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6. Applications

The model described in Chap. 4 predicts the number of audible harmonics in complex tones, the multiplicity and tonalness (and hence consonance) of musical tone simultaneities (tones, dyads and chords), the various possible pitches of simultaneities and their perceptual importances (saliences), and the roots of chords. It also predicts the strength of harmonic and melodic relationships between sequential musical sound (pitch commonality and pitch proximity). It quantifies sensory and cultural aspects of the tonality of chord progressions: repetition, sequential harmonic relationships (including the roots of broken chords), consonance, and implication (of triads, scales and keys). It enables "objective" psychoacoustical analysis of harmonic progressions, and may be applied in composition.

6.1 Simultaneities

6.1.1 Masking

The number of audible harmonics in a complex tone is limited by masking (Sect. 4.3). According to Plomp [1964], Marsenne, in 1636, reported hearing out 7 harmonics from a complex tone; Helmholtz [1863] reported hearing between 12 and 16; and Brandt, in 1861, heard 8 harmonics in gut strings and 13 in brass strings. Schouten [1940] reported 12 audible harmonics. Schenker [1906], in his discussion of the origins of harmony, assumed that only 6 harmonics were clearly audible. These differences are due to variations in physical spectra, in theoretical approach, and in the ability to listen analytically.

Plomp [1964] performed experiments to determine the number of directly audible harmonics in complex tones comprising harmonics of equal sound pressure level (SPL). He found that listeners could reliably distinguish between 5 and 8 harmonics, depending on the fundamental frequency of the tone. According to the masking algorithm of Terhardt [1979a] and of Terhardt et al. [1982b], up to 11 harmonics may be audible in such sounds. Equations (4.12–16) of the present model (Chap. 4) make similar predictions in the case of "typical" musical tones (4.8). These results and calculations are compared with each other in Table 6.1.

Plomp's experimental values are lower than Terhardt's theoretical ones. This is to be expected, as the degree to which a listener can ignore complex tone

Table 6.1. Number of audible harmonics in typical full complex tones

Note	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
P	12	24	36	48	60	72	84	96	108
N_1		7	8	7	6	5	5		
N_2	1	10	11	9	7				
N_3	8	10	11	10	10	10	7	3	1

P: Pitch category
N_1: Measured number of distinguishable harmonics in complex tones of loudness 60 phon and with 12 harmonics of equal SPL [Plomp 1964]
N_2: Calculated number of audible harmonics in complex tones with many harmonics of 60 dB SPL each [after Terhardt 1979]
N_3: Calculated number of audible harmonics, according to (4.16), of full complex tones [spectra specified by (4.8)]

sensations and concentrate only on pure tone sensations by analytical listening is limited. But Terhardt's calculated values lie near the upper ends of Plomp's error bars, indicating that Plomp's listeners sometimes heard all the harmonics Terhardt calculated to be audible.

The masking algorithm described in Sect. 4.3 differs from that of Terhardt [1979a] in three main ways.

(i) Specified values of critical bandwidth at low frequencies in Terhardt's algorithm are too high (Sect. 4.3.1), so masking at low frequencies is overestimated. According to his algorithm, for example, a full complex tone with fundamental frequency 33 Hz (C_1) and harmonics of equal amplitude has only one audible component (the fundamental). Specified critical bandwidths in the present model (4.12) are smaller, but not as small as some published measurements of critical bandwidth (Table 4.1).
(ii) The masking pattern of a pure tone is assumed in Terhardt's model to be triangular when SPL is plotted against pure tone height (critical-band rate). In the present model, the masking algorithm is simplified by assuming that the pattern is triangular when auditory level (level above the threshold of audibility) is plotted against pure tone height.
(iii) The high-frequency side of the masking pattern of a pure tone is normally less steep (in decibels per critical band) than the low-frequency side. This means that low-frequency pure tone components mask high-frequency components more than the other way around. For simplicity, this effect is neglected in the present model, and the magnitudes of the two gradients are set equal (4.14).

It was considered preferable in designing the current model to underestimate masking rather than to overestimate it. If masking is underestimated, sensitivity of calculated pitch properties to changes in overall level is reduced. This makes the model more suitable for music-theoretical applications, in which the overall level (loudness) of a chord is normally unimportant (Sect. 2.2.1). Underestimation of masking also allows for the possibility that slightly subthreshold tone components contribute to the formation of complex tone sensations: residue pitches can sometimes be heard in sounds containing only high, adjacent harmonics, which (due to masking) cannot be heard out separately [Moore and Rosen 1979].

According to the present model, isolated musical tones in the central musical pitch range have ten audible harmonics. Similarly, the harmonic template for the simulation of complex tone perception (Sect. 4.4.1) is limited to ten components. This is consistent with the theory that the template is acquired from experience of complex tones (Sect. 2.4.3).

6.1.2 Spectral Dominance

Spectral dominance [Fletcher and Galt 1950; Ritsma 1967] is an effect by which the precise pitch of a complex tone is most likely to be determined by tone components in a particular range of frequencies. Terhardt's model [Terhardt et al. 1982b] accounts for this effect explicitly by means of a *spectral frequency weight* function with a broad maximum at 700 Hz (F_5). Various versions of this function were tried out in the current model, but none significantly improved the correlation between calculations and the results of the pitch analysis experiment (Sect. 5.3), an experiment designed to be sensitive to effects of spectral dominance on tone saliences. The reason for this is not clear. It could involve the limited frequency range of the sounds used in the experiment, problems with intonation of the probe tones, or inadequacy of the simplified masking algorithm. Whatever the reason, it was clear that the effect of spectral dominance on the salience of pure and complex tone sensations was relatively small, and could safely be neglected for music-theoretical purposes.

The threshold of audibility limits the auditory levels of pure tone components of very low and very high frequency in a way roughly similar to the way in which the spectral frequency weight function modifies the saliences of corresponding pure tone sensations. The threshold of audibility for pure tones [Terhardt 1979a] falls below 10 dB SPL in the range C_4-C_9 (260 Hz to 8.4 kHz); and Terhardt's spectral frequency weight function exceeds 0.8 in the range C_4-C_7 (260 Hz to 2.1 kHz). By neglecting spectral dominance, the current model may place undue emphasis on pure tone sensations in the range C_7-C_9 (one octave either side of the top note of the piano). This range was of negligible importance in the pitch analysis experiment (Fig. 5.3). Some compensation for this effect is provided in musical applications of the model by the falloff in specified auditory levels of pure tone components at high frequencies and high harmonic numbers (4.8).

The most salient tone sensation evoked by a complex tone with a fundamental frequency above about C_6 (1 kHz) is normally the pure tone sensation at the fundamental: the upper components of such a tone do not contribute to its pitch [Flanagan and Guttman 1960]. This is accounted for in

Terhardt's model by a *fundamental frequency weight*. It, too, is neglected in the present model. As a result, the model predicts that the most salient tone sensations of "typical" musical tones in the range C_6-C_8 (the top two octaves of the piano) are complex tone sensations when, in fact, they are pure tone sensations. This is not a serious drawback, as the difference between pure and complex tone sensations is not directly perceptible, and does not directly affect pitch relationships (pitch commonality and pitch distance).

6.1.3 Multiplicity

Calculated multiplicities (Sect. 4.5.1) of selected musical tone simultaneities are shown in Fig. 6.1.

The example of fourth-spaced chords in part (a) of the figure is appropriate: the fourth is a fairly typically sized interval, and fourth-spaced chords spanning less than five octaves contain no octave doublings (which tend to reduce multiplicity). Calculated multiplicity exceeds the number of notes for single notes and dyads. Here, the variable M is best interpreted as a measure of *pitch ambiguity*. The pitch ambiguity of a single complex tone is about two: its pitch may be heard either at the fundamental or at a number of other pitches (especially the upper and lower octaves; see Fig. 6.3 and Terhardt et al. [1986]). In unaccompanied melody and two-part counterpoint, context and streaming (coherent amplitude or frequency modulation of harmonics: [McAdams 1984]) generally indicate the actual number of voices. In three-part music, multiplicity corresponds closely to the actual number of voices, making three perhaps the ideal number of voices for a contrapuntal passage in which all voices are important. Counterpoint in four or more voices is harder to write or listen to, as it is hard to follow (segregate) more than two or three voices

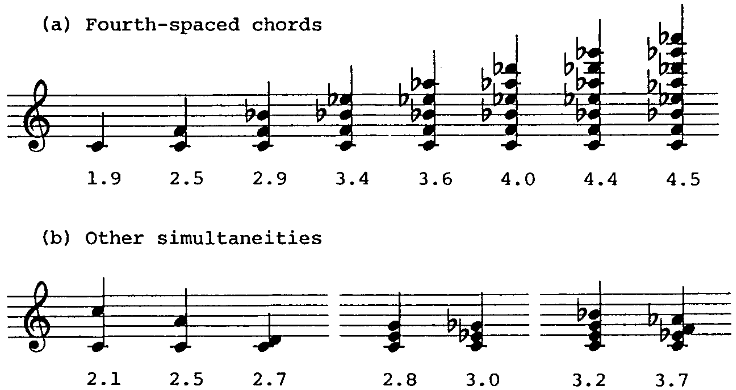


Fig. 6.1. Calculated multiplicity M, (4.24), of simultaneities of full complex tones (4.7, 8)

at once [Bregman and Campbell 1971]. For this reason, only two or three of the voices in a contrapuntal passage such as a Bach fugue are normally active at a given time; the others fill in the harmony.

Of course there is a big difference between picking out the voices of a chord and picking out the melodies of a contrapuntal passage. In the case of a progression, the task is made easier by streaming, i.e. by the horizontal context of each tone sensation. But it is hard to analyze and understand the structure of two melodies simultaneously [Sloboda 1985, p. 167]. The calculated multiplicity of a chord gives only a rough (and relative) indication of the number of separately perceptible melodies to which its tones may belong.

The musical examples in part (b) of the figure show the dependency of multiplicity on the intervals between notes in dyads and chords. Dyads typically have calculated multiplicities in the range 2.1 – 2.7, close-position triads in the range 2.8 – 3.0, and sevenths in the range 3.2 – 3.7. More consonant combinations of notes tend to produce lower calculated multiplicities, as they blend or "fuse" more easily.

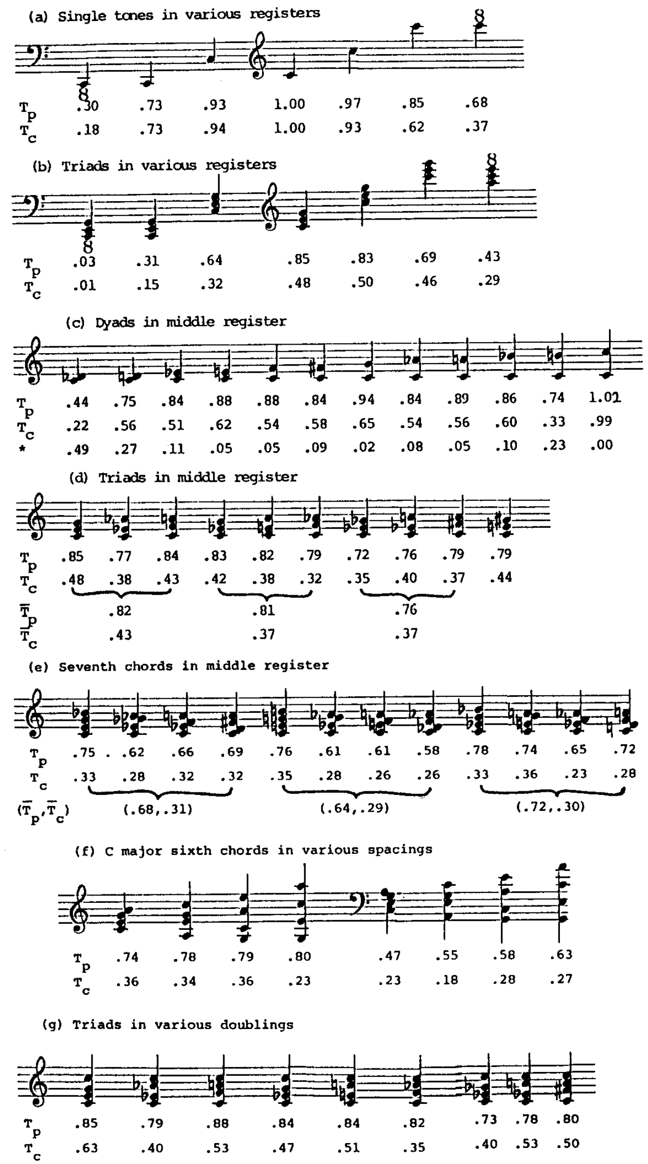
6.1.4 Tonalness

The tonalness of a sound depends on the audibility of its (pure/complex) tone components (Sect. 3.2.2). *Pure tonalness* depends on the number and audibility of a sound's pure tone components; *complex tonalness*, on the audibility of a sound's most audible complex tone component. The two parameters are defined quantitatively in Sect. 4.4.3 in such a way that they both equal one for a "typical" harmonic complex tone at middle C. In most musical applications, the parameters are closely correlated with each other, so that references to "calculated tonalness values" in the following may be regarded as referring to either pure or complex tonalness, or both.

Calculated tonalness values for single musical tones in different registers are shown in Fig. 6.2a. Each tone has 16 harmonics with "typical" auditory levels (4.8). Middle C (C_4) has the highest calculated tonalness. Tones in the middle of the pitch range of music (and speech) have higher tonalness than very high or very low tones. In general, tones with higher tonalness are more common in music than tones with lower tonalness.

Calculated tonalness values for close position triads (Fig. 6.2b) are highest in registers 4 and 5. Values drop sharply in low registers due to masking, which reduces the audibilities of pure tone components. According to the model, the triad in register 1 (the first one analyzed) is almost completely pitchless: only a few pure tone sensations (corresponding to upper harmonics of the tones of the chord) are (barely) audible. This may be confirmed by playing the triad on the piano.

Calculated tonalnesses of dyads (simultaneous tone pairs) are set out in Fig. 6.2c. Calculated roughnesses [Hutchinson and Knopoff 1978] are also listed. The octave has easily the highest calculated tonalness – physically and perceptually, it is almost the same as a single complex tone. The fifth has the



second highest value. The minor second has the lowest tonalness (and the highest roughness, see Sect. 3.2.2). The major second and minor seventh intervals also have quite low tonalness (and high roughness) values. The other intervals vary relatively little with respect to their calculated tonalness. This suggests that, in two-part writing where homogeneity of consonance is desirable, all intervals beside the seconds, the major seventh and the octave may be freely used, including the tritone.

Malmberg [1918] investigated the consonance of dyads such as those in Fig. 6.2c by presenting them to and discussing them with a panel of eight academics (five musicians and three psychologists). The "blending", "smoothness", "fusion" and "purity" of dyads, produced by both tuning forks and the piano, were assessed. The rank order obtained after averaging over these various conditions, from least to most consonant, was m2, M7, M2, m7, T, m3, m6, P4, M3, M6, P5, P8. This order is similar to the order of calculated tonalnesses and roughnesses of the intervals in the figure. It may be concluded that the rank order of consonance of dyads of complex tones within the octave is relatively stable with respect to the way in which consonance is defined (blending, tonalness, etc.) and typical changes in the spectral content of the tones making up the dyads (Malmberg did not specify the spectral composition of the tones used in his experiment).

Figure 6.2d shows calculated tonalnesses for various close position triads in register 4. Values vary relatively little over this set, in accordance with the small range of consonance ratings of these chords given by untrained listeners [Roberts 1986]. The values nevertheless show some consistent and musically sensible trends. Triads including perfect fifth or fourth intervals have higher pure tonalness than triads including tritones or minor sixths (augmented fifths). Major triads have higher complex tonalness than minor triads. Root position major and minor triads usually have higher tonalness than their inversions; the reverse is the case for the diminished triad.

The dissonance of the augmented triad in music theory is not reflected by its calculated tonalness; it appears to have cultural rather than sensory origins. The triad does not fit into "overlearned" diatonic scale patterns (Sect. 3.4.3); it is not part of a commonly used tetrad (whereas the diminished triad is part of the dominant seventh chord); and its root, and therefore its harmonic function in context, is highly ambiguous. Consequently, the triad is relatively uncommon, and unfamiliar. The role of roughness in this argument could be investigated by applying the model of Aures [1984], possibly in an appropriately simplified form.

Figure 6.2e presents calculated tonalnesses of common tetrads (seventh chords). Overall, tetrads are less tonal than triads. Values for different tetrads, like the values for different triads above, cover only a small range. The tetrad

◄

Fig. 6.2. Calculated pure tonalness T_p, (4.21), and complex tonalness T_c, (4.22), of simultaneities of full complex tones (4.7, 8). *: Roughness [after Hutchinson and Knopoff 1978]. $\bar{T}_p$, $\bar{T}_c$: Means over groups of chords

with the highest pure tonalness is the minor seventh chord in root position, and that with the highest complex tonalness, the minor seventh in first inversion (or major sixth in root position). With the possible exception of the minor seventh, root position seventh chords are more tonal than their inversions. The relatively low calculated tonalness of the major-minor sevenths (the first four chords in the set) is due to the tritone interval between third and seventh [Hindemith 1940]. The major-minor seventh's importance in Western music appears to be due to its relative lack of root ambiguity (all notes correspond to harmonics of the root) and its key-defining property (because it includes the interval of a tritone, it is diatonic – dominant, in fact – in only one key [Browne 1981]).

The calculated tonalness values listed in Fig. 6.2f are consistent with music-theoretical conventions (Sect. 1.1.2) that close spacing should be avoided in low and wide spacing in high registers (cf. also Fig. 6.2b). Other variations in calculated tonalness in this set depend in a less straightforward way on the intervals between the notes in the chords.

The effect on calculated tonalness of doubling different voices of a triad at a distance of one octave is shown in Fig. 6.2g. The best note to double in a major triad, according to calculated complex tonalness values, is the root (the first chord in the figure); either the root or the third may be doubled in the minor triad (the fourth and fifth chords). The lowest calculated (pure and complex) tonalness values result from doubling the third of the major triad (the second chord) and the fifth of the minor (the sixth chord). The calculated tonalness of the diminished triad is lowest when the conventional root is doubled. On the whole, these results are consistent with conventional doublings in Western harmony (Sect. 1.1.2).

Complex tonalness values in Fig. 6.2 tend to cover a wider relative range than pure tonalness values. This is to be expected, as pure tonalness depends only on the audibility of pure tone components, whereas complex tonalness additionally depends on the presence of harmonic pitch patterns between audible pure tone components. Theoretically, complex tonalness may be a more appropriate measure of musical consonance than pure tonalness, as almost all noticed (or perceptually relevant) tone sensations are of the complex kind.

6.1.5 Pitch Analyses

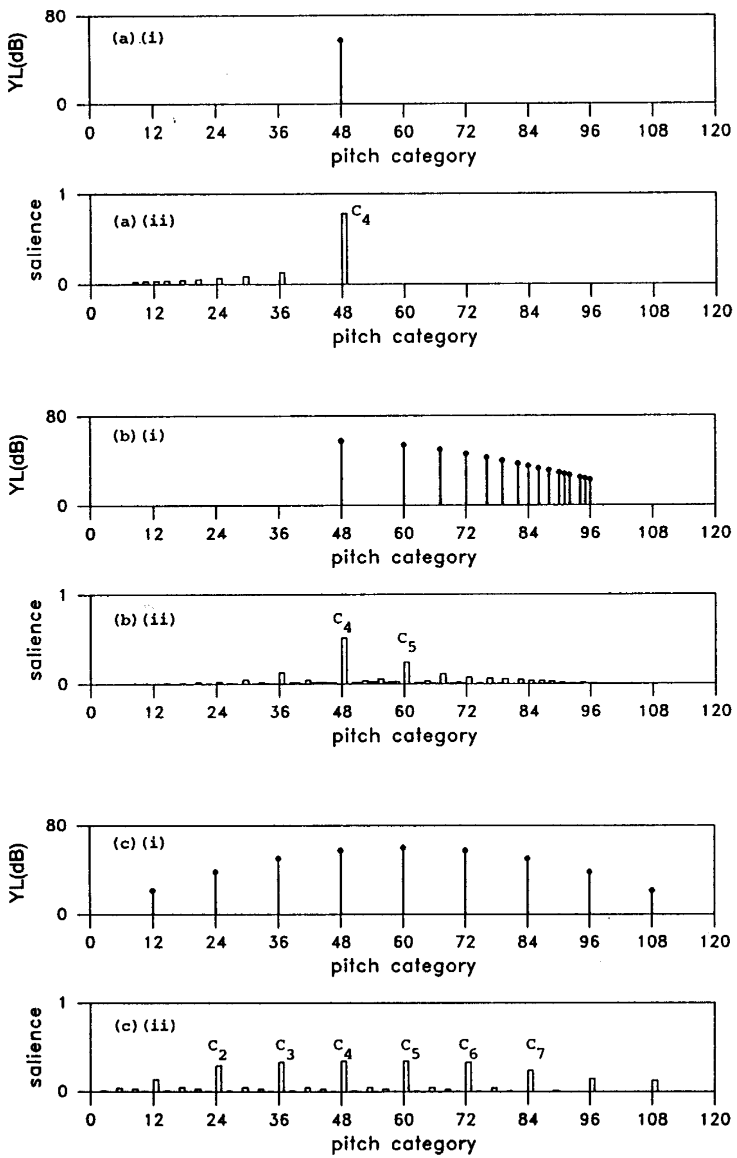
According to the model, a pure tone evokes weak complex tone sensations at subharmonic pitches (Sect. 2.4.5). The calculated pitch configuraton of a pure tone is graphed in Fig. 6.3a. The calculated salience (probability of noticing) of the main (pure) tone sensation (C_4) is 0.78, that of the suboctave (complex) tone sensation (C_3), 0.13. The calculated multiplicity of the tone (1.3) applies only when the tone appears in isolation; in context, its multiplicity may be set equal to one, and calculated saliences may accordingly be divided by 1.3.

The complex tone (Fig. 6.3b) has 16 harmonics whose auditory levels decrease with increasing harmonic number according to (4.8). According to the model, only the lower 10 harmonics are audible (Sect. 6.1.1), so only these influence the calculated salience of complex tone sensations; the irregular spacing of the harmonics 11 – 16 (caused by pitch categorization) has no effect on the calculation. The two highest calculated tone saliences are 0.55 (at the fundamental, C_4) and 0.26 (at the upper octave, C_5). The calculated salience of the upper fifth (G_4, a residue tone sensation) is only 0.06. So the calculated octave-ambiguity of the tone is much more pronounced than its calculated fifth-ambiguity. This is consistent with assumptions concerning octave equivalence and fifth relationships in Western music theory. As before, the calculated multiplicity of the tone (1.9) applies only when it appears in isolation; when the tone is perceived as a single whole, calculated saliences may be divided by 1.9.

According to the model, the main tone sensation of the pure tone in Fig. 6.3a is more salient than that of the complex tone in part (b) of the figure, due to the relative absence of distracting subsidiary tone sensations in the pure tone. This is consistent with experimental results [Fastl and Stoll 1979]. However, the calculated *audibility* of the main tone sensation of the complex tone is much greater than that of the pure tone, as more sensory cues (harmonics) point to its existence. This distinction between salience and audibility resolves conceptual difficulties with the term *pitch weight* in Terhardt's model: pitch weight (as he defines it) is a measure of audibility (as defined here), not of salience (as defined here).

The octave-spaced tone (Fig. 6.3c) has pure tone components at all the C's in the range of hearing, from C_1 (32 Hz) to C_9 (8.4 kHz). The tone sensations it evokes are mainly C's, although some weak (residue) tone sensations on F's, and some still weaker residue tone sensations on A*b*'s, are also output by the model. (This is because the third and sixth harmonics of an F, and the fifth and tenth harmonics of an A*b*, are all C's.) The experiments described in Sects. 5.3.3 and 5.6.3 provided tentative evidence for the existence of subfifth, but not subthird, tone sensations in octave-spaced tones; the latter may simply be too weak to be detected experimentally. The main tone sensation evoked by an octave-spaced tone, according to the model, has an equivalent frequency in the neighbourhood of 300 Hz, in agreement with Terhardt et al. [1986]. This absolute pitch effect is associated with spectral dominance (Sect. 6.1.1), and explains experimental results [Deutsch et al. 1984; Deutsch 1987] in which one of two octave-spaced tones a tritone apart is consistently heard to be higher or lower than the other. The calculated multiplicity of the tone is 2.9, i.e. considerably greater than that of an ordinary complex tone (Sect. 5.2.3). This suggests that octave-spaced tones are less likely than full complex tones to be heard as single entities, even in context.

According to the model, the most salient tone sensation of the minor triad C_4 – Eb_4 – G_4 corresponds to its lowest note, middle C (Fig. 6.3d). Next in order of calculated salience are the tone sensations corresponding to the other two notes [Terhardt et al. 1982a]. The root of the chord (C) corresponds to the most prominent chroma in the bass region. The chroma probability profile



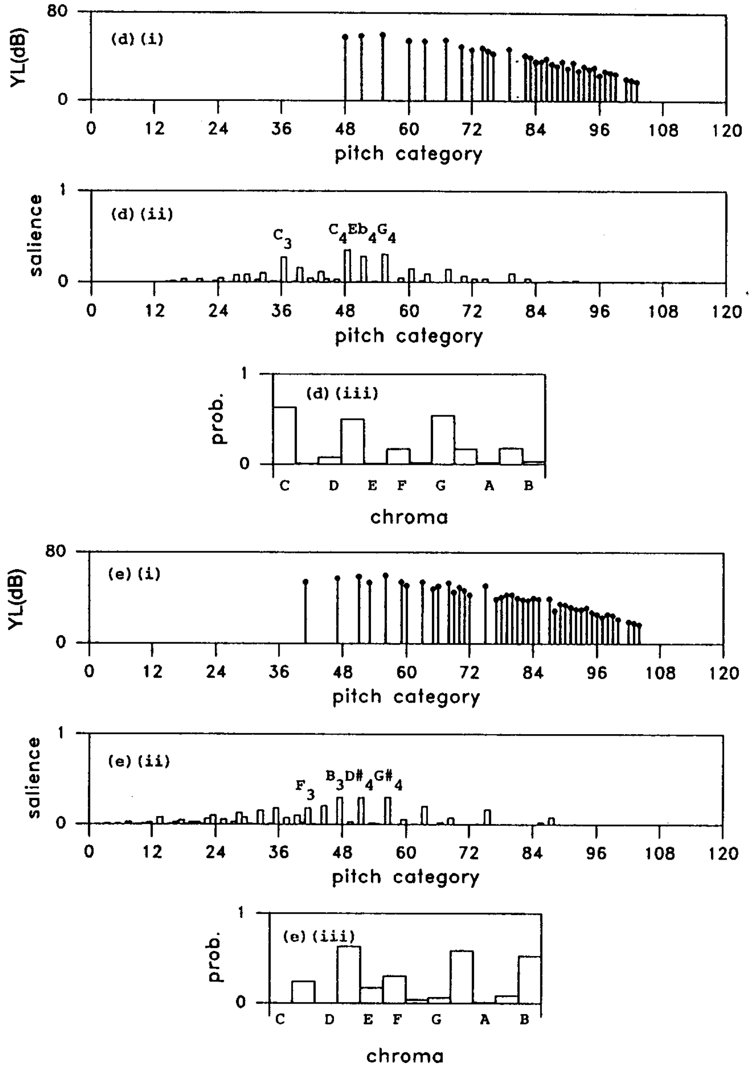


Fig. 6.3. Calculated pitch analyses of tones and chords. Auditory levels YL according to (4.6, 8, 10); saliences according to (4.26). **(a)** Pure tone on C_4. **(b)** Full complex tone on C_4. **(c)** Octave-spaced tone on C. **(d)** Minor triad C_4–Eb_4–G_4. **(e)** *Tristan* chord F_3–B_3–$D\#_4$–$G\#_4$. *(i)* Pure tone components. *(ii)* Tone sensations. *(iii)* Chroma probability profile

of the chord is slightly different from the chroma probability profile of its octave-generalized equivalent (Table 6.2b): it depends to a small extent on voicing (inversion, spacing, doubling).

The final sound simultaneity analyzed in this section is the musicologically infamous *Tristan* chord (Fig. 6.3e). Terhardt [1982] has suggested that the root of the Tristan chord is C#, as all its notes correspond to harmonics of C#. However, C# does not figure at all prominently in the present analysis. The three most salient tone sensations of the chord (corresponding to its upper three tones) are almost equally salient, suggesting that the root of the chord is quite ambiguous. The most salient tone sensations evoked by the chord in the bass region are E_2, $G\#_2$ and B_2 (pitch categories 28, 32 and 35). While these enhance the ease with which the Tristan chord resolves onto the E^7 chord which follows it (by enhancing the pitch commonality of the two chords), none is salient enough to represent the root of the chord: the most salient of the three, B_2, has a calculated salience of 0.18, which is relatively small in comparison to the salience (0.27) of the tone sensation evoked by the minor triad C_4–Eb_4–G_4 at C_3 (Fig. 6.3d), and not much bigger than the calculated salience (0.15) of the nearby tone sensation at $G\#_2$.

In this light, it is not surprising that music theorists disagree about the root and function of the *Tristan* chord. The chord is traditionally described as a French augmented sixth chord in A minor (F–B–D#–A) with a rising chromatic appoggiatura to the A (G#); it may also be interpreted as a chromatic alteration of the diminished seventh F–B–D–G# [Mitchell 1967; Knupp 1984]. In a psychoacoustical approach, it is a chord in its own right: a half-diminished seventh chord with no clearly defined root, set in an unusual context.

6.1.6 Chroma Salience and the Root

A *chord class* is specified by the intervals between the root and the other pitch classes of the other notes of the chord. For example, a major triad is specified by intervals of 4 and 7 semitones above the root. The root is normally understood in music theory to depend only (or primarily) on chord class: it is almost independent of voicing (inversion, spacing, doubling). By contrast, the pitch properties of different voicings of the same chord class (e.g. the saliences of particular tone sensations) can differ markedly [Terhardt et al. 1982a].

In order to investigate thoroughly the relationship between the root of a chord class and its calculated pitch properties, it would be necessary somehow to average the pitch properties of many different voicings of a particular chord class. A convenient and appropriate short-cut to this procedure is to build the chord out of *octave-spaced tones* (4.10).

Hindemith [1940] pointed out that *dyads* have roots, just as chords (triads, sevenths, etc.) do, and began his discussion of the roots of chords accordingly. Although there are twelve different dyads of ordinary complex tones within the

Table 6.2. Chroma probability profiles of simultaneities of octave-spaced tones. (Boldface values correspond to actual notes)

(a) Dyads

		C		D		E	F		G		A		B
m2	C D*b*	**0.80**	**0.77**	0.03	0.03	0.00	0.20	0.19	0.00	0.12	0.12	0.03	0.02
M2	C D	**0.79**	0.00	**0.81**	0.00	0.03	0.17	0.00	0.17	0.10	0.00	0.19	0.00
m3	C E*b*	**0.78**	0.02	0.03	**0.79**	0.00	0.34	0.00	0.00	0.50	0.00	0.02	0.12
M3	C E	**0.85**	0.00	0.08	0.00	**0.69**	0.15	0.02	0.00	0.09	0.15	0.02	0.00
P4	C F	**0.63**	0.09	0.02	0.02	0.00	**0.88**	0.00	0.02	0.08	0.00	0.23	0.00
T	C F#	**0.80**	0.00	0.27	0.00	0.13	0.20	**0.81**	0.00	0.26	0.00	0.02	0.21

(b) Triads

		C		D		E	F		G		A		B
maj.	C E G	**0.83**	0.00	0.05	0.05	**0.43**	0.15	0.01	**0.44**	0.05	0.15	0.01	0.00
min.	C E*b* G	**0.77**	0.01	0.02	**0.69**	0.00	0.27	0.00	**0.55**	0.28	0.02	0.01	0.06
aug.	C E G#	**0.73**	0.12	0.06	0.00	**0.74**	0.11	0.06	0.00	**0.74**	0.11	0.06	0.00
dim.	C E*b* G*b*	**0.68**	0.02	0.20	**0.64**	0.02	0.27	**0.69**	0.00	0.53	0.00	0.02	0.40
sus.	C F G	**0.76**	0.05	0.02	0.10	0.00	**0.77**	0.00	**0.51**	0.07	0.01	0.15	0.00

(c) Tetrads

		C		D		E	F		G		A		B
M7	C E G B*b*	**0.77**	0.00	0.04	0.18	**0.40**	0.10	0.09	**0.36**	0.07	0.13	**0.38**	0.00
m7	C E*b* G B*b*	**0.61**	0.01	0.01	**0.74**	0.00	0.17	0.03	**0.39**	0.25	0.01	**0.38**	0.04
M7	C E G B	**0.70**	0.01	0.04	0.06	**0.68**	0.09	0.02	**0.61**	0.03	0.23	0.01	**0.29**
Ø7	C E*b* G*b* B*b*	**0.46**	0.01	0.11	**0.69**	0.01	0.14	**0.63**	0.00	0.41	0.00	**0.48**	0.25
d7	C E*b* G*b* B*bb*	**0.58**	0.02	0.46	**0.59**	0.02	0.46	**0.59**	0.02	0.46	**0.58**	0.02	0.46

(Ø denotes "half-diminished")

octave in Western music, there are only six different kinds of dyad made up of octave-spaced tones. A dyad of octave-spaced tones a fifth apart, for example, is simply a transposition of a dyad of octave-spaced tones a fourth apart. The six kinds of dyad correspond to the six *interval classes* (1, 2, 3, 4, 5 and 6 semitones) referred to by Forte [1964]. Calculated chroma probability profiles of the six different dyads of octave-spaced tones are presented in Table 6.2a.

According to the model, the only two of these for which one chroma is clearly more important than all the others are the dyads spanning the intervals of a major third (4 semitones) and a perfect fourth (5 semitones). Their main chroma correspond to their musical roots. In the case of the major second (2 semitones), the weak emphasis on the top note suggested by the model is in accord with an interpretation of the dyad as part of a dominant seventh chord. The tendency for the minor third (3 semitones) to be interpreted as part of a minor triad (i.e. for the lower note to be the root) is not reflected by the calculations. It is not clear whether this effect is due to cultural conditioning or to the role of harmonic overtones: the third harmonic of E*b* (omitted when the chord is built from octave-spaced tones) is octave-equivalent to the seventh harmonic of C. For the other intervals – the minor second (1 semitone) and the tritone (6 semitones) – the calculated saliences of the two chroma are about the same. Accordingly, these intervals are not assigned roots in music theory.

Calculated chroma probability profiles of some *triad* classes are shown in Table 6.2b. There are 20 possible triad classes [Forte 1964]; for reasons of space, only the five most common (or consonant) ones are analyzed here.

The calculated root of the C major triad, like its music-theoretical root, is clearly C. The supremacy of the root (C) of the C minor triad is jeopardized, but not overwhelmed, by the salience of the third (E*b*). The augmented, diminished and suspended triads are more ambiguous with respect to their roots. An additional candidate for the root of the C diminished triad (besides its actual tone chroma) is A*b* – a tone which, if physically present, would turn the triad into a major-minor seventh chord. This explains why the diminished triad on the leading note normally functions as a dominant seventh with missing root. The suspended triad ("sus") could be interpreted either as a suspended fourth chord (with root C) or a suspended second chord (with root F).

Cuddy and Badertscher [1987] asked children and adults with varying amounts of musical training and experience to rate how well octave-spaced tones on all twelve degrees of the chromatic scale went with broken (melodic) major and diminished triads. For the major triad, measured profiles peaked clearly at the conventional root; subsidiary peaks occurred at the other two notes, and at other notes in the associated major scale (with the exception of the leading note). For the diminished triad, profiles were essentially flat. These results agree qualitatively with the calculations presented in the table, suggesting that listeners familiar with Western music can extrapolate tonal implications of simultaneous tones (chords) to those of sequential tones (broken chords), see Sect. 3.4.3.

Schenker [1906] asserted that the diatonic scales derive from their tonic triads, and not vice versa. On the whole, the presented data are consistent with this view. The five most prominent chroma evoked by the C major triad (C–E–G) are C, E, F, G and A; the other notes of the C major scale, D and B, are not prominent. In the case of the C minor triad (C–E*b*–G), C, E*b*, F, G and A*b* are prominent; again, the second and seventh degrees of the scale, according to the model, are not particularly strong. On the other hand, scale degrees II and VII are the closest in pitch to the tonic. In short, scale degrees IV and VI are primarily *harmonically* related to the tonic *triad*, whereas VII and II are *melodically* related to the tonic *note*.

Chroma probability profiles of tetrads of octave-spaced tones are presented in Table 6.2c. Like the major triad, the major-minor seventh chord has an unambiguous root, according to both the model and music theory; this explains why it is so important in music theory, in spite of its relatively low tonalness (Sect. 6.1.4).

The calculated root of the minor seventh chord C–E*b*–G–B*b* is E*b*, suggesting that the chord is best regarded as an E*b* sixth chord. This conflicts with music theory and practice, in which C and E*b* are about equally likely to serve as functional roots of the chord. The minor seventh chord was just as common as the major sixth chord (e.g. IV^6 in a major key) during the Baroque, classical and romantic periods; both commonly preceded dominant harmony at perfect cadences. One might expect a slight bias in favour of the added sixth chord, in accordance with its more frequent appearance as a tonic chord (examples from Mahler and Debussy, as well as jazz and popular styles come to mind) but not as much as suggested by the calculations. The calculations could be improved by omitting the masking section of the model: when the chord is constructed from octave-spaced tones, C is masked more than E*b*, as it is closer to B*b* than E*b* is to C.

Like the minor seventh chord, the major seventh chord C–E–G–B has two possible roots: the conventional root (C) and the major third (E). However, E seldom acts as the root of this chord in music. Again, the masking algorithm is largely responsible for the model's underestimation of the salience of the conventional root (C).

The root of the half-diminished seventh chord C–E*b*–G*b*–B*b* is calculated to be E*b* rather than C. This agrees with music theory, in which IV^6 is preferred to II^7 in preparation for V^7 in a minor key (e.g. in Bach chorales); the half-diminished seventh chord is regarded as more stable in first inversion than root position, because the fifth above the bass is perfect rather than diminished. Note also that the minor sixth chord (e.g. E*b*–G*b*–B*b*–C) can function as a tonic (e.g. in Gershwin's *Summertime*), but (as far as I know) a half-diminished seventh cannot. Another possible root of the half-diminished seventh is the sub-major-third (A*b* in the example); this explains why the chord functions as a dominant (more precisely, as a dominant ninth with missing root) when it precedes the tonic triad in the key of D*b* major.

The diminished seventh is the most ambiguous chord of all, having potential roots not only at, but also just below, all its note chroma. The addition of a note a semitone (or minor ninth) below any of the notes of a diminished seventh turns it into a dominant minor ninth chord.

In summary, the root of a chord class is found to correspond well with the chroma with the highest calculated salience, when the chord is constructed from octave-spaced tones. This supports Terhardt's [1974a] claim that the root of a chord – although it is defined in cultural terms – has a strong sensory basis.

6.2 Progression

6.2.1 Pitch Commonality

The pitch commonality of a pair of simultaneities is the extent to which they evoke tone sensations falling in the same pitch categories (Sect. 3.2.3). It is quantitatively defined (Sect. 4.6.1) such that it equals 1 for two identical sounds.

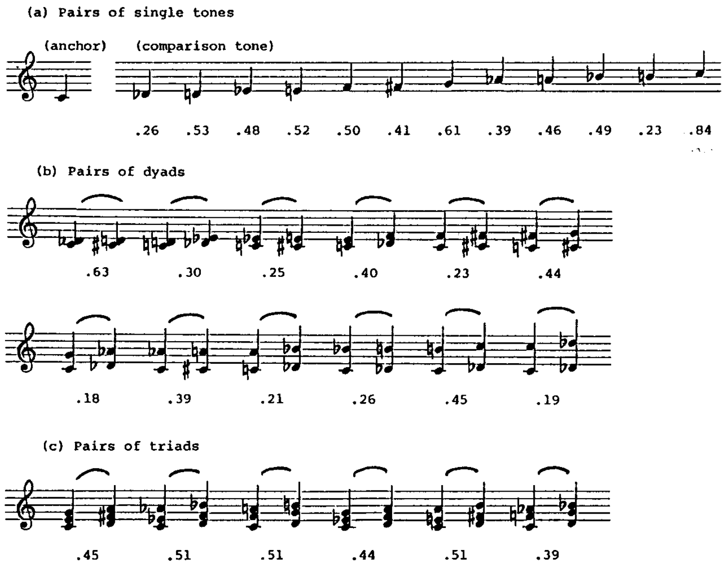


Fig. 6.4. Calculated pitch commonality C (4.29) of pairs of simultaneities of full complex tones (4.7, 8)

Figure 6.4a shows calculated pitch commonalities of sequential complex tones. The rank order of intervals obtained by this method, from lowest to highest pitch commonality, is M7, m2, m6, T, M6, m3, m7, P4, M3, M2, P5 and P8 (where M stands for major, m for minor, P for perfect and T for tritone). This order is similar to that obtained by ranking distances on the cycle of fifths: T, m2/M7, m3/M6, M3/m6, M2/m7, P4/P5, P8. The biggest differences between the two ranks involve the perfect fourth, which is stronger according to the cycle of fifths method, and the tritone, which is stronger according to the pitch commonality method.

In Fig. 6.4b, pairs of dyads in which voices move exactly in parallel over an interval of one semitone are compared for pitch commonality. The resultant rank order of intervals, from lowest to highest pitch commonality, is P8/P5, M6, P4, m3/m7, M2, m6/M3, T/M7, m2. This order resembles a rank from closest to farthest on the cycle of fifths. More interestingly, the intervals for which parallel motion through a semitone produces the lowest pitch commonality are the octave and fifth, because the salience of octave and fifth dyads is concentrated in relatively few pitch categories. This suggests a link between pitch commonality and the conventional avoidance of parallel fifths and octaves in mainstream Western harmony. Other factors associated with this convention include the fusion of tones in parallel fifths and octaves, which reduces their definition and contrapuntal independence, and a reaction against Medieval *organum* styles.

Figure 6.4c presents pitch commonalities of six pairs of triads moving in exact parallel motion through the interval of a major second. The pairs separate into two groups: those of relatively high pitch commonality (the second, third and fifth pairs) and those of relatively low pitch commonality (the others). Included in the second group are two combinations incorporating parallel fifths (the first and fourth).

A progression from one chord class to another may be *voiced* in different ways, i.e. in different inversions, spacings and doublings. In music theory, certain progressions are assumed to be strong regardless of how they are voiced, providing parts move through small steps, and parallels are avoided. Ideally, a psychoacoustical analysis of specific chord class relationships would involve averaging over a large number of acceptable voicings. This process may be short-cut by building chords from octave-spaced tones (as in Sect. 6.1.6).

Table 6.3a shows calculated pitch commonalities of pairs of major triads of octave-spaced tones at all possible chromatic intervals. One of the chords in each pair (the "anchor") is C major; the other is a major triad whose root is indicated in the top row of the table. The rank order of increasing pitch commonality is G*b*, D*b*, D, E*b*/E, F, C. The rank order of strength of harmonic relationships according to the cycle of fifths is G*b*, D*b*, E, E*b*, D, F, C. The relationship between C and D is weaker according to pitch commonality than it is according to the cycle of fifths, as triads a second apart have no notes in common. The major third relationship is stronger according to pitch commonality: major triads as major third apart have a common note (E in this

Table 6.3. Calculated pitch commonality of chords of octave-spaced tones

(a) Pairs of major triads (anchor: C major)												
Comparison chord (major)	C	D*b*	D	E*b*	E	F	G*b*					
Pitch commonality	1.00	0.26	0.39	0.51	0.52	0.63	0.21					
Rank: G*b* D*b* D E*b* E F C												
(b) Pairs of minor triads (anchor: C minor)												
Comparison chord (minor)	c	c#	d	d#	e	f	f#					
Pitch commonality	1.00	0.27	0.39	0.51	0.54	0.63	0.21					
Rank: f# c# d d# e f c												
(c) Mixed pairs of triads (anchor: C minor)												
Comparison chord (major)	C	D*b*	D	E*b*	E	F	F#	G	A*b*	A	B*b*	B
Pitch commonality	0.78	0.35	0.27	0.81	0.30	0.60	0.26	0.59	0.75	0.24	0.45	0.45
Rank: A F# D E D*b*/B G F A*b* C E*b*												
(d) Seventh chord and major triads (anchor: G^7, G – B – D – F)												
Comparison chord (major)	C	D*b*	D	E*b*	E	F	F#	G	A*b*	A	B*b*	B
Pitch commonality	0.61	0.42	0.59	0.57	0.48	0.60	0.32	0.92	0.33	0.41	0.69	0.51
Rank: F# A*b* A D*b* E B E*b* D F C B*b* G												
(e) Seventh chord and minor triads (anchor: G^7, G – B – D – F)												
Comparison chord (minor)	c	c#	d	e*b*	e	f	f#	g	g#	a	b*b*	b
Pitch commonality	0.61	0.29	0.74	0.35	0.74	0.48	0.39	0.76	0.47	0.47	0.48	0.70
Rank: c# e*b* f# g#/a f/b*b* c d d/e g												

case). The results for pairs of minor triads (part b of the table) are similar to those for pairs of major triads.

Three of the above comparison chords have no notes in common with the anchor chord: D*b*, D and G*b*. As these chords are constructed from octave-spaced tones, they do not have any frequencies in common, either. Yet, the D chord clearly stands in a stronger harmonic relationship to the C chord than D*b* or G*b* do. Bharucha and Stoeckig [1987] confirmed this experimentally, interpreting it as "evidence for activation spreading via a cognitive structure that links related chords" (p. 523). In a psychoacoustical approach, such harmonic relationships are initially explicable in purely sensory terms (pitch commonality), and are reinforced by cultural conditioning (Sect. 3.1.1).

Table 6.3c presents calculated pitch commonalities for pairs of triads in which one is major, the other minor. The strongest relationship is found between the C minor triad and its relative major (E*b*). Next come the parallel major relationship (C), the submediant relationship (A*b*), and the major dominant (G) and the major subdominant (F). Other relationships, according to the model, are much weaker. (Note that the G–c cadence is more fundamental than A*b*–c in music theory, because only G–c clearly defines a key.)

Table 6.3d lists the pitch commonality of a G major-minor seventh chord (the dominant seventh of C) and all twelve major triad classes. Calculated pitch commonality is strongest for three common notes (G^7–G); these chords are so similar that they hardly invoke a feeling of progression. The second strongest pitch commonality is predicted for two common notes (G^7–B*b*). This progression is quite common (e.g. II^7–IV or IV–II^7 leading to V^7 in the key of F major). The major triads F, D and C have about equal pitch commonality with G^7. This is consistent with musical practice: in the key of C, F and D often precede G^7 in perfect cadences, and C often follows G^7. The coherence of the progressions *b*VI–V^7 (A*b*–G^7) and *b*II–V^7 (D*b*–G^7) is not accounted for by these pitch commonality rankings. It may be explained instead by pitch commonality with other chords usually occurring in the same context (e.g. the pitch commonality of *b*VI with i and iv, and of *b*II with iv), and by pitch distance: the root of *b*VI is a semitone away from the tonally important scale degree V, and the root of *b*II is a semitone away from I.

Table 6.3e compares the chord G^7 with each of twelve minor triads. Apart from the parallel minor triad g, the most closely related minor triads to G^7 belong to the key of C major (d, e). Next comes b: G^7 functions as a German augmented sixth chord when it resolves to this chord, and is notated G–B–D–E#. Despite their chromatic relationship, G^7 and b have high pitch commonality, because they have two notes in common. The pitch commonality of G^7 and c (V^7 and *i* in C minor) is surprisingly high, considering that these chords have only one tone in common; the pitch commonality of G^7 with other minor triads with which it shares one common note (f, b*b* and g#) is much lower.

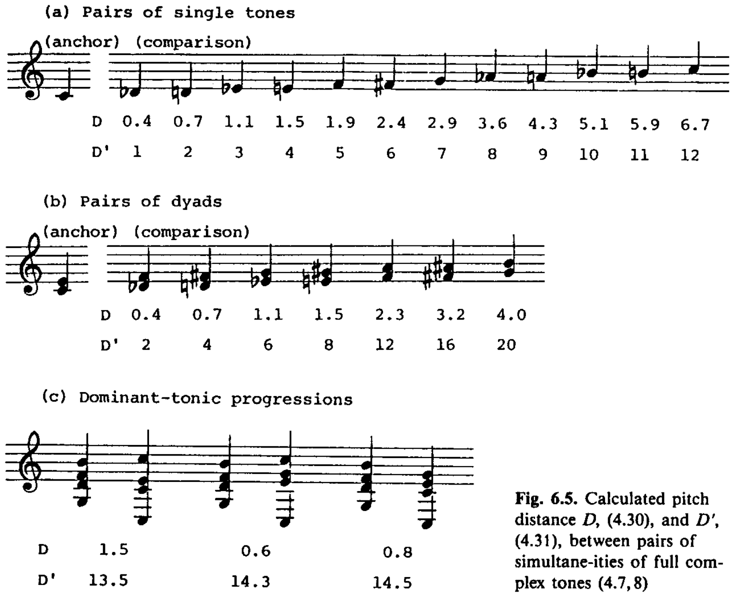


Fig. 6.5. Calculated pitch distance *D*, (4.30), and *D'*, (4.31), between pairs of simultane-ities of full complex tones (4.7, 8)

6.2.2 Pitch Distance

Two different formulations for the apparent overall pitch distance between sequential sounds are described in Sect. 4.6.2. The first, (4.30) for *D*, takes into account all tone sensations in each sound, and their calculated saliences. The second, (4.31) for *D'*, takes into account only the actual complex tone components (notes) of a musical chord. Values of *D* and *D'* are compared for specific sound pairs in Fig. 6.5.

Figure 6.5a deals with pairs of single complex tones (notes) in musical intervals. Values of *D'* are simply equal to interval size in semitones. Values of *D* are smaller, due to overlapping of pitch configurations. For small intervals, *D* is less than half of the interval size in semitones. With increasing interval size, the magnitude of *D* accelerates relative to interval size, until the pitch distance between tones an octave apart exceeds half of the interval size.

Figure 6.5b shows calculated pitch distances between major third dyads. Here, the effect of overlapping pitch configurations is more pronounced: both formulations of pitch distance suddenly accelerate as the interval between the dyads exceeds a major third. These data are consistent with the conventional avoidance of this kind of part-crossing.

Figure 6.5c deals with three different voicings of a dominant seventh to tonic progression. Conventional theory recommends that the third of the dominant should rise by a semitone to the root of the tonic, and the seventh of the dominant should fall to the third of the tonic. Only the first of the three progressions conforms to both of these conventions; and only (4.31) for D' is consistent with music theory in this case. This confirms that D' is a better indicator of good voice leading (Sect. 4.6.2).

Values of D' in part (c) of the figure seem too large in comparison to values in parts (a) and (b). This could be rectified by reducing the effective salience of each tone component in (4.31) for chords of more than three notes, in line with the observation that it is difficult to attend to more than three tones at once (Sect. 6.1.3).

6.2.3 Tonicity

Section 3.4.3 dealt with the tonality of scales and melodies. The present section specifically investigates the tonality of *chord progressions*. The investigation is based on sensory properties of the chords according to the model in Chap. 4.

Musical experience suggests that the tonality of a Western chord progression is influenced by the following four factors. (i) The tonic pitch occurs more often than other pitches in a progression. (ii) The tonic pitch is the root of the chord formed by superposing the roots of chords in the progression (e.g. it is the root of the fifth between V and I: Sect. 3.4.3). (iii) The tonic chord stands in a strong harmonic relationship with other chords in the progression. (iv) The tonic chord is relatively consonant.

In my Ph.D. thesis [Parncutt 1987a], I attempted to determine the tonic of typical, octave-generalized chord progressions by means of systematic and (as far as possible) non-arbitrary procedures based upon each of the above criteria. None of the four criteria was found to predict the tonic with any generality when applied alone, for the following reasons.

Point (i) is not entirely true: the *dominant* is normally the note which occurs most often, as it is common to both tonic and dominant harmony. The tonic nevertheless sounds more important than the dominant, apparently because it is the root of the perfect fifth dyad formed by the tonic and dominant when they are heard simultaneously (Sect. 3.4.2). Point (ii) is inadequate by itself: the roots of suspended triads such as II–V–I and IV–V–I are ambiguous (Sect. 6.1.6), yet the corresponding progressions have relatively unambiguous tonics. Point (iii) produces similar ambiguities. Regarding (iv), consonance enhances the sense of finality associated with the tonic chord (see *time-span reduction preference rule 2* of Lerdahl and Jackendoff [1983]); but this criterion cannot by itself determine the tonic in progressions where all chords have about the same consonance (e.g. progressions of major and minor triads). In any case, the resolution of dissonance is not necessarily sufficient for the finality of a cadence [Matthews and Pierce 1980]. Evidence for this is the use of quite dissonant final (tonic) chords in jazz.

Predictions improved as more criteria were taken into account in the one procedure. It eventually became apparent that all four criteria needed to be accounted for, if the tonic was to be predicted with any reliability, and the following procedure was developed. First, configurations of salience against pitch category were calculated for each chord in the sequence. Criterion (i) was accounted for by combining the configurations across time to yield the overall salience of each pitch in the chromatic scale ($P = 0, \ldots, 120$). Saliences were expressed as probabilities of noticing each pitch category at any time during the progression [after (4.28)]. Criterion (ii) was accounted for by applying the harmonic pattern-recognition procedure represented by (4.19) to the progression's overall configuration of salience against pitch. The output consisted of potential candidates for the "root of the progression", i.e. the root of both simultaneous and sequential pitches, as if they had all been heard simultaneously. Next, criterion (iii) was accounted for by calculating the pitch commonality of each chord in the progression with the array of candidates for the "root of the progression". Finally, criterion (iv) was accounted for by multiplying these pitch commonality values by tonalness values for the corresponding chords. The resultant value assigned to each chord was called its *tonicity*.

Table 6.4. Calculated tonicity of chords in octave-generalized progressions

1st chord	Weight	2nd chord	Weight	3rd chord	Weight
C	0.67	G^7	0.54		
c	0.67	G^7	0.52		
C	0.66	F	0.64	G	0.60
c	0.64	f	0.62	G	0.54
C	0.64	G	0.63	d	0.62
c	0.62	d^o	0.58	G	0.57
C	0.63	G^7	0.53	D^7	0.52
c	0.63	Ab^7	0.52	G^7	0.49
F	0.63	G	0.62		
f	0.61	G	0.58		

Uppercase letters: chords with major thirds. Lowercase: minor. d^o denotes "d diminished": D – F – Ab

Results are shown in Table 6.4. The chord with the highest calculated tonicity corresponds to the tonic for all tested chord progressions whose tonics are clearly defined in music theory (but not for ambiguous progressions such as F–G and f–G). Moreover, progressions with relatively high maximum tonicity have relatively stable tonal structure.

The order of presentation of the chords of each example in the table is not specified. If the predictions of the described method were to be tested experimentally, they would need to be compared with results averaged over all possible orders of presentation of each progression. In order to account theoretically for order of presentation in specific cases, it would be necessary to consider such things as the decay of the "sensory trace" of each chord with time, and interference effects due to the presentation of successive chords (Sect. 2.2.3).

The described procedure takes both sensory and cultural aspects of Western tonality into consideration, suggesting that both sensory and cultural effects contribute to the perception of tonality and tonics in Western musics. Sensory effects include tone salience, tonalness and pitch commonality. Culture-specific aspects of the procedure include the assumption in point (iii) that listeners are familiar with the sounds of triads and the relationship between a triad and its root, and the particular way in which sensory effects are selected and combined in the procedure.

The above procedure is not completely reliable. For example, the tonics of the progressions c – f – G and C – G – d (rows 4 and 5 of the table) are relatively unambiguous (c and C respectively), but calculated tonicity values for these progressions do not clearly distinguish candidates for the tonic. The tonic of such progressions may have been disambiguated during the historical development of Western tonality, as the major and minor scales became familiar during the 16th and 17th centuries.

6.2.4 Implied Triad/Scale

A *key* may be regarded as a tonal system centred either on a chord class with a clear root (the tonic triad) or on a scale with a clear tonic (the tonic note) (Sect. 3.4.3). All twelve note chroma (pitch classes) may occur (as notes) in a given musical key [Goldman 1965]. What distinguishes one key from another is how often particular chroma occur. Chroma belonging to the prevailing scale occur more often than others (non-diatonic, or chromatic, chroma). Of the diatonic chroma, the most commonly occurring are the tonic and dominant.

Tonic triads and scales are not always played in their complete or typical forms: they need only be *implied* in order to produce a sense of key [Browne 1981]. Such implication may only be understood by a listener with extensive experience of music based on diatonic scales and triadic harmony. Several different triads and scales may be implied by a passage, so several different triads and scales may compete for the status of tonic.

Consider first the possibility that the tonic is that triad class which is most strongly implied by the progression. The pitch configurations of typical major and minor triads are familiar to the Western listener. Given octave equivalence, these pitch configurations may adequately be specified by *chroma salience profiles* (4.28), calculated directly from pitch analyses of major and minor triads of octave-spaced tones (Table 6.2). These profiles may be used as templates in a pattern recognition model (Sect. 2.3.3). If the tonic of a chord progression is an implied triad, the progression's chroma salience profile should correlate better with the chroma salience profile of the tonic triad than with that of any other major or minor triad. According to this hypothesis, however, the tonic of the progression C d G is A minor or E minor (left part of Table 6.5). These two triads go well with the triads in the progression, but only act as tonics in "modal" harmonic styles, without leading notes.

Table 6.5. Correlation of chroma probability profiles of octave-generalized progressions with profiles of triads and scales

Chords in progression	"Implied triad" method			"Implied scale" method		
C G^7	C (0.77)	G (0.65)	e (0.60)	C (0.76)	F (0.64)	G (0.49)
c G^7	c (0.84)	G (0.57)	C (0.51)	c (0.82)	E*b* (0.68)	B*b* (0.55)
C F G	C (0.65)	F (0.56)	G (0.51)	C (0.88)	F (0.75)	B*b* (0.54)
c f G	c (0.76)	f (0.54)	A*b* (0.47)	c (0.86)	E*b* (0.77)	A*b* (0.70)
C d G	a (0.61)	e (0.60)	C (0.58)	C (0.95)	F (0.82)	G (0.61)
c d^o G	g (0.47)	G (0.47)	c (0.30)	E*b* (0.86)	c (0.82)	A*b* (0.62)
C D^7 G^7	C (0.62)	G (0.60)	D (0.43)	C (0.82)	G (0.73)	F (0.68)
c A*b*^{7} G^7	c (0.80)	A*b* (0.55)	E*b* (0.43)	c (0.86)	E*b* (0.68)	A*b* (0.53)
F G	F (0.61)	G (0.60)	d (0.60)	C (0.80)	F (0.64)	G (0.59)
f G	f (0.61)	G (0.50)	F (0.37)	c (0.68)	E*b* (0.59)	f (0.59)

Uppercase: major chords and scales. Lowercase: minor. d^o denotes "d diminished": D – F – A*b*. Numbers in brackets are correlation coefficients

The role of leading notes in determining tonics may be accounted for by correlating the chroma salience profile of a progression with *scale* templates, in which each chroma is assigned a value of 1 (if it belongs to the scale) or 0 (if it does not). For example, the C major scale may be represented by the sequence 101011010101; C minor by 101101011000l. (The last digit in each case corresponds to the leading note, B.) Results for representative chord progressions appear in the right part of Table 6.5.

On the whole, the implied scale method is successful. Occasional failures are due to the failure of the method to account for the different relative importances of different scale degrees. For example, the method assumes the leading note to be just as important as the tonic, and so predicts that the progression c – d^o – G (where the superscript o means "diminished") is in E*b* major instead of C minor. The leading note of both major and minor keys actually has relatively low harmonic importance, as reflected in music theory by its distance from the tonic along the cycle of fifths.

6.2.5 Key Profile

Krumhansl and Kessler [1982] measured the relative importances of the twelve chroma in a musical key, by comparing representative chords and chord progressions in a given key with single probe tones, presented just after each passage. The effect of pitch register was minimized by building both chords and probe tones from octave-spaced tones. Listeners, all of whom regularly played musical instruments, were "instructed to rate on a 7-point scale how well, in a musical sense, each probe tone fit into or went with the musical element just heard" (p. 342). Results (Fig. 6.6) were averaged over four different musical elements: an isolated tonic triad (I); the progression IV (subdomi-

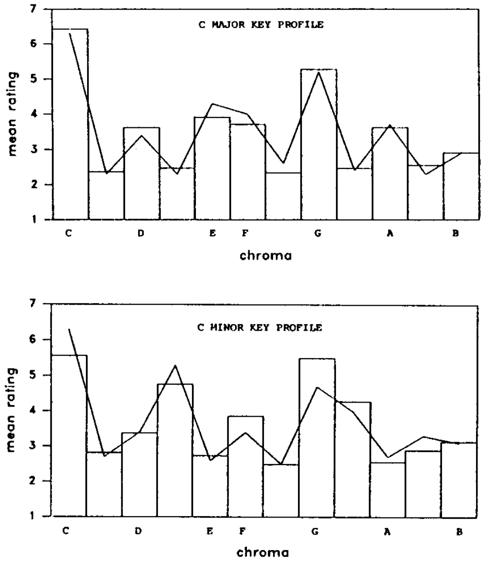


Fig. 6.6. Modelling of Krumhansl and Kessler's [1982] key profiles. *Zig-zag lines*: experimental results (see "Expt." rows in Table 6.6). *Histogram*: calculations (see "Calc." rows in the table) fit to experimental results by linear regression

nant), V (dominant), I; the progression II (supertonic), V, I; and the progression VI (submediant), V, I.

Krumhansl and Kessler supposed that "the hierarchy of tonal stability is acquired through experience with the structural relations that obtain in the music itself" (p. 364). They hypothesized two main factors which could lead to the acquisition of such a hierarchy. First, some chromatic scale degrees occur more often than others in diatonic music. These tend to be the same tones which are judged by listeners to "go well" with a musical passage. Second, the structure of tonal hierarchies bears some resemblance to the harmonic structure of single tones. This suggests that the acoustical properties of musical tones have influenced the evolution of harmony in Western music.

These explanations are not fully satisfactory. First, the most commonly occurring tone in a major or minor key is more often the dominant than the tonic (Sect. 6.2.3). Second, the relationship between the minor key hierarchy and the harmonic series is not clear (Sect. 1.2.3).

It turns out that the experimental results of Krumhansl and Kessler may be modelled quite closely by chroma tally profiles (Table 6.6). The effect of all the chords in the progressions on the experimental results may be taken into account by averaging over all four musical elements listed above, giving each element equal weight. This is achieved by adding up the calculated chroma salience profiles of six I's, three V's, one II, one IV and one VI triad. The

Table 6.6. Modelling of Krumhansl and Kessler's [1982] key profiles. (Boldface values correspond to actual notes)

(a) Calculation of major key template

		I					IV		V				
I	(maj.)	**1.57**	0.00	0.05	0.05	**0.55**	0.16	0.01	**0.54**	0.06	0.16	0.01	0.00
ii	(min.)	0.01	0.06	**1.30**	0.01	0.02	**1.06**	0.00	0.31	0.00	**0.73**	0.32	0.02
IV	(maj.)	**0.54**	0.06	0.16	0.01	0.00	**1.57**	0.00	0.05	0.05	**0.55**	0.16	0.01
V	(maj.)	0.16	0.01	**0.54**	0.06	0.16	0.01	0.00	**1.57**	0.00	0.05	0.05	**0.55**
vi	(min.)	**1.06**	0.00	0.31	0.00	**0.73**	0.32	0.02	0.01	0.06	**1.30**	0.01	0.02

Comparison of calculation (6 I + ii + IV + 3 V + vi) with experiment [Krumhansl and Kessler 1982]:

Calc.	**11.5**	0.2	**3.7**	0.5	**4.5**	**3.9**	0.1	**8.3**	0.5	**3.7**	0.7	**1.7**
Expt.	**6.4**	2.2	**3.5**	2.3	**4.4**	**4.1**	2.5	**5.2**	2.4	**3.7**	2.3	**2.9**

Correlation coefficient: $r = 0.986$

(b) Calculation of minor key template

		I					IV		V				
i	(min.)	**1.30**	0.01	0.02	**1.06**	0.00	0.31	0.00	**0.73**	0.32	0.02	0.01	0.06
ii	(dim.)	0.02	0.48	**1.07**	0.02	0.22	**0.96**	0.02	0.31	**1.03**	0.00	0.71	0.00
iv	(min.)	**0.73**	0.32	0.02	0.01	0.06	**1.30**	0.01	0.02	**1.06**	0.00	0.31	0.00
V	(maj.)	0.16	0.01	**0.54**	0.06	0.16	0.01	0.00	**1.57**	0.00	0.05	0.05	**0.55**
VI	(maj.)	**0.55**	0.16	0.01	**0.54**	0.06	0.16	0.01	0.00	**1.57**	0.00	0.05	0.05

Comparison of calculation (6 i + ii + iv + 3 V + IV) with experiment [Krumhansl and Kessler 1982]:

Calc.	**9.6**	1.1	**2.8**	**7.1**	0.8	**4.3**	0.0	**9.4**	**5.6**	0.3	1.3	**2.1**
Expt.	**6.3**	2.7	**3.5**	**5.4**	2.6	**3.5**	2.5	**4.8**	**4.0**	2.7	3.3	**3.2**

Correlation coefficient: $r = 0.941$

results correlate well with Krumhansl and Kessler's measured tone profiles (kindly supplied by Dr. Krumhansl).

The histogram in Fig. 6.6 is obtained when the calculated key profiles are mapped by linear regression onto experimental profiles. Assuming that the calculated tone profiles well reflect the sensory properties of major and minor keys, discrepancies between calculations and experimental results reflect the influence of cultural conditioning. This is expected to have been considerable due to the musical orientation of both the listeners and the experimental task in the experiments of Krumhansl and Kessler. Bigger discrepancies occur for the minor key, supporting the music-theoretical idea than the minor is somehow less "natural" (i.e. less sensorially based) that the major, or that it represents a "distorted" version of the major (Sect. 3.4.2). The supremacy of the tonic over the dominant, which has a clear sensory basis in the case of the major key, appears to be largely cultural in the minor case: the sensory basis for the root of the minor triad is relatively weak (Table 6.2b). The mean experimental response for the third degree of the minor scale considerably exceeds the calculated response, perhaps because the minor third of the minor key is the most important chroma distinguishing it from the major key. The

Table 6.7. Correlation of chroma probability profiles of octave-generalized progressions with key profiles

Chords	Correlations		
C G^7	C (0.93)	G (0.70)	c (0.62)
c G^7	c (0.93)	C (0.63)	E*b* (0.50)
C F G	C (0.89)	F (0.76)	G (0.62)
c f G	c (0.92)	f (0.57)	A*b* (0.53)
C d G	C (0.83)	G (0.75)	F (0.67)
c d° G	c (0.81)	g (0.53)	E*b* (0.51)
C D^7 G^7	C (0.85)	G (0.81)	F (0.52)
c A*b*^{7} G^7	c (0.92)	A*b* (0.56)	E*b* (0.47)
F G	F (0.73)	C (0.67)	G (0.64)
f G	c (0.64)	f (0.60)	C (0.50)

Uppercase: major chords and keys. Lowercase: minor. d° denotes "d diminished": D – F – A*b*

low salience of the leading note of both major and minor keys is consistent with its distance from the tonic on the cycle of fifths, and with rules preventing its doubling in harmony.

A key profile may be regarded as a cross between a triad template and a scale template: key profiles account not only for the importance of the tonic triad, but also for the importance of the prevailing scale, in the determination of musical keys. Krumhansl and Kessler determined the key of specific passages of music by correlating their experimentally determined key profiles with profiles of the passages (obtained by the same experimental method). They also used their key profiles to produce a multidimensional map of relationships between musical keys (generalizing the music-theoretical idea that scales are perceived to be related if they have tones in common), and traced the tonal movement of specific passages through the space.

Predicted tonics of typical chord progressions according to the method of implied keys, using only calculated versions of Krumhansl and Kessler's tone profiles, are shown in Table 6.7. The results are correct in all cases where the tonic is clearly defined in music theory. This is not surprising, as the key templates were themselves calculated from chroma profiles of similar progressions.

The method of implied key is restricted to chord progressions, as the term "key" itself presupposes triadically based harmony. The method does not always correctly determine the key of unaccompanied melodies. The key of an unaccompanied melody in Western music may instead be found by the method of implied scales (Sect. 6.2.4). Alternatively, the key of a melody may be regarded as the key of its implied accompaniment, i.e. the key of an implied chord progression.

T_c	.45	.21	.68	.32	.42	.25	.53	.33
C_t	1.00	.94	.74	.42	.49	.95	.72	.37
C_s	.94	.77	.65	.58	.57	.76	.72	
D'	2.2	3.1	6.9	5.2	3.1	1.0	3.0	

Fig. 6.7. Analysis of the opening eight bars of Beethoven's Sonata Op. 27 No. 2. T_c: Complex tonalness (4.22) of each chord. C_t: Pitch commonality (4.29) of each chord with the first chord (the tonic). C_s: Pitch commonality of successive chords. D': Pitch distance (4.31) between successive chords

6.3 Pieces

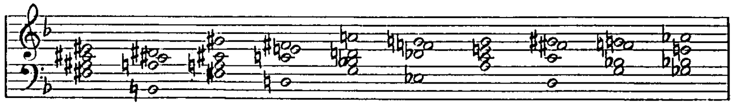
6.3.1 Analysis

The model in Chap. 4 may be applied to the analysis of specific chord progressions, including underlying chord progressions of polyphonic passages.

An appropriate harmonic reduction of the first four bars of Beethoven's *Moonlight Sonata* is notated in Fig. 6.7. Complex tonalness (a measure of the consonance of a chord, see Sect. 6.1.4) varies considerably during the progression. It is lowest for the second chord. The unexpectedly low apparent dissonance of this chord may be explained either psychoacoustically, in terms of the relatively high pitch commonality and low pitch distance between the chord and its neighbours (Sect. 3.2.1), or music-theoretically, in terms of voice leading (the B in the bass being merely a passing note). Complex tonalness is highest for the third chord, being the only root position major triad in the progression.

Pitch commonality between each chord and the tonic chord (defined here to be the opening chord) is lowest for chords whose notes do not all belong to the standard heptatonic scale defined by the key signature: the fourth chord in the progression (a Neapolitan sixth) and the fifth and last chords (different voicings of the dominant seventh, with sharpened thirds). In all these cases, low pitch commonality is associated with dissonance, tension, and expectation of resolution.

Pitch commonality between successive chords is consistently high ($C > 0.5$), reflecting the general harmonic cohesion of the passage. Pitch distances between successive chords increase to a maximum toward the middle of the phrase, as the progression moves away from the tonal centre, and then fall back to low values in preparation for the return of the tonic (after the last chord in the example).



T_c .30 .28 .32 .29 .33 .29 .31 .28 .44 .30

C_t .49 .46 .29 .53 .65 .65 .70 .52 .67 .32

C_s .58 .76 .54 .50 .54 .53 .46 .41 .43

D' 11.5 12.2 12.2 12.2 12.2 9.9 13.7 10.4 3.1

Fig. 6.8. Analysis of an arrangement of a progression from *The Girl From Ipanema* by Jobim, de Moraes and Gimbel. T_c: Complex tonalness (4.22) of each chord. C_t: Pitch commonality (4.29) of each chord with $F_3-A_3-C_4-F_4$ (the tonic). C_s: Pitch commonality of successive chords. D': Pitch distance (4.31) between successive chords

According to the progression's chroma probability profile (Table 6.8), the chroma most likely to be noticed in the progression are the tonic and dominant, C# and G#. Next in line are the minor third (E) and perfect fourth (F#); the minor sixth (A) is also well represented. The key profile of C# minor, in which C#, G#, E, A and F# rate highly, therefore correlates well with the chroma probability profile of the progression, in spite of the roughly equal saliences of I and V (C# and G#), see Fig. 6.6. The high correlation coefficient ($r > 0.8$) reflects the clarity of the tonal structure of the passage; apparently, the Neapolitan sixth chord (chord 4) does not disrupt the prevailing C# minor tonality. The key of E major is predicted to be the second most strongly implied key of the passage; the piece modulates (in passing) to E major a few bars after the end of this phrase.

The excerpt analyzed in Fig. 6.8 is an arrangement (by the author) of a progression from *The Girl From Ipanema* by Jobim, De Moraes and Gimbel. The analysis shows that the techniques developed in this study can be applied to quite complex harmonies and could be useful in jazz research.

Complex tonalness (and hence consonance) is generally lower, and varies considerably less, in this example than in the Beethoven. Instead, musical unity seems to be maintained by *homogeneity* of texture and consonance, suggesting a kind of trade-off between homogeneity and consonance. Complex tonalness is highest for the second-last chord, the G minor seventh: this is the only chord with octave doubling. The progression could perhaps be made more homogeneous by reducing the consonance of this chord, e.g. by shifting the upper voice of the chord up a tone.

The tonic of the sequence is defined for the purpose of analysis to be the chord $F_3-A_3-C_4-F_4$, a chord which nicely precedes and follows the sequence. Pitch commonality with the tonic chord is generally lower than in the Beethoven example. Again, the last chord has low pitch commonality with the tonic, increasing expectation that it will resolve to the tonic (which it in fact

Table 6.8. Octave-generalized analysis of progression in Fig. 6.7

	C		D		E	F		G		A		B
Chroma probability profile of entire progression:	0.73	0.99	0.70	0.74	0.95	0.26	0.95	0.23	1.00	0.91	0.45	0.75
Correlation of chroma probability profile with diatonic templates:												
major	−0.39	+0.32	+0.14	−0.42	+0.59	−0.62	+0.39	−0.39	+0.11	+0.56	−0.74	+0.43
minor	−0.45	+0.81	−0.37	−0.05	+0.02	−0.25	+0.52	−0.57	+0.39	+0.13	−0.29	+0.10

Rank: c# E A f# B F#/g# C# D a G# b e d# f a# d C/G Eb c g F

Table 6.9. Octave-generalized analysis of progression in Fig. 6.8

	C		D		E	F		G		A		B
Chroma probability profile of entire progression:	0.94	0.98	0.95	0.89	0.93	0.93	0.97	0.97	0.93	0.92	0.95	0.72
Correlation of chroma probability profile with diatonic templates:												
major	+0.04	+0.37	+0.06	−0.07	−0.41	+0.06	+0.34	+0.04	−0.06	−0.07	+0.14	−0.44
minor	+0.05	+0.20	+0.29	−0.23	−0.51	+0.31	+0.13	+0.28	−0.45	−0.04	+0.28	−0.31

Rank: Db f bb/g c#/d Bb D/F c G/C Ab Eb/A eb b B E g# e does). The fifth (G minor-add-9), sixth (Eb^9, seventh (Am7) and ninth (Gm7) chords have the highest pitch commonality with the tonic: all these commonly occur in jazz in the key of F major. The third chord (F# minor-add-9) is quite remote from the key of F major, and also has the lowest pitch commonality with the tonic.

Pitch commonality between successive chords is lower than in the Beethoven example, but never falls below 0.4, despite large distances traversed on the cycle of fifths. The lowest value occurs between the third-last and second-last chords. The relationship between these chords is maintained in other ways than by pitches in common: the pitch distance between them is relatively small (no voice moves through more than a semitone), and the rising fourth in the bass is familiar to musical listeners. Pitch distance between successive chords is higher, but more consistent, than in the Beethoven example; again, pitch distance is lower just before the return to the tonic.

The consistently high chroma probabilities in the profile (Table 6.9) indicate that all twelve chroma are normally noticed at some time during the progression. The chroma least often noticed is B, lying diametrically opposite to F (the song's tonic) on the cycle of fifths. The correlation between the profile of the calculated key of this progression (D*b* major) and the chroma probability profile of the passage is low ($r<0.4$), implying a weak tonal structure (the passage is clearly not in D*b*!). F minor ranks highly in the list of calculated keys, suggesting that contact with the tonic key of the piece (F major) is not completely lost during this tonal adventure. A more interesting and relevant picture of the changing tonality of the passage (including passing modulations to F# major/minor and G minor) could be obtained by looking at a few chords at a time, or by allowing the effect of previously heard chords to decay exponentially with a time constant comparable to the duration of auditory sensory memory (Sect. 2.2.3).

6.3.2 Composition

The twentieth century is unique in the history of Western music. It is the only century in which the most commonly heard art music was composed in previous centuries. Despite many decades of exposure of Western audiences to a range of non-diatonic styles, old-fashioned diatonic tonality remains the most popular way of organizing musical pitch (Sect. 1.1.4).

Diatonic tonality is in many ways an arbitrary, culture-specific system, but (as this study has shown) it also has a strong sensory basis. This basis appears to be responsible for the survival of diatonic tonality over four centuries, in spite of considerable and repeated changes and developments in Western musical styles. It is also responsible, at least in part, for the introduction of Western tonal harmony to non-Western cultures [Nettl 1986].

The current study extends the work of Terhardt and his colleagues in isolating and quantifying sensory aspects of music perception. The point has been reached where it should be possible to apply this knowledge in consistent

ways in order to create new tonal styles – styles whose sensory basis is similar to that of existing tonalities, but whose arbitrary, 'cultural' aspects are different.

Consider the composition of chords for a particular group of instruments or voices. The first limitation on such chords is the pitch range of each instrument or voice. Another limitation (built into Western notation, and into the model in Chap. 4) is that pitches are confined to semitone categories.

These two limitations, taken together, allow for a very large number of different chords. This number may be reduced to a manageable level by limiting the sensory consonance (roughness, tonalness) of chords (or other simultaneities), resulting in a kind of *vocabulary* from which homorhythmic progressions may be composed. The size of the vocabulary could be further reduced either by further manipulation of psychoacoustical parameters, or by the composer's personal choice. Relatively consonant vocabularies of this kind would resemble existing diatonic chord vocabularies; vocabularies of relatively low consonance would be less familiar, and so more likely to have interesting applications in composition.

A possible way of composing a progression from a given vocabulary of simultaneities might be to select the first simultaneity at random, and then to select subsequent simultaneities so that the pitch commonality and pitch distance between sequential pairs fell within prescribed limits. In order to give progressions more unity, it may be useful to prescribe a kind of tonal centre, defined as an array of salience against pitch category (Sect. 6.2.5), and to specify limits on pitch commonality between individual simultaneities and the tonal centre.

These techniques are similar to mathematical (including serial) techniques advocated by some twentieth century composers (e.g. Xenakis, Boulez), in that they make the composer's job easier: part of the compositional process becomes more or less automatic. The techniques described here have the additional advantage that they are based not on arbitrary or intuitive principles but on established psychoacoustical theory and data.

To be successful, new styles should grow out of existing, accepted styles, and should be introduced gradually to their audiences. New music needs to be introduced in appropriate contexts; for example, atonal styles may succeed in film soundtracks but fail in the concert hall. New music needs to be performed on appropriate instruments; for example, atonal electronic styles may succeed where atonal orchestral styles fail. These considerations hold regardless of the extent to which new styles may be based on universal aspects of auditory perception.

The techniques described above are not supposed to have any particular aesthetic value. As pointed out by Lerdahl and Jackendoff [1983], "Both tonality and contemporary techniques have produced masterpieces; both have produced trash" (p. 301). The primary source of the quality of a piece of music has always been (and, hopefully, will always remain) the personal contribution of the composer.