Consonance and Dissonance in Theory, Practice and Science
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(Read by Graham Hair, accompanying Powerpoint by Richard Parncutt)

At last year’s Conference on Interdisciplinary Musicology in Sheffield we presented a paper on consonance and dissonance in music theory and psychology which is now in press with the journal of interdisciplinary studies. Today we will summarise this revised concept and invite four colleagues, Johanna Devaney, Valerie Ross, Emery Schubert and Rytis Ambrazevicius, to comment on it.

The project began when Richard asked me to respond to his scientific approach to consonance and dissonance, which we abbreviate to C/D, from my point of view as a composer, performer and occasional theorist. Scientific approaches to C/D include those of Helmholtz, Stumpf, Cazden, Leonard Meyer, Terhardt and Krumhansl. We compare these with the composer’s viewpoint of James Tenney in his 1988 book “The history of consonance and dissonance”. We will also consider relevant writings of Heinrich Schenker and Arnold Schoenberg, and more recently of Karol Berger, Lawrence Dreyfus, and Pieter van der Merwe.

Tenney (1988) identified five different C/D concepts or CDCs. Four of them refer to individual sonorities, which we call vertical C/D. The exception is CDC-1, which applies to relationships between successive tones in unaccompanied melody – octaves, fifths and fourths – which we call horizontal C/D. These can be explained by tunability based on ratios of string or pipe lengths in musical instruments, consistent with Wolfe and Schubert (in press), who argued that musical scales comprising discrete pitch categories must have arisen from the constraints of tunable musical instruments. We will argue that CDC-1 also involves perception, in particular something called pitch commonality, which we will return to later.

In this paper we will push two related ideas. First, in a humanities approach, C/D has many different aspects. This inherent complexity tends to be neglected in scientific approaches. Second, C/D is primarily about music, not sound. It is originally and fundamentally a holistic idea that applies to extended musical passages rather than individual sonorities. This is true both in music theory, for example when theorists analyse extended passages, and in music psychology, where in an ecological approach we are interested in perception in real musical contexts.

The main contribution of Helmholtz to music theory was his theory of roughness. We experience roughness when we play C and Db together on the piano keyboard. Frequencies that are more than a few semitones apart can be clearly separated by the ear, but at smaller intervals the ear has trouble. Roughness is the auditory analog of rubbing your fingertips across a piece of sandpaper. The grains of sand
correspond to fluctuations in the amplitude envelope (beats), which cannot be individually heard at frequencies above about 20 cycles per second.

Terhardt analysed C/D into roughness and “harmony”. The origin of roughness is physiological, but harmony, in Terhardt’s concept, is learned from exposure to speech and music. Specially, he assumed that harmony is based on the ability to recognize harmonic patterns of audible partials, which also allows us to perceive a pitch at the fundamental of a harmonic complex tone. In his theory this ability is acquired by exposure to speech. Terhardt’s “harmony” concept overlaps with Stumpf’s “fusion” and Riemann’s “harmonic function”.

A theory of the C/D of individual sonorities that is based on roughness and fusion can explain central aspects of musical structure. Why, for example, are major and minor triads so important in common-practice tonality? Of all sets of three pitch classes, major and minor triads are the only ones to combine low roughness with high fusion. It’s as simple as that. If this explanation is correct, it renders all competing theories such as harmonic dualism obsolete.

A similar approach can explain why music theorists from the Middle Ages to the 19th Century regarded the P4 interval as much more dissonant than the P5, and the m6 as much more dissonant than the M6, which contradicts a Pythagorean approach based on number ratios. We propose that the root of an isolated sonority is determined by a combination of interval relationships and voicing. The interval relationships come from the harmonic series. If we assume octave equivalence, the first ten harmonics lie at five intervals above the fundamental: P1, P5, M3, m7 and M2. These intervals also determine the roots of musical chords (a simple algorithm has been presented and tested elsewhere). Regarding voicing, other things being equal, the lowest tone is more likely to be the root. This theory predicts roots not only for chords of three or more tones, but also for isolated harmonic intervals. The vertical P4 and m6 are the only intervals in the chromatic scale whose root is often perceived in the upper voice. Perceptual fusion, and hence consonance, is reduced when the upper and lower tones of a harmonic interval compete for the listener’s attention.

Chord progressions are also emotional. If they were not, we would not care about them. Leonard B. Meyer suggested that as we listen to music we are constantly expecting given structural continuations. These expectations may be either fulfilled, which may evoke positive emotion such as the feeling of success, or denied, which may produce negative emotions such as disappointment, frustration or yearning.

Many different structures can act as implications or realizations. A rising leap in a melody can imply a falling step, because rising leaps are often followed by falling steps. Harmonic implications may be driven by the bass line in both its melodic shape and harmonic foundation. In this way, the implication-realisation idea can explain the
feeling that tonal music "progresses". It can also explain the different functions of authentic and interrupted cadences.

Meyer's idea of implication and realization is compatible with our approach to C/D because it balances music-theoretical and music-psychological ways of thinking. It also enables C/D to be contextualized within a broader theoretical treatment of musical emotion. We have incorporated it into a theory of chord progression based on the pitch commonality of successive sounds.

Pitch commonality is the degree to which two successive sounds have pitches in common, independent of context. By "pitch" we mean a subjective experience. Pitch is not physical frequency, nor is it a note in a score. The pitch that we experience at the missing fundamental is a pitch in this sense. It is not a physical frequency and often not a musical note either. So pitch commonality is not the same as the number of notes or frequencies in common.

The theory of pitch commonality aims to explain the origin of harmonic relationships between successive tones and chords. For example, the chords C major and D minor have moderate pitch commonality but no common notes. A C major triad has missing fundamentals at D, F and A, while a D minor triad has missing fundamentals at G and Bb. So the two chords have pitches D, E, F, G and A in common.

The chord CEG implies A, because E corresponds the 3rd harmonic of A, and G corresponds to the 7th. CEG also implies F, of which C is the 3rd harmonic and G is the 9th. CEG also implies D, of which C is the 7th harmonic and E is the 9th. Repeating the same procedure for D minor, DFA implies Bb, of which F is the 3rd harmonic and D is the 5th. It implies also G, of which D is the 3rd harmonic, F is the 7th, and A is the 9th.

In this process, the auditory system tolerates quite large mistunings between partials and fundamentals. For example, the difference between the so-called natural seventh with a ratio of 7:4 and the equally tempered minor seventh is about a 1/3 semitone. We know from psychoacoustical experiments that mistunings of this order have little effect on the salience of derived virtual pitches. So enharmonic spellings are also irrelevant. It doesn't matter whether a pitch is notated C# or Db, and it doesn't matter whether a pitch is an audible partial or a missing fundamental. Pitch commonality depends only on those pitches that fall in the same semitone categories, and their perceptual salience.

The theory of pitch commonality is an attempt to answer simple music-theoretic questions such as: Why do successive isolated tones spanning octave, fifth and fourth intervals sound more related than successive tones spanning major seventh or tritone intervals? Why is a C-major triad perceived to go with a D-minor triad although...
the two have no tones in common? Why are the chords C7 and F#7 considered harmonically remote although they have two tones in common?

The theory can also explain why a C major triad more often progresses to F major than to G major, and why it more often progresses to A minor than to E minor. In general, falling fifths and thirds between roots are more common than rising. Meyer’s theory of implication and realization offers a possible explanation. Chord progressions are preferred in which missing fundamentals in the current chord are realized in the following chord. So if a C chord implies the pitch F, there is a tendency for F to be included as a note in the following chord.

This implication-realization idea is not merely theoretical and perceptual - it also about performance. Chords with high pitch commonality are easier for choristers to sing in tune. If the next tone in a given voice is a missing fundamental in the current chord, it is easier to imagine its pitch in advance of singing it.

In all such arguments we must consider the nature-nurture dichotomy. We may simply have learned these relationships through exposure to tonal music. But that does not explain why the music became like this in the first place. Pitch commonality can contribute to an explanation both of the origin of familiar musical structures and their real-time perception.

Pitch commonality may also be applied to the relationship between a sound and its context. The consonance of a chord in context depends not only on its vertical consonance in isolation but also on the pitch commonality between the chord and its context. This can be modeled mathematically as the correlation coefficient between the pitch profile of the chord and the tone profile of the local key according to Krumhansl. In this way the concept of pitch commonality can be expanded to include interkey relationships.

Music in C major moves more often and effortlessly toward the key region of G major than to F major. Generally, modulations to flat-side keys seem more harmonically distant than modulations to sharp-side keys. Cuddy and Thompson (1992) confirmed this asymmetry in an empirical study. The asymmetry can be explained if we assume that flats relative to a given key signature sound more salient than sharps, because they tend to lie at major third and perfect fifth intervals below the tones represented by the key signature, which gives them more perceptual salience. That can also explain why sharps relative to a key signature are more common than flats.

In general, the global C/D of a chord progression involves both the vertical C/D of individual sonorities and the horizontal C/D of their relationships to context. Vertical C/D has two components, roughness and fusion. Horizontal C/D also has two components, pitch commonality and linear pitch proximity. Linear pitch proximity is about voice leading: in good voice leading, pitch proximity tends to be high. There
seems to be a trade-off between pitch commonality and pitch proximity: If the pitch commonality between successive sonorities is high, their pitch proximity can be low, and vice-versa. So for example it is more important to adhere to conventions of voice leading in complex chromatic progressions than in simple diatonic ones.

A theory that separates C/D into different aspects can be used to characterize different musical styles. In Renaissance and Baroque polyphony and classical tonality, as well as modern popular music, sonorities are predominately major and minor triads, which have high fusion and low roughness. Harmonic progressions are often confined to diatonic scales or are otherwise strong (high pitch commonality) and each voice tends to move to the closest pitch in the next chord (high pitch proximity). Exceptions can be found in music of Gesualdo and Liszt, in which triadic chord progressions proceed in unexpected directions (low pitch commonality). The chromatic tonal music of the late 19th and early 20th Centuries, from Wagner to Debussy, explored more dissonant sonorities (medium fusion and smoothness). The atonal music of Schoenberg and his followers tended to disregard or systematically avoid all so-called natural aspects of consonance including fusion, smoothness, pitch commonality and in some cases even pitch proximity.

One of the most interesting tonal developments of the 20th Century was the emergence of bebop harmony during the 1950s. Its complex and often bitonal chords fuse to a medium degree; fusion is an important element of the style, but is compromised by harmonic complexity. Roughness is either irrelevant or desirable, and hence tends to be high (i.e. smoothness is low). Harmonic relationships between successive chords and between each chord and the prevailing tonic are perceptible but complex, so pitch commonality is medium. Linear pitch proximity is high, which compensates for other forms of dissonance.

We now turn to humanities approaches. Many ideas from music theory, history, philosophy and practice have a bearing on these scientific ideas.

First, concepts of C/D have changed throughout music history, as explained by James Tenney but also by Karol Berger, Pieter van der Merwe, and Lawrence Dreyfus. We are inheritors of particular historically contingent ways of thinking about consonance and dissonance.

Second, C/D are experienced holistically. That suggests that the idea of creating “a harmonious whole” in a piece of music involves more than merely constructing chords and chord progressions. The idea of creating “a harmonious whole” might be related to the idea that a work should exemplify an integrated totality or a unified organism. Schenker and Schoenberg agreed on this point but talked about it in very different ways. The goal of creating an organic whole suggests that factors additional to C/D might be involved, for example motivic relationships.
Before the Enlightenment, the idea of a judicious and balanced use of C/D to create a harmonious whole was the basis of the craft of harmony. According to Berger,

By 1825, the understanding of harmony seems already to have been reduced to its narrow, modern sense ... the craft of constructing chords and chord progressions ... [But earlier, in] a somewhat broader, still musical-technical, sense harmony also included the counterpoint, the craft of combining diverse, simultaneous melodic lines. In its broadest sense the audible harmony produced by musicians participated in the intelligible harmony of creation. (Berger, 2007, pp. 121–122)

The locus classicus of this pre-Enlightenment attitude is the music of Bach, as exemplified with particular force in his fugues. Berger describes fugue as “Bach’s demonstrations of what can be done with the subject contrapuntally. In the C-major fugue there are seven such demonstrations, each designed to show how the subject can be combined with itself in imitation ... ”

In the C-major fugue from the first book of Bach’s Well-Tempered Keyboard, there are 22 complete statements of the subject, pitched on all seven degrees of the C-major scale. The fugue begins with the traditional series of statements (subject, answer, answer, subject) on C, G, G and C respectively. In the following series of statements, there are (as Berger asserts) seven demonstrations of “how the subject can be combined with itself in imitation”. The pairs vary in voice combinations and intervals between entries. Limited by the time-frame of a typical fugue, Bach presents only a selection from a large number of theoretically-possible combinations.

A fugue is an open-ended chain of demonstrations, judiciously balancing various different types of combinations. It is a paratactical form - an assortment of compatible things of equal importance. By contrast, hypotactical forms focus on the build-up and release of harmonic tension, and on a heightened sense of drive towards cadence-points; a typical example is sonata form.

Parataxis also imposes a particular means of ending of a fugue, as Berger notes:

A reader of Bach’s two sets of preludes and fugues The Well-Tempered Keyboard (WTC) will be struck by the emphatic gestures by which the composer often announces the approaching end of the fugue ... For the greater part of its duration it is impossible to predict when or how soon the fugue will come to an end. Then quite suddenly ... it becomes apparent that Bach is wrapping things up ... Because the nature of the genre is essentially atemporal, because one never knows in advance how many demonstrations there will be or in which order they will be introduced, the end is in danger of seeming arbitrary and abrupt. Hence the need for
emphatic gestures to announce that the end is imminent (Berger, 2007, pp. 89–91)

This pre-Enlightenment atemporal conception of a harmonious whole is also embodied in the dance forms of the Baroque and earlier periods, whose ubiquitous antecedent-consequent phrase pairs often comprised a phrase or section cadencing on the dominant (dissonance) answered by an antithetical phrase or section cadencing on the tonic (consonance).

This holistic attitude to composition can be compared and contrasted with the post-Enlightenment conception of harmony, focusing more narrowly on the contrast between dissonant and consonant chords and chord-progressions, on the build-up and release of harmonic tension, and on a heightened sense of drive towards cadence-points. Berger writes:

One of my central claims ... is that at some point between the early and late eighteenth century, between Bach and Mozart, musical form became primarily temporal ... earlier phases must not only precede but also cause the appearance of later ones ... one-after-another must become one-because-of-the-other (Berger, 2007, pp. 179-180)

Pieter van der Merwe explained the historical shift with a linguistic analogy:

One of the fundamental changes in nineteenth-century music is that climax gradually replaced antithesis as the chief organising principle. I must immediately add here that the word “climax” is here used in its original [dictionary] sense of “a figure in which a number of propositions or ideas ... [are] ... set forth so as to form a series in which each rises above the preceding in force or effectiveness of expression ...” the stock example being Caesar’s “I came; I saw; I conquered” (Van der Merwe, 2004, pp. 311–312)

Climax is a hypotactical relation. In contradistinction to a paratactical relation, it implies materials that are primary or subordinate relative to each other, and a cumulatively- and hierarchically-ordered sequence of things (Berger’s “one-because-of-the-other”). Berger devotes a chapter (“Mozart at Play”) to illustrating hypotactical form by analysing the hierarchy of cadences in the Allegro of the Mozart Concerto in C minor, K491, in order to show how the cumulative effect of which Van der Merwe speaks is achieved (Berger 2007: 197–198).

We have already referred to the concept of musical organicism and its most influential early 20th-century protagonists, Schenker and Schoenberg. The ambition of organicism is to create works in which everything is an outgrowth of a basic (work-specific) idea. Organicism gives a musical work unity (including “the unity of musical
space”; Schoenberg 1975: 223), which – in the broadest sense – could be considered an aspect of consonance. From a psychological viewpoint, organicism can only be perceived if recent musical events are perceived to be similar to earlier musical events that have been stored in memory.

In the aforementioned 19th-century examples, organicism refers to melodic and rhythmic shapes and their evolution. Schoenberg’s radical step was to include harmony in this scenario (via “unity of musical space” – the companion principle to “emancipation of dissonance”). Again, 19th-century examples might be interpreted as precursors. The Tristan chord (and the idea of leitmotif in general) can be construed as a move in the organicist direction, but it is also a functional (pre-dominant) dissonance. A totally organicist piece might treat the Tristan chord exclusively in terms of its unique intervallic characteristics, as Schoenberg does with motifs (melodic, rhythmic and harmonic) in his later atonal pieces. The focus on intervallic content culminated in Forte’s (1973) categorisation of all the 234 possible chords according to their unique intervallic characteristics with no mention of C/D; Carter (2002) proceeded along similar lines, sweeping thematic and harmonic entities under a single (motivic) umbrella, combining emancipation of dissonance with unity of musical space.

For the later Schoenberg, the creation of a harmonious whole meant the creation of a comprehensively integrated totality, based on a limited pool of work-specific motifs (melodic, rhythmic and harmonic) presented in hundreds of guises. One can see why dissonance had to be emancipated to achieve this organicist utopia. In the world of unemancipated dissonance, a motif presented melodically might be capable of becoming a harmony (e.g. if it were triadic), but not if it were a Bartókian motif of (say) 6 adjacent semitones. Totally dissonant harmony was the engine by means of which thematic and harmonic domains could be unified into an organic whole.

But this brings us to a paradox: a harmonious whole in Schoenberg’s sense needed totally dissonant harmony for its realization. Schoenberg disregarded traditional rules for the treatment of dissonance, but not arbitrarily, because his combination of a motivic sense of harmony and the unity of musical space could only be achieved if dissonance was “emancipated” from its role in functional harmony. This paradox forces us to consider the extents and limits of our C/D-dichotomies because it shows that one interpretation of the same general concept of a harmonious whole can conflict with another.

Our holistic concept of C/D includes the popular conception of C/D as pleasant/unpleasant and the history of C/D in Western music and music theory. It includes many contrasting terms and references in musical and psychological discourse. We compare and contrast dichotomies that overlap or interact with the C/D concept such as tense/relaxed, primary/subordinate, centric/acentric, diatonic/chromatic, stable/unstable, close/distant, similar/different, rough/smooth,
fused/segregated, related/unrelated, familiar/unfamiliar, implied/realized, tonal/atonal; our perception of these dichotomous pairs often intensifies, parallels or stands in for our perception of C/D. Details of these dichotomies are presented in our paper in press.

We turn now to the concept of tonality. When we talk of “tonality” we tend to bundle together terms such as consonance, diatonicism (use of tones from a diatonic scale, e.g. white piano keys) and centricity (the use of tonal centres or reference pitches) when it might be better to distinguish between them. The same applies to their “almost-opposites”, dissonance, chromaticism and acentricity when talking of “atonality”. Composer Anthony Payne has commented that “My music is not tonal, but I often use tonal centres”. We understand that his music is not diatonic or consonant, but often centric.

We often assume that music which exemplifies a high degree of consonance is likely also to exemplify a high degree of diatonicism and centricity, and that music which exemplifies a high degree of dissonance is likely also to exemplify a high degree of chromaticism and acentricity. Most of today’s popular music exemplifies consonance, diatonicism and centricity as an integrated bundle (the same applies to early Mozart minuets). But a middle-period Bartók piece, while characterised by adherence to tonal centres, also embodies a high degree of dissonance and chromaticism. Moreover, certain passages of “chromatic” harmony in the 16th and early 17th centuries (e.g. celebrated works by Gesualdo) draw on unusual juxtapositions of triads, creating an uncertain or ambiguous sense of tonal centre – at least on the small and medium scale – which led musicologists such as Lowinsky (1961) to describe them as “atonal”. They may be vertically consonant and horizontally dissonant.

In conclusion, C/D is a multi-faceted phenomenon that can best be addressed by bringing together humanities and sciences. A humanities approach acknowledges the richness of detail and refers repeatedly to specific examples. A scientific approach searches for general principles and refers to statistical regularities. We cannot understand C/D without considering both its simplicity and its complexity. We need to consider both our spontaneous “gut reactions” to music and the diversity of possible reactions to the same musical event and the kaleidoscope of ineffable experiences that can accompany music.

Thinkers and researchers of the past have tended to emphasize one or the other aspect of C/D, leading to conflicts or cognitive dissonance. Wikipedia defines cognitive dissonance as:

... an uncomfortable feeling caused by holding conflicting ideas simultaneously. The theory of cognitive dissonance proposes that people have a motivational drive to reduce dissonance. They do this by changing their
attitudes, beliefs, and actions. Dissonance is also reduced by justifying, blaming, and denying (1 Nov 2010; our emphases)

We are similarly motivated to reduce the cognitive dissonance between the traditional approaches of the humanities and the sciences to C/D in order to come closer to its essence. This involves questioning entrenched and cherished attitudes and beliefs (conventional wisdom) of both sides, from generalities about the nature of truth and how it can be acquired right down to specific details of the analysis of musical passages. It involves breaking through hidden walls of justification, blaming and denying, since each side has developed sophisticated verbal means of justifying its own approach, indirectly blaming the other side for its failure to address central issues, and denying that its own limited approach may be part of the problem. We are trying to open up these hidden agendas by acknowledging that a long history of conflict between humanities and sciences exists in this area, and emphasizing that these conflicts tend to be denied or ignored by both sides. Our solution is to place the arguments in the open and constructively to pit them against each other.