestrial planets or very small ice giants - we can assume that these massive exoplanets are also mainly made of gas. Then, according to our theory of planetary formation, we are looking at an impossibility. As discussed in the previous section, gas planets only are able to form beyond the Ice Line, since there is just not enough condensed matter inside of it to reach the masses needed for capturing the gaseous components of the disk. But as [Ida and Lin, 2004] discussed, many found exoplanets circle their star at less than one AU (some as close as 0.05 AU) while having masses ranging between ten and thousand earth masses. As this paper was published quite a while ago, many more exoplanets have been discovered since then. Because detection techniques have improved and a longer time of observation made it possible to also detect planets with longer orbital periods, our picture of the distribution of exoplanetary masses and semi-major axes is not looking as odd as it did back then. But this does not make these seemingly impossible 'Hot Jupiters' [1] disappear. The question is now how to modify our theory of planet formation to prevent Hot Jupiters from being its full stop.

We solve this problem by enhancing our theory and postulating planetary migration. All giant planets start their life outside the Ice Line. Probably quite close to it because the disk's gas density decreases with distance, which is why our ice giants Uranus and Neptune have much less gas than Jupiter and Saturn. Then, when the right circumstances are met, they migrate inwards through the disk, stopping only when the disk dissipates. [Trilling, 2002] were able to confirm that such a mechanism would produce the distribution of semi-major axes we are able to observe. There are three types of planetary migration. We now will take a look at each of them.

## I. Type I Migration

Type I migration was first postulated long before any exoplanets have been found; [Goldreich and Tremaine, 1980] being one of the first papers about it. The concept behind Type I migration is quite simple: A solid body moves through a gas disk and exchanges angular momentum with the gas. The planet's gravitational potential produces density waves in the gas. Gas on the inner side of the planet travels faster, thus the inner density waves pull the planet forward - it receives angular momentum and migrates outwards. Density waves on the outside of the planet move slower, pull it back, and so the planet loses momentum and migrates inwards. The resulting direction of migration depends on the sum of these effects. Usually, the inwards pull dominates. To calculate this effect we only have to look at gas which is in a resonance with the planet

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[1] Technically, 'Hot Jupiter' is the term for only the closest Jupiter-mass planets, orbiting at about 0.05 AU and experiencing tidal locking with their star. For the purpose of planetary migration we do not have to distinguish between these very close planets and such which are further away, but still unexpectedly close, and therefore we will call all of them Hot Jupiters.

(everywhere else the torque the gas exerts on the planet cancels itself out over time). Lindblad resonances with the gas occur where the angular velocity is:

$$\Omega = \Omega_p \pm \frac{\kappa_e}{m} \,, \tag{5}$$

where $\Omega_p$ is the planet's angular velocity, $\kappa$ is the epicyclic frequency of the gas, and $m$ is an integer number $m > 0$. For a nearly Keplerian disk, $\kappa \approx \Omega$ due to the eccentricity of the gas' orbit [Goldreich and Tremaine, 1980]. This means that for every orbit of the planet the gas completes a whole number of epicycles. The second but less important form of resonance is corotational resonance, where gas and planet have the same orbital period. For a disk where the surface density is proportional to $r^{-1.5}$, the torque of corotational resonances is zero [Papaloizou et al., 2007]. The torque that one Lindblad resonance exerts on a planetin a Keplerian disk is

$$T_m = sign(\Omega - \Omega_p)\frac{\pi^2\Sigma}{3\Omega_p\Omega}\left(\frac{rd\phi_m}{dr} + \frac{2m^2(\Omega-\Omega_p)}{\Omega}\phi_m\right)^2 |_{r=r_L} \,, \tag{6}$$

according to [Papaloizou et al., 2007], with $\Omega$ and $\Omega_p$ the angular velocity of the gas and planet, $r_L$ the radius of the Lindblad resonance, $\Sigma$ the surface density, and $phi_m$ the components of the planet's gravitational potential expanded into Fourier modes:

$$\phi(r,\theta,t) = \Sigma_{m=0}^{\infty}\phi_m(r)cos(m(\theta - \Omega_p t)) \,. \tag{7}$$

This can also be expressed via a different formula, taken from [Goldreich and Tremaine, 1980]

$$T_m \approx m^2 r^4 \Omega_p^2 \Sigma \left(\frac{M_p}{M_s}\right)^2 \,. \tag{8}$$

Here, $M_p$ and $M_s$ are the masses of the host star and the planet. For a disk with varying temperature and scale height one has to modify the torque equation a little bit. This leads to a torque cutoff, which means that the torque reaches a maximum at a certain $m$ but decreases rapidly afterwards [Papaloizou et al., 2007]. This cutoff resonance has the value

$$m = \frac{r\Omega_p}{c_s} \,, \tag{9}$$

with the sound speed $c_s$. One can calculate the net torque on a planet by adding up all $T_m$ up to this point. The torques exerted by the inner Lindblad resonances are overall weaker than those from the outer ones, which means that the net torque is negative and makes the planet wander inwards. This net torque, corresponding to a change in angular momentum, can be expressed as a change of the planet's semi-major axis:

$$\frac{da}{dt} = -(2.7 - 1.1x)\frac{M_p}{M_s}\frac{\Sigma a^2}{M_s}\frac{v_k}{h^2} \,, \tag{10}$$

where $h$ is the aspect ratio - the ratio of the scale height to the semi-major axis $h = \frac{H}{a}$. $x$ is the power of $r$ in the surface density formula; usually, $x = -1.5$ [Tanaka et al., 2002].
If one evaluates this equation, we face another impossibility: planets in the gaseous disk migrate inward very fast. In fact, so fast that every giant planet's planetary core disappears in a matter of $10^5$ years. What does this mean?

Naturally, this means that we have ignored something we shouldn't have ignored. One of these things are - again - turbulences. If the gas in the disk has been ionized it is not stable anymore in the presence of a magnetic field. This is called the magnetorotational instability [Armitage, 2017]. This can cause magnetohydrodynamic turbulences that alter the surface density distribution of the disk and make Type I migration become a random walk and not a steady march into doom.

Another thing is assuming that the disk is isothermal. If it isn't, corotation resonances can give a planet a significant amount of angular momentum, therefore make it migrate outwards or to certain stable distances. The reason for this is a bigger surface density gradient. In a simple Keplerian disk, we only see this at the edges, but if there are turbulences, vertically non-isothermal temperature distributions, and dead zones present, there are more such regions.

The last one probably is my favourite explanation (and not just because it's the title of a Cronenberg film). For MHD turbulences to happen, the gas has to be at least partially ionized. This is the case close to the host star, in the inner disk, and further outside via cosmic rays that can penetrate the disk here quite well. Somewhere in the disk's midplane, where the host star is too far away and the disk too thick for cosmic rays, a dead zone forms. Here, one finds very steep surface density gradients which lead to strong corotational torques. It is possible that planets and planetary cores wander towards the border of a dead zone where they achieve torque balance.

## II. Type II Migration

Type I migration applies only to low-mass planets, while Type II is relevant for planets with a higher mass. The reason for this is that more massive planets possess a larger Hill radius:

$$R_H = a\sqrt[3]{\frac{M_p}{3M_s}}\,. \tag{11}$$

When the Hill radius is big enough gas can not flow around the planet and instead is accreted by it. Thus, the planet begins to clear its vicinity of gas and forms a gap in the protoplanetary disk where there is almost no gas. Type I migration isn't possible anymore: if the planet moves inwards it receives positive torque, since the inner disk still pushes it outwards while the missing outer disk can't work against it. Therefore, the planet always stays within its gap. Type II migration becomes possible because the gap itself is able to move through the disk.

For a gap to form, the gap-opening time, which is the time in which the planet can deliver enough angular momentum to clear a ring in the disk, has to be shorter than the time the disk needs to fill the gap up again due to its viscosity and pressure [Goldreich and Tremaine, 1980].

$$t_{open} = \frac{\Delta L}{T}, \quad \Delta L = \Sigma\Omega_p(r\Delta r)^2, \quad t_{close} = \frac{(\Delta r)^2}{\nu}\,. \tag{12}$$

Here, $\Delta r$ is the width of the gap forming at distance $r$, $\Delta L$ is the delivered angular momentum, and $\nu = \alpha c_s H$ with $\alpha$ a viscosity parameter is the disk viscosity with the speed of sound $c_s$ and the scale height $H = \frac{c_s}{\Omega_p}$ [Armitage, 2017]. By using (8) and (9), we get the condition:

$$\frac{M_p}{M_s} \geq \left(\frac{c_s}{r\Omega_p}\right)^2 \sqrt{\alpha}\,. \tag{13}$$

This expression gives us the minimum mass of a planet to form a gap. Inserting typical disk parameters into this condition tells us that Jupiter is able to clear a gap in its path and therefore avoid Type I migration [Armitage, 2017]. This is important since we can assume that, after the gap has been formed, only negligible growth can happen and giant planets do not become more massive during migration. Saturn almost reaches this and is only able to produce a partial gap. Its migration rate is slowed, at least if one applies Type II on Saturn. On the other side, we will see that Type III migration is faster than Type II - an important key for the Grand Tack. [Chambers, 2009] presents a very simple equation for the rate of migration:

$$\frac{da}{dt} = -\frac{3\nu}{2a} \,. \tag{14}$$

Although the planet mass has no direct correlation to its Type II migration rate, it strongly affects how the planet migrates, and at which position it will end up. The more massive a planet is the bigger its gap becomes. Massive planets need longer to form their gap and during this time, they experience no migration [Trilling, 2002]. This means that they stay longer at more or less the same position before starting to migrate towards their star. Also, if a planet is too massive its migration rate again depends on its mass, since the disk can not provide enough angular momentum to move the planet as fast, and so its migration rate decreases further [Papaloizou et al., 2007]. Another effect to consider is the interaction of more than one giant planet with the disk. Migration still is an effect of exchange of angular momentum between disk and planet. Since giant planets remove parts of the disk, planets change each other's circumstances. For example, let us look at two planets big enough to create gaps. More than two is a quite unusual scenario, and with Jupiter and Saturn (supposed it is able to create a non-partial gap after all) it is a situation important for our own solar system. For two planets, the outer one migrates inwards quite undisturbed while the inner one changes its semi-major axis only slightly [Kley, 2000]. Statistical evaluations of planet survival under different disk parameters done by [Trilling, 2002] reveal that 30 % of forming planets survive migration, and that 76 % of these have semi-major axes greater than 1 AU - the presence of a second planet could increase both of these numbers significantly. If two giant planets come very close to each other by this mechanism, this can explain the quite big eccentricities some exoplanets are showing. If the disk dissipates before this, a system similar to our own remains. [Kley, 2000] also comes to the conclusion that the two planets, if close enough, reach a resonance of 2:1 - Since this is another key element of the Grand Tack, a continuation of the simulation by [Kley, 2000] might have already led to new insights.

## III. Type III Migration

Type III is the middle course of planetary migration. It applies only to planets which are too massive for Type I, but not massive enough to form a full gap. Additionally it only works for disks which are more massive than the MMSN. It is also called 'Runaway Migration' due to its very fast nature: Type-III-migrating planets can reduce their axes to a third over only about 50 periods [Pepliński et al., 2008]. The details of Type III depend strongly on the disk's behaviour around the migrating planet, and not even the direction of the migration is known beforehand.

The driving forces behind Type III migration are corotation torques. Since the planet has formed at least a partial gap, Lindblad resonances which would lie inside the gap are weakened. Gas is still able to cross the gap and move between the inner and outer disk, and gas in corotation resonance can do so on horseshoe orbits [Papaloizou et al., 2007]. Horseshoe orbits have their name from the appearance they possess when watched from a rotating frame. They can only occur around planets (or other objects that are heavier than the orbiting particles) because they require an exchange of angular momentum. A particle on a horseshoe orbit spends half of its orbit inwards of its parent body, where it orbits faster than it. When it catches up, gravitation of the parent lifts it up onto an outwards lying orbit where it orbits slower and, after some time, gets overtaken by the planet. When in its vicinity again, the planet pulls the horseshoe object

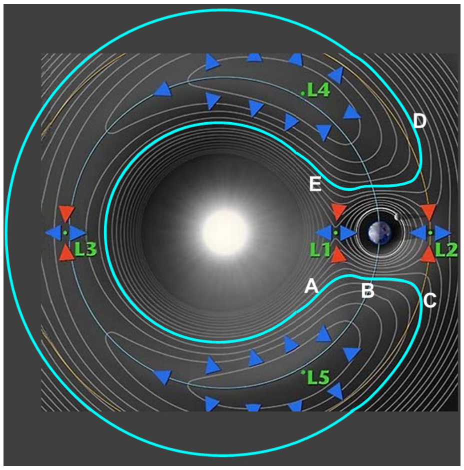

**Figure 1:** *Illustration of a horseshoe orbit. The points marked L1 to L5 are the Lagrange points, the lines are regions of the same total energy (potential plus kinetic energy) and therefore possible orbits. As one can see, objects can orbit around L4 and L5, which makes those two Lagrange points stable. Increasing the energy of such an orbiting object brings them into a horseshoe orbit, where it orbits around both L-points, but not the associated body (Here: the Earth). Horseshoe orbits look like this in a moving inertial system, so Earth is standing still. Source: NASA, public domain.*

back onto the inner part of its orbit. The two crossings of the planet's orbit are when angular momentum is exchanged. For a stable disk-free system of planets the total torque would be zero over one orbital period. The same would be the case had the disk a homogenous surface density. But because this is not the case, we have a net torque. Corotation torques are back now, since the density gradient is steeper in a partial gap, and vortices can amplify this effect. See our discussion of Type I migration and how turbulences can reduce the drift rate. All this could imply that disks are much more turbulent than we - out of convenience - assume.

Another effect in runaway migration is the inertial limit [Masset and Snellgrove, 2001]. The planet initially migrates already fast enough to be out of the gap region it would have formed after one orbit.
The migration rate induced by Type III takes the form of a harmonic oscillator [Masset and Papaloizou, 2003], [Pepliński et al., 2008]

$$\alpha(M_p - \delta m)\frac{da}{dt} + \beta\delta m\frac{d^2a}{dt^2} = T \tag{15}$$

with $T$ the lindblad torque, $\alpha = \frac{a\Omega_p}{2}$, $\beta = \frac{\pi a^2}{3x_s}$ with $x_s$ the half width of the horseshoe region, $M_p$ the mass of the planet itself plus an amount of gas that is gravitationally bound to the planet (the circumplanetary disk) and that has no influence on the migration other than migrating with the planet. We can see in this formula how the initial migration rate $\frac{da}{dt}$ affects the acceleration. $\delta m$ is the co-orbital mass deficit. It is the difference between the mass that would be in the region if it had the constant surface density of the more dense side, and the actual mass. For a homogenous disk $\delta_m$ would be zero.
If we look at our equation again, we see that the behaviour of its solution depends on $M_p$ and $\delta m$. If $M_p$ is the higher one, $\frac{da}{dt}$ is exponentially suppressed and the planet follows other migration mechanisms. If $\delta m$ is higher, we have a very high rate of migration [Masset and Papaloizou, 2003].

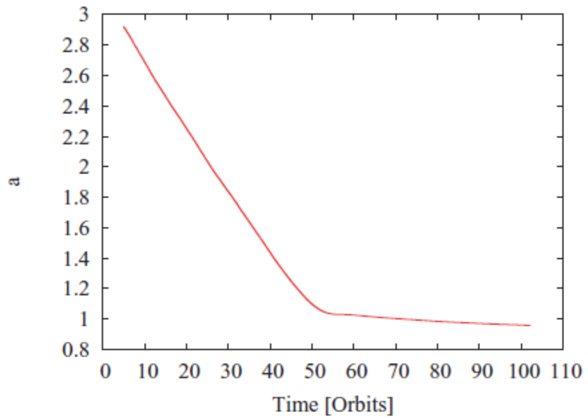

**Figure 2:** *Semi-major axis development of a Saturn-sized planet during Type III migration. We note the abrupt ending of the fast Type III migration regime at* $\approx 52$ *orbits. [Pepliński et al., 2008]*

If we would find an exoplanet that has fallen victim to Type III Migration, we could observe it losing AUs of distance over one's lifetime. Astronomical events taking place over human time spans are rare and usually lead to the destruction of everything involved. Type III is an exception: Although a potential ecosystem of such a planet would be drastically changed, contrary to Type I, Type III poses little danger for the planet to fall into its star. The reason for this is that Type III migration is strongly dependent on the planet mass. Type II planets have created a gap and accrete almost no gas after this, but Type III planets do. The fast migration rate is equivalent to a fast rise in planet mass. After some time, the planet wanders out of the runaway domain into the slower Type II migration domain by forming a real gap. This can be seen in Figure 2.

## IV. Migration mode summary

Type I was the starting point of planetary migration theories, generally applicable to all planets unless special circumstances are met. Still, it is the most troublesome because it indicates that all planets should disappear in their host stars and further research in the nature of protoplanetary disks has to be made to explain why this isn't the case in at least thousands of cases where we indeed can observe (exo)planets. Type II applies for massive planets that form a gap, which frees it from the most Lindblad resonance torques and lets it wander through the disk slower. Type II migration timescales are of the same order as disk lifetimes, so a lot of massive planets survive and occupy a wide range of possible semi-major axes around their stars. Type III is quite an obscure migration mechanism, only applying to a small domain of planets and disks but probably including Saturn. It is by far the fastest of these three modes but doesn't pose the same danger Type I does. If we assume that the Grand Tack model applies to our solar system, we can assume that the Sun's protoplanetary disk was much more dense and turbulent than the simple models assume - this can explain the survival of the terrestrial planets and giant planet cores via stochastic migration and/or turbulences, as well as enable Saturn to migrate via Type III. Now that we have collected all the elements needed to build our own Grand Tack, let us do exactly that.

# IV. The Grand Tack

The Grand Tack is a play in three acts. In Act One, the situation unfolds. A disk and giant planets form, together with a multitude of planetesimals in the inner disk. The rules - disk behavior, migration types - have been established in the last chapters. We come to Act Two, where Jupiter migrates inwards, truncating the disk and scattering planetesimals. Act Three brings the pay-off to the set up Type III migration - Saturn catches up very quickly, bringing Jupiter back on track and thus enabling the terrestrial planets to form in peace.

In this chapter we will discuss in detail every step of the Grand Tack, before continuing with the aftermath in the next chapter.

## I. The State of the Disk

At the start the giant planets Jupiter and Saturn have already formed. Jupiter is assumed to be as massive as it is today, while Saturn has about half of its mass as it will be accreting significant amounts of gas during its migration. We ignore Uranus and Neptune because they are not involved in our theory. Jupiter's initial position is at 3.5 AU, Saturn's at 4.5 AU [Brasser et al., 2016]. The inner planet's disk still consists of planetary embryos and protoplanets and so is still in the oligarchic phase of planet formation. The embryos only possess a fraction of their isolation mass $M_{iso}$ and are equally spaced. In some models the planetary embryos haven't grown via oligarchic growth but through pebble accretion; in that version they all also have the same mass. Because calculations are easier this way, most simulations, e.g. [Brasser et al., 2016] or [Jacobson and Morbidelli, 2014], use these equal-mass embryos.

The protoplanetary disk has a surface density with the same power law as the MMSN, but with a higher density constant of $\Sigma_0 = 2272 \;\; gcm^{-2}$ according to [Brasser et al., 2016], who published the latest simulations about the Grand Tack. [Walsh et al., 2011], who first proposed the Grand Tack scenario, used a much less dense disk and different power law exponents in their simulations. This approach is quite unusual and we won't employ them here either. Viscous heat transfer leads to higher temperatures than in comparable models, and the disk is assumed to be vertically isothermal as an approximation. The Ice Line lies at 2.7 AU. At this point, the amount of solids in the disk triples. These values vary of course between different papers and authors. Those reported here come from [Brasser et al., 2016] and we will try to stay close to this paper.

The rest (the most important ones, of course) of the parameters are hard to pin down. There is a lot we do not know about the formation of Jupiter, Saturn, and the terrestrial planets, and what we know often isn't really specific when it comes to absolute values. So in simulations these parameters are varied and the outcomes can be compared to what we see today. The most important parameter is time. Jupiter starts to migrate inwards after it has formed and then cleared a gap. Depending on when this happens the terrestrial planetary embryos have more or less time to grow, changing their mass $M_e$ as well as the ratio between total embryonic and planetesimal mass in the inner disk, $\Sigma M_e : \Sigma M_p$. These two parameters were most often varied in [Brasser et al., 2016] and [Jacobson and Morbidelli, 2014]. Also, the disk dissipates over time, and a later formation means that its density is reduced. Additionally we can vary the point in which Saturn starts migrating and therefore vary the tacking location and so the point at which the inner disk is truncated.

The migration rate of Jupiter has a negligible effect on the simulation outcome, so this is set to a certain value dependent on the assumed viscosity of the disk. The viscosity in turn is dependent on the scale height, the sound speed in the gas, and a constant factor $\alpha$ which is about $10^{-2}$.

### I.1 Intermission: Asteroids

The planetesimals in the disk will not only become part of our terrestrial planets, some of them will also form the Asteroid Belt. A big success of the Grand Tack model was that it was able to successfully explain the composition and mass of the Asteroid Belt. Classical formation theories failed to reproduce especially the mass. The Asteroid Belt consists of two main types of bodies with quite different composition. S-Type asteroids are lacking water and other volatile substances. They are a big group of many differently composited types of asteroids sharing their common origin in the inner disk up to the Ice Line at around 3.0 AU [Walsh, 2012]. C-Type asteroids, again a big group of diverse asteroids, have a more primitive composition and contain more water and volatiles. Naturally, they originate from the outer disk beyond the Ice Line [Walsh, 2012]. We will further discuss asteroids in the next chapter, when we look at the consequences arising from the Grand Tack.

## II. Inward Migration of Jupiter and tacking point

[Hansen, 2009] first proposed the idea of a truncated disk producing the terrestrial planets. The inner disk ranges from 0.7 to 1.0 AU - roughly the positions of Venus and Earth today. Numerical simulations then produce two big earth-mass planets equivalent to Venus and Earth inside this annulus. Mars only accretes a small fraction of this. The same applies to Mercury on the inner side of the disk. The Grand Tack doesn't attempt to explain the inner truncation, and models often ignore Mercury. Simulations often fail to produce Mercury-like planets since the planetary embryos at the start of the simulation already have a big portion of Mercury's mass. Since collisions are treated as perfect mergers with no scattered material, most resulting planets are more massive than Mercury [Jacobson and Morbidelli, 2014].

We now look at Jupiter. The giant planet starts to form his gap at his current mass and at a distance from the sun of roughly 3.5 AU. When its gap has fully formed, Jupiter starts his inwards migration of Type II. We might speculate if Jupiter has formerly undergone Type III migration and thus might originate from farther outside, but this makes no difference for the Grand Tack. As mentioned above, migration speed doesn't change the outcome of the simulations much, but usually a migration time of 0.1 Myr is given [Brasser et al., 2016]. In this time, Jupiter moves from his initial location at 3.5 AU to the tacking position at 1.5 AU [Walsh et al., 2011]. This is equivalent to a disk truncated at 1 AU and therefore in accordance to [Hansen, 2009]. If we include Type I migration of our embryos and employ a more realistic disk model ([Brasser et al., 2016] called the one from [Walsh et al., 2011] 'artificial') we see that all our planets are too close to the Sun. This means that Jupiter's tacking location must be farther out, probably at 2 AU. Also of importance is the age of the disk. As the disk becomes older, more and more planetesimals become lost and dynamical friction gets weaker. So, the formation time of Jupiter, the time it takes for Jupiter to reach the inner disk respectively has an influence on the eccentricity and inclination of the embryos and planetesimals.

When Jupiter marches inwards, what happens to the objects (S-type planetesimals/asteroids, embryos) it encounters? Most of these are being transported inwards. [Walsh et al., 2011] explain this with resonance trapping by Jupiter, with excitation of their eccentricity, which brings them closer inside and then lets these objects stay there as the gas damps their eccentricity, and with gas drag. A fraction of the matter gets scattered outside Jupiter's orbit when it is gravitationally disturbed by Jupiter. Meanwhile, the higher concentrated mass in the inner disk leads to a sustainably accelerated planet formation process.

## III. Inward migration of Saturn and resonance motion

Giant planets form faster if they are more massive because their gravitational cross-section enhances their mass accretion rate. The higher accretion rate increases the planet mass further. So, when Jupiter has formed and is on its way towards the inner disk, Saturn has still less than the mass it has today. The exact value of Saturn's mass is not known since the minimum mass for Type III migration depends on the disk parameters which in turn are also not well constrained. [Walsh et al., 2011] and [Brasser et al., 2016] give a value of approximately half of today's Saturn mass. [Masset and Papaloizou, 2003] put it at one Saturn mass, making it accrete almost no matter during its migration. This is an unrealistic assumption, and in the most simulations Saturn actually gains a big portion of its mass.

Saturn's migration happens really fast, and can be taken as instantaneous compared to the slow migration of Jupiter (a few hundred years opposed to about 100 kyr). As Saturn approaches Jupiter it crosses a few lines at which they fall into mean-motion resonance to each other. At these points their eccentricity increases rapidly, but their inwards migration does not stop. This is because the eccentricities need to be higher than a certain value to couple the two planet's orbits together [Masset and Snellgrove, 2001]. This only happens when they finally reach the 2:3 resonance which is the most cited resonance, although sometimes 1:2 is also considered. The 1:2 resonance applies for disks with low overall mass and low viscosity, implying a low aspect ratio of disk height to radius [Pierens et al., 2014]. This also is the resonance for which the Nice model, that explains the later evolution of the solar system millions of years after the Grand Tack, works better. We will use the value of 2:3 here since we are assuming a more massive disk.

When Jupiter and Saturn are in their final resonance, their orbital period ratio is 2:3 and therefore, according to Kepler's laws, their semi-major axes have a ratio of $a_S : a_J = 1.31$. These ratios are conserved until the gas disk dissipates and Jupiter's and Saturn's migration stops.

Mean-motion resonance can let planets move together as one. And not just two planets, but even a whole chain of planets. It is assumed that Uranus and Neptune also formed nearer to the sun than they are today. As Jupiter and Saturn move outwards, they fall into resonance with Uranus and then Neptune too, producing a line of planets moving outwards together and enlarging the scale of the planetary system [Walsh et al., 2011].

But, how does this work? To avoid complexity we take a look only at our more important planets, Jupiter and Saturn. Consider Type II migration again. As the gap drives the gas out of those points in the disk where the strongest Lindblad resonances lie, their torques vanish and only the weaker ones farther away from the planet remain. We also note that, according to (8), the torque on a planet is proportional to its mass squared. Jupiter, as it has more mass, experiences stronger torques than Saturn at every possible Lindblad resonance. The resonance locked planets can be thought of as two planets sharing one large gap in the disk. Would there be only one planet, lets say Jupiter, it would rest in the middle of the gap. Approaching the inner edge would increase the outwards torques, since then stronger Lindblad resonances would appear and would push Jupiter back out again. The reverse would happen when the planet approaches the outer gap edge - it would be pushed inwards.

Now there are two of them. As Jupiter lies on the inner side, it experiences a positive net torque proportional to $M_J^2$. Saturn on the contrary receives negative net torque proportional to $M_S^2$ [Masset and Snellgrove, 2001]. Because Jupiter is much more massive than Saturn, the positive torque overweighs and the system of planets migrates outwards. The absolute torque acting on the system has to be corrected for the different distances of Jupiter and Saturn to the inner, outer edge respectively since different masses cause different gap widths, but this is only a small correction to the effect the mass difference between the planets causes ($\left(M_J/M_S\right)^2 \approx 10$).

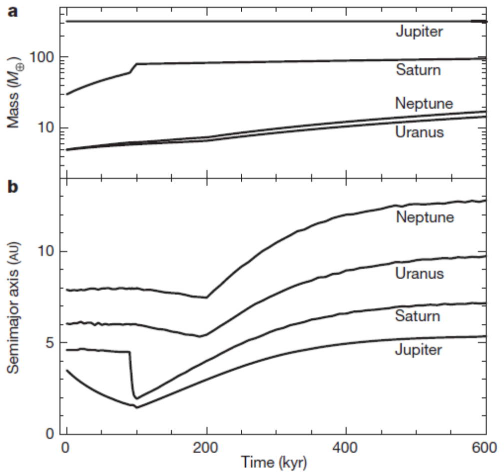

**Figure 3:** *An overview of the development of the giant planets in terms of their masses (top) and semi-major axes (bottom). They start further in than they are today, and begin migrating inwards. Jupiter is the first as it forms earliest, then the others follow. As Jupiter has fully cleared a gap around it, it accretes almost no mass during this. Fulfilling the requirements for Type III migration, Saturn catches up very rapidly and subsequently gets in resonance with Jupiter and they migrate outwards. After 200 kiloyears, they reach Uranus and Neptune and all four planets are in resonance together. (From [Walsh et al., 2011])*

Although the gaps are fully cleared, gas can pass through it. We do not have Type II migration at hand: Even though there is a gap, the planets do not move together with the disk, which moves inwards. Contrary, the coupled planets and their shared gap are not dependent on the viscosity as Type II migration is, but move in the direction and with the speed that their mass ratio dictates.
On their way out Jupiter and Saturn encounter planetesimals again. Outside of their orbits we have the primitive C-type planetesimals as well as the 15 % [Walsh, 2012] of S-type planetesimals that have been scattered outwards earlier by Jupiter. A percentage of them gets scattered back in again, together with a similar fraction of C-types. Models verify that the S-types that are encountered first by the outwards migrating planets occupy more inner orbits than the C-types that get scattered inwards later [Walsh, 2012]. The point at which C-type asteroids begin dominating the Belt is at roughly 2.8 AU [Walsh, 2012], which matches the experimental data very well. Still, the eccentricity and orbital distribution of the asteroids pose a problem. As the asteroids get scattered inwards by close encounters with the giant planets, naturally they show very eccentric orbits. Today's Asteroid Belt's eccentricity distribution peaks at 0.1, while the one derived from Grand Tack simulations lies around 0.4. Additionally, the orbits are uniformly distributed instead of showing the typical gaps seen today [Deienno et al., 2017].
Outward migration of Jupiter and Saturn ends when the disk dissipates. At this point, they are at roughly the same positions they occupy today. Simulations of the Grand Tack usually continue for 150 Myr after the disk dissipates to observe the further evolution after the tacking event: the terrestrial planets finally form, accreting the rest of the planetesimals slowly. The Asteroid Belt changes its structure slightly, and the giant planets, which are still in resonance, start becoming more unstable. The terrestrial planets need around 50 Myr to form from the first solids [Brasser et al., 2016]. Most of the way to fully formed planets has been covered while the gaseous disk was still present by forming the planetary embryos. After the tacking event, most mass is concentrated in the embryos, which each have masses between the ones of Moon and Mars, and collide with each other sooner or later to usually form between 3 and 4 planets. The remaining planetesimals either get absorbed in these protoplanets, are scattered out of the system, or fragment into dust and then dissipate like the gas disk. After 150 Myr nothing really happens anymore, and the system can be assumed to be stable until further repositioning takes place during the events described by the Nice model.

## IV. The Nice Model

The giant planets need a little more time to settle. Their state of resonance (together with Uranus and Neptune) is not stable, and so after 450 - 600 Myr they start changing their orbits. This is the Nice model, the missing link between the Grand Tack and the current solar system.

The Nice model describes the events that eventually lead to the Late Heavy Bombardment (LHB) and that begin with the giant planets becoming temporarily unstable and falling out of resonance later. The giant planets then begin to restructure their orbits and to gravitationally perturb smaller planets and asteroids. In some simulations, Neptune and Uranus switch places, or a fifth gas giant gets ejected out of the system. In all cases Kuiper Belt objects get perturbed and scattered inwards, which leads to Jupiter migrating a little bit inwards again. This can be seen in Figure 4.

When we tie the Grand Tack and the Nice model together via numerical simulations that use the first's outcome as input for the second, we get a pretty satisfying model of the solar system. The Grand Tack can explain a lot of observations in our solar system, and fortunately is able to do so over a variety of initial conditions, since we lack detailed knowledge of many of them. By analyzing the fine differences between simulations made with different initial conditions, we can try to determine how the solar system must have been at its beginning.

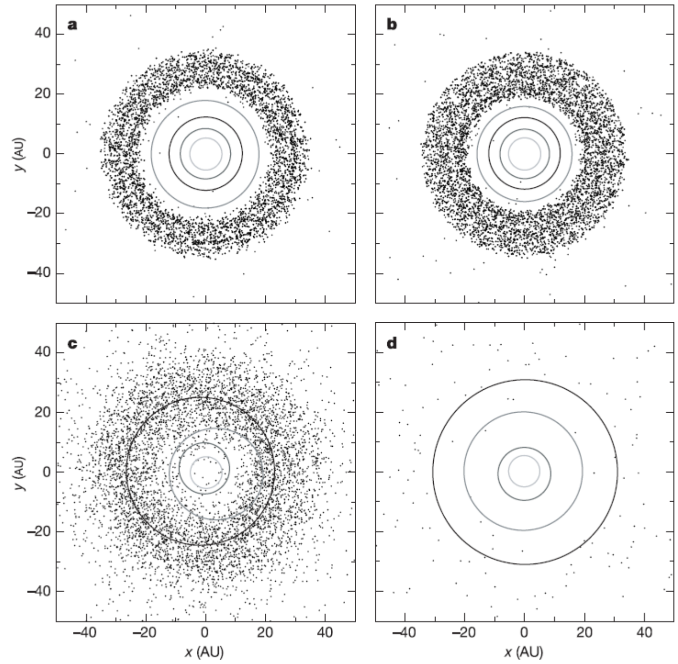

**Figure 4:** *The orbits of the four outer planets and the positions of the outer planetesimal disk particles at different times of the nice model, measured from beginning of planetary formation. a) The start of the simulation at 100 megayears. b) The beginning of the LHB at 879 megayears. c) during the LHB, only three million years later at 882 megayears. d) After the end of the LHB at 1082 megayears. At this point, 97 % of the planetesimal disk mass has been depleted, either through accretion or through scattering. Taken from one of the three papers that first proposed the Nice model in 2005, [Gomes et al., 2005].*

## V. Constraining initial conditions

Looking at the time needed for planets to form, we can determine the ratio of embryo to planetesimal mass. More planetesimals mean that dynamical friction plays a more important role, therefore damping the eccentricity and inclination of the planetary embryos. As shown by (4), lower relative velocities (a direct consequence of increased dynamical friction) lead to a bigger gravitational focusing factor, therefore making it easier for bodies to gain mass. This implies that planets need less time to form if more planetesimals are present, since this increases dynamical friction.

One problem with this is that formation time varies widely between the terrestrial planets. Earth needed about 10 times longer than Mars [O'Brien, 2014]. One theory addressing this is that Mars has more or less skipped the collisional phase, and in fact comes from a single planetary embryo that got scattered out of the truncated disk of solids onto its current orbit where it accreted only little mass. The same might apply to Mercury. This might give us an idea of the embryonic mass [Jacobson and Morbidelli, 2014] by saying that embryos at the moment of tacking are a bit lighter than Mars, Mercury respectively. So, they only accreted planetesimals but never collided with other embryos. The different masses of Mars and Mercury (by a factor of two) indicate that not all embryos have the same mass, but that oligarchic growth leads to a distribution of embryonic masses. [Brasser et al., 2016] also calculated an equal-mass scenario following pebble accretion, but ruled it out due to other reasons (as mentioned, most Grand Tack simulations ignore Mercury or fail to produce an equivalent planet).

Another way to test our initial conditions on plausibility is the implementation of different statistics onto the terrestrial planets. The first parameter is obviously the number of planets, $N$. The next is the obliquity statistics:

$$S_O = \frac{1}{N} \sum_k |cos\epsilon_k| \,, \tag{16}$$

with $\epsilon_j$ the obliquity of the k-th planet. Rotation axes fully parallel to the orbit axis would give $S_O = 1$, fully randomly orientated ones $S_O = 0.5$ [Chambers, 2001]. The next two parameters are the percentage of mass that is concentrated in the most massive planet, $S_m$, and a mean spacing parameter $S_H$:

$$S_H = 2 \sum_{k=1}^{N-1} \frac{a_{k-1} - a_k}{a_{k-1} + a_k} \sqrt[3]{\frac{3}{m_{k+1} - m_k}} \,, \tag{17}$$

with $a_k$ the semi-major axis of the k-th planet, and $m_k$ its mass [Brasser et al., 2016], [Chambers, 2001]. $S_m$'s meaning is self-explanatory. In our solar system, Earth is the most massive terrestrial planet with $S_m = 0.505$. The spacing parameter gives the average distance of the planets to each other and usually is given in multiples of the Hill radius. Values should lie around $45 \pm 12$ with a $2\ \sigma$ error [Brasser et al., 2016]. A successful simulation should be able to produce a planet with a mass share of $S_m \approx 0.5$ at a distance of 1 AU; $S_m$ and $S_H$ provide a measure for this criterion.

The remaining two parameters are the radial mass concentration (RMS) $S_C$ and the angular momentum deficit (AMD) $S_d$:

$$S_C = max\left(\frac{\sum_j m_j}{\sum_k m_k log_{10}^2(a/a_k)}\right) \,, \tag{18}$$

$$S_d = \frac{\sum_j m_j \sqrt{a_j(1-e_j^2)} cos(i_j) - \sum_k m_k \sqrt{a_k}}{\sum_l m_l \sqrt{a_l}} \,, \tag{19}$$

with $e_j, i_j$ the eccentricity and inclination of the j-th planet. $S_C$ is the maximum of the bracketed term over all $a$ [Brasser et al., 2016], [O'Brien, 2014], [Chambers, 2001]. One can use these parameters to compare different disk and mass concentration conditions with each other and to estimate which one is the most plausible. The RMC quantifies the concentration of mass in the inner disk region. A higher value corresponds to more mass lying in a smaller region (a single planet would produce an infinite RMC). Since, with Venus and Earth carrying almost 92 % of it, almost all of the mass is localized entirely within a small annulus ranging from 0.723 AU to 1.0 AU. This gives a big RMC value of $S_C = 89.9$ [O'Brien, 2014]. The AMD gives the difference between the angular momentum of the real simulated system and an 'ideal' one where all eccentricities and inclinations are zero. This difference is then normalized to values between 0 and -1, with more negative values meaning that the system is more eccentric and inclined ('dynamically hot'). Today's values lie around -0.0018 [O'Brien, 2014].

While the other three parameters do not change very much after the Grand Tack, the AMD is expected to decrease during the Late Heavy Bombardment 500 Myr later. A lot of planetesimal mass originating in the outer regions of the solar system is scattered inwards. Signatures of this event can be found as numerous impacts on the Moon which consistently are dated to originate around 3.95 Gyr ago. The Nice model explains this event to have been triggered by instabilities of the giant planets' orbits. These instabilities, although Nice model simulations mostly ignored the terrestrial planets, are believed to dynamically excite the inner planets, and therefore lower the AMD value from around -0.001 to its current value. So, although it would be a very important factor for constraining our model parameters, we cannot provide a definite value for it.

[Brasser et al., 2016] applied different embryonic masses, mass ratios between planetary embryos and planetesimals, and oligarchic and equal-mass mass distributions to simulate the formation of terrestrial planets over 150 Myr. Comparing the four parameters $S_m, S_H, S_C, S_d$ with their real-life counterparts, they were able to determine time and place of the tacking event as well as to confirm that oligarchic embryos fit the data better than equal mass ones. They concluded that the Grand Tack happened 2 Myr after formation of the first solids, a typical timescale for formation of giant planets. Incorporating Type I migration into their simulation, they were able to show that a tacking point at 2.0 AU was more probable than one at 1.5 AU, as the original models expected [Walsh et al., 2011]. The embryonic and planetesimal masses were harder to pinpoint, since all their parameters produced reasonable results. [Jacobson and Morbidelli, 2014] on the other hand presented an embryonic mass of 0.08 Earth masses (a little less than Mars indeed) and a ratio of total embryonic to planetesimal mass of 8:1 as their best-fitting parameters. For this result, they used the formation time of Mars [Nimmo and Kleine, 2007] as a deciding criterion.

# V. Consequences

The Grand Tack was an idea to explain the truncation of the disk which produced our small Mars. What made it so successful was the multitude of other phenomena it was able to explain. In theory, these consequences also qualify to test our initial conditions and parameters, but often these consequences arise independently from and over a big range of parameter values.

## I. A Small Mars

The first to present the idea of a truncated planetesimal disk was [Hansen, 2009], initially with no explanation for this truncation but with the result that it would fit observations really well. Simulations using [Hayashi, 1981] for the disk surface density such as [Chambers, 2001] successfully produced Earth and Venus equivalents, but ended up with Mercury and Mars having too much mass due to the steady distribution of material in the disk. Also, eccentricity and inclination often were too high due to neglecting or underestimating dynamical friction. [Hansen, 2009] started the simulation mid-oligarchic to avoid having too heavy planetary embryos: Since the mass is so concentrated, the isolation mass would be comparable to a planet mass. In comparison, in the Grand Tack scenario, we already have almost all mass in the embryos. But as they are distributed over a wider region, they all are less massive than if they were forming in a small annulus. In this way, [Hansen, 2009] matches Grand Tack initial conditions quite well. A big difference are the missing planetesimals, which are an important point in the Grand Tack.

Simulations from a truncated disk, together with a bimodal mass distribution of embryos and planetesimals, lead to correct masses, eccentricities, and inclinations in the Grand Tack [Walsh et al., 2011]. Mercury equivalents emerge, but are still rare; this might indicate that there is as much work to do on the inner edge of the disk as on the outer, where Mars equivalents almost always form [Hansen, 2009], [O'Brien, 2014].

## II. Depletion of the Asteroid Belt

With about $3 \cdot 10^{21}$ kg the Asteroid Belt is much less massive than the terrestrial planets. Without the Grand Tack and with a non-truncated disk, just like Mars it should be much heavier, in this case by a factor of thousand. This would raise its mass from estimated 5 percent of Moon's mass to 50 percent of Earth's. But despite Mars we cannot solve this discrepancy in the mass with a simple disk truncation: Since Jupiter crosses the Belt region (twice, even) it should be bare and devoid of any mass that initially must have been there. The Asteroid Belt we see today consists of the scattered planetesimals from inside and outside the Ice Line. Grand Tack simulations roughly reproduce the distribution of S- and C-Type asteroids, but fall short in explaining other details.

The Asteroid Belt resulting from the Grand Tack consists of two classes of planetesimals on dynamically excited orbits, with high eccentricities and inclinations of up to 25° [Walsh, 2012] stemming from the chaotic scattering process. The mass of the Asteroid Belt is much smaller than in other formation models, but still bigger by a magnitude than it actually is today [Brasser et al., 2016]. Theories suggest that a huge part of the mass possibly has been lost over time due to unstable orbits [Minton and Malhotra, 2010]. Mainly affected are highly eccentric asteroids, which by their removal also reduce the mean eccentricity in the Belt, solving another problem at the same time [Deienno et al., 2017]. This can be seen in figure 5, were the resulting distribution fits the current one, besides having additional asteroids which are more eccentric and will get lost over time.

Also, the higher mass might be an artifact from the simulation's limitations: A more diverse initial mass for the planetesimals could decrease it as well as the implementation of non-perfect merger collisions, in which at every collision some part of the planetesimal mass turns into dust that easily can escape the Belt. Besides all these speculations the Asteroid Belt also slowly loses some of its excess mass during the Nice model events. This final inwards migration of Jupiter causes the orbital restructuring in the Asteroid Belt by destabilizing and therefore depleting the resonance regions in the Belt which, together with the migrating Jupiter, wander through the Belt and can remove 25 to 50 % of the Belt mass [Walsh, 2012].

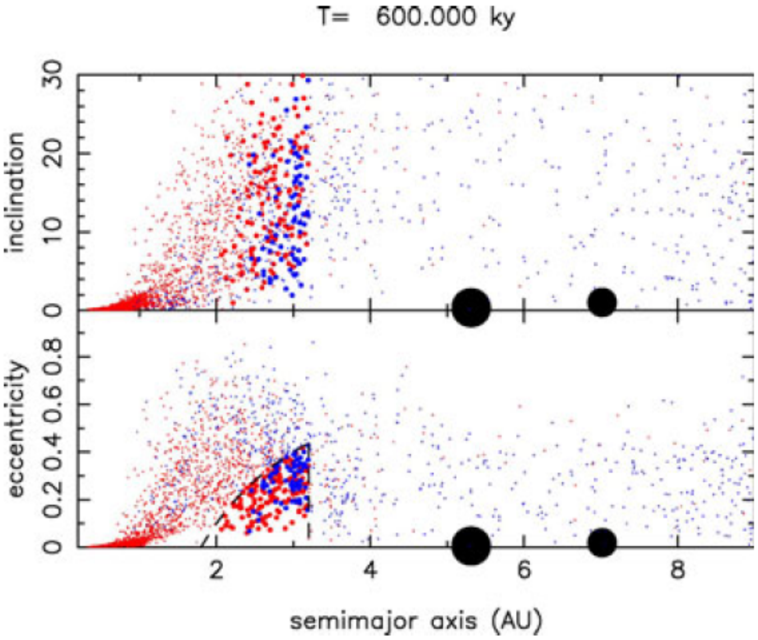

**Figure 5:** *Final planetesimal distribution after Jupiter and Saturn finish their outward migration 600 kyr after the start of the simulation at the beginning of planetary formation. Red and blue dots represent S- and C-type asteroids. The black dots indicate Jupiter and Saturn's final positions. In the upper and lower panel the inclinations, eccentricities respectively of the asteroids are depicted. The bigger blue and red dots indicate asteroids that lie within today's Belt parameters. It is obvious that the resulting post-Grand-Tack Belt differs significantly. Eccentricity as well as inclination increase with semi-major axis in the solar system, and are overall bigger in the simulation result. [Walsh et al., 2011]*

## III. Formation of the Moon

Earth's moon formed by the latest impact of a planetary embryo on the proto-Earth. This impact produced a lot of debris, parts of which then formed the Moon. This explains the strikingly similar isotope ratios of Earth and Moon. However, radiometry produces inconclusive ages for the Moon depending on the isotopes used (for example by Hf/W and $^{182}W/^{184}W$ measurements [Jacobson et al., 2014]). A method affiliated with the Grand Tack is the different abundance of Highly Siderophile Elements (HSE) on the Moon and in Earth's mantle. An impact by a planetesimal embryo carries enough energy to completely melt the mantle and therefore allows differentiation of the molten material. Siderophile elements should therefore have, together with Iron, diffused into the core when the latest giant impact happened. Planetesimals which hit the Earth after the latest impact can not diffuse anymore and remain in the mantle. Assuming that the planetesimals all have the same (chondritic) composition, one can calculate the mass that has been accreted by Earth after the last impact. An uncertainty arises from bigger, already differentiated planetesimals. Their core could sink down and merge with the Earth's core without a direct trace, leaving lesser HSEs on the surface than undifferentiated impactors. This would mean that there is more lately accreted mass than HSEs would indicate. In the simplest case with no differentiated planetesimals, we have a lately accreted mass of $M_{LA} = 0.0048 M_E$, or about 0.48 % of Earth's total mass, according to simulations of [Jacobson et al., 2014]. Grand Tack simulations then can determine the point during planet formation when the Moon was formed (i.e. when the last big impact on Earth happened) by measuring the amount of mass that was accreted after it. [Jacobson et al., 2014] produces an estimated time for the moon-forming event of $95 \pm 32$ Myr after the formation of the first solids. [Brasser et al., 2016] can confirm this, but admit that their results show a great uncertainty, ranging from 30 Myr to 120 Myr. Overall, Grand Tack models are able to reproduce the time of the Moon-forming impact.

## IV. Water Delivery

Finally, we take a look at water delivery to the inner planets. As they lie within the Ice Line, it is assumed that the major fraction of volatile elements and molecules did not come in solid form or from the gas disk (as it has long dissipated at this point), but from comets from the outer solar system impacting on the fully formed planets. This theory is supported by the ratio of Deuterium to Hydrogen that is the same in C-type asteroids and in water on Earth [Raymond and Morbidelli, 2014]. As the accretion timescale for planetesimals from between and beyond the giant planets is longer than for those from the inner disk, water-rich bodies are accreted over a longer timespan and at a later epoch [O'Brien, 2014]; thus, we can assume that most of their volatile mass remains on Earth after impact. Although the amount of water on Earth is quite uncertain (the Earth's mantle is evaluated to contain between a third and ten times the mass of the crust's water and even more in earlier times), we can say that the Grand Tack model delivers at least the minimum mass of water and most likely more. As [Raymond and Izidoro, 2017] point out, C-type asteroids are scattered inwards during giant planet formation independently of the form of migration or formation, so we have to correct the amount of delivered water upwards since our Grand Tack model starts with a fully formed Jupiter. Still, the mass of water and other volatiles (the water mass fraction, or WMF) is lower than in the classical model, where the water is assumed to come from chondritic meteorites scattered from closer regions of the disk inside Jupiter's orbit, from where there is a much higher probability for them to impact on Earth [O'Brien, 2014].

# VI. Alternatives

The Grand Tack is only one of a few different models which try to explain the formation of our terrestrial planets. We want to take a quick look on other popular competing hypotheses: The Sweeping Secular Resonance model and the EEJS - the Extra Eccentric Jupiter and Saturn model, two theories that, as the Grand Tack, try to identify and explain our solar system by interactions of Jupiter and Saturn. We also will look at a model with a more detailed surface density model and at the possibility of the whole planetesimal disk migrating inwards.

These models attempt the same things as the Grand Tack: Produce a small Mars, deliver water to Earth, deplete the Asteroid Belt from most of its mass and from all of its protoplanets, and provide the right timescales. To find out how the Grand Tack model performs against these other models, let us first summarize its comparable advantages and disadvantages.

## I. The Grand Tack's Pros and Cons

Pros:

- On the plus side, the Grand Tack explains all of the above discussed phenomena. Its initial proposal was made to explain the small Mars mass, but it also explained the mass and roughly the distribution of the Asteroid Belt, something unforeseen which brought the model many followers.
- It also provides a method to deliver water into the inner solar system, something which happens naturally when Jupiter clears out the planetesimals in the disk.
- Additionally, the Grand Tack can produce just the right timescales of formation. The moon-forming incident takes place at the right time. Earth needs about 150 Myr to form while Mars, as it is only a single scattered planetary embryo, forms much faster over about 10 Myr.

Cons:

- The biggest problem of the Grand Tack is that it is widely unconstrained. We have a variety of different mechanisms in place which all have to fit together, and which all depend on different parameters. Especially the protoplanetary disk gives us many unknown variables since such disks appear to exist in many different varieties. The two important migration modes for the giant planets depend strongly on surface density and viscosity of the disk. If we take Type I into account for the planetary embryos, we have to add the disk's density power law and magnetohydrodynamic behaviour to our list of unconstrained parameters. (On the other hand, simulations showed that the Grand Tack works over a broad range of these parameters.)
- Jupiter and Saturn have to form and then meet at the right time. This fact is, of course, used as an initial/boundary condition in the simulations, and we do not know enough about giant planet formation to be sure that the parameters that produce the right Tack also produce giant planets at the right time and position.
- Although the Asteroid Belt's composition can be explained, the asteroids' orbits are far more eccentric than observed, and the initial distribution of asteroids follow a very simplified model. Last but not least the Grand Tack cannot explain why the inner edge of the disk too is truncated.

## II. The Sweeping Secular Resonance Model

A secular (or precession) resonance means that the precession rate of two bodies' perihelia is the same. Secular resonances between a small and a large object can increase the eccentricity of the small one, even to the point where it reaches $e = 1$ and gets ejected from the system. The precession rate of a planet depends on the sum of its gravitational perturbations, one term depending on the disk and its surface density. As the disk dissipates and the precession rates change, the point of resonance between a planet and Jupiter/Saturn moves inwards, and the resonance sweeps across the disk.

The SSR model assumes that these resonances are responsible for the localized mass distribution of our terrestrial planets. Each planetary embryo that gets caught in the resonance increases its eccentricity, which means that it crosses orbits with embryos lying further inside where this leads to collisions and mergers. This is in contrast to the conventional formation theory, where these collisions are thought to come from the gravitational perturbations solely between the embryos [Thommes et al., 2008], [Nagasawa et al., 2005].

Pros:

- The SSR model produces a small Mars and a depleted Asteroid Belt. Like in the Grand Tack, most material ends up in a small annulus, where in most simulations only little mass gets scattered outside to form a small Mars.
- Because much of the material comes from beyond the Ice Line, the terrestrial planets are provided with a lot of water. Since the most massive protoplanets are the ones farthest out, the resulting planets possess more water if they are more massive.
- The two most important parameters from [Chambers, 2001], the Radial Mass Concentration and the Angular Momentum Deficit, are in agreement with the observations. As in the Grand Tack, the AMD is higher in value, but is believed to decrease during the Late Heavy Bombardment.

Cons:

- The accretion rate in the Sweeping Secular Resonance model is faster and therefore leads to a shorter timescale between 20 and 30 Myr [Thommes et al., 2008]. Therefore it is still in the range suggested from Hf/W and other isotope ratio measurements, although quite far on the shorter side.
- The giant planets do not undergo orbital migration. In [Thommes et al., 2008] it is argued that Jupiter and Saturn have not undergone much migration due to their large orbits, and so the effect of the disk on the giant planets is simply ignored. This is problematic, as migration timescales are similar to the timescales in the SSR, and would significantly modify the sweeping rate.

## III. The Extra Eccentric Jupiter and Saturn Model (EEJS)

The EEJS is one out of a variety of similar models (and the one which provides the best results) proposed and simulated by [Raymond et al., 2009]. They tested different positions and eccentricities of Jupiter and Saturn that may have led to their initial configurations of the Nice model and their effect on the building blocks of the terrestrial planets after the gas disk has dissipated. The EEJS avoids the migration problem of the SSR model by starting after the dissipation of the gas disk.

Since gravitational interactions between the planets and planetary embryos lead to a decrease of their eccentricity. This means that the giant planets must have had higher eccentricities before, if one assumes that they ended up at the same positions they occupy today. Nowadays, Jupiter and Saturn possess an eccentricity of around 0.05. The EEJS uses initial values of 0.07 to 0.10 [Raymond and Morbidelli, 2014]. These higher eccentricities lead to a stronger interaction of planetary embryos caught in a secular resonance with Jupiter or Saturn. Especially, the $\nu_6$ resonance that lies at 2.1 AU (The '6' meaning the resonance with the sixth planet, Saturn) plays an important role. It separates the inner part of the planetesimal disk from the rest. The terrestrial planets thus form quite undisturbed in a somewhat truncated inner disk.

Pros:

- [Raymond et al., 2009] and [Morishima et al., 2010] ran simulations on the EEJS which showed that an eccentricity of 0.10 is too much for the model to work. As it still appeared promising, [Raymond et al., 2009] ran more simulations with eccentricities of 0.07 and 0.08 for Jupiter and Saturn, respectively, and got better fitting results. In fact, they were able to reproduce the formation times of Earth and Mars as well as Mars' small mass.
- The AMD in most cases was about twice the observed value, although one can argue that the late heavy bombardment can decrease it.

Cons:

- The values that could not be reproduced were the RMC and the WMF. The innermost planet usually lies too far inside, and the RMC was thus usually around half of the actual value. And since the $\nu_6$ resonance separates the inner disk from the water-rich regions of the disk, the WMF is 10 to 100 times too low.
- Again, we face various problems with the initial conditions. When Saturn and Jupiter are at their final positions, the EEJS cannot reasonably be linked to the Nice model. The much harder problem with it however is that there is no mechanism that could produce these high eccentricities in the first place. Before the disk dissipated, the gas planets should have already formed, and the residual gas would have damped their eccentricities to a low value.

## IV. A More Detailed Surface Density

[Jin et al., 2008] explored the effects a less-steady surface density could have on planet formation. The normal approach on disk viscosity is a direct proportionality to radius, even though this viscosity can have many sources. The hydrodynamic viscosity, $\alpha_{hydro} \approx 10^{-3}$, originates in standard hydrodynamic processes, while a higher viscosity originates in magnetorotational instabilities (MRI). This higher viscosity (called just $\alpha$ in the here cited paper) is about one magnitude higher and applies to regions in the disk where the gas is ionized and MHD turbulences occur: in the inner disk where the temperature is high enough due to the proximity to the sun, and in the outer disk where the density is low enough for cosmic radiation to substantially ionize the gas.

Therefore, we end up with a distribution where there is a middle region of significantly lower viscosity. This leads to a decrease in surface density at the border between middle and inner region, where high viscosity gas flows inwards and low viscosity gas cannot follow fast enough. For the right starting conditions this minimum is at the right place when Mars forms so that Mars can accrete a suitably small amount of mass.

Pros:

- Again, the model solves the initial problem with Mars. A big advantage of this model is that it uses a quite interesting surface density profile that is probably closer to reality than the simple models usually used. Just as non-turbulent disks simplify simulations but ultimately pose problems for Type I migration rates, the small Mars problem could also just be a problem coming from a simplified surface density.
- [Izidoro et al., 2014] argue that this model can provide water on Earth in the same way the conventional theories do.

Cons:

- The simulations of [Izidoro et al., 2014] produce the giant planets on the same semi-major axes and with the same eccentricities they have today. This seems unlikely when one considers that the Nice model should also be applicable to explain the LHB and the Kuiper Belt.
- The Asteroid Belt remains unexplained. The minimum in the surface density is highly localized, and would anyway be still too dense to produce such a highly depleted Belt. This means that there has to be (at least) one other event that depletes the Asteroid Belt, but which doesn't affect neither the terrestrial planets nor the giant planets - a highly unlikely case.

## V. The Inward Migration of the Solid Disk

In oligarchic growth there is a link between the single masses of the planetesimals and the final masses of the planetary embryos. When planetesimals collide with embryos they produce fragments. Bigger fragments come from bigger planetesimals and can be accreted more easily onto a protoplanet, contrary to the small fragments from smaller planetesimals as they feel the disk's gas drag more and slowly get removed from the disk. Therefore, less mass remains for the embryos. In [Kobayashi and Dauphas, 2013], realistic assumptions on the solid disk surface density and the planetesimal size have been made that lead to embryos with the size of Mars. Beyond the Ice Line, the increase in solid mass leads to bigger planetesimals that are able to produce the giant planets. Type I migration leads to planetary embryos wandering inwards, to about 1 AU as in [Hansen, 2009]. The Mars protoplanet is the outermost one, since there are just the giant planets beyond it. After the oligarchic phase these concentrated embryos start merging with each other, scattering Mars outside onto its current orbit and leaving the Asteroid Belt free of any embryos.

Pros:

- The Disk Migration Model can explain the masses of Mars and the Asteroid Belt by quite simple mechanisms.
- Because of gas drag of the small fragments and the migration of outwards lying protoplanets, there also exists a mechanism to deliver water into the inner solar system.

Cons:

- Since the Asteroid Belt is composed of the remaining planetesimals after most of the solids migrated inwards, it must have remained undisturbed since the dissipation of the gas disk. This means that the Belt's asteroids should all be on highly circular and non-inclined orbits. Resonance effects of the giant planets may clear certain gaps of the Belt that we can observe today, but they're not able to perturb the asteroids enough to explain their orbits.

# VII. CONCLUSIONS

In the last section we discussed some different models for our solar system and saw that they all have one fundamental problem: they explain one feature, usually the small Mars, but leave others unaccounted for, or pose constraints that make it nearly impossible to realize them. The EEJS for example produces the right results, but makes it hard to link it to what happened before and after it: What caused the giant planets' eccentricities initially? And how can we link it to the Nice model, or alternatively to some other theory for the formation of the rest of the solar system?

The Grand Tack on the other hand manages to explain all the things we are looking at. It starts from reasonable initial conditions, produced by established formation processes, and ends up with Jupiter and Saturn in a mean-motion resonance - exactly what we need for the Nice model.

## I. What about Mercury?

Together with the Nice model the Grand Tack explains almost everything from Venus outwards, but Mercury is another topic. We always looked at what happens on the outside, how our disk of planetesimals got truncated on the outer side, but we do not care for the inner side where Mercury could have formed similar to Mars. The reason for this is this: We don't really know what to look for. Mercury's small mass contrasts with its very big core, which makes up around 70 % of its mass. Probably Mercury had a bigger mass when it formed, but lost its mantle at a late collision with a protoplanet. This event could have been similar to the Moon-forming event that occurred to Earth, but with a steeper incident angle so that the mass got lost instead of ending up in an orbit around Mercury to form a moon (Or maybe, Mercury lost its moon afterwards).

This means that probably Mercury should have been more massive after all, and the small (but nonetheless higher than observed) Mercury masses in Grand Tack simulations would be accurate. In this case, we would only need to know when this impact happened and how massive the impactor was, and compare this with the last big impact on Mercury in Grand Tack simulations.

Another possibility would be that this close to the sun less minerals would have solidified, as metals condensate earlier than rocks. This would mean that we have to modify our solid disk surface density distribution and introduce a 'Stone Line' in our simulations.

There are more theories about this but they all don't really modify the Grand Tack scenario, as they usually do not care about the outer parts of the system. This means that we probably can integrate whatever theory we want.

## II. How probable is the Grand Tack?

One of the points I noted as a disadvantage of the Grand Tack model is that it depends on a lot of variables. Although it does not really matter how big the disk's aspect ratio is or where exactly Jupiter tacks, as they all produce similar outcomes, we still have a few fundamental constraints. If Jupiter and Saturn have the wrong mass ratio, or form too fast, or if the disk viscosity isn't right (as we have seen earlier, there is a lot of uncertainty in the viscosity due to MRIs), the Grand Tack does not work at all.

But this is really just a problem if we look at only one stellar system. But there's billions of them only in our galaxy, and so the probability for at least one where this mechanism was realized is really high. As our simulations become more elaborate and more and more exoplanets are being discovered, we might soon have another way to verify or falsify our theories: Statistics.

Planetary migration only got attention after Hot Jupiters were discovered and people realized it was really happening. Now, if we could find enough exoplanetary systems similar to our own, we could approach this problem statistically. By comparing the distribution of planetary systems that a model predicts with the actually found systems, we'd have a new method of testing formation models. How many systems look like ours? How many times has the Grand Tack failed, where we find terrestrial planets with a more uniformly distributed mass? How many times do we find young systems which are just in the middle of a Tack, with two giant planets in resonance? With a little luck, we could even find a Saturn-equivalent undergoing rapid (but for a human being still slow) Type III migration. Unfortunately, this is not possible at the moment: To test the Grand Tack, we would have to find many systems with two big gas planets and a few smaller terrestrial ones. Also, we could only really look at the planetary orbits and masses, as most of the other parameters such as the formation time would be hidden to us.

We could also take a look at different systems and try to explain them on the same basis as the Grand Tack explains ours. For example, the absence of giant planets would mean that the solid disk would not have been truncated and therefore more terrestrial planets would have formed - which might explain systems like Trappist-1.

### III. Implications on the Rare-Earth-Hypothesis

This brings us to a quite popular question: Life in the Universe.

If we say that the circumstances of the Grand Tack are quite special and happened maybe only a handful of times in our galaxy, and that life is able to evolve everywhere in the universe, what are the chances to live in such a system instead of one with a more common history of planetary formation? Following the Anthropic Principle, one might conclude that (at least intelligent) life could only exist under the special circumstances that the Grand Tack provides, because this would improve the probability of us living in a habitable post-Grand Tack system.

But when we think what the reasons for this could be, there is a more optimistic outlook. The Grand Tack produces terrestrial planets in the Habitable Zone and provides them with water, so of course it makes a system more life-friendly than a system devastated by a single Hot Jupiter. And there have to be more mechanisms to build habitable planets, as we have found numerous planets in Habitable Zones, even without any giant planets at all. Also, almost every formation model provides water in some way.

To come back to the probability of a Tacking event, we have to go back, to a point in time before the Grand Tack happened, and look at the formation of giant planets. There are many uncertainties in this regarding their formation times and even mechanisms, and of course how many of them form. This in turn leads us back to the composition of the protoplanetary disk, of which we can only speculate even more. Only when we have answered these questions will we be able to say how common the Grand Tack is, and ultimately be able to understand it.

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