

Spezialvorlesung zur  
Theoretischen Physik:  
Ab-initio Methoden in der  
Festkörperphysik  
(2 VO)  
653.534  
Peter Puschnig

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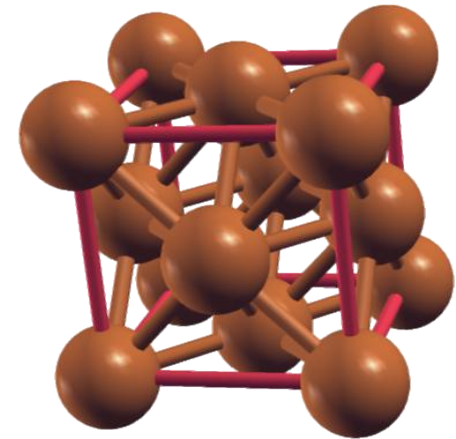
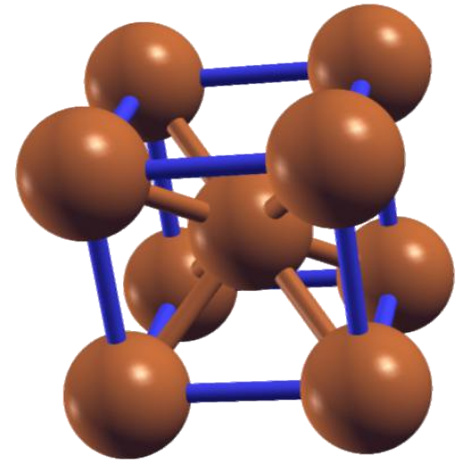
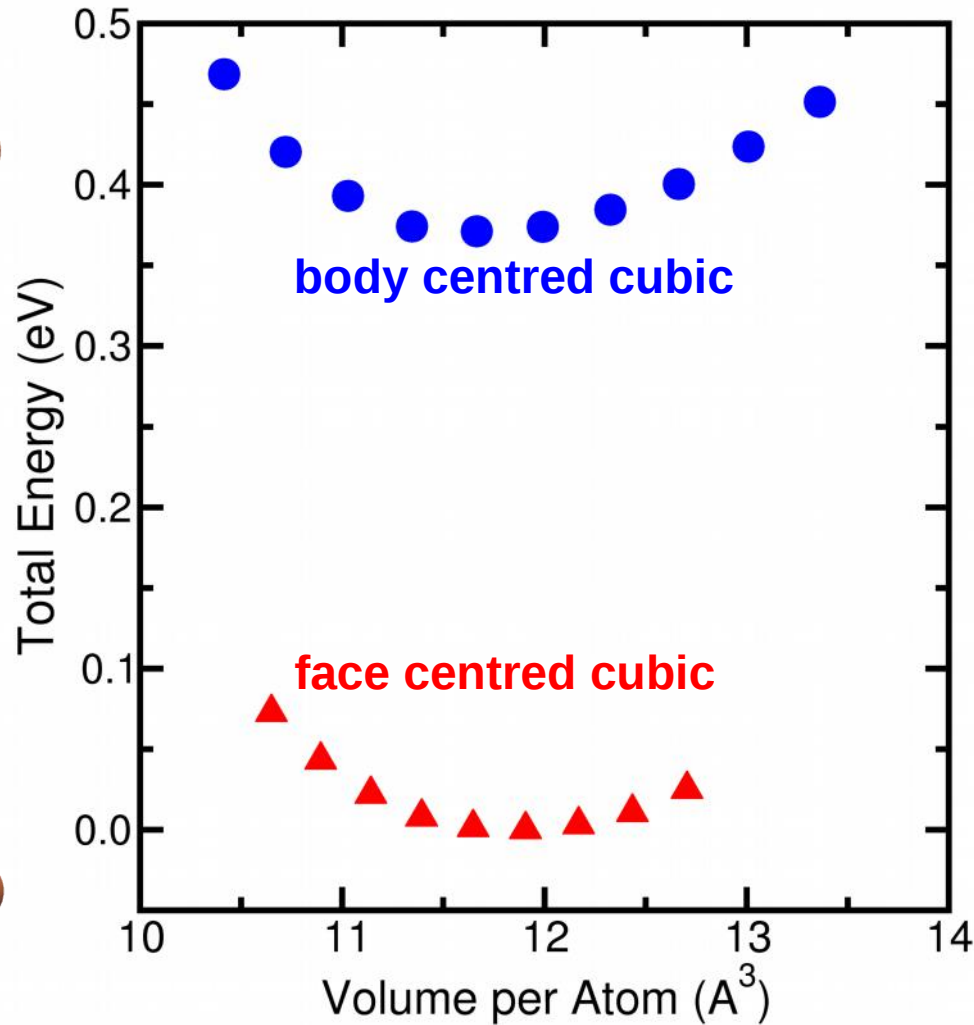
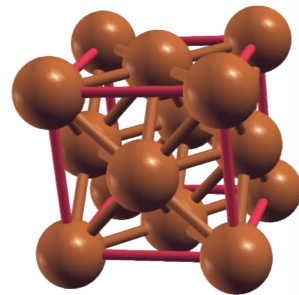
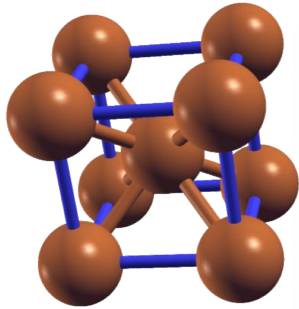
## **4. Outlook**

- 4.1 Known Problems of DFT
- 4.2 Current Trends in DFT
- 4.3 Beyond DFT

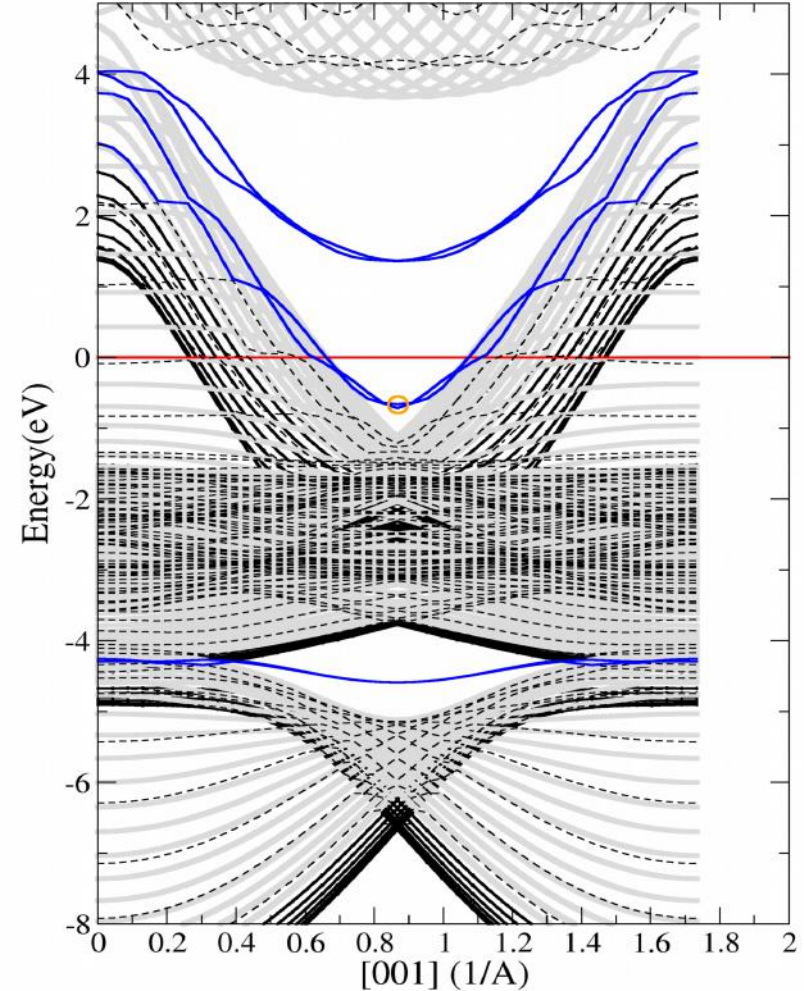
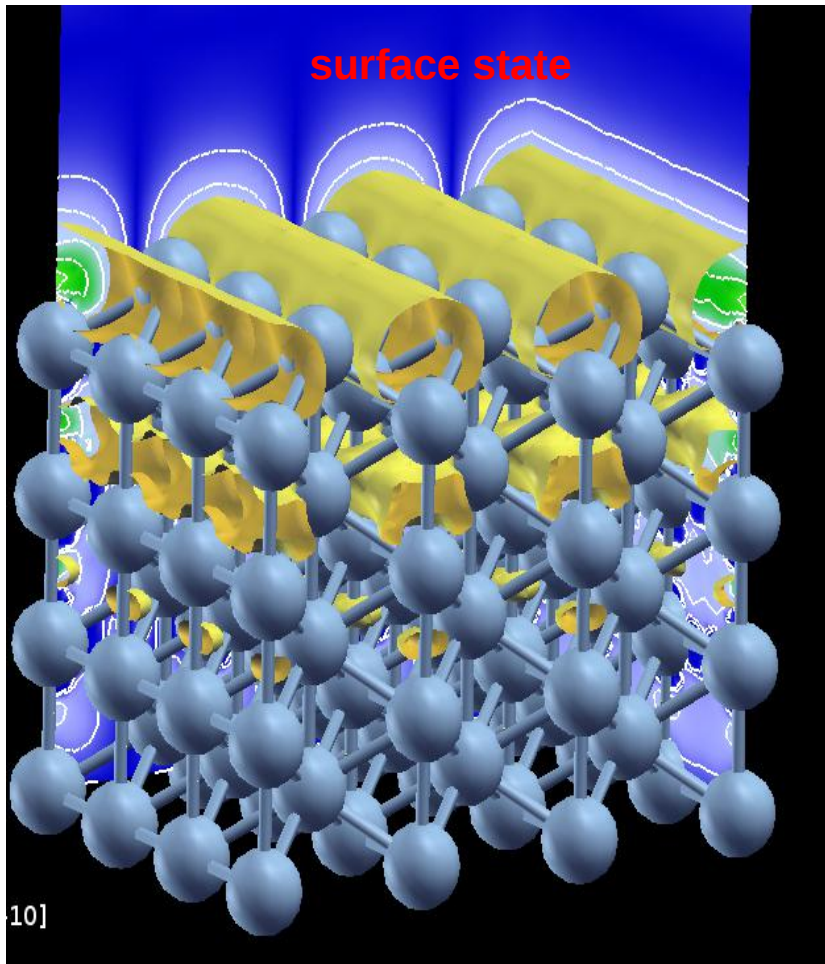
# Literature

- [1] A Primer in Density Functional Theory, Lecture Notes in Physics, C. Fiolhais, F. Nogueira, M. Marques (Eds.), Springer (2003).
- [2] Density Functional Theory – A Practical Introduction, David S. Sholl, Janice A. Steckel, Wiley (2009).
- [3] Walter Kohn, Nobel Lecture: Electronic structure of matter-wave functions and density functionals, Rev. Mod. Phys. 71, 1253 (1999).
- [4] R. O. Jones, O. Gunnarsson, The density functional formalism, its application and prospects, Rev. Mod. Phys. 61, 689 (1989).
- [5] Peter Hohenberg, Walter Kohn, Inhomogeneous Electron Gas, Phys. Rev. 136, B864 (1964).
- [6] W. Kohn, L. J. Sham, Self-Consistent Equations Including Exchange and Correlation Effects, Phys. Rev. 140, A1133-A1138 (1965).

# Elastic Properties: Bulk Modulus

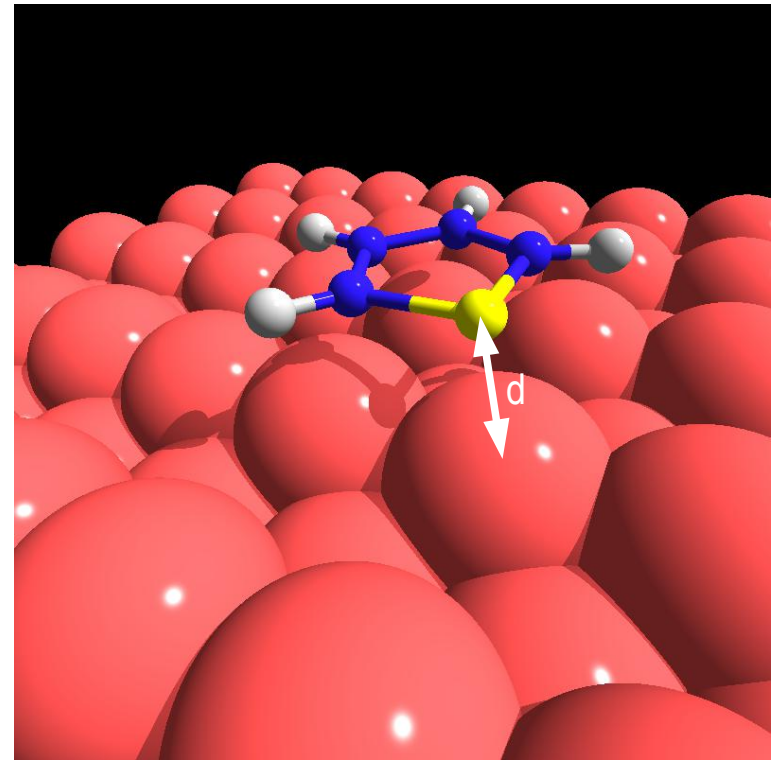
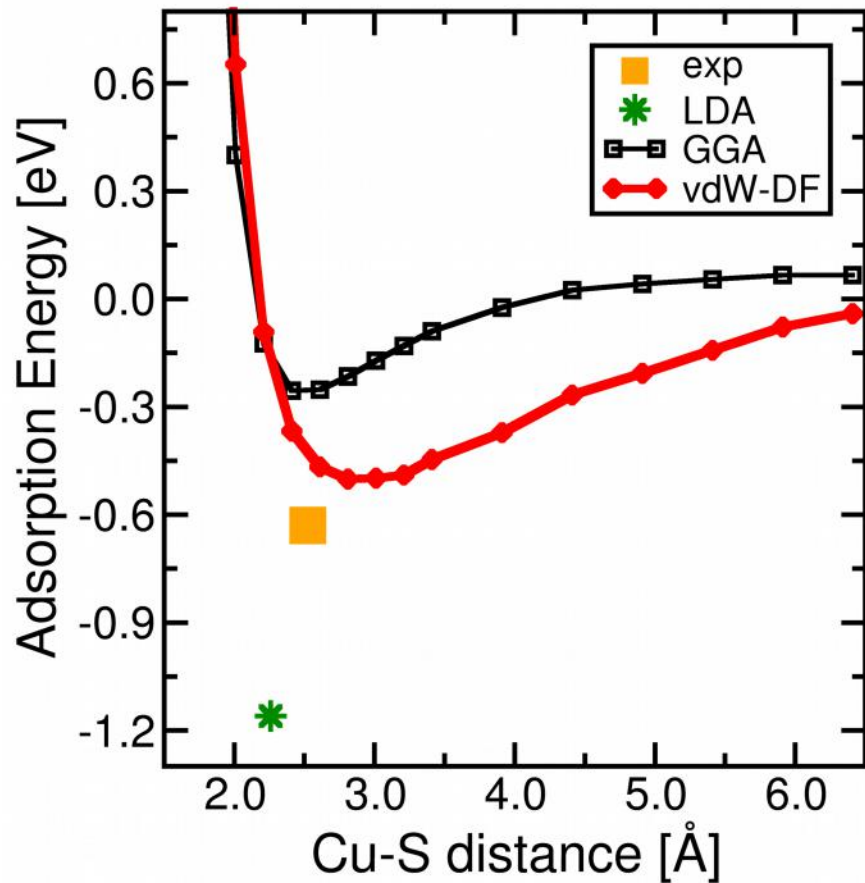


# Surface State at a Cu(110) Surface



Surface state (left) and electronic band structure of a Cu(110) surface

# Molecules Adsorbed at Surfaces

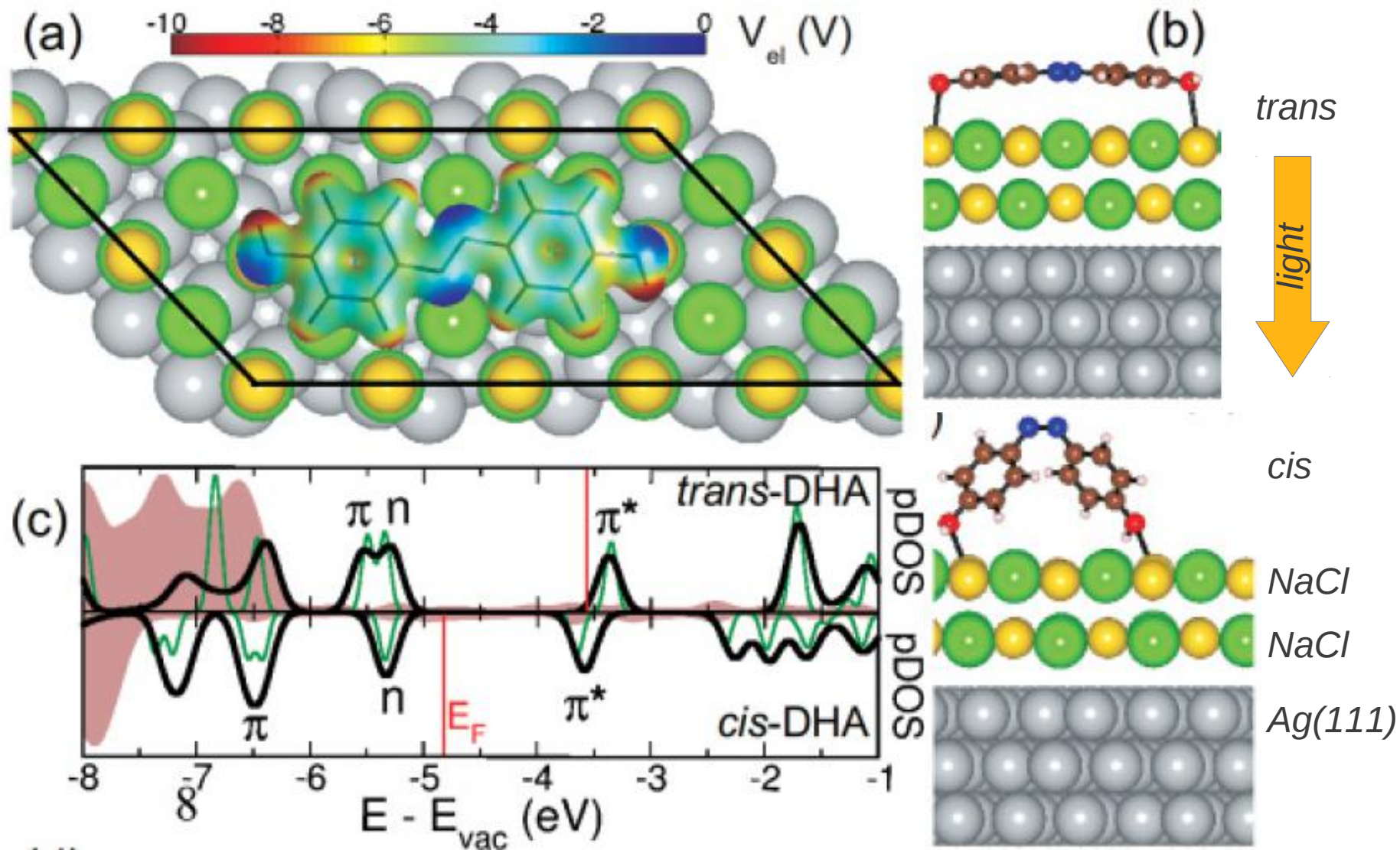


Thiophene@Cu(110): Sony et al., *Phys. Rev. Lett.* **99**, 176401 (2007).

PTCDA@Cu,Ag,Au(111): Romaner et al., *New. J. Phys.* **11**, 053010 (2009).



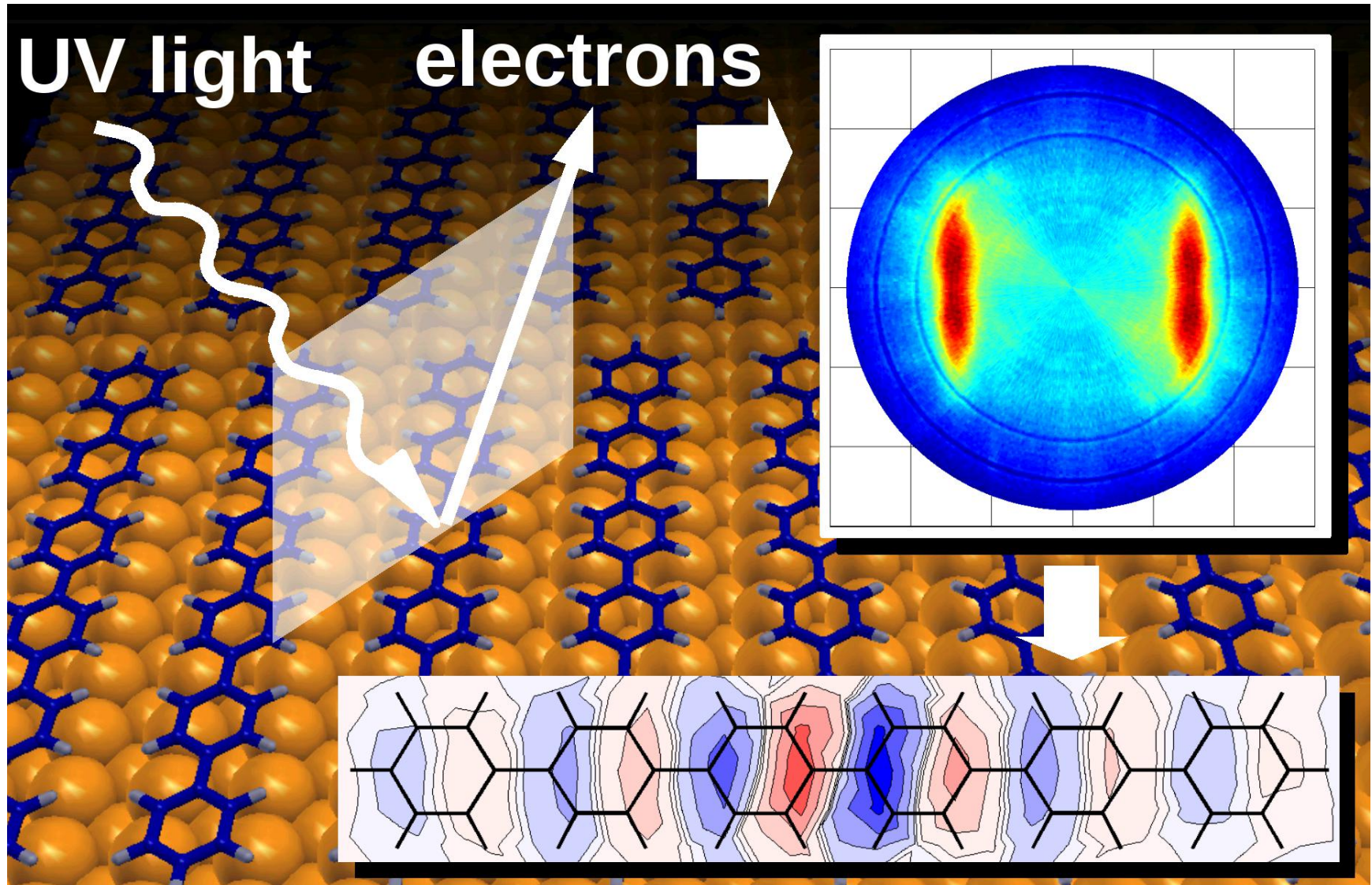
# Molecules Adsorbed at Surfaces



Henzl et al., *Phys. Rev. B* **85**, 035410 (2010).

## 1.1 Introduction

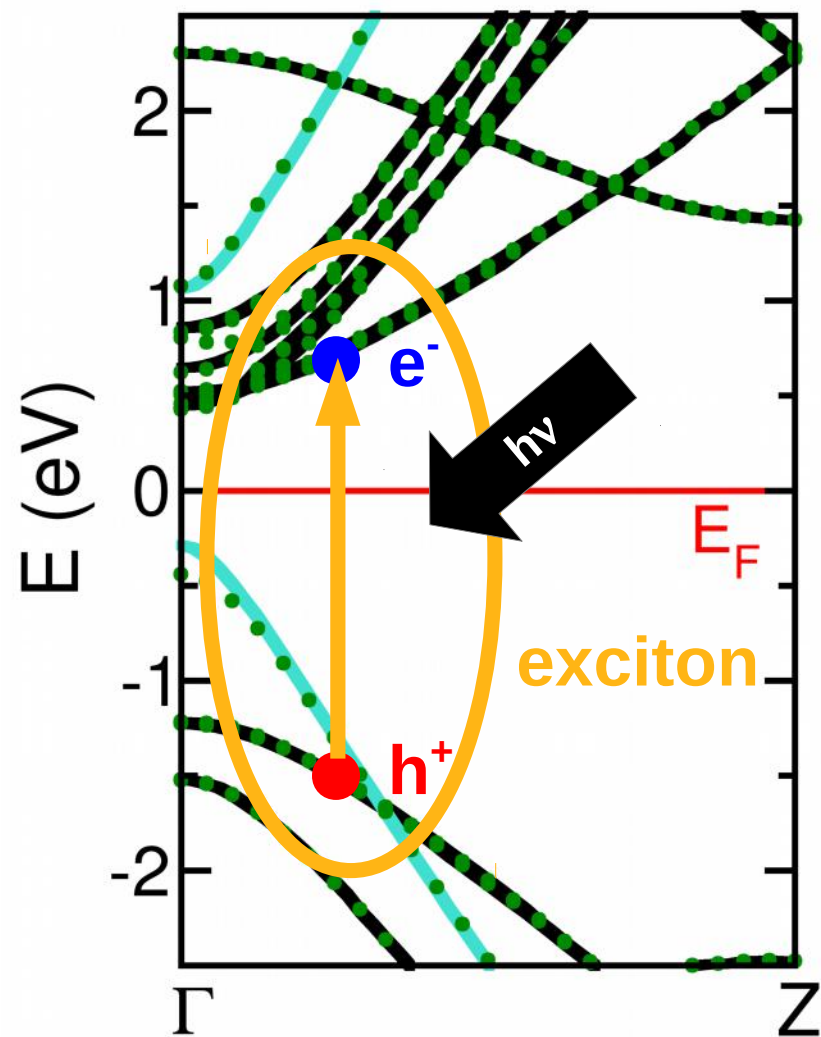
# Simulation of Photoemission Process



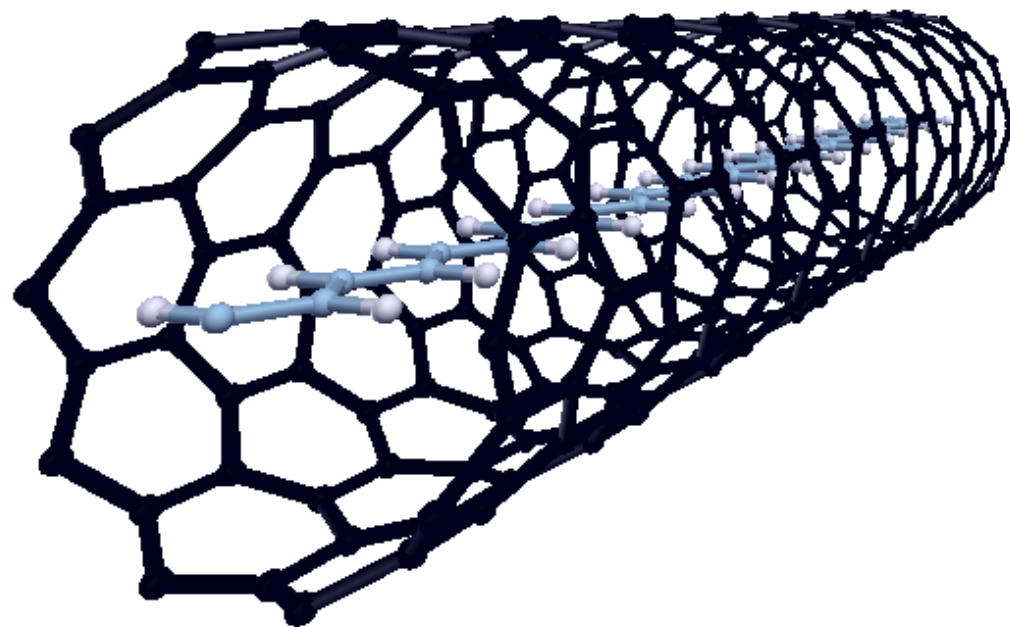
Puschnig et al., *Science* **326**, 702 (2009).



# A Polymer Inside a Carbon Nanotube

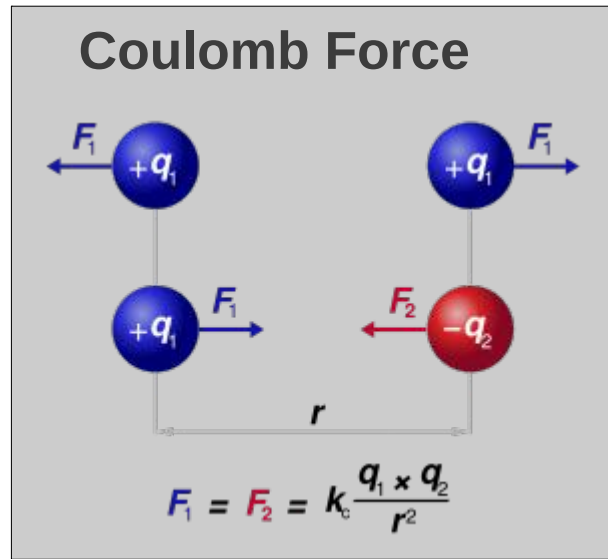


Puschnig, unpublished



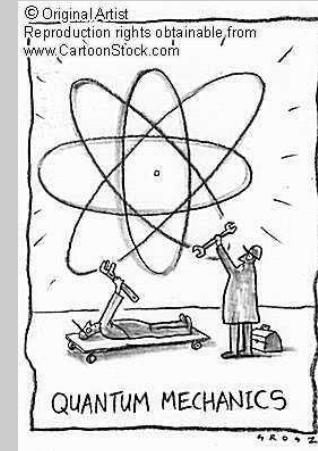
*trans*-polyacetylene  
inside a  
(10,0) CNT

# The Quantum Mechanical Many-Body Problem



+

## Quantum Mechanics



Band  
structure

Elastic  
properties

vibrations

Magnetic  
properties

Optical  
properties

# The Quantum Mechanical Many-Body Problem

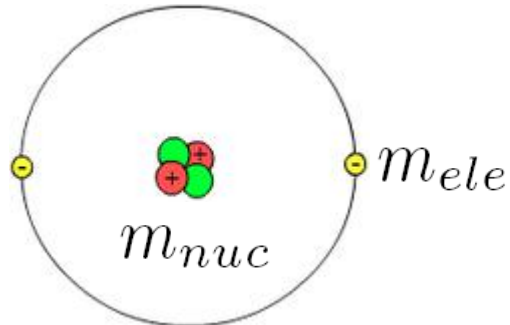
The material world of everyday experience, as studied by chemistry and condensed-matter physics, is built up from electrons and a few (or at most a few hundred) kinds of nuclei. The basic interaction is electrostatic or Coulombic: An electron at position  $\mathbf{r}$  is attracted to a nucleus of charge  $Z$  at  $\mathbf{R}$  by the potential energy  $-Z/|\mathbf{r} - \mathbf{R}|$ , a pair of electrons at  $\mathbf{r}$  and  $\mathbf{r}'$  repel one another by the potential energy  $1/|\mathbf{r} - \mathbf{r}'|$ , and two nuclei at  $\mathbf{R}$  and  $\mathbf{R}'$  repel one another as  $Z'Z/|\mathbf{R} - \mathbf{R}'|$ . The electrons must be described by quantum mechanics, while the more massive nuclei can sometimes be regarded as classical particles. All of the electrons in the lighter elements, and the chemically important valence electrons in most elements, move at speeds much less than the speed of light, and so are non-relativistic.

In essence, that is the simple story of practically everything. But there is still a long path from these general principles to theoretical prediction of the structures and properties of atoms, molecules, and solids, and eventually to the design of new chemicals or materials. If we restrict our focus to the important class of ground-state properties, we can take a shortcut through density functional theory.

*Quoted from: J. Perdew, "A Primer in DFT"*

## 1.1 Introduction

# Electrons and Atomic Nuclei



He atom (schematically)

$$m_{nuc} \gg m_{ele}$$

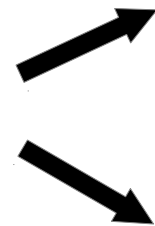


**Born-Oppenheimer approximation**

$$\Psi_{total} = \psi_{electronic} \times \psi_{nuclear}$$

**de Broglie wavelength**

$$\lambda = \frac{h}{p}$$

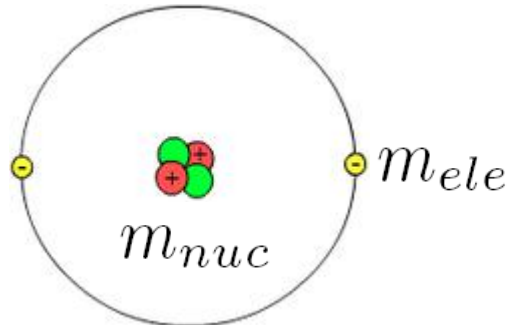


**electrons QM**

**nuclei classical (Newton)**



# The Adiabatic Approximation



*He atom (schematically)*

$$m_{nuc} \gg m_{ele}$$

## The Born-Oppenheimer (=adiabatic) Approximation

- The electrons can follow the much heavier nuclei instantaneously
- Electronic and nuclear motion can be separated
- The nuclear coordinates can be regarded as parameters for the electronic problem
- The electronic energy provides a potential for the nuclear motion

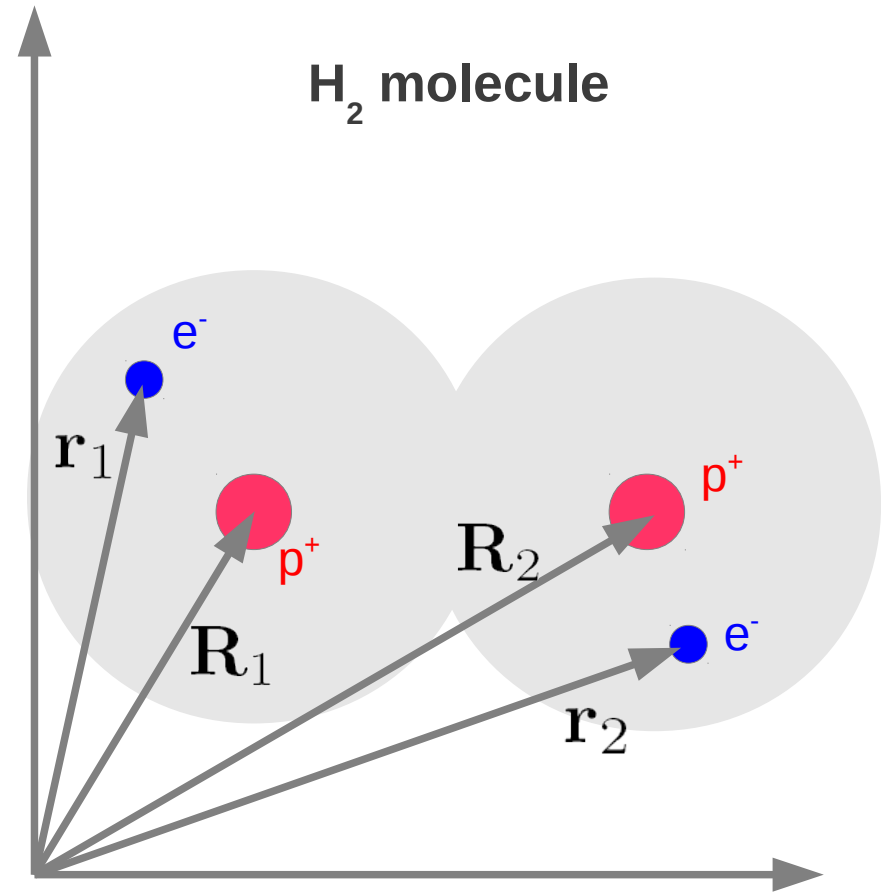
# Towards the Many-Electron Problem

## Kinetic Energy

$$T = T_e + T_n$$

$$\begin{aligned} T_e &= \frac{p_1^2}{2m_e} + \frac{p_2^2}{2m_e} \\ &= \sum_{i=1}^2 \frac{p_i^2}{2m_e} \end{aligned}$$

~~$$T_n = \frac{P_1^2}{2M_n} + \frac{P_2^2}{2M_n}$$~~



# Towards the Many-Electron Problem

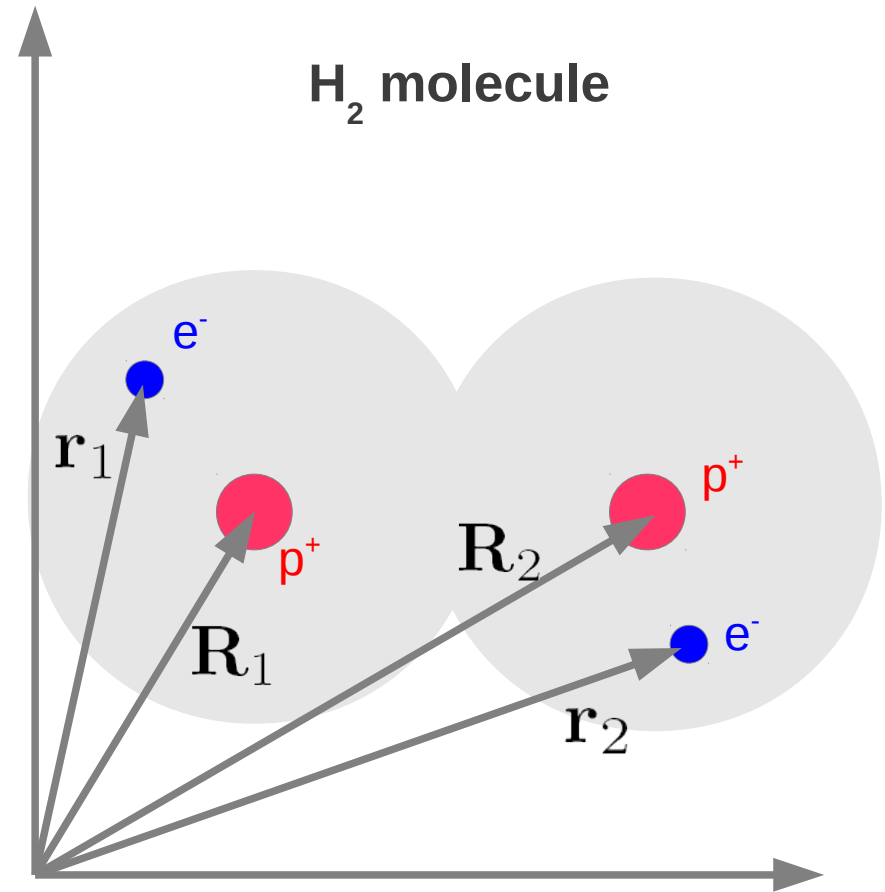
## Potential Energy

$$V = V_{n-n} + V_{n-e} + V_{e-e}$$

$$V_{n-n} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{R}_1 - \mathbf{R}_2|}$$

$$V_{e-e} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

$$\begin{aligned} V_{n-e} &= -\frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_1 - \mathbf{R}_1|} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_1 - \mathbf{R}_2|} \\ &\quad - \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_2 - \mathbf{R}_1|} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_2 - \mathbf{R}_2|} \\ &= -\sum_{i=1}^2 \sum_{j=1}^2 \frac{1}{4\pi\epsilon_0} \frac{Z_j e^2}{|\mathbf{r}_i - \mathbf{R}_j|} \end{aligned}$$



# Towards the Many-Electron Problem

## Total Electronic Hamiltonian

$$H = T_e + V$$

$$= \sum_{i=1}^2 h_i + V_{e-e}$$

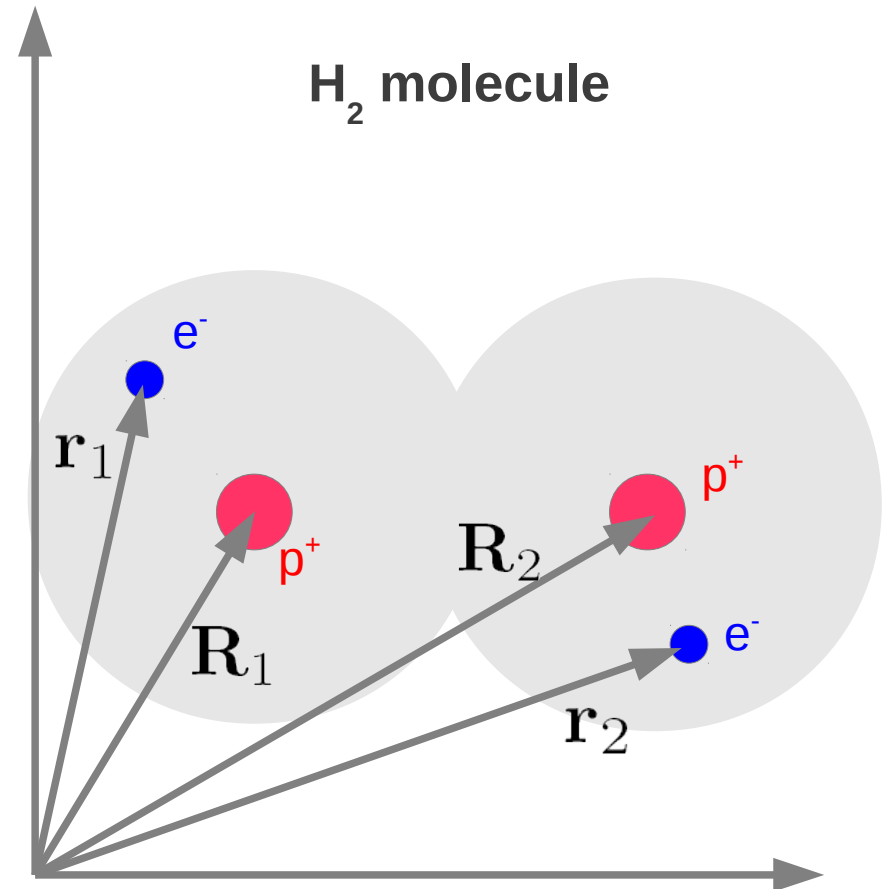
$$h_i = -\frac{\hbar^2}{2m_e} \Delta_i + \sum_{j=1}^2 \frac{1}{4\pi\epsilon_0} \frac{Z_j e^2}{|\mathbf{r}_i - \mathbf{R}_j|}$$

$$V_{e-e} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$



*2-electron Schrödinger equation*

$$H\Psi(\mathbf{r}_1, \mathbf{r}_2) = E\Psi(\mathbf{r}_1, \mathbf{r}_2)$$





# The Many-Electron Problem

## Total Electronic Hamiltonian

$$H = \sum_{i=1}^n h_i + V_{e-e}$$

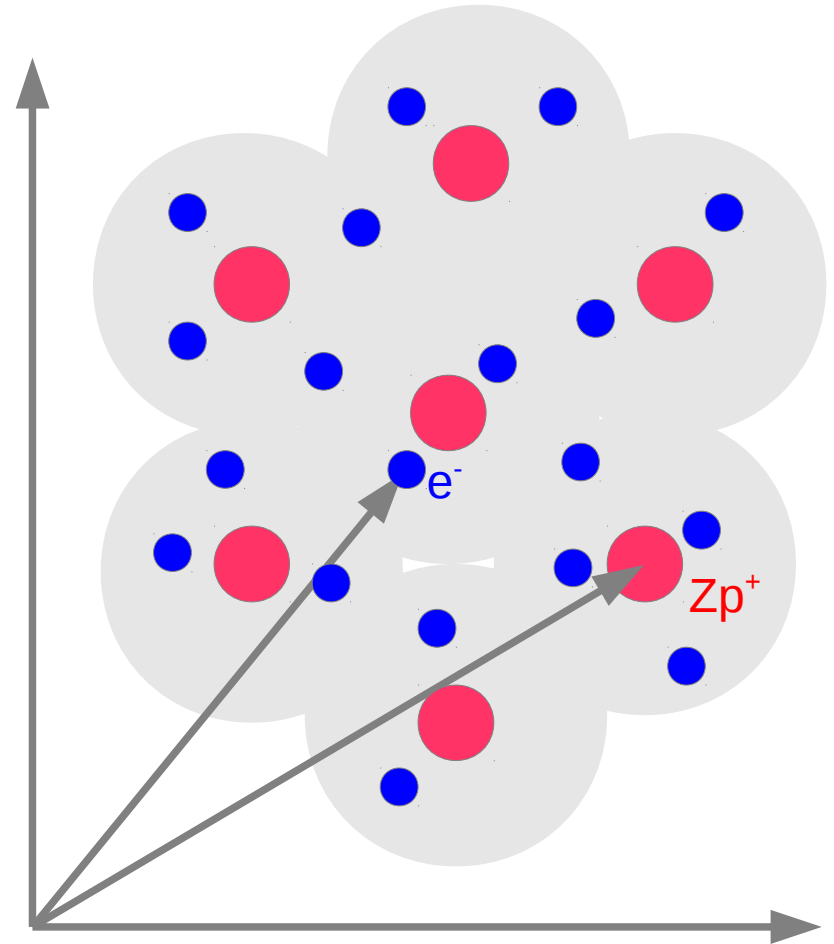
$$h_i = -\frac{\hbar^2}{2m_e} \Delta_i + \sum_{j=1}^N \frac{1}{4\pi\epsilon_0} \frac{Z_j e^2}{|\mathbf{r}_i - \mathbf{R}_j|}$$

$$V_{e-e} = \sum_{i=2}^n \sum_{j=1}^{i-1} \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|}$$



*Many-electron Schrödinger equation*

$$H\Psi(\mathbf{r}_1, \dots, \mathbf{r}_i, \dots, \mathbf{r}_n) = E\Psi(\mathbf{r}_1, \dots, \mathbf{r}_i, \dots, \mathbf{r}_n)$$



# Van Vleck Catastrophe

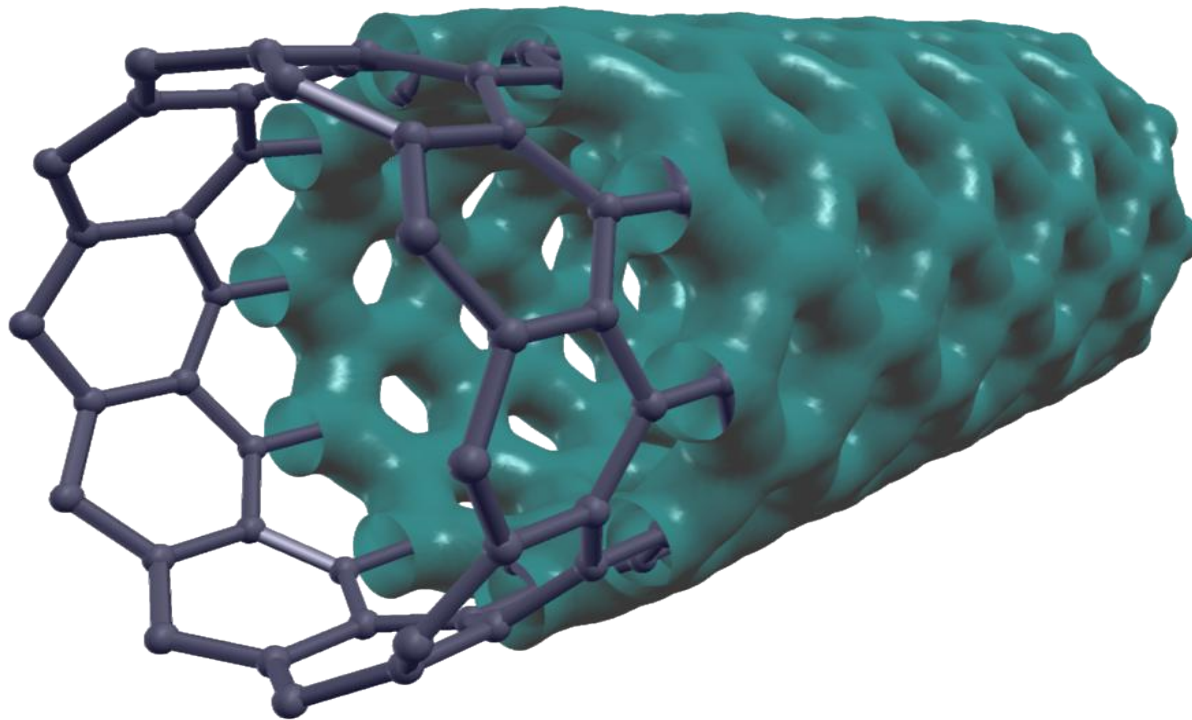
- Small molecules

- ◆ Wave function methods (HF, configuration interaction, ...) give excellent results
- ◆ Number of parameters in many-electron wave function:  $M = p^{3N}$

- Large molecules and solids

- ◆ „Many electron wave function is not a legitimate concept when  $N > 100$ “
- ◆ Number of parameters in wave function:  $M = p^{3 \cdot 100} = 10^{150}!!$
- ◆ Accuracy of the wave function becomes a problem!
- ◆ Storage of the results:  $B = q^{3 \cdot 100} = 10^{150}$  bits required !!

# Electron Density as a Loophole



- Electron density  $n(\mathbf{r})$  is the basic variable
- Density Functional Theory (DFT) provides rigorous framework
- All microscopic and macroscopic properties depend on  $n(\mathbf{r})$

# DFT in a Nutshell: Hohenberg-Kohn-Theorems

1. If two systems of electrons, one trapped in a potential  $v_1(\mathbf{r})$  and the other in  $v_2(\mathbf{r})$ , have the same ground-state density  $n(\mathbf{r})$  then necessarily  $v_1(\mathbf{r}) - v_2(\mathbf{r}) = \text{const.}$

*Corollary:* the ground state density  $n(\mathbf{r})$  uniquely determines the potential and thus all properties of the system, including the many-body wave function. In particular, the "Hohenberg-Kohn" functional, defined as

$$F[n] = T[n] + U[n]$$

is a universal functional of the density (not depending explicitly on the external potential).

2. For any positive integer  $N$  and potential  $v(\mathbf{r})$  the density functional

$$E[n] = F[n] + \int n(\mathbf{r})v(\mathbf{r})d^3r$$

obtains its minimal value at the ground-state density of  $N$  electrons in the potential  $v(\mathbf{r})$ . The minimal value  $E[n]$  is then the ground state energy of this system.

Peter Hohenberg, Walter Kohn, Inhomogeneous Electron Gas, Phys. Rev. 136, B864 (1964).



# DFT in a Nutshell: Kohn Sham Equations

$$\left[ -\frac{1}{2} \nabla^2 + V_{\text{ext}}(\mathbf{r}) + V_H(\mathbf{r}) + V_{xc}(\mathbf{r}) \right] \psi_i(\mathbf{r}) = \varepsilon_i \psi_i(\mathbf{r})$$

$$-\frac{Z}{r}$$

$$\int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 r'$$

$$\frac{\delta E_{xc}[n(\mathbf{r})]}{\delta n(\mathbf{r})}$$

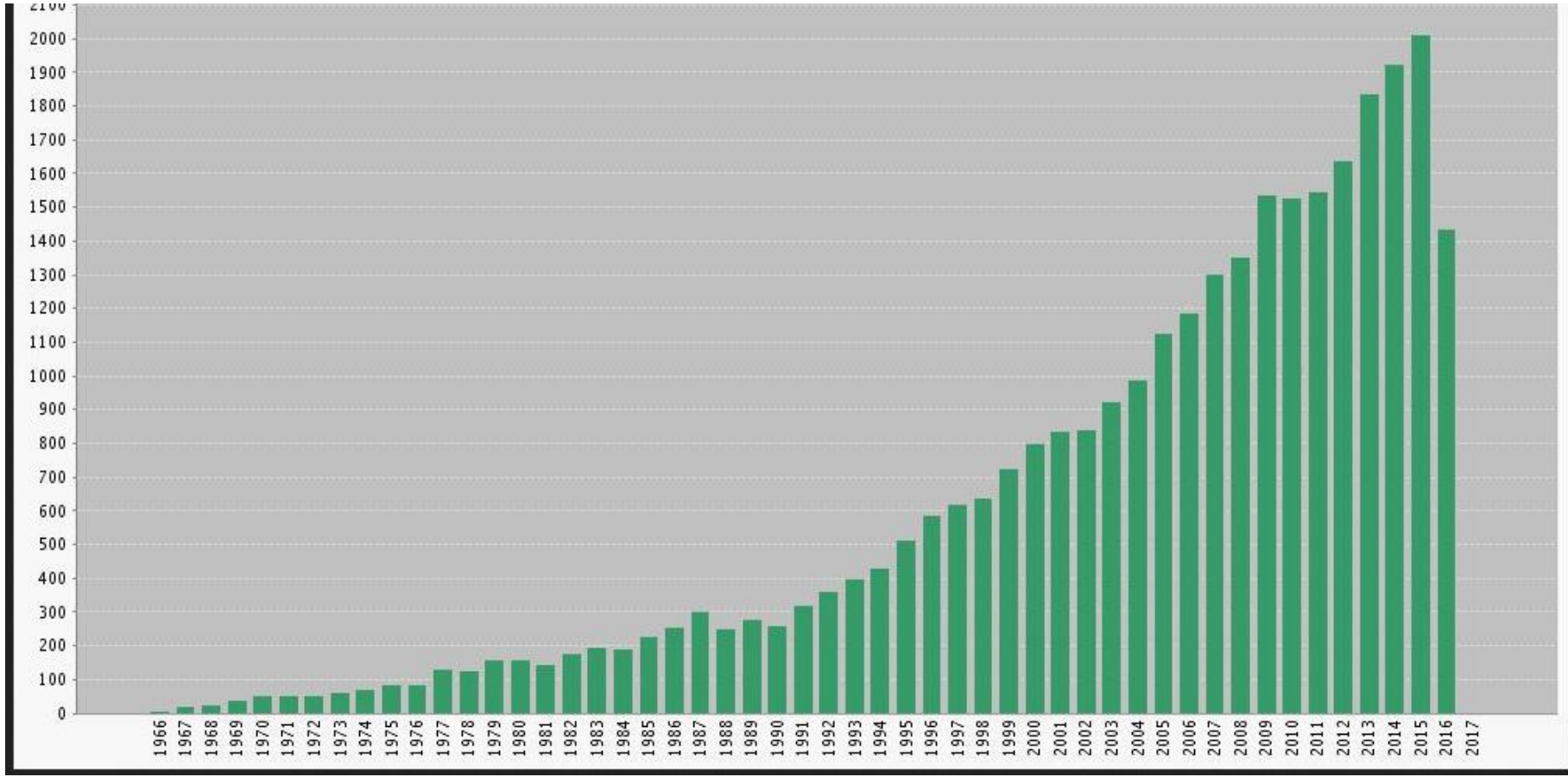
Self-consistency

**Approximations:**  
e.g.: LDA, GGA, ...

$$n(\mathbf{r}) = \sum_i^{\text{occ}} |\psi_i(\mathbf{r})|^2$$

W. Kohn, L. J. Sham, Self-Consistent Equations Including Exchange and Correlation Effects, Phys. Rev. 140, A1133-A1138 (1965)

# Citation Report of Kohn-Sham paper from 1965



> 30000 citations

W. Kohn, L. J. Sham, Self-Consistent Equations Including Exchange and Correlation Effects, Phys. Rev. 140, A1133-A1138 (1965)

# DFT in a Nutshell: Total Energy

$$E = T_s[n_\uparrow, n_\downarrow] + \int d^3r n(\mathbf{r})v(\mathbf{r}) + U[n] + E_{xc}[n_\uparrow, n_\downarrow]$$

*kinetic energy*      *electron-ion interaction*      *Hartree Energy (self-repulsion of electrons)*      *Exchange-Correlation-energy*

$$T_s[n_\uparrow, n_\downarrow] = \sum_{\sigma} \sum_{\alpha} \theta(\mu - \varepsilon_{\alpha\sigma}) \langle \psi_{\alpha\sigma} | -\frac{1}{2} \nabla^2 | \psi_{\alpha\sigma} \rangle$$

$$U[n] = \frac{1}{2} \int d^3r \int d^3r' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$E_{xc}$  Defined to include everything else omitted from the first three terms ...

$$E_{xc}^{\text{LSD}}[n_\uparrow, n_\downarrow] = \int d^3r n(\mathbf{r})e_{xc}(n_\uparrow(\mathbf{r}), n_\downarrow(\mathbf{r})) \quad \text{Local spin density approximation}$$

# Wave Function Theory

## One Electron

$$\hat{h} = -\frac{1}{2}\nabla^2 + v(\mathbf{r})$$

Hamiltonian

$$\hat{h}\psi_\alpha(\mathbf{r}, \sigma) = \varepsilon_\alpha\psi_\alpha(\mathbf{r}, \sigma)$$

Stationary  
Schrödinger  
Equation (SE)

$$\sum_\sigma \int d^3r |\psi_\alpha(\mathbf{r}, \sigma)|^2 = \langle \psi | \psi \rangle = 1$$

Normalization

## Many Electrons

$$\hat{H} = -\frac{1}{2} \sum_{i=1}^N \nabla_i^2 + \sum_{i=1}^N v(\mathbf{r}_i) + \frac{1}{2} \sum_i \sum_{j \neq i} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Hamiltonian

$$\hat{H}\Psi_k(\mathbf{r}_1\sigma_1, \dots, \mathbf{r}_N\sigma_N) = E_k\Psi_k(\mathbf{r}_1\sigma_1, \dots, \mathbf{r}_N\sigma_N)$$

SE

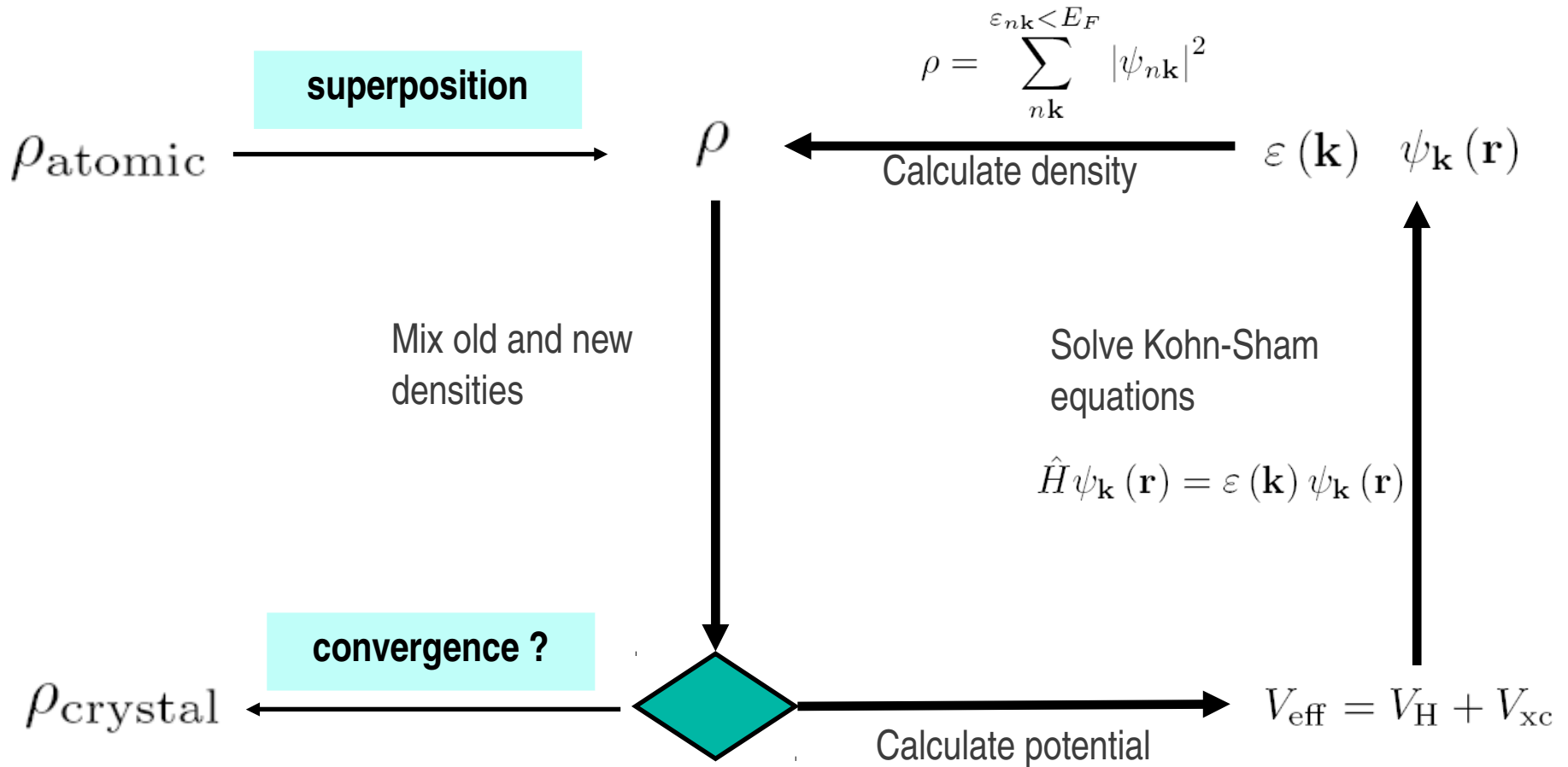
$$\begin{aligned} \Psi(\mathbf{r}_1\sigma_1, \dots, \mathbf{r}_i\sigma_i, \dots, \mathbf{r}_j\sigma_j, \dots, \mathbf{r}_N\sigma_N) = \\ - \Psi(\mathbf{r}_1\sigma_1, \dots, \mathbf{r}_j\sigma_j, \dots, \mathbf{r}_i\sigma_i, \dots, \mathbf{r}_N\sigma_N) \end{aligned}$$

Pauli Principle

$$\frac{1}{N!} \sum_{\sigma_1 \dots \sigma_N} \int d^3r_1 \dots \int d^3r_N N! |\Psi(\mathbf{r}_1\sigma_1, \dots, \mathbf{r}_N\sigma_N)|^2 = \int |\Psi|^2 = \langle \Psi | \Psi \rangle = 1.$$

Normalization

# 1.2 Wave Function Theory



## 2.1 Self Consistency

Kohn-Sham equation  
(differential eigenvalue equation)

$$\hat{H}\psi_{\mathbf{k}}(\mathbf{r}) = \varepsilon(\mathbf{k})\psi_{\mathbf{k}}(\mathbf{r})$$

Linear expansion in  
known basis functions

$$u_{\mathbf{k}}(\mathbf{r}) = \sum_j c_j(\mathbf{k})\phi_j(\mathbf{r})$$

$$\phi_j(\mathbf{r} + \mathbf{R}) = \phi_j(\mathbf{r})$$



Kohn-Sham equation  
(matrix eigenvalue equation)

$$\sum_j H_{ij}(\mathbf{k})c_j(\mathbf{k}) = \varepsilon(\mathbf{k})\sum_j S_{ij}(\mathbf{k})c_j(\mathbf{k})$$

Hamilton matrix

$$H_{ij}(\mathbf{k}) = \int_{\Omega} d^3r e^{-i\mathbf{k}\mathbf{r}} \phi_i^*(\mathbf{r}) \hat{H} e^{i\mathbf{k}\mathbf{r}} \phi_j(\mathbf{r})$$

Overlap matrix

$$S_{ij}(\mathbf{k}) = \int_{\Omega} d^3r \phi_i^*(\mathbf{r}) \phi_j(\mathbf{r})$$

## 2.2 Basis Functions



# Plane Wave Basis

Plane wave basis:

$$\phi_j(\mathbf{r}) \rightarrow \phi_{\mathbf{G}}(\mathbf{r}) = \frac{1}{\sqrt{\Omega}} e^{i\mathbf{G}\mathbf{r}}$$

Lattice periodicity

$$\phi_{\mathbf{G}}(\mathbf{r} + \mathbf{R}) = \phi_{\mathbf{G}}(\mathbf{r})$$

Orthogonality

$$\int_{\Omega_0} \phi_{\mathbf{G}}^*(\mathbf{r}) \phi_{\mathbf{G}'}(\mathbf{r}) = \delta_{\mathbf{G}\mathbf{G}'}$$

Completeness

$$\sum_{\mathbf{G}} \phi_{\mathbf{G}}^*(\mathbf{r}) \phi_{\mathbf{G}}(\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$$

Fourier expansion of lattice periodic functions:

$$V(\mathbf{r}) = \frac{1}{\Omega_0} \sum_{\mathbf{G}} \tilde{V}(\mathbf{G}) e^{i\mathbf{G}\mathbf{r}}$$

$$\tilde{V}(\mathbf{G}) = \int_{\Omega_0} d^3r V(\mathbf{r}) e^{-i\mathbf{G}\mathbf{r}}$$

# Plane Wave Basis

Overlap matrix  $S_{ij}(\mathbf{k}) \rightarrow S_{\mathbf{G}\mathbf{G}'} = \delta_{\mathbf{G}\mathbf{G}'}$

Hamilton matrix  $H_{ij}(\mathbf{k}) \rightarrow H_{\mathbf{G}\mathbf{G}'}(\mathbf{k}) = \underbrace{\frac{1}{2} (\mathbf{k} + \mathbf{G})^2}_{\text{kinetic energy}} \delta_{\mathbf{G}\mathbf{G}'} + \underbrace{\frac{1}{\Omega_0} \tilde{V}(\mathbf{G} - \mathbf{G}')}_{\text{effective potential}}$

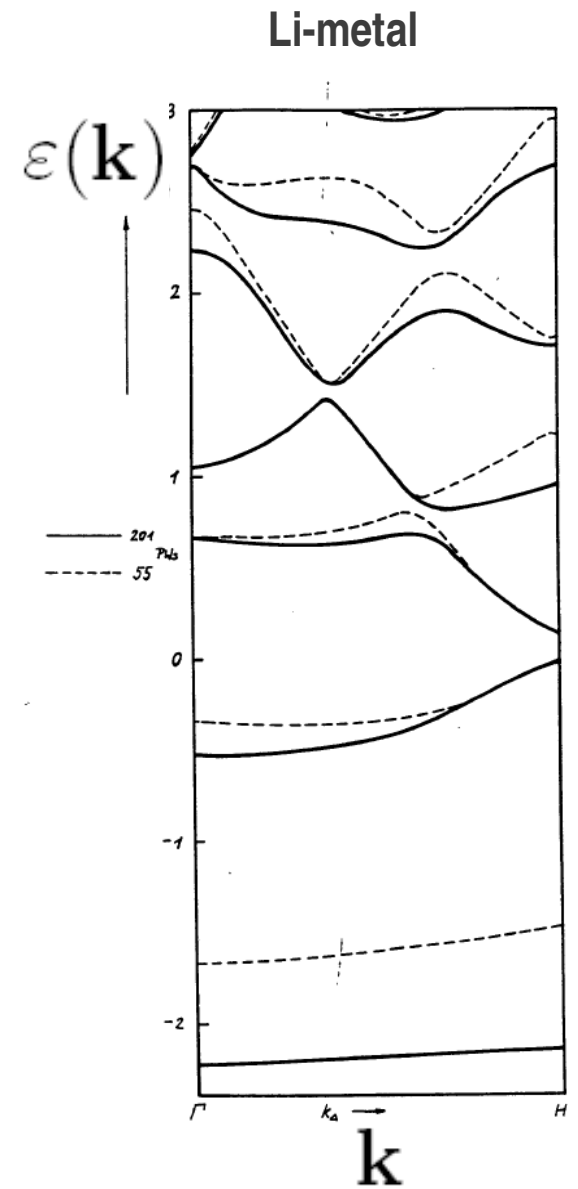
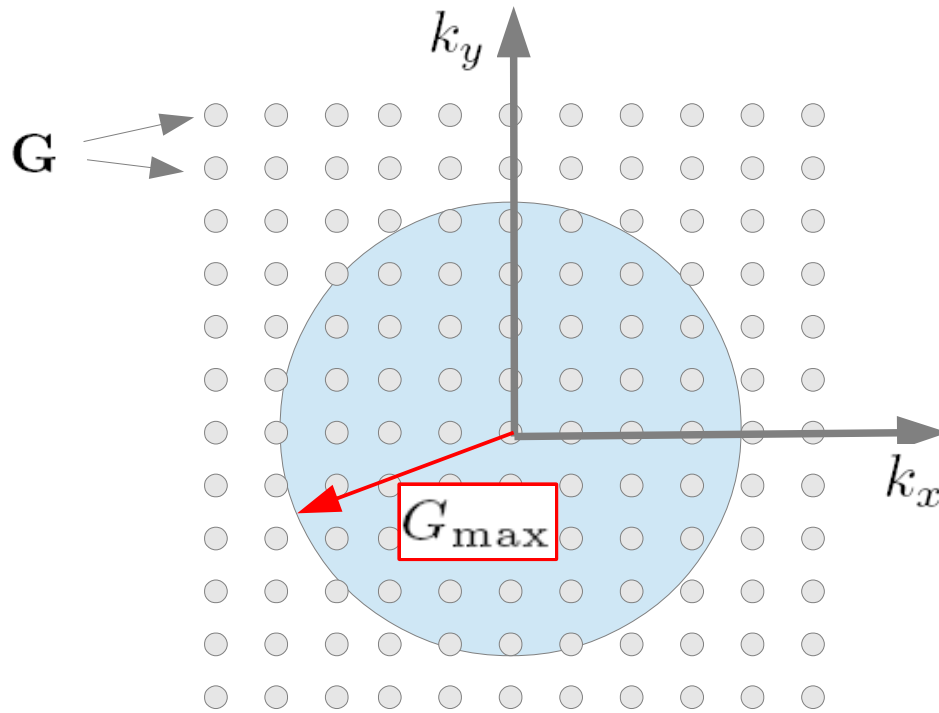
Secular equation = Matrix eigenvalue equation

$$\sum_{\mathbf{G}'} \underbrace{H_{\mathbf{G}\mathbf{G}'}(\mathbf{k})}_{\text{matrix}} \underbrace{\tilde{u}_{\mathbf{k}}(\mathbf{G}')}_{\text{vector}} = \underbrace{\varepsilon(\mathbf{k})}_{\substack{\text{eigen} \\ \text{value} \\ = \text{"band structure"}}} \underbrace{\tilde{u}_{\mathbf{k}}(\mathbf{G})}_{\text{vector}}$$

# Convergence Problems

Plane wave cut-off

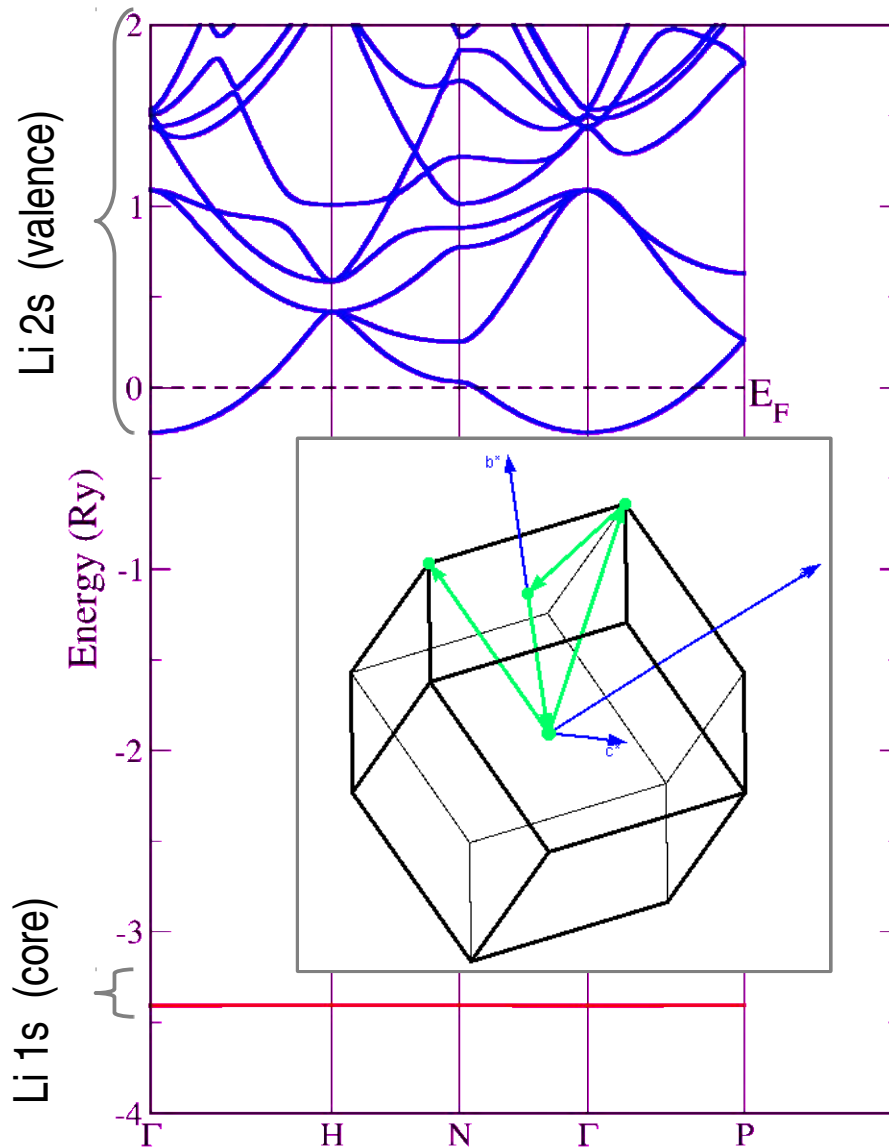
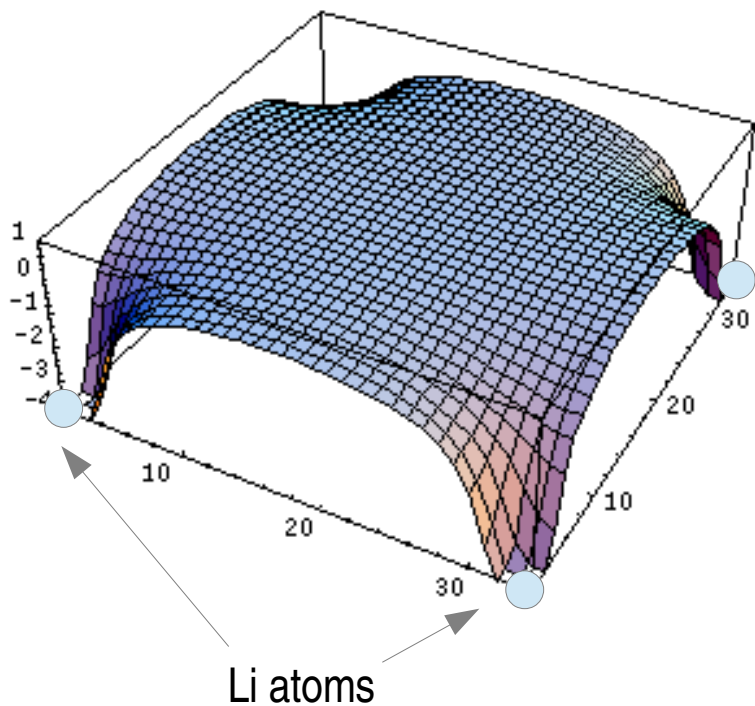
$$u_{\mathbf{k}}(\mathbf{r}) = \sum_{|\mathbf{k}+\mathbf{G}| \leq G_{\max}} \tilde{u}_{\mathbf{k}}(\mathbf{G}) e^{i\mathbf{G}\mathbf{r}} \quad E_{\text{cutoff}} = \frac{1}{2} G_{\max}^2$$



# Core States

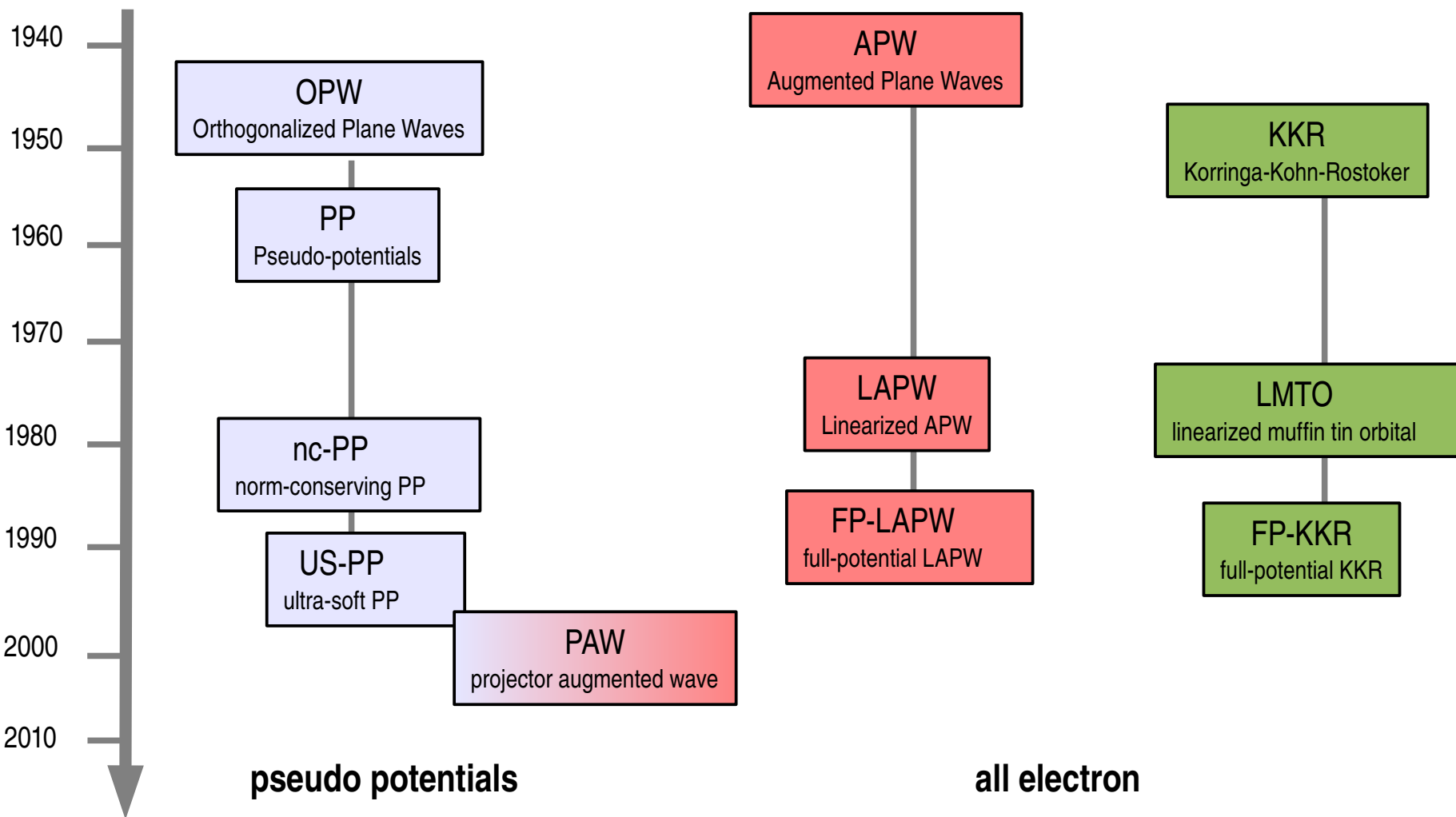
bcc-Li: accurate band structure

Potential (2D-section)



## 2.2 Basis Functions

# Other Basis Sets - Overview



# Some State-of-the-Art Program Packages

**PW-PP**

plane wave pseudo potentials

**PAW**

projector augmented wave

**FP-LAPW**

full-potential LAPW

**PWscf**

<http://www.pwscf.org/>

**CP-PAW**

<http://www2.pt.tu-clausthal.de/atp/>

**WIEN2k**

<http://www.wien2k.at/>

**ABINIT**

<http://www.abinit.org/>

**EXC!TING**

<http://exciting.sourceforge.net/>

**VASP**

<http://cms.mpi.univie.ac.at/vasp/>

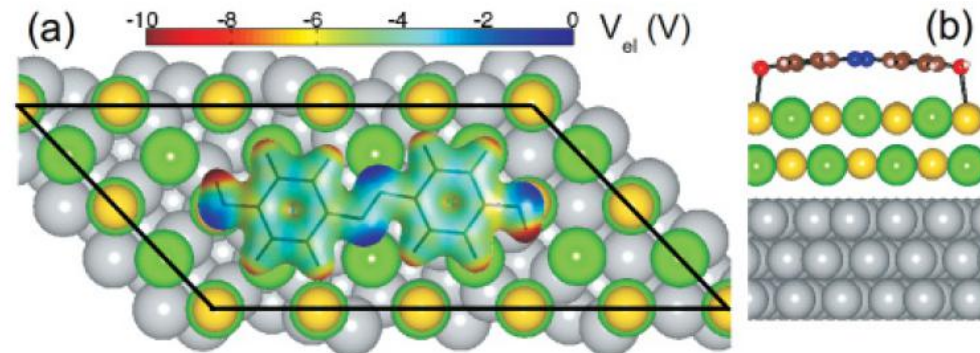
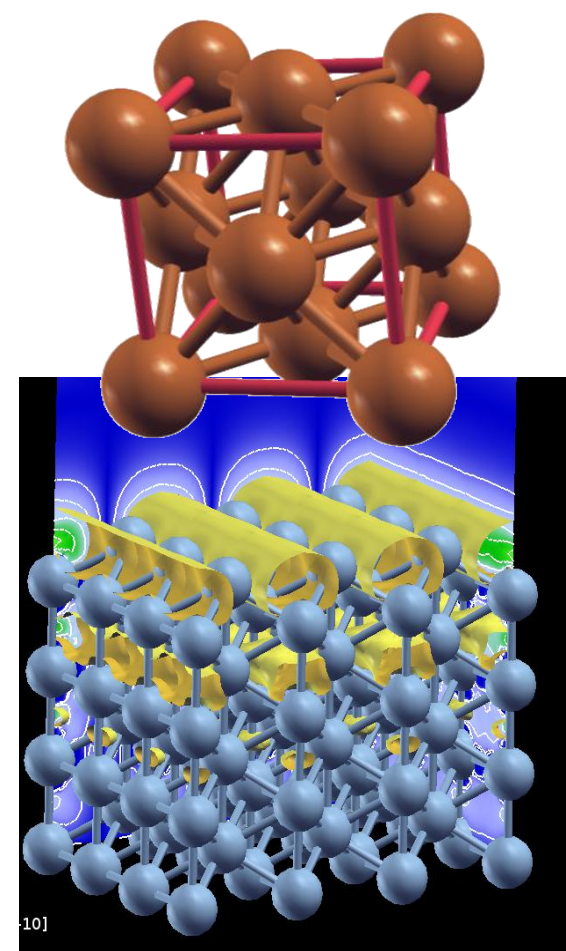
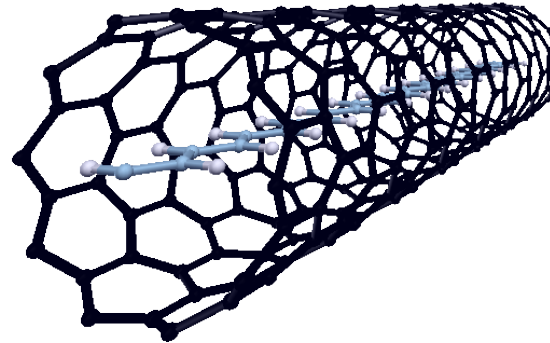


# Welche Eigenschaften?

- **Strukturelle Eigenschaften**
  - Gitterkonstanten
  - Phasenstabilität
  - Atomare Kräfte
  - Oberflächenrelaxationen
  - Defekte
- **Gitterdynamik**
  - Schwingungsfrequenzen
  - Phasenübergänge, Thermodynamik
- **Elastische Eigenschaften**
- **Elektronenverteilung**
  - Elektrische Feldgradienten
- **Elektronische Eigenschaften**
  - Bandstruktur
  - Zustandsdichte
- **Spektroskopie**
  - Dielektrische Funktion
  - Optische Absorption
  - Optische Bandlücke
  - Photoemissionsspektren
  - Core level - Spektren
  - Ramanstreuung
  - Comptonstreuung
  - Elektronenspektren
  - Positronstreuung

# Welche Systeme?

- **Kristalline Festkörper**
  - Isolatoren
  - Halbleiter
  - Metalle
  - Supraleiter
- **Defekte in Festkörpern**
  - Oberflächen
  - Interfaces
  - Versetzungen
  - Punktdefekte
- **Nichtperiodische Systeme**
  - Moleküle, Cluster, ...



# Calculation of the Kohn-Sham Bandstructure

Self-consistent solution of the Kohn-Sham equations

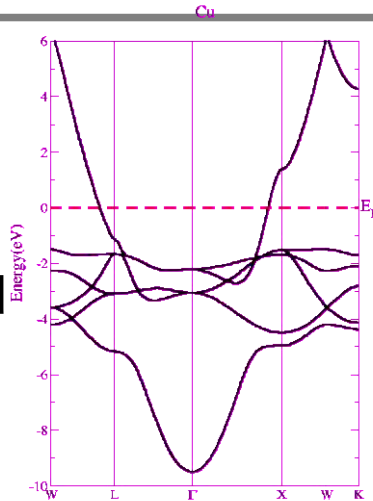
$$\hat{H}\psi_{\mathbf{k}}(\mathbf{r}) = \varepsilon(\mathbf{k})\psi_{\mathbf{k}}(\mathbf{r})$$



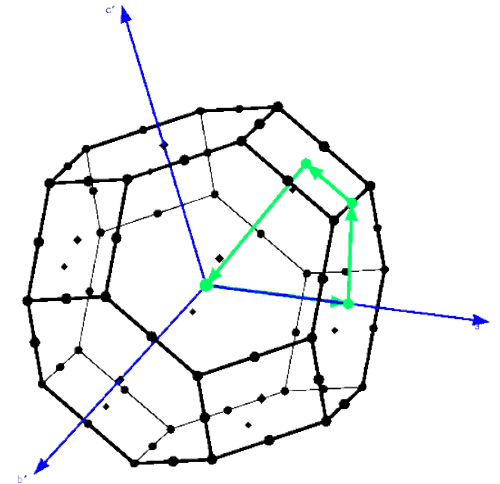
Self-consistent electron density, total energy, Fermi energy, ...



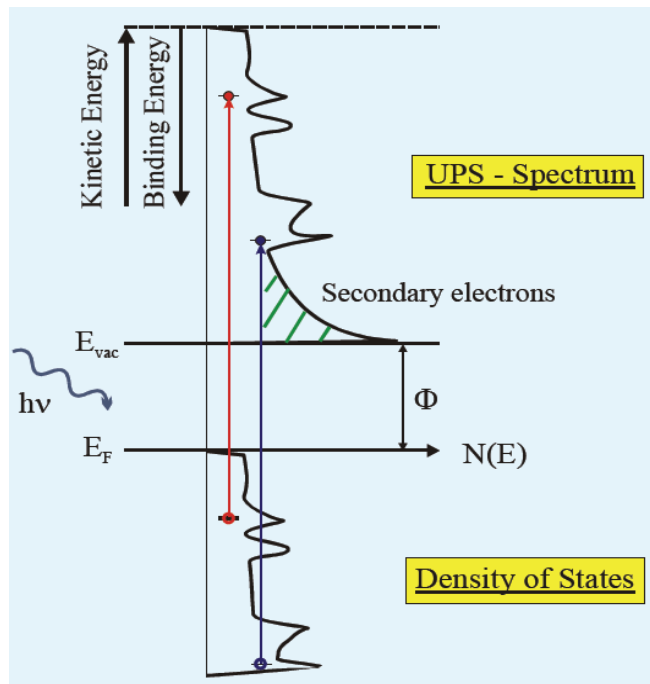
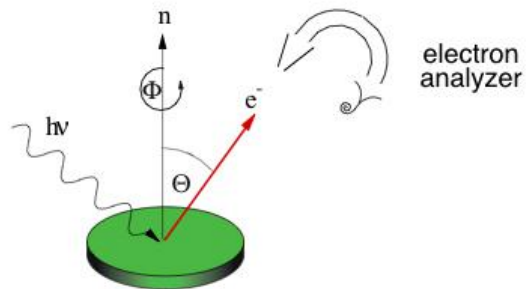
Solve KS-equations for k-points along selected k-path



Choose k-path within Brillouin zone

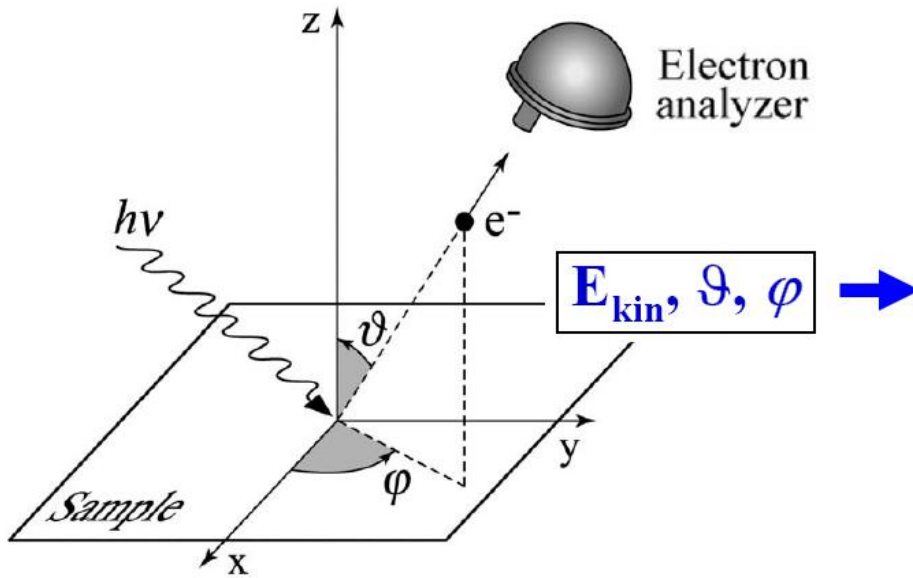


# How to measure band structures?



## 3.1 Band Structures

# Angle Resolved PhotoEmission Spectroscopy



$$\mathbf{K} = \mathbf{p} / \hbar = \sqrt{2mE_{kin}} / \hbar$$

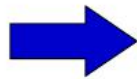
$$K_x = \frac{1}{\hbar} \sqrt{2mE_{kin}} \sin \vartheta \cos \varphi$$

$$K_y = \frac{1}{\hbar} \sqrt{2mE_{kin}} \sin \vartheta \sin \varphi$$

$$K_z = \frac{1}{\hbar} \sqrt{2mE_{kin}} \cos \vartheta$$

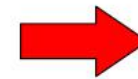
Vacuum

$$\begin{matrix} E_{kin} \\ \mathbf{K} \end{matrix}$$



Conservation laws

$$\begin{matrix} E_f - E_i = h\nu \\ \mathbf{k}_f - \mathbf{k}_i = \cancel{\mathbf{k}_{h\nu}} \end{matrix}$$



Solid

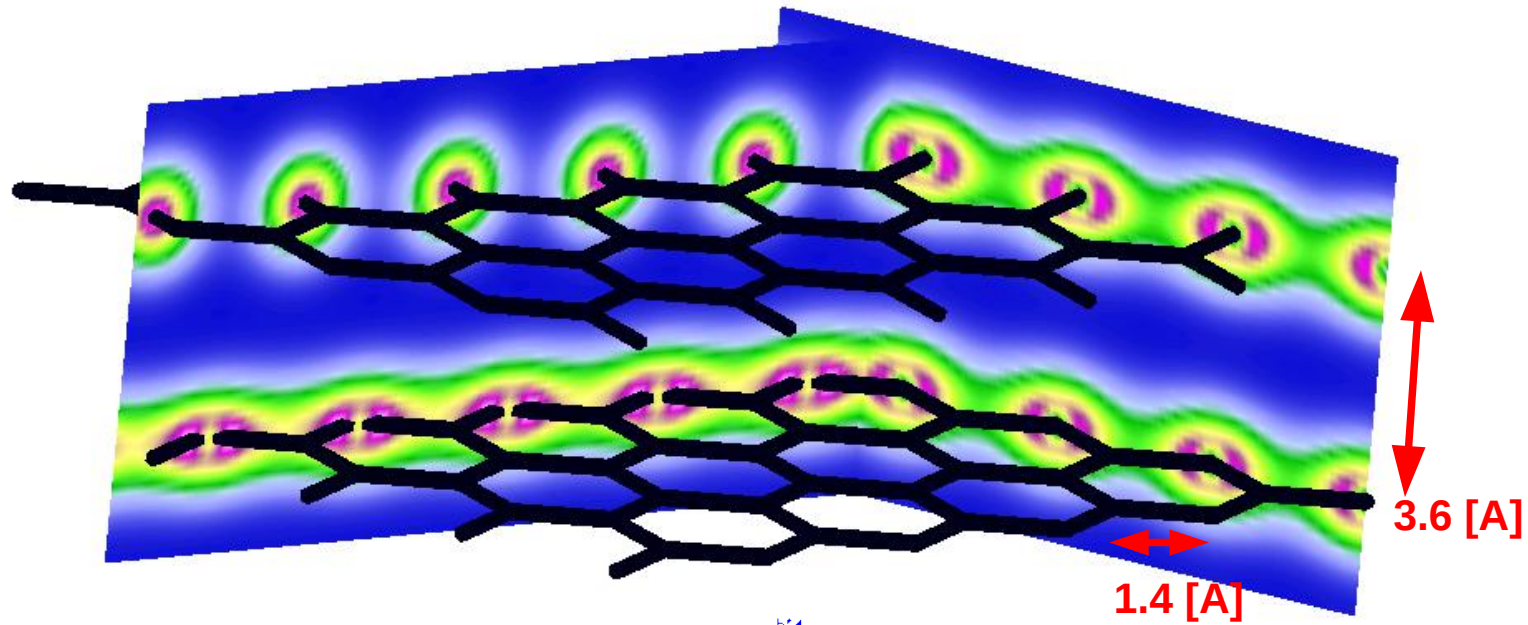
$$\begin{matrix} E_B \\ \mathbf{k} \end{matrix}$$

## 3.1 Band Structures

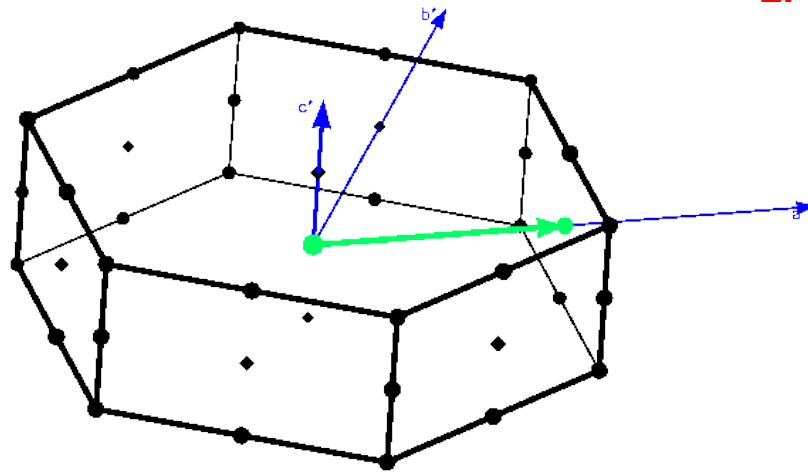


# Band Structure of Graphite

Layered  
crystal  
structure



Brillouin  
Zone





# ABINIT: Input File for Graphite

```
graphite.in (modified) - /home/pep/LEHRE/AbInitio/sample_inout/graphite/
File Edit Search Preferences Shell Macro Windows Help
/home/pep/LEHRE/AbInitio/sample_inout/graphite/graphite.in byte 1021 of 1661 L: 28 C: 0
1 # C in graphite structure
2 # Dataset # 1 = scf calculation
3 # Dataset # 2 = wave functions on a dense mesh: 16x16x8
4
5 #Definition of the unit cell
6 acell 4.639114 4.639114 12.676098
7
8 rprim 1.000000 0.000000 0.000000 # hexagonal lattice (to be scaled by ace
9 -0.500000 0.866025403 0.000000
10 0.000000 0.000000 1.000000
11
12 ixc 14 # GGA, revPBE of Y. Zhang and W. Yang, Phys. Rev. Lett. 80, 890 (199
13 ecut 25.0
14 nstep 25
15 diemac 3.0
16 diemix 0.5
17 #Definition of the structure
18 ntypat 1
19 znucl 6
20 natom 4 # There are two atoms
21 typat 1 1 1 1 # They both are of type 1, that is, Carbon.
22 xred 0.00 0.00 1/4 # Triplet giving the REDUCED coordinate of atom 1.
23 0.00 0.00 3/4 # Triplet giving the REDUCED coordinate of atom 2.
24 1/3 2/3 1/4 # Triplet giving the REDUCED coordinate of atom 3.
25 2/3 1/3 3/4 # Triplet giving the REDUCED coordinate of atom 4.
26
27
28]
```

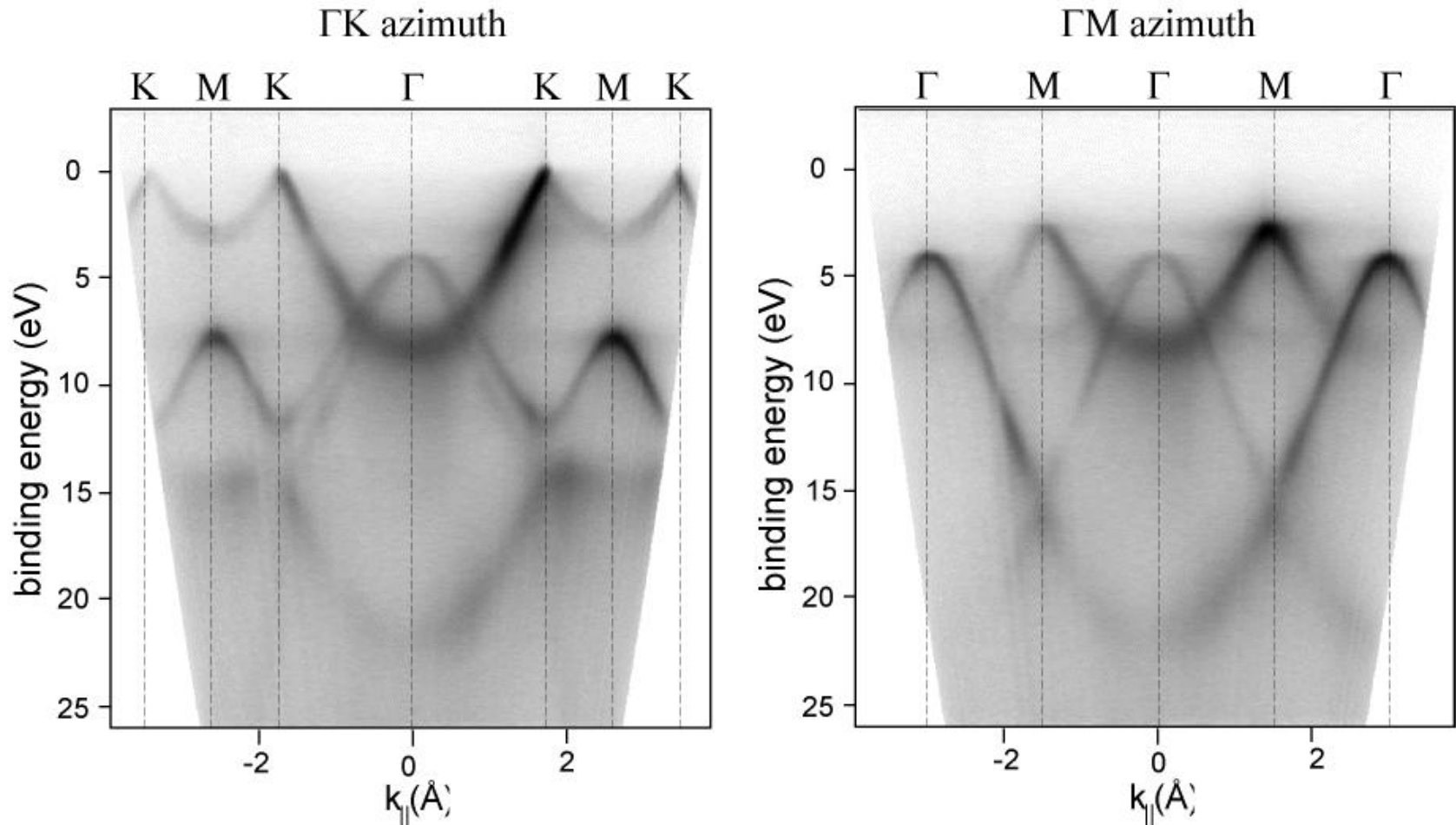
... continued on next slide ...

# ABINIT: Input File for Graphite

... continued from previous slide ...

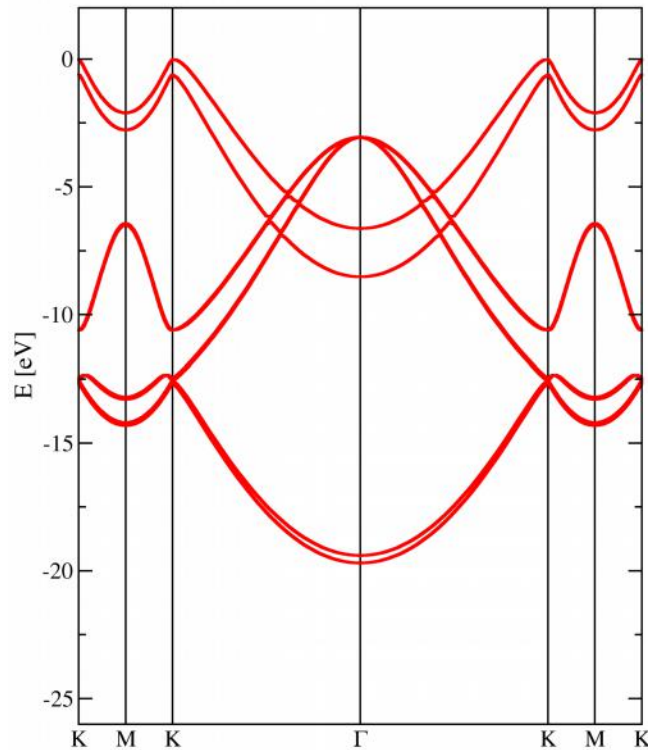
```
28[
29ndtset 2
30
31#Dataset 1 : self-consistent calculation
32kptopt1 1
33ngkpt1 10 10 4
34nshiftk1 1
35shiftk1 1 1 1
36prtden1 1 # Print the density, for use by dataset 2
37toldfel 1.0d-10
38
39#Dataset 2 : the band structure
40iscf2 -2
41getden2 -1
42kptopt2 -4
43nband2 16
44ndivk2 40 40 40 20
45kptbounds2 0.000000000 0.000000000 0.000000000
46 0.500000000 0.000000000 0.000000000
47 0.333333333 0.333333333 0.000000000
48 0.000000000 0.000000000 0.000000000
49 0.000000000 0.000000000 0.500000000
50
51tolwfr2 1.0d-12
52enunit2 1 # Will output the eigenenergies in eV
53prteig2 1
54
55
```

# Experimental Band Structure of Graphite

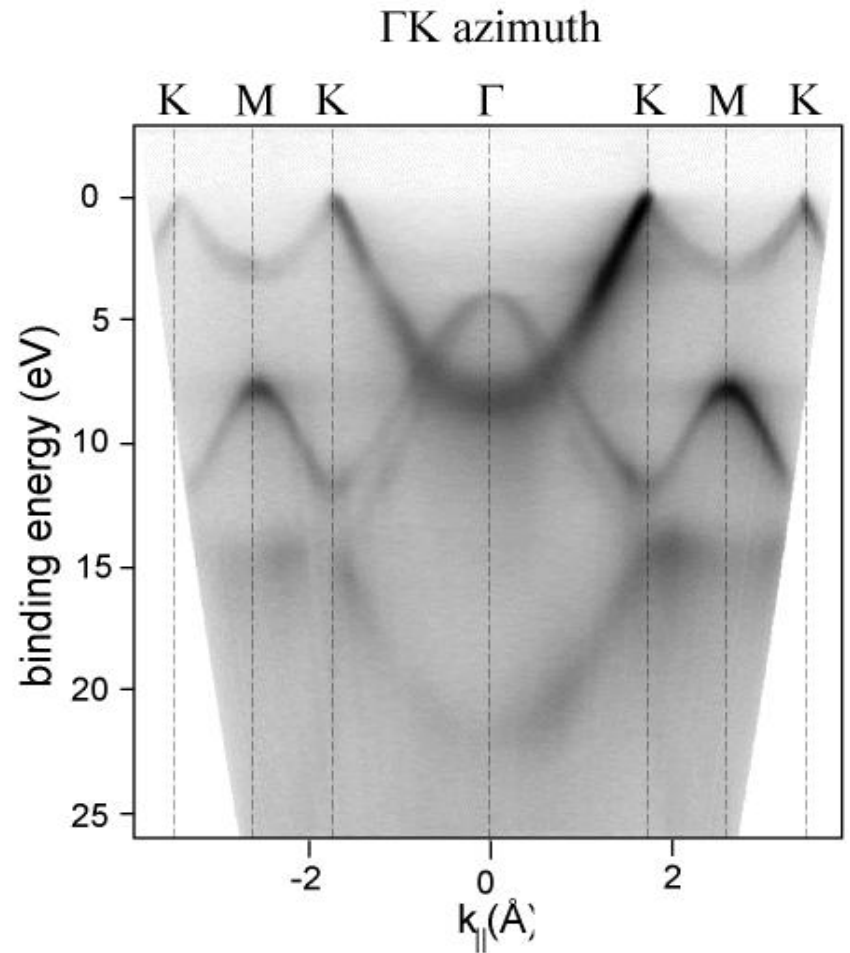


The band structure of graphite along two high symmetry lines Gamma-K and Gamma-M is given below. (measured at the New Toroidal Electron Spectrometer for Angle-Resolved Photoelectron Spectroscopy with Synchrotron Radiation at BESSY II)

# DFT vs. Experiment

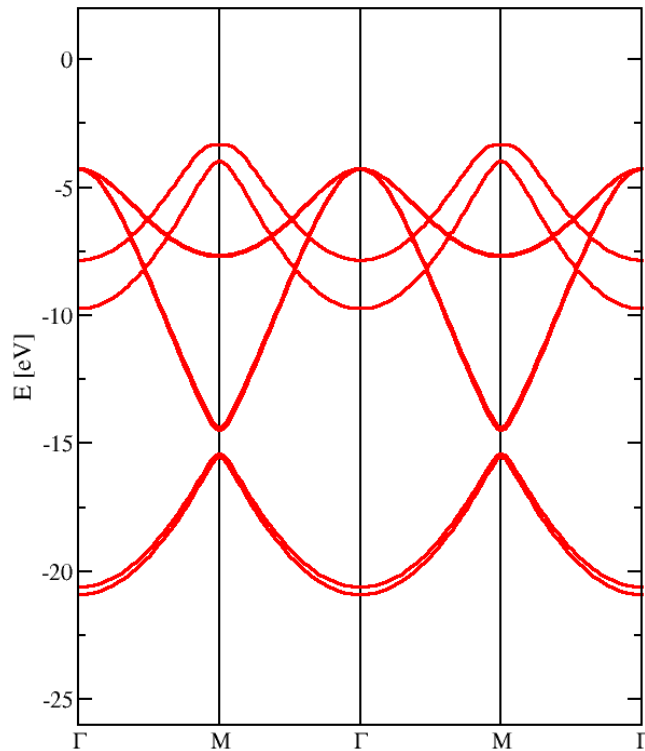


Theory: DFT

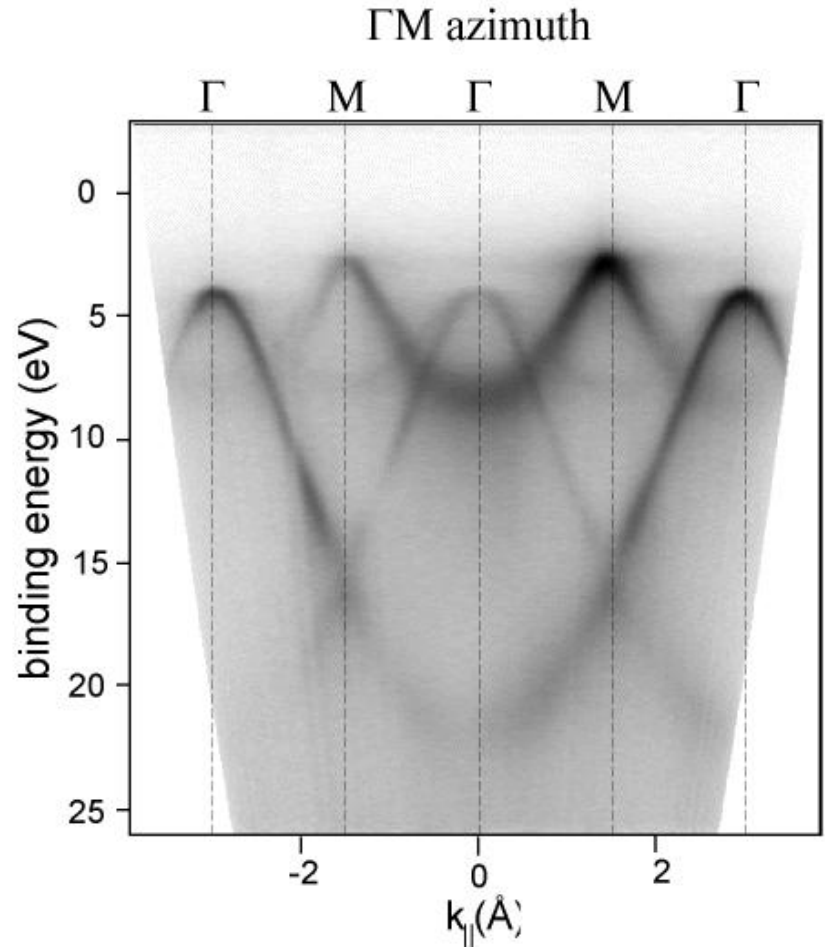


Experiment: ARUPS

# DFT vs. Experiment



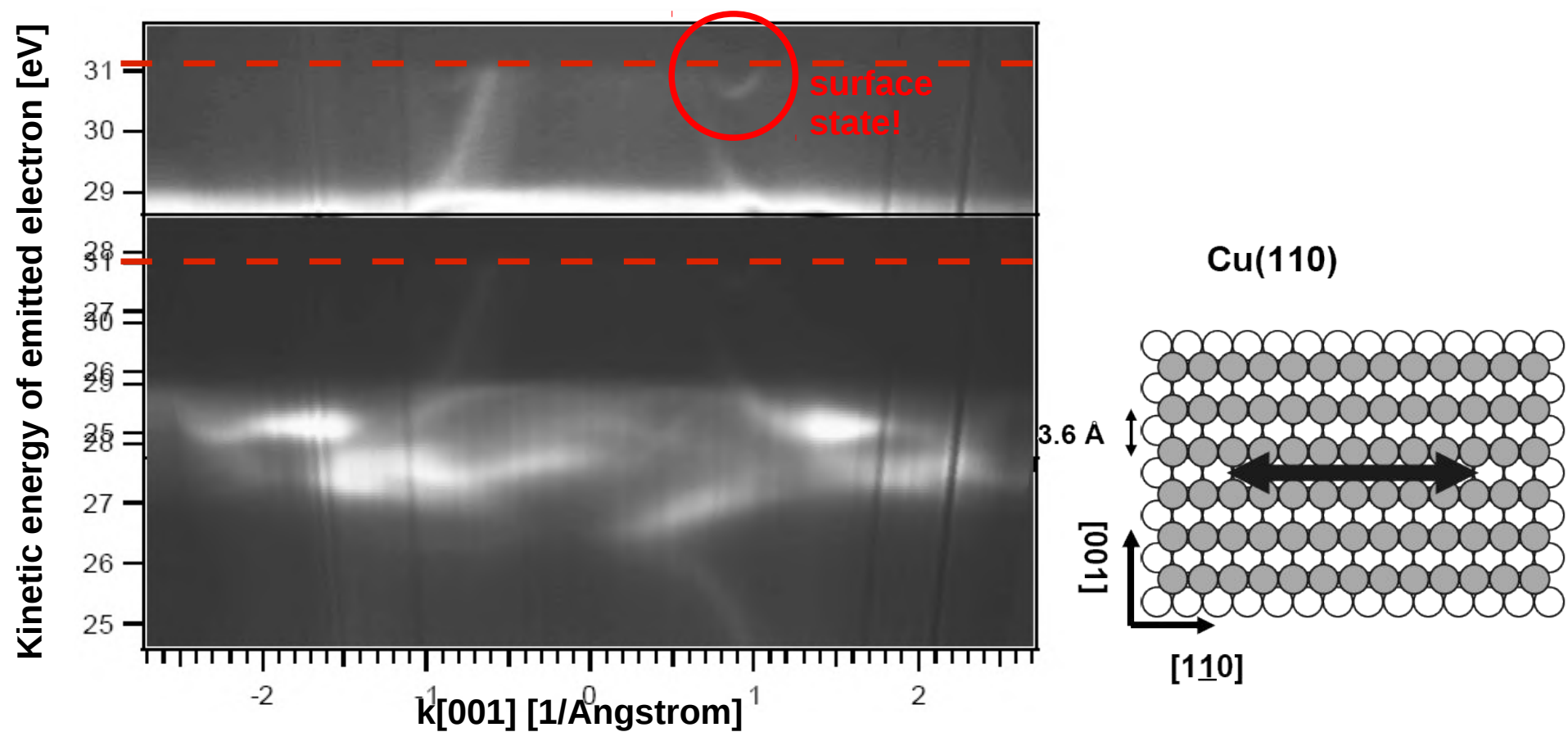
Theory: DFT



Experiment: ARUPS

## 3.1 Band Structures

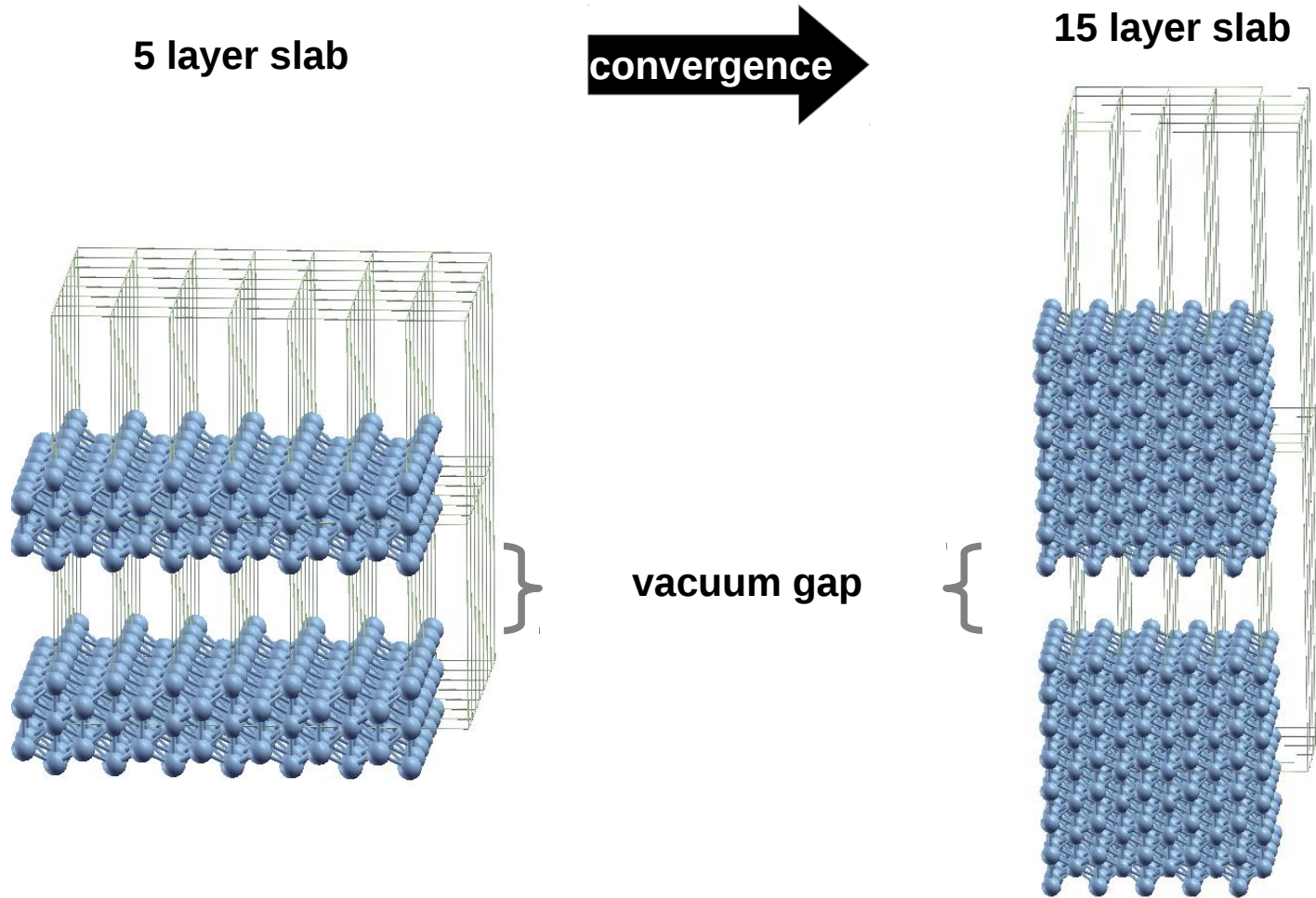
# Band Structure of Cu(110) Surface



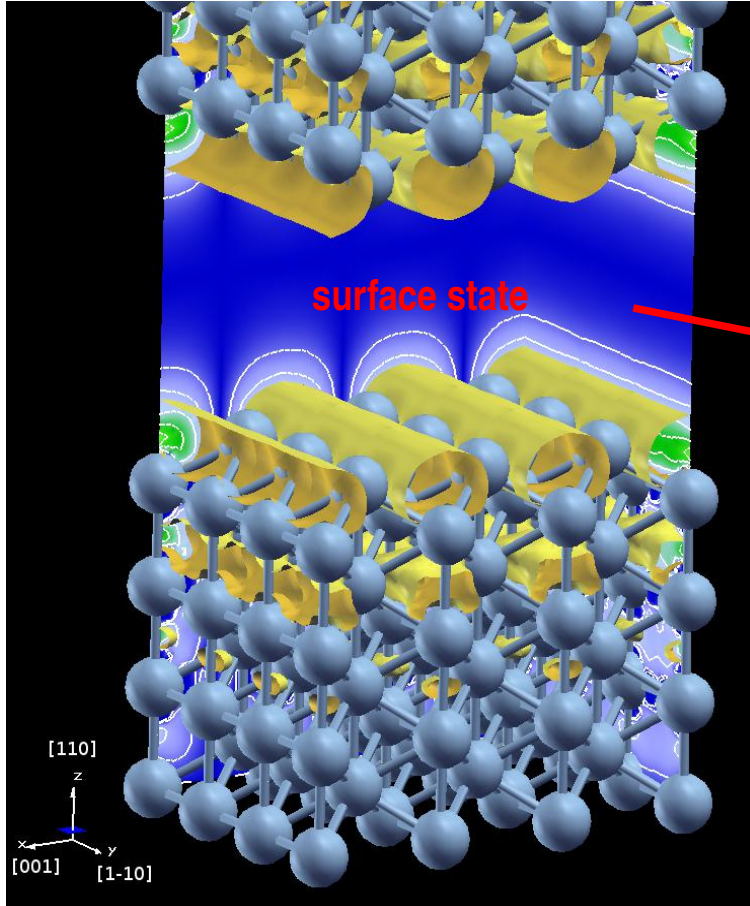
ARUPS band structure for Cu(110) surface measured along the [001] direction (data taken by Stephen Berkebile at BESSY II in Berlin.)



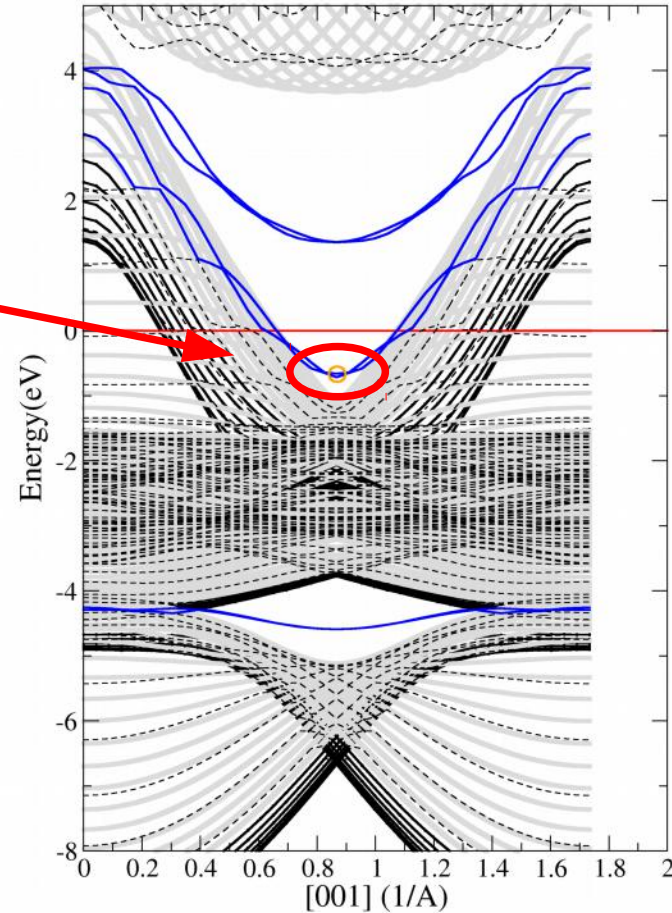
# Surface Calculations – Repeated Slab Approach



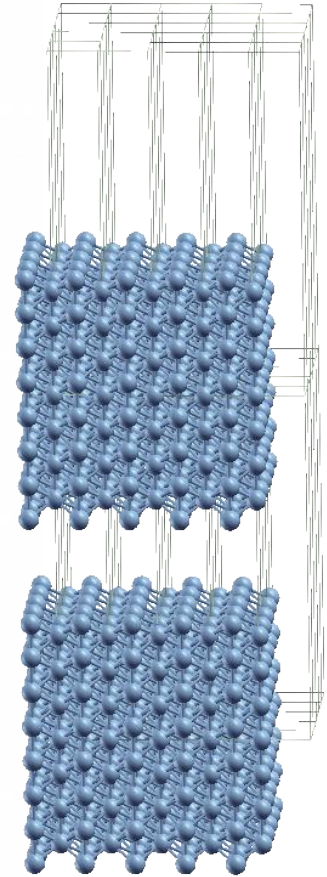
# Surface Calculations – Repeated Slab Approach



Surface states on Cu(110) along [001] direction  
bulk Cu = grey lines, 15-layer-slab = black lines + surface states = blue lines



15 layer slab



# Total Energy from DFT

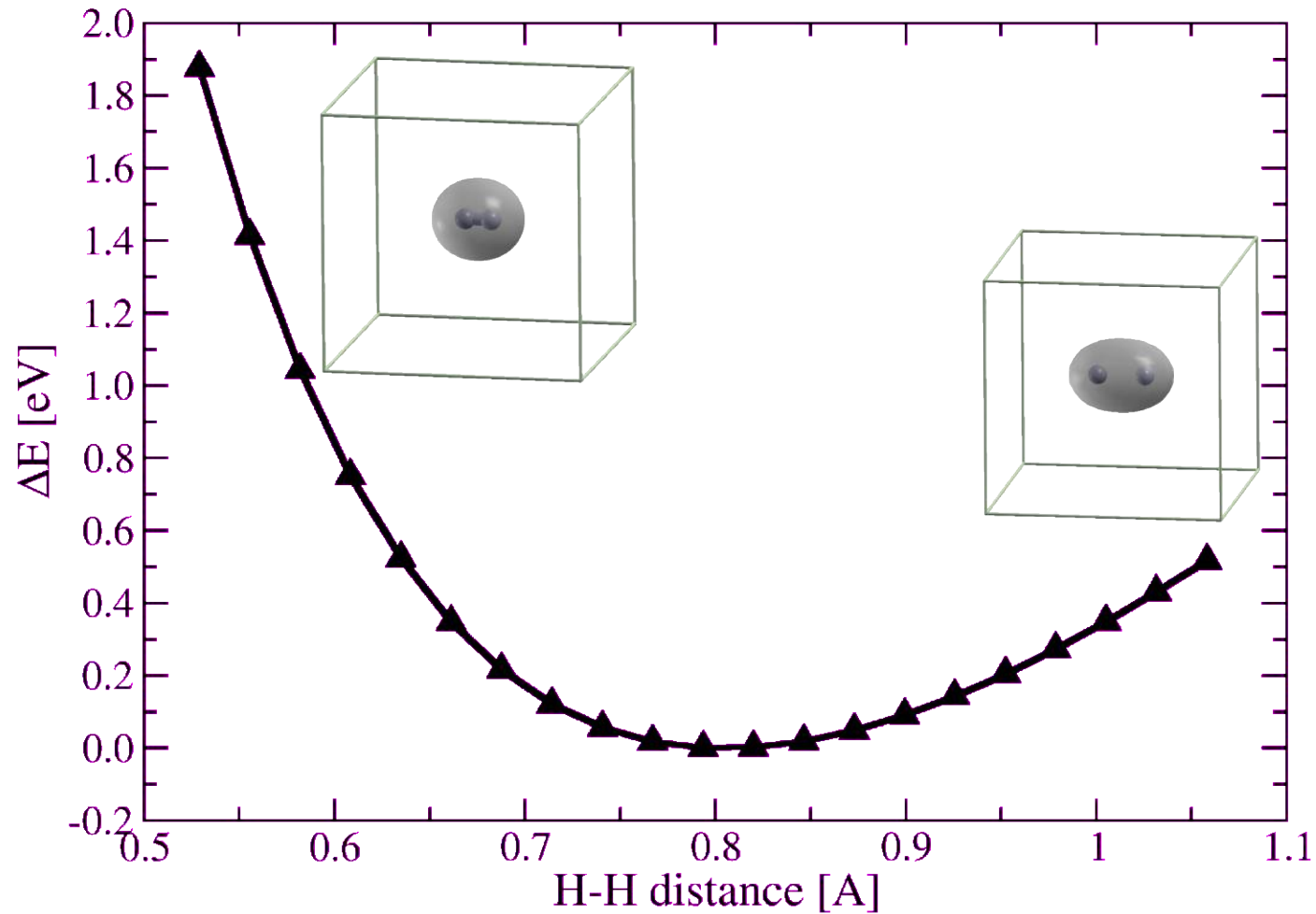
Kohn-Sham wave functions

$$E[\{\phi\}, \{\mathbf{R}\}] = \underbrace{-\frac{1}{2} \sum_{i=1}^n \int \phi_i^*(\mathbf{r}) \nabla^2 \phi_i(\mathbf{r}) d^3r}_{\text{Kinetic}} + \underbrace{\frac{1}{2} \int \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r d^3r'}_{\text{Hartree}} + \underbrace{E_{xc}[n]}_{\text{xc}} + \int V_{\text{ext}}(\mathbf{r}; \mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N) n(\mathbf{r}) d^3r$$

electron-nuclei electrostatic energy

positions of atomic nuclei

# Simple Example: H<sub>2</sub> Molecule

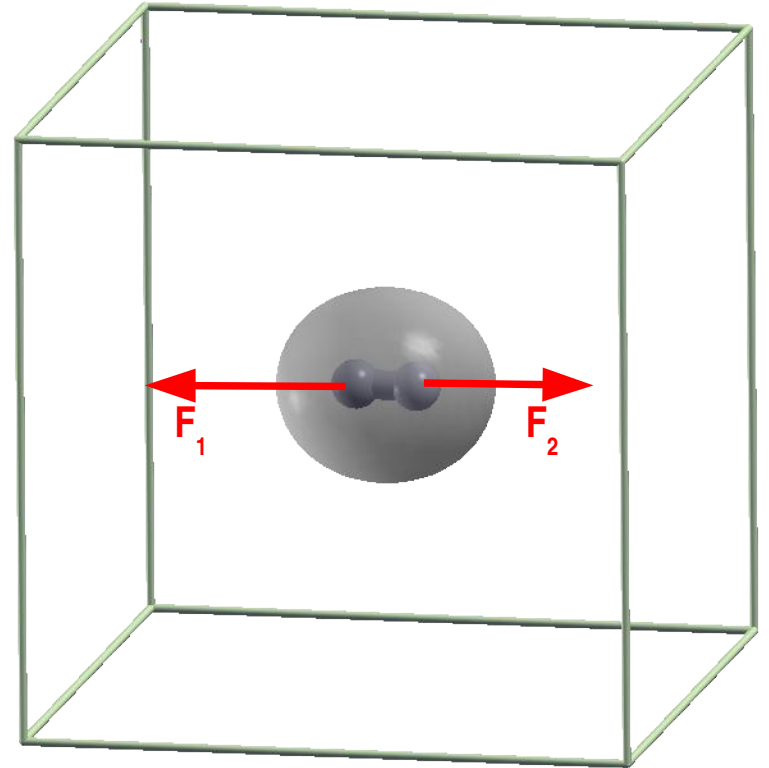


## 3.2 Structural Properties

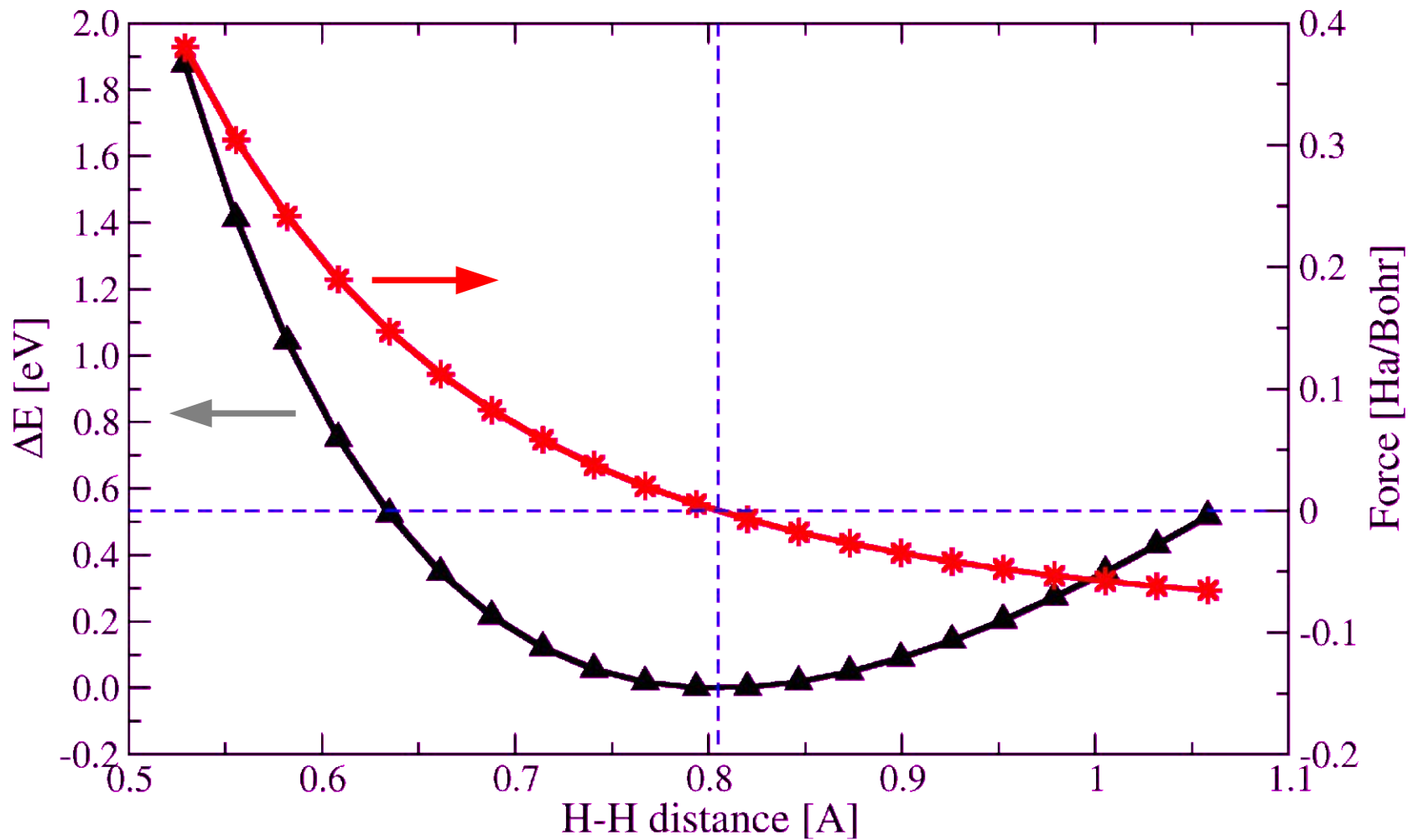
# The Hellmann-Feynman Theorem

$$\mathbf{F}_I = -\frac{\partial}{\partial \mathbf{R}_I} E [\{\phi\}, \{\mathbf{R}\}]$$

*The force  $\mathbf{F}_I$  acting on the atom at position  $\mathbf{R}_I$  can be calculated from the ground state energy and Kohn-Sham orbitals*



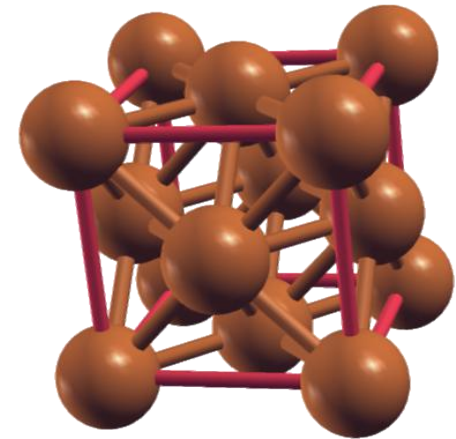
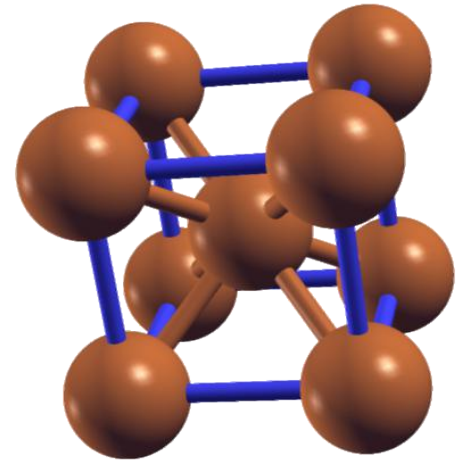
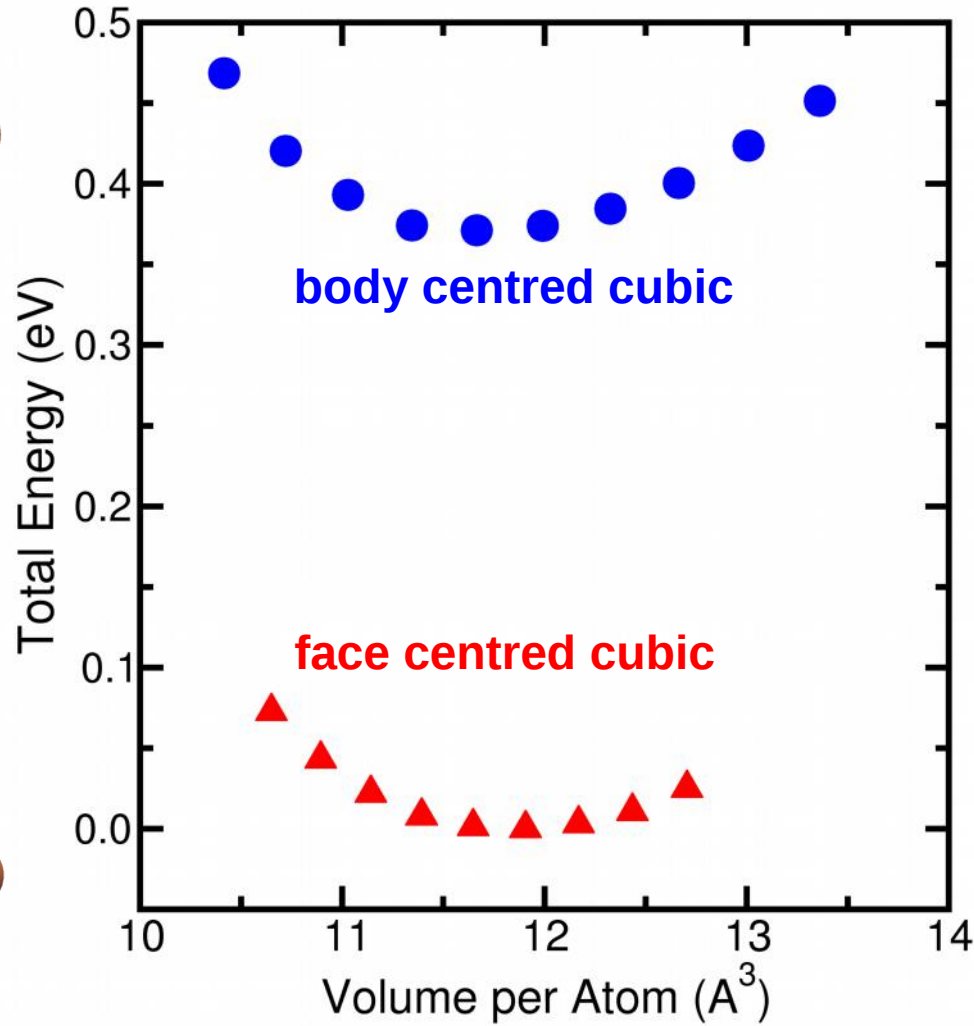
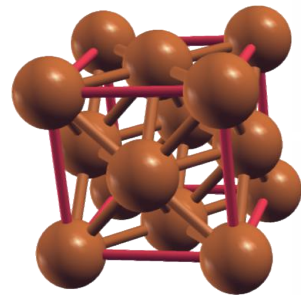
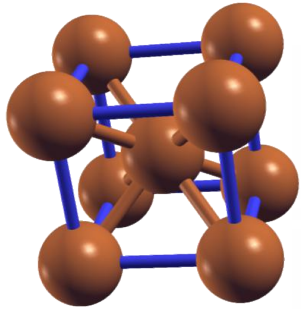
# Simple Example: H<sub>2</sub> Molecule



## 3.2 Structural Properties



# Elastic Properties: Bulk Modulus





# Elastic Properties: Bulk Modulus

**Definition:**

$$B = -V \left( \frac{\partial P}{\partial V} \right)_T$$

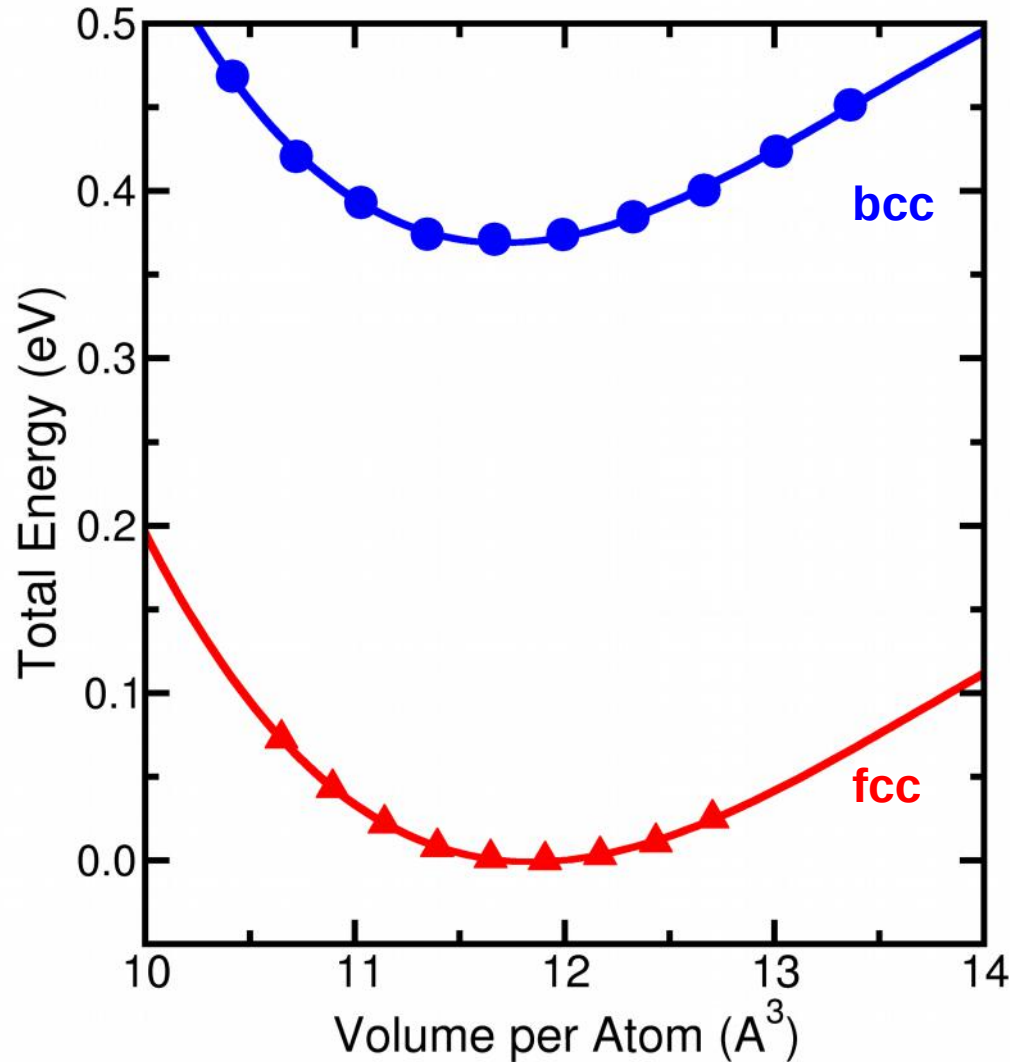
$$P = - \left( \frac{\partial E}{\partial V} \right)_S$$

**Equation of State according to Murnaghan**

$$B' = \left( \frac{\partial B}{\partial P} \right)_T \rightarrow \text{const.} \quad \Rightarrow \quad V(P) = V_0 \left( 1 + B'_0 \frac{P}{B_0} \right)^{-1/B'_0}$$

$$E(V) = E_0 + \frac{B_0 V}{B'_0} \left( \frac{(V_0/V)^{B'_0}}{B'_0 - 1} + 1 \right) - \frac{B_0 V_0}{B'_0 - 1}$$

# Elastic Properties: Bulk Modulus



$$a_{\text{exp}} = 3.60 \text{ \AA}$$

$$a_{\text{DFT}} = 3.63 \text{ \AA}$$

$$P = - \left( \frac{\partial E}{\partial V} \right)_S$$

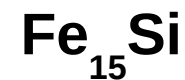
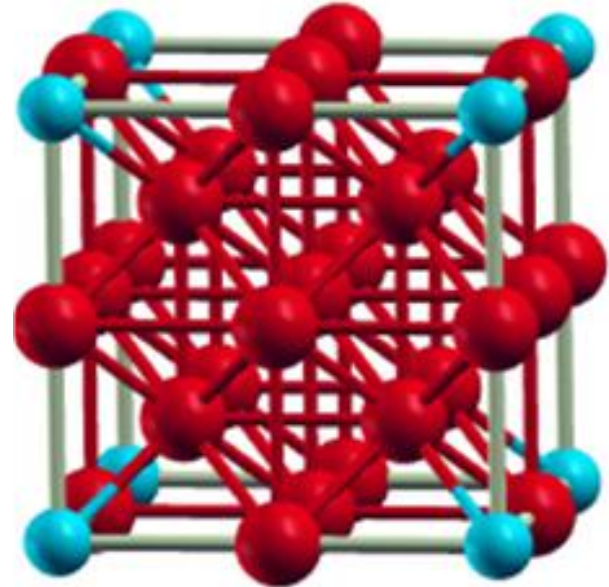
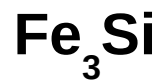
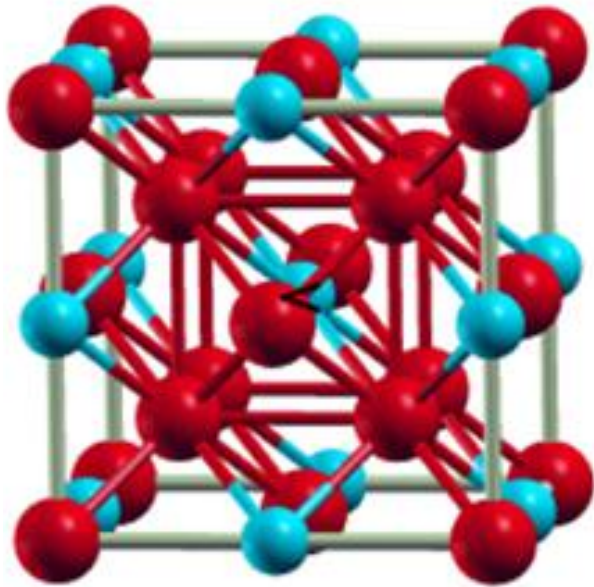
$$B = -V \left( \frac{\partial P}{\partial V} \right)_T$$

$$B_{\text{exp}} = 142 \text{ GPa}$$

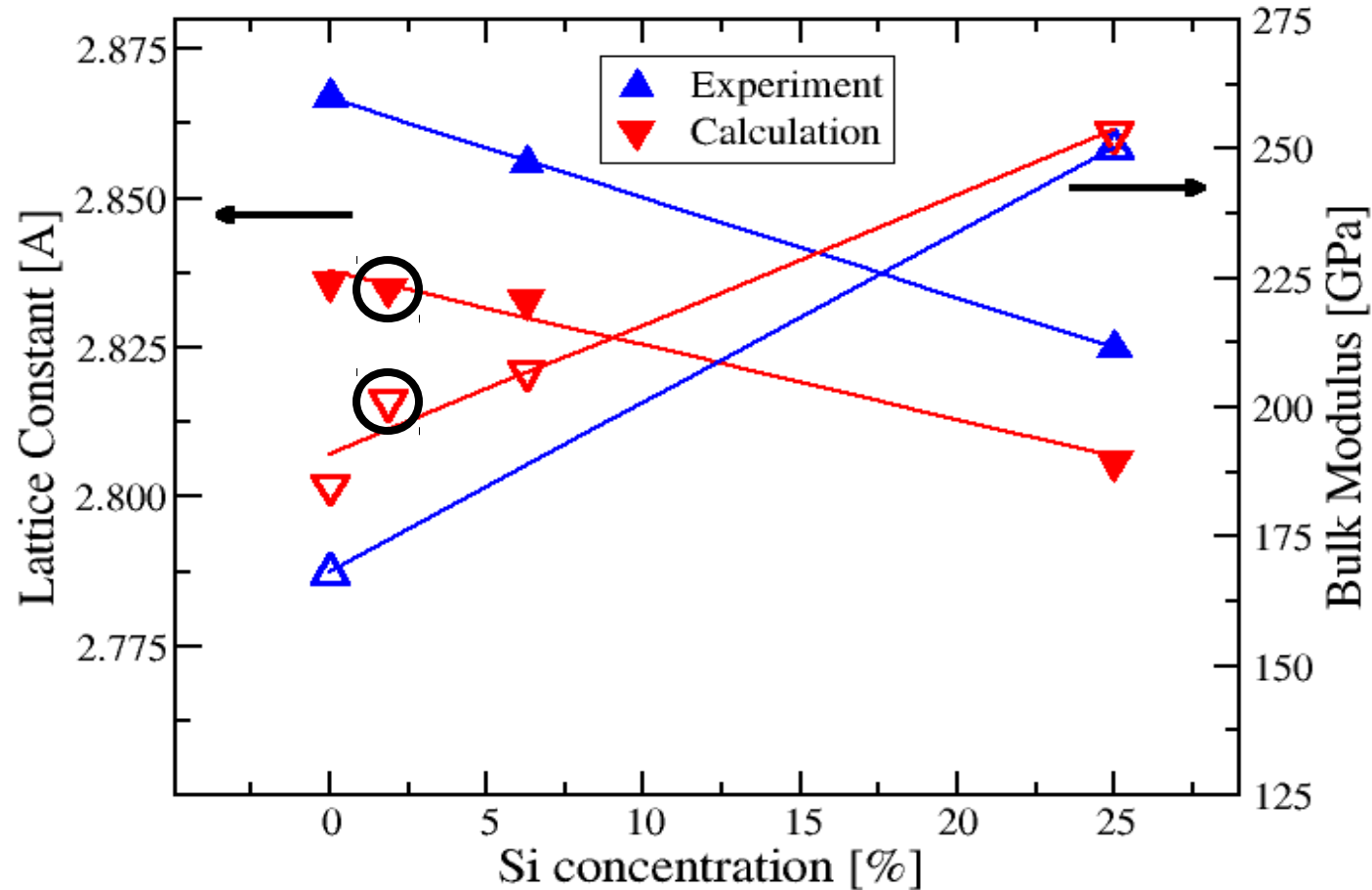
$$B_{\text{DFT}} = 139 \text{ GPa}$$

# Example: Fe-Si Alloy

*Supercell Calculations:*

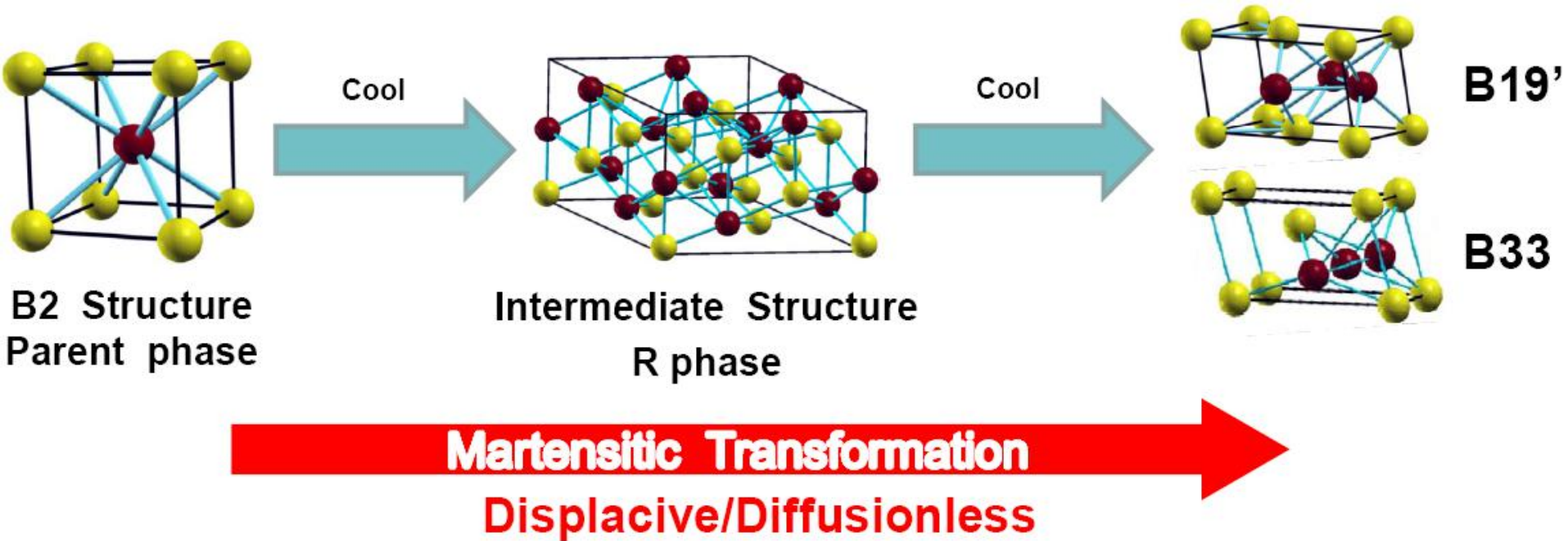


# Example: Fe-Si Alloy



## 3.2 Structural Properties

# Example: NiTi Shape Memory Alloys



## 3.2 Structural Properties

# Elastic Constants

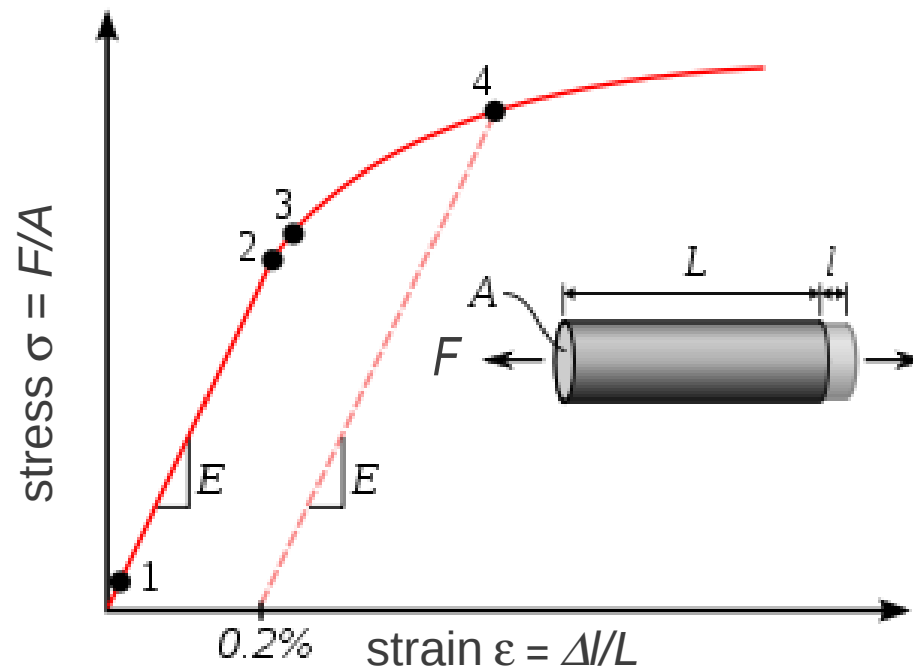
Hooke's Law

„ut tensio sic vis“ (1678)

stress ↓

$$\sigma_i = C_{ij} \varepsilon_j$$

strain ↓



# Elastic Constants

Hooke's Law

„ut tensio sic vis“ (1678)

$$\begin{array}{ccc} \text{stress} & & \text{strain} \\ \downarrow & & \downarrow \\ \sigma_i & = & C_{ij} \varepsilon_j \end{array}$$

**Cubic  
Crystal**

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{11} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix}$$

3 different components



# Elastic Constants

Hooke's Law

„ut tensio sic vis“ (1678)

$$\begin{array}{ccc} \text{stress} & & \text{strain} \\ \downarrow & & \downarrow \\ \sigma_i & = & C_{ij} \varepsilon_j \end{array}$$

Monoclinic  
Crystal

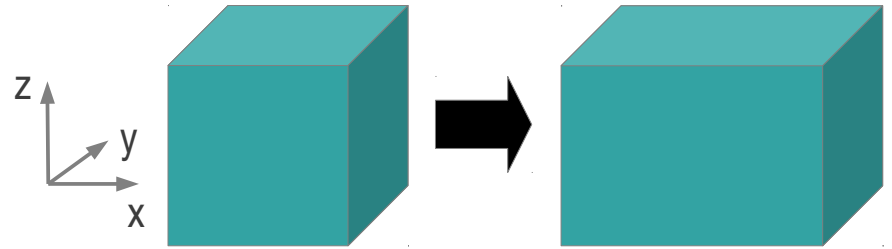
$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & C_{15} & 0 \\ C_{12} & C_{22} & C_{23} & 0 & C_{25} & 0 \\ C_{13} & C_{23} & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & C_{46} \\ C_{15} & C_{25} & C_{35} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & C_{46} & 0 & C_{66} \end{pmatrix}$$

13 different components

# How to Calculate Elastic Constants?

Special  
Strains

$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Total Energy

$$E = E_0 + V_0 \left( \sigma_1 \varepsilon + \frac{1}{2} C_{11} \varepsilon^2 \right)$$

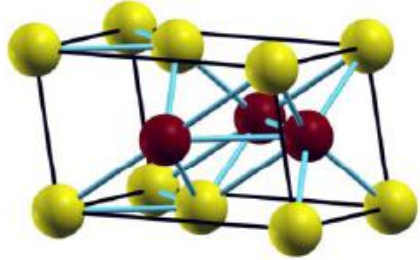
Monoclinic  
Crystal

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & C_{15} & 0 \\ C_{12} & C_{22} & C_{23} & 0 & C_{25} & 0 \\ C_{13} & C_{23} & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & C_{46} \\ C_{15} & C_{25} & C_{35} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & C_{46} & 0 & C_{66} \end{pmatrix}$$

*13 different components*

# Results for NiTi B19' Phase

**B19':**



	a (a.u)	b (a.u)	c (a.u)	$\beta$
Experimental (3)	8.780	7.763	5.476	97.8
WIEN2k	8.910	7.668	5.557	100.6

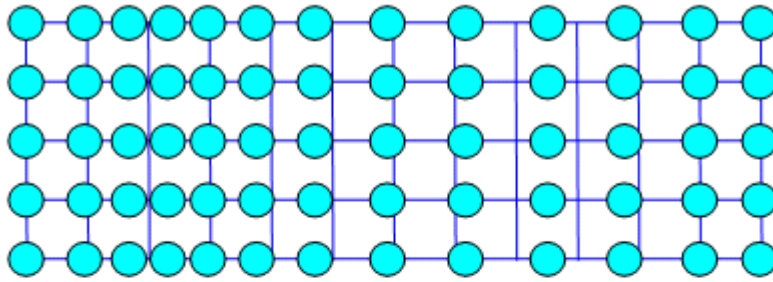
[3] Acta MetallMater, 33, 2094, 1985

Elastic Constant (GPa)	C11	C12	C13	C15	C22	C23	C25	C33	C35	C44	C46	C55	C66
Elastic - WIEN2k	186	131	114	-3	239	121	-6	226	28	86	-3	23	80

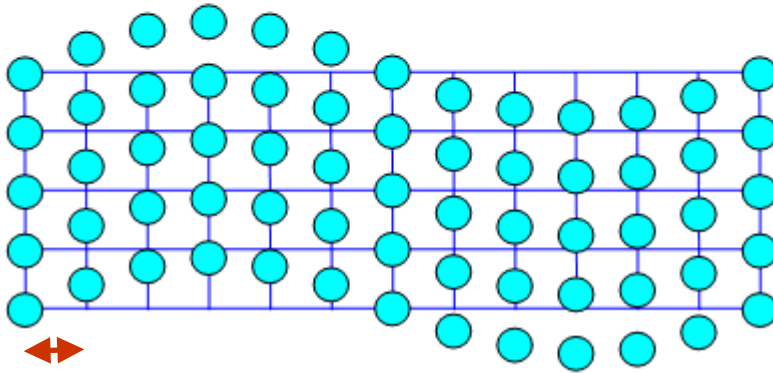
Golesorkhtabar, Spitaler, Puschnig, Ambrosch-Draxl, *to be published*

## 3.2 Structural Properties

# Frozen Phonon Approach



longitudinale Welle



transversale Welle

←—————→  
Schwingungsperiode

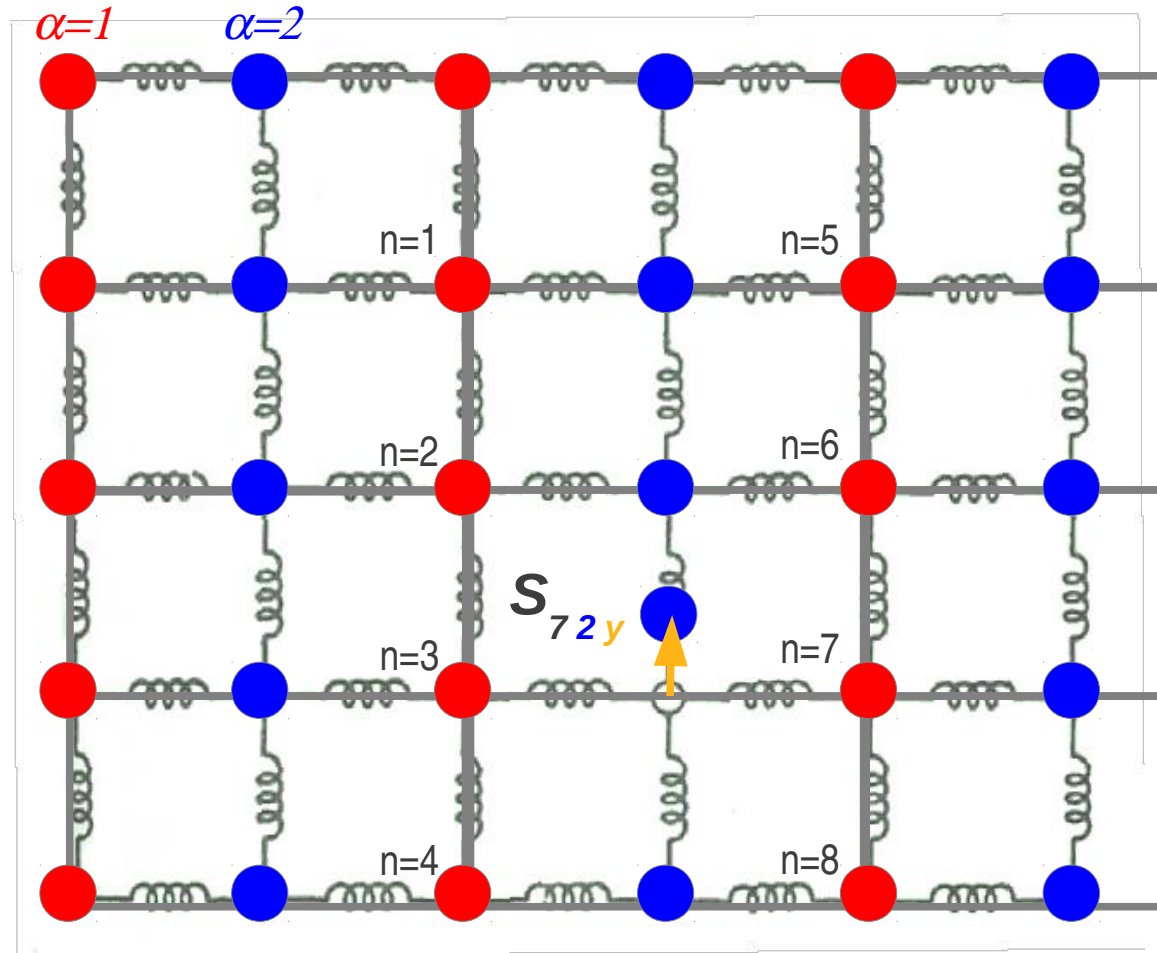
[www.tf.uni-kiel.de](http://www.tf.uni-kiel.de)

# Super Cell Approach

- Atom type A
- Atom type B

$$s_{n\alpha i}$$

displacement  $s$  of  
atom type  $a$   
in unit cell  $n$   
in direction  $i$



## 3.3 Vibrational Properties

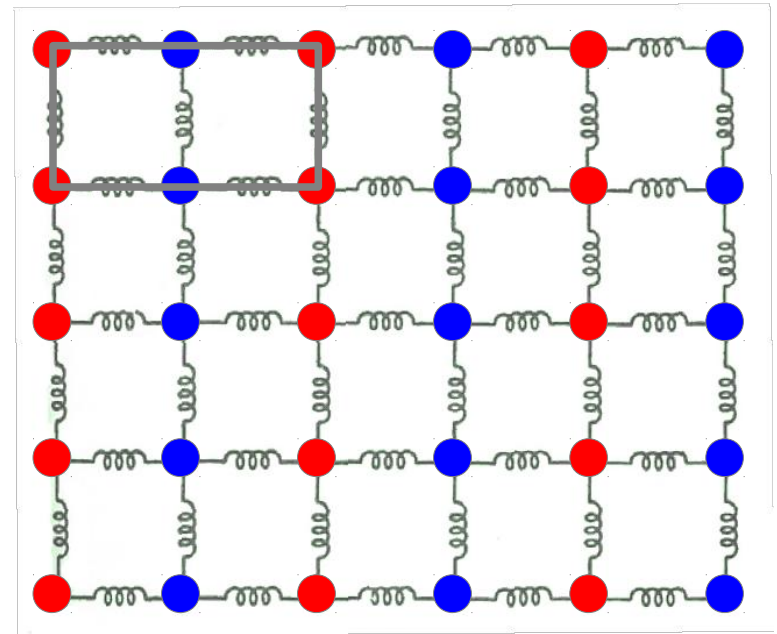
# Equation of Motion

Kinetic Energy  $T$

$$T = \sum_{n=1}^N \sum_{\alpha=1}^r \sum_{i=1}^3 \frac{M_{\alpha}}{2} \left( \frac{ds_{n\alpha i}(t)}{dt} \right)^2$$

Potential Energy  $W$

$$W = W(\mathbf{R}_{n\alpha}) + \underbrace{\sum_{n\alpha i} \left[ \frac{\partial W(\mathbf{x})}{\partial s_{n\alpha i}} \right]_{\mathbf{x}=\mathbf{R}_{n\alpha}}}_{=0} s_{n\alpha i} + \underbrace{\sum_{n\alpha i} \sum_{n'\alpha'i'} \left[ \frac{\partial^2 W(\mathbf{x})}{\partial s_{n\alpha i} \partial s_{n'\alpha'i'}} \right]_{\mathbf{x}=\mathbf{R}_{n\alpha}}}_{=\Phi_{n\alpha i}^{n'\alpha'i'}} s_{n\alpha i} s_{n'\alpha'i'} + O(s^3)$$



## 3.3 Vibrational Properties

# Dynamical Matrix

Equation of motion

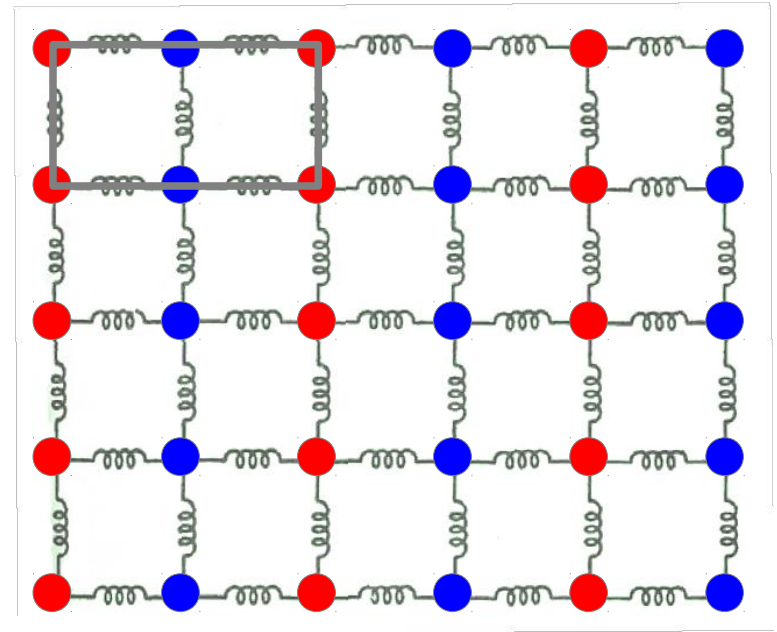
$$M_\alpha \frac{d^2 s_{n\alpha i}}{dt^2} = - \sum_{n'\alpha'i'} \Phi_{n\alpha i}^{n'\alpha'i'} s_{n'\alpha'i'}$$

Ansatz for solution

$$s_{n\alpha i}(t) = \frac{1}{\sqrt{M_\alpha}} u_{n\alpha i} e^{-i\omega t} \quad u_{n\alpha i} = c_{\alpha i} e^{i\vec{q}\vec{R}_n}$$

Eigenvalue equation for phonon frequencies  $\omega(\vec{q})$

$$\omega^2 c_{\alpha i} = \sum_{\alpha'i'} \underbrace{\left[ \sum_{n'} \frac{\Phi_{\alpha i}^{\alpha'i'}(n')}{\sqrt{M_\alpha M_{\alpha'}}} e^{i\vec{q}\vec{R}_{n'}} \right]}_{D_{\alpha i}^{\alpha'i'}(\vec{q})} c_{\alpha'i'}$$





# Dynamical Matrix

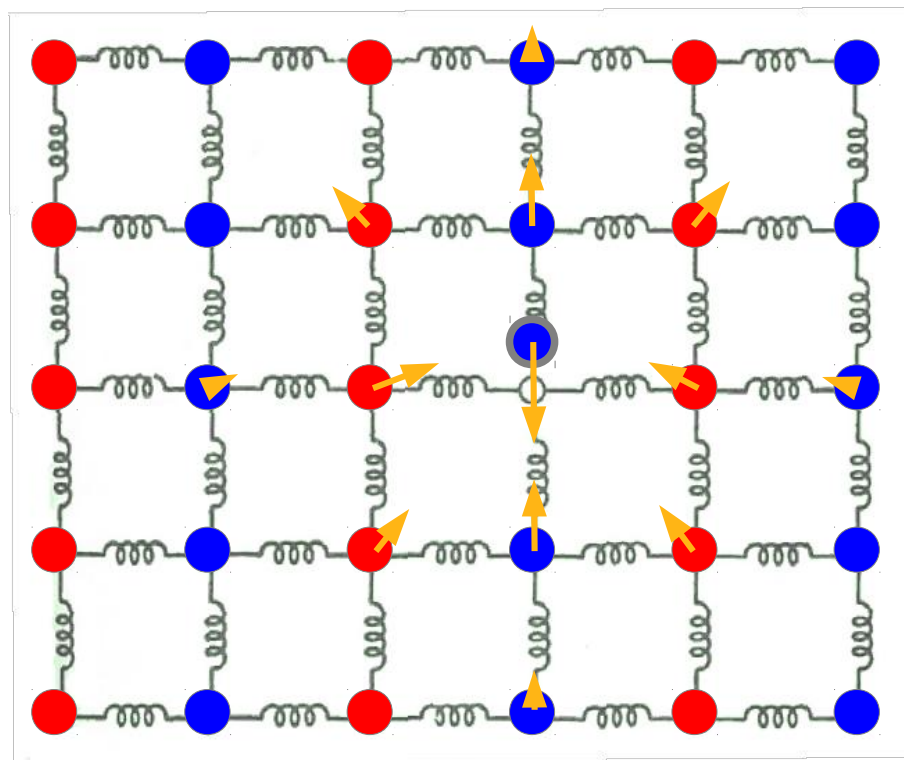
## Hellmann-Feynman Theorem

$$\mathbf{F}_I = -\frac{\partial}{\partial \mathbf{R}_I} E [\{\phi\}, \{\mathbf{R}\}]$$

The force  $\mathbf{F}_I$  acting on the atom at position  $\mathbf{R}_I$  can be calculated from the ground state energy and Kohn-Sham orbitals

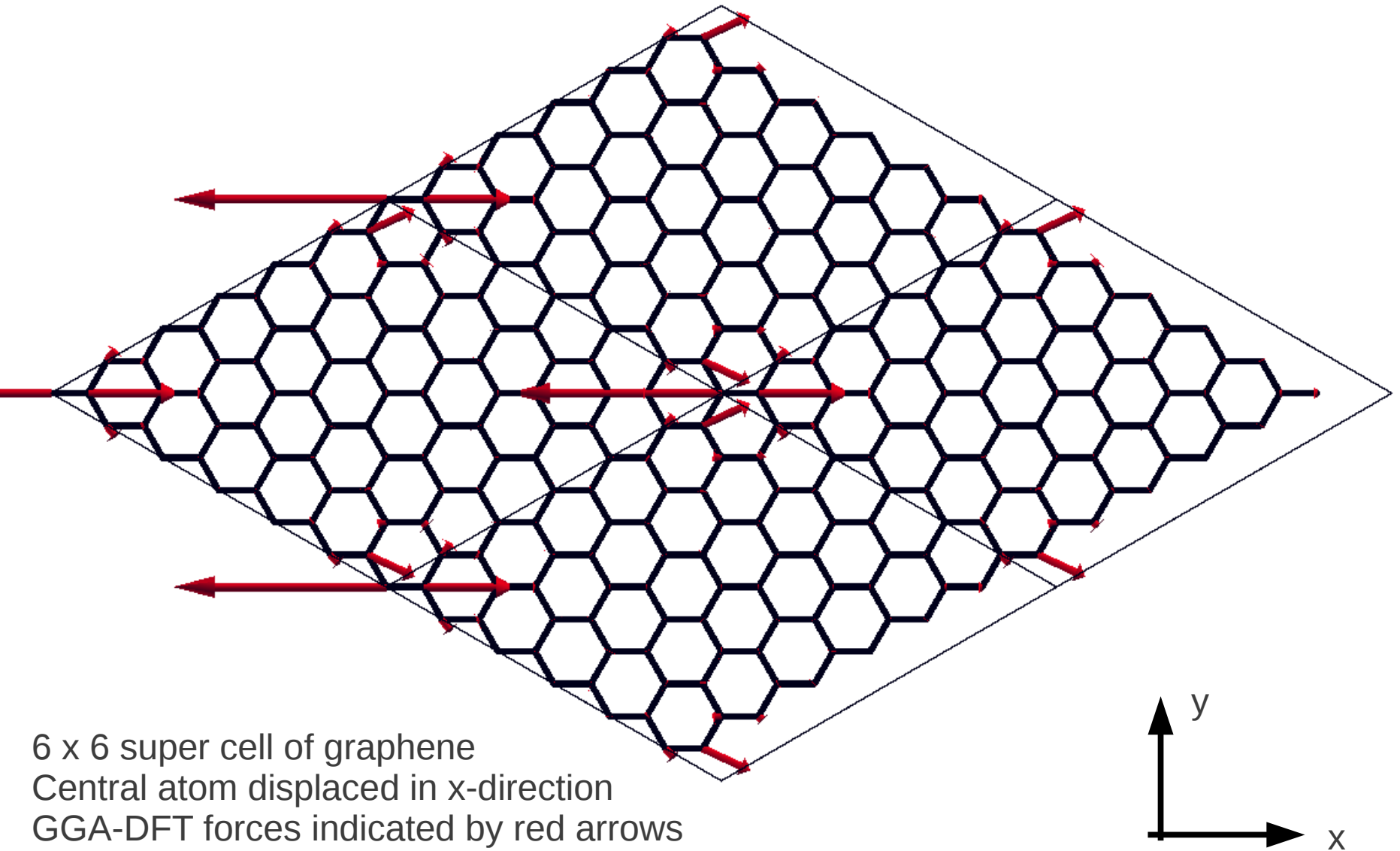
## Force constant matrix

$$\Phi_{I\mu, \nu} = \frac{\partial^2 F^{\text{el}}}{\partial u_{I\mu} \partial u_\nu} = \frac{\partial \tilde{F}_{I\mu}^{\text{HF}}}{\partial u_\nu}$$



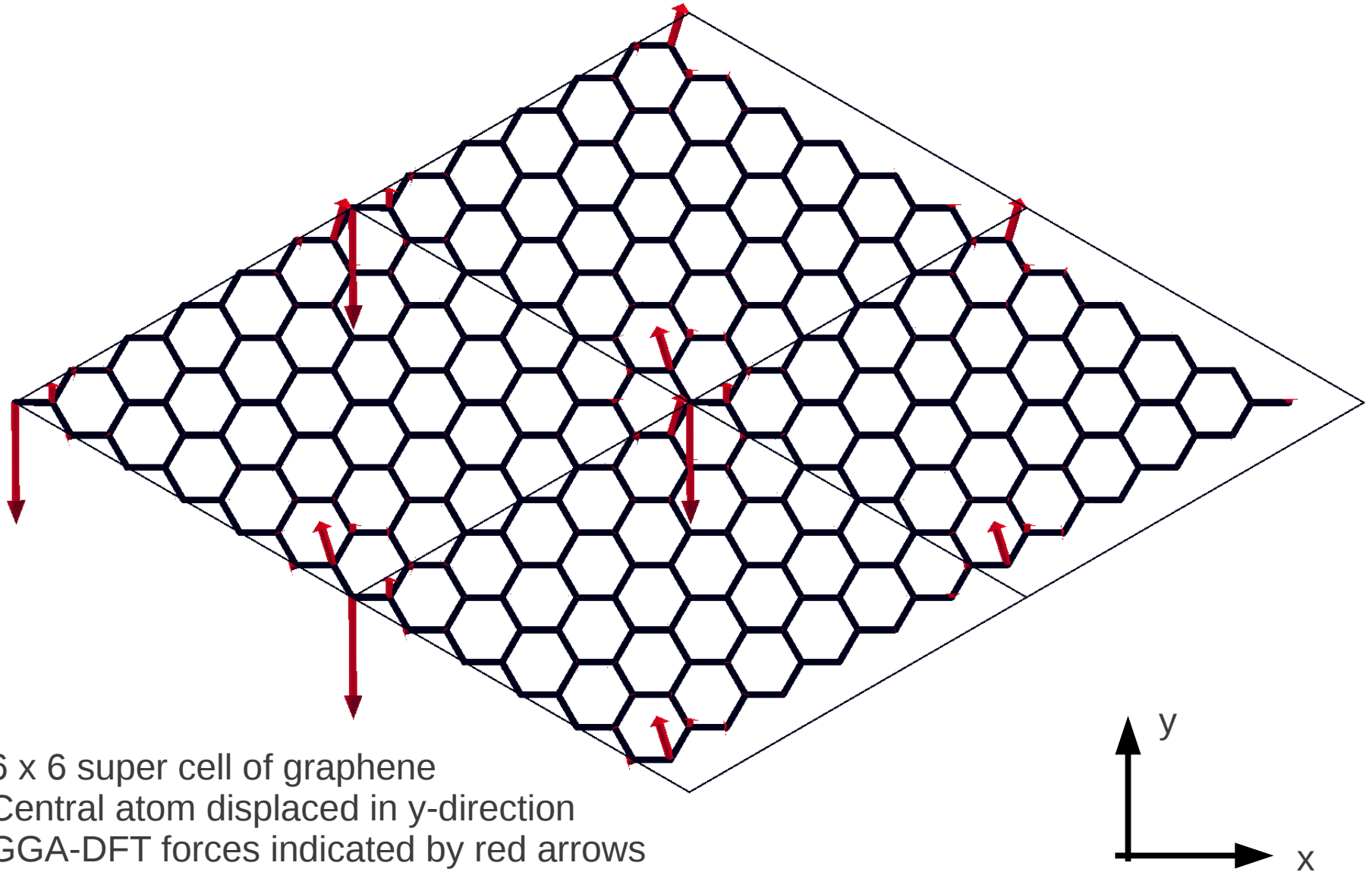
## 3.3 Vibrational Properties

# Force Constants in Graphene



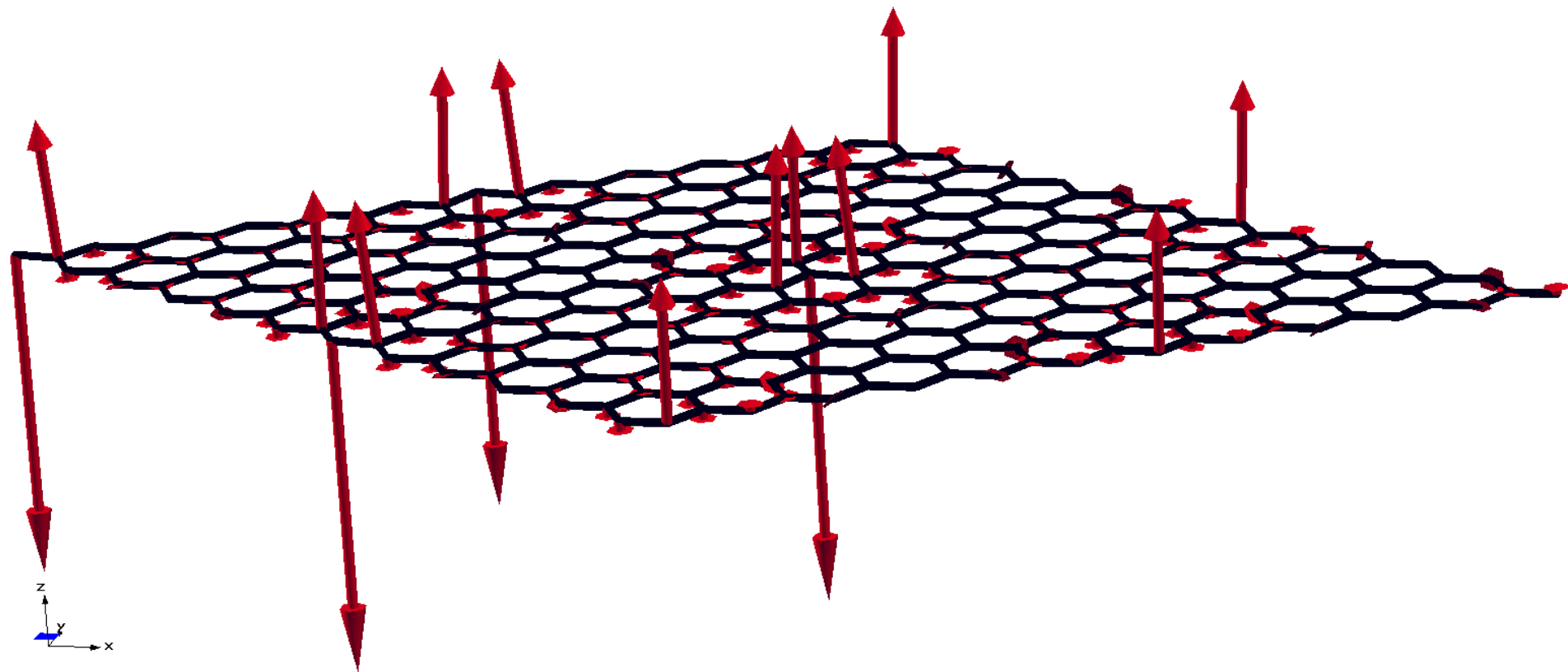
## 3.3 Vibrational Properties

# Force Constants in Graphene



## 3.3 Vibrational Properties

# Force Constants in Graphene



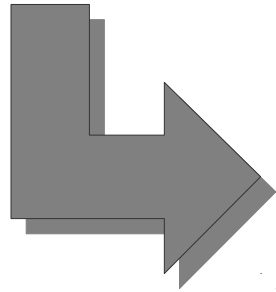
6 x 6 super cell of graphene  
Central atom displaced in z-direction  
GGA-DFT forces indicated by red arrows

## 3.3 Vibrational Properties

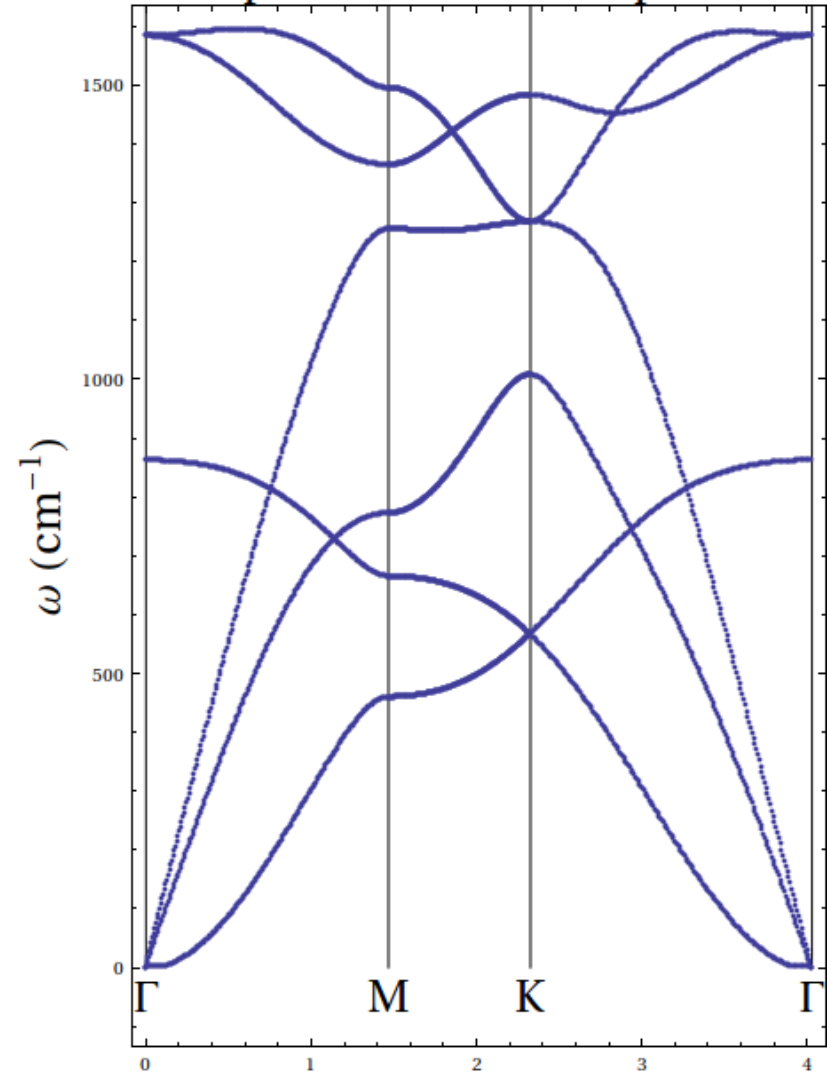
# Phonon Dispersion in Graphene

The force constants are used to set up the dynamical matrix  $D(\mathbf{q})$  whose eigenvalues are the squares of the phonon frequencies  $\omega(\mathbf{q})$

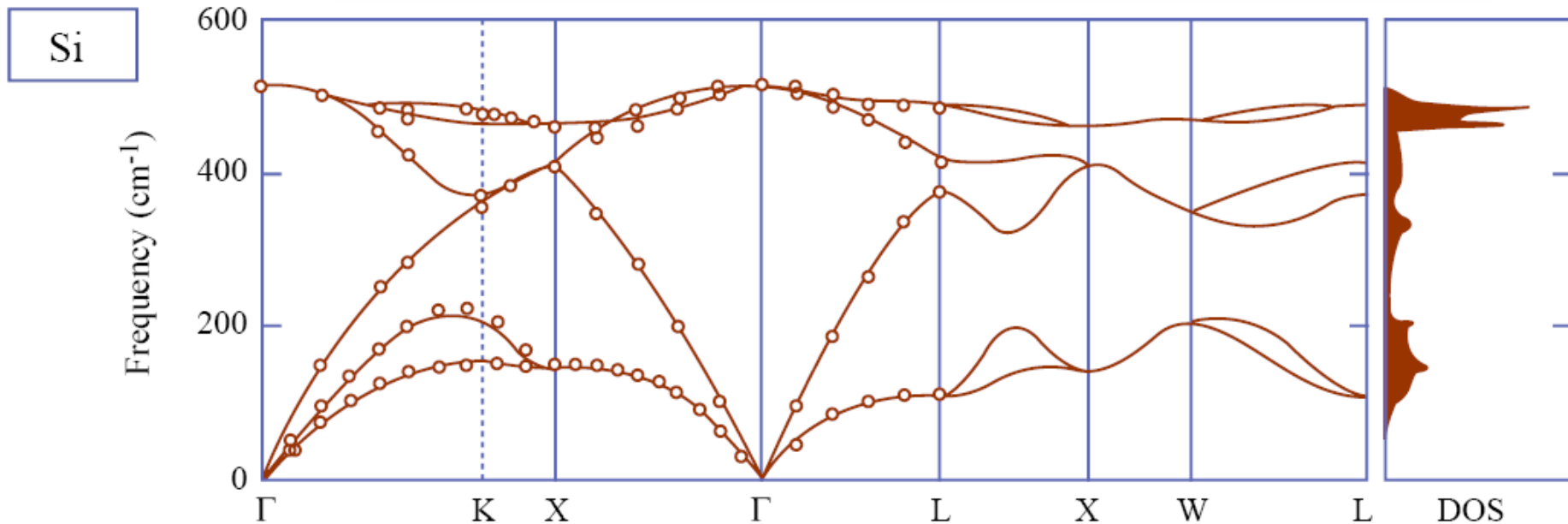
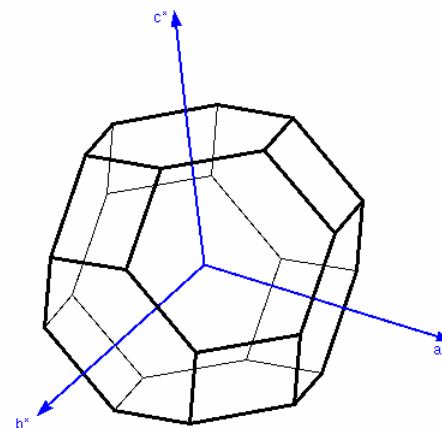
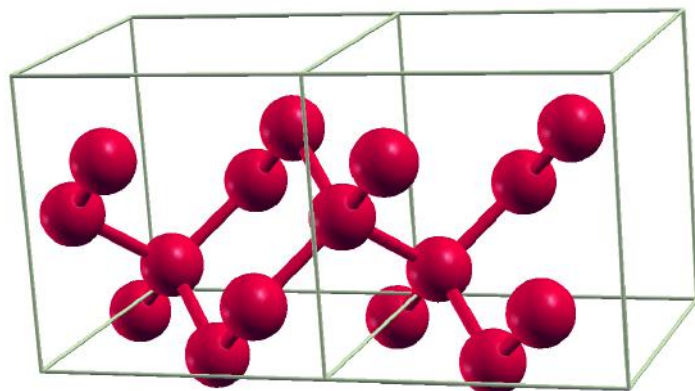
$$\omega^2 c_{\alpha i} = \sum_{\alpha' i'} \underbrace{\left[ \sum_{n'} \frac{\Phi_{\alpha i}^{\alpha' i'}(n')}{\sqrt{M_\alpha M_{\alpha'}}} e^{i\vec{q}\vec{R}_{n'}} \right]}_{D_{\alpha i}^{\alpha' i'}(\vec{q})} c_{\alpha' i'}$$



Graphene Phonon Dispersion

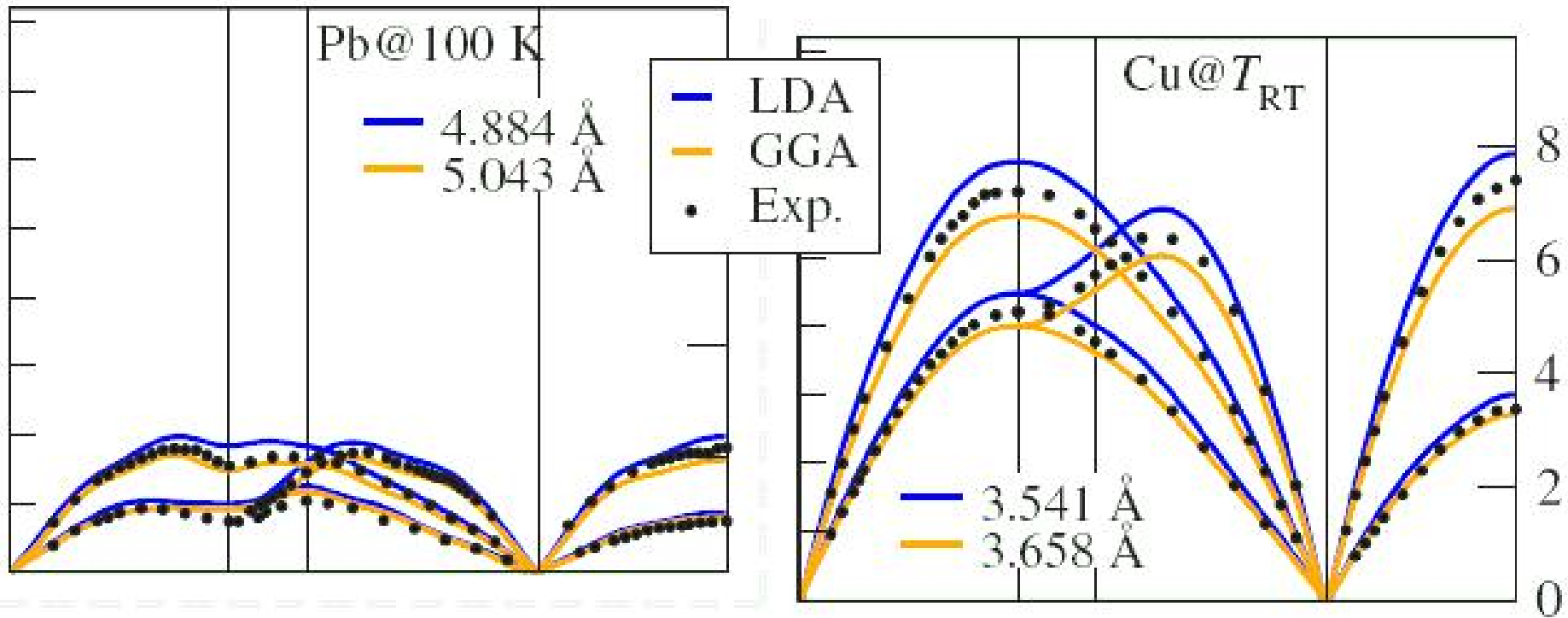


# Phonon Dispersion in Silicon



## 3.3 Vibrational Properties

# Phonon Band Structure of fcc-Metals

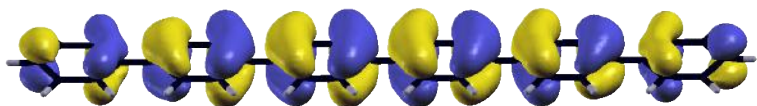
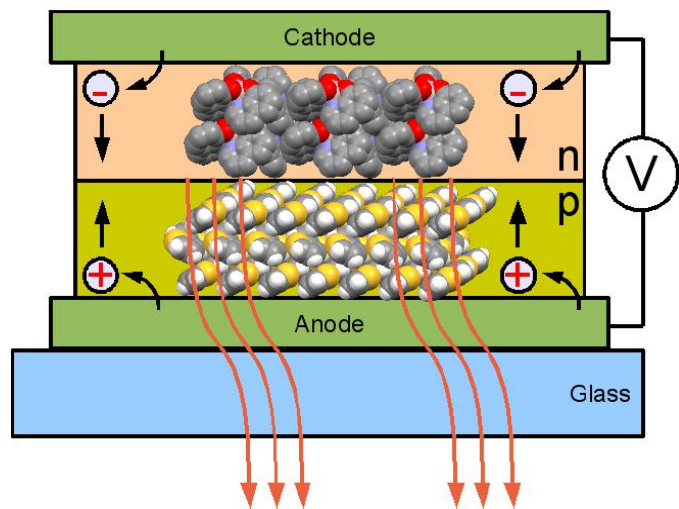


DFT-calculated phonon dispersion in Pb (left) and Cu (right) compared to experiment (dots)

Graphs are taken from Grabowski et al, *Phys. Rev. B* **76**, 024309 (2007).

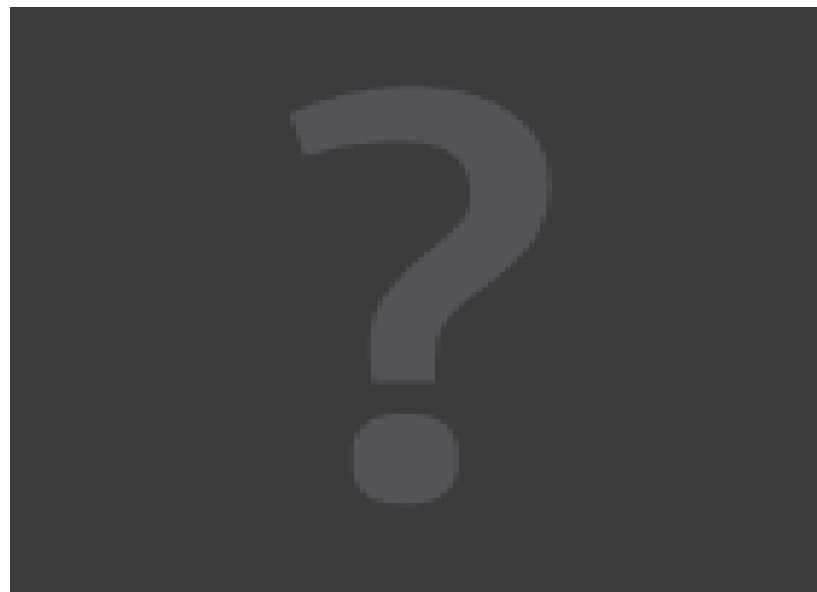


## Organic-LED



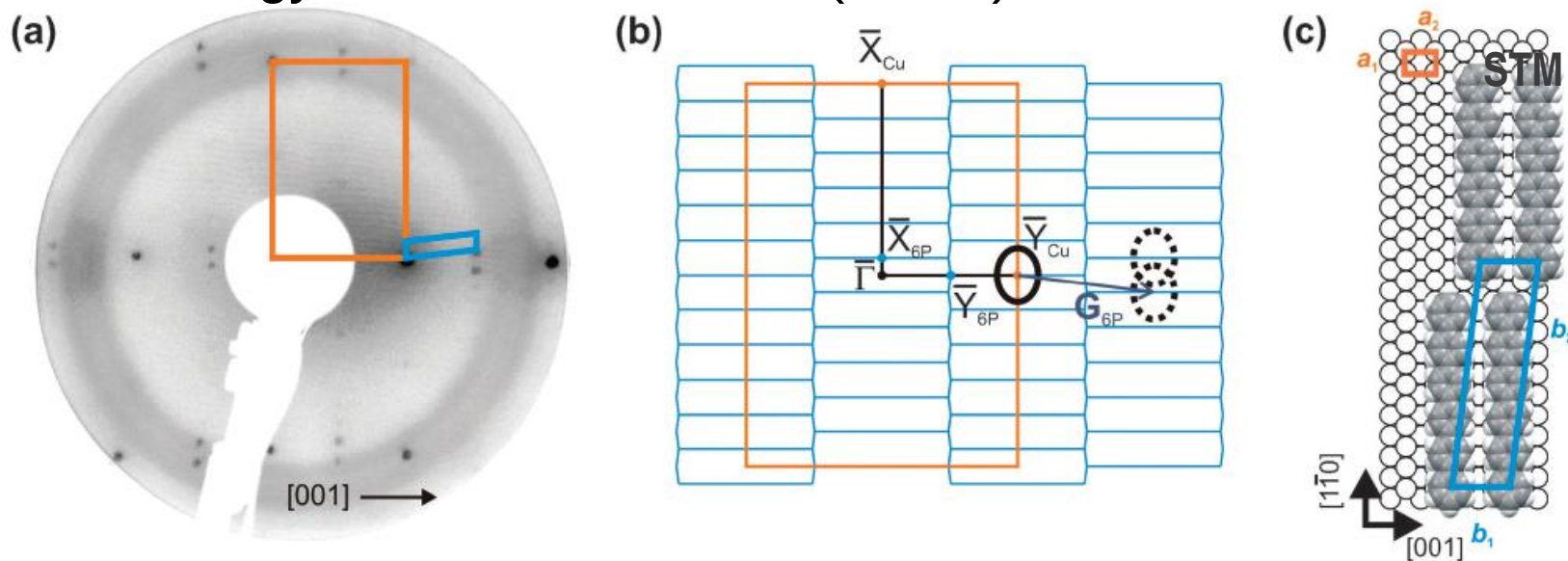
Highest Molecular Orbital (HOMO)  
of para-Sexiphenyl (6P) ( $C_{36}H_{26}$ )

## Monolayer 6P/Cu(110)

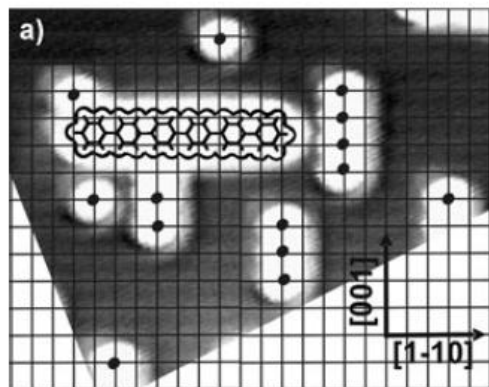


# Experimental Evidence

## Low Energy Electron Diffraction (LEED)

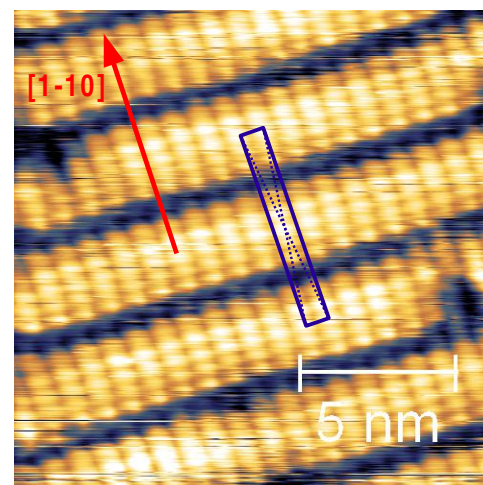


## Scanning Tunneling Microscopy (STM)

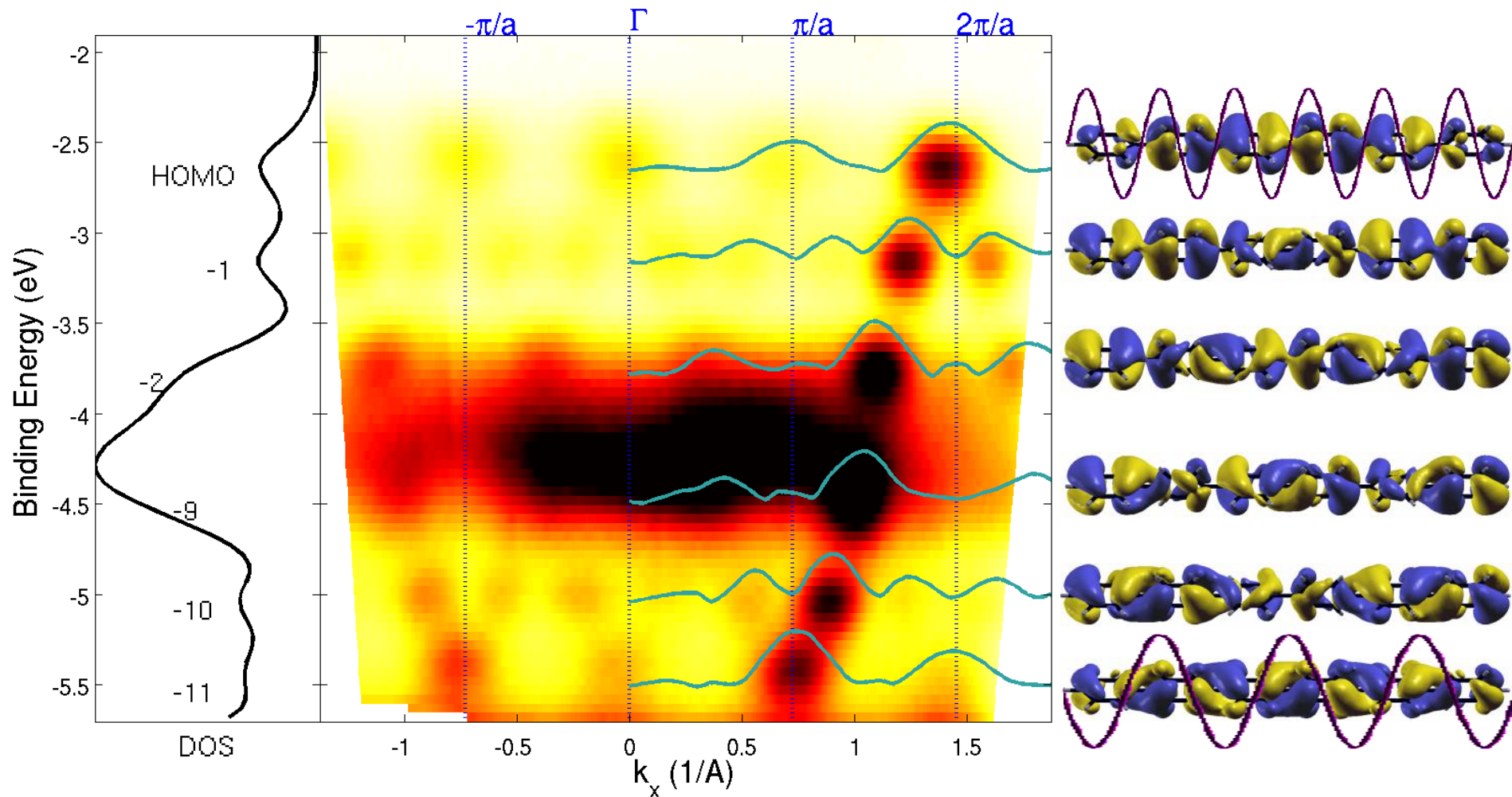


Berkebile et al.,  
*Phys. Chem. Chem. Phys.*,  
**13**, 3604–3611 (2011).

M. Oehzelt et al.,  
*ChemPhysChem* **8**,  
1707 – 1712 (2007).



# Angle-Resolved Photoemission: Multilayer

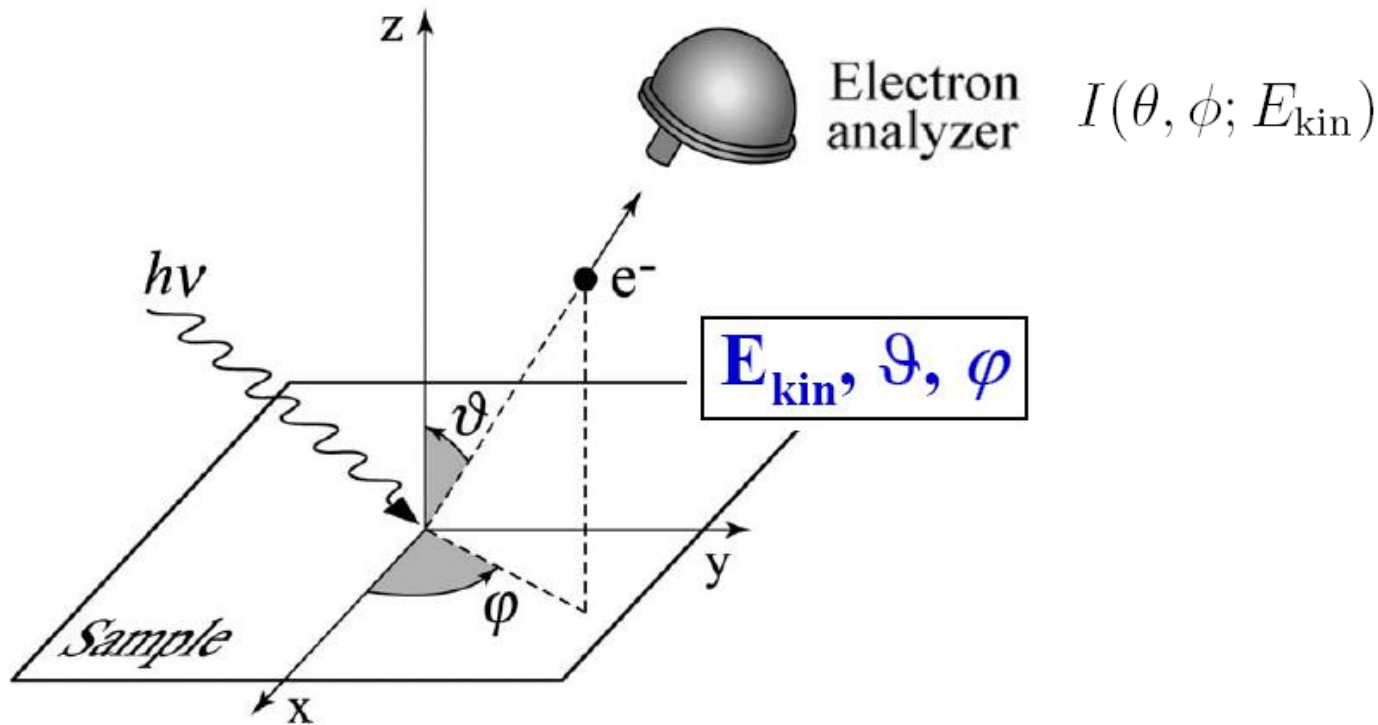


G. Koller et al., *Science* **317**, 351 (2007).

# Photoemission Intensity

## One Step Model

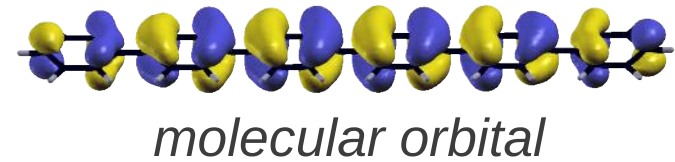
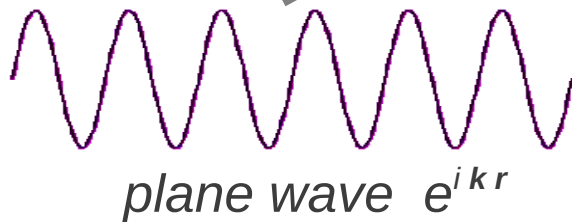
$$I(\theta, \phi; E_{\text{kin}}) \propto \sum_i \left| \langle \psi_f^*(\theta, \phi; E_{\text{kin}}) | \mathbf{A} \cdot \mathbf{p} | \psi_i \rangle \right|^2 \times \delta(E_i + \Phi + E_{\text{kin}} - \hbar\omega)$$



# Photoemission Intensity

## One Step Model

$$I(\theta, \phi; E_{\text{kin}}) \propto \sum_i \left| \langle \psi_f^*(\theta, \phi; E_{\text{kin}}) | \mathbf{A} \cdot \mathbf{p} | \psi_i \rangle \right|^2 \times \delta(E_i + \Phi + E_{\text{kin}} - \hbar\omega)$$



**Approximation:** final state = plane wave

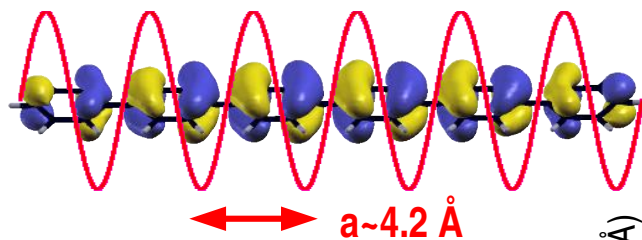
$$I_i(\theta, \phi) \propto |(\mathbf{A} \cdot \mathbf{k})|^2 \times \left| \tilde{\psi}_i(\mathbf{k}) \right|^2$$

Fourier Transform of Initial State Orbital

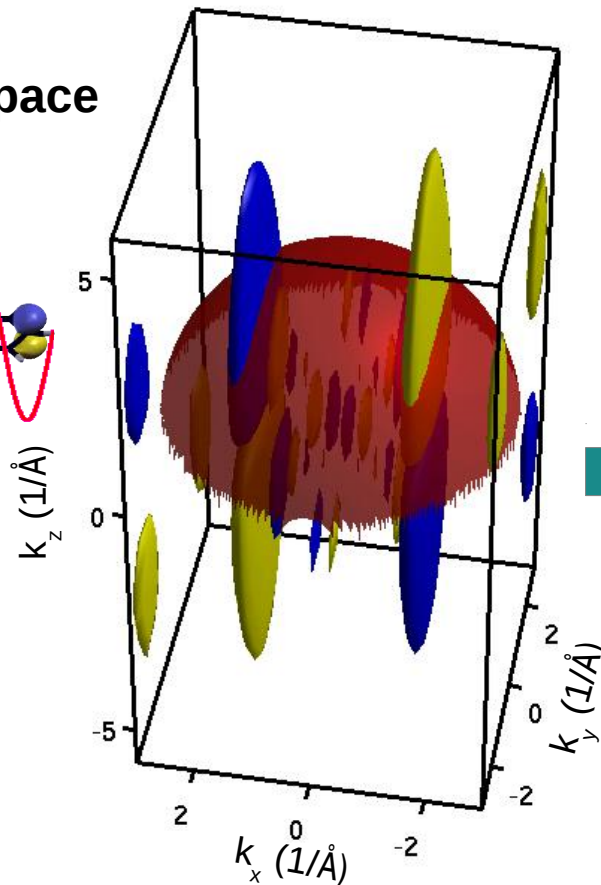
[Feibelman and Eastman, *Phys. Rev. B* **10**, 4932 (1974).]

# Photoemission Intensity

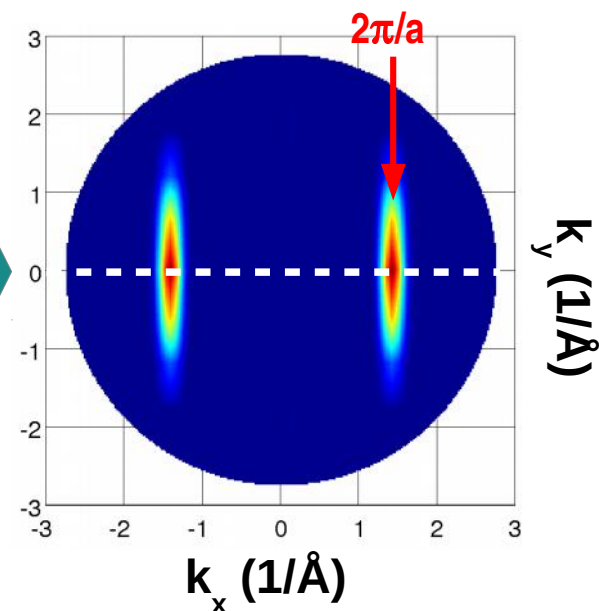
Molecular Orbital in Real Space



Calculation of  
the Fourier Transform

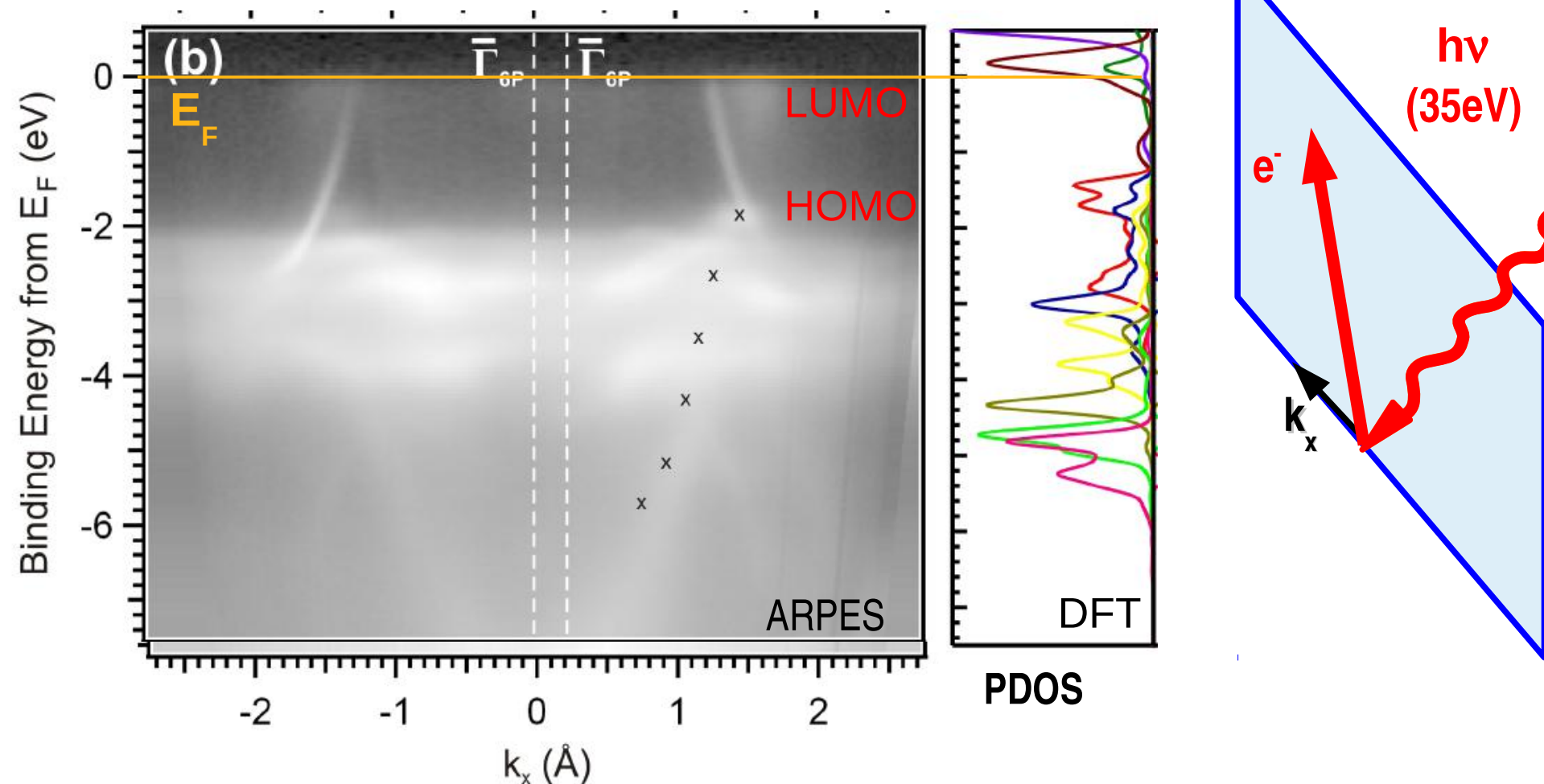


Hemispherical Cut Through  
3D Fourier Transform





# Angle-Resolved Photoemission: Monolayer

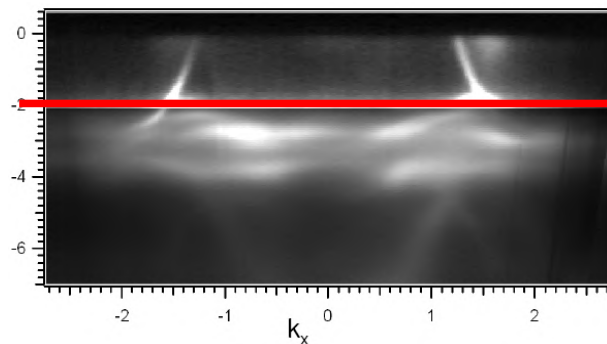


Berkebile et al., PCCP 13, 3604 (2011)

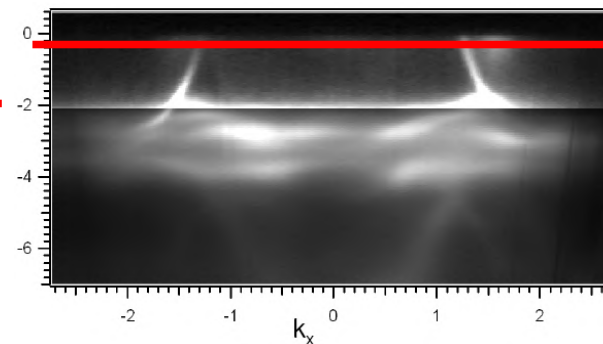


# 2D-Momentum Maps

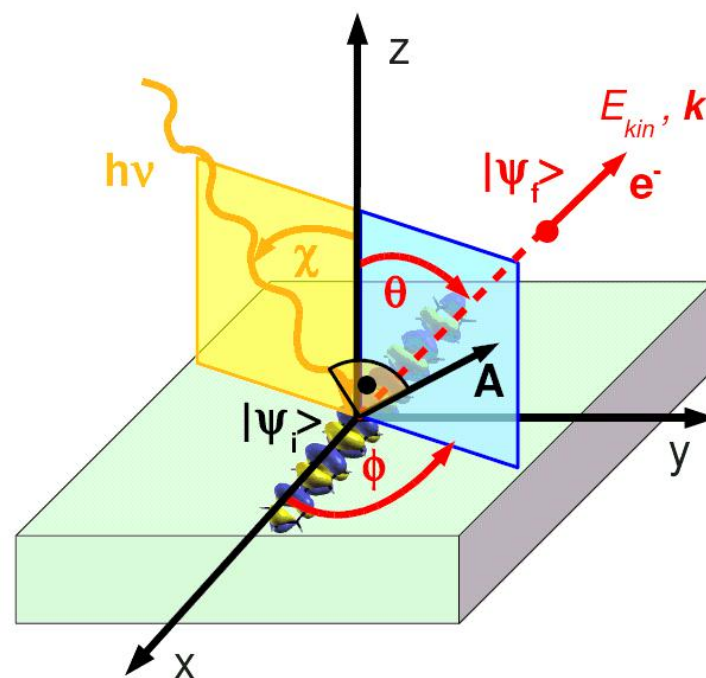
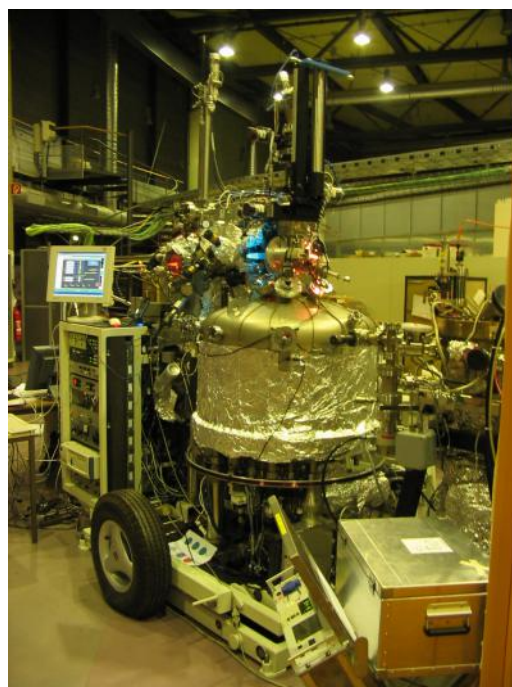
HOMO



filled  
LUMO



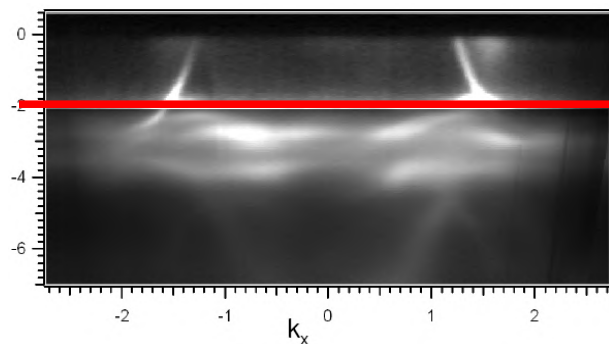
*The Toroidal Electron Spectrometer for Angle-Resolved Photoelectron Spectroscopy with Synchrotron Radiation at BESSY II*



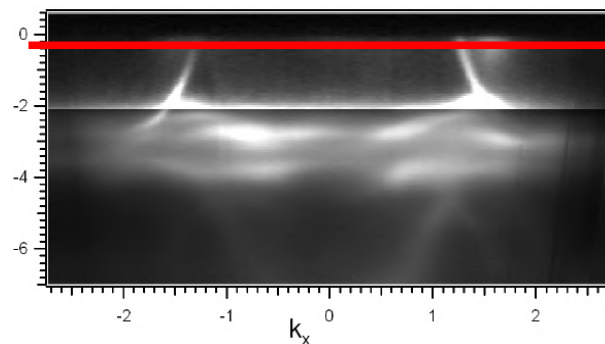
## 3.4 Surfaces and Interfaces

# 2D-Momentum Maps

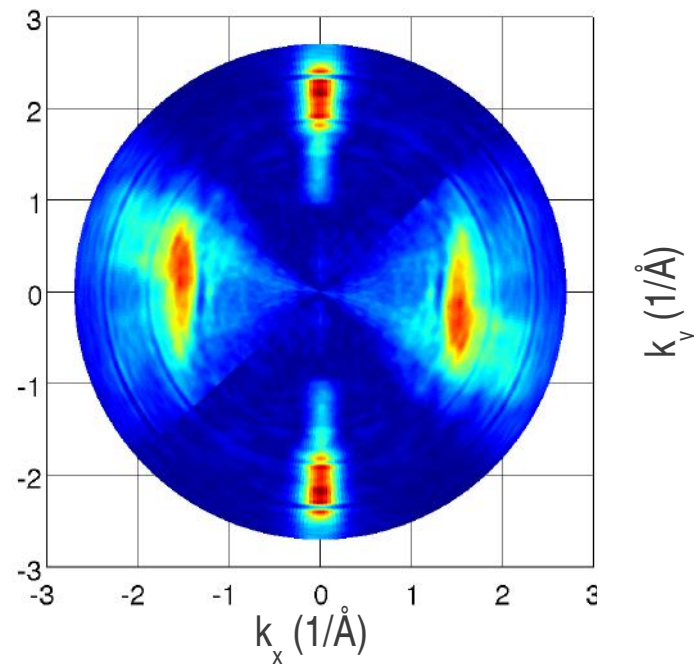
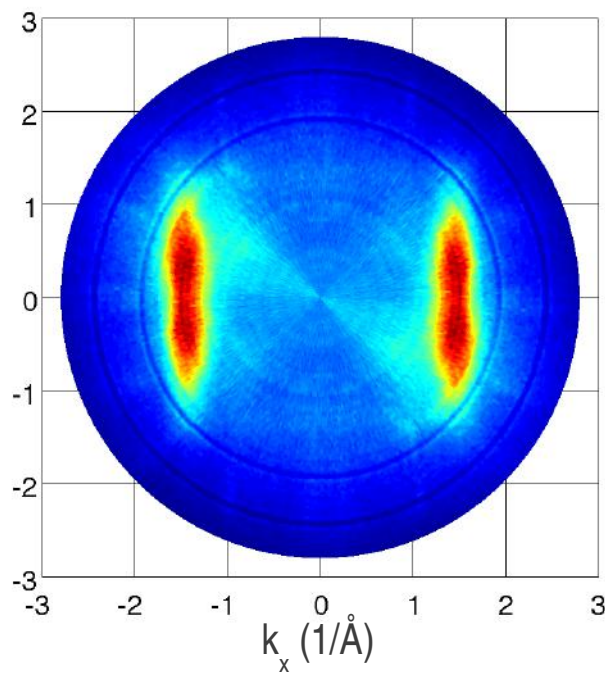
**HOMO**



**filled  
LUMO**

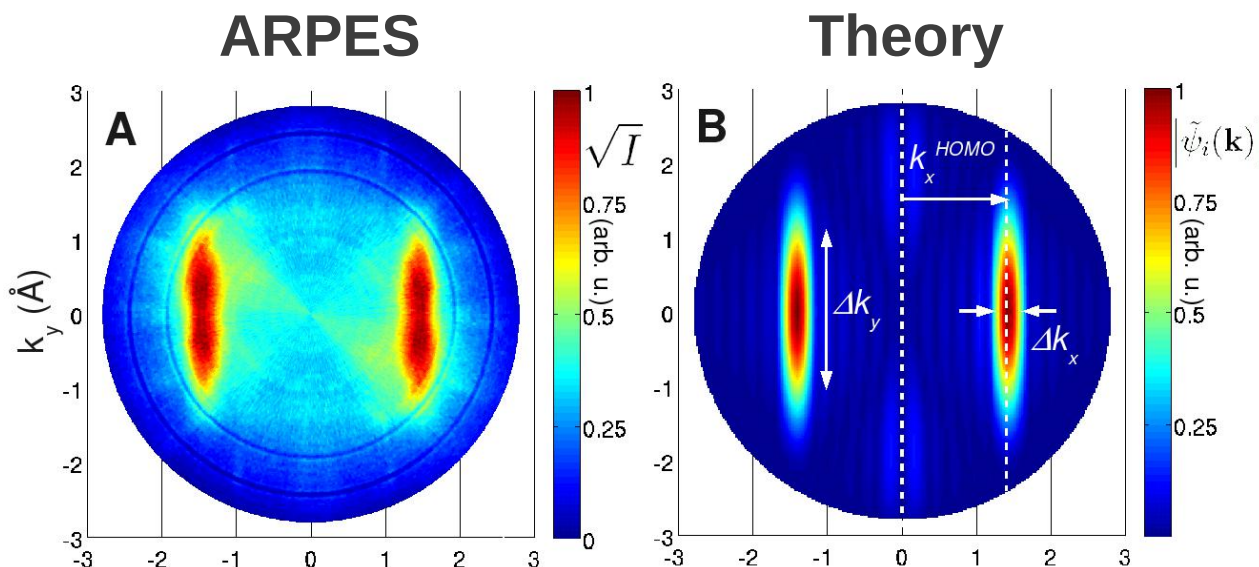


**ARPES**  
data for a  
monolayer of  
6P / Cu(110)

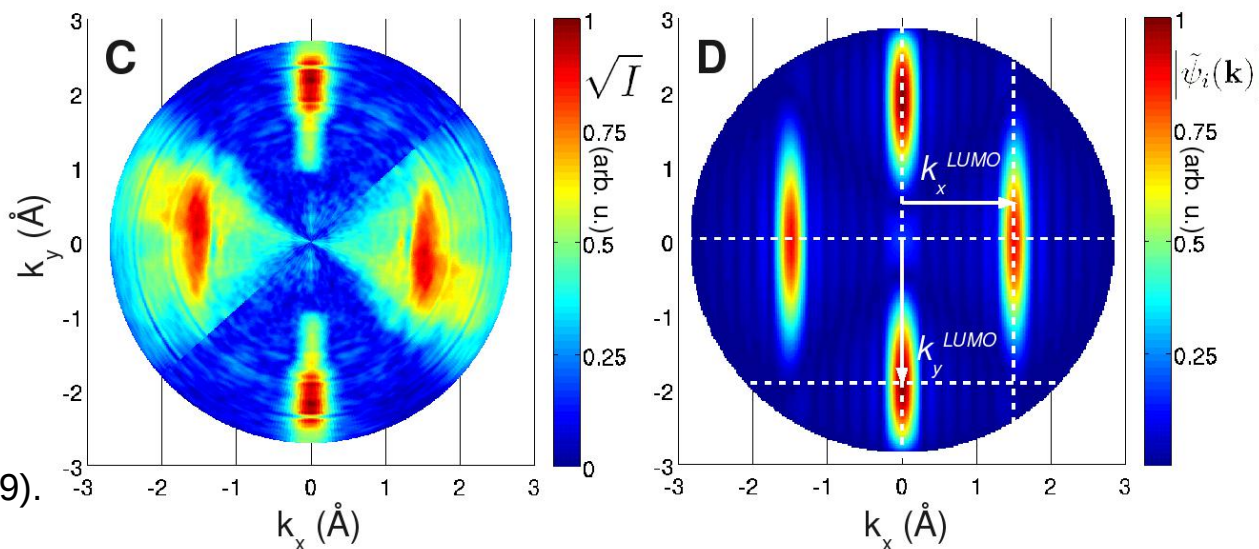


# 2D-Momentum Maps

**HOMO**



**LUMO**



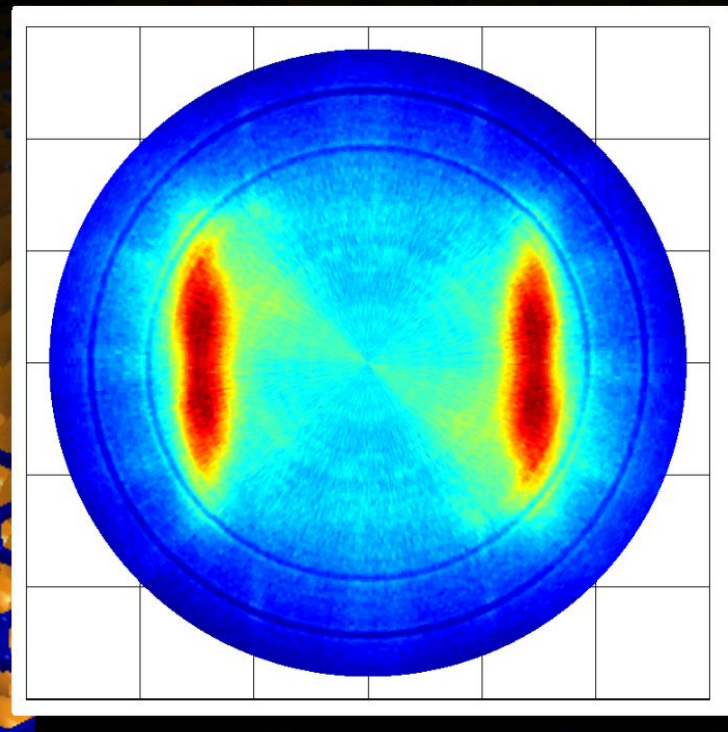
Puschnig et al.,  
*Science* **326**, 702 (2009).



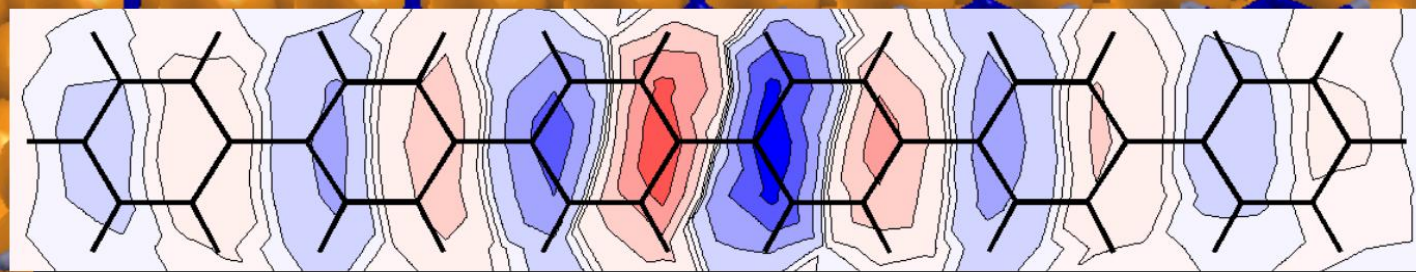
# Reconstruction of Orbitals

UV light electrons

ARPES



IFT

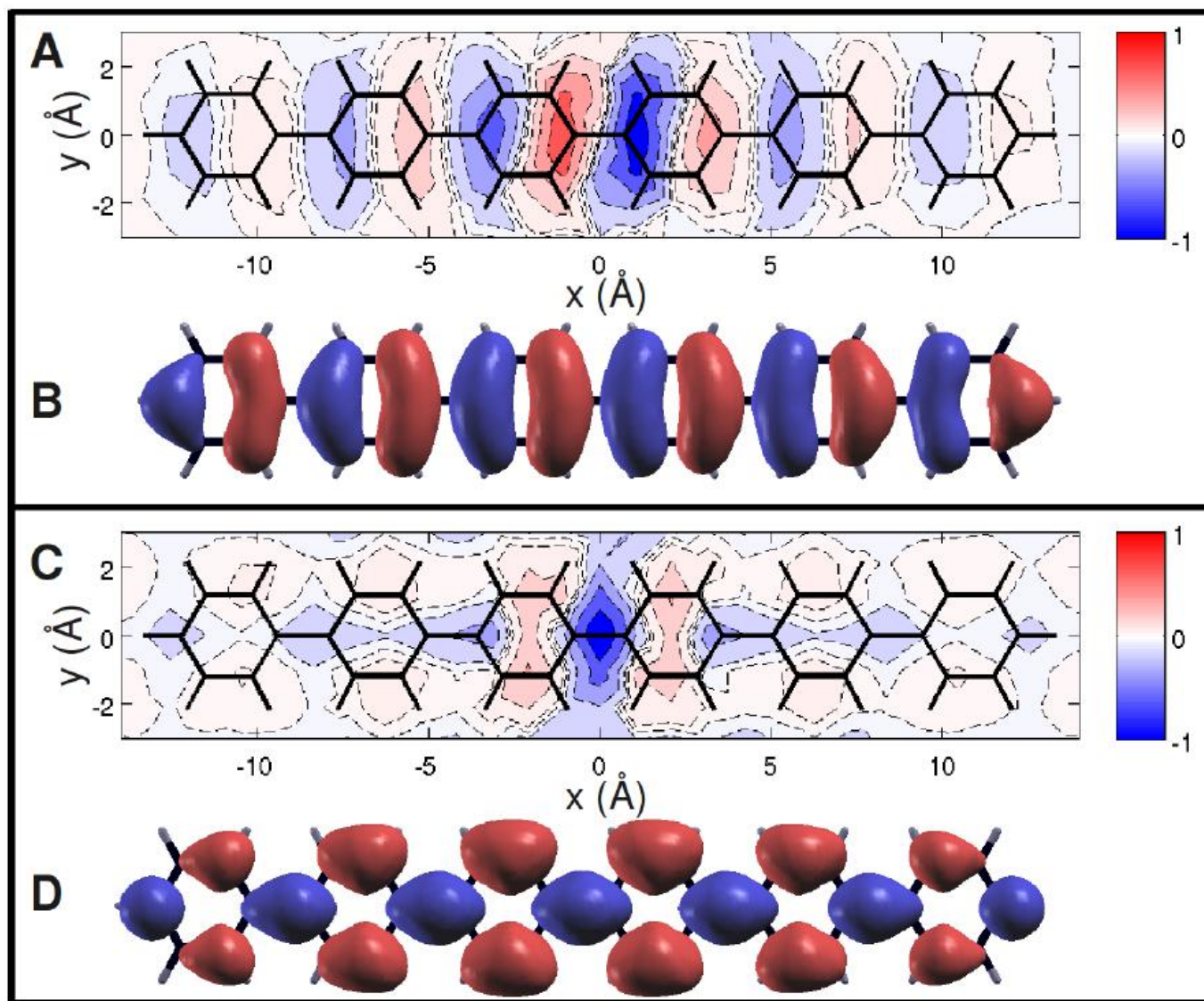


## 3.4 Surfaces and Interfaces



# Reconstruction of Orbitals

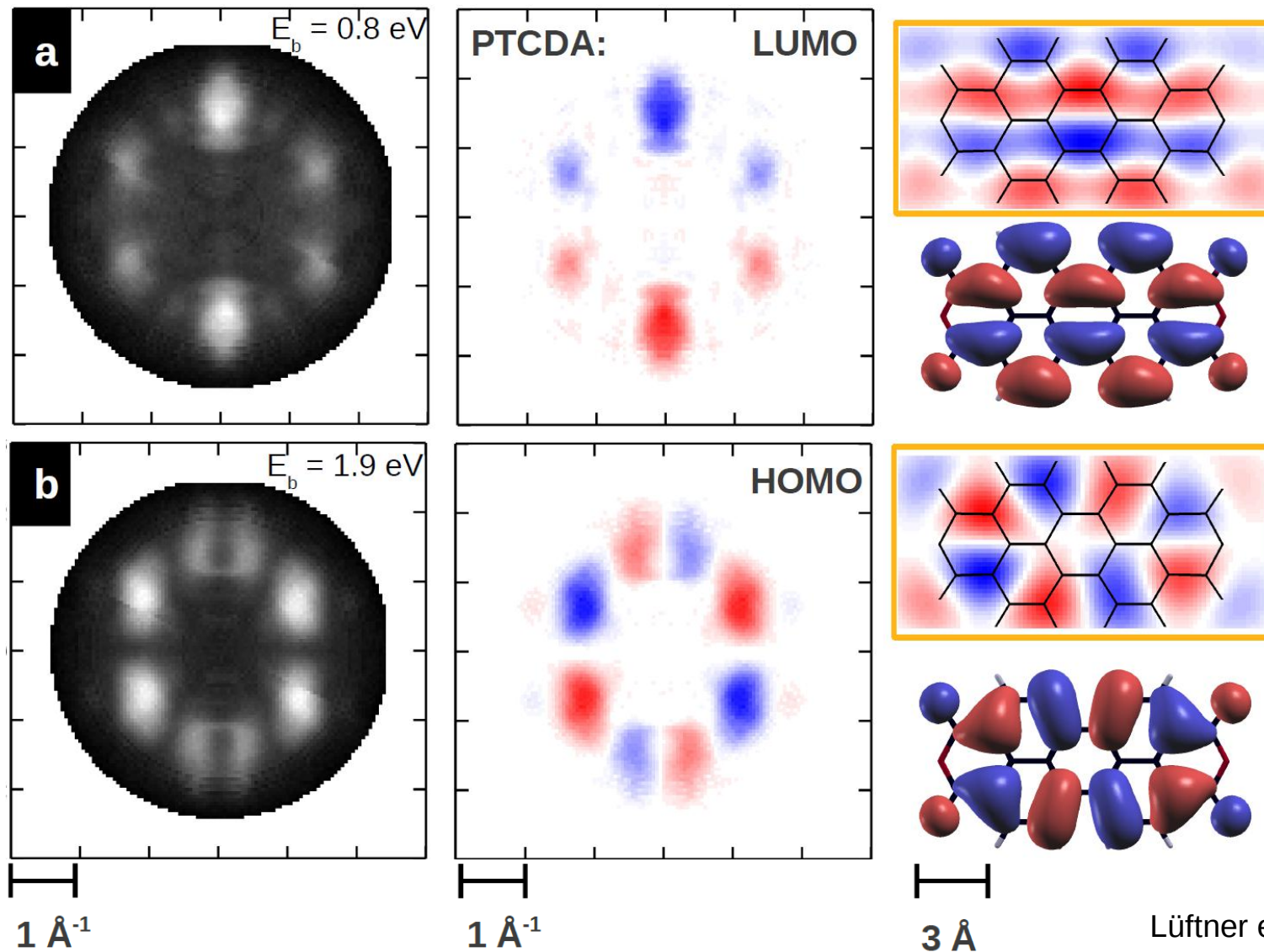
**HOMO**



**LUMO**

Puschnig et al.,  
*Science* **326**, 702 (2009).

# Reconstruction of Orbitals



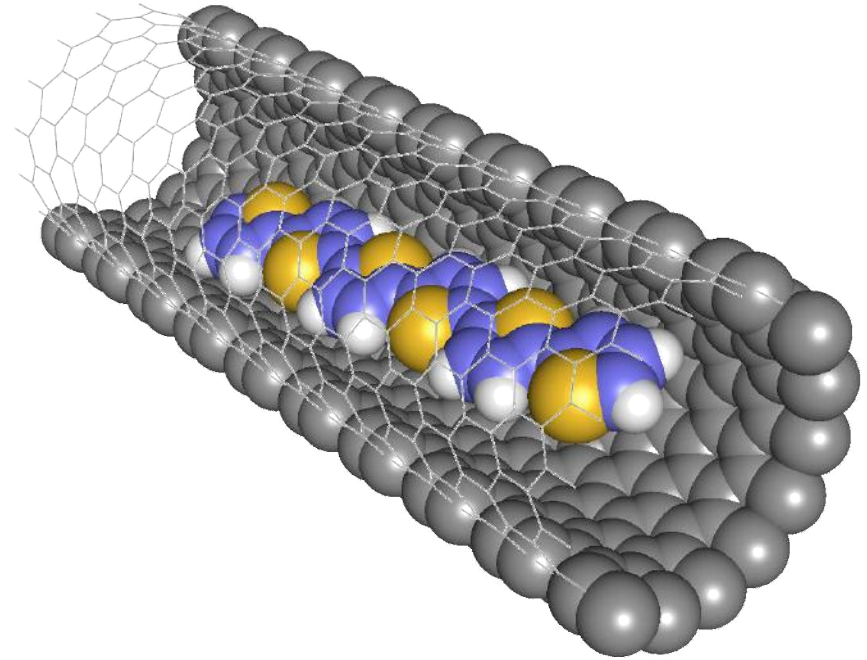
Lüftner et al., *PNAS* (2013),  
[www.pnas.org/cgi/doi/10.1073/pnas.1315716110](http://www.pnas.org/cgi/doi/10.1073/pnas.1315716110)

## 3.4 Surfaces and Interfaces

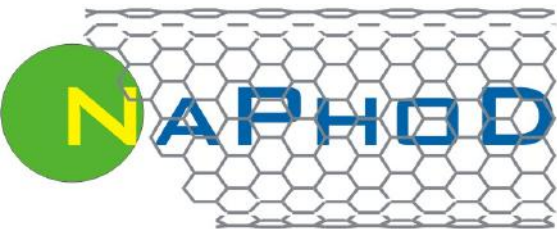
# Example: Nano-Peapods



*peas in a pod*



*organic molecules  
in a  
carbon nanotube*



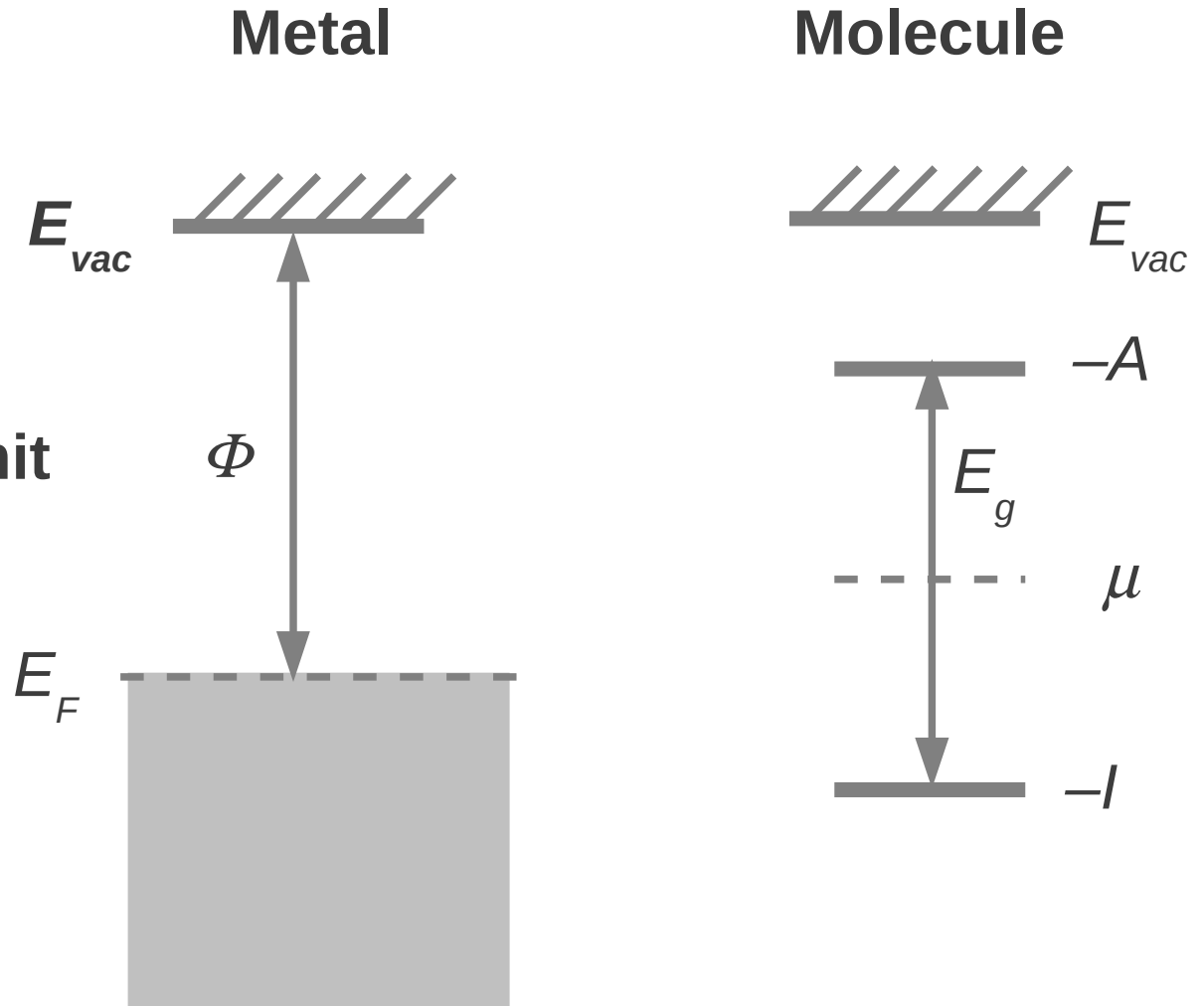
EU-project  
Nano-Hybrids for **Photonic Devices**



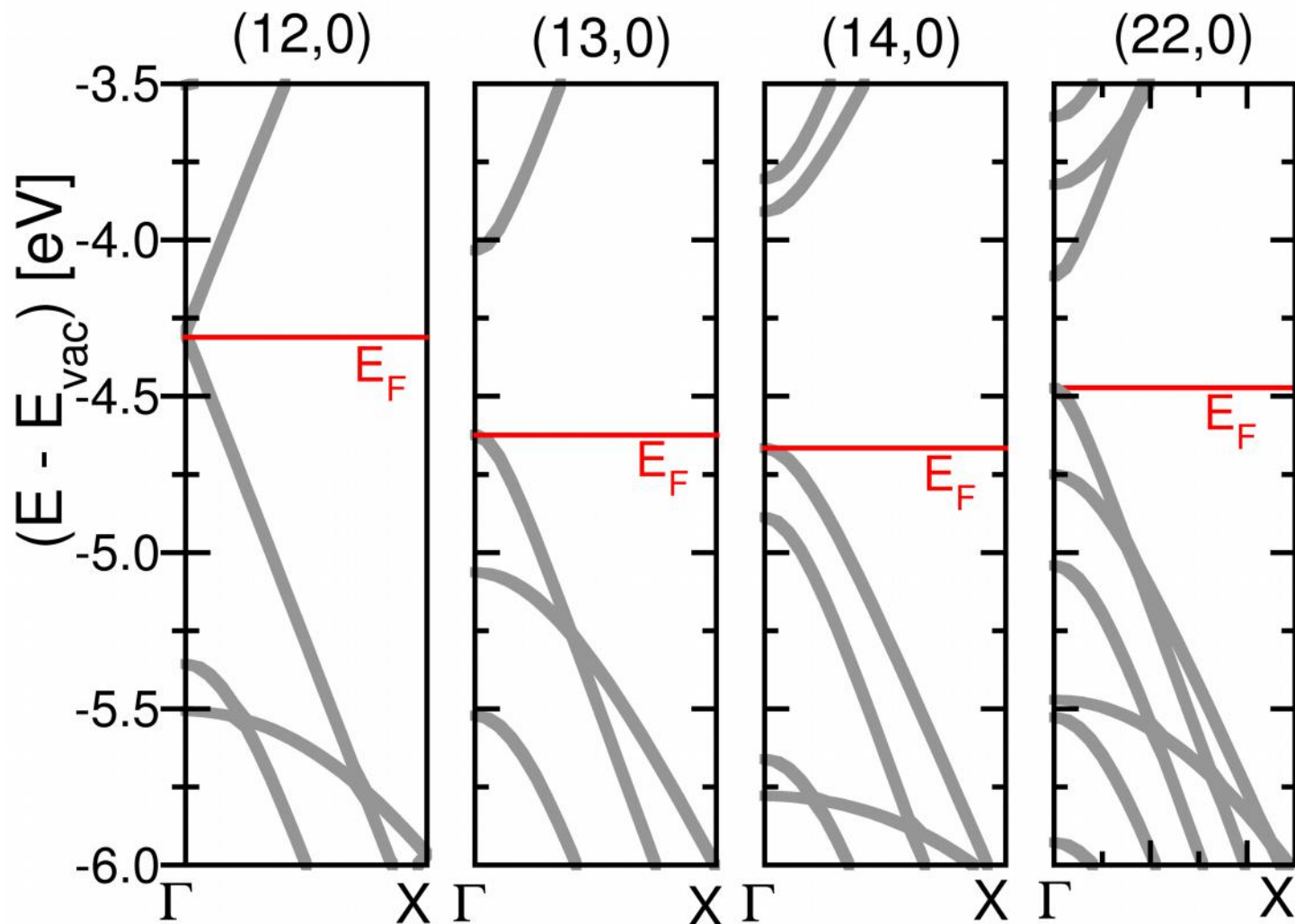
# Level Alignment

Weak interaction  
(physisorption)

**Vacuum Level  
Alignment  
= Schottky-Mott Limit**



# Band Structure: CNT



# Single-Wall Carbon Nanotubes

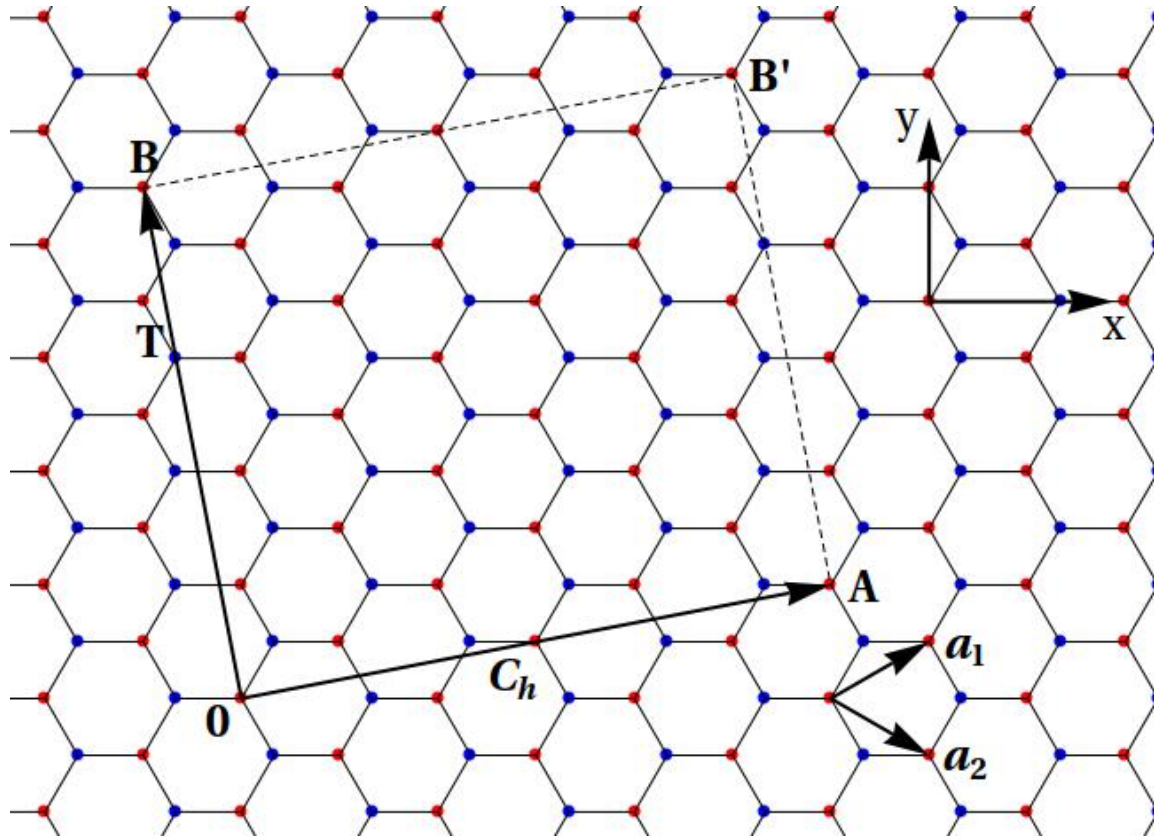
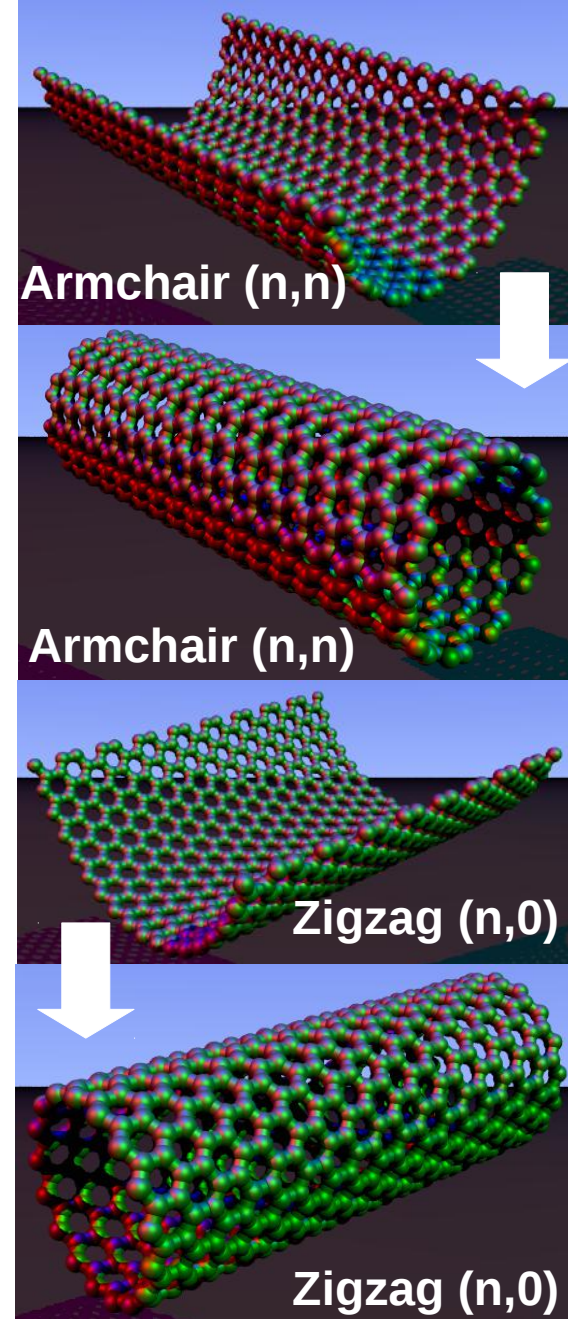
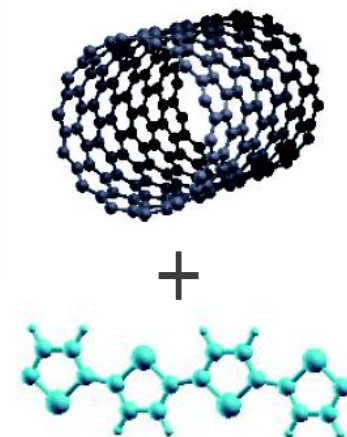
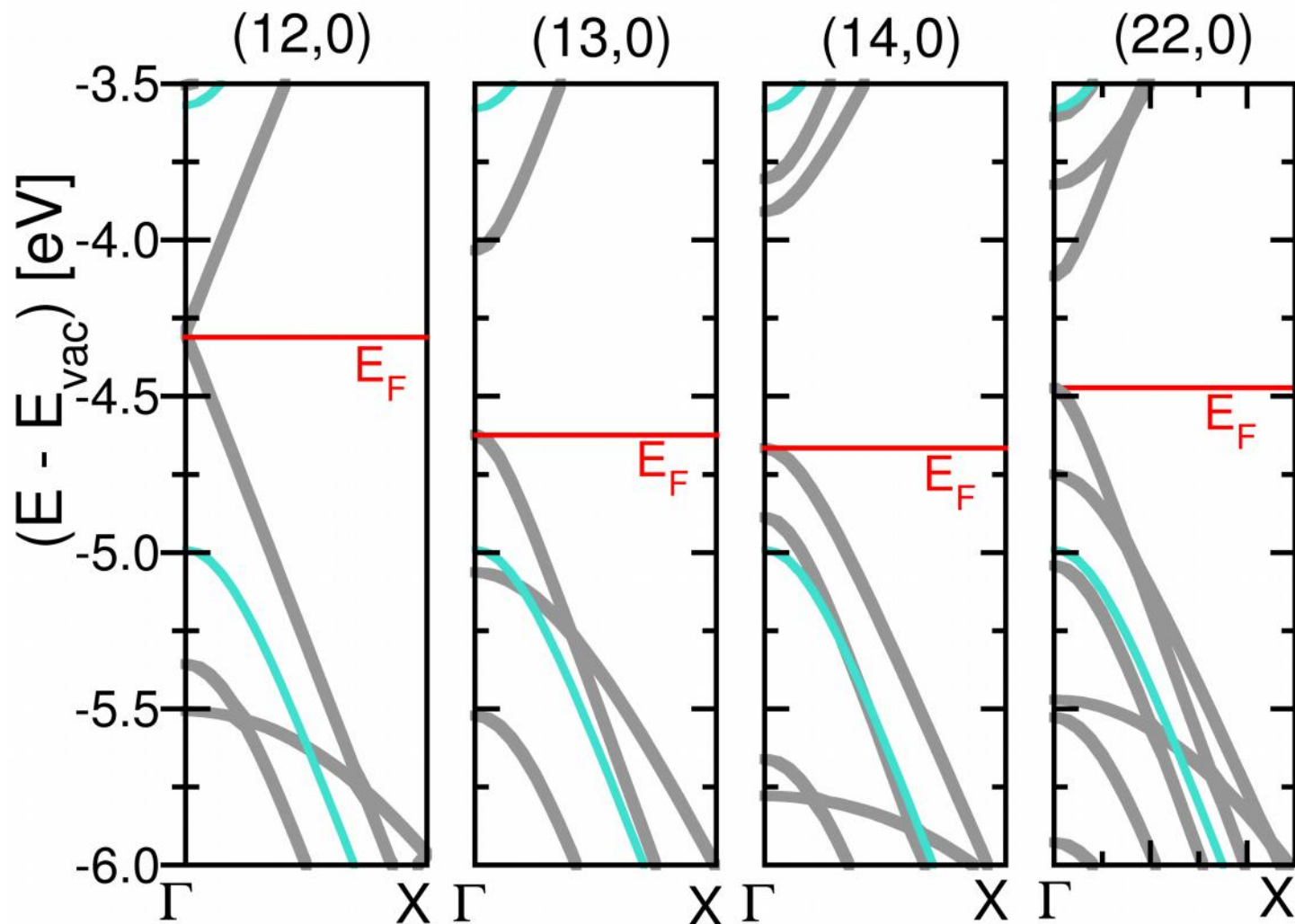


Figure 1: Aufrollen des Graphengitters zu einer Kohlenstoffnanoröhre (CNT). Das Rechteck  $0AB'B$  definiert die Einheitszelle der CNT, die durch Verbinden der Punkte  $0$  und  $A$  sowie  $B$  und  $B'$  entsteht. Der chirale Vektor  $C_h$  beschreibt dabei den Umfang der CNT, und der Vektor  $T$  definiert den Translationsvektor entlang der CNT-Achse. Die in der Figur gezeichnete CNT entspricht  $C_h = (4, 2)$  und  $T = (4, -5)$ .

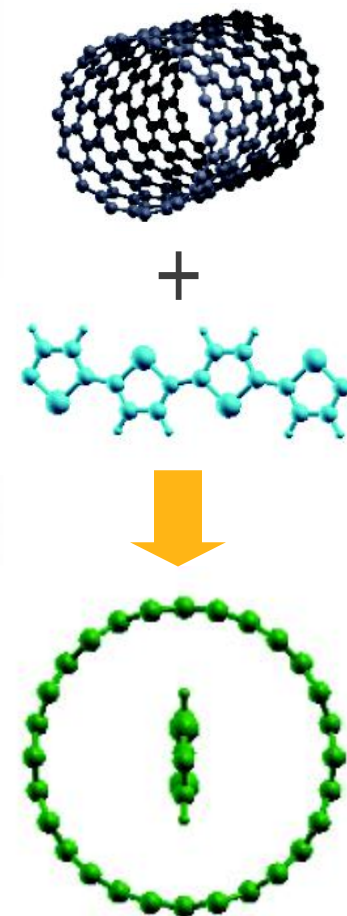
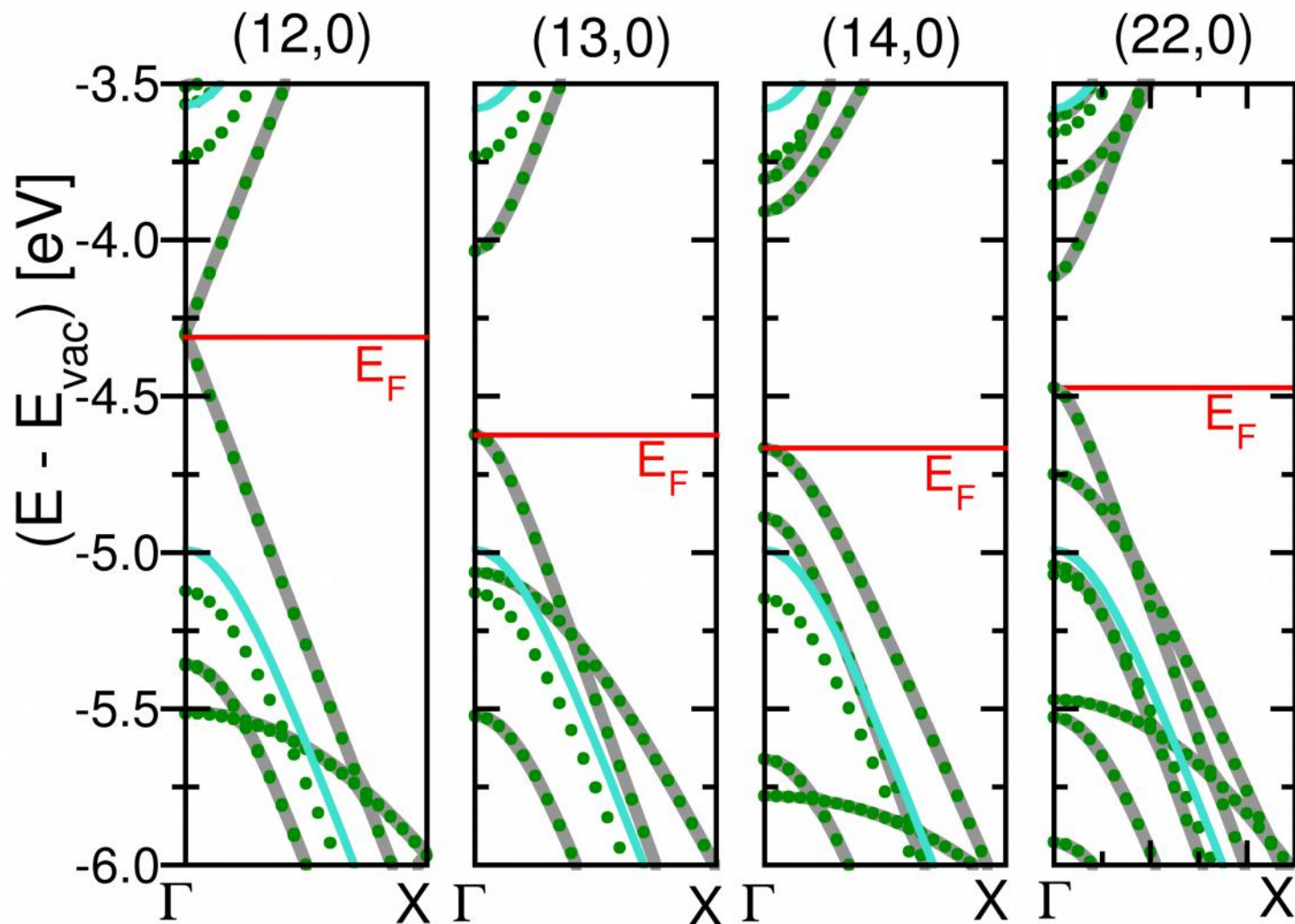


# Band Structure: CNT + Polymer

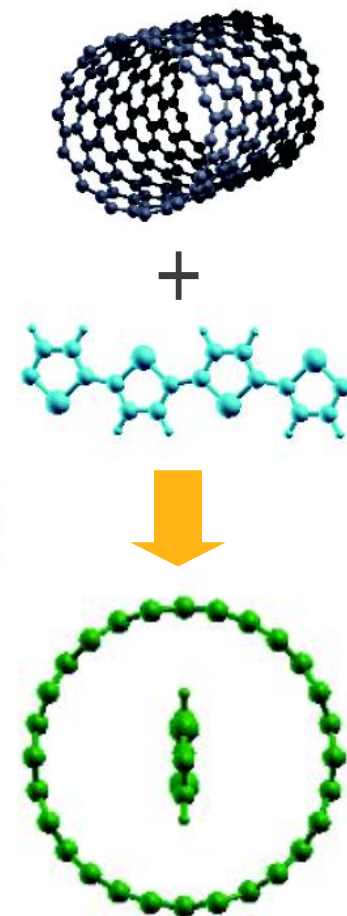
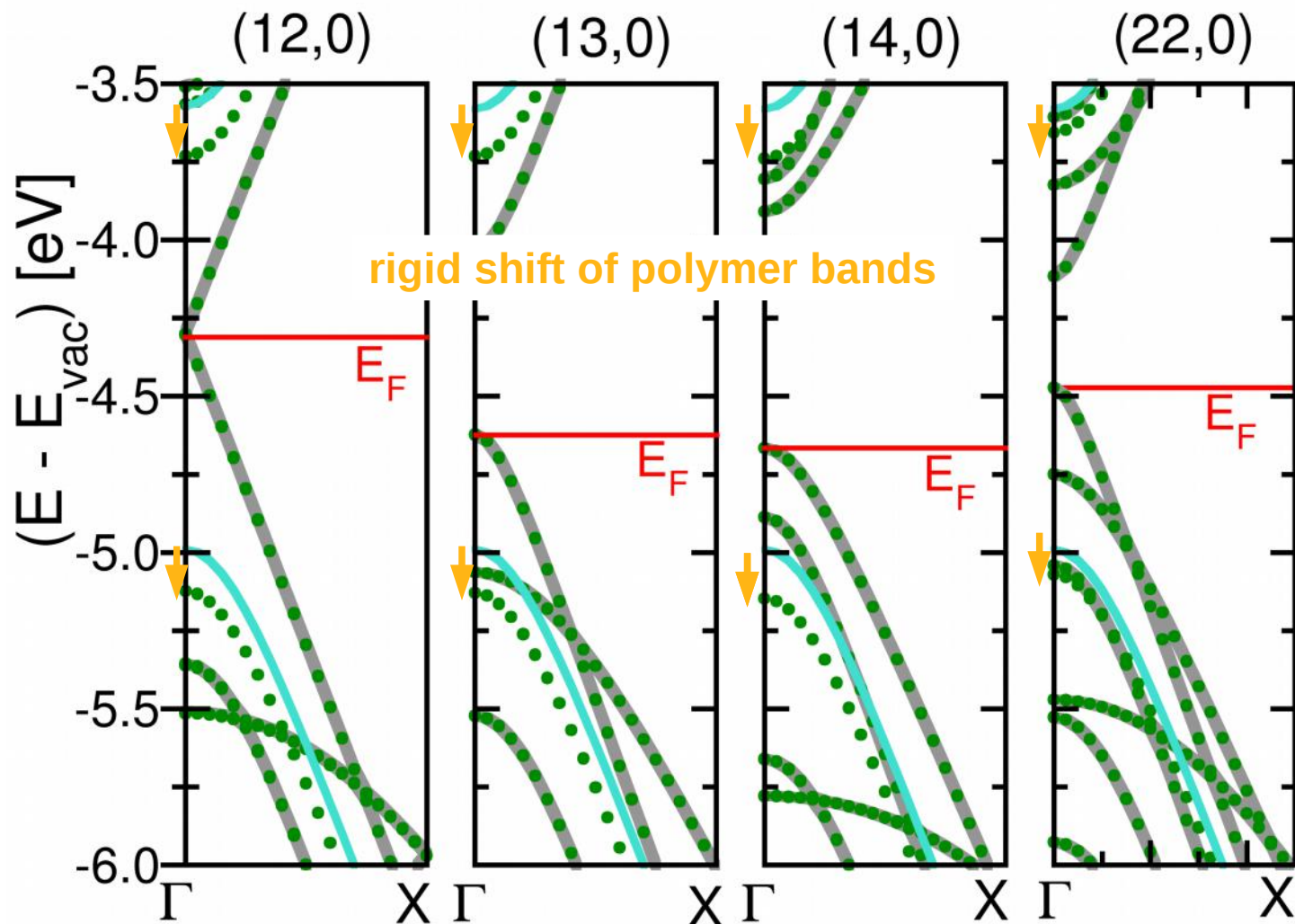




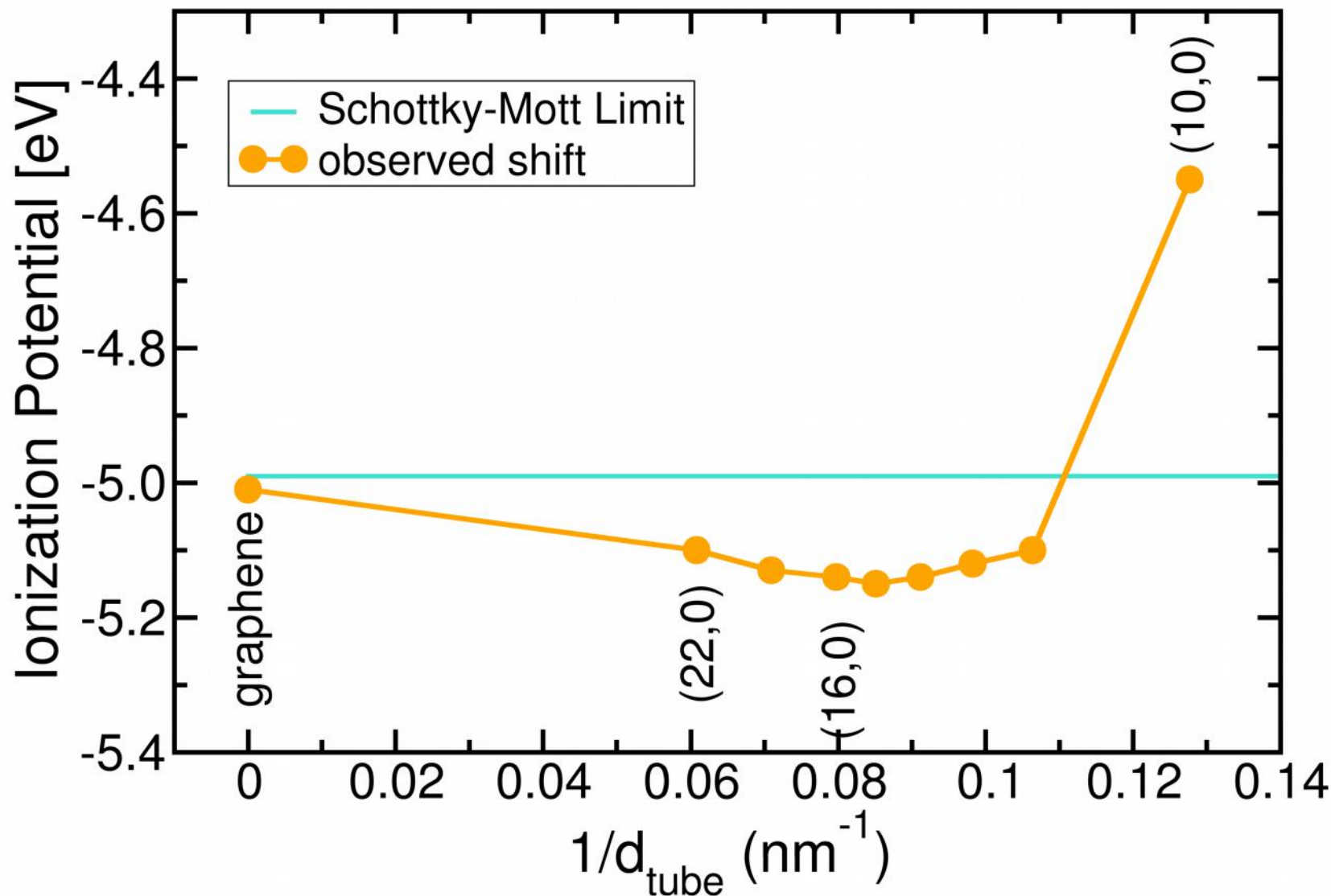
# Band Structure: Peapod



# Band Structure: Level Alignment



# Ionization Potential of Polymer

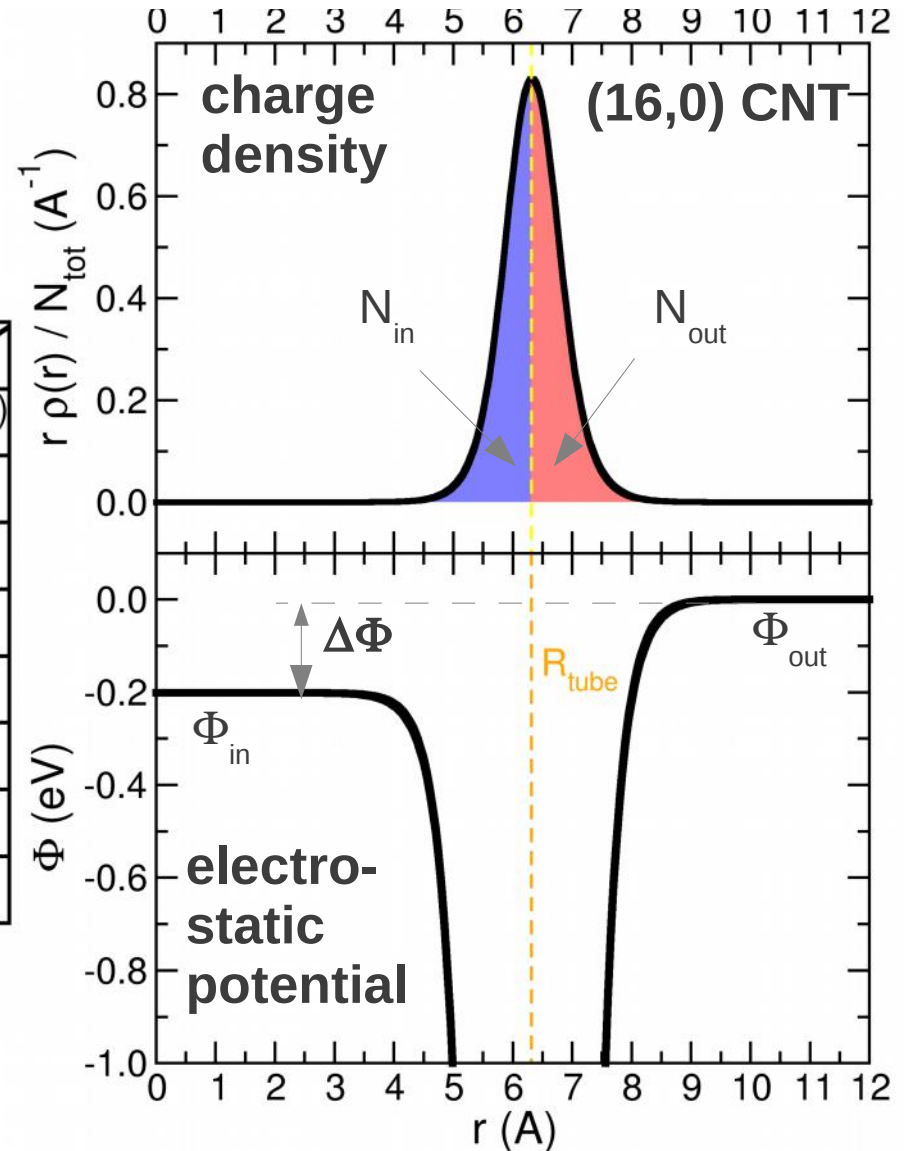
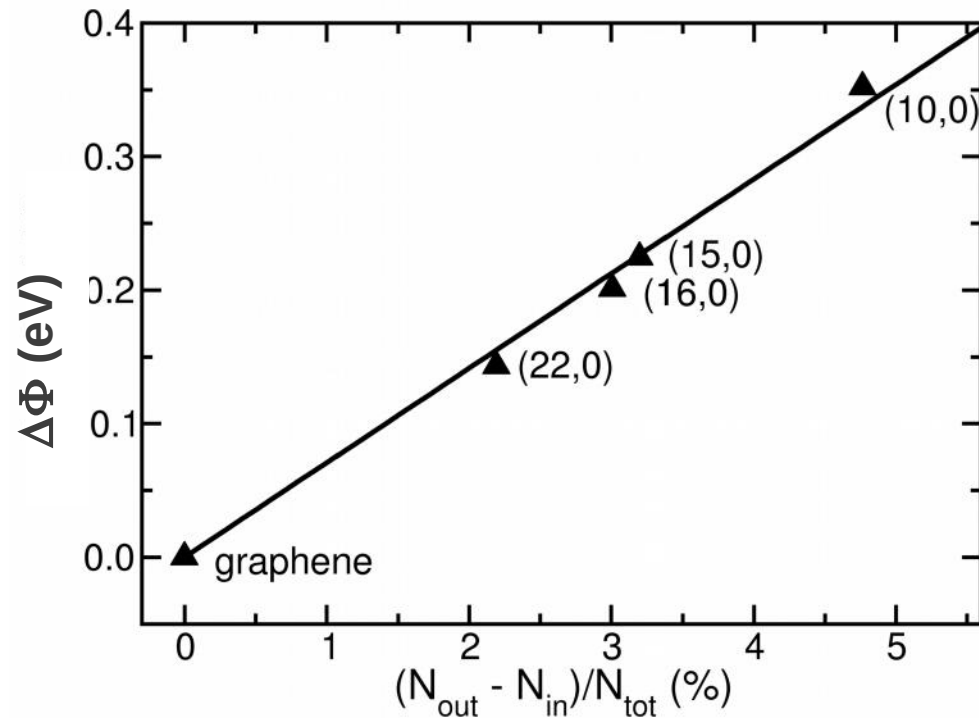




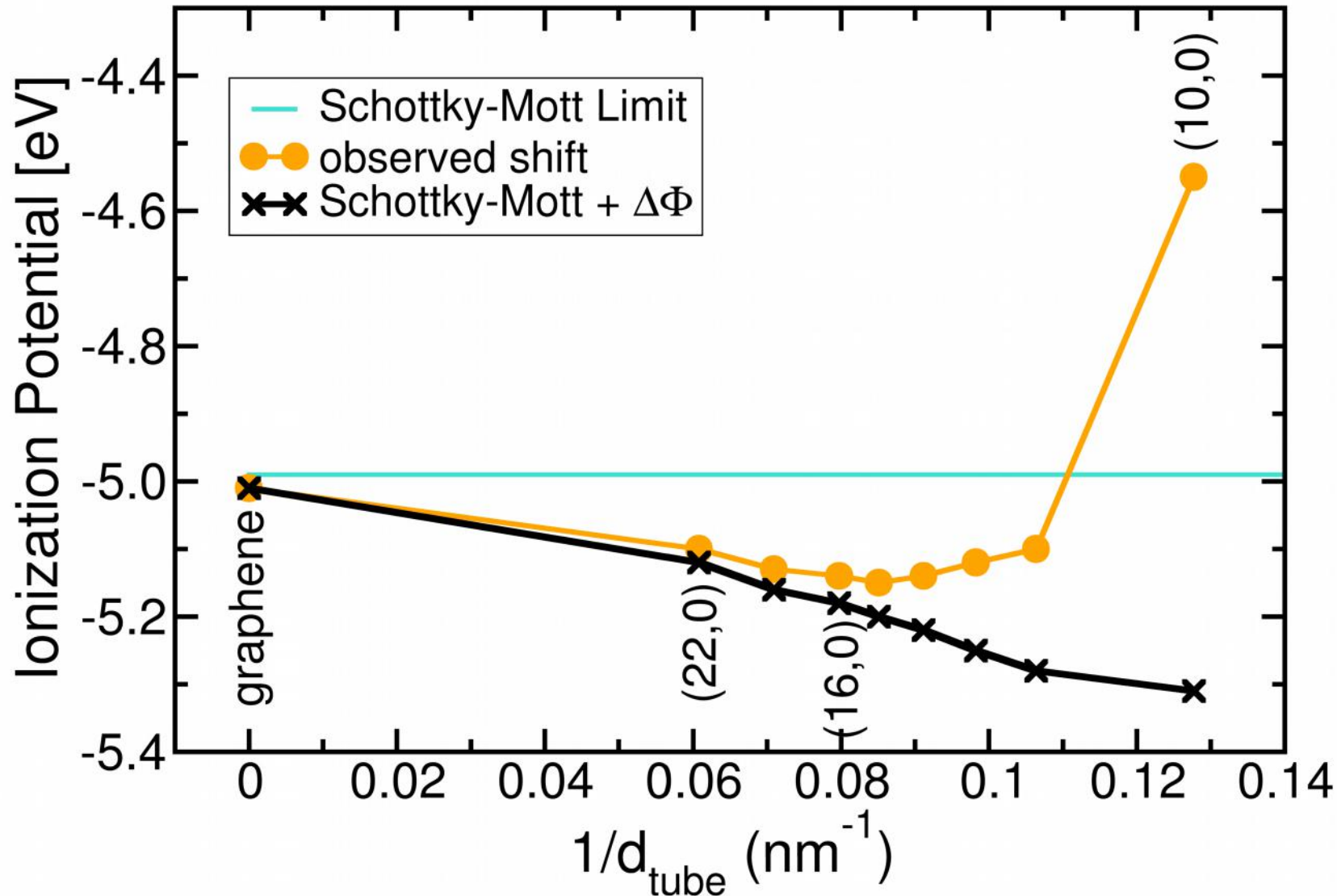
# Ionization Potential of Polymer

## 1. Charge asymmetry in CNTs

→  $\Delta\Phi$



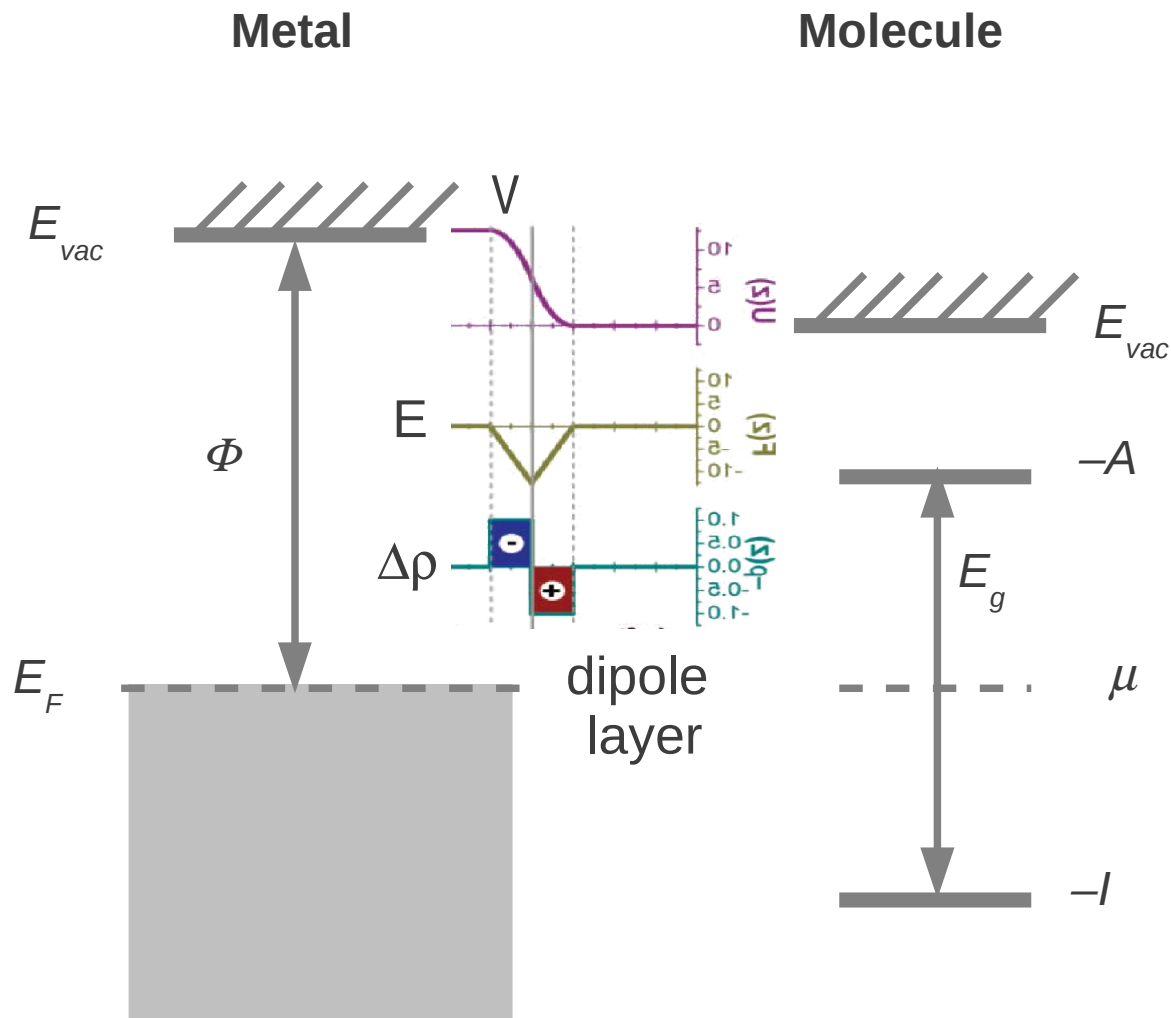
# Level Alignment



# Level Alignment

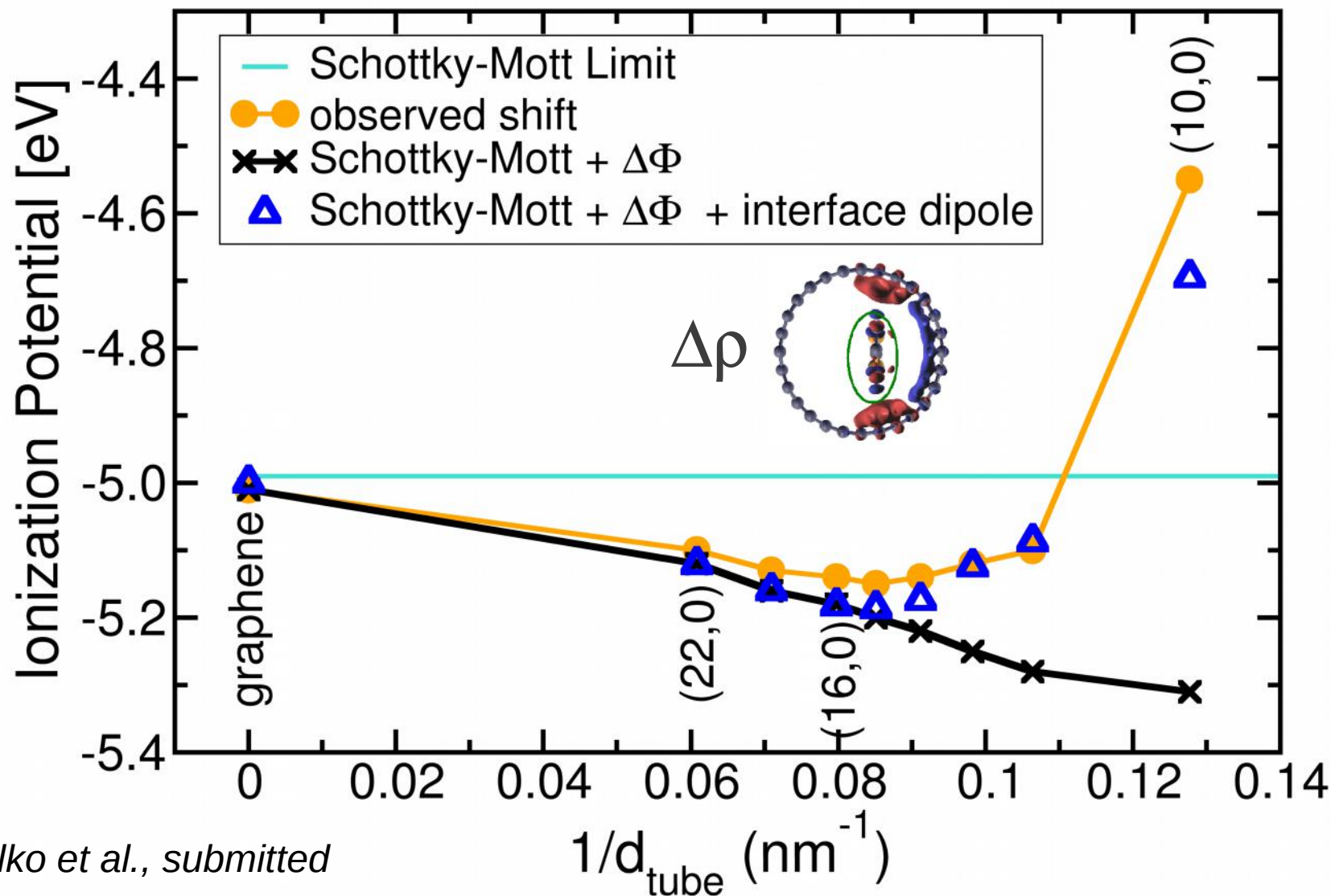
Weak interaction  
(physisorption)

## 2. Interface dipole (push-back effect)



See for instance: Heimel et al., *Acc. Chem. Res.* **41**, 721-729 (2008).

# Level Alignment



Milko et al., submitted

# Free Energy from Statistical Physics

$$H \rightarrow E_r(V, N) \rightarrow Z(T, V, N) \rightarrow F(T, V, N)$$

Microscopic Hamiltonoperator

$$H|\Psi\rangle = E_r|\Psi\rangle$$

Partition Function for canonical ensemble


$$Z(T, V, N) = \sum_r \exp[-\beta E_r(V, N)]$$

Free Energy

$$F(T, V, N) = -k_B T \ln [Z(T, V, N)]$$

## 3.6 Thermodynamic Properties

# Coarse-Graining Procedure

$$H|\Psi\rangle = E_r|\Psi\rangle$$


Should contain all electronic, magnetic, vibrational and configurational degrees of freedom!  
Unfortunately, this is an extremely challenging – if not impossible – task :-)

But we can make use of different time-scales of excitations

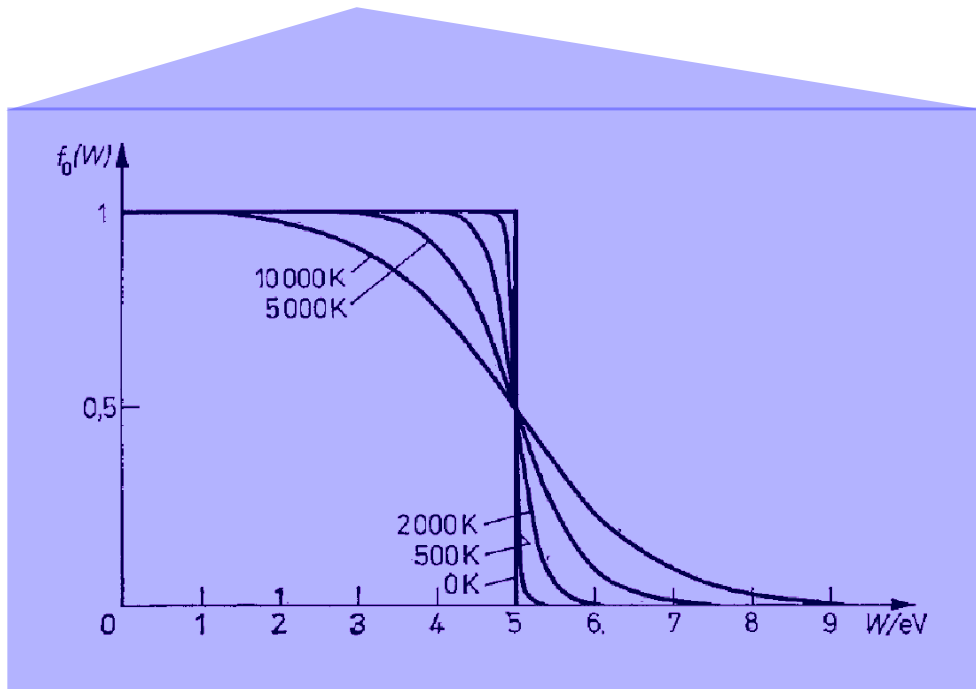
- Electronic :  $10^{-15}$  sec
- Magnetic :  $10^{-13}$  sec
- Vibrational :  $10^{-12}$  sec
- Configurational :  $>10^{-10}$  sec

**➡ Can be calculated separately**

# Entropy Contributions

$$F(T, V, N) = E_0(V, N) - TS$$

$$S_{el} + S_{mag} + S_{vib} + S_{conf}$$



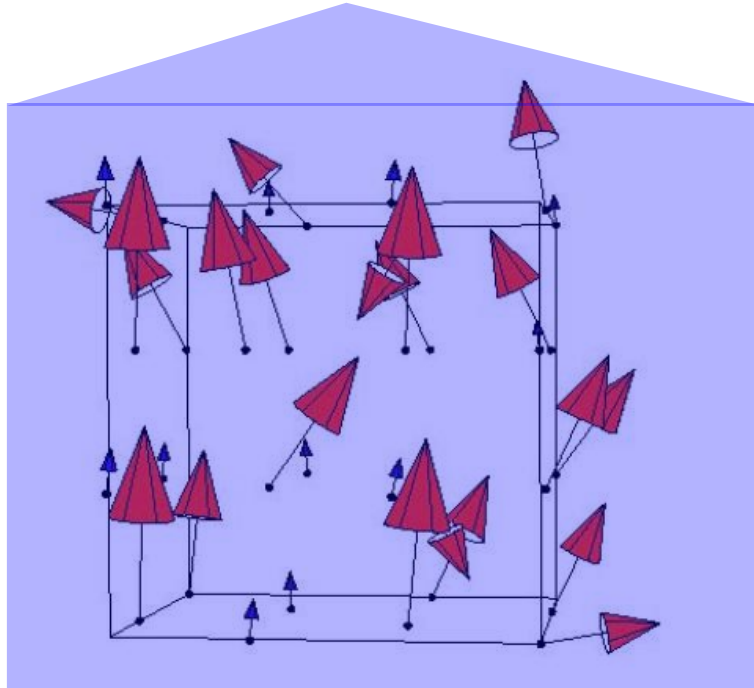
## 3.6 Thermodynamic Properties



# Entropy Contributions

$$F(T, V, N) = E_0(V, N) - TS$$

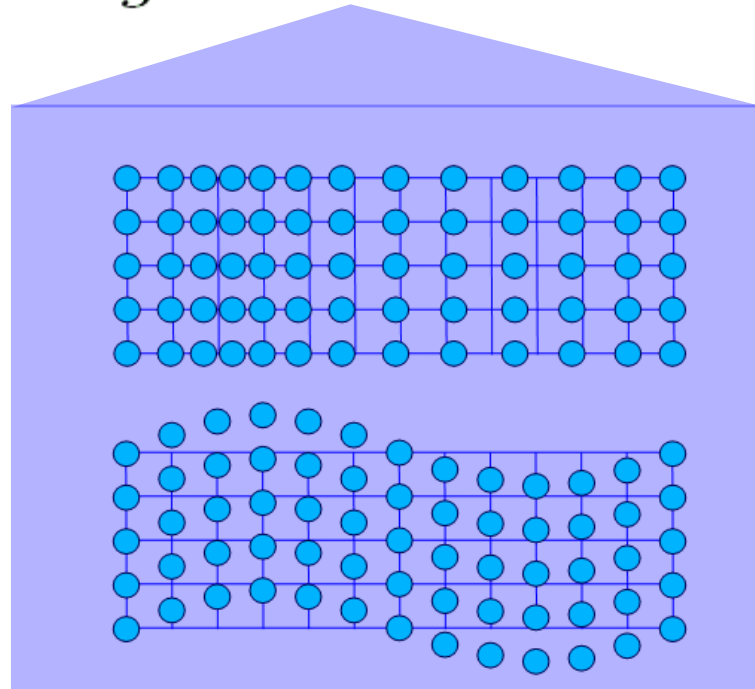
$$S_{el} + S_{mag} + S_{vib} + S_{conf}$$



# Entropy Contributions

$$F(T, V, N) = E_0(V, N) - TS$$

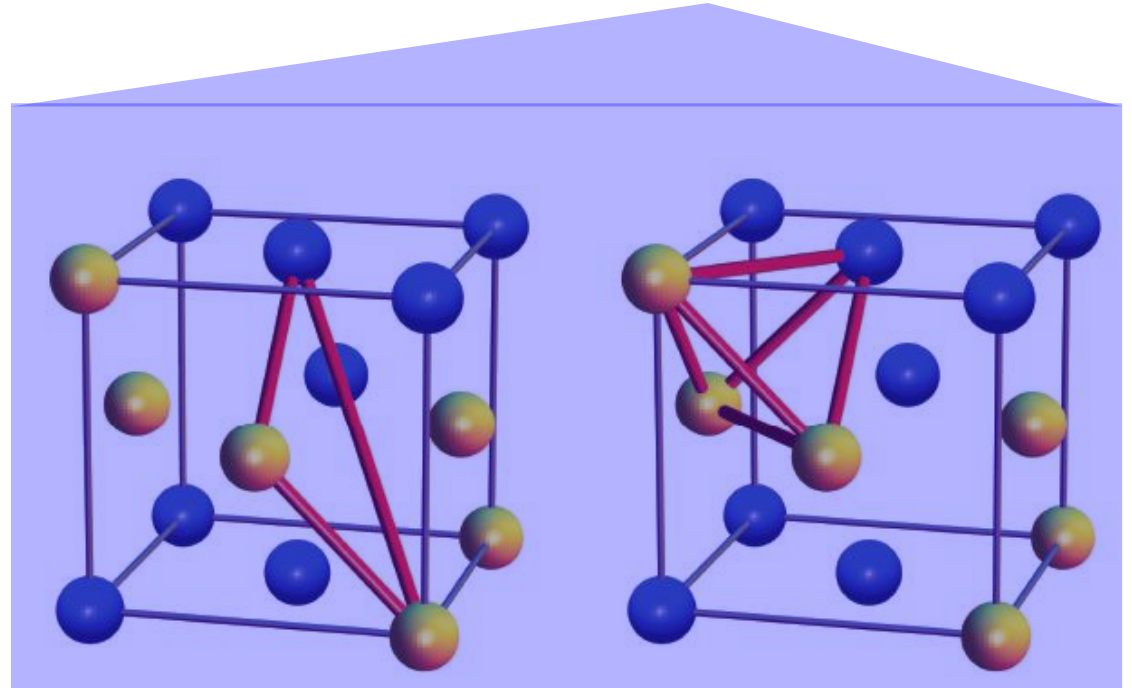
$$S_{el} + S_{mag} + S_{vib} + S_{conf}$$



# Entropy Contributions

$$F(T, V, N) = E_0(V, N) - TS$$

$$S_{el} + S_{mag} + S_{vib} + S_{conf}$$



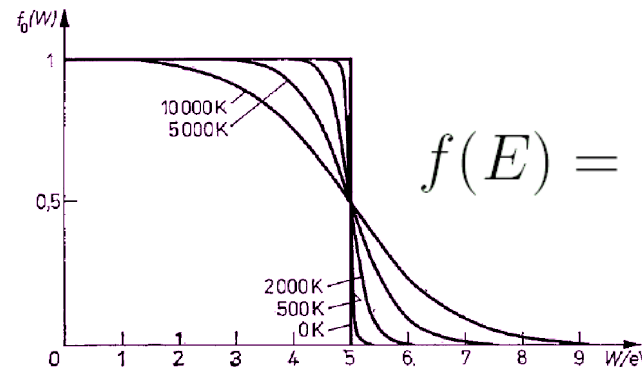
# Electronic Entropy

**Electronic entropy:** easy to evaluate from DFT, but usually small contribution to the free energy

$$F^{\text{el}} = E^{\text{tot}}(T) - TS^{\text{el}}$$

$$S_{\text{el}} = - \int dE N(E) [f(E) \ln f(E) + (1 - f(E)) \ln(1 - f(E))]$$

Density of States  
(DOS)



$$f(E) = \left[ 1 + e^{\beta(E-\mu)} \right]^{-1}$$

# Vibrational Entropy

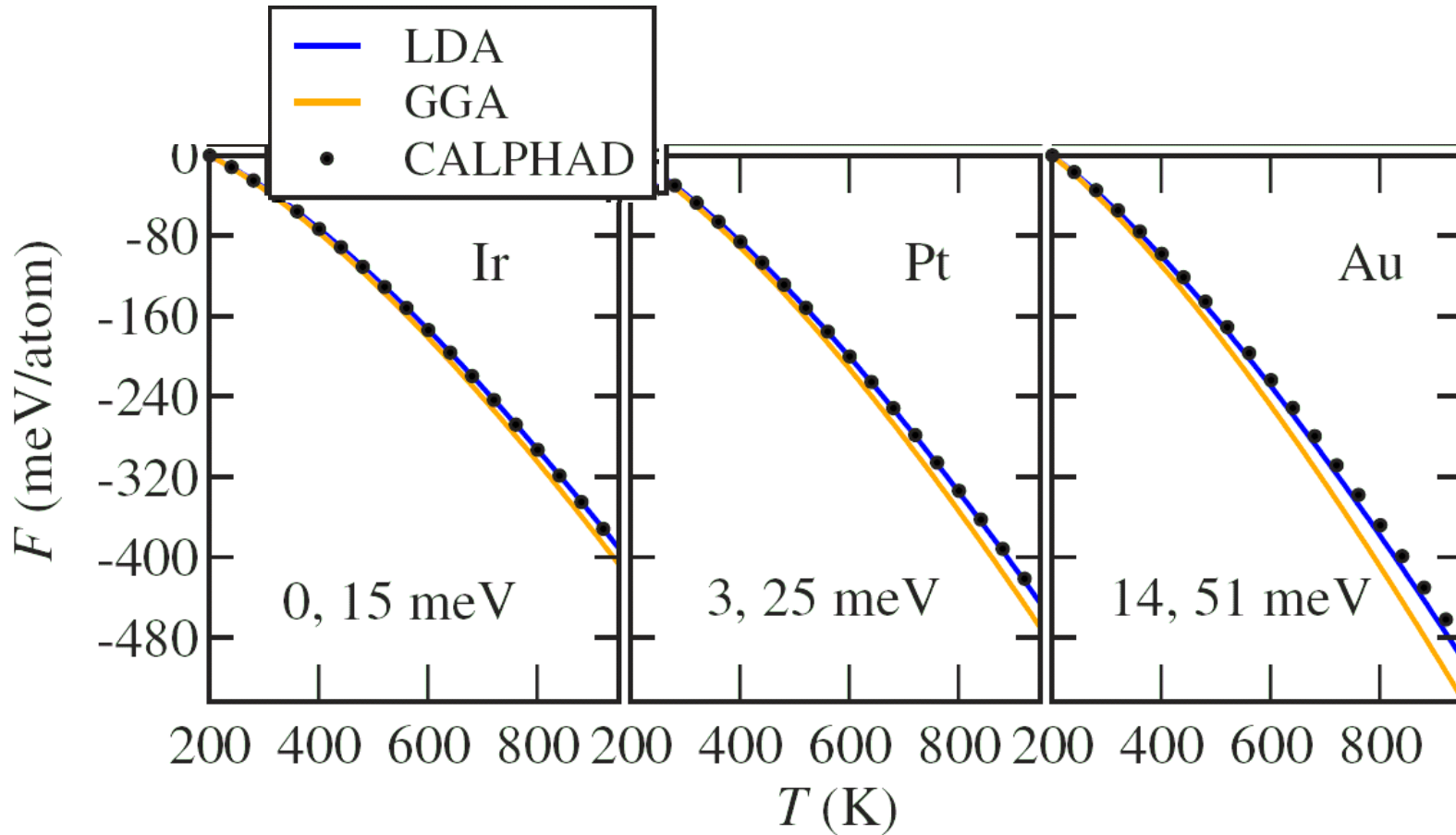
**Vibrational entropy:** Phonon density of states is required which is usually evaluated with the quasi-harmonic approximation. Important for thermal expansion and phase equilibria.

$$F^{\text{vib}} = \frac{1}{N} \sum_i^{3N} \left\{ \frac{1}{2} \hbar \omega_i + k_B T \ln \left[ 1 - \exp\left(-\frac{\hbar \omega_i}{k_B T}\right) \right] \right\}$$

↑  
sum over all  
degrees of  
freedom

↑  
phonon  
frequencies

# Free Energy



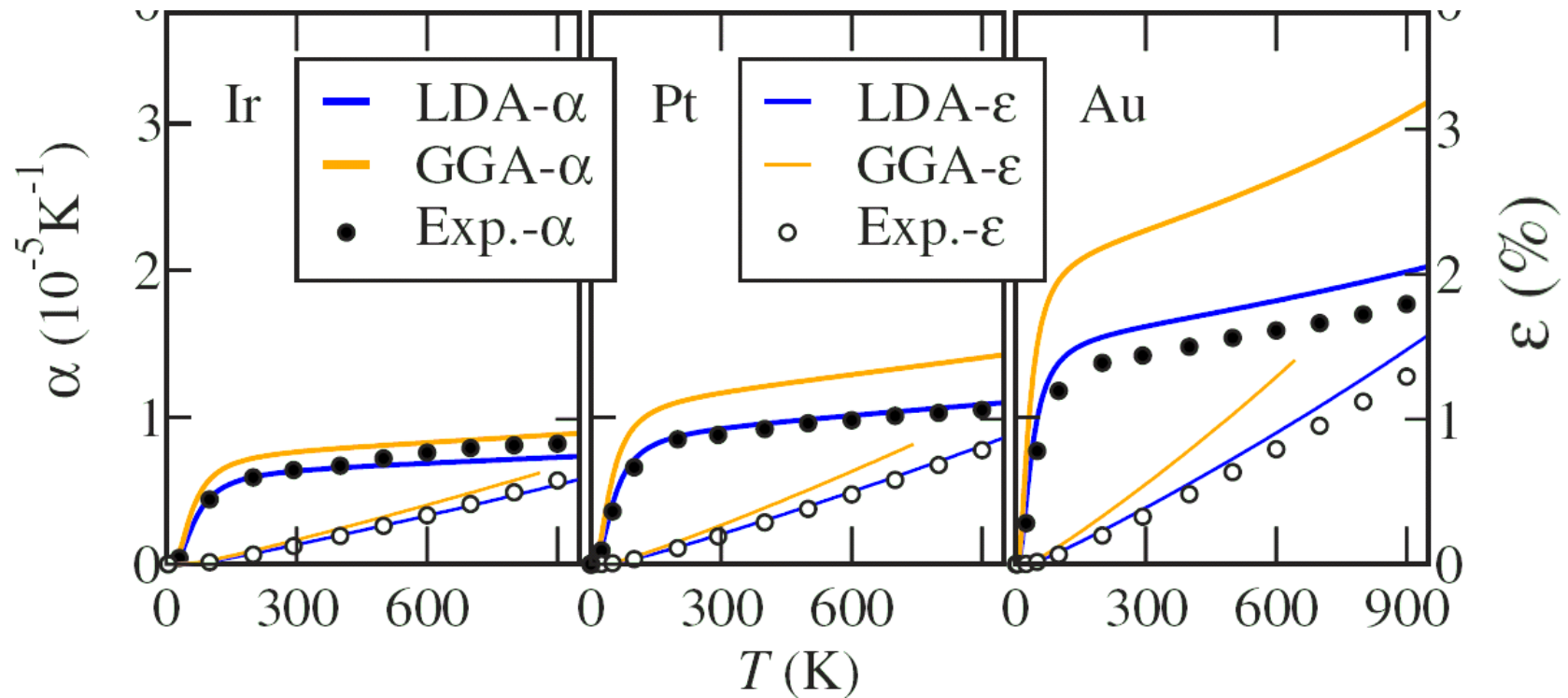
DFT-calculated free energy  $F$  for Ir, Pt, Au

Graphs are taken from Grabowski et al, *Phys. Rev. B* **76**, 024309 (2007).

## 3.6 Thermodynamic Properties

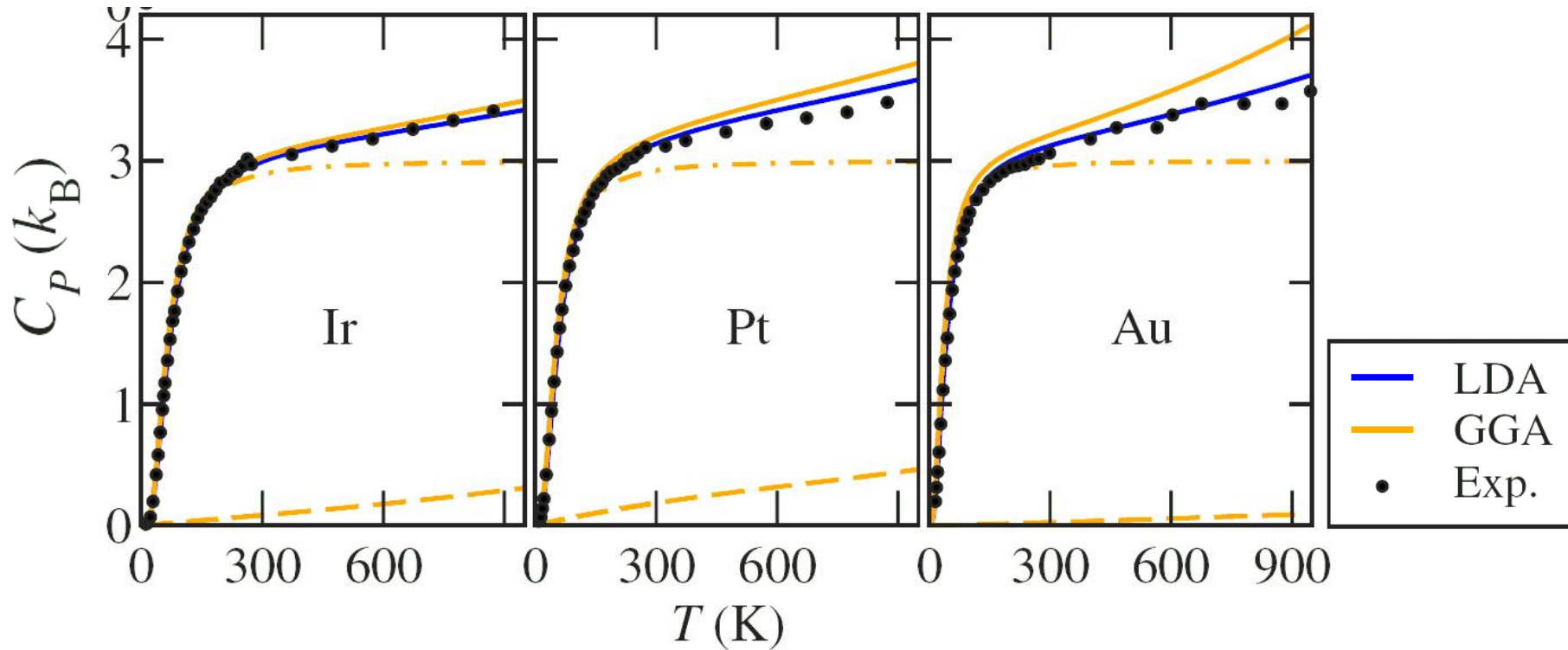


# Thermal Expansion



DFT-calculated linear thermal expansion,  $\epsilon$ , and the thermal expansion coefficient,  $\alpha$ , for Ir, Pt, Au  
Graphs are taken from Grabowski et al, *Phys. Rev. B* **76**, 024309 (2007).

# Heat Capacity



DFT-calculated specific heat capacity  $C_p$  for Ir, Pt, Au

Graphs are taken from Grabowski et al, *Phys. Rev. B* **76**, 024309 (2007).

# What can go wrong?

## I. Total Energy Related:

- a) Typical error in equilibrium geometries: 2-5% (GGA) or 5-10% in LDA
- b) No Van-der-Waals Bonding in LDA or GGA

## II. Band Structure Related:

- a) Band Gaps of Semiconductors / Insulators are too small (50 %)
- b) Band width of localized states (e.g. d- or f-bands) too large in LDA / GGA
- c) Wrong relative order of electronic states
- d) LDA or GGA yield metallic state although real material is insulator

# Van der Waals Forces

Van der Waals force is the sum of the attractive or repulsive forces between molecules other than those due to covalent bonds, the hydrogen bonds, or the electrostatic interaction of ions with one another or with neutral molecules. It includes:

- (1) force between two permanent dipoles (ok in LDA/GGA)
- (2) force between a permanent dipole and a corresponding induced dipole (ok in LDA/GGA)
- (3) force between two instantaneously induced dipoles (London dispersion force)**

It is also sometimes used loosely as a synonym for the totality of intermolecular forces. Van der Waals forces are relatively weak compared to covalent bonds, but play a fundamental role in fields as diverse as supramolecular chemistry, structural biology, polymer science, nanotechnology, surface science, and condensed matter physics.

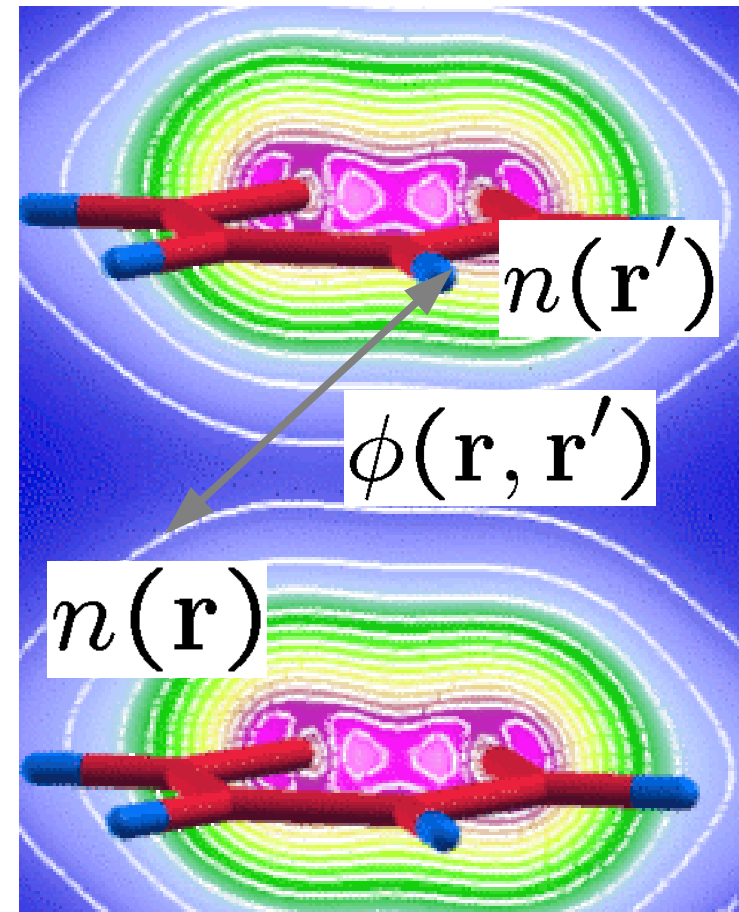
**How Geckos Stick on der Waals:**

<http://news.sciencemag.org/sciencenow/2002/08/27-02.html>



**Non-overlapping densities:**

**Dispersion forces arise from a truly non-local correlation effect**



# Van-der-Waals Density Functional

Total energy in DFT:

$$E[n] = T_s[n] + V_{ee}[n] + V_{ion}[n] + E_{xc}[n]$$

Exchange-Correlation in vdW-DF according to Langreth-Lundqvist (Phys. Rev. Lett. 92, 246401 (2004)).

Exchange in GGA

$$E_{xc}[n] = E_x^{\text{GGA}}[n] + E_c^{\text{new}}[n]$$

Correlation split into local and non-local part (nl)

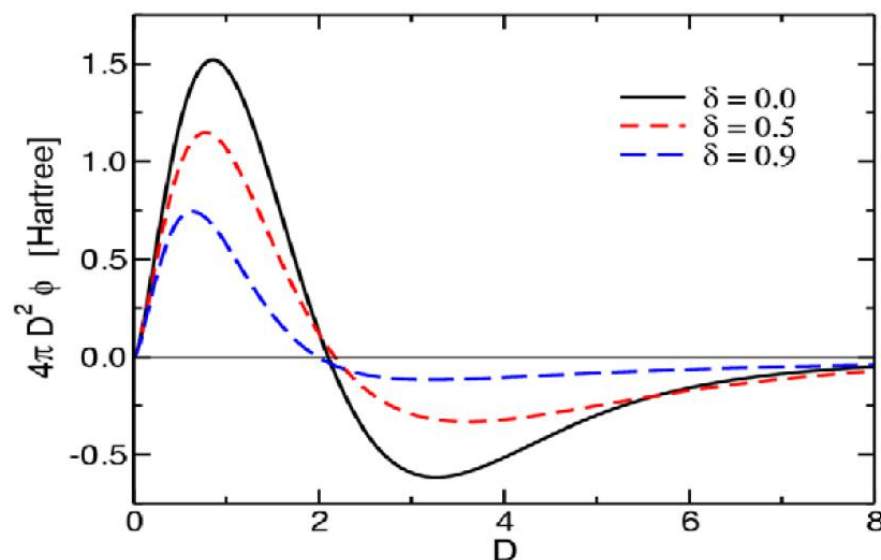
$$E_c[n] = E_c^o[n] + E_c^{\text{nl}}[n]$$

Non-Local Correlation Energy:

$$E_c^{\text{nl}} = \frac{1}{2} \int d^3r \int d^3r' n(\mathbf{r}) \Phi(\mathbf{r}, \mathbf{r}') n(\mathbf{r}')$$

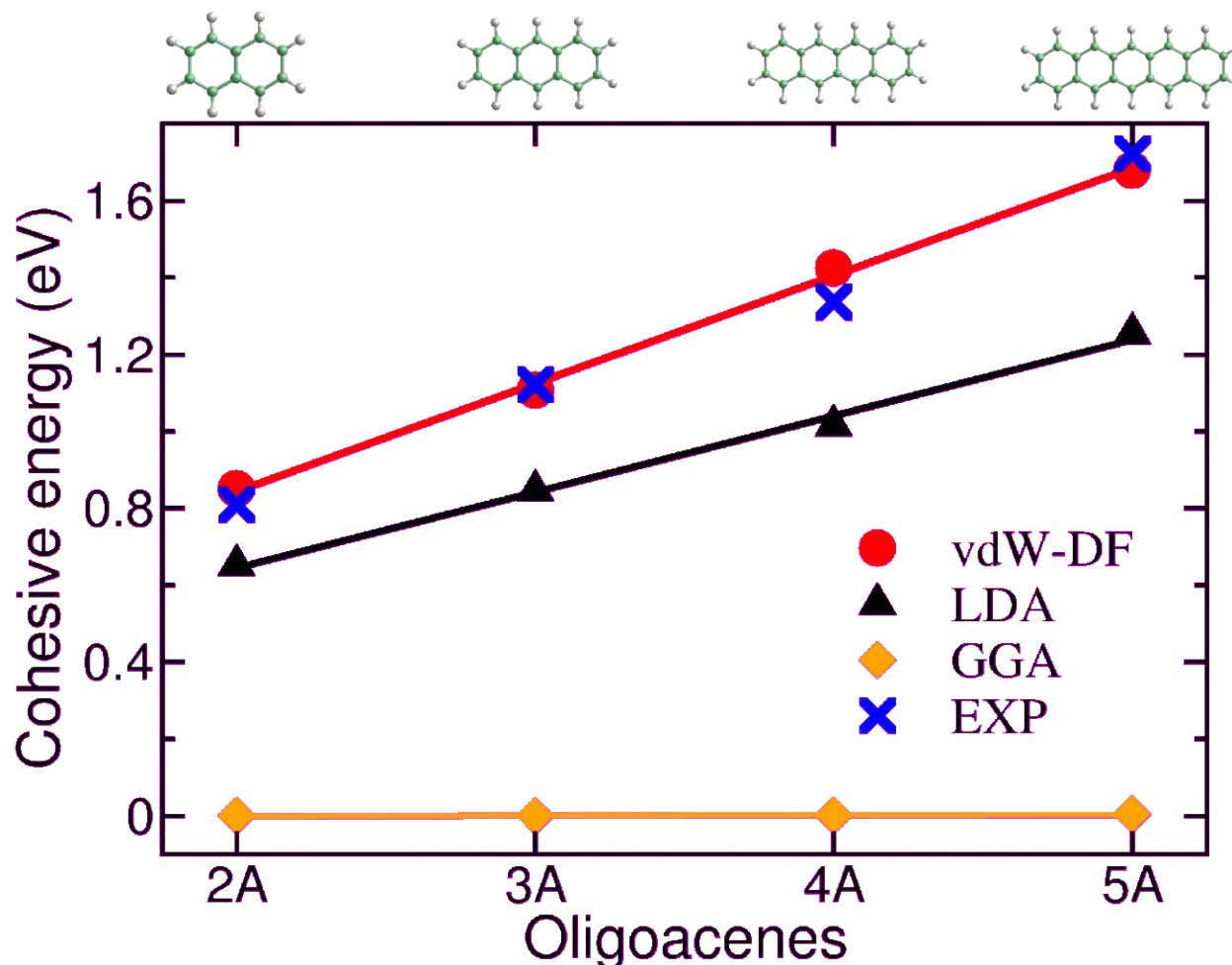
where  $\Phi(\mathbf{r}, \mathbf{r}') = \Phi(q(\mathbf{r}), q(\mathbf{r}'))$ . The interaction kernel depends on the density and its gradient,  $q(\mathbf{r}) = q(n(\mathbf{r}), \nabla n(\mathbf{r}))$ , and can be given a scalable form via

$$D = \frac{q + q'}{2} |\mathbf{r} - \mathbf{r}'|, \quad \delta = \frac{1}{2} \frac{q - q'}{q + q'}$$
$$q = q(\mathbf{r}), \quad q' = q(\mathbf{r}').$$



Langreth et al., A density functional for sparse matter J. Phys.: Condens. Matter 21, 084203 (2009).

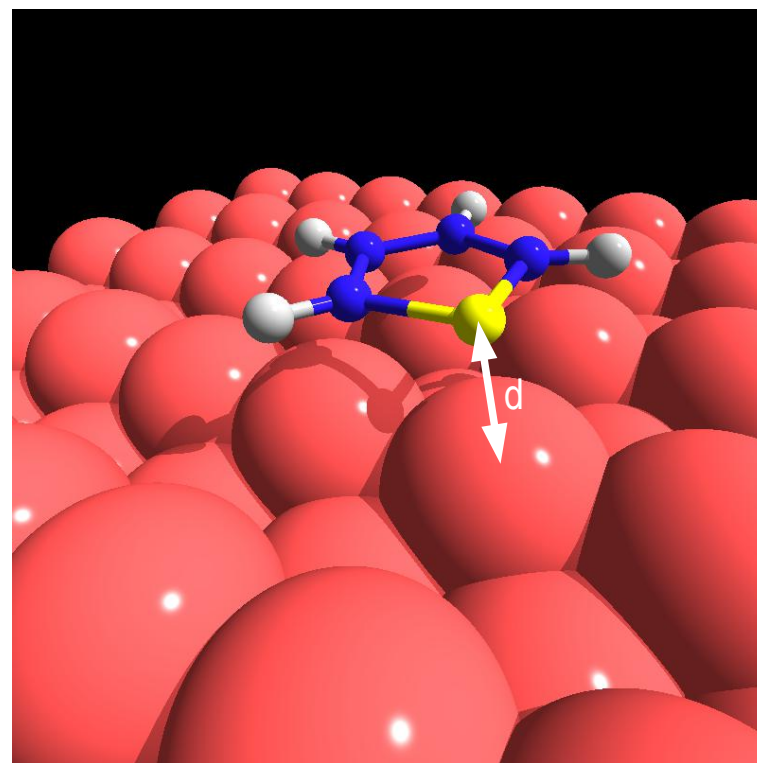
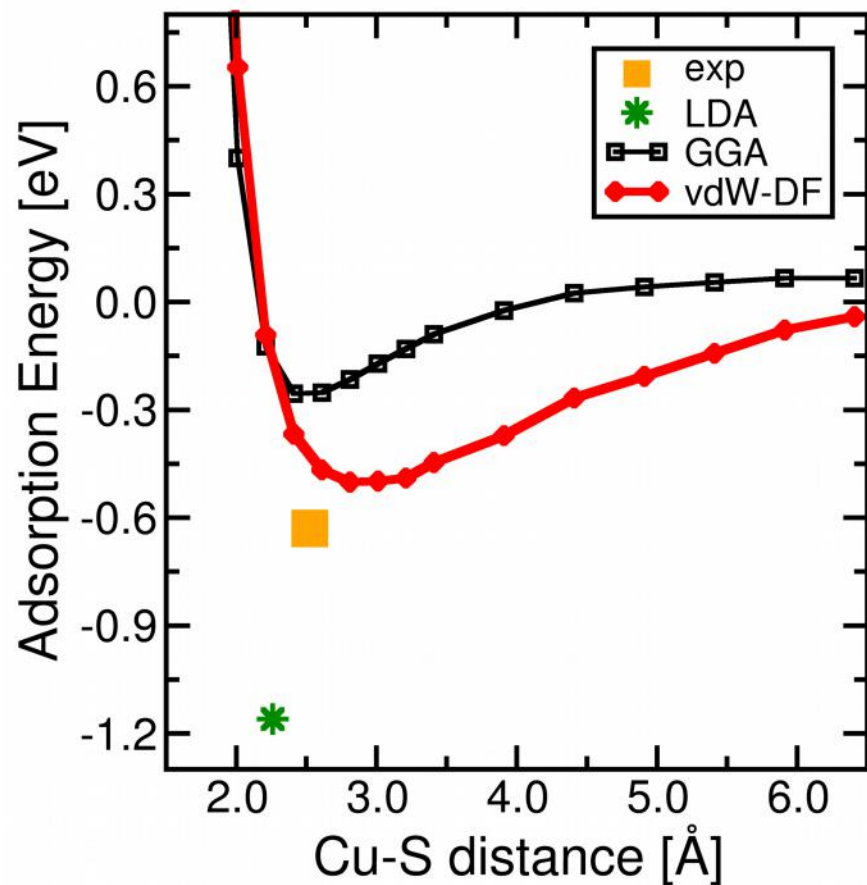
# Cohesive Energy of Molecular Crystals



Nabok, Puschnig, Ambrosch-Draxl, *Phys. Rev. B* **77**, 245316 (2008).



# Thiophene Molecule / Cu(110)



Thiophene@Cu(110): Sony, Puschnig, Nabok, Ambrosch-Draxl, *Phys. Rev. Lett.* **99**, 176401 (2007).

PTCDA@Cu,Ag,Au(111): Romaner et al., *New. J. Phys.* **11**, 053010 (2009).

# Alternative Approaches

## Ab-initio vdW-Density Functional Theory

### Theory

Dion et al, *Phys. Rev. Lett.* **92**, 246401 (2004).  
Thonhauser et al., *Phys. Rev. B*, **76**, 125112 (2007)  
Vydrov et al., *Phys. Rev. Lett.*, **103**, 063004 (2009)  
Lee et al. *Phys. Rev. B* **82**, 081101 (2010)

### Efficient Implementations

Roman-Perez et al. *Phys. Rev. Lett.* **103**, 096102 (2009).  
Vydrov et al., *J. Chem. Phys.* **132**, 164113 (2010).  
Lazic et al. *Comp. Phys. Commun.* **181**, 371 – 379 (2010).  
Nabok et al. *Comp. Phys. Commun.* **182**, 1657-1662 (2011).

### Applications

... huge number ...

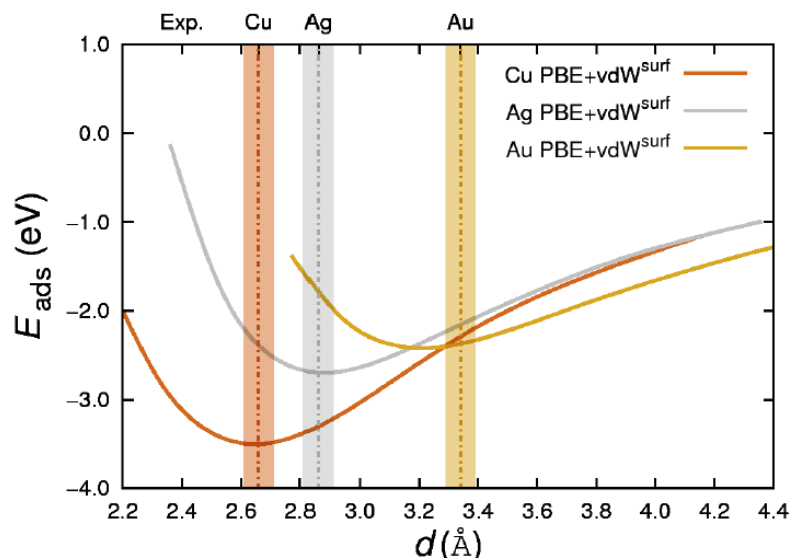
Brief Review see e.g.:  
Langreth et al., *J. Phys.: CM* **21**, 084203 (2009)

## Semi-Empirical Correction

Grimme, *J. Comput. Chem.*, **25**, 1463 (2004).  
Grimme, *J. Comput. Chem.*, **27**, 1787 (2006).  
Tkatchenko et al. *PRL* **102**, 073005 (2009).

Ruiz et al. *PRL* **108**, 146103 (2012):

### PTCDA / Coinage-Metal(111)-Surfaces



# Alternative Approaches

## Adiabatic Connection Fluctuation-Dissipation (ACFD)

*Theory*