Asymmetries in Business Cycles and the Role of Oil Prices

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Abstract

We estimate asymmetries in innovations to Solow residuals for eleven OECD countries using Stochastic Frontier Analysis. Likelihood ratio statistics and variance ratios imply that all countries with net energy imports have significant negative asymmetries, while other countries do not. We construct a simple theoretical model in which the measured Solow residual combines effects from technology, factor utilization, and the terms of trade. For oil importers, the model implies an asymmetric response of measured TFP to oil price increases and decreases. When we condition Solow residuals separately on positive and negative oil price changes to allow asymmetric responses, evidence for remaining negative asymmetric innovations to the Solow residuals vanishes for all countries except Switzerland. Switzerland's relatively dominant financial sector suggests that their asymmetries could be due to a financial crisis, a hypothesis that we test and fail to reject.

Keywords: Solow residuals, stochastic frontier analysis, oil prices, financial crises
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1 Introduction

There is considerable evidence that business cycles are asymmetric. Recessions tend to be short and deep, while expansions are mild and protracted. Macroeconomists seek to understand this asymmetry.

A standard practice in macroeconomics is to model output as dependent on inputs of capital and labor with a residual, representing total factor productivity (TFP). This residual is typically measured using the Solow residual from a Cobb-Douglas production function. It captures technology as well as anything which affects output other than capital and labor inputs. In a seminal paper, Prescott (1986) demonstrated that innovations to TFP, drawn from a symmetric distribution, could explain more than half of the variance of output over business cycles. This result focused attention on symmetric innovations to TFP as a driving force in business cycles.

We are interested in the possibility that asymmetric business cycles could be characterized by asymmetric innovations to TFP, motivated by Barro (2006). In an attempt to explain the equity premium puzzle created by DSGE modeling, he postulates an asymmetric time series process for total productivity. Specifically, he assumes that the log of total factor productivity evolves as a first order autoregressive process with a constant mean and an error with two components. One component is i.i.d normal, capturing symmetric productivity shocks, as in the standard Dynamic Stochastic General Equilibrium (DSGE) model. The second component is asymmetric and has a distribution containing only negative values. It captures rare disasters, which Barro defines to include war, natural disasters, financial crises – any event severe enough to create at least a fifteen percent cumulative fall in GDP.

By now it is well known that TFP, measured by the Solow residual, is not entirely an exogenous shock, but instead is a combination of an exogenous shock and endogenous responses to other exogenous shocks. For example, an exogenous shock other than productivity, could create an endogenous response of capacity, thereby affecting the Solow residual.[1] Ramey (2016) surveys a growing literature which seeks to separate the endogenous and exogenous components of the Solow residual with the objective of using the exogenous component as the business cycle driver. However, she notes that the alternative measures of the exogenous components in recent articles are not highly correlated with each other and that they have conflicting business cycle effects. She concludes that

[1] Work on the relationship between the Solow residual and capacity utilization includes Shapiro (1993), Burnside and Rebelo (1995), and Basu and Kimball (1997).

we do not yet have a consensus yet on how to do the separation.

We do not attempt a separation of the Solow residual into exogenous and endogenous components. Instead, we ask whether innovations to the Solow residual, itself a combination of an exogenous shock and the endogenous response to exogenous shocks, are drawn from an asymmetric distribution. In particular, if the underlying distribution has negative skewness, then the probability of a large adverse innovation exceeds the probability of a large favorable innovation, potentially attributing deep recessions to the asymmetric behavior of the innovation to the Solow residual. And since the mode innovation is positive in a distribution with negative skewness, then expansions characterized by these innovations should be longer than contractions. However, a finding of asymmetry alone provides no information on whether the asymmetry is due to an asymmetric exogenous shock to TFP or to asymmetry in the endogenous component.

The first goal of this paper is to determine whether OECD countries have symmetric or asymmetric innovations to TFP, measured by Solow residuals. We assume that total factor productivity, measured by the Solow residual, has a symmetric i.i.d. component and an asymmetric component with only negative values, as in Barro (2006). In contrast to Barro, we test for the presence of asymmetry in TFP instead of measuring it as large negative GDP outcomes. We compute the innovation in the Solow residual[2] and use Stochastic Frontier Analysis (SFA) to estimate the model. SFA is typically employed in the micro literature to study firm-level productivity in a cross-section. We adapt it to estimate the extent of asymmetry in aggregate time series. Furthermore, we extend the model to allow for arbitrary signs of skewness following the approach suggested in Hafner et al. (2016). We compare the restricted model without asymmetries to the unrestricted model using a likelihood ratio test. Additionally, we estimate the ratio of variances for the two components of the innovation to compare the extent of asymmetries across countries. Our sample consists of eleven OECD countries which had quarterly data available back at least to the early 1980's on GDP, employment, and investment.

We reject the null of symmetry in favor of negative asymmetry for most countries in our sample, generally confirming Barro's hypothesis on asymmetry in TFP. The rejection itself is atheoretical, providing no information on the determinates of the asymmetry, particularly whether it is due to asymmetry in the exogenous innovation or in the endogenous response of TFP to other exogenous shocks. However, there is a notable distinction

[2]To compute the innovation, we first difference the logarithm of the Solow residual to generate a stationary time series. Then we regress the log differenced Solow residual on sufficient lagged values of itself to produce a residual without autocorrelation.

dividing countries with asymmetries from others, which suggests a hypothesis we can test. All countries with negative asymmetries are net energy importers while none of the other countries are, leading us to consider the role of energy prices in creating the asymmetry.

There is a large empirical literature on output responses to oil prices.[3] Hamilton (1983) demonstrated that all but one post World War II recession in the US were preceded by a large rise in oil prices. Mork (1989) and Hamilton (2011) have argued that business cycles exhibit asymmetric responses to oil price shocks, but Kilian and Vigfusson (2011a,b) fail to reject the null of no asymmetry in a VAR with oil prices and GDP. Kilian (2008) finds negative responses in the US to oil events, measured as international supply disruptions. Engemann et al. (2011) argue that the probability of switching to a recession regime in a Markov switching model depends on the price of oil for the US and several other OECD countries. Hall (1988) explicitly addressed TFP instead of output and found dramatic falls in productivity with oil price increases.

We present a simple model to illustrate how the endogenous response of the Solow residual for oil importers to symmetric oil price changes could be asymmetric across positive versus negative price changes. In the model, the measured Solow residual is the product of exogenous technology, endogenous capacity utilization, and an endogenous terms-of-trade component.

Finally, we consider whether conditioning measured TFP, not only on its own lags but also on the lags of oil price changes, separated into positive and negative values to allow the asymmetric response, can reduce the asymmetry in the residual. We find that this conditioning reduces negative asymmetries, measured by variance ratios, for all energy-importing countries except France. Additionally, it implies that we can no longer reject the null hypothesis that the measured Solow residuals are symmetric in favor of the alternative of negative asymmetry for all countries except Switzerland. Given the large role of the financial sector in Switzerland, we hypothesize that its asymmetries could be due to financial crises and find that conditioning on financial crises leaves us unable to reject the null of symmetry.

Our work testing for asymmetry in TFP is related to other empirical work on business cycles which either seeks to confirm asymmetry or to characterize it econometrically. One approach is to measure amplitude and frequency of expansions and recessions. McKay and Reis (2008) confirm that contractions are briefer and more violent when measured

[3]Our focus is on asymmetry in TFP, measured with Solow residuals, not in total output, but the two concepts are closely related since TFP is one component of the production function.

by employment but not when measured by GDP. NBER business cycle dating generates recessions with smaller duration and frequency than booms for the US. Another approach postulates Markov switching between expansion and recession regimes. Markov switching models (Hamilton 1989, Raymond and Rich 1997, Clements and Krolzig 2002, Engemann et al. 2011) similarly find that recession regimes are systematically different from expansion regimes. These approaches tend to find asymmetry, and like ours, none require specification of potential determinants of asymmetry as a component of the empirical tests.

The rest of the paper is organized as follows. In Section 2 we introduce our econometric methodology. Section 3 presents the data and the baseline empirical results. Section 4 develops a theoretical model to understand the contribution of oil price changes to the negative asymmetries for oil importers. Section 5 presents additional results on asymmetries for oil price importers after conditioning on oil positive versus negative price changes. Section 6 concludes.

2 Econometric Methodology

In this section, we present an econometric model to investigate and test whether Solow residuals have an asymmetric component. Asymmetry is a key issue in the literature on production efficiency. The classical model of stochastic frontier analysis (SFA), proposed simultaneously by Meeusen and van den Broek (1977) and Aigner et al. (1977) to estimate inefficiencies across individual firms, incorporates a composite error term which, under the null hypothesis of efficiency, is symmetric. We modify this framework to include additional dynamic effects that take into account potential serial correlation of Solow residuals and use it to estimate the ratio of variances of the symmetric and asymmetric error components. Furthermore, we extend the stochastic frontier model to allow for both positive and negative residual skewness, which allows for standard asymptotics when testing for asymmetry.

2.1 Model specification

We follow Barro (2006) and assume that the first difference of logged productivity is determined by a composite error. We use the data described below to construct a measure of the Solow residual (SR_{Mt}), and express the log-differenced Solow residual (q_t) as

$$q_t = \log(SR_{Mt}) - \log(SR_{Mt-1}) = \mu_t + w_t, \tag{1}$$

where, conditional on the information set generated by lagged Solow residuals, μ_t is the conditional mean of q_t, and w_t is a composite error term with mean zero. We assume that this error term can be decomposed as

$$w_t = v_t - u_t, \tag{2}$$

where v_t is a Gaussian random variable, $v_t \sim N(\mu_u, \sigma_v^2)$ with mean μ_u. The normal-exponential stochastic frontier model assumes the second term u_t to be an exponentially distributed (positive) random variable with mean $\mu_u > 0$. Note that by construction the mean of w_t is restricted to be zero. Other typical choices for the distribution of the one-sided error term u_t are half-normal, truncated normal, or gamma (Kumbhakar and Lovell 2000, Murillo-Zamorano 2004). For our data, described in Section 3, we found that the exponential distribution provides the best fit, so we restrict attention to this model. The presence of the term u_t implies that the distribution of the composite error w_t is asymmetric, except for the degenerate case in which $\mu_u = 0$ and the distribution reduces to a normal with variance σ_v^2. However, this model restricts the skewness of w_t to be negative, which can cause problems for inference.

Estimation of this model is problematic when the sample skewness is positive. Aigner et al. (1977) demonstrated that theoretically in such situations the MLE of μ_u will converge to zero, and Lee (1993) showed that in this case the information matrix is singular, which implies that maximum likelihood standard errors cannot be calculated. In practice, when the residuals have positive skewness, the maximum likelihood estimator will either fail to converge or will converge to a local maximum. A solution to this problem is proposed in Hafner et al. (2016), which we adapt to our setting. We extend the distribution of u_t to allow for negative values and extend its parameter μ_u to lie in $\mathbb{R}$. To be specific, when $\mu_u < 0$ we define the density of u_t to be an exponential distribution mirrored onto the negative axis by reflecting it at zero. This mirrored exponential distribution has mean μ_u and standard deviation $|\mu_u|$. The distribution of w_t therefore always has mean zero by construction, but allows for positive and negative skewness depending on the sign of μ_u.

The specification of the conditional mean μ_t depends on potential serial correlation of the growth rates of Solow residuals q_t. We consider autoregressive models of order p, AR(p), given by

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + w_t, \tag{3}$$

where the usual stationarity conditions are assumed to be satisfied, and w_t is the composite error term defined by equation (2).[4]

2.2 Estimation and evaluation

Estimation of model (3), including the autoregressive coefficients and parameters in the distributions for u_t and v_t can be conveniently done by maximum likelihood.

A closed form expression for the density of the composite error $w_t = v_t - u_t$ in the standard normal-exponential stochastic frontier model (i.e. when $\mu_u > 0$) exists and is given by (see Kumbhakar and Lovell 2000)

$$f(w_t; \mu_u, \sigma_v) = \frac{1}{\mu_u} \Phi\left(-\frac{w_t - \mu_u}{\sigma_v} - \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right), \tag{4}$$

where Φ denotes the standard normal distribution function and μ_u is the parameter of the exponential distribution, which has mean equal to μ_u and variance equal to μ_u^2. The mean of u_t is subtracted from w_t to account for the fact that, unlike in the classical stochastic frontier model, w_t is assumed to have mean zero. The log-likelihood function of the joint model, including the autoregressive dynamics, can be obtained in a straightforward way.

For the extended distribution to deal with cases in which the sample skewness of the residuals of the autoregressive model in equation (3) is allowed to take any sign we obtain the composite density of w_t,

$$\begin{aligned} f(w_t; \mu_u, \sigma_v) \;\; = \;\; & \frac{1}{\mu_u} \Phi\left(-\frac{w_t - \mu_u}{\sigma_v} - \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right) I(\mu_u \geq 0) \\ - \;\; & \frac{1}{\mu_u} \Phi\left(\frac{w_t - \mu_u}{\sigma_v} + \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right) I(\mu_u < 0), \end{aligned} \tag{5}$$

which for $\mu_u > 0$ corresponds to the one given in (4) with negative skewness, and for $\mu_u < 0$ is the mirrored version with positive skewness. $I(\cdot)$ denotes the indicator function.

The density of the extended model is shown in Figure 1. The mean of the density is equal to zero for all three cases. However, for the distribution with negative skewness, the one represented by the long dashed curve, there is a higher probability of a severely adverse outcome compared to the distribution with no skewness, the one with the solid

[4]In our application we experimented with MA terms and found them unnecessary to capture dynamics.

line. Additionally, with negative skewness, there is a higher probability of moderately positive outcomes compared with the distribution without skew.

Substituting the model (5) for equation (4) avoids convergence problems of the estimation algorithm due to positive sample skewness. A further advantage of using this model compared with (4) is that testing for symmetric error terms in our framework is equivalent to testing $H_0 : \mu_u = 0$, which can be done with a likelihood ratio test. With the restriction $\mu_u \geq 0$, however, the parameter to test is on the boundary of the parameter space under the null hypothesis. Lee (1993) showed that, in this case, the likelihood ratio statistic asymptotically follows a mixture of a χ^2 distribution with one degree of freedom and a point mass of 1/2 at zero. In the case of the extended model (5), Hafner et al. (2016) show that, despite the fact that the information matrix is still singular at $\mu_u = 0$, the likelihood ratio statistic follows a standard χ^2 distribution because the parameter μ_u is no longer on the boundary under the null hypothesis.

The relative importance of u_t can be measured by computing the variance ratio (VR) of the asymmetric component relative to the symmetric component as

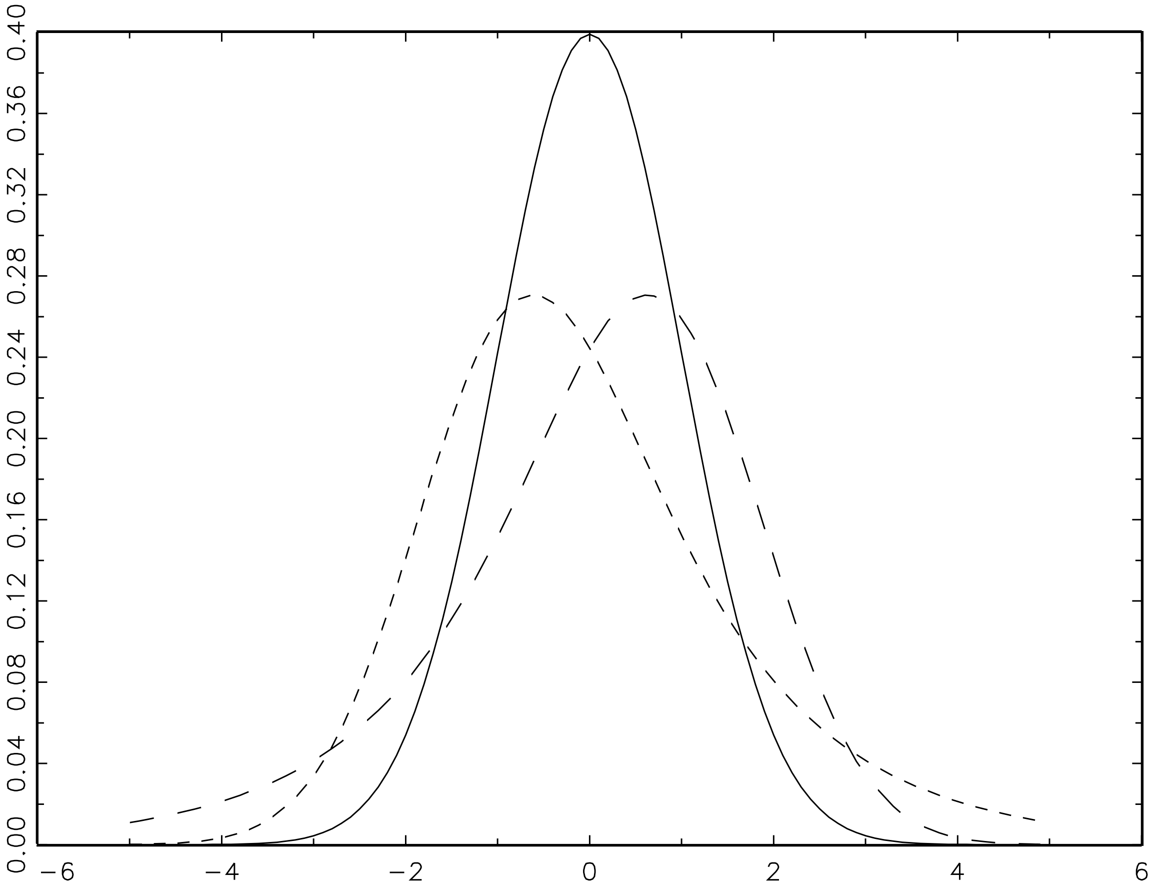
$$VR = \frac{\mu_u^2}{\sigma_v^2}, \tag{6}$$

which is estimated using the parameter estimates for the two error components. Note that the variance ratio measures the degree of asymmetry in the data with an increase in the ratio denoting more asymmetry. This is in contrast to the traditional stochastic frontier literature where the inefficiency of a firm is measured by *technical efficiency* (TE) defined by Battese and Coelli (1988) as $E[\exp(-u)|w]$. TE measures the distance from the efficient frontier or actual output divided by optimal efficient output. However, in our context TE cannot be interpreted, because the expectation of u is absorbed by the mean of v and consequently only the variance of u relative to the variance of v is relevant.

3 Empirical Analysis of Asymmetries

The data set consists of seasonally-adjusted quarterly observations on constant-price GDP, investment, and employment for all OECD countries which had data dating back at least to the early 1980's. Data is from Main Economic Indicators (MEI) from the OECD. The OECD does not have data on labor hours, a preferable measure for the labor input, or on capital stock. We include the following countries: Australia, Canada, France, Germany, Italy, Japan, Korea, Norway, Switzerland, United Kingdom and United States.

Figure 1: Density function (5) with $\sigma_v = 1$. Solid curve: $\mu_u = 0$ (Gaussian), long dashed curve: $\mu_u = 1.5$ (negative skewness), short dashed curve: $\mu_u = -1.5$ (positive skewness)



The sample begins in 1973:Q1, with the exception of Korea (1983:Q1) and Switzerland (1976:Q1), and ends in 2011:Q3. We let the sample begin in 1973 because, looking forward to our hypothesis about oil prices, Kilian and Vigfusson (2011b) argue that due to the regulation of nominal oil prices prior to 1973, the dynamic properties of oil prices in that period contain no information about the period following 1973. We do not use a panel since we want to allow behavior to differ across countries.[5] Since we are interested in business cycle properties, quarterly frequencies are essential. Therefore, we estimate

[5]Additionally, the data set does not constitute a panel because units of measurement for each country's output and investment differ since they are measured in country-specific units of 2005 GDP. Data from the Penn World Tables does adjust cross-country data to comparable units, using purchasing power parity measures of relative prices, but that data exists only at annual frequency.

eleven separate equations, decomposing the variance of each country's Solow residual into a symmetric and an asymmetric component.

To construct Solow residuals, we first construct measures of the capital stock. We use the perpetual inventory method, letting the initial value of the capital stock (K_0) be the steady-state equilibrium value with the growth rate (g), equal to the average of growth over the first ten years of the sample, annual depreciation (δ) at 0.07, following Easterly and Levine (2001), and initial investment equal to its initial value (I_0).[6] Subsequent values for capital are computed using the equation for the adjustment of the capital stock,

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{7}$$

To compute Solow residuals, we use employment as the measure of labor input and set capital's share at 0.35, following Stock and Watson (1999).

Augmented Dickey-Fuller tests with and without a trend do not reject the null hypothesis of a unit root in Solow residuals for all countries. Therefore, we compute the first differences of the logarithm of the series. Before estimating our models we standardize the difference of log Solow residuals to have mean zero and variance equal to one. This standardization does not affect the estimation of the quantities of interest such as the skewness or the variance ratio. Empirical autocorrelations and partial autocorrelations (not reported) suggest that low order autoregressive models capture the dynamics in the data and that no significant autocorrelation is present for many series[7]. We estimate simple AR(1) models for all countries but our results are robust with respect to that choice using, e.g., four lags for all countries. The estimated mean equations can be found in Table 3 in Appendix A.

We present the estimation results from our baseline dynamic stochastic frontier (DSF) model, defined in equations (1) to (3) and the error density given by (5), in Table 1 . The table contains the estimated parameters of the composite error term, the sample skewness of the residuals w_t, the log-likelihood of both the model restricted to symmetry (LL sym) and our dynamic stochastic frontier model (LL DSF), the likelihood ratio statistic (LR stat) for the null of symmetry along with its p-value, and the variance ratio (VR) implied by the estimated model.

The likelihood ratio test rejects the null hypothesis of symmetry in favor of the alternative of negative asymmetry for all countries except Australia, Canada and Norway.

[6]The initial capital stock becomes $K_0 = \frac{I_0}{g+\delta}$. Since the sample begins with 1973Q1, I_0 is investment in this first period.

[7]We do not report detailed results to preserve space but these are available upon request.

For Australia the symmetric model is rejected, but in favor of the model with positive skewness. For Canada and Norway, we fail to reject the null.

Our finding of asymmetry is completely independent of what might be creating it. We propose a hypothesis based on characteristics of the countries which exhibit the asymmetries. Countries for which we fail to reject the null of symmetry, are net energy exporters. Both Canada and Norway are oil exporters. Australia is a net oil importer, but a net exporter of energy. The UK's experience is mixed, with net exports from 1981 to 1988 and from 1993 to 2003, and net imports in other periods (Bolton 2010). These results suggest that oil prices could have a role in creating asymmetries in Solow residuals for countries which are net importers of energy.

Table 1: Estimation results of the DSF model

	Aus	Can	Fra	Ger	Ita	Jap	Kor	Nor	Swi	UK	US
μ_u	-0.57***	-0.26	0.49***	0.58***	0.59***	0.69***	0.56***	-0.56***	0.60***	0.70***	0.60***
σ_v	0.80***	0.93***	0.82***	0.80***	0.69***	0.70***	0.78***	0.80***	0.61***	0.68***	0.79***
skewness	0.318	0.020	-0.161	-0.590	-0.635	-0.825	-0.675	0.269	-2.631	-0.340	-0.222
LL AR	-214.4	-210.9	-211.5	-215.6	-203.6	-216.8	-159.8	-212.8	-191.9	-216.7	-215.3
LL DSF	-212.5	-210.9	-210.0	-211.9	-199.1	-210.4	-154.7	-211.8	-172.6	-212.8	-213.2
LR stat.	3.838	0.001	3.017	7.421	9.010	12.869	10.228	1.918	38.515	7.616	4.1868
p-val.	0.050†	0.976	0.082	0.006	0.003	0.000	0.001	0.166	0.000	0.006	0.041
VR	0.513	0.079	0.361	0.537	0.740	0.965	0.516	0.495	0.963	1.063	0.575

Note: Table 1 reports the estimation results of model (5) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively. A † represents a rejection of the null hypothesis of symmetric innovations when the estimated model implies positive skewness.

4 Oil Prices and Asymmetric Solow Residuals

We present a model in which the Solow residual combines an exogenous technology component with endogenous terms-of-trade and capacity utilization components. Oil prices do not directly affect technology. However, an increase in the world price of energy, created by an increase in the price of oil, could affect measured TFP through the terms of trade and capacity utilization.

We construct a simple model of a representative firm, operating in an open economy, to show that the Solow residual for the representative firm responds non-linearly and asymmetrically to an increase in the relative price of imported energy. Our model complement's Hamilton's (1996, 2011) demand-side argument for an asymmetric output response to oil price changes.

4.1 Model setup

We assume that all countries have some ability to produce their own energy, and that this ability varies across countries. The representative firm in each country chooses how much energy to produce based on the price of imported energy and its own production costs, and imports the remainder. We assume that the price of imported energy is exogenous to the representative firm.

Since imported energy's share is empirically increasing in price, we follow Hassler et al. (2012) and specify a CES production function for output as

$$Y_t = \left[(1-\gamma) \left[A_t \tilde{K}_t^{\alpha} \tilde{L}_t^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \gamma \left[A_t^E E_t \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{8}$$

where γ is imported energy's share in the event that $\epsilon = 1$, E_t is imported energy, A_t^E is technology in energy, ϵ is the elasticity of substitution between the capital-labor composite and energy, and $\tilde{K}$ and $\tilde{L}$ represent capital and labor utilization, respectively. We assume $\epsilon < 1$ to be consistent with empirical evidence that imported energy's share is increasing in price. The value for γ determines the curvature of production with respect to imported energy. A country which produces all of its own energy domestically will have $\gamma = 0$. We think about γ as a structural decision made by the representative firm based on its own production technology for energy and the imported price. This decision cannot be changed in the short run. However, if the price of energy gets too high, even with the current value for γ, the firm could be better off producing all of its own energy and could choose to do so. This puts an upper bound on the price for which the firm will import.

We assume that the energy price is low enough that the representative firm chooses to import. In this case, the firm chooses the quantity of imported energy input to maximize

$$\left[(1-\gamma)\left[A_t\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}\right]^{\frac{\epsilon-1}{\epsilon}}+\gamma\left[A_t^E E_t\right]^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}-p_t E_t,$$

where p_t is the relative price of energy in terms of production. First order conditions require that the marginal product of imported energy in production equal its relative price

$$\frac{\partial Y_t}{\partial E_t}=p_t,$$

yielding an expression for optimal imported energy as

$$E_t = Y_t p_t^{-\epsilon}\gamma^{\epsilon}\left(A_t^E\right)^{\epsilon-1}. \tag{9}$$

Using equation (9), imported energy's share can be expressed as

$$\frac{p_t E_t}{Y_t}=\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}. \tag{10}$$

Therefore, with $\epsilon < 1$, imported energy's share of production depends positively on its relative price. In the case with $\epsilon = 1$, imported energy's share is simply γ, independent of price.

Substituting equation (9) for E_t into equation (8) for production yields an expression for production as a function of the relative price of energy,

$$Y_t = A_t\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}(1-\gamma)^{\frac{\epsilon}{\epsilon-1}}\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}}. \tag{11}$$

Using equations (9) and (11), we can solve for imported energy demand as a function of price and the utilized capital-labor composite $\left(\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}\right)$ as

$$E_t = (1-\gamma)^{\frac{\epsilon}{\epsilon-1}}\frac{A_t}{A_t^E}\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}\left[\left(\frac{p_t}{\gamma A_t^E}\right)^{\epsilon-1}-\gamma\right]^{\frac{\epsilon}{1-\epsilon}}. \tag{12}$$

The firm will choose to import positive quantities of energy only if the term in square brackets is positive, implying an upper bound on the energy price for which it chooses to import. For $\epsilon < 1$, derivatives of equations (11) and (12) illustrate that an increase in the price of energy reduces energy imports, thereby reducing production for a given value for capital and labor.

Kehoe and Ruhl (2008) explain that we must calculate the Solow residual from value added, not from production. Value added, not total production, is the GDP concept. Value added (GDP) is given by production minus the value of imports in terms of production. Using equations (10) and (11) yields

$$GDP_t = Y_t - p_t E_t = Y_t \left(1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right) = A_t \tilde{K}_t^{\alpha} \tilde{L}_t^{1-\alpha} \left(1-\gamma\right)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{13}$$

An increase in the relative price of energy is a deterioration in an energy-importer's terms of trade, and this reduces the value of GDP measured in terms of domestic goods.

4.2 Solow Residual

We define the Solow residual as GDP divided by the capital-labor composite yielding

$$SR_t = \frac{A_t \tilde{K}_t^{\alpha} \tilde{L}_t^{1-\alpha} \left(1-\gamma\right)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{K_t^{\alpha} L_t^{1-\alpha}} = A_t \mu_t \left(1-\gamma\right)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},$$

where

$$\frac{\tilde{K}_t^{\alpha} \tilde{L}_t^{1-\alpha}}{K_t^{\alpha} L_t^{1-\alpha}} = \mu_t = \mu_{Kt}^{\alpha} \mu_{Lt}^{1-\alpha}$$

with μ_K and μ_L representing utilization factors for capital and labor respectively, where both are choice variables for the firm. This representation implies that the Solow residual is the product of three terms, technology (A), a capacity utilization factor (μ_t), and a terms-of-trade factor given by

$$SR_{TOT,t} = \left(1-\gamma\right)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{14}$$

We view the technology term as purely exogenous and consider the other two, each in turn. Begin with the terms of trade factor.

4.2.1 Terms of Trade and the Solow Residual

The terms-of-trade component of the Solow residual is given by equation (14). Taking the derivative of equation (14) with respect to the energy price and computing the elasticity yields

$$\frac{p_t}{SR_{TOT,t}} \frac{\partial SR_{TOT,t}}{\partial p_t} = -\gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon} < 0,$$

which is imported energy's share from equation (10). Therefore, an increase in the price of imported energy reduces imports, thereby reducing the Solow residual, reducing output for given capital and labor. The derivative of the elasticity with respect to price is negative, implying that the logarithm of the Solow residual is falling at an increasing rate in the logarithm of the energy price. These two derivatives imply a negative and non-linear response of the Solow residual to an increase in the energy price. The same percentage change in price has a larger absolute effect on the percentage change in the Solow residual if the price change is positive.

We can understand the effect of energy price increases and decreases using a diagram. Figure 2 graphs production, equation (8), as a function of imported energy for a given value of the utilized capital labor composite $\left(\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}\right)$ as Y, and the cost of imported energy for a given price, P_j, as P_jE. Under the assumption that energy price is low enough for the firm to prefer imports, the profit maximizing use of energy occurs at E_0 where the slope of the production function equals the price of energy; equivalently, where the slope of Y equals the slope of P_0E. The value for GDP_0 is given by the vertical distance between Y and PE at the point where the slopes are equal. The terms-of-trade component of the Solow residual is the value of GDP_0 divided by the utilized capital-labor composite $\left(\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}\right)$.

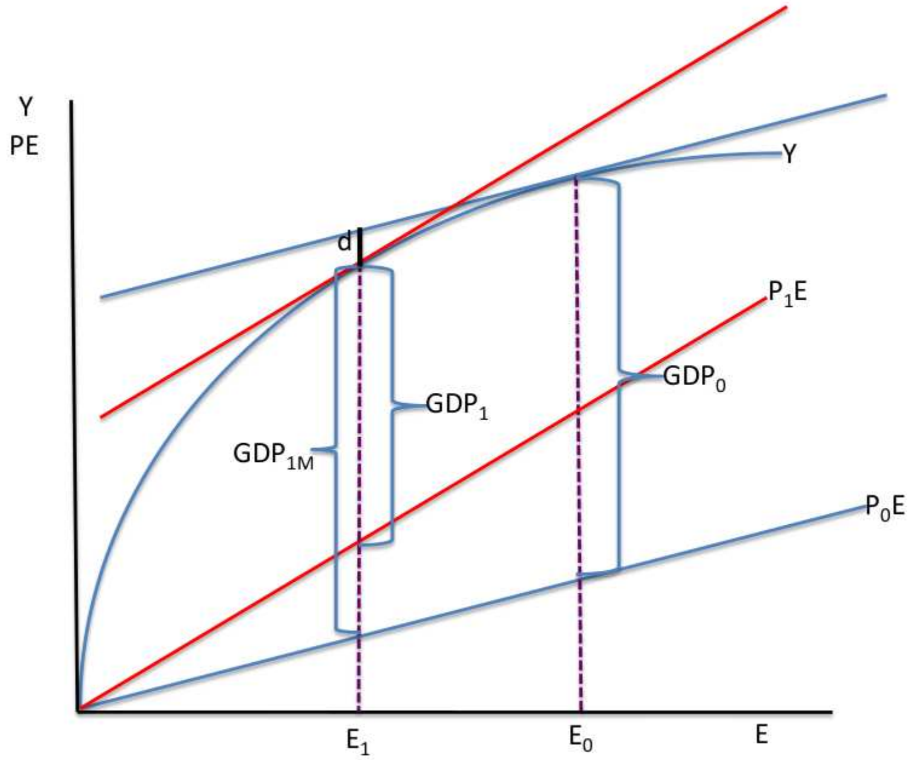
Now, consider the effect of an increase in the price of imported energy (P_j) on GDP, holding the utilized capital-labor composite constant, under the assumption that the price increase is small enough that the representative firm continues to import energy. As price increases to $P_1 > P_0$, the slope of P_jE rises, and equilibrium occurs where the slope of the production function is higher to match the higher energy price, at E_1. Therefore, holding the utilized capital-labor composite constant, energy use falls and GDP falls from GDP_0 to GDP_1. Since GDP is lower for a given value for $\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}$, the terms-of-trade component of the Solow residual is lower. A decrease in price would reduce the slope of the P_jE line, increasing GDP. Therefore, the terms-of-trade component of the Solow residual is decreasing in the relative price of energy.

4.2.2 Capacity Utilization and the Solow Residual

Now, consider the effect of a change in the energy price on capacity utilization. Using equations (13) and (14), we can rewrite the equation for GDP as

$$GDP_t = A_t\tilde{K}_t^{\alpha}\tilde{L}_t^{1-\alpha}SR_{TOT,t} = A_tK_t^{\alpha}L_t^{1-\alpha}\mu_tSR_{TOT,t}.$$

Figure 2: Real GDP and Measured Real GDP



An increase in $SR_{TOT,t}$ raises the marginal products of utilized capital and labor. In equilibrium, the firm responds to the increase in marginal products by raising both the quantities of capital and labor and their utilization rates.[8] Therefore, the capacity utilization component of the Solow residual (μ_t) is an increasing function of $SR_{TOT,t}$ $(\mu_t'(SR_{TOT,t}) > 0)$, implying that an increase in the price of oil reduces the terms of trade component of the Solow residual, in turn reducing the capacity utilization component according to

$$\frac{\partial \mu_t}{\partial p_t} = \mu_t'(SR_{TOT,t}) \frac{\partial SR_{TOT,t}}{\partial p_t} < 0. \tag{15}$$

[8]The model needs to have reasons that the firm might respond with changing factor utilization instead of changing factor quantities. For capital, we generally assume that it is fixed in the short-run, but its utilization is not. For labor, hiring and firing costs can yield labor hoarding as reflected in changing labor utilization.

It appears that an increase in the price of oil reduces the Solow residual on two counts, one based on the terms of trade effect and one based on a capacity utilization effect. But, the section below illustrates that this is not quite correct.

4.2.3 Constant Dollars and the Solow Residual

Although optimal firm decisions on inputs and capacity utilization are based on measurement using the relative price of imported energy in terms of production, Kehoe and Ruhl (2008) explain that this measure of the Solow residual is not the measure in the national income accounts. The measure of real value added in the data is not in terms of a numeraire (production in our example) but in terms of constant dollars.[9] Letting $\bar{p}$ denote the dollar price of imported energy in the base year with the dollar price of output at unity, and using equation (9), we can express value added in constant dollars, which we call measured GDP ($GDP_{M,t}$) as

$$GDP_{M,t} = Y_t - \bar{p}E_t = Y_t \left[1 - \bar{p}p_t^{-\epsilon}\gamma^{\epsilon}\left(A_t^E\right)^{\epsilon-1}\right] = Y_t \left[1 - \gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\frac{\bar{p}}{p_t}\right]. \qquad (16)$$

Using the first equality in equation (16), the derivative of value added with respect to p_t is given by

$$\left(\frac{\partial Y_t}{\partial E_t} - \bar{p}\right)\frac{\partial E_t}{\partial p_t} = (p_t - \bar{p})\frac{\partial E_t}{\partial p_t}.$$

Given that a firm optimally equates the marginal product of energy with the price, there is no effect of a marginal increase in the price of energy on measured value added if the price equals the baseline price of $\bar{p}$. However, when price exceeds its baseline value an increase in energy price takes on the negative sign of the effect of an increase in price on energy imports. And when the price is below the baseline, the effect of an increase in energy price on measured GDP is positive.

The effect of energy prices on the Solow residual mirrors their effect on the value added measure of GDP. Using equations (11) and (16), the measured Solow residual in the data can be expressed as

$$SR_{M,t} = \frac{GDP_{M,t}}{K_t^{\alpha}L_t^{1-\alpha}} = A_t\mu_t^{\frac{\epsilon}{\epsilon-1}}SR_{MTOT,t},$$

[9]Kehoe and Ruhl show that there would be no effect of the price of energy on the Solow residual if we measured it in chained dollars. However, there are no chained-dollar measures of the Solow residual and since chained dollar measures do not have additive and multiplicative properties (Whelan 2000), we cannot construct them from chained measures of investment and output. Furthermore, even if chained values were available, they have the counterintuitive property that the change in the relative price of an input has no effect on 'real' profit, holding other inputs constant.

where the terms-of-trade component is given by

$$SR_{MTOT,t} = (1-\gamma)\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}}\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\frac{\bar{p}}{p_t}\right].$$

The price elasticity of the terms-of-trade component of the measured Solow residual is given by

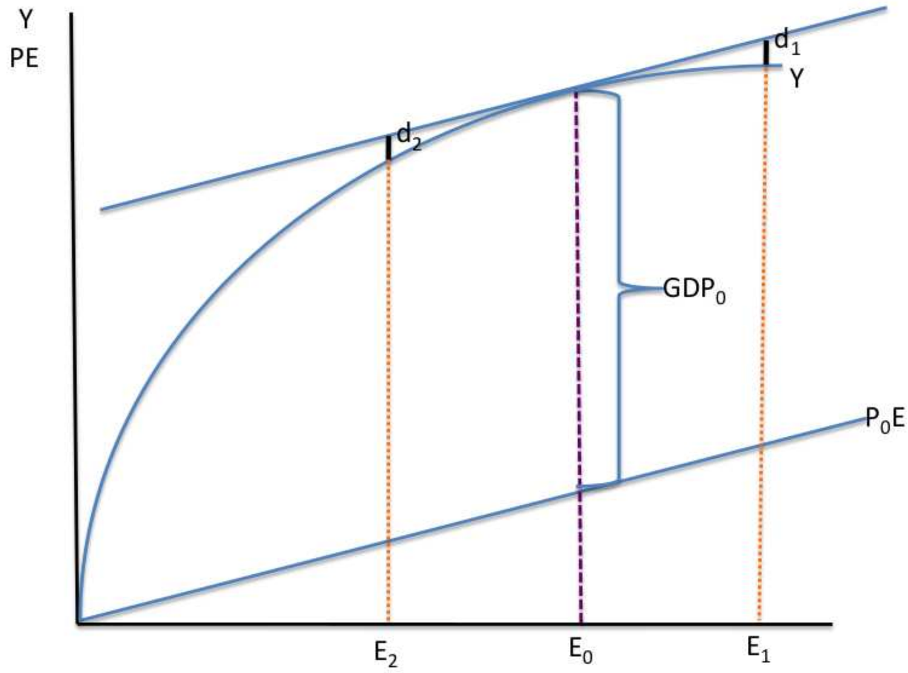
$$\begin{aligned} \frac{p_t}{SR_{M,t}}\frac{\partial SR_{M,t}}{\partial p_t}|\mu_t &= -\left(\frac{p_t-\bar{p}}{p_t}\right)\frac{(1-\gamma)^{\frac{\epsilon}{\epsilon-1}}\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}}{\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\frac{\bar{p}}{p_t}\right]} \qquad (17)\\ &= \left(\frac{p_t-\bar{p}}{p_t}\right)\left(\frac{p_t}{SR_{TOT,t}}\frac{\partial SR_{TOT,t}}{\partial p_t}\right)\frac{(1-\gamma)^{\frac{\epsilon}{\epsilon-1}}}{\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\frac{\bar{p}}{p_t}\right]}. \end{aligned}$$

The price elasticity is proportional to the actual terms-of-trade component when when price is above the baseline value. Otherwise, it takes the opposite sign. The logarithm of the terms-of-trade component of the measured Solow residual is concave in the log of price with the peak occurring at $p_t = \bar{p}$.

To facilitate intuition for this very asymmetric result, we illustrate the behavior of the terms-of-trade component of the measured Solow residual in constant dollars using Figure 2. After the increase in price, the equilibrium quantity of imported energy is allowed to change, but since price is in constant dollars, the price is fixed. Therefore, the measured value for GDP after the price increase equals production at the new optimal energy input of E_1 less energy inputs valued at P_0, not at P_1, yielding a value for measured GDP as GDP_{1M}. GDP falls only by the vertical distance d. Therefore, the effect of the increase in the price of energy on GDP and on the Solow residual is much smaller in the data, and would be zero if there were no concavity in production with respect to imported energy. Indeed, an infinitesimally small price change, as with a derivative, implies no change in measured GDP.

Now compare the effect of a price increase and a price decrease on the terms-of-trade component, using Figure 3. We assume that the quantity of imported energy is initially optimal at E_0. As price changes, E moves in the opposite direction. GDP is the difference between the new value of production and the new value of energy imports measured at the initial unchanged price. Therefore, a price decrease raises imported energy to E_1 and decreases GDP by d_1, whereas a price increase reduces imported energy to E_2 and reduces GDP by d_2. Consequently, both positive and negative changes in price reduce GDP for a

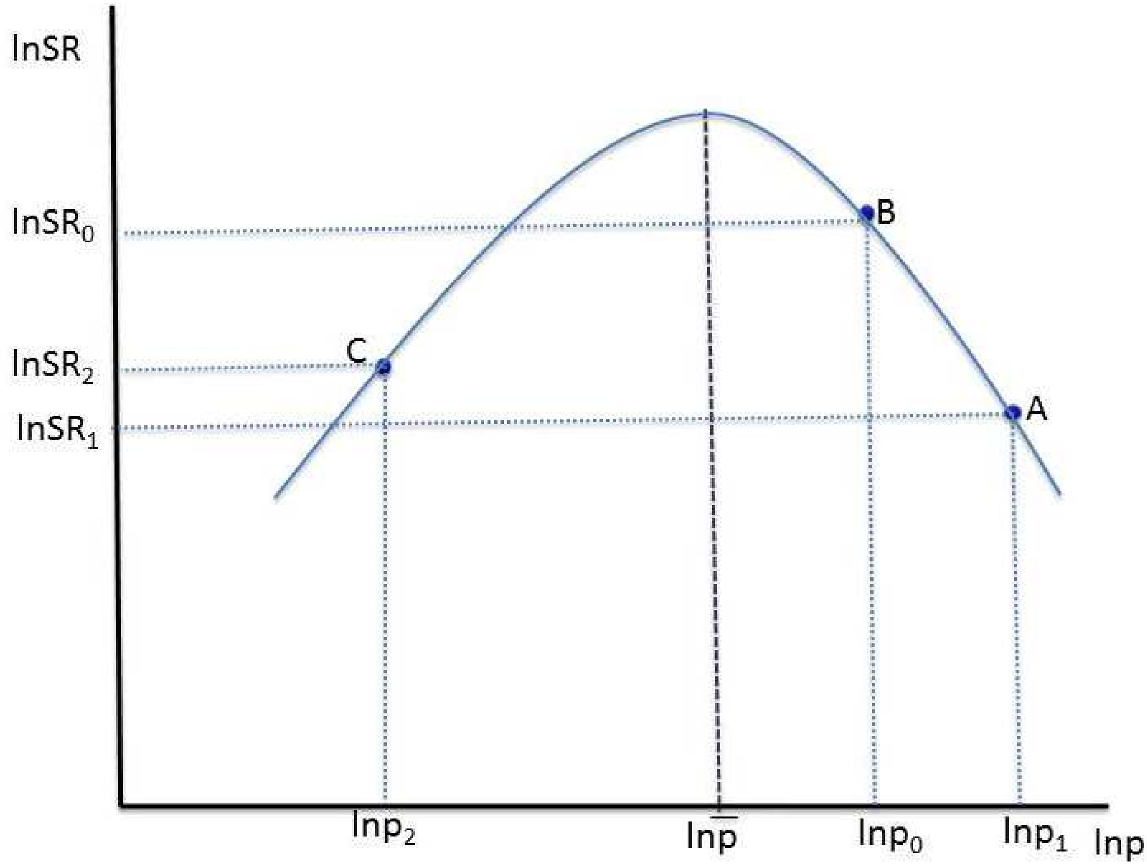
Figure 3: Measured GDP and Energy Price Changes



given capital-labor composite, thereby reducing the measure of the Solow residual in the data. Measurement in constant dollars creates an extreme asymmetric response of the terms-of-trade component of the Solow residual to energy price changes at the baseline price, with any non-infinitesimal price change having a negative effect.

The logarithm of the terms-of-trade component of the measured Solow residual is a hump-shaped function of the logarithm of price, given by equation (17), and illustrated in Figure 4. The peak of the hump occurs at the baseline price of $\ln \bar{p}$ where an infinitesimally small change in the price has no effect on the Solow residual, and a larger change reduces the Solow residual irrespective of the sign of the change. When price is above the baseline, say at $\ln p_1$, placing the system at point A along the curve, then an increase in price moves the economy further from the optimal, reducing the value for the Solow residual. Due to concavity in production, the response to a given-sized percentage increase in price is

Figure 4: Response of Solow Residual to Energy Price



larger the greater the initial distance from the baseline, with the response from $\ln p_1$ at point A exceeding that from $\ln p_0$ at point B. Additionally, concavity implies that the response is increasing at an increasing rate in the magnitude of the price increase and that when the price is above the baseline, the response to a price increase exceeds that for a price decrease. This pattern is opposite when price is initially below the baseline. And due to the effect of constant dollars in creating the hump-shaped response, a reduction in price from $\ln p_0$ to $\ln p_2$ moves the economy from point B to point C, reducing the Solow residual from SR_0 to $SR_{2.}$

Allowing capacity utilization to change, the elasticity of the measured Solow residual with respect to the price of imported energy is given by

$$
\begin{aligned}
\frac{p_t}{SR_{M,t}}\frac{\partial SR_{M,t}}{\partial p_t} &= \frac{p_t}{SR_{TOT,t}}\frac{\partial SR_{TOT,t}}{\partial p_t} \\
&\times \left\{ \frac{\partial \mu_t}{\partial SR_{TOT,t}}\frac{SR_{TOT,t}}{\mu_t} + \left(\frac{p_t-\bar{p}}{p_t}\right)\frac{(1-\gamma)^{\frac{\epsilon}{\epsilon-1}}}{\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]\left[1-\gamma\left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\frac{\bar{p}}{p_t}\right]} \right\}
\end{aligned}
$$

Compare the response of the measured Solow residual to an oil price increase and decrease under the assumption that the capacity utilization effect dominates. An increase in the price of oil reduces $SR_{TOT,t}$ reducing the capacity-utilization component of the measured Solow residual (μ_t). Since the log of $SR_{TOT,t}$ is falling in the log of price at an increasing rate, due to concavity, price increases cause changes of greater magnitude in the log of $SR_{TOT,t}$ than price decreases do, implying larger movements in capacity utilization. For $p_t > \bar{p}$, the increase or decrease in price creates a terms-of-trade effect which augments the effect on capacity utilization. Due to concavity, the price increase has a larger augmenting effect than the price decrease. When $p_t < \bar{p}$, the price increase or decrease creates a terms-of-trade effect which diminishes the capacity utilization effect. And the offsetting effect is smaller for price increases than for price decreases due to concavity. Therefore, whether price is above or below $\bar{p}$, price increases have a larger absolute effect on the measured Solow residual than price decreases do, due to concavity in production.

These results imply that conditioning measured Solow residuals for oil importers separately on oil price increases and decreases should reduce evidence for negative asymmetries.

5 Evidence on Determinants of Asymmetries

We have established that net oil importers have asymmetric innovations to TFP, as measured by Solow residuals. The above model has shown that to account for the asymmetric effects of oil price changes on measured Solow residuals, it is necessary to divide price changes into positive and negative.

We measure the oil price by deflating the spot price of oil by the US CPI to obtain the real oil price.[10]

[10] We use the spot price of West Texas Intermediate from the FRED data base for the nominal oil price. The US CPI is the consumer price index for all urban consumers from BLS. We use this real measure of

We assume that the conditional mean of our TFP variable depends on its own lags, on the lags of the log difference in the real price, and on the positive oil price changes, following the arguments in Kilian and Vigfusson (2011a) on how to properly specify nonlinear models of oil prices changes. Let x_t be the log difference in the real price of oil and let x_t^+ be the positive percentage changes in real oil price. Therefore, we augment the baseline model (3) by lags of these variables, yielding

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + \sum_{i=1}^{p'} \alpha_i x_{t-i} + \sum_{i=1}^{p'} \alpha_i^+ x_{t-i}^+ + \tilde{w}_t, \tag{18}$$

where α is a vector of coefficients. This is equivalent to the first equation in a VAR in which TFP is ordered first. Equivalently, we are assuming that contemporaneous changes in oil prices take at least one quarter to affect TFP. The equation is valid whether the variables in x_t are endogenous or exogenous to q_t. That is, this equation is consistently estimated with OLS irrespective of the coefficients in the regression of x_t on q_t and lagged values of both.[11] The conditional mean specifications for all countries are summarized in Table 3 in the appendix. We exclude Australia, Canada and Norway from the analysis as these countries did not show any evidence of negative asymmetries initially and our model in Section 4 only justifies asymmetric effects of oil price changes for oil importing countries.The UK was a net energy importer for some periods and a net exporter for others. Therefore, we define a dummy variable, identifying the periods in which it was a net importer (1973Q1-1980Q2, 1988Q3-1992Q4 and 2003Q3-2011Q3). The dummy as a multiplier on x_t and x_t^+ to allow the effect of oil prices to differ across the two regimes.

The estimation results of the models containing asymmetric oil price shocks in equation (18) are shown in Table 2. Statistics for skewness of the residuals and the estimated variance ratio demonstrate that the negative asymmetry is reduced for all countries that initially showed negative asymmetry. In fact, the p-values of the likelihood ratio test reveal that the negative asymmetry is not significant for all reported countries, except for Switzerland. For two countries, France and the UK, the asymmetry is significant but now the innovations show positive asymmetry. For Switzerland the asymmetry is slightly reduced when conditioning on oil, but is still present.

oil price for the US and other countries. An alternative for other countries would have been to convert the dollar price of oil into local currency price, using the exchange rate, and then deflate by the relevant foreign consumer price index. We did not take this route because this method would identify exchange rate crises as periods of large oil price increases.

[11] Therefore, the large literature on whether oil price changes are endogenous or exogenous is not relevant to our analysis.

Table 2: Estimation results of the DSF model with oil

	Fra	Ger	Ita	Jap	Kor	Swi	Swi (crisis)	UK	US
μ_u	-0.53***	0.37*	0.41***	0.15***	0.48***	0.55***	0.12***	-0.45***	0.45***
σ_v	0.70***	0.85***	0.71***	0.88***	0.79***	0.58***	0.69***	0.65***	0.79***
skewness	0.655	-0.057	-0.226	-0.008	-0.143	-2.359	0.091	0.863	-0.106
LL AR	-198.3	-204.4	-187.4	-199.4	-152.3	-176.2	-149.8	-187.3	-204.1
LL DSF	-194.5	-204.2	-186.6	-199.4	-150.8	-164.0	-149.7	-180.8	-203.2
LR stat.	7.585	0.499	1.673	0.057	3.009	24.488	0.316	13.148	1.670
p-val.	0.006†	0.480	0.196	0.811	0.083	0.000	0.574	0.000†	0.196
VR	0.578	0.192	0.329	0.031	0.365	0.880	0.032	0.478	0.327

Note: Table 2 reports the estimation results of model (5) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The conditional mean equation includes four lags of oil price returns and four lags of positive oil price returns. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively. A † represents a rejection of the null hypothesis of symmetric innovations when the estimated model implies positive skewness.

To explain the results for Switzerland, note that a source of business cycle asymmetries could be extreme events such financial crises. Barro (2006) included financial crises in his list of rare events. Therefore, for the case of Switzerland we augment our model by conditioning on a dummy for the first quarter of 1991 to account for a banking crisis following a sharp decline in real estate prices, effectively removing that observation. The null hypothesis of symmetric innovations can no longer be rejected, implying that this single outlying observation was responsible for the remaining negative asymmetries after adjusting for oil prices.

As a robustness check we considered an alternative asymmetric oil price variable. We consider the variable defined by Hamilton (2003, 2011) that becomes effective only when the oil price attains a new 3-year high. With X_t denoting the logarithm of the real oil price in period t, net oil price increase is defined by

$$ham_t = \max(0, X_t - \max(X_{t-1}, \dots, X_{t-12})). \tag{19}$$

The results when replacing x_t^+ by ham_t can be found in Table 4 in Appendix B. All our findings continue to hold in this case.

The results of this section and the previous ones imply that positive and negative changes in oil prices have asymmetric and non-linear effects on the measured Solow residual for countries with net energy imports. There are no negative asymmetries for countries which are net exporters of energy. The model implies that negative asymmetries are not due to asymmetric technology shocks, but rather to the asymmetric response of firms to the price of imported energy. Concavity in production is responsible for the asymmetry, with price increases having larger adverse effects than price decreases. These results do not require that oil price changes be exogenous for any particular country. Firm response to oil price changes, whatever their origin, is responsible for the asymmetry.

6 Conclusion

We investigate business cycle asymmetry by estimating the degree of asymmetry present in total factor productivity, as measured by Solow residuals, for eleven OECD countries. Barro (2006) introduced the idea that total factor productivity could have two components, one which is distributed normal i.i.d., as in DSGE models, and one which has only negative realizations. Barro identified his asymmetric component with events which reduced real GDP by at least fifteen percent. In contrast to Barro, we estimate the extent of asymmetry present in Solow residuals using Stochastic Frontier Analysis.

We perform likelihood ratio tests to determine whether the Solow residual contains a negative asymmetric component and compute variance ratios to estimate the extent of the asymmetry. We find that eight of the eleven OECD countries in our sample have significant negative asymmetries. Additionally, we use the pattern of asymmetry across countries to understand its cause. All countries with significant negative asymmetries are net energy importers and all countries without are net energy exporters. This pattern leads us to consider whether oil prices can explain the asymmetry.

We introduce a simple theoretical model to understand the effects of oil prices on the Solow residual for an oil importing country. Allowing for a terms-of-trade effect and a capacity utilization effect, we find that concavity in production implies an asymmetric response to oil price increases and decreases.

We condition Solow residuals on positive and negative percentage changes in the real price of oil and determine whether asymmetries are reduced for net oil importers. We find that negative asymmetries are eliminated for all net oil-importing countries except Switzerland. Asymmetry for Switzerland is eliminated when conditioning on a major Swiss financial crisis in 1991. These results support the hypothesis that oil prices are a significant determinant of business cycle asymmetry for oil importers.

A Conditional mean equations

Table 3: Conditional mean equations

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_1^+$	$\hat{\alpha}_2^+$	$\hat{\alpha}_3^+$	$\hat{\alpha}_4^+$
Aus-AR	0.002	-0.14								
	(0.079)	(0.08)								
Can-AR	0.003	0.26								
	(0.084)	(0.09)								
Fra-AR	-0.002	0.26								
	(0.077)	(0.08)								
Fra-Oil	0.311	0.22	2.83	-0.59	0.58	0.22	-3.01	-0.57	0.35	-3.45
	(0.127)	(0.08)	(0.86)	(0.89)	(0.87)	(0.85)	(1.30)	(1.31)	(1.31)	(1.28)
Ger-AR	0.000	-0.05								
	(0.080)	(0.08)								
Ger-Oil	0.329	-0.05	2.25	2.27	0.03	-0.53	-2.70	-2.88	-0.16	-1.45
	(0.133)	(0.08)	(0.90)	(0.92)	(0.94)	(0.89)	(1.36)	(1.37)	(1.39)	(1.35)
Ita-AR	-0.018	0.30								
	(0.073)	(0.07)								
Ita-Oil	0.270	0.34	1.30	1.77	-1.57	0.94	-2.42	-1.33	1.96	-4.25
	(0.118)	(0.07)	(0.80)	(0.81)	(0.84)	(0.79)	(1.21)	(1.22)	(1.23)	(1.20)
Jap-AR	-0.005	-0.05								
	(0.080)	(0.07)								
Jap-Oil	0.398	-0.09	3.72	2.16	-1.12	0.65	-6.60	-3.28	2.56	-1.47
	(0.130)	(0.08)	(0.87)	(0.91)	(0.90)	(0.86)	(1.31)	(1.40)	(1.33)	(1.31)

Table 3 – continued

Kor-AR	-0.009	0.01								
	(0.090)	(0.09)								
Kor-Oil	0.012	-0.02	2.94	-1.97	-1.08	0.18	-3.61	2.51	3.32	-2.60
	(0.170)	(0.09)	(0.97)	(0.99)	(0.98)	(0.96)	(1.86)	(1.89)	(1.88)	(1.86)
Nor-AR	-0.002	-0.25								
	(0.079)	(0.08)								
Swi-AR	-0.007	0.18								
	(0.072)	(0.06)								
Swi-Oil	0.220	0.17	2.64	0.85	-0.34	0.29	-4.07	0.33	0.65	-0.95
	(0.114)	(0.07)	(0.72)	(0.76)	(0.76)	(0.71)	(1.32)	(1.35)	(1.34)	(1.36)
UK-AR	-0.077	0.10								
	(0.081)	(0.08)								
UK-Oil	0.302	-0.05	3.80	0.25	-1.33	-0.24	-8.94	0.23	1.36	-2.42
	(0.084)	(0.08)	(1.08)	(1.09)	(1.06)	(1.02)	(1.44)	(1.57)	(1.44)	(1.38)
US-AR	0.000	0.15								
	(0.080)	(0.08)								
US-Oil	0.354	0.04	2.74	-0.86	0.72	-1.34	-5.13	0.06	-2.14	0.18
	(0.136)	(0.08)	(0.91)	(0.92)	(0.91)	(0.88)	(1.38)	(1.40)	(1.35)	(1.34)

Note: Table 3 reports the estimated conditional mean equations for our dynamic stochastic frontier model defined by equations (2) to (5) denoted by 'AR' and for the corresponding model conditioning on log differences of real oil prices and positive oil price changes given in equation (18). Standard errors are given in parentheses.

B Robustness check using Hamilton's oil variable

Table 4: Estimation results of the DSF model with Hamilton's oil variable

	Fra	Ger	Ita	Jap	Kor	Swi	Swi (crisis)	UK	US
μ_u	-0.45***	0.47***	0.37**	0.56***	-0.52***	0.57***	-0.35**	-0.44***	-0.36***
σ_v	0.73***	0.80***	0.74***	0.69***	0.82***	0.59***	0.63***	0.69***	0.81***
skewness	0.316	-0.253	-0.148	-0.396	-0.261	-2.389	0.224	0.649	-0.008
LL AR	-194.7	-204.8	-187.4	-196.8	-154.3	-180.4	-153.1	-190.8	-200.2
LL DSF	-193.3	-203.7	-187.0	-195.4	-154.1	-166.6	-152.9	-185.9	-199.5
LR stat.	2.777	2.022	0.785	2.813	0.430	27.683	0.424	9.874	1.384
p-val.	0.096†	0.155	0.376	0.094	0.512	0.000	0.515	0.002†	0.239
VR	0.378	0.351	0.248	0.665	0.406	0.920	0.308	0.404	0.201

Note: Table 2 reports the estimation results of model (5) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The conditional mean equation includes four lags of oil price returns and four lags Hamilton's oil variable defined in equation (19). The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively. A † represents a rejection of the null hypothesis of symmetric innovations when the estimated model implies positive skewness.

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