

# Gauge invariance and Observables in Particle Physics

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Switzerland



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Natural Sciences

**FWF** Österreichischer  
Wissenschaftsfonds



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  - Path integral formulation and symmetries

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    - Qualitative changes

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  - Standard Model
    - Experimental signatures
  - Beyond the Standard Model
    - Qualitative changes
  - Quantum (super)gravity

What's the deal?

-

Gauge symmetry

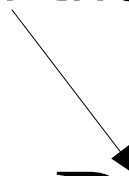


# Path integral

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Classical action as weight factor (yields classical limit when dominating)

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# Path integral

$$\langle \phi(x) \dots \phi(z) \rangle = \int_{\Omega} D\phi \phi(x) \dots \phi(z) e^{iS[\phi]}$$

Expectation values are weighted averages  
over space-time histories



# Path integral

Dependencies on special events  
is only due to external choices

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# Path integral and global symmetries

[Review: Maas'17]

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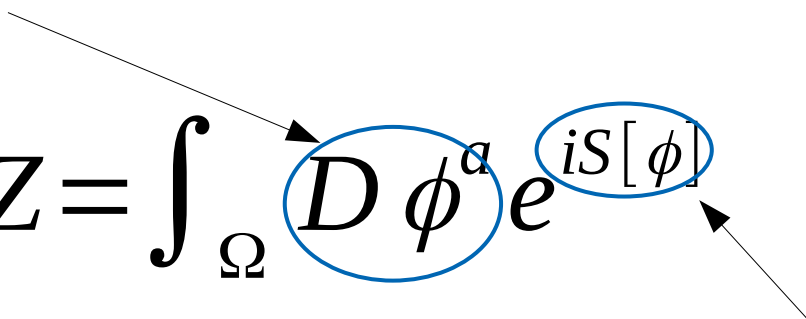
Field - transforms linearly under a group  $\phi^a \rightarrow G^{ab} \phi^b$


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# Path integral and global symmetries

[Review: Maas'17]

Measure is invariant  
- no anomalies

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Action is invariant  
 $S[\phi] = S[G\phi]$

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Integration range  
- contains all orbits  $G\phi$

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  - There is no absolute orientation/frame in the internal space
  - Does not change when averaging over position
  - There is no absolute charge

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$$\langle \phi^b(x) \rangle = \int_{\Omega} D\phi^a e^{iS[\phi]} \phi^b(x) = 0$$

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- Relative charge measurement averaged over all possible starting point
  - Vanishes because no preferred absolute starting point

# Path integral and global symmetries

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$$\begin{aligned} & \langle \delta_{bc} \phi^b(\mathbf{x}) \phi^c(\mathbf{y}) \rangle \\ &= \int_{\Omega} D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(\mathbf{x}) \phi^c(\mathbf{y}) \end{aligned}$$

- Group-invariant quantity
  - Measures relative orientation
  - Created from an invariant tensor  $\delta_{ab}$

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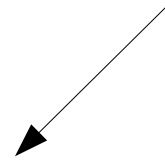
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- No longer invariant under gauge transformations
  - Vanishes just as any other non-invariant quantity

# Path integral and local symmetries

[Review: Maas'17]

Transporter



$$\langle \phi^b(x) U^{bc}(x, y) \phi^c(y) \rangle$$
$$= \int_{\Omega} D\phi^a DU e^{iS[\phi]} \phi^b(x) U^{bc}(x, y) \phi^c(y)$$

- Transporter compensates gauge transformations
  - Implemented by gauge fields

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# Path integral and local symmetries

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Reduced integration range

$$\langle \phi^b(x) \phi^c(y) \rangle = \int_{\Omega_c} D\phi^a DU W(U, \phi) e^{iS[\phi]} \phi^b(x) \phi^c(y) \neq 0$$

- Reduction of integration region by gauge fixing
  - Arbitrary choice of coordinates
  - Weight factor to keep gauge-invariant quantities the same

Brout-Englert-Higgs Physics

-

The Standard Model

# A toy model

# **A toy model: Higgs sector of the SM**

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- Consider an  $SU(2)$  with a fundamental scalar



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- Essentially the standard model Higgs

$$L = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + gf_{bc}^a W_\mu^b W_\nu^c$$

- $W_s$   $W_\mu^a$  

- Coupling  $g$  and some numbers  $f^{abc}$



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- **Ws**  $W_\mu^a$  
- **Higgs**  $h_i$  
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- Parameters selected for a BEH effect

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- Global SU(2) custodial (flavor) symmetry

- Acts as (right-)transformation on the scalar field only

$$W_\mu^a \rightarrow W_\mu^a \quad h \rightarrow h \Omega$$

# Textbook approach

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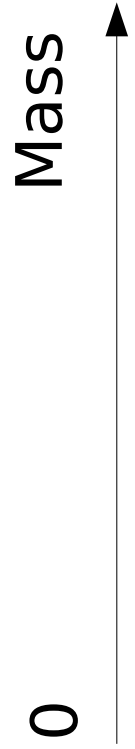
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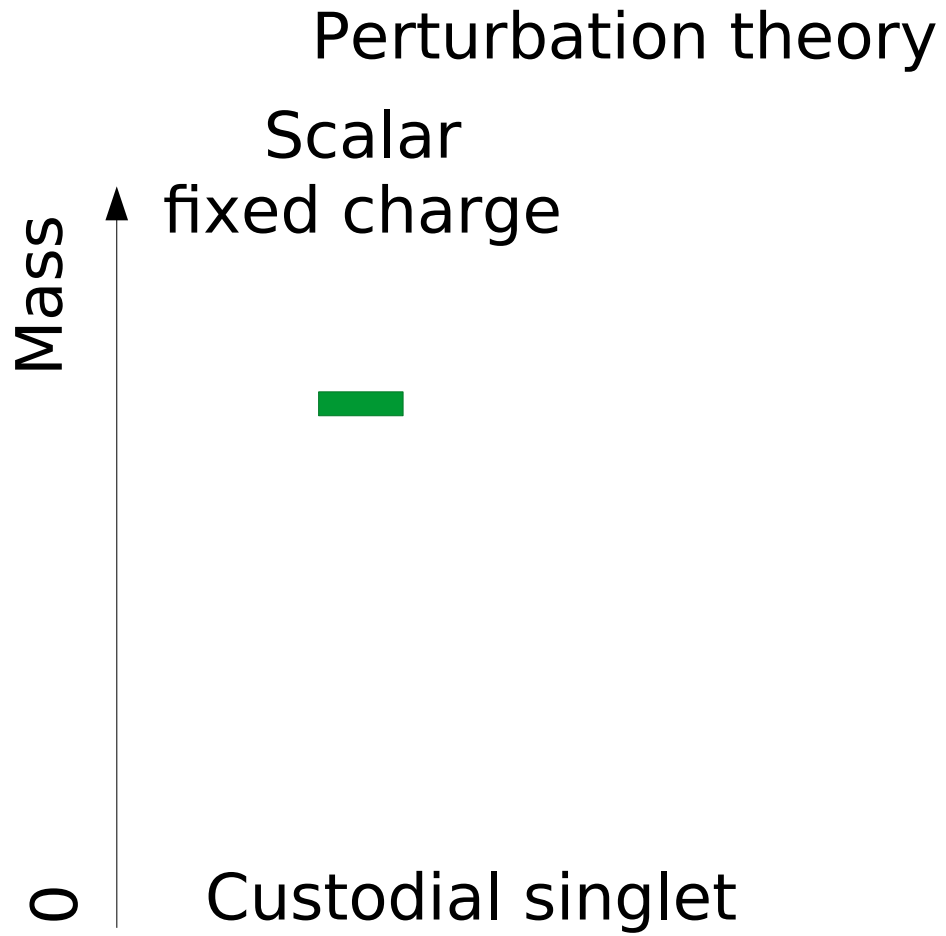
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- Get masses and degeneracies at tree-level
- Perform perturbation theory

# Physical spectrum

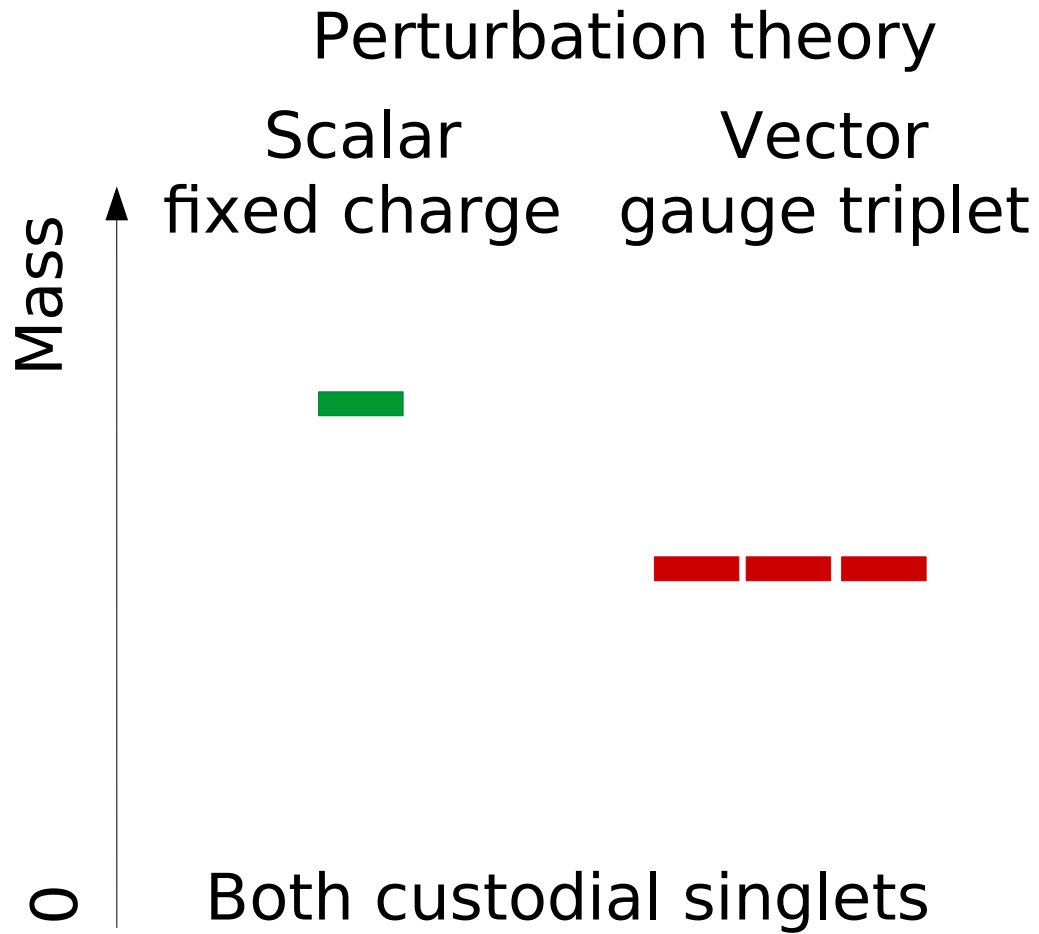
Perturbation theory



# Physical spectrum



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# The origin of the problem

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  - And this includes non-perturbative aspects...
  - ...even at weak coupling [Gribov'78, Singer'78, Fujikawa'82]

# Physical states

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- Need physical, gauge-invariant particles



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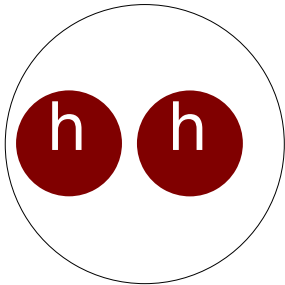
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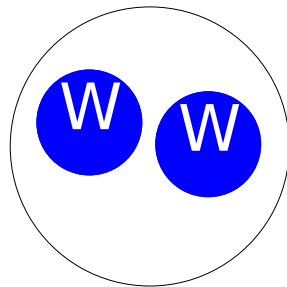
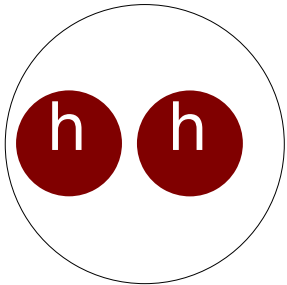
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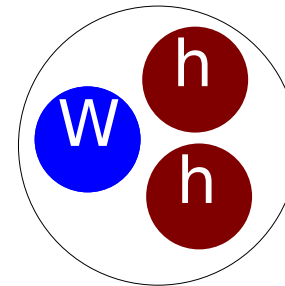
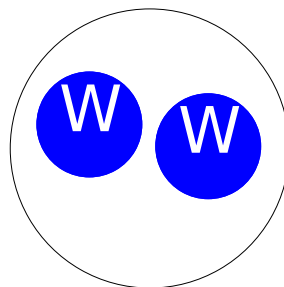
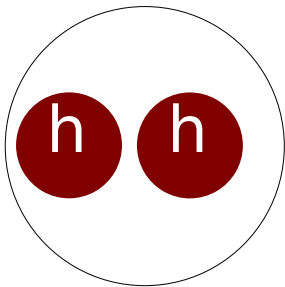
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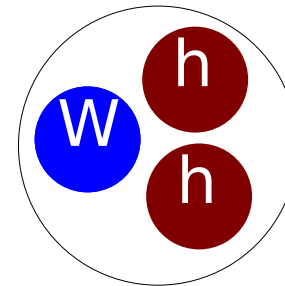
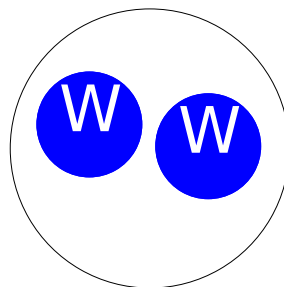
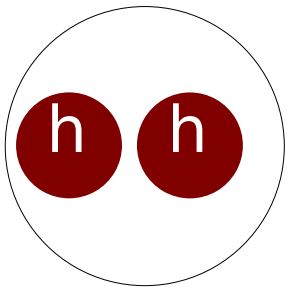
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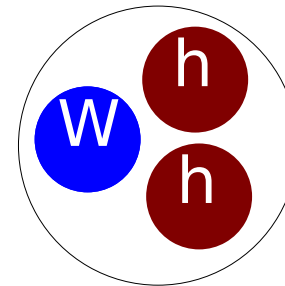
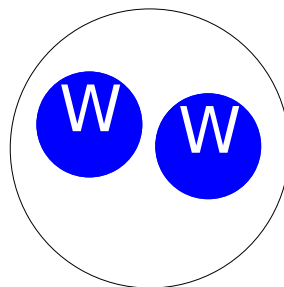
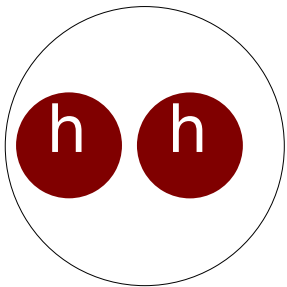


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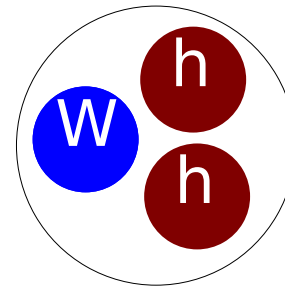
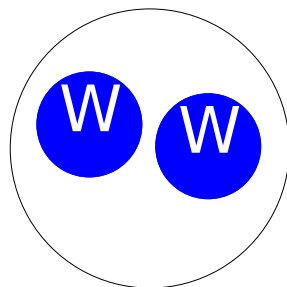
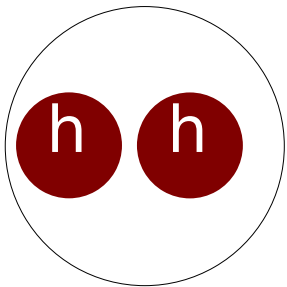


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  - Think QED (hydrogen atom!)
- Can this matter?



# How to make predictions

[Fröhlich et al.'80,'81,  
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- $J^{PC}$  and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states

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  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators

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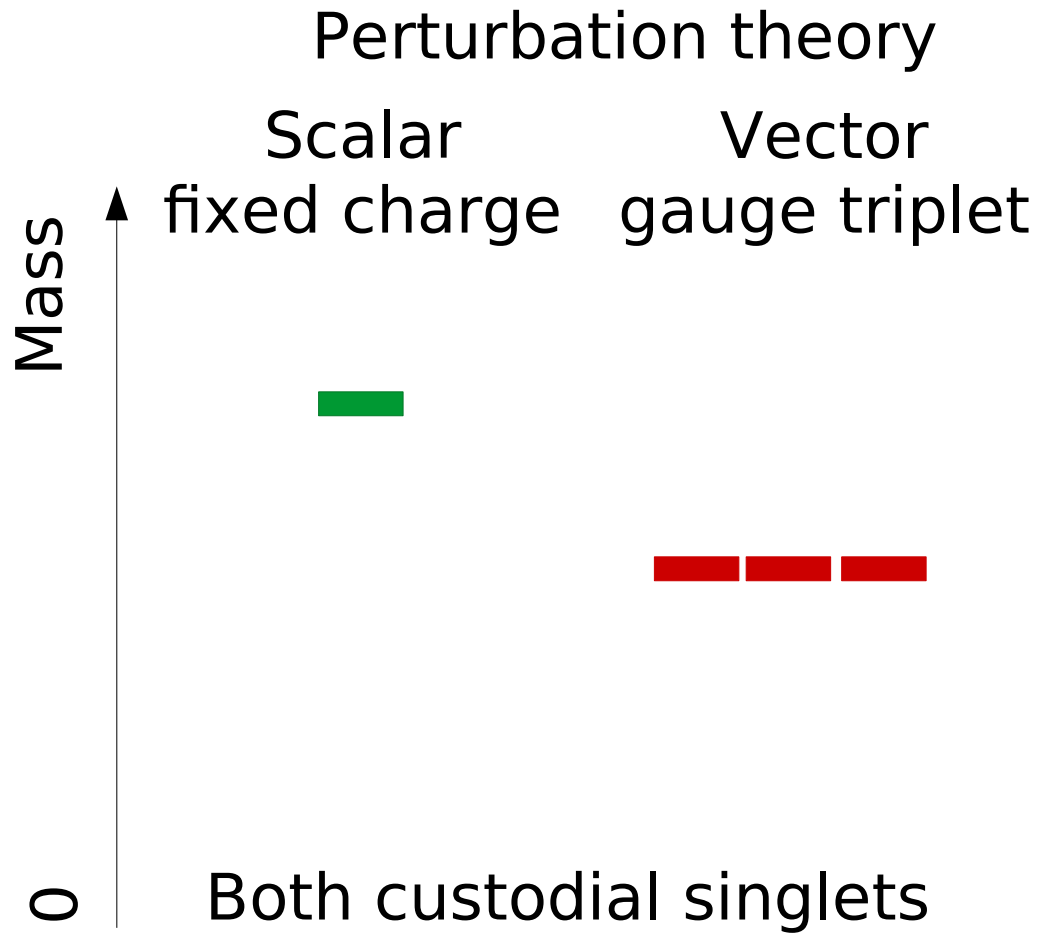
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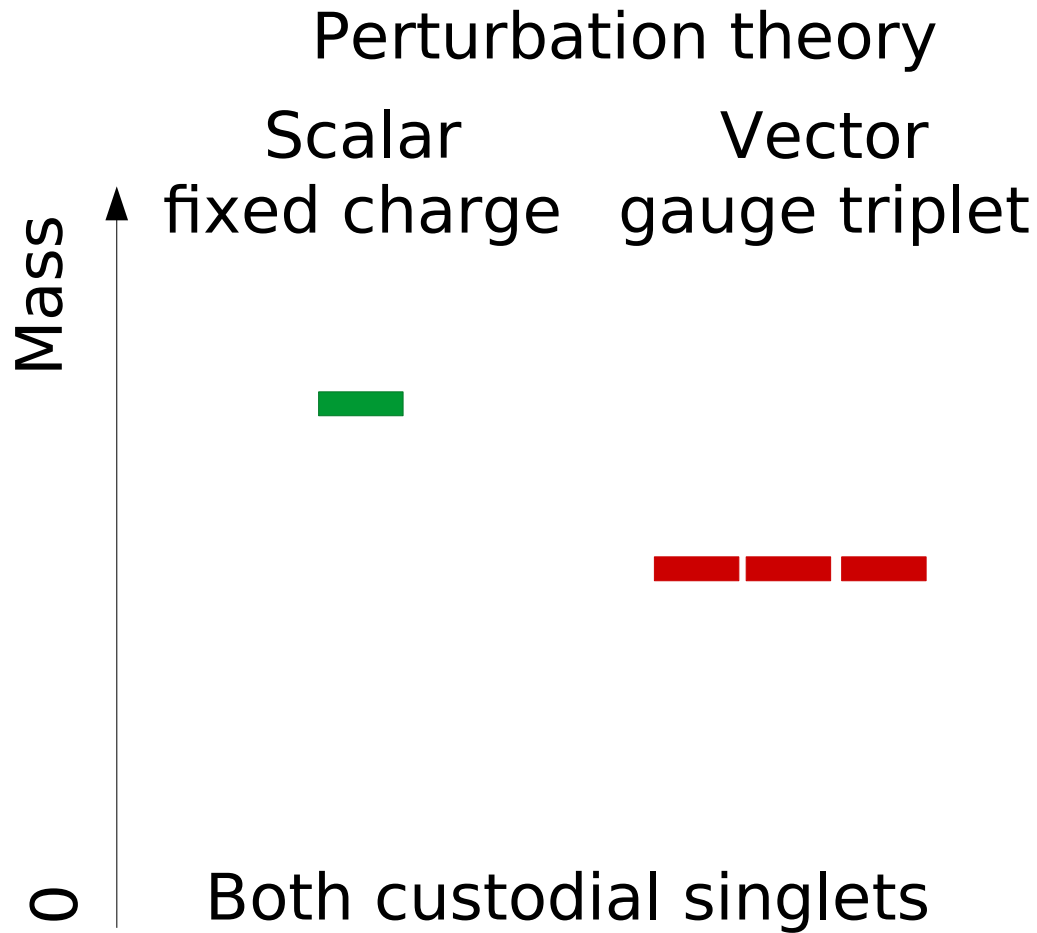
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# Physical spectrum

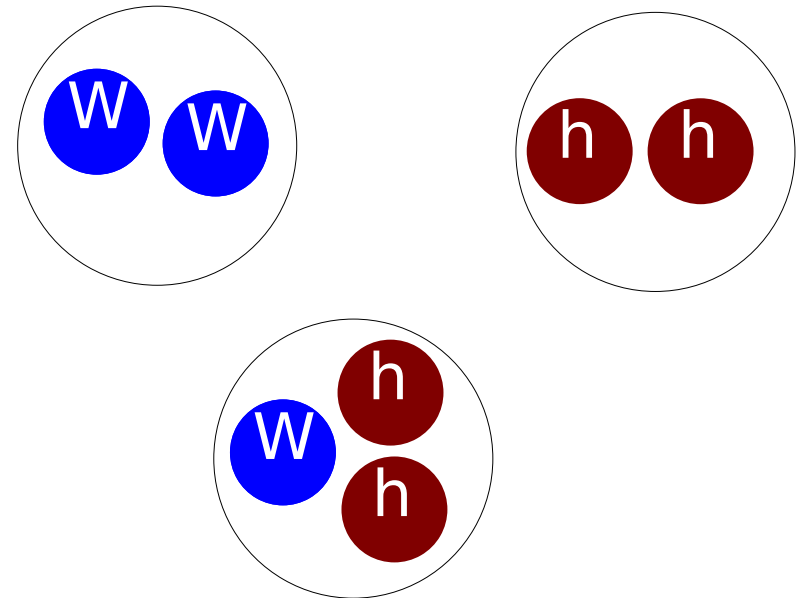


Experiment tells that somehow the left is correct

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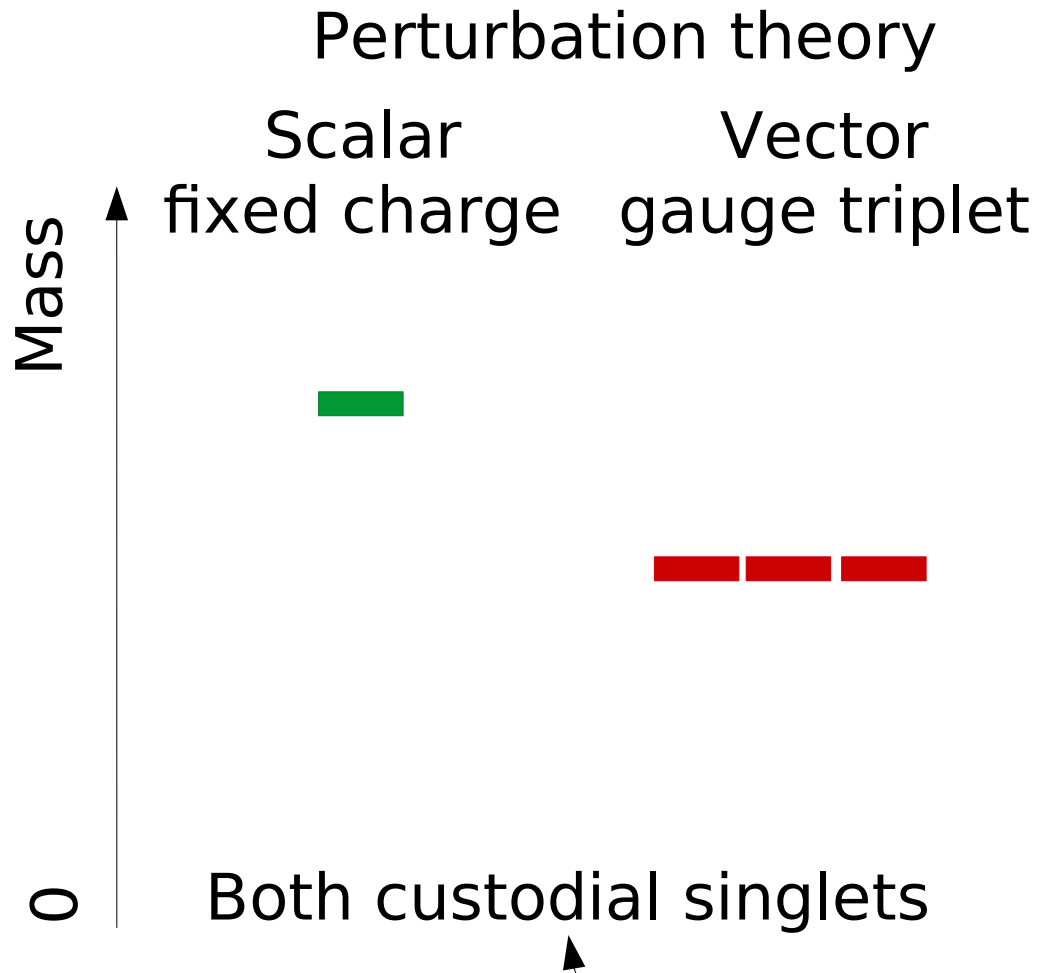
Composite (bound) states



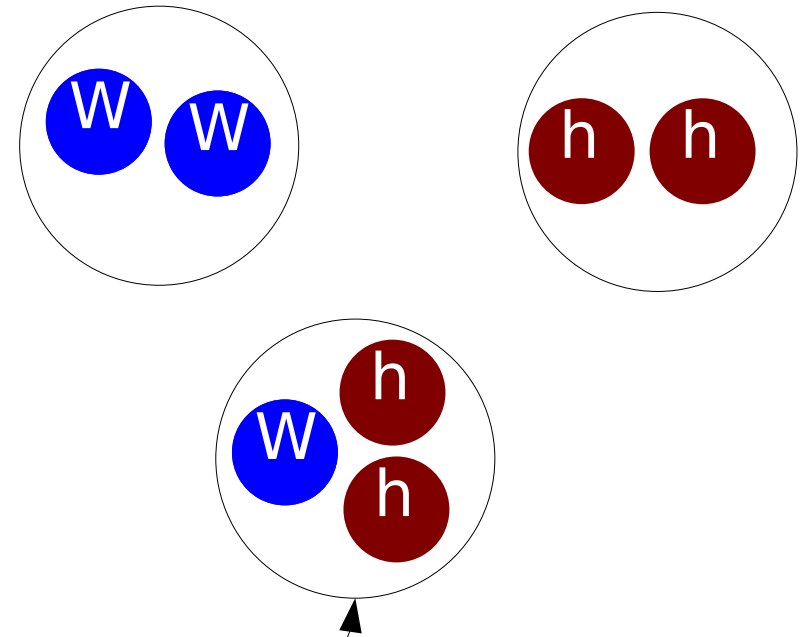
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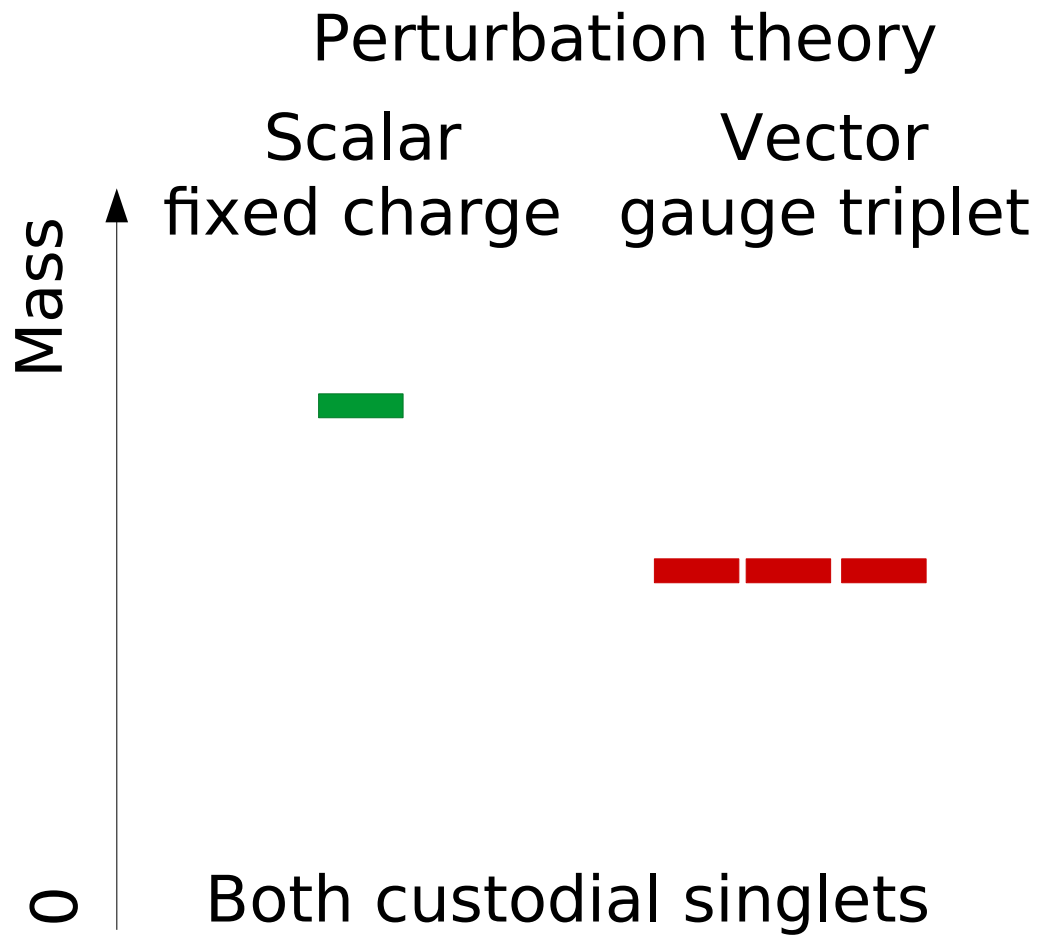
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Experiment tells that somehow the left is correct  
Theory say the right is correct  
There must exist a relation that both are correct

# Physical spectrum

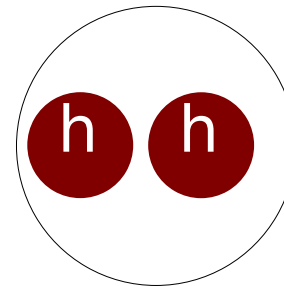
[Maas'12, Maas & Mufti'14]



Gauge-invariant

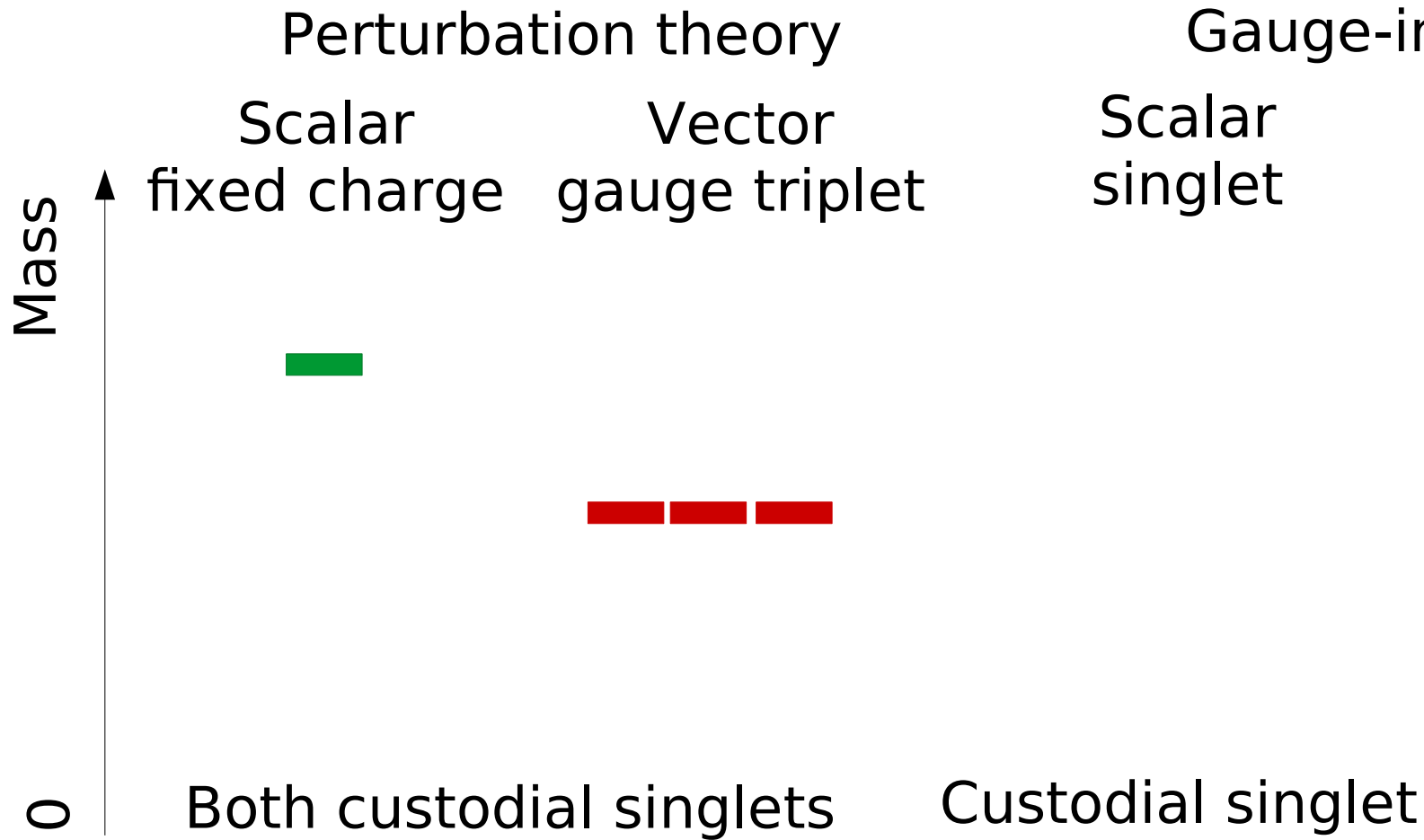
Scalar singlet

$$h(x)^+ h(x)$$

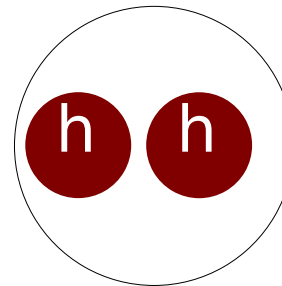


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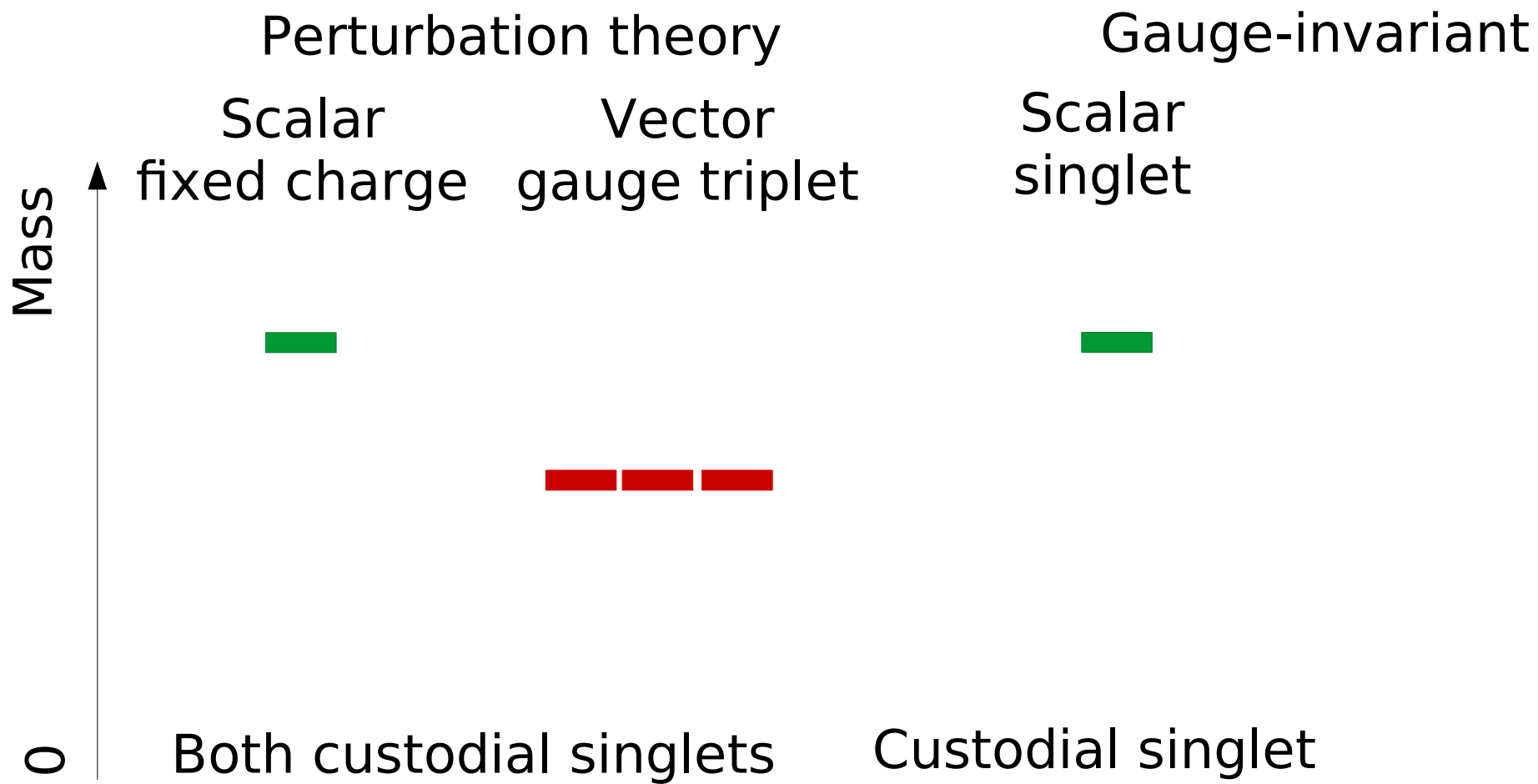


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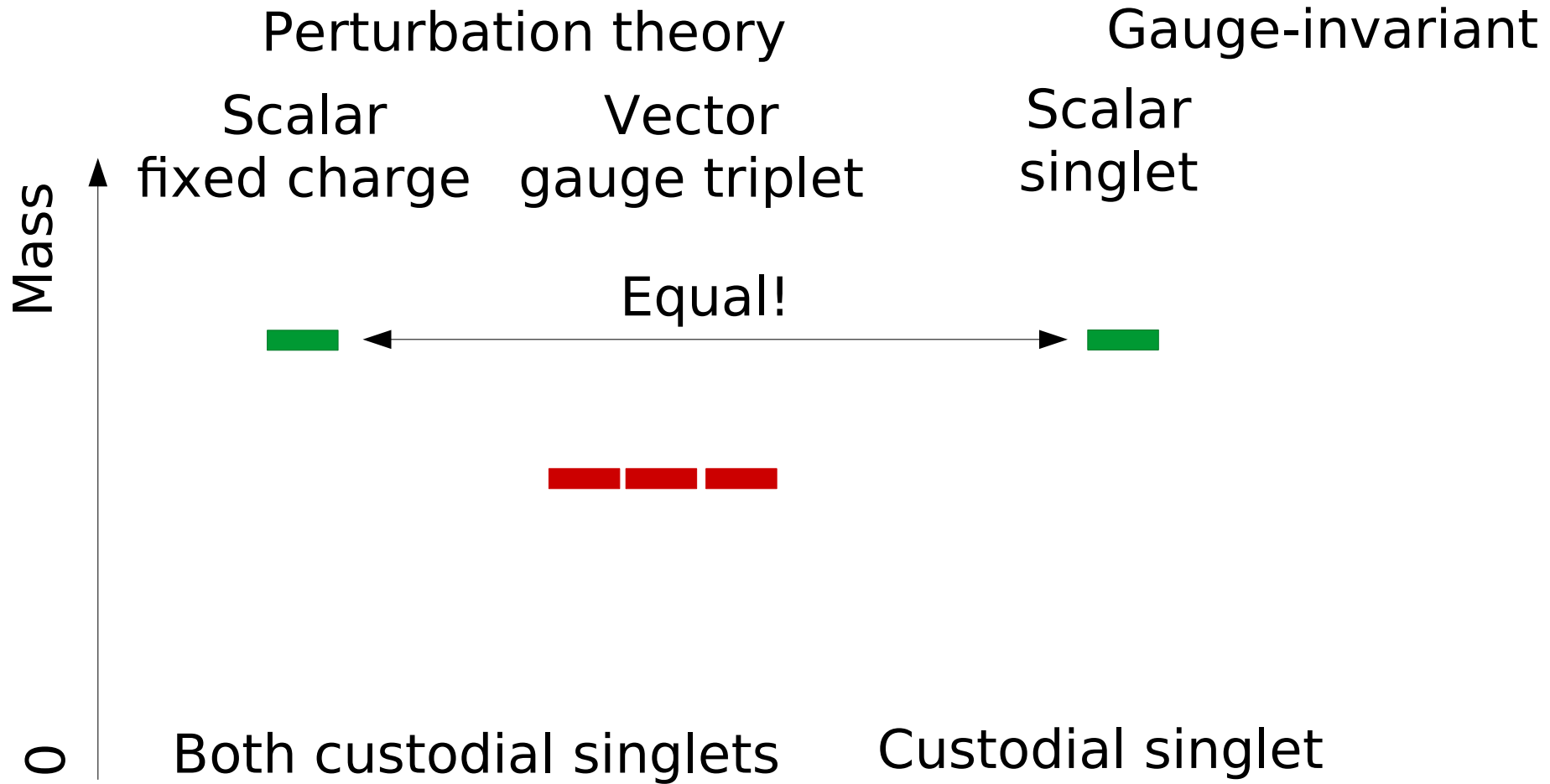
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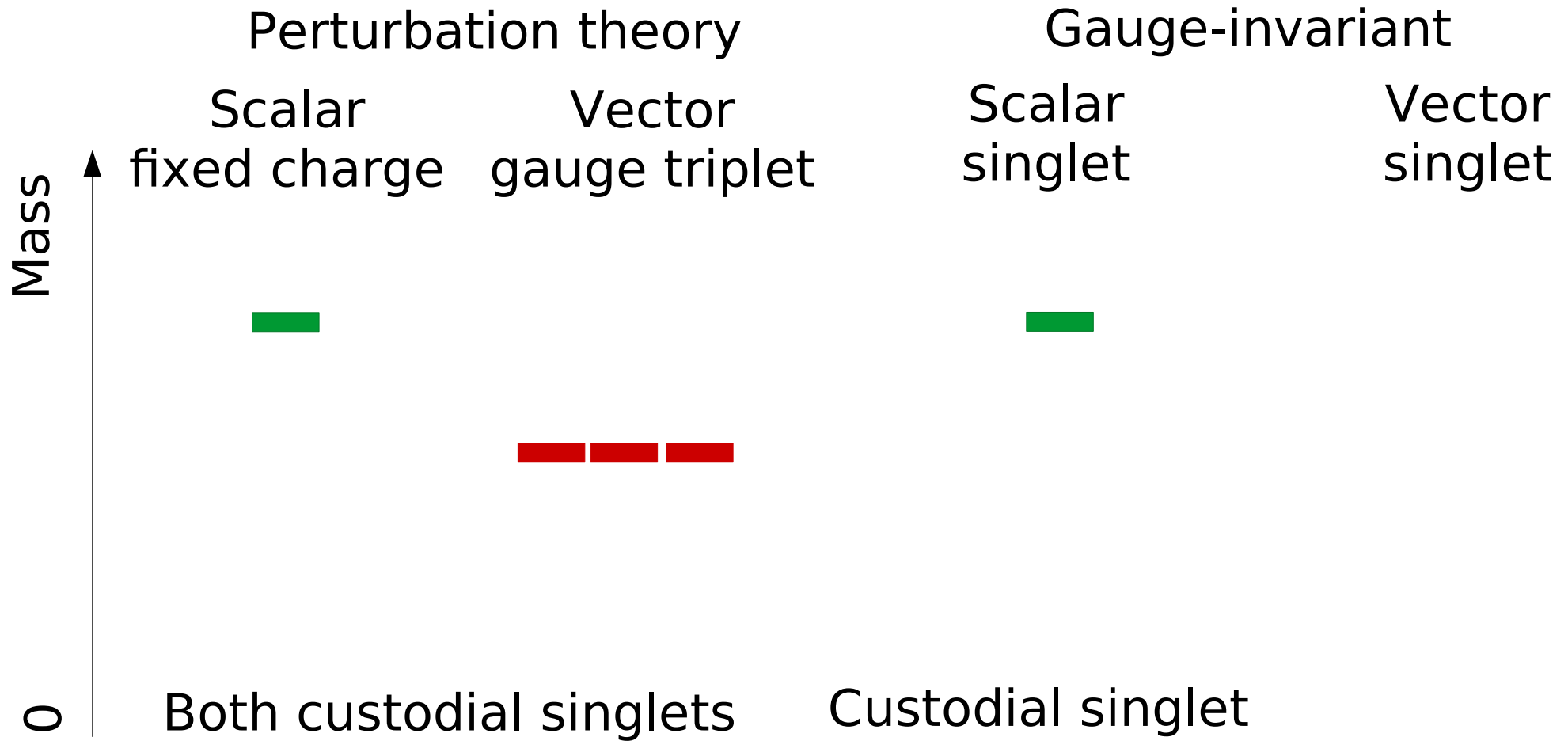
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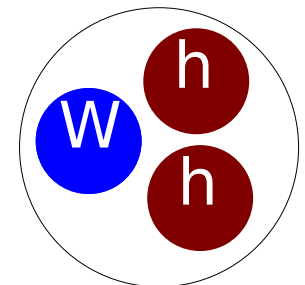


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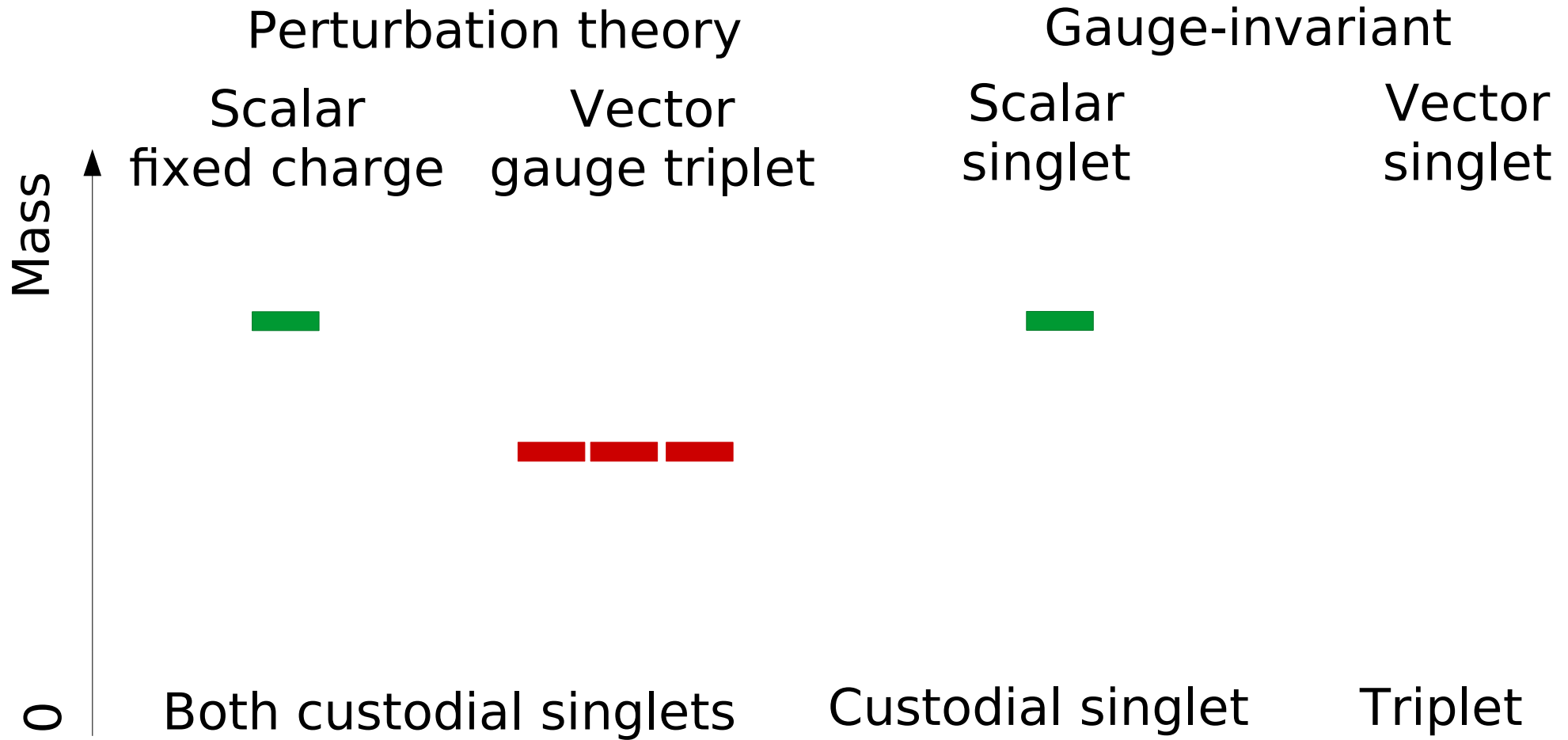


$$\text{tr } t^a \frac{h^+}{\sqrt{h^+ h}} D_\mu \frac{h}{\sqrt{h^+ h}}$$

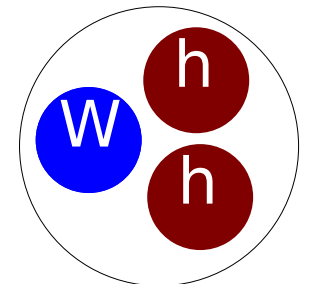


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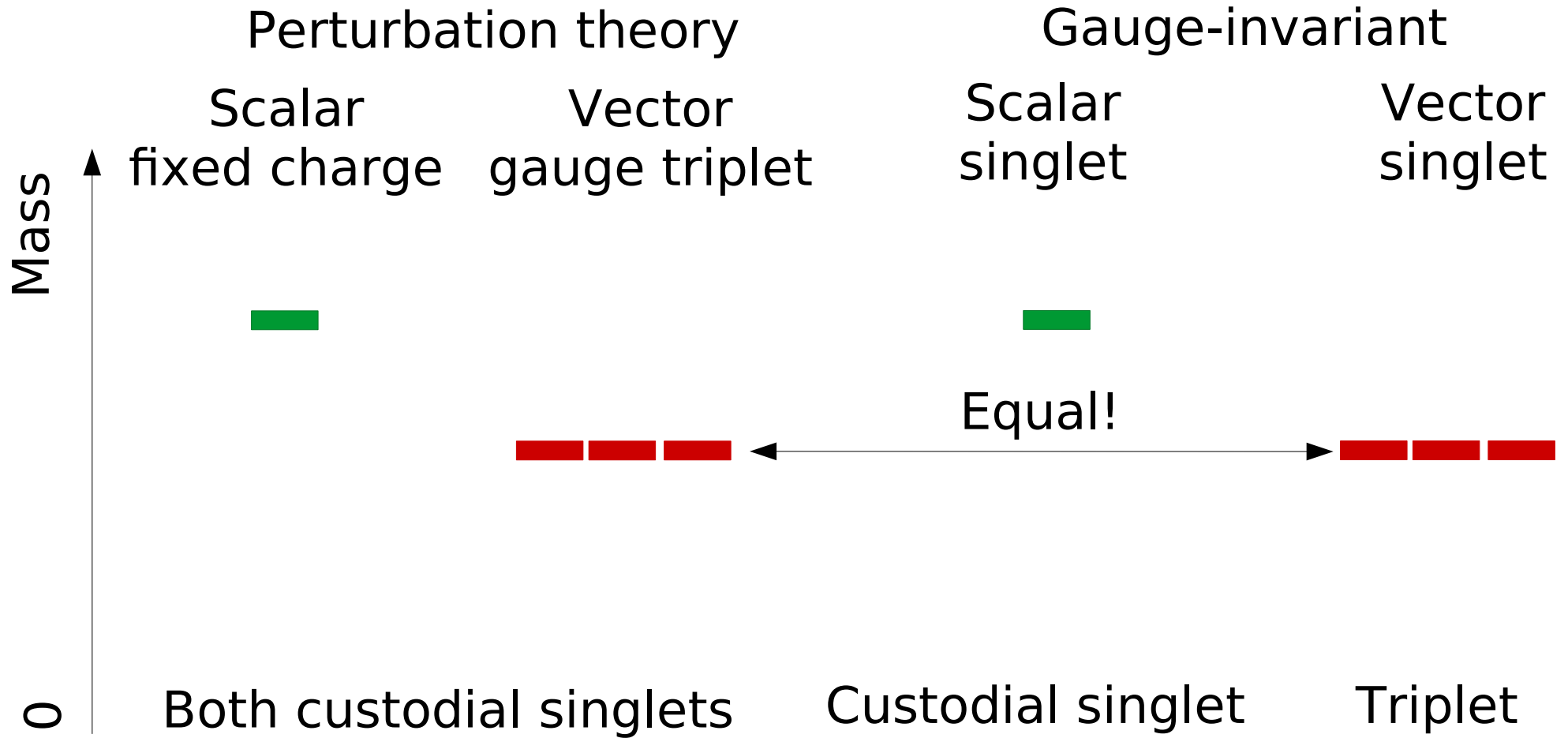


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# Physical spectrum

[Maas'12, Maas & Mufti'14]





A microscopic origin

-

Fröhlich-Morchio-Strocchi  
mechanism

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  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas'12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

# Augmented perturbation theory

[Fröhlich et al.'80,'81  
Maas'12,'17]

1) Formulate gauge-invariant operator

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Higgs field

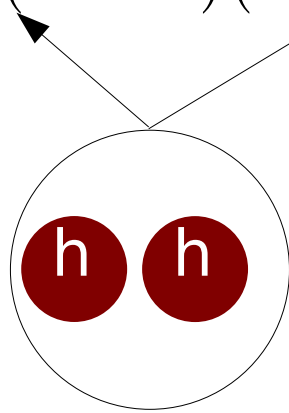


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Trivial two-particle state

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Standard  
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What about  
this? →

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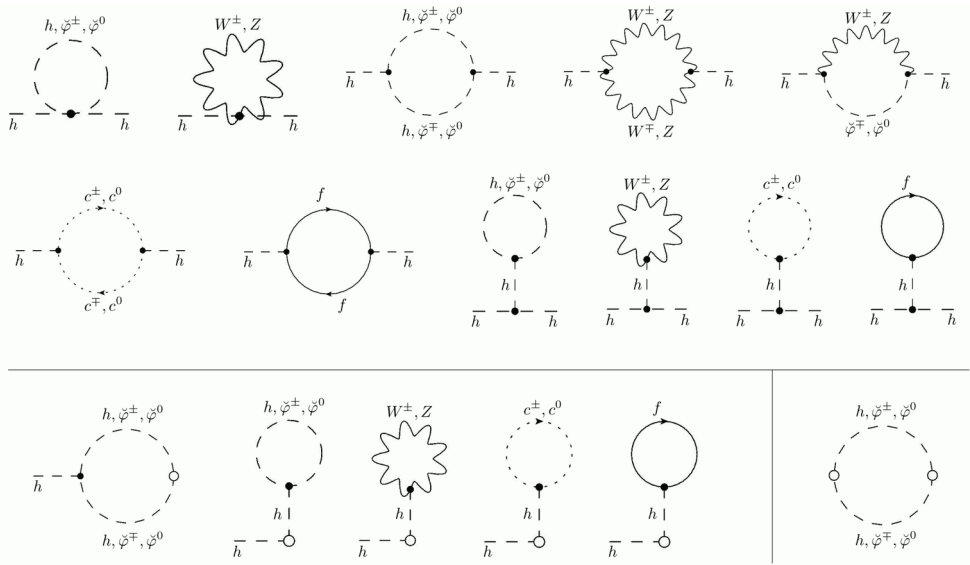
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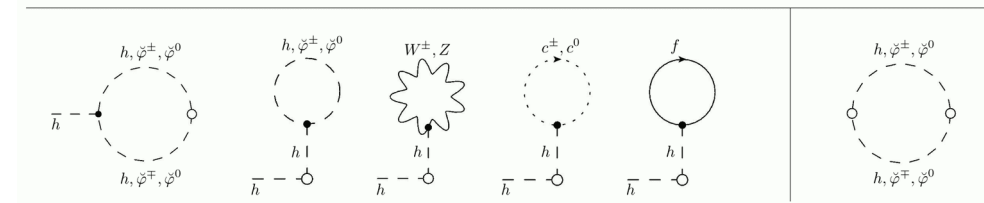
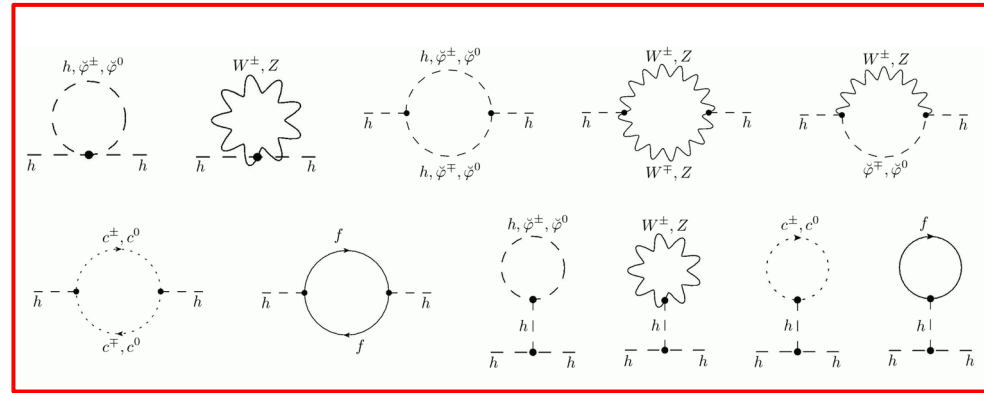
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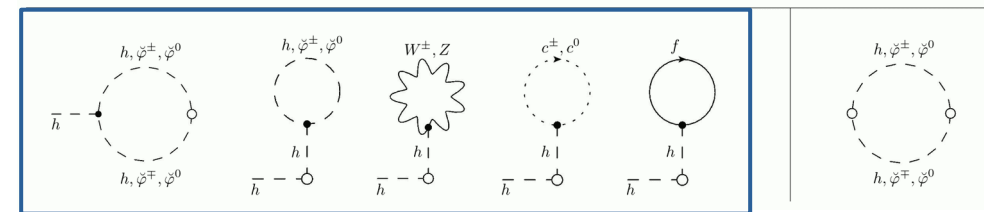
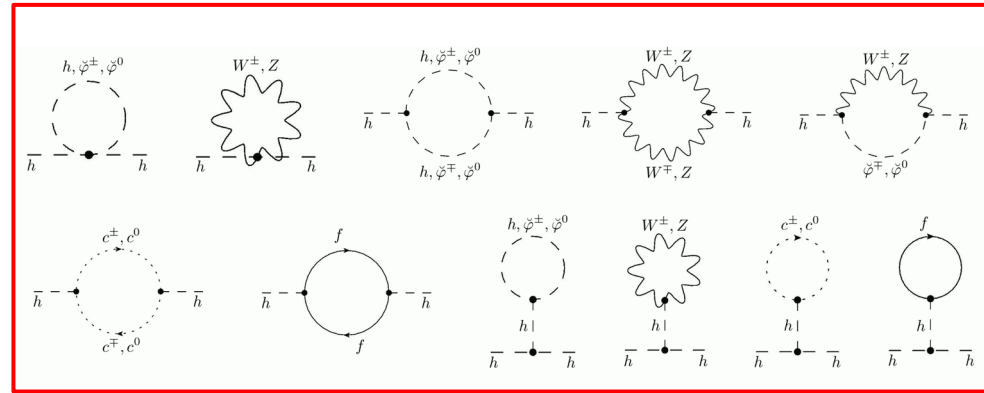
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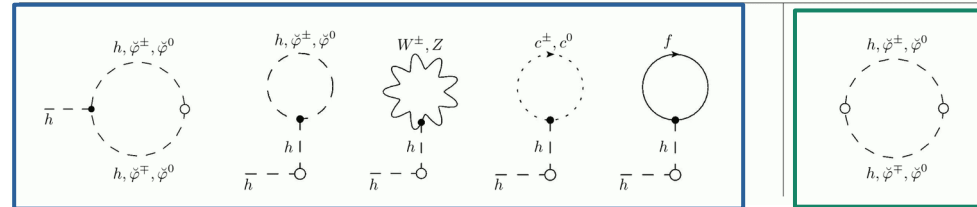
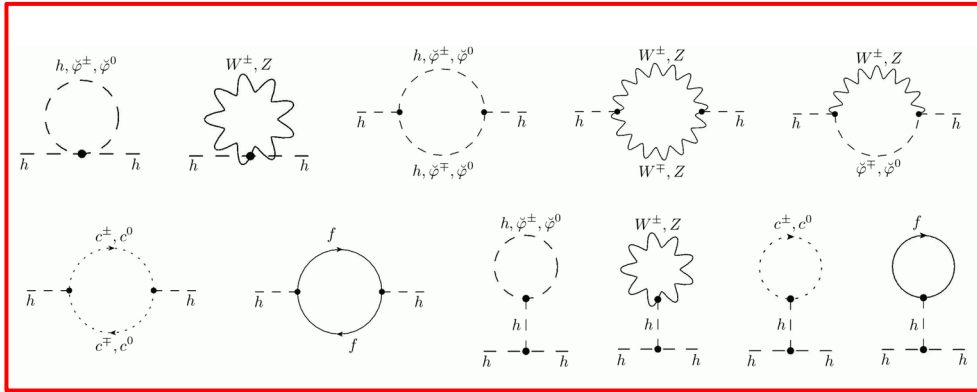
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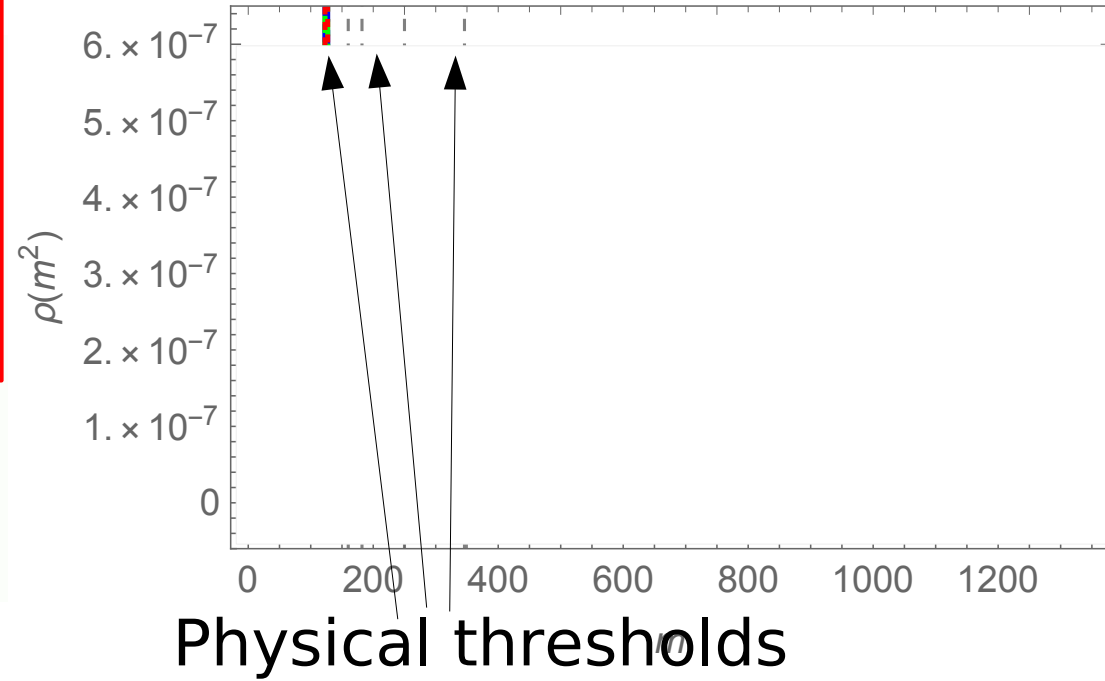
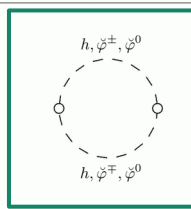
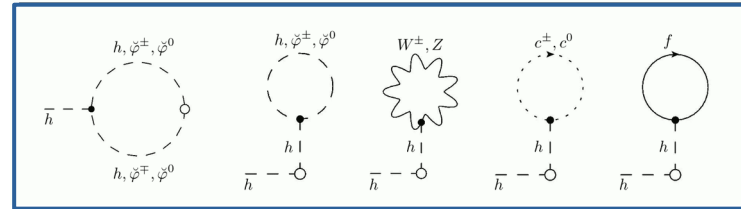
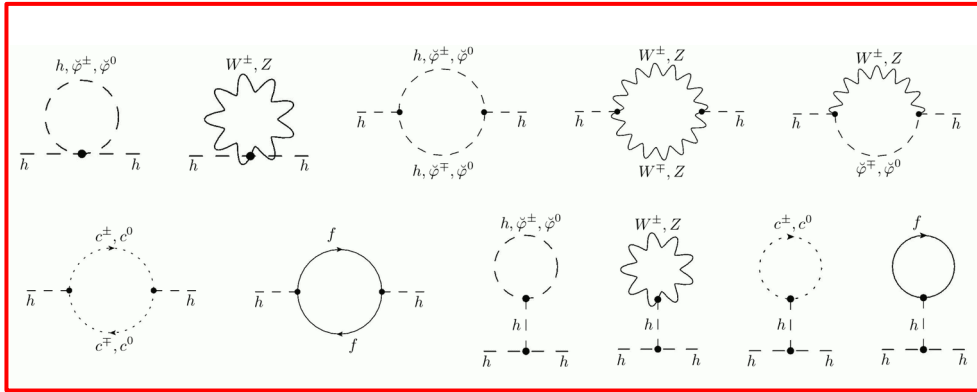
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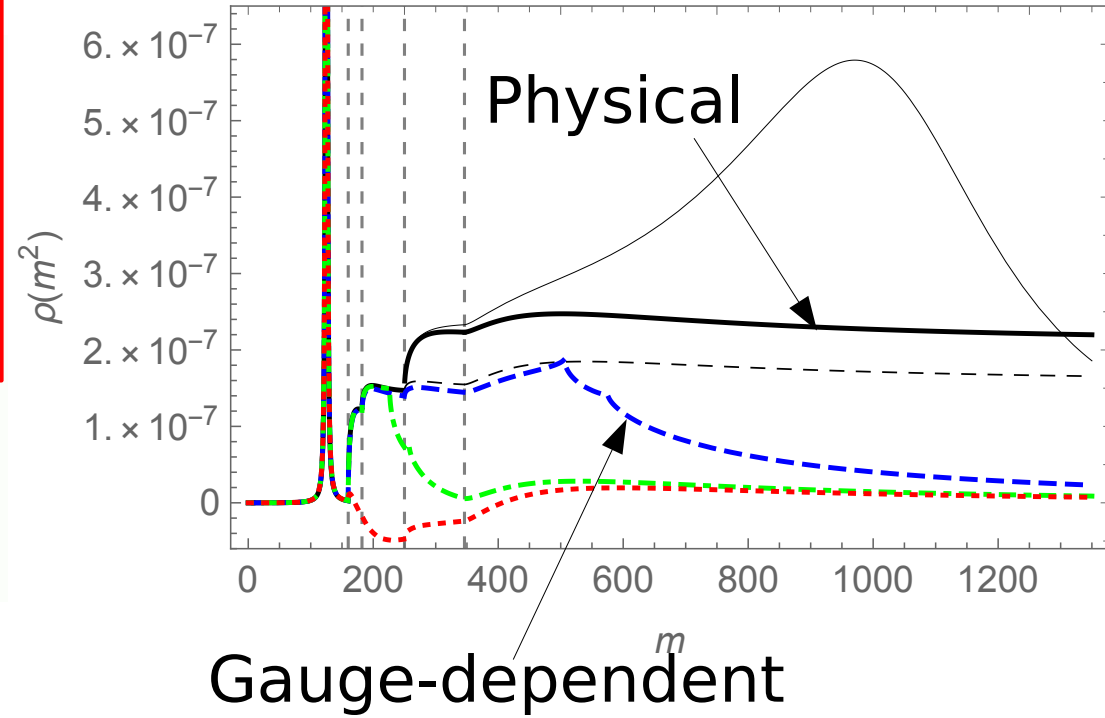
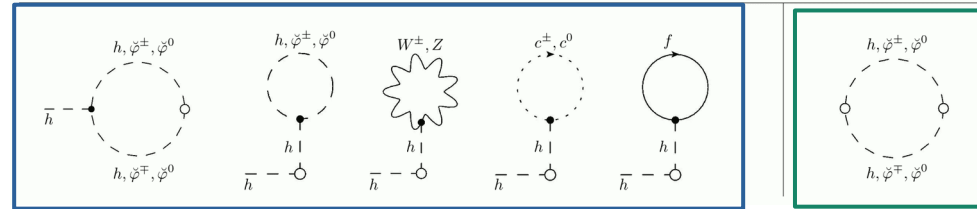
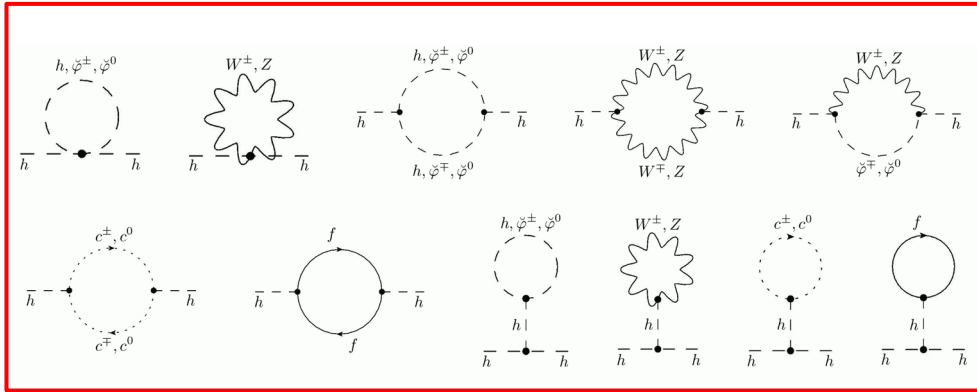
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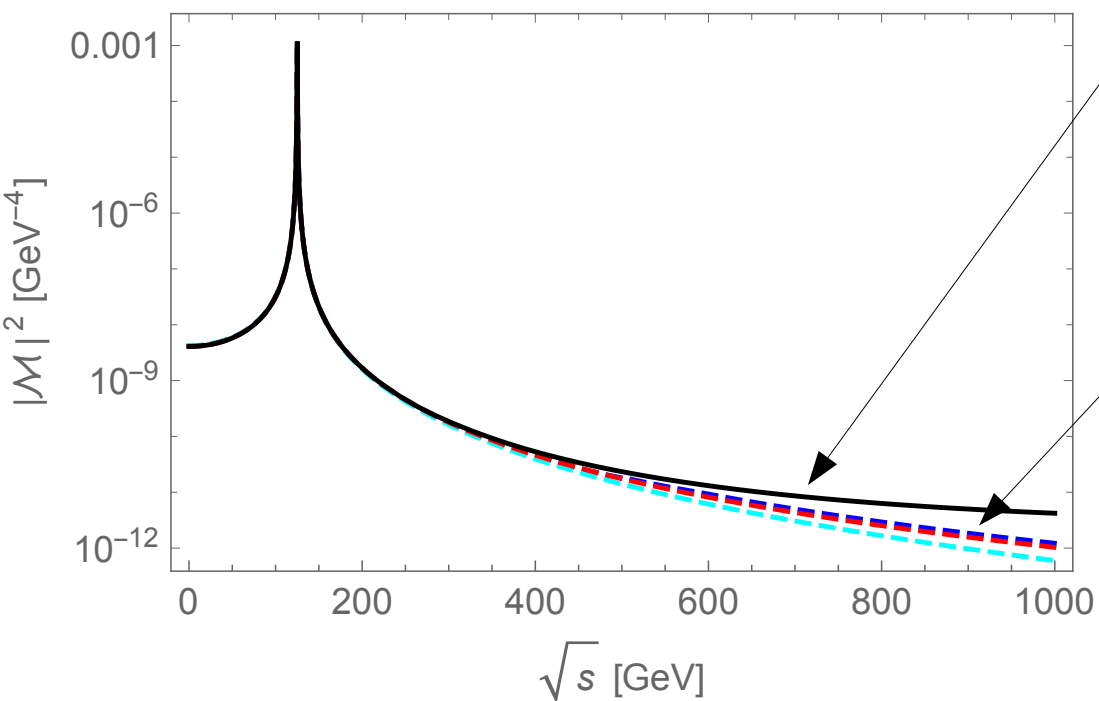
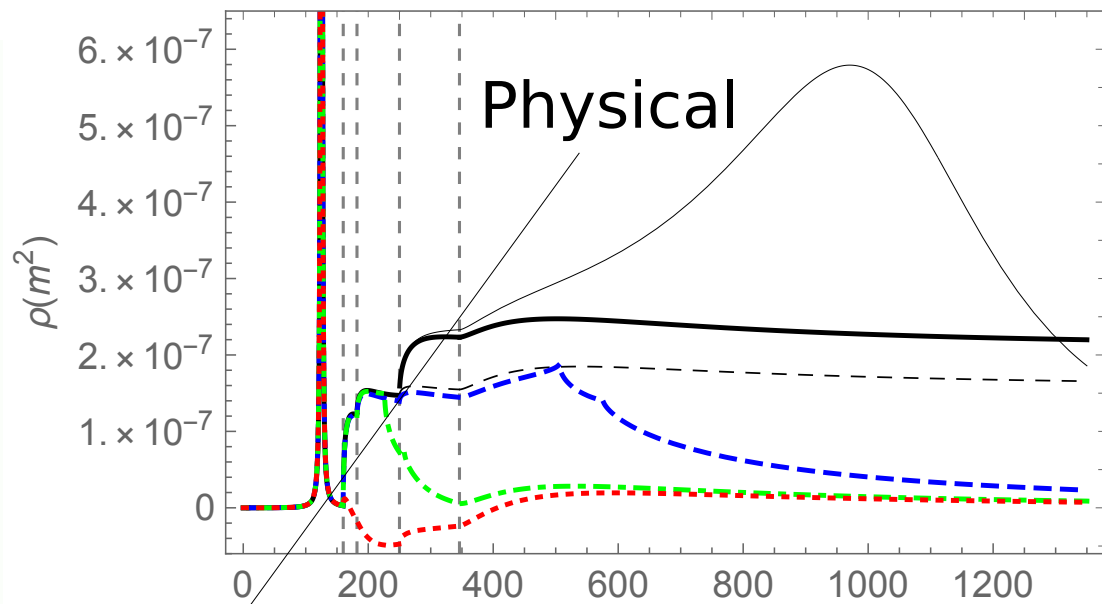
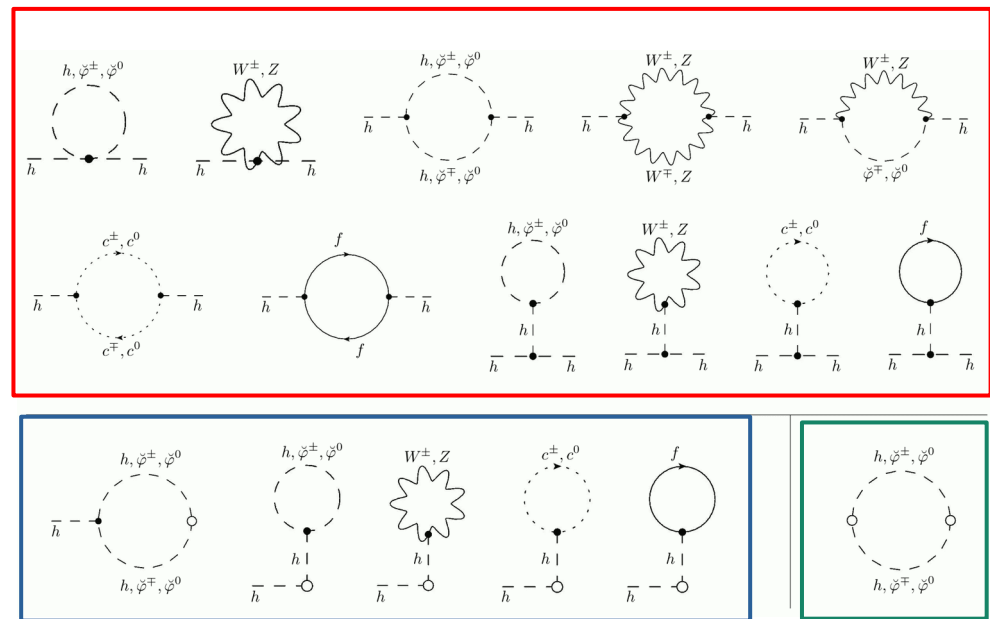
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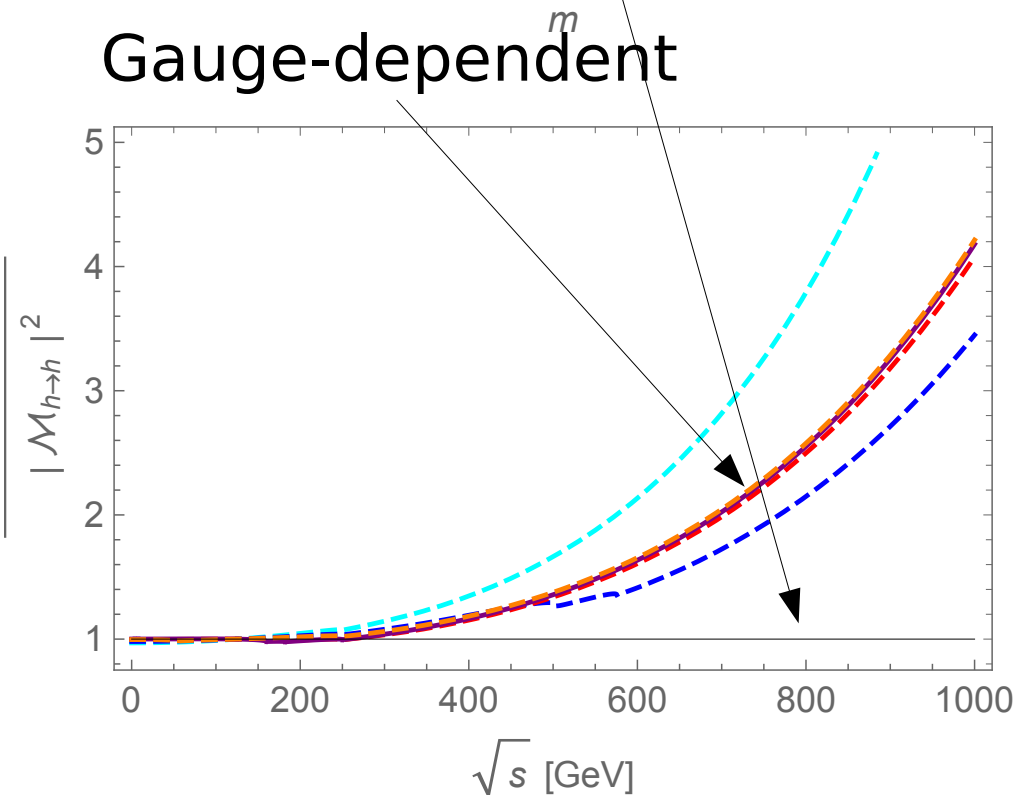
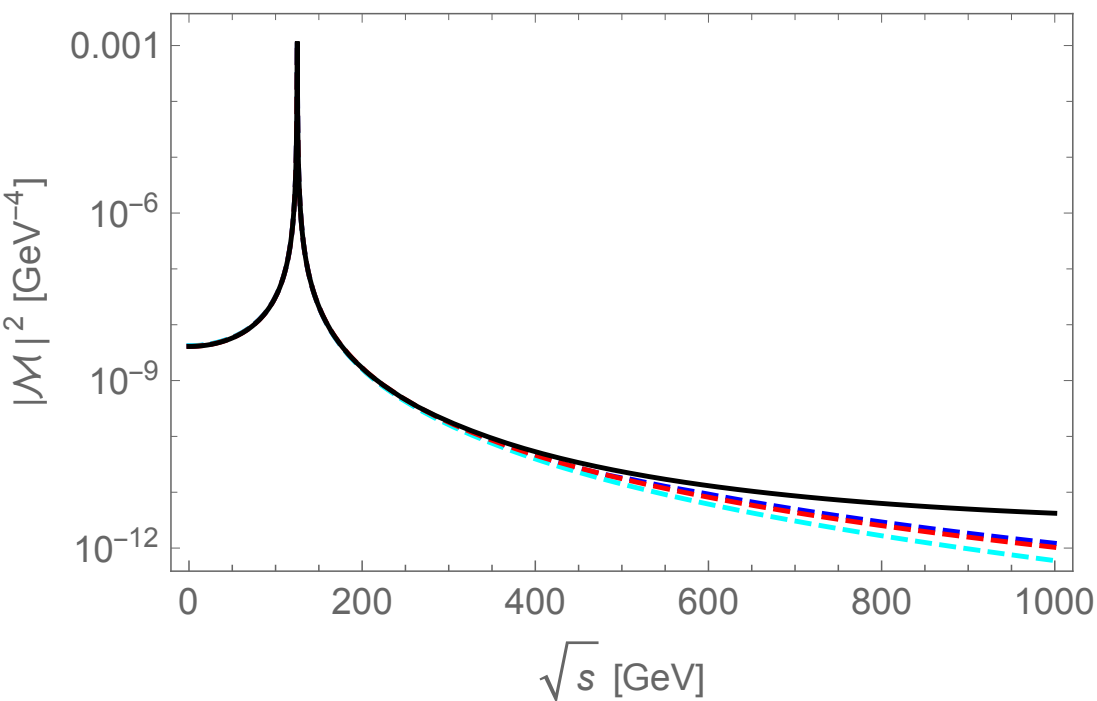
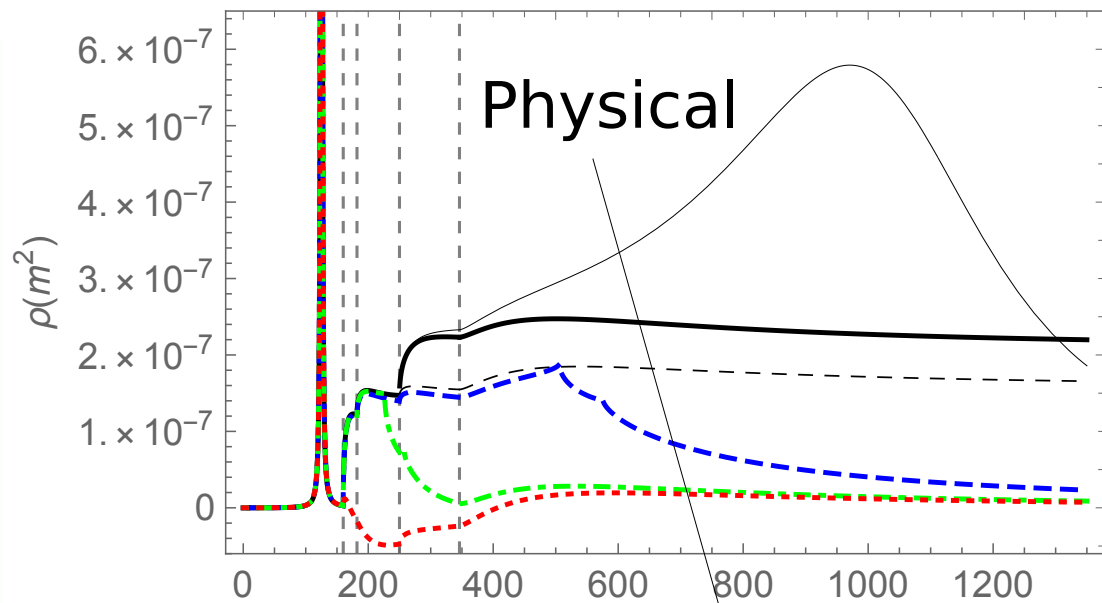
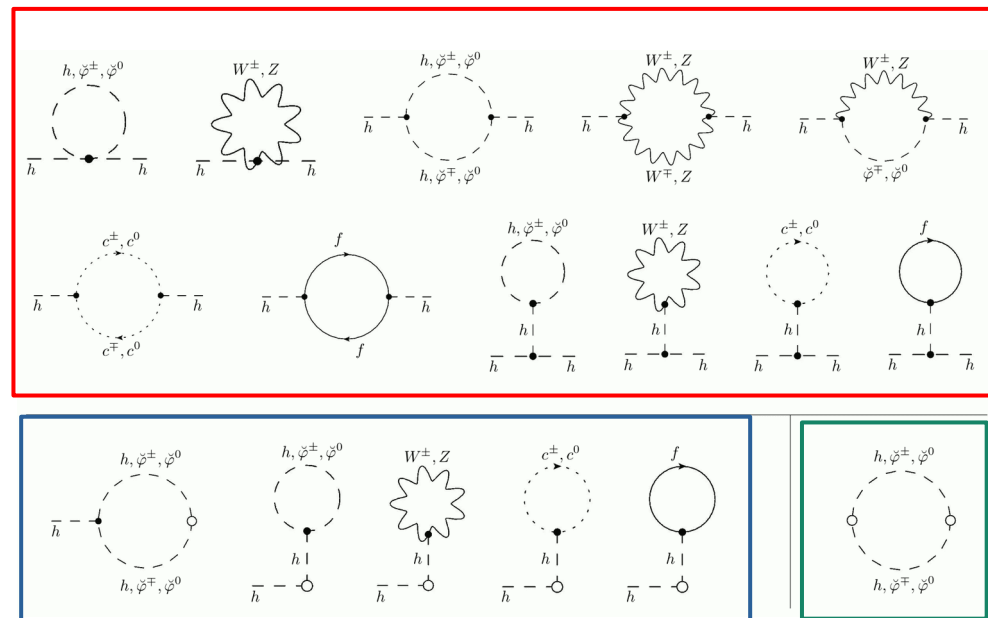
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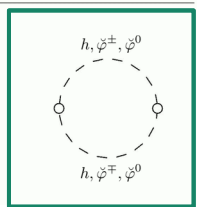
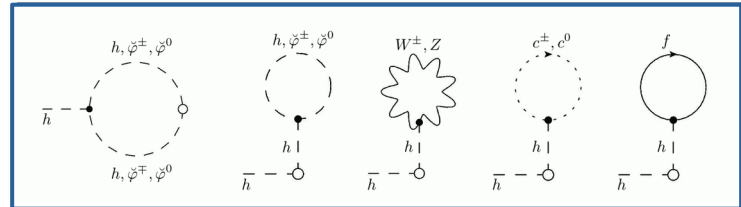
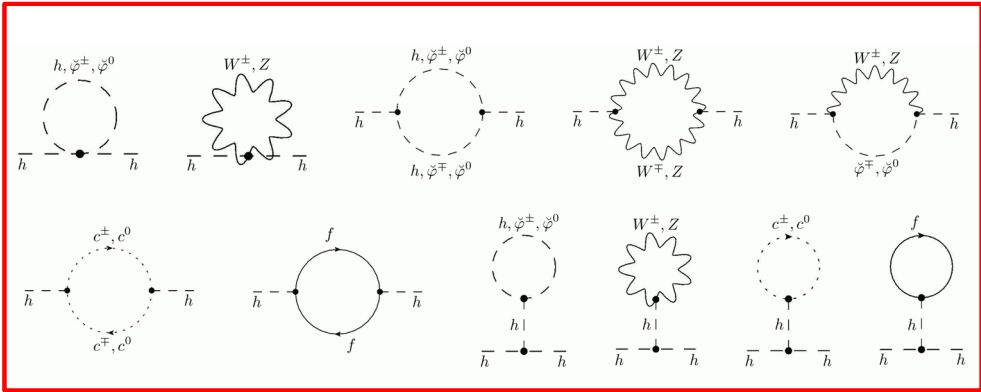
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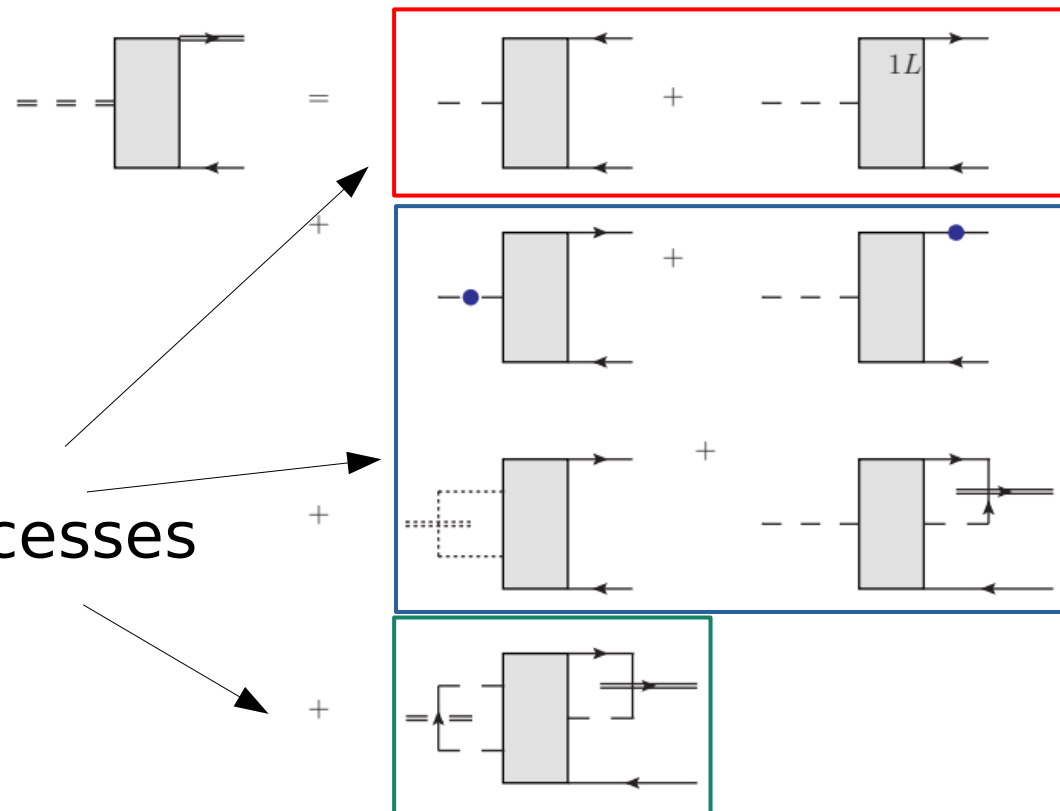


# Consequences: The Higgs

[Maas'12,'17  
Maas & Sondenheimer'20,  
Maas et al. unpublished]



Same structure repeats itself  
For decays and scattering processes



# What about the vector?

[Fröhlich et al.'80,'81  
Maas'12]

# What about the vector?

[Fröhlich et al.'80,'81  
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1) Formulate gauge-invariant operator

1- triplet:  $\langle (\tau^i h + D_\mu h)(x) (\tau^j h + D_\mu h)(y) \rangle$

# What about the vector?

[Fröhlich et al.'80,'81  
Maas'12]

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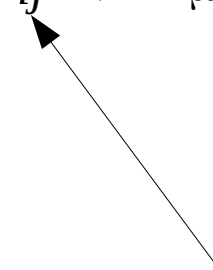
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Exactly one gauge boson  
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Matrix from  
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# Phenomenological Implications

-

Can we measure this?

# **Bound states as extended objects**

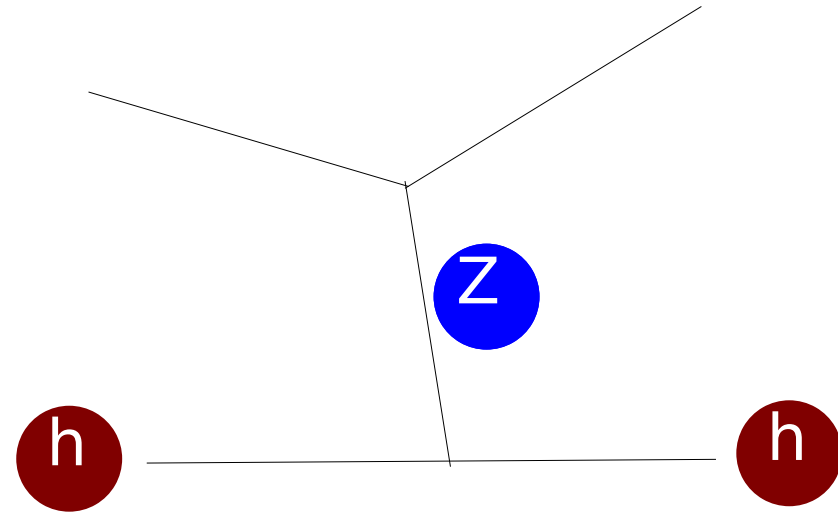
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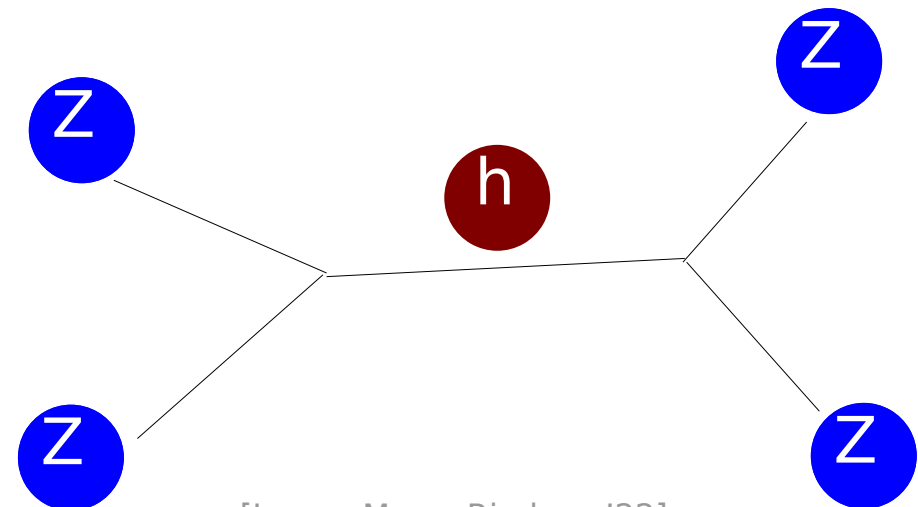
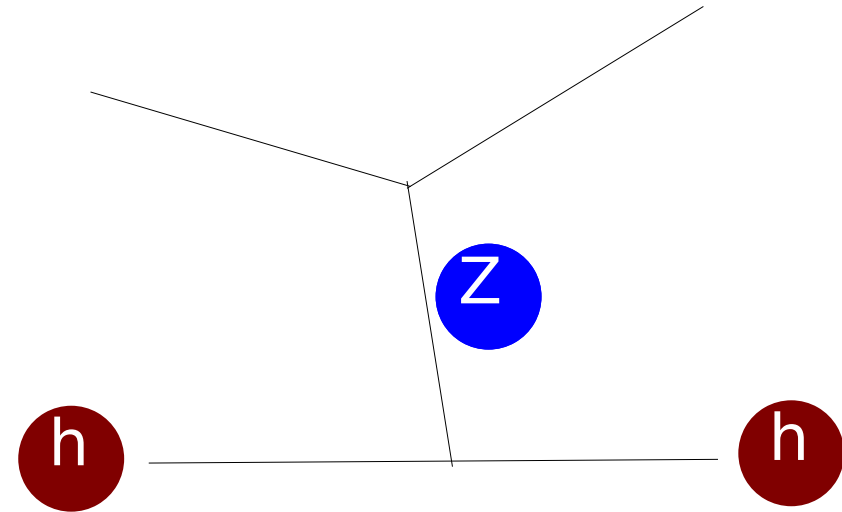
[Maas, Raubitzek, Törek'18]



# Bound states as extended objects

- Two possibilities to measure extension
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process
  - Elastic scattering
    - Standard vector boson scattering process at low energies
    - Use this one

[Maas, Raubitzek, Törek'18]



[Jenny, Maas, Riederer'22]

# Radius from elastic scattering in VBS

- Elastic region:  $160/180 \text{ GeV} \leq \sqrt{s} \leq 250 \text{ GeV}$ 
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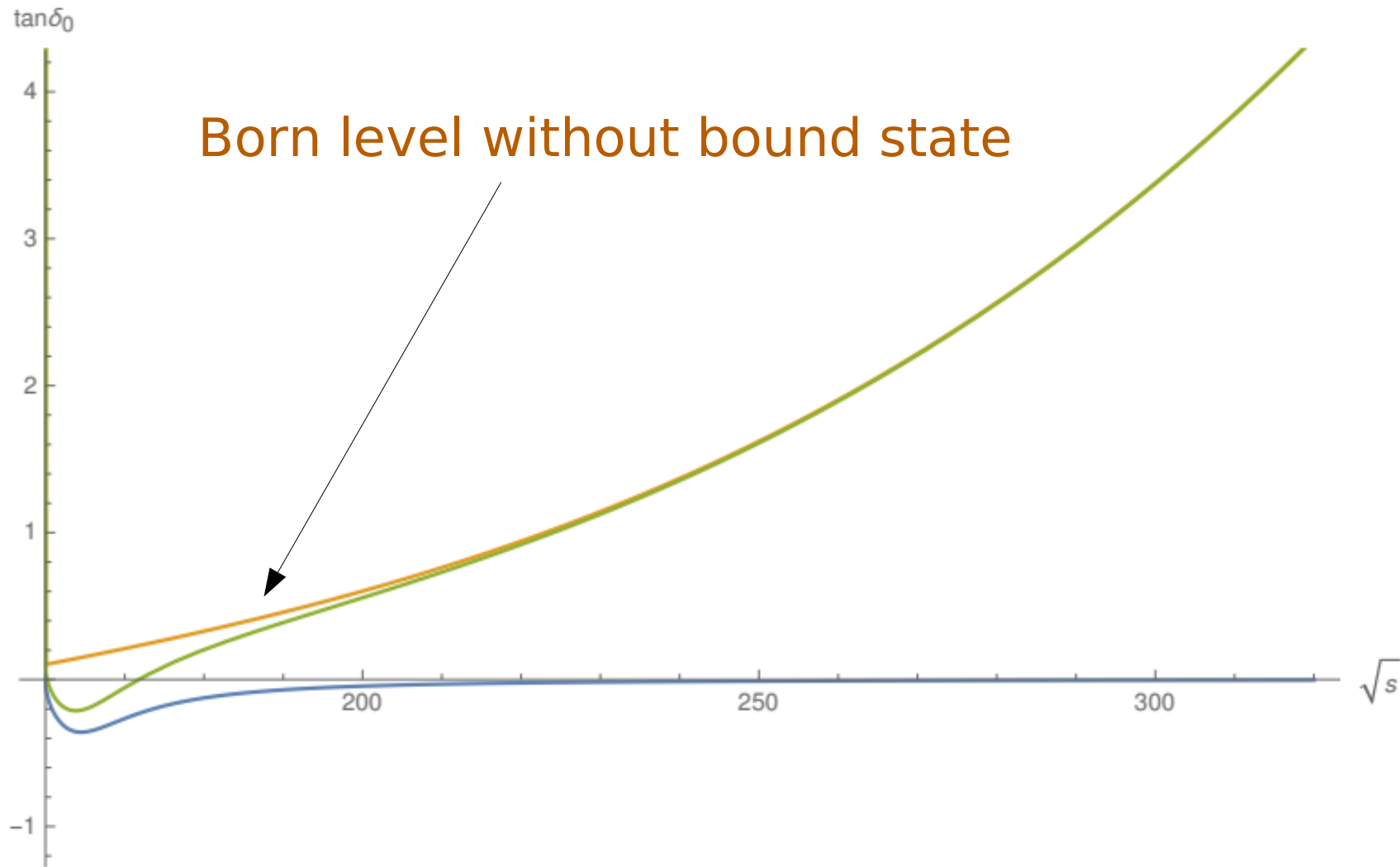
Scattering length ~ "size"

Phase shift

# Impact of a finite size of the Higgs

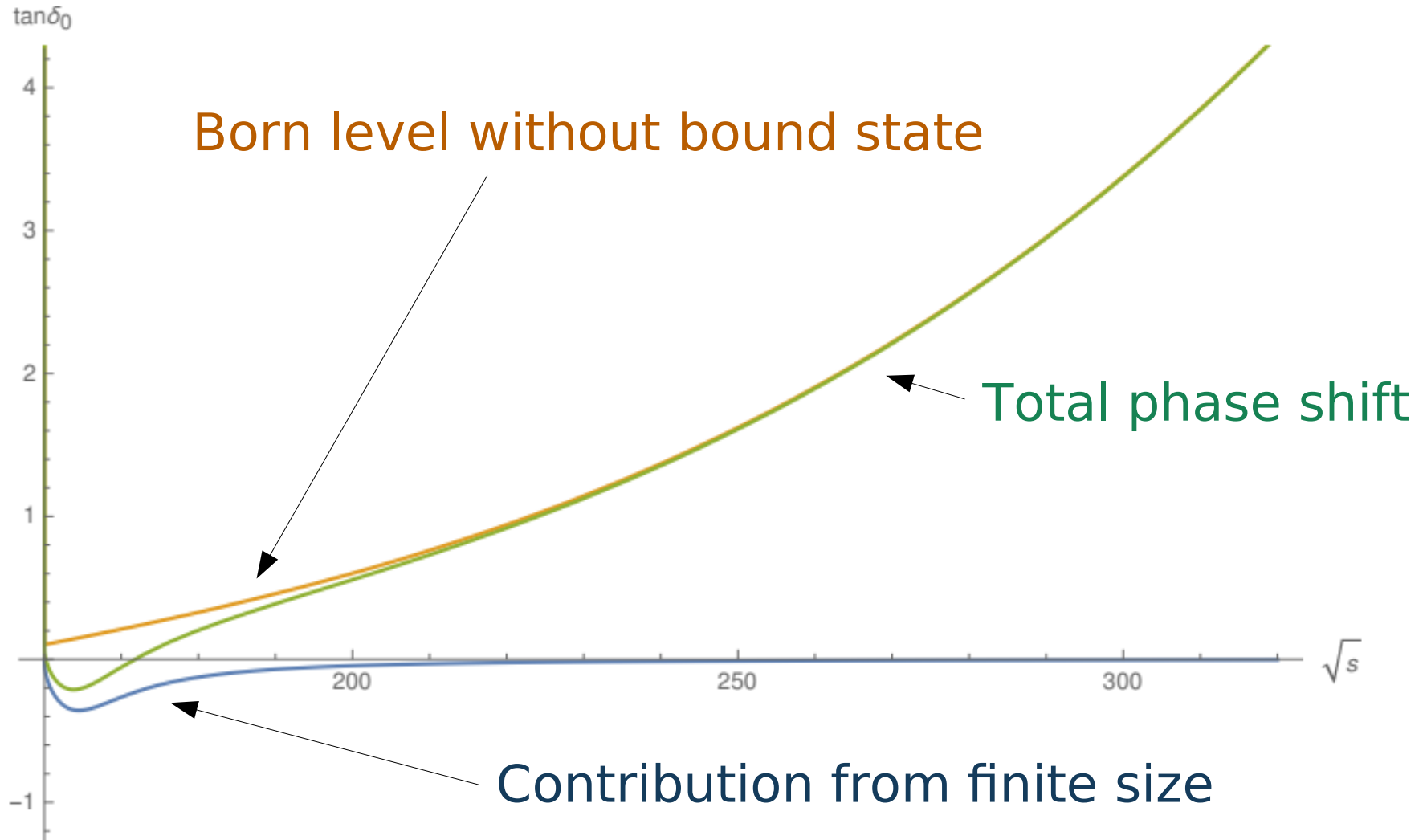
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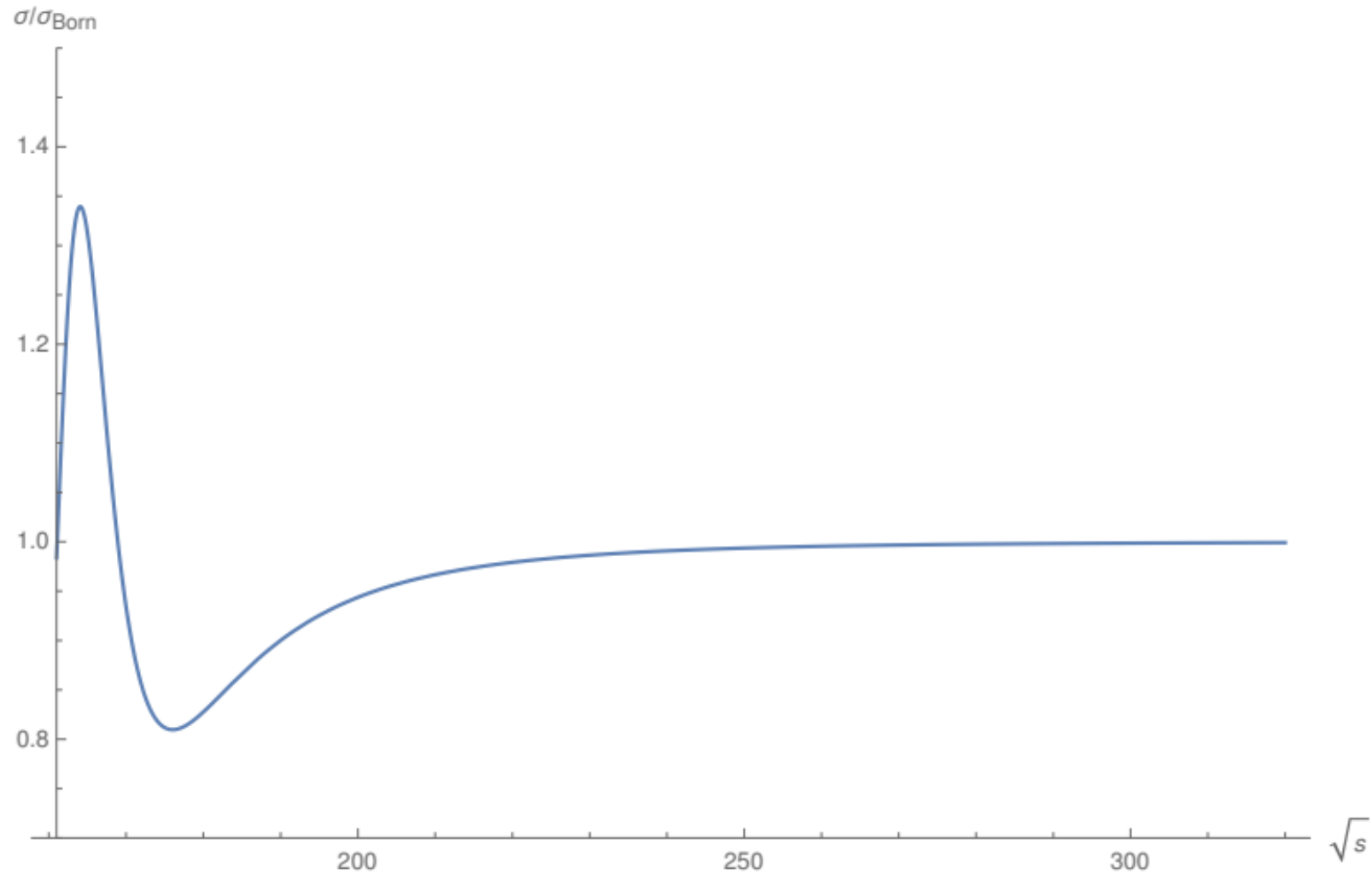
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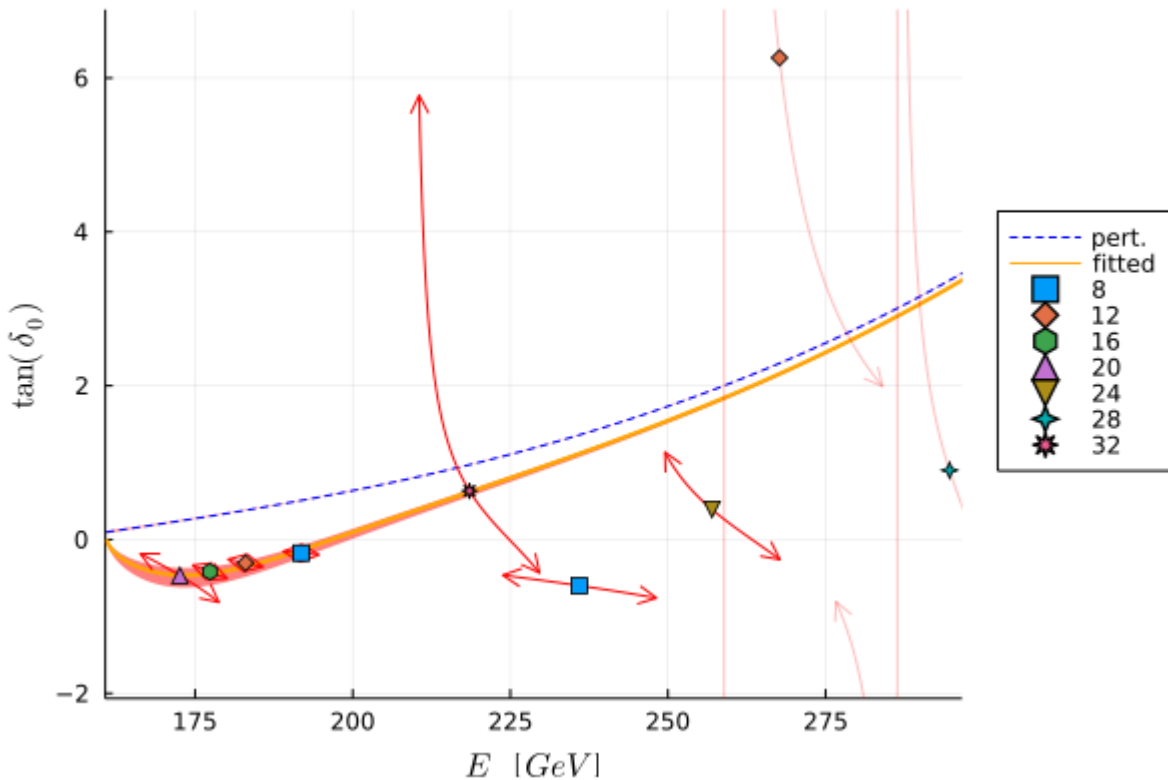


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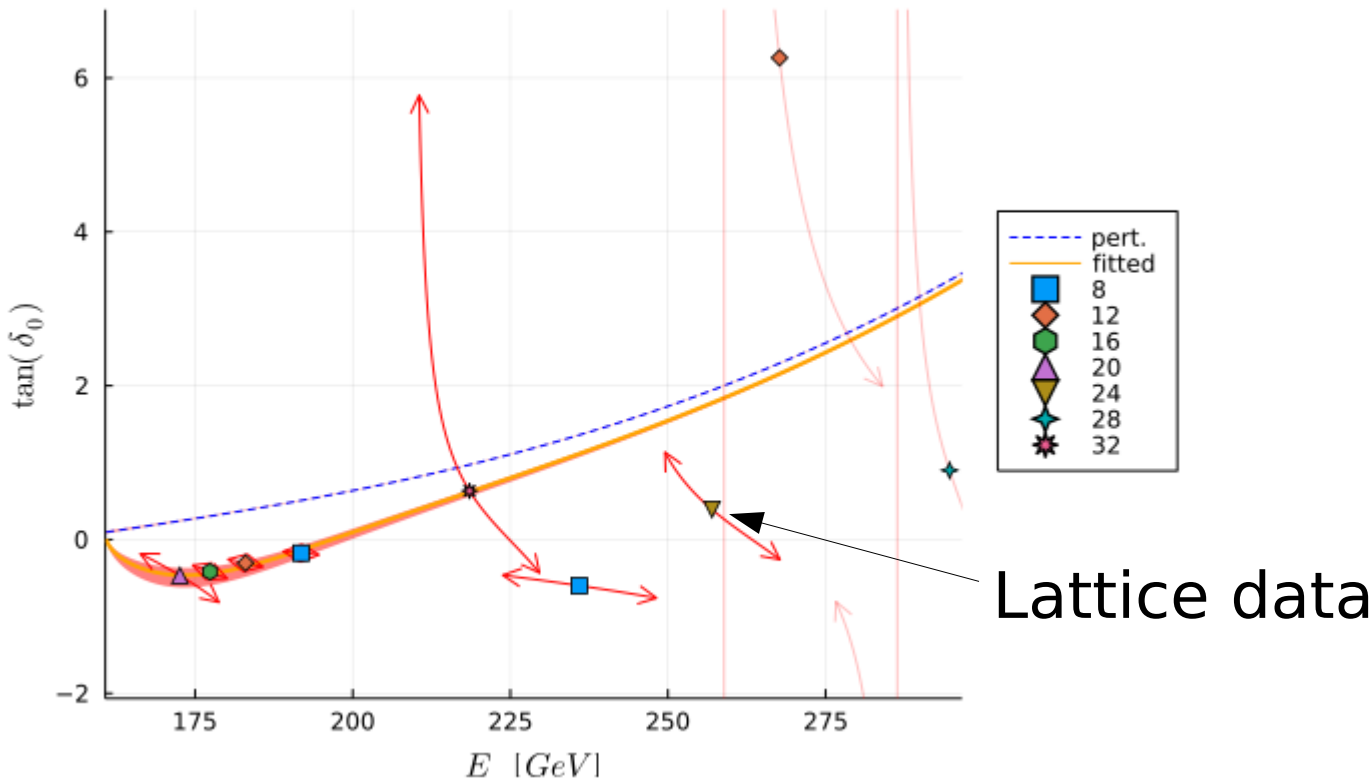
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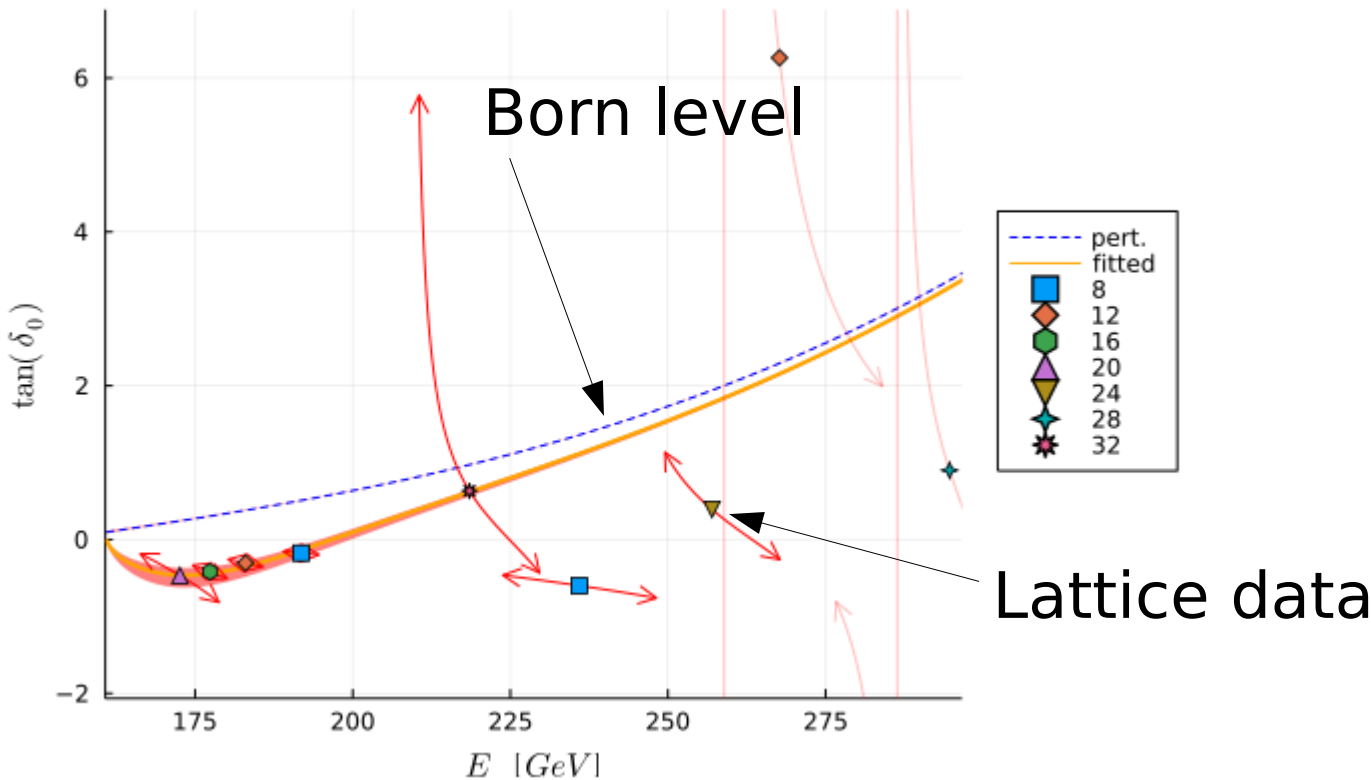
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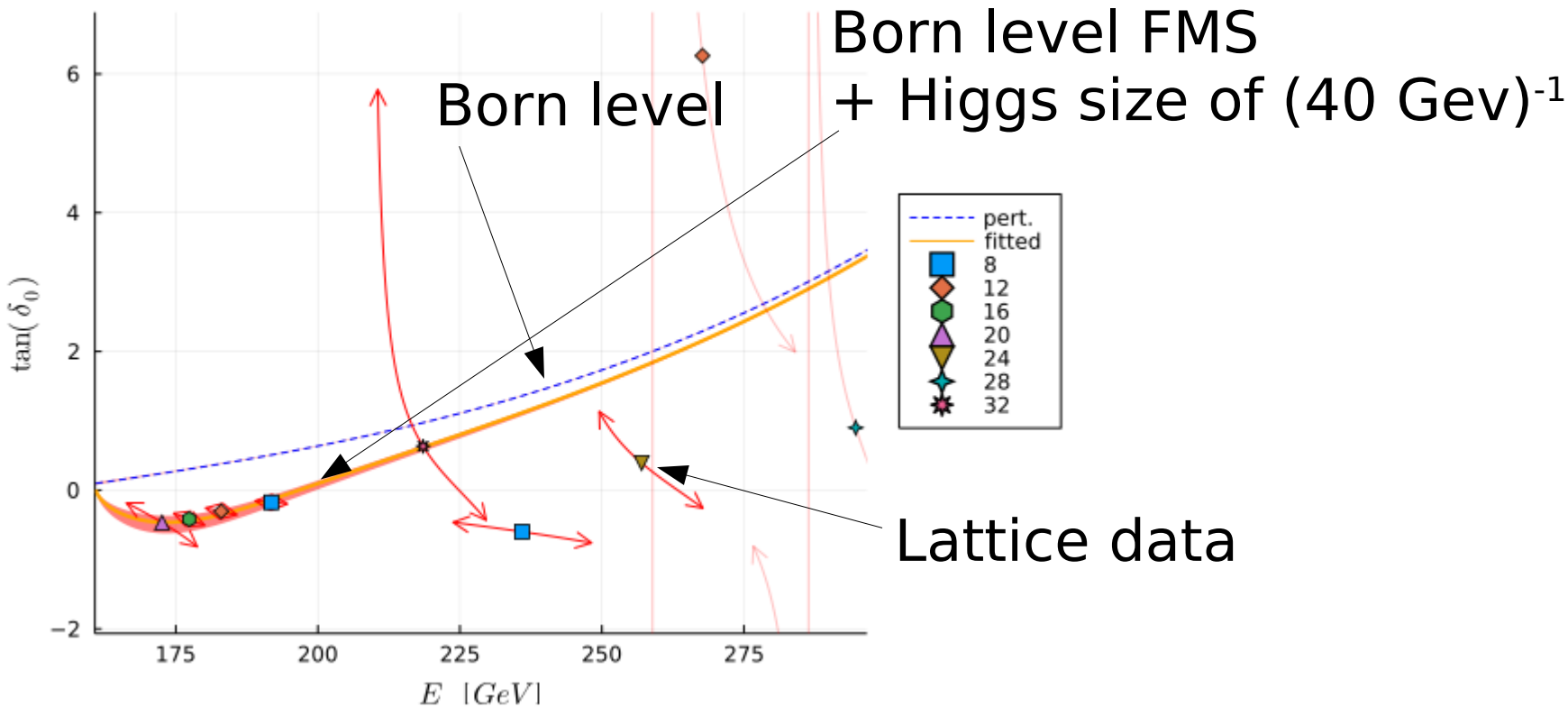


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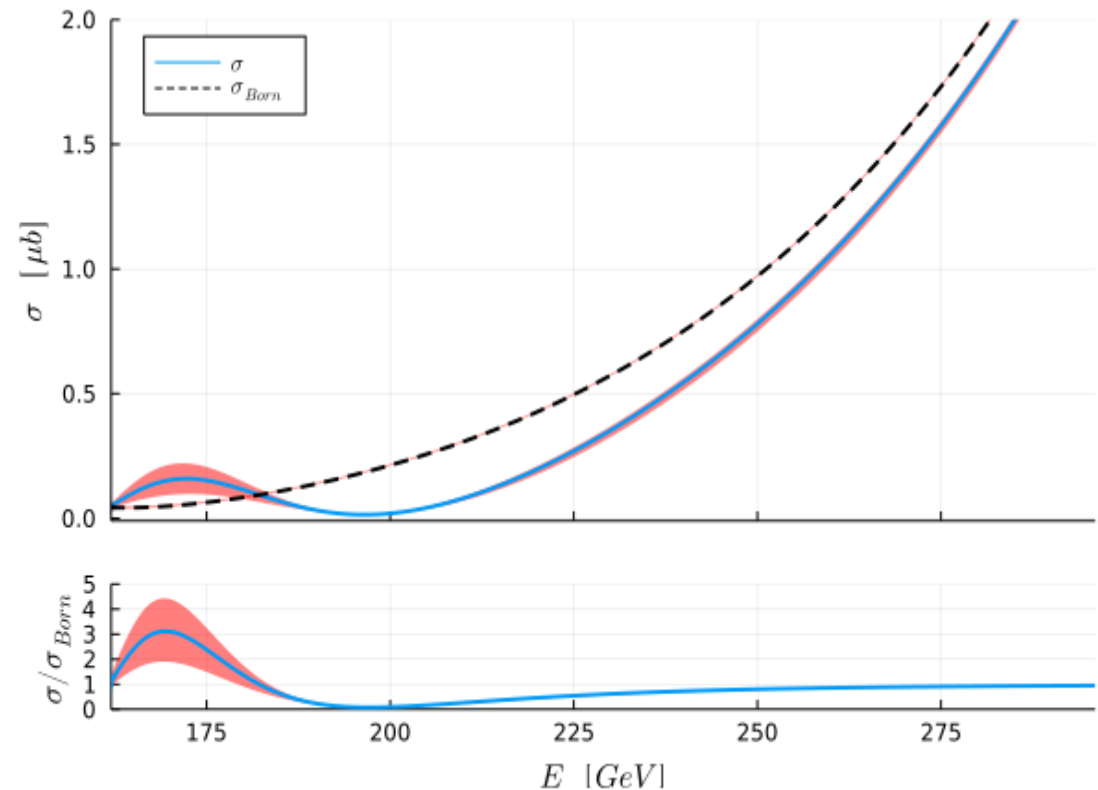
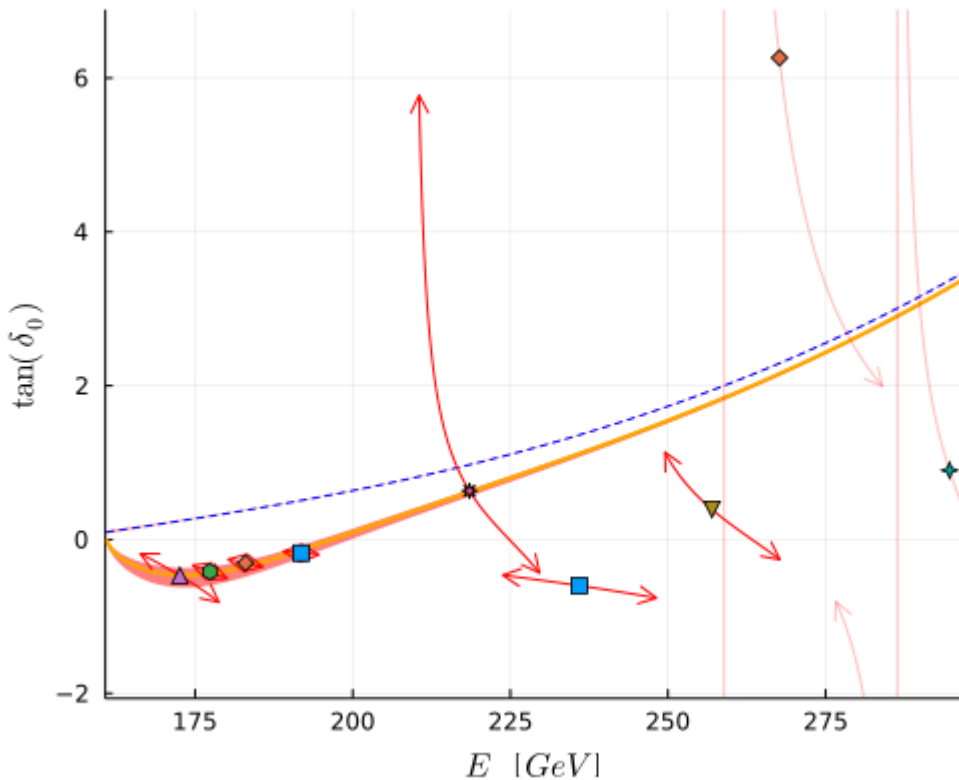
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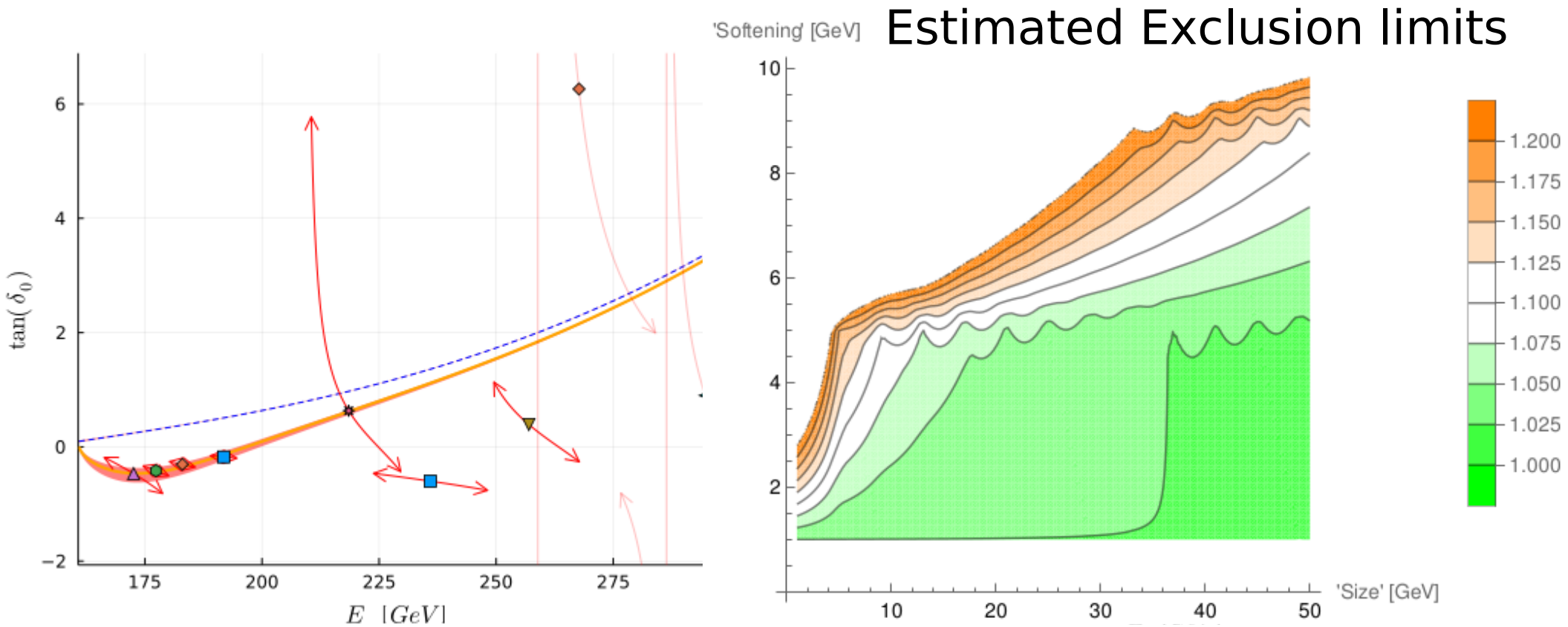
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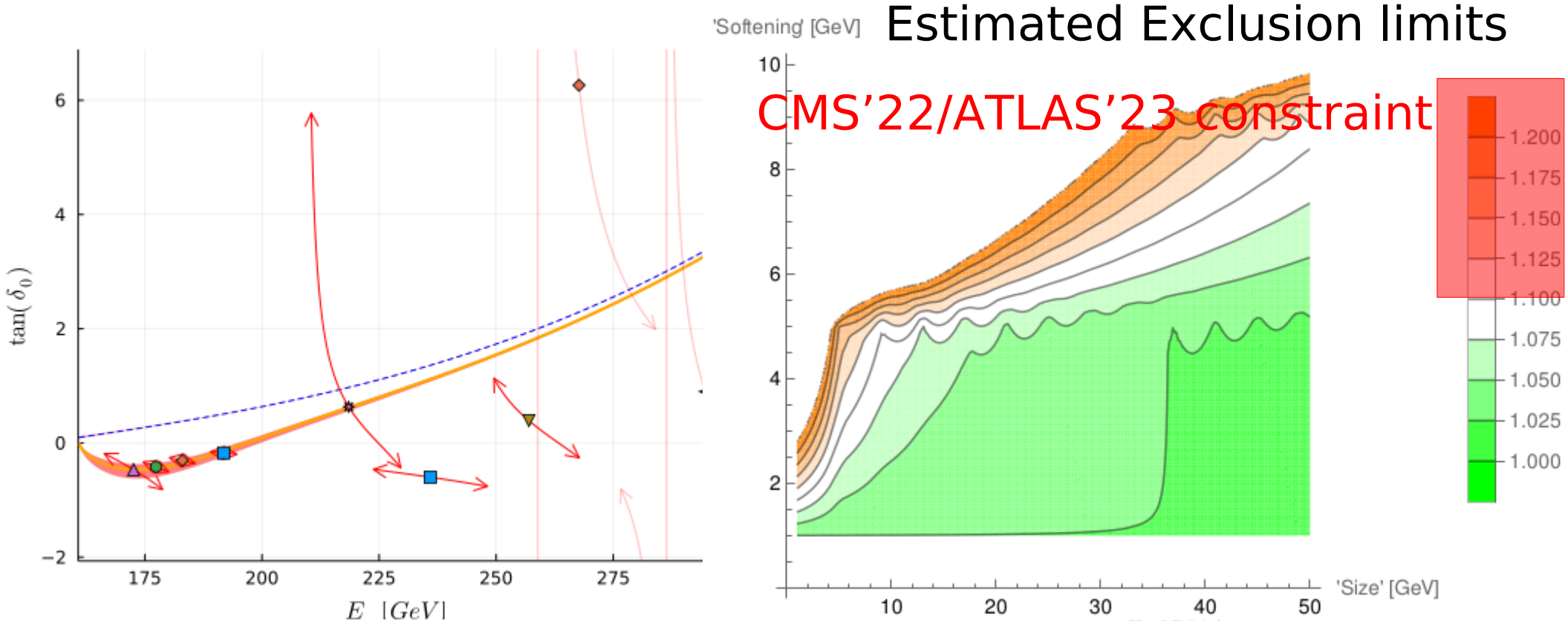
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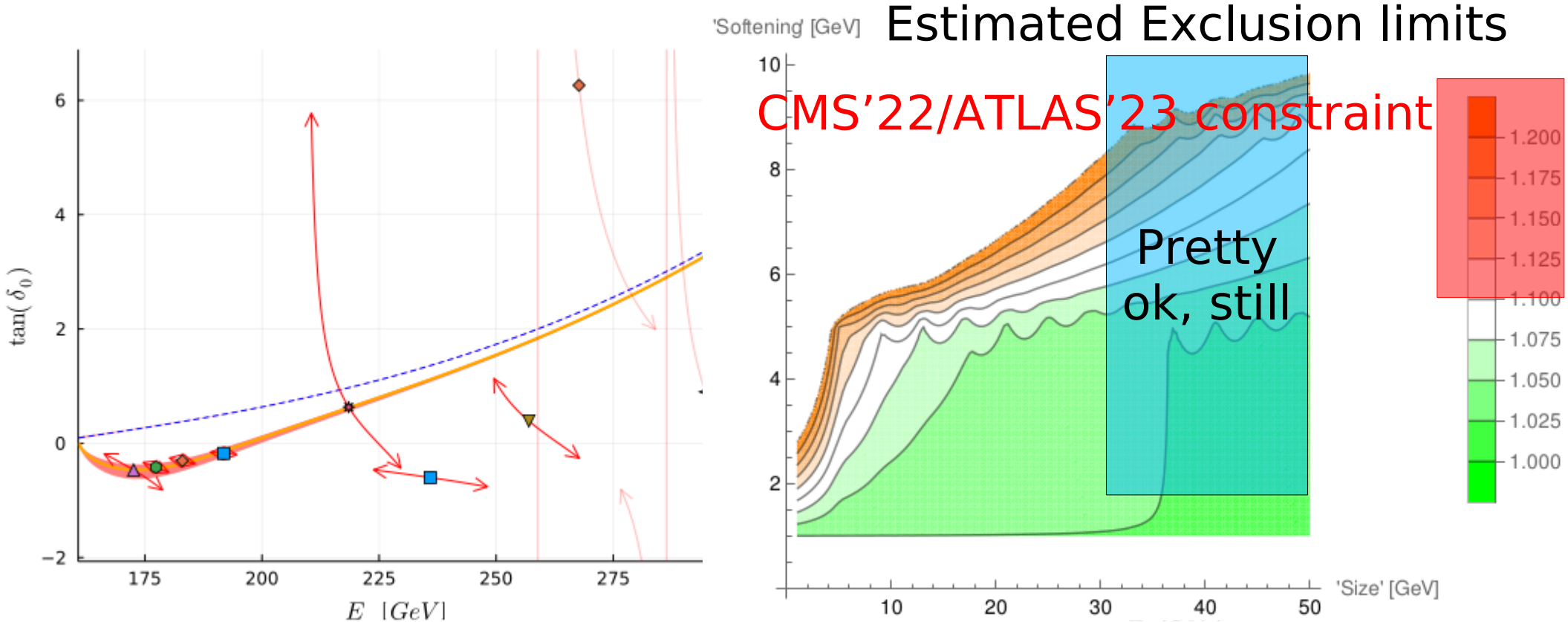
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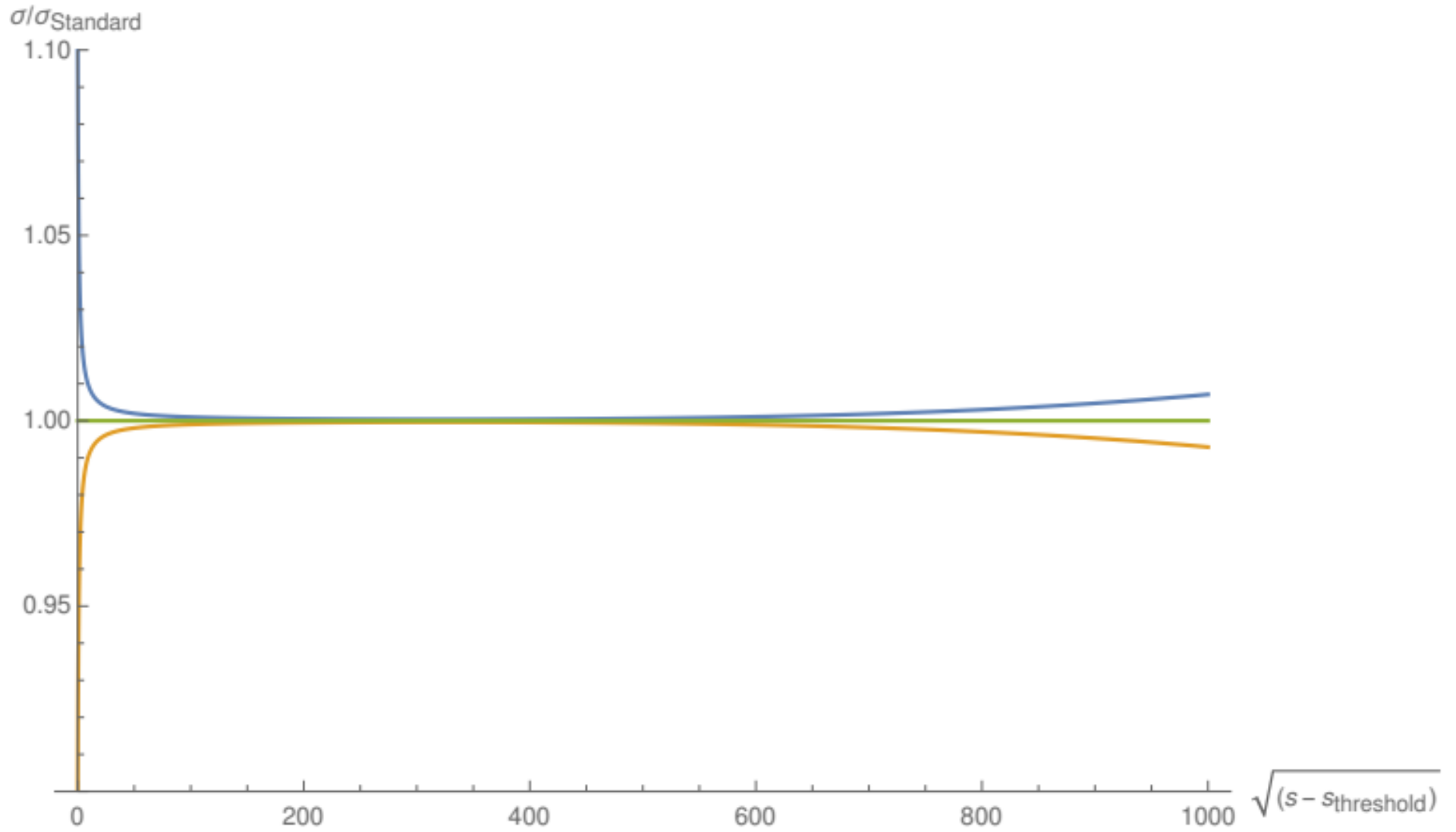
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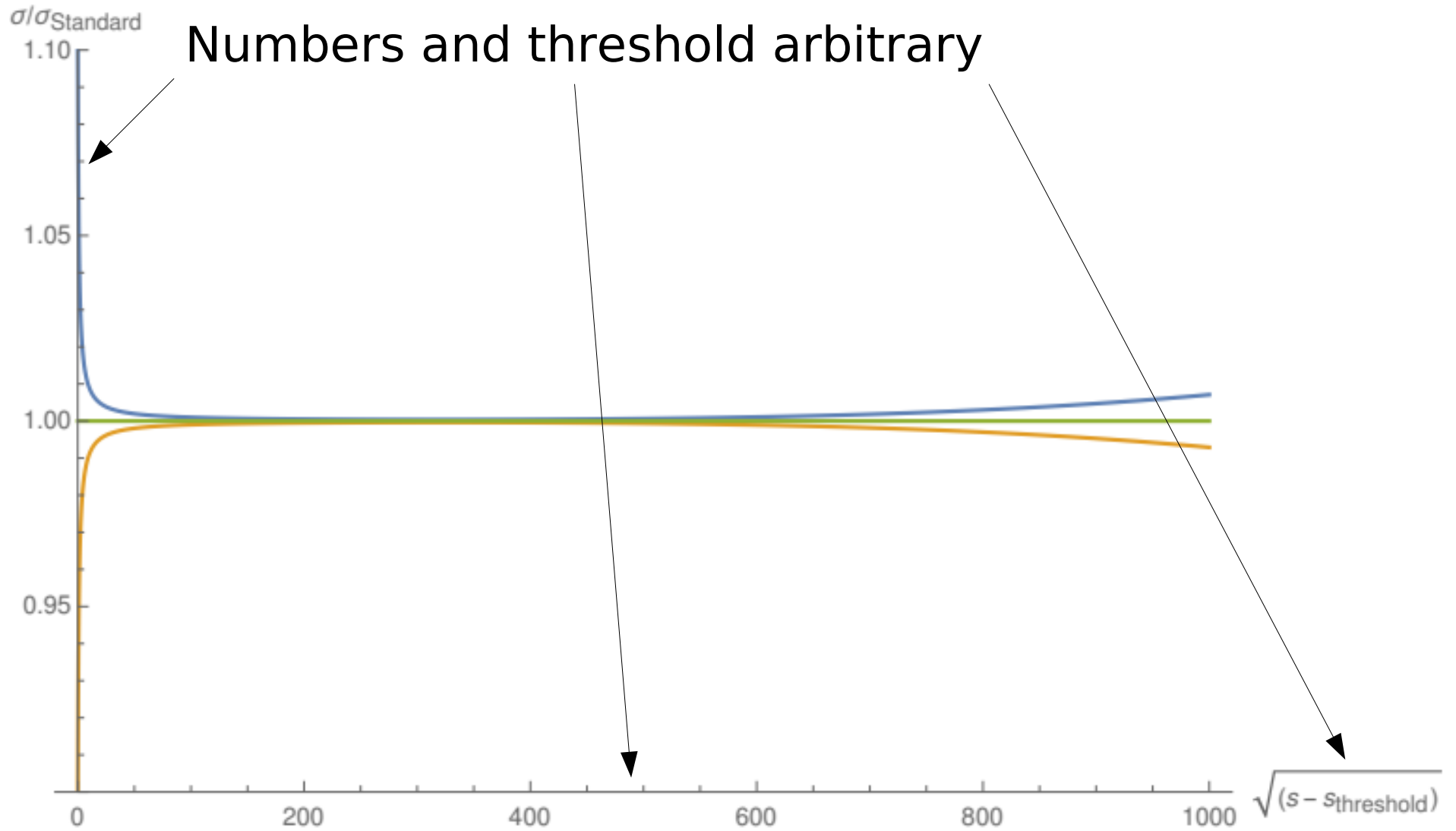


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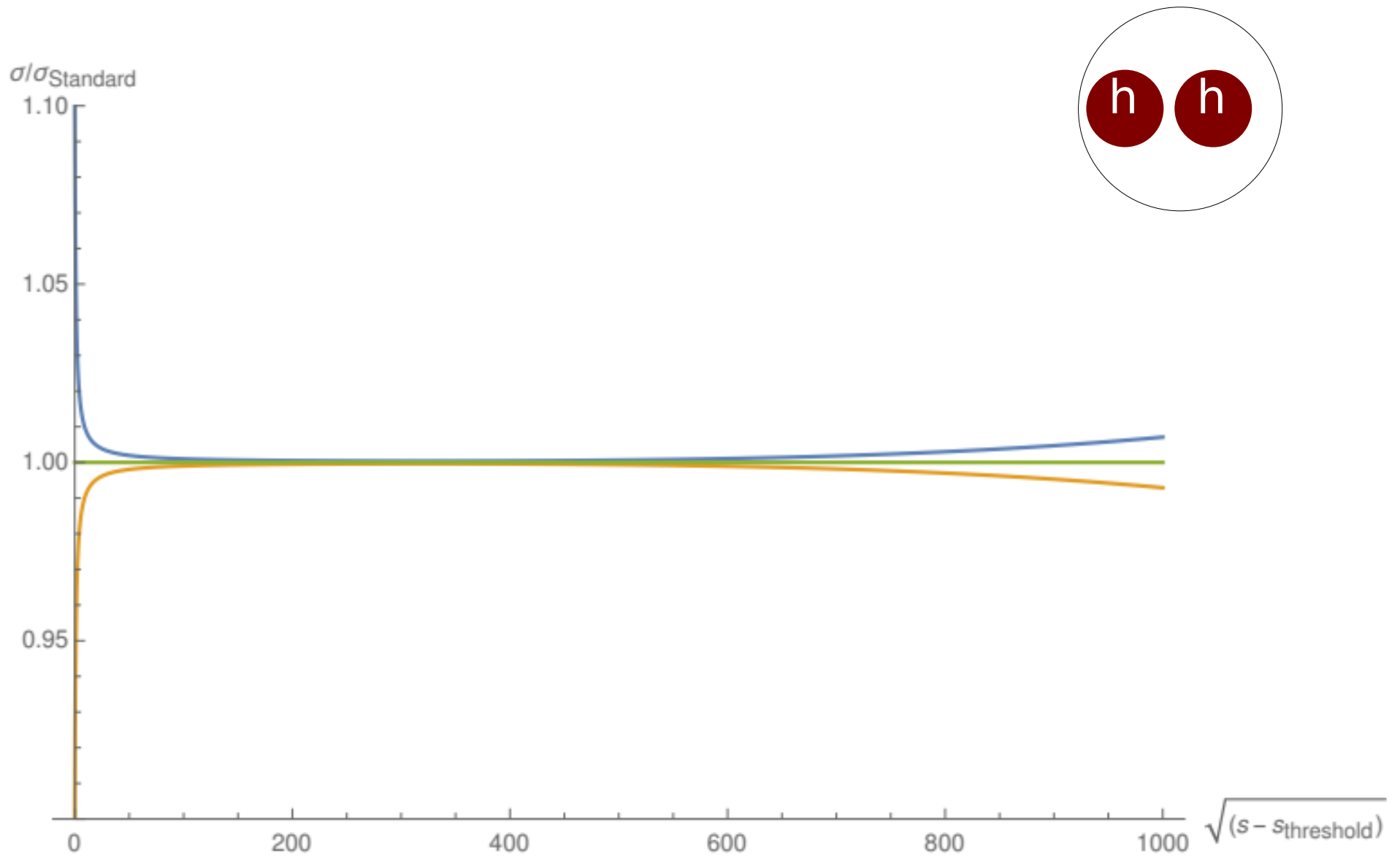


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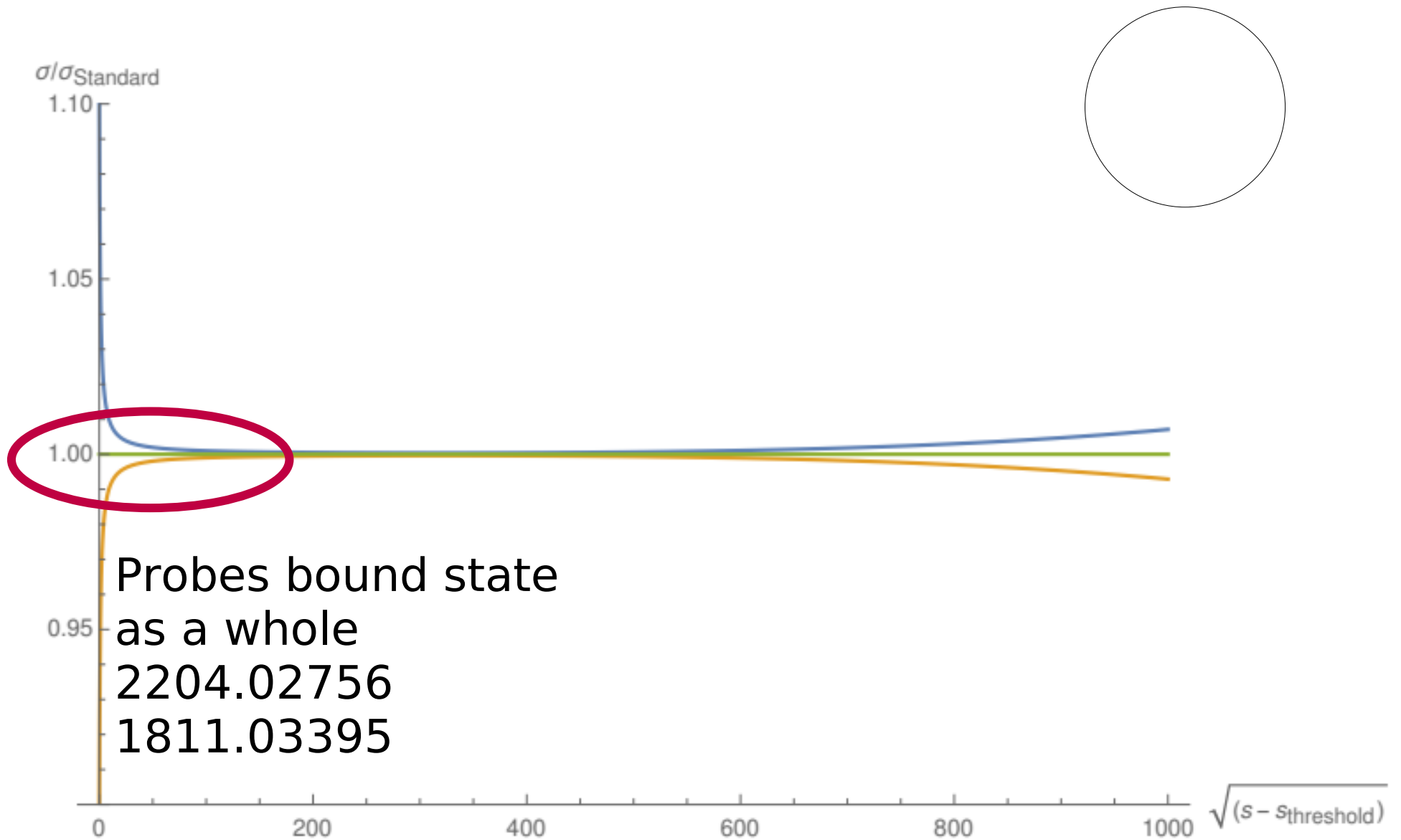




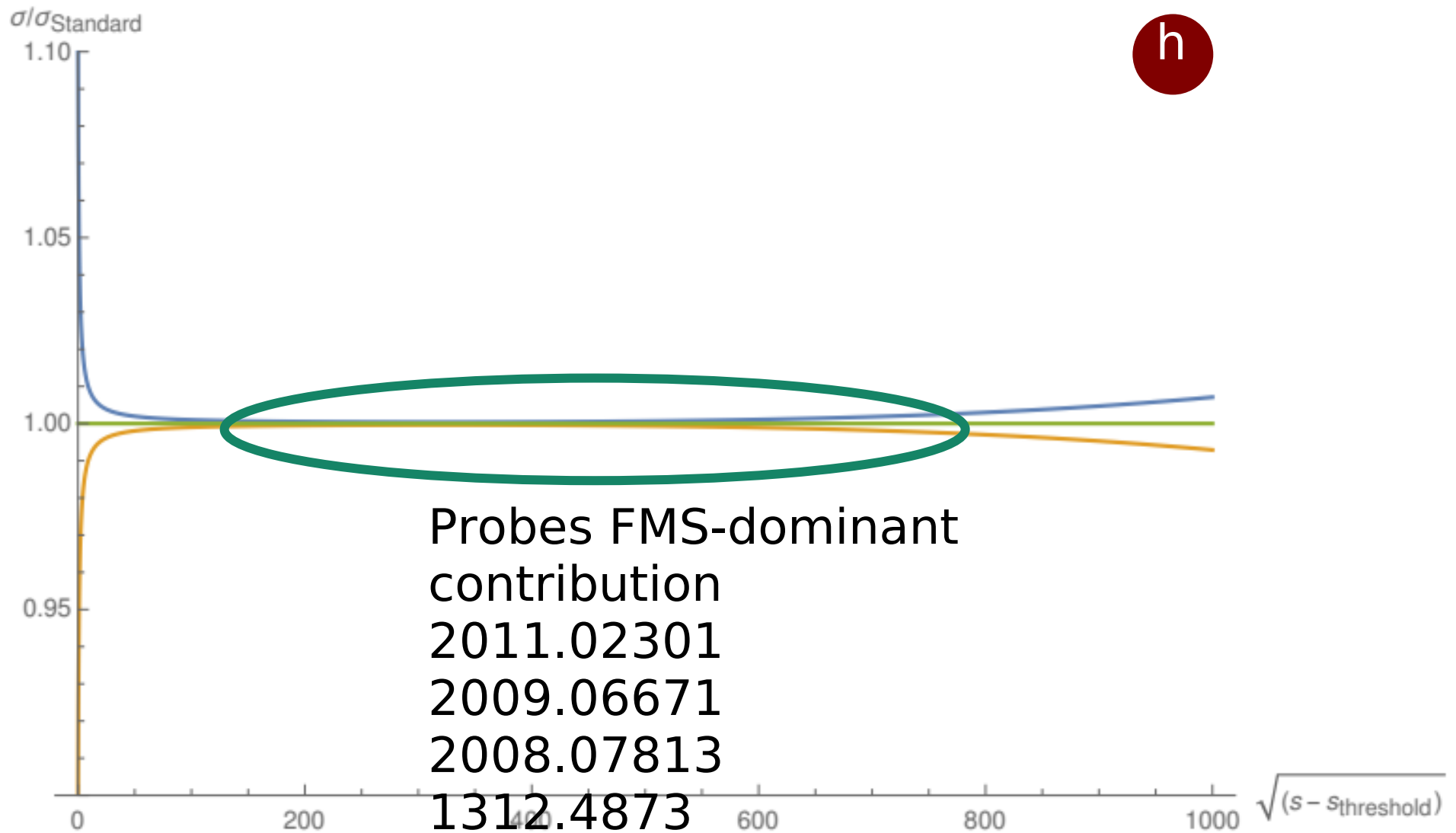
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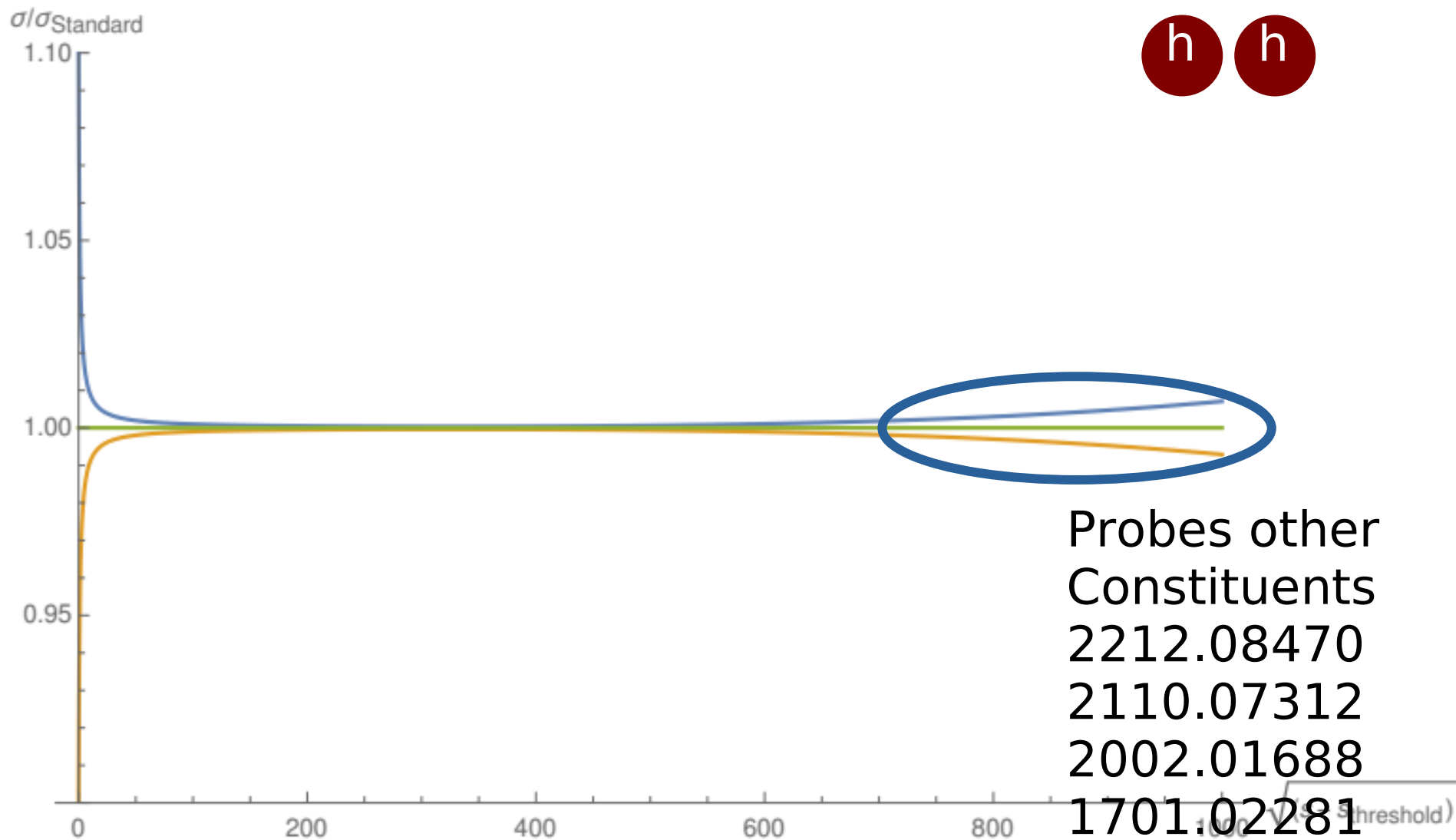
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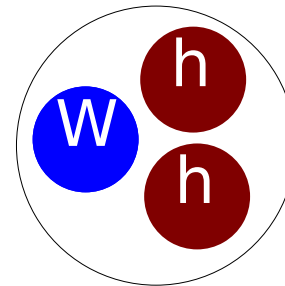
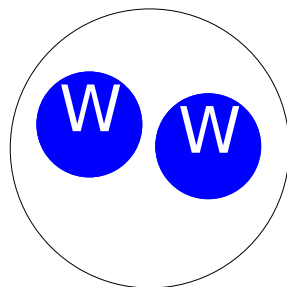
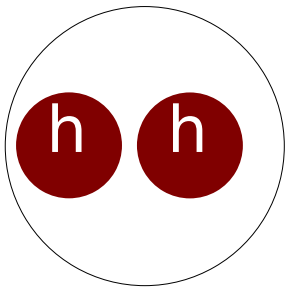
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[Fröhlich et al.'80,  
Banks et al.'79]

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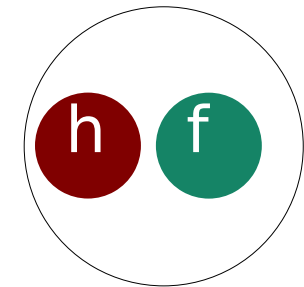
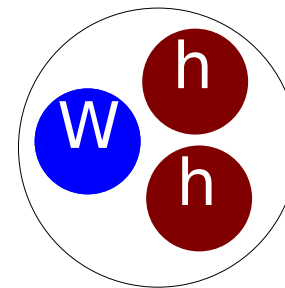
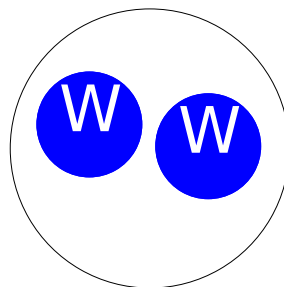
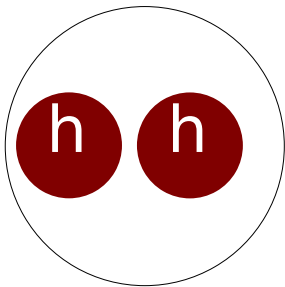


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# Flavor on the lattice

- Only mock-up standard model
  - Compressed mass scales
  - One generation
  - Degenerate leptons and neutrinos
  - Dirac fermions: left/right-handed non-degenerate
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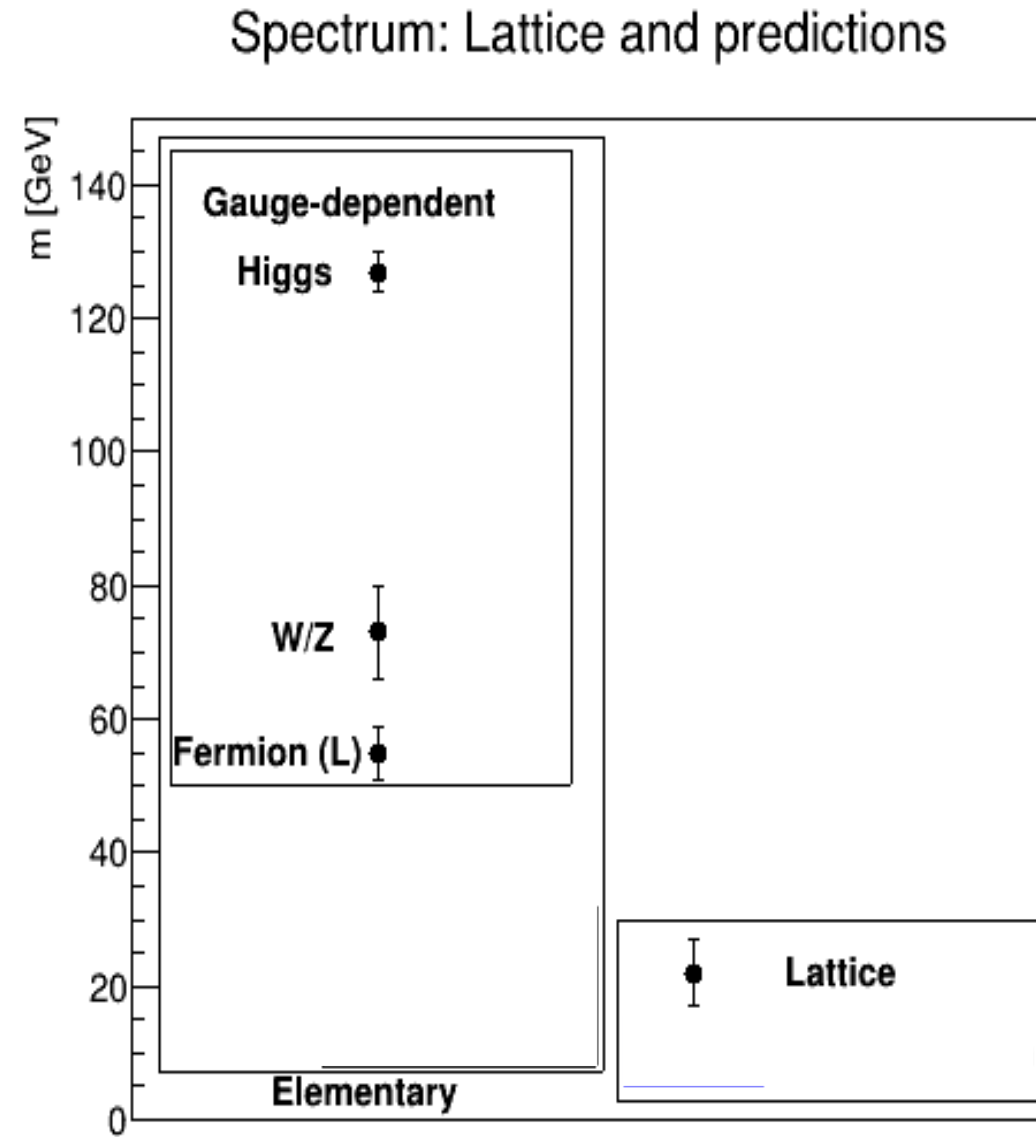


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- Same qualitative outcome
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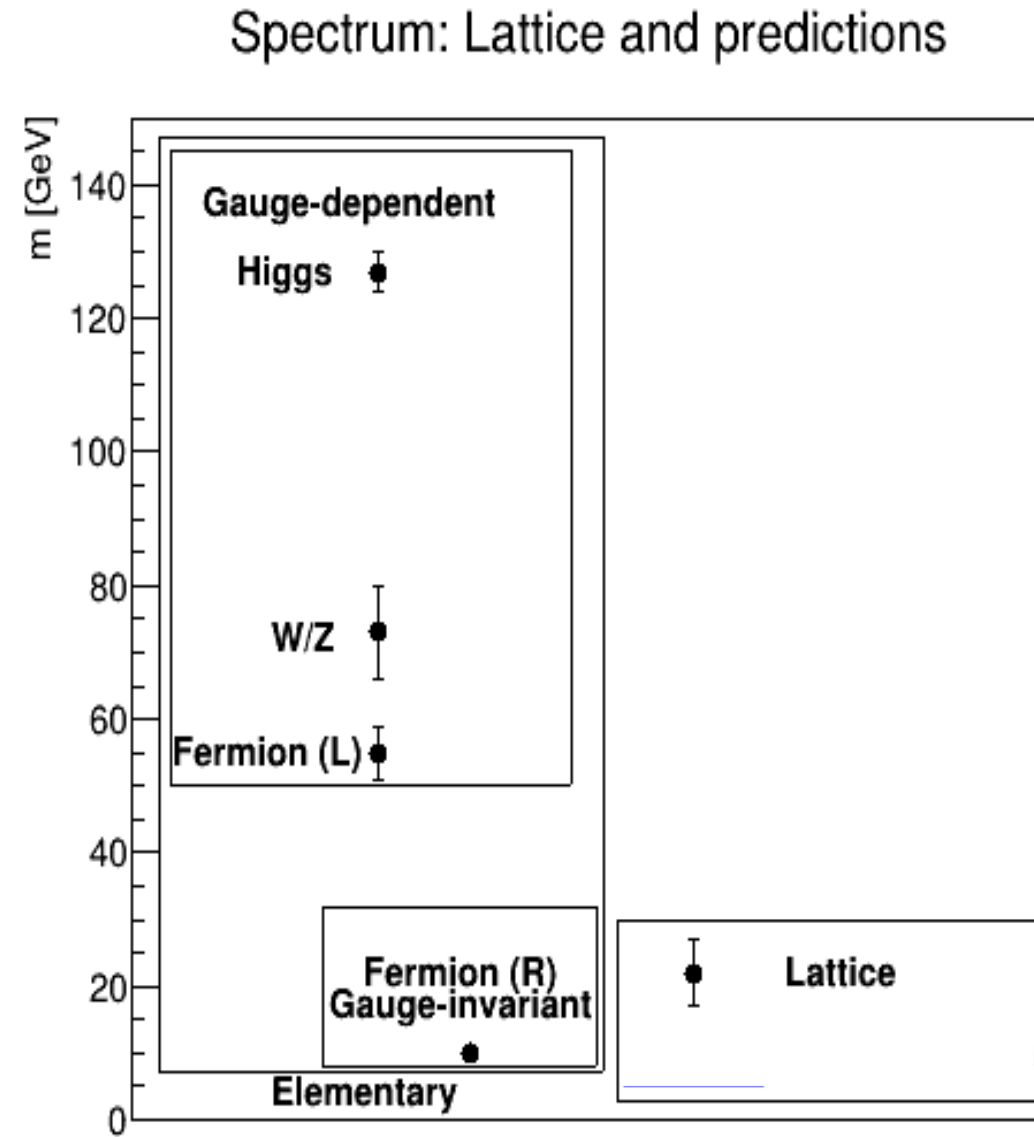
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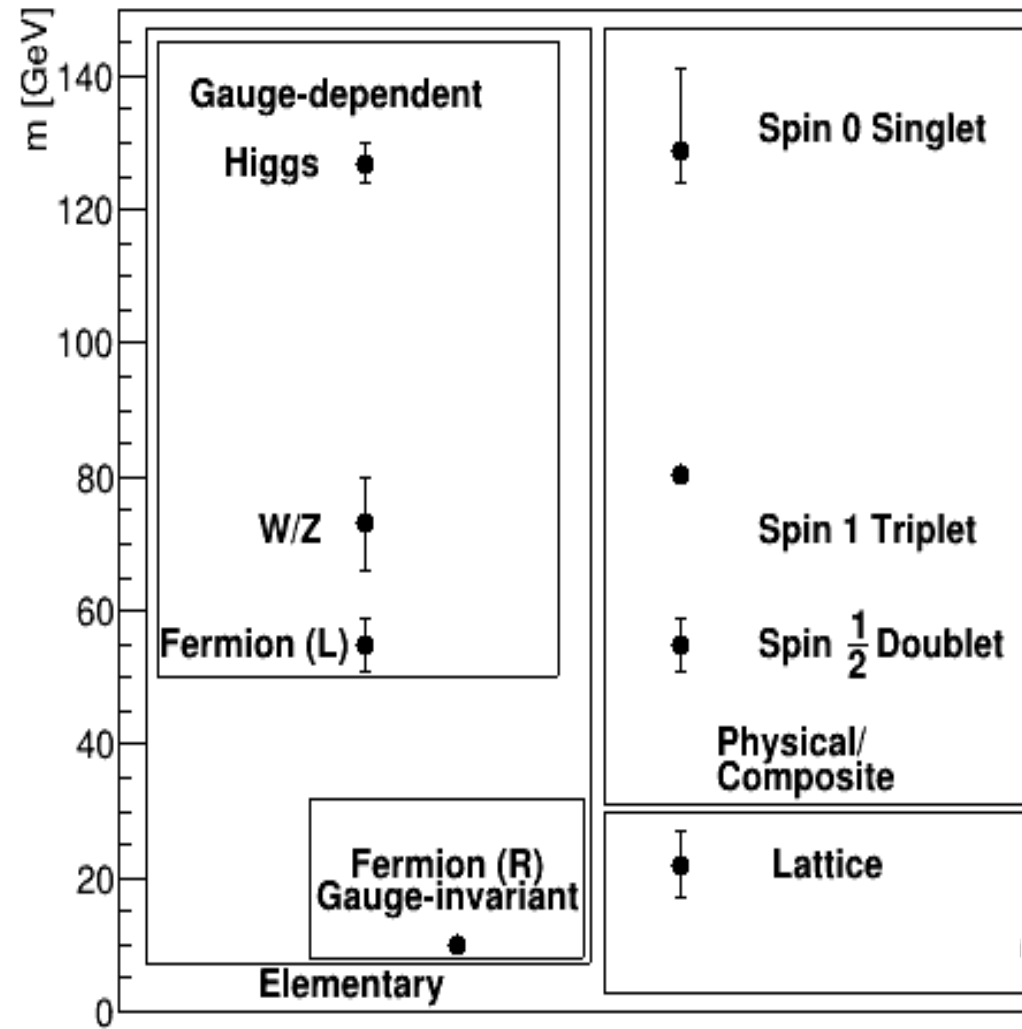
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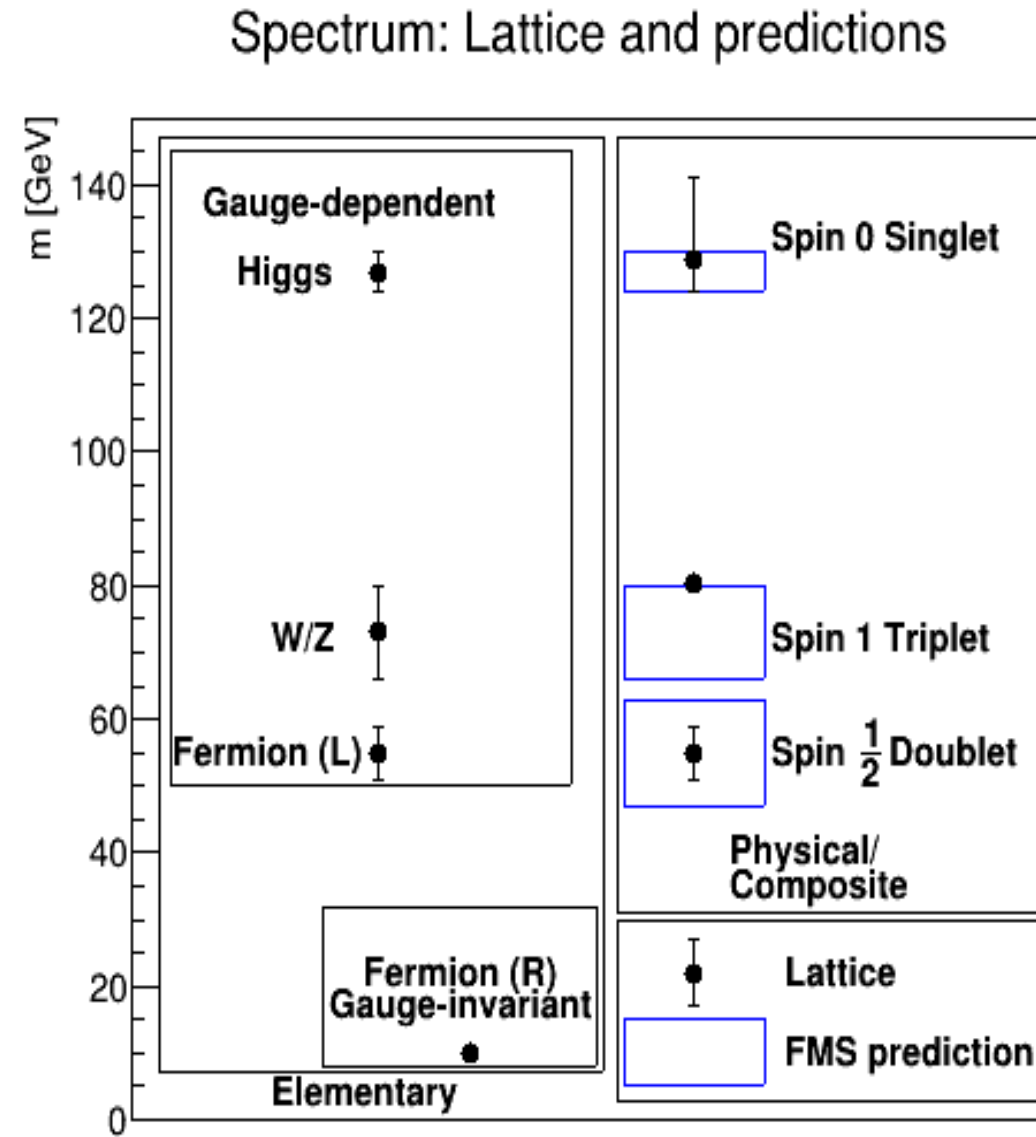
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Spectrum: Lattice and predictions



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  - Flavor and custodial symmetry patterns
- Supports FMS prediction



New physics

-

Qualitative changes

# Beyond the standard model

[Maas'15  
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- Global U(1) custodial (flavor) symmetry

- Acts as (right-)transformation on the scalar field only

$$W_\mu^a \rightarrow W_\mu^a \qquad h \rightarrow \exp(ia) h$$

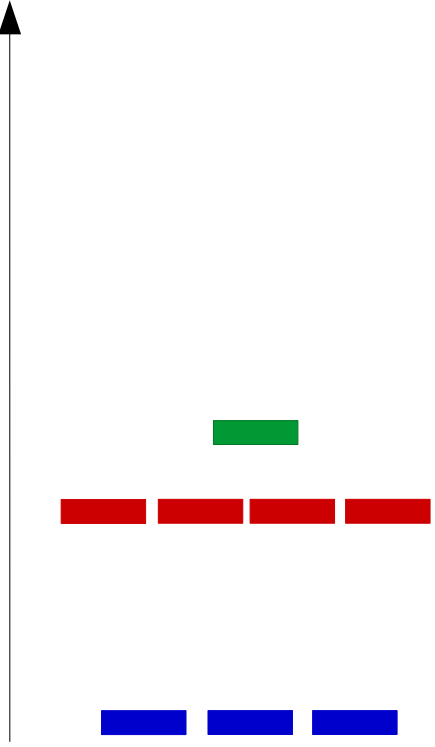
# Spectrum

Gauge-dependent  
Vector

Mass

0

'SU(3) → SU(2)'



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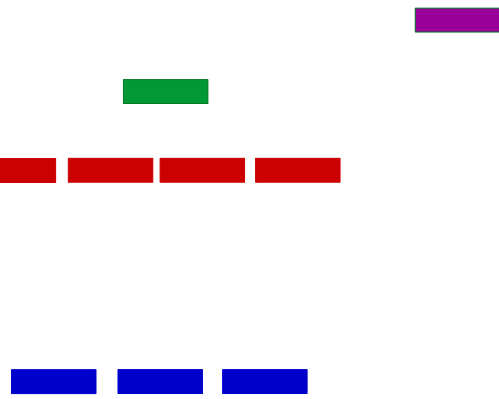
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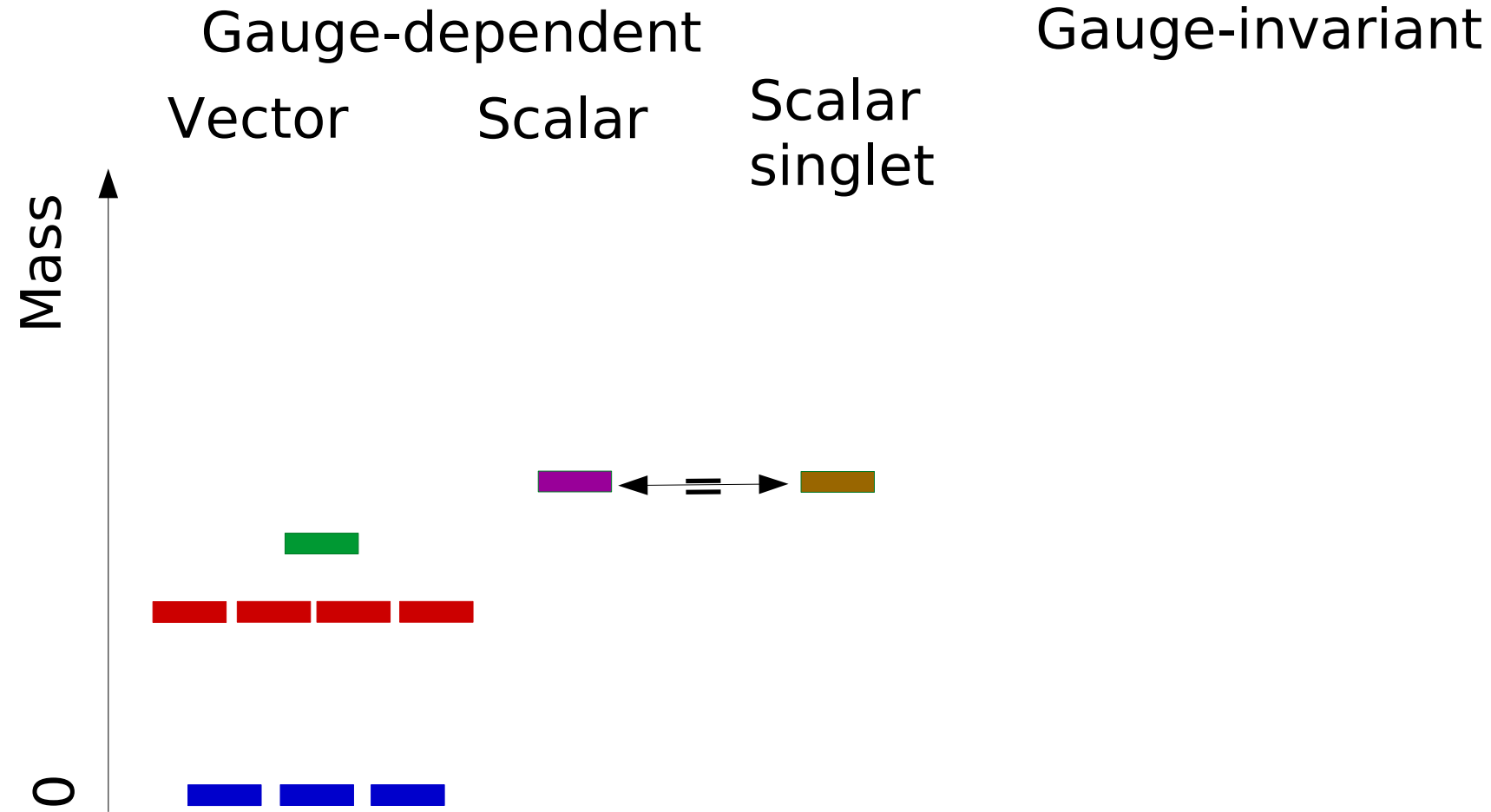
0



Confirmed in gauge-fixed  
lattice calculations [Maas et al.'16]

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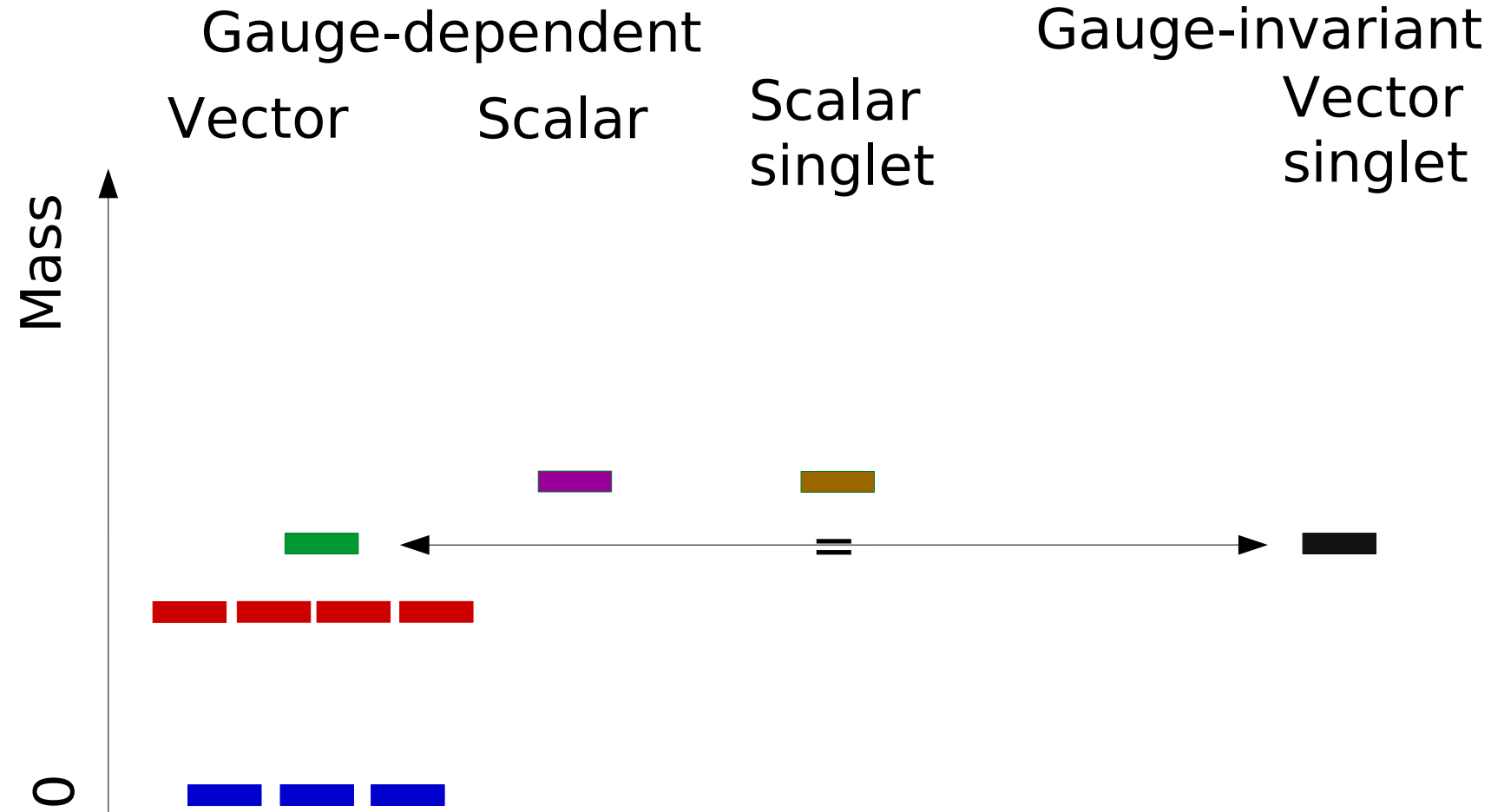
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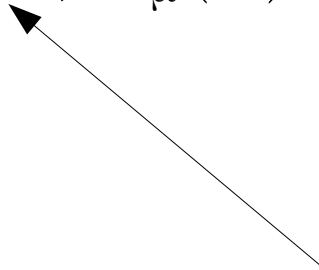
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Only one state remains in the spectrum  
at mass of gauge boson 8 (heavy singlet)

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- Now: States without elementary analogue
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue -  $U(1)$  acts as baryon number
  - Lightest must exist and be absolutely stable

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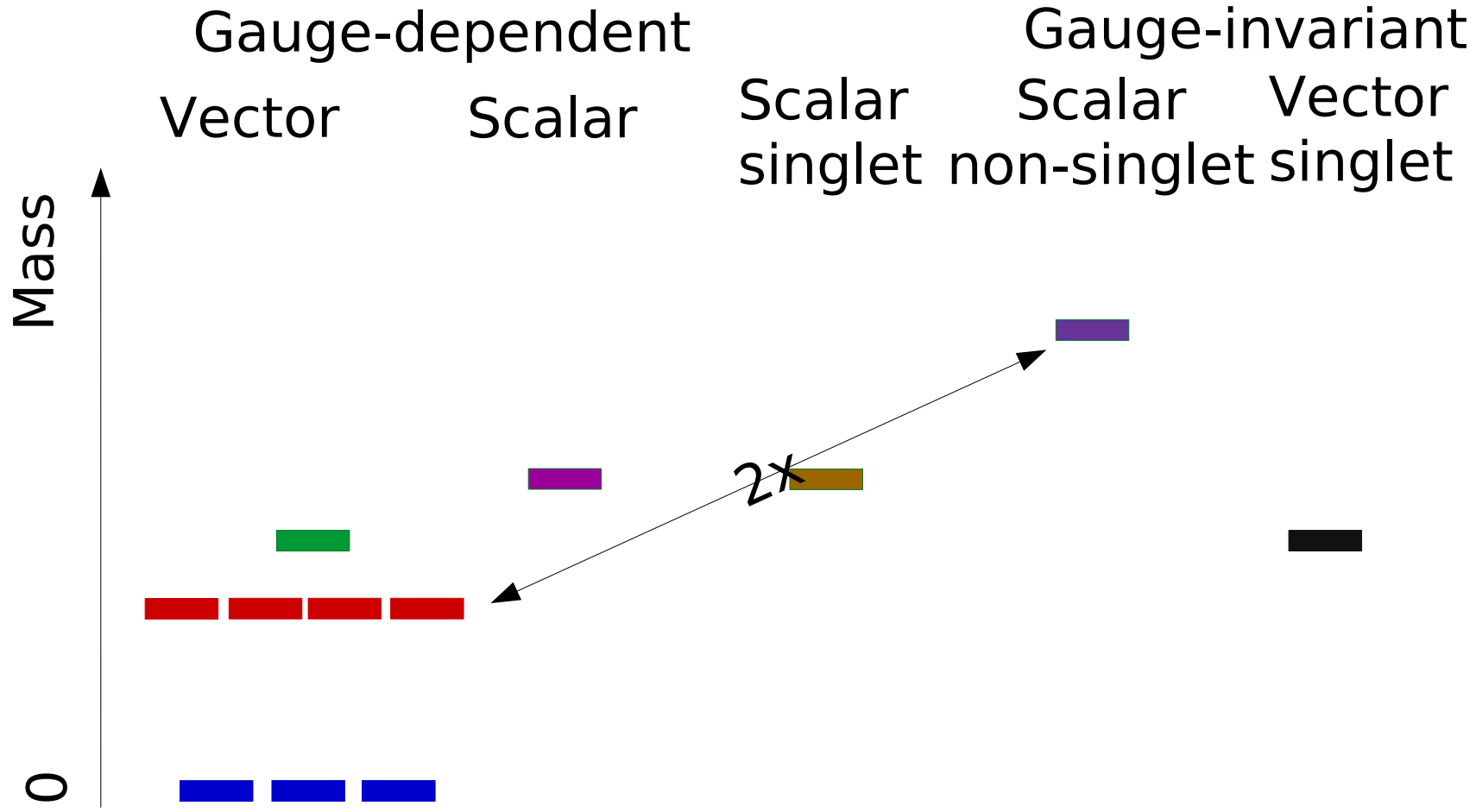
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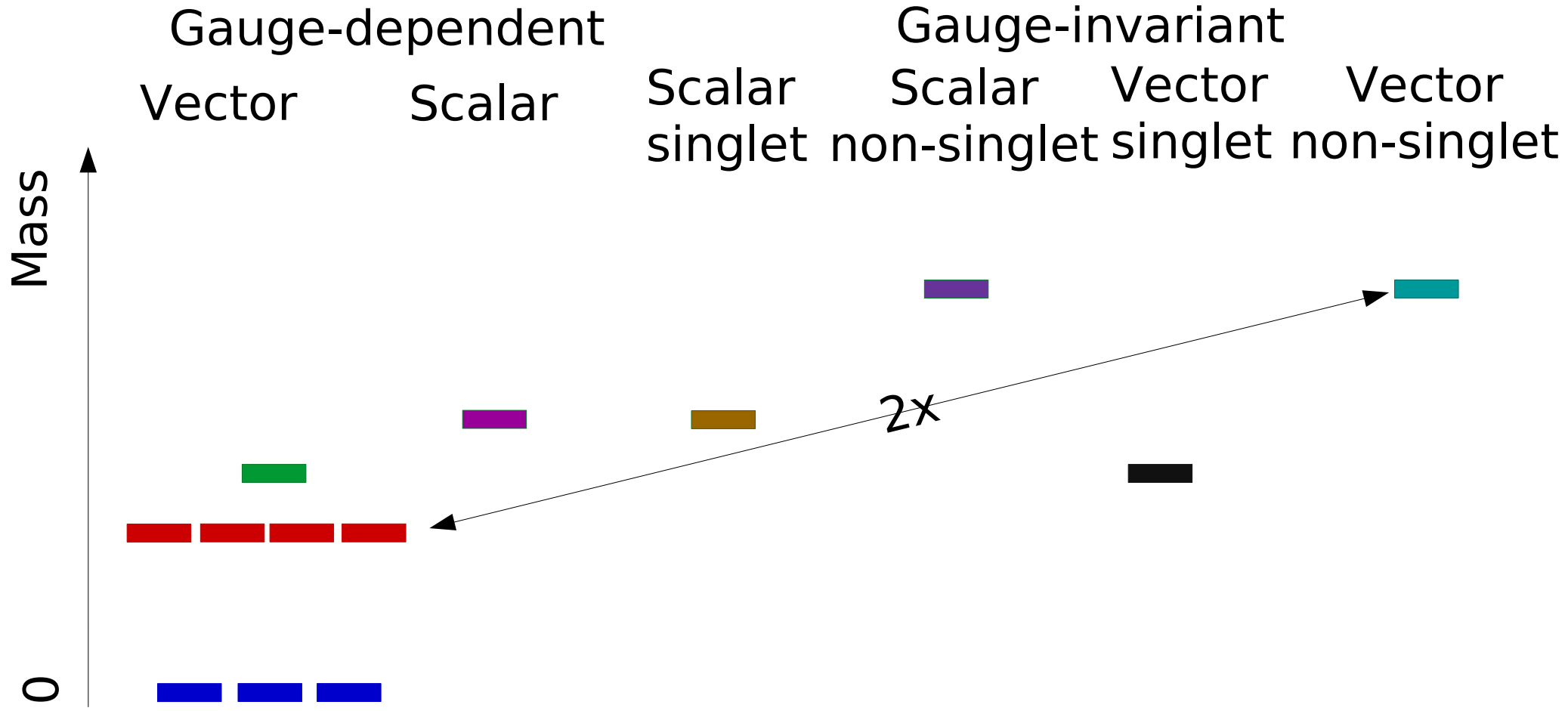
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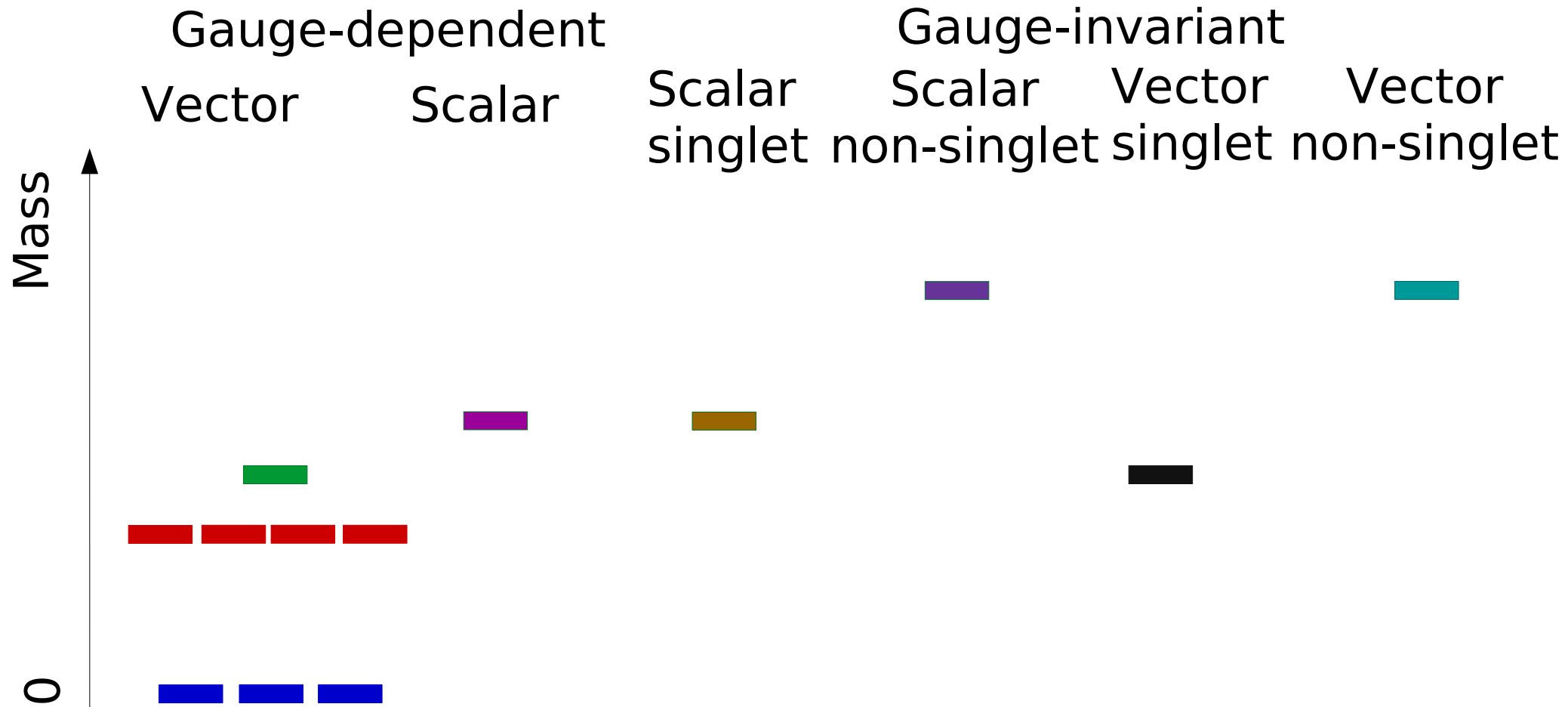
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- Qualitatively different spectrum
- No mass gap!

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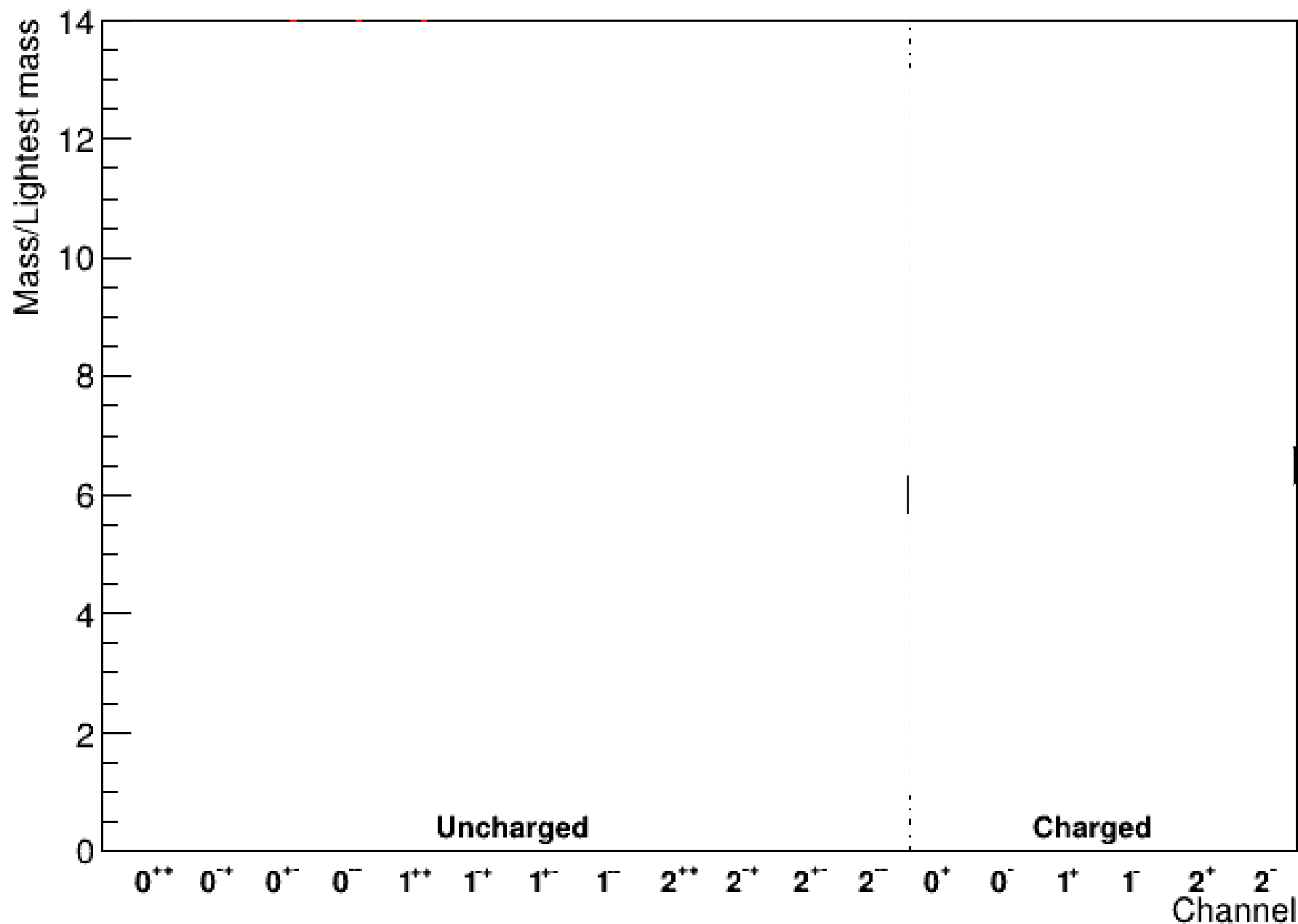
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  - All channels:  $J < 3$
  - Aim: Ground state for each channel
    - Characterization through scattering states

# Spectrum

**PRELIMINARY**

[Dobson et al.'21]

8.433600-0.488003-9.544000

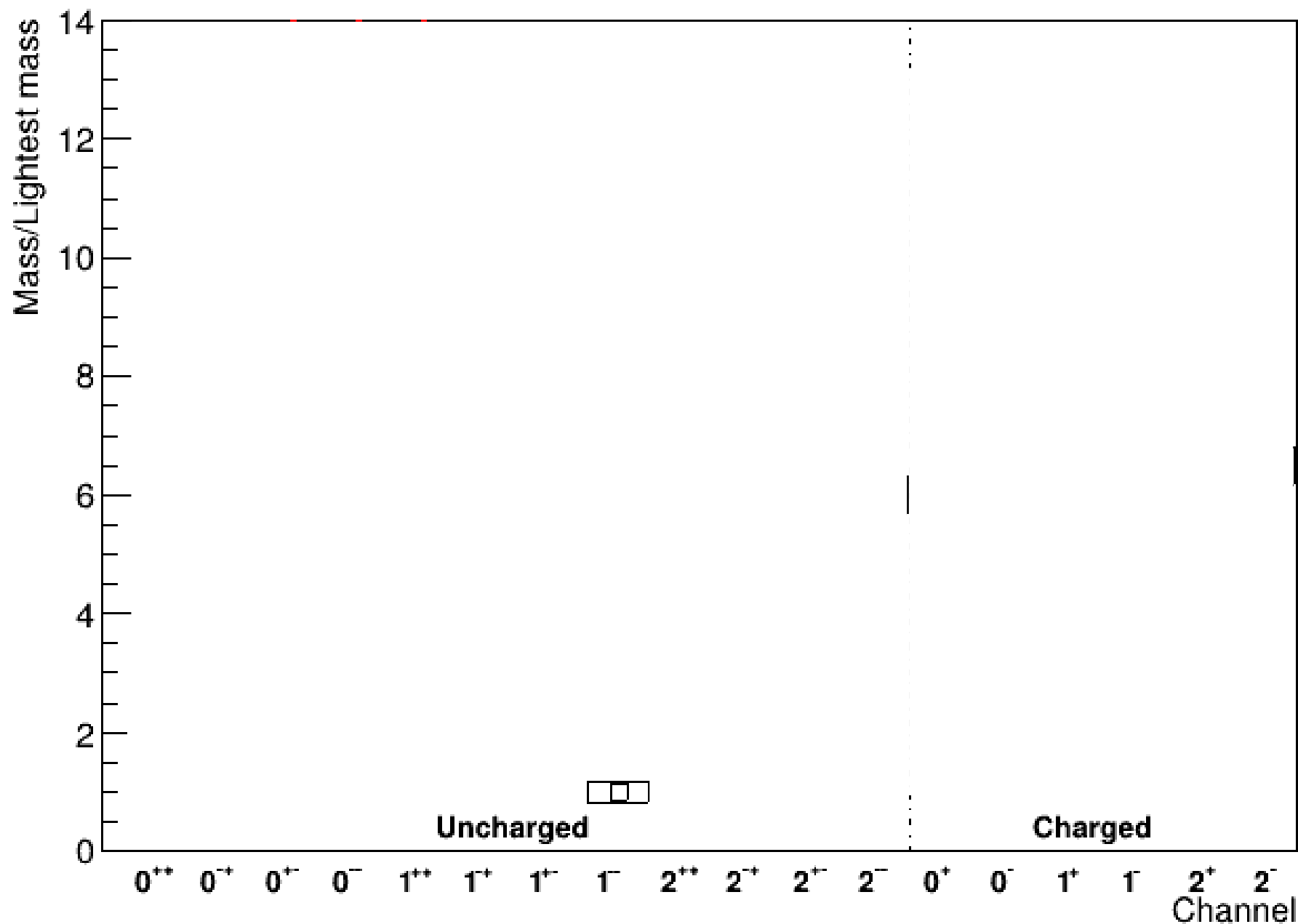


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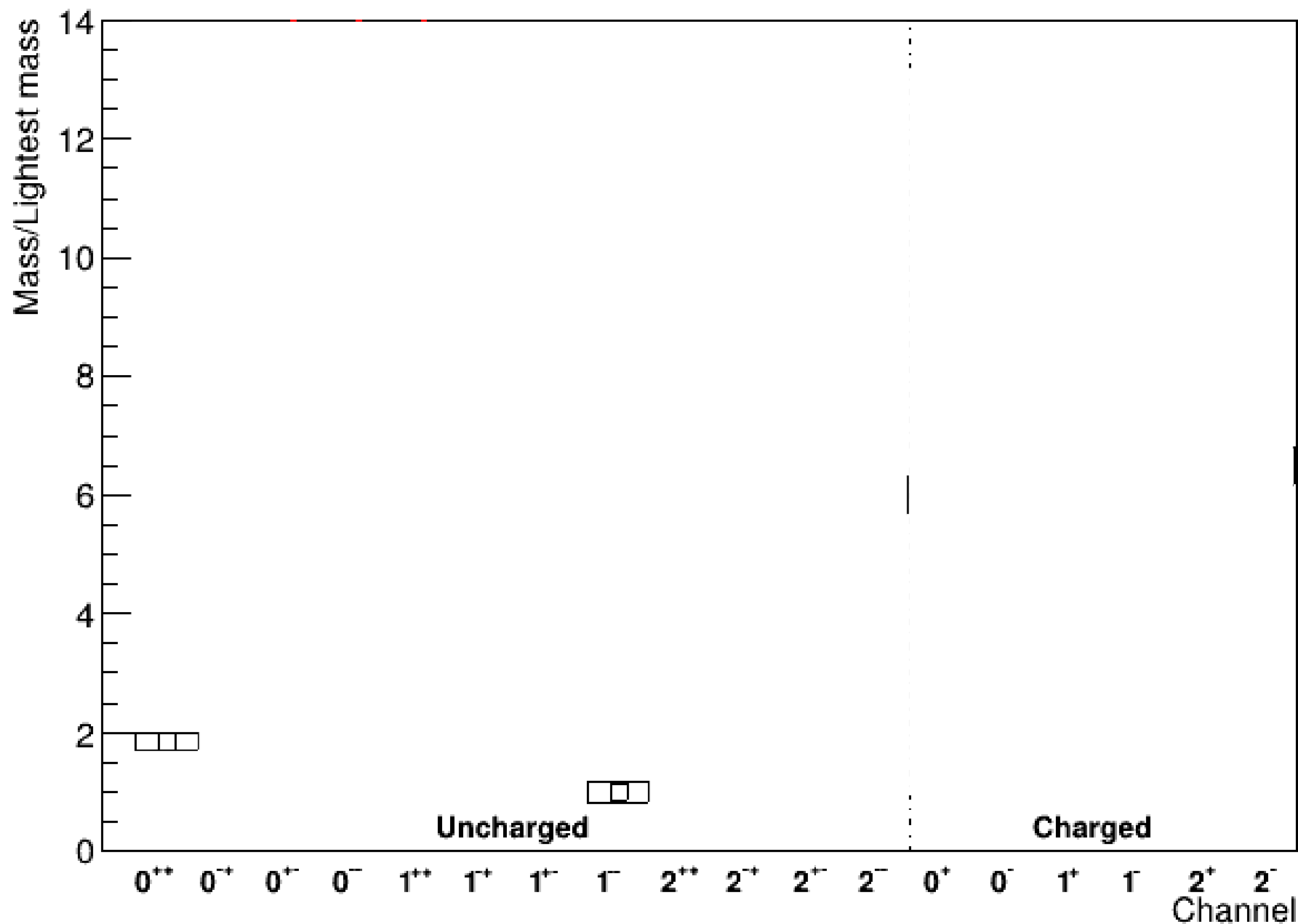


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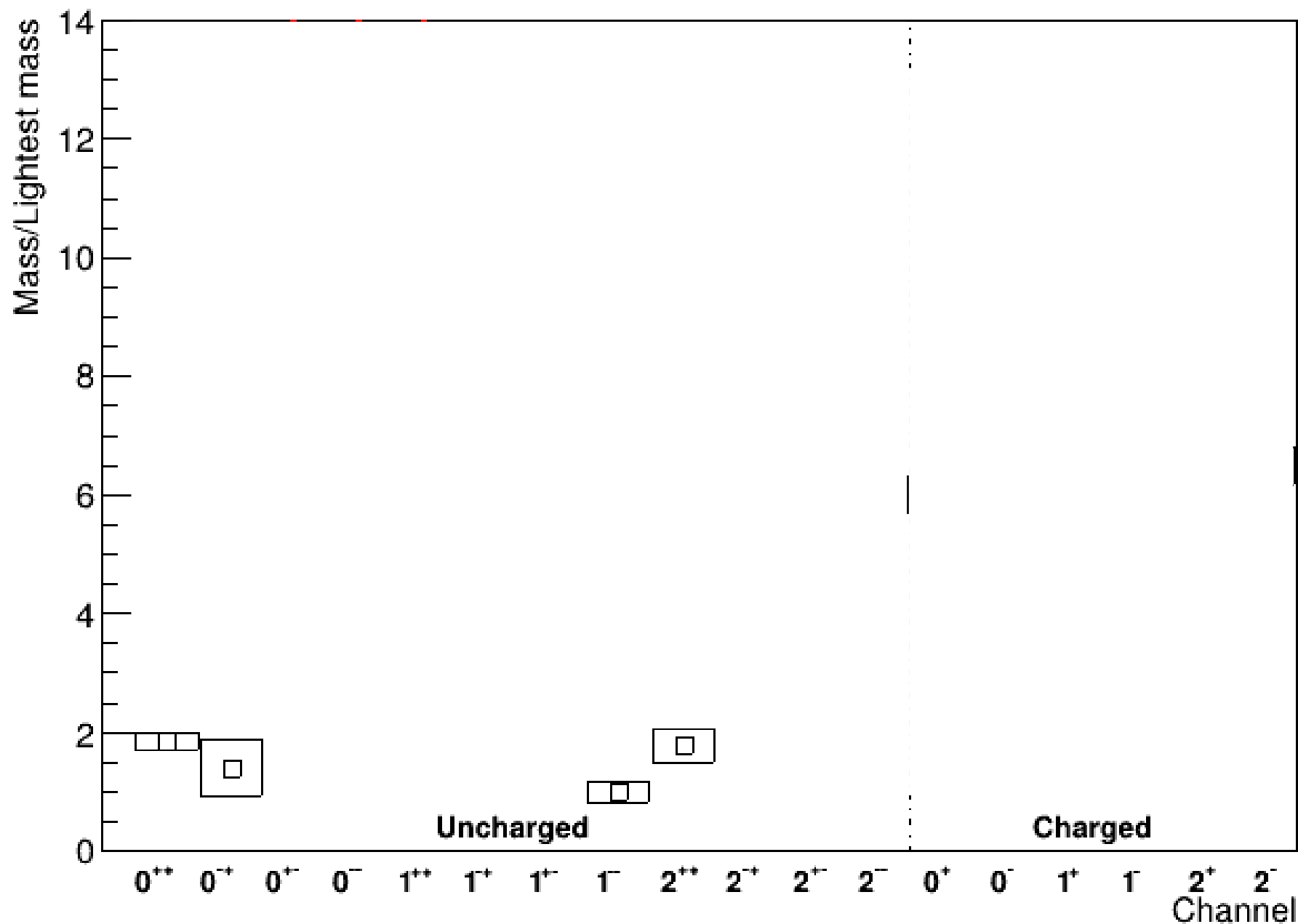


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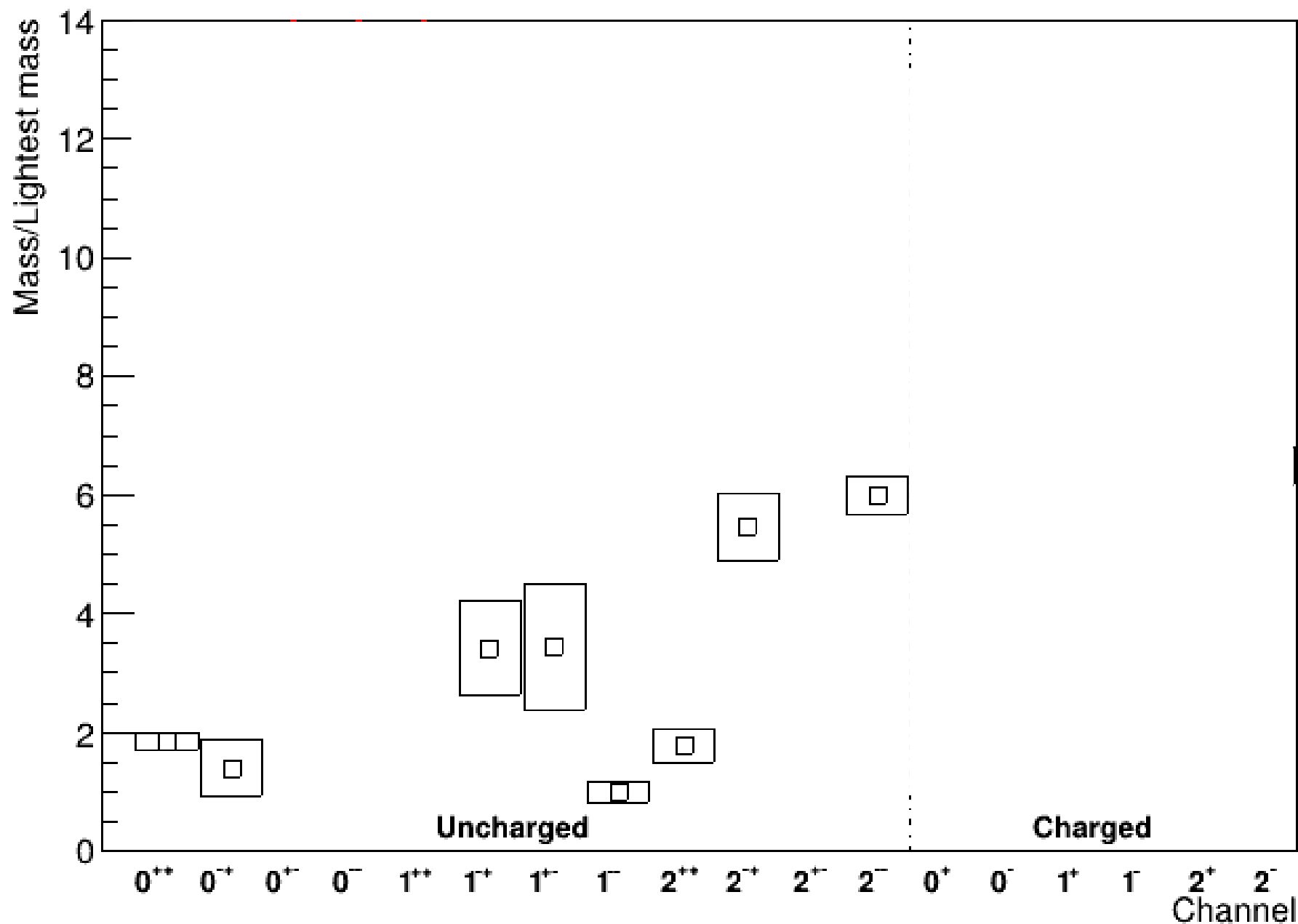


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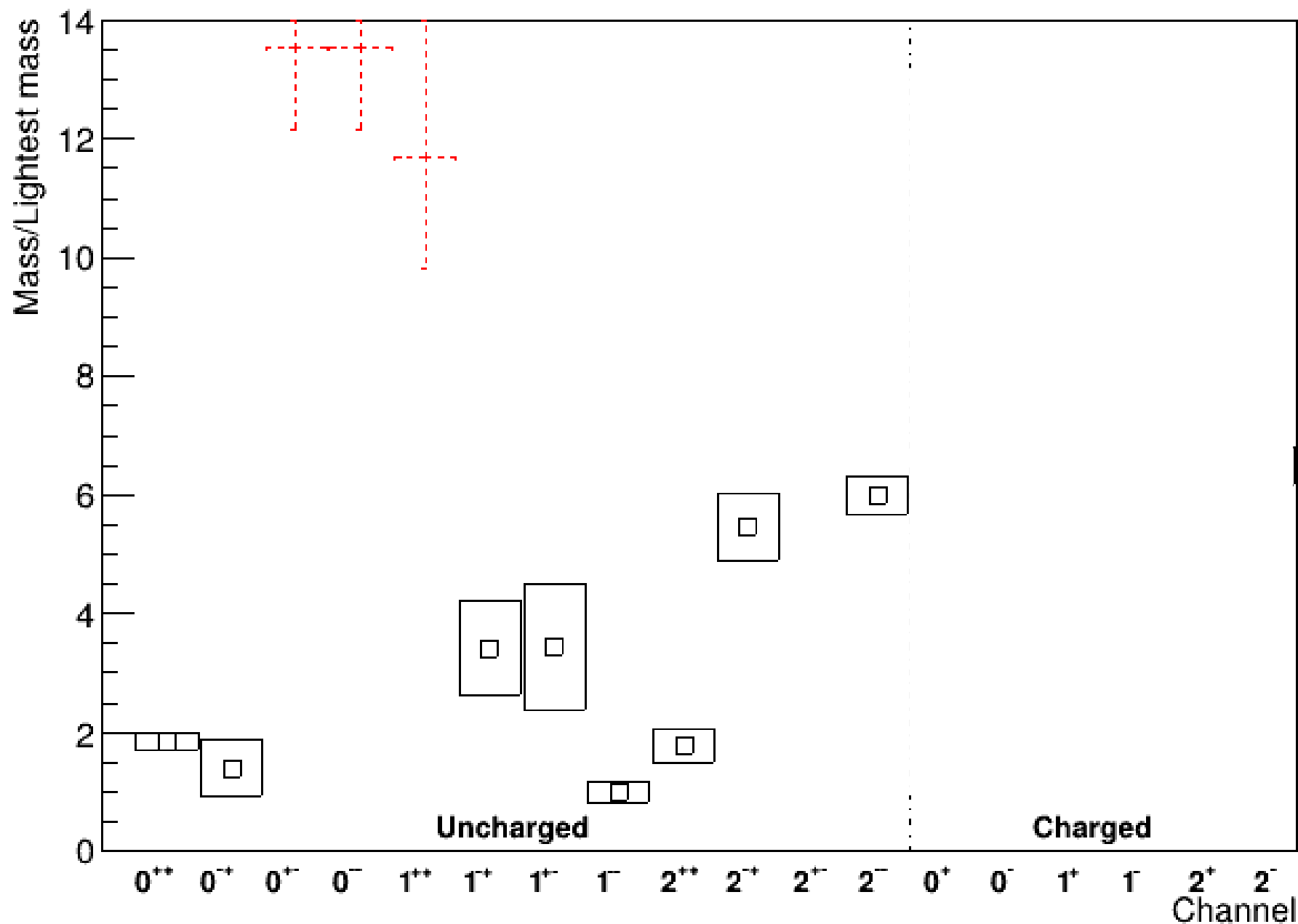


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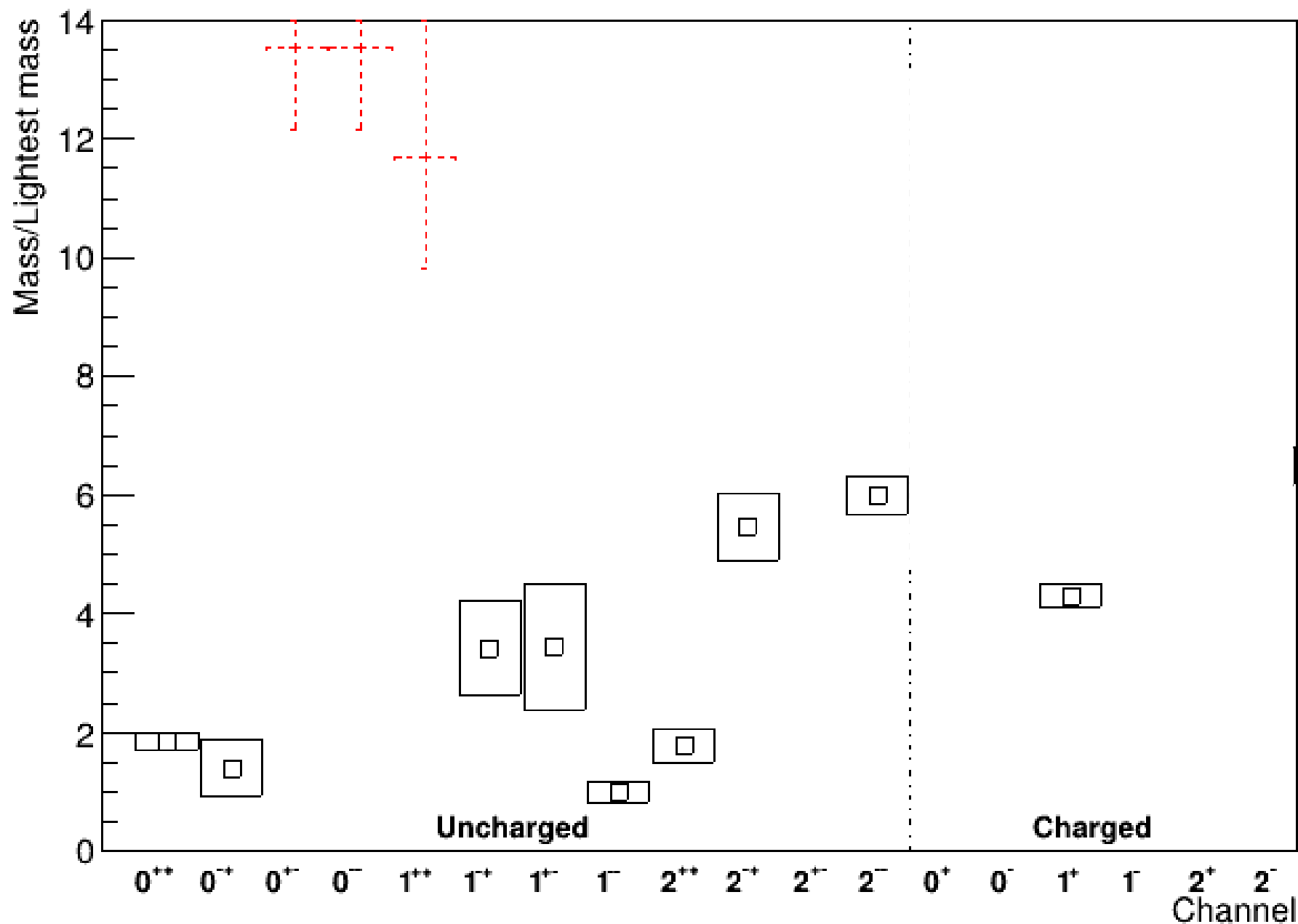


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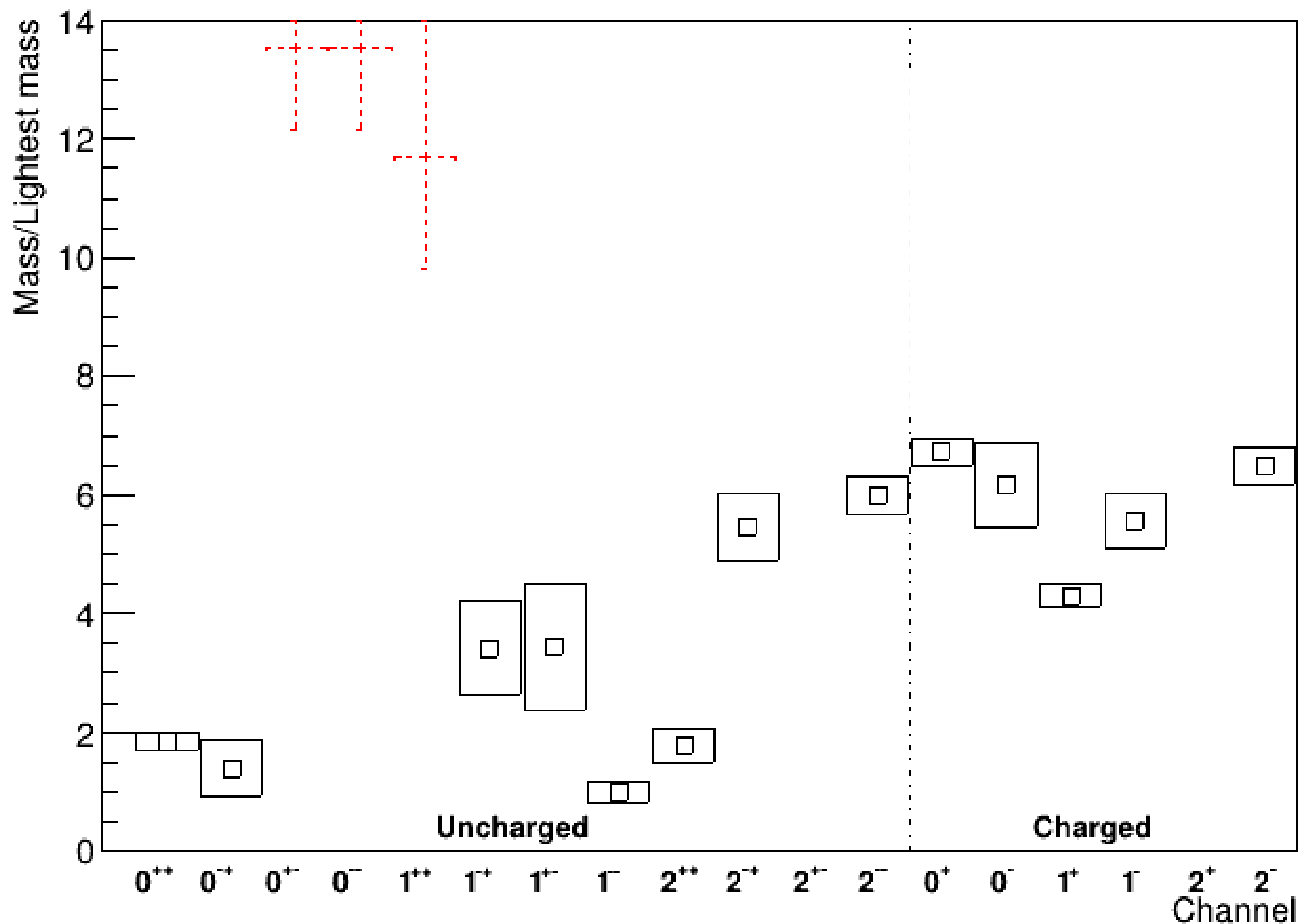


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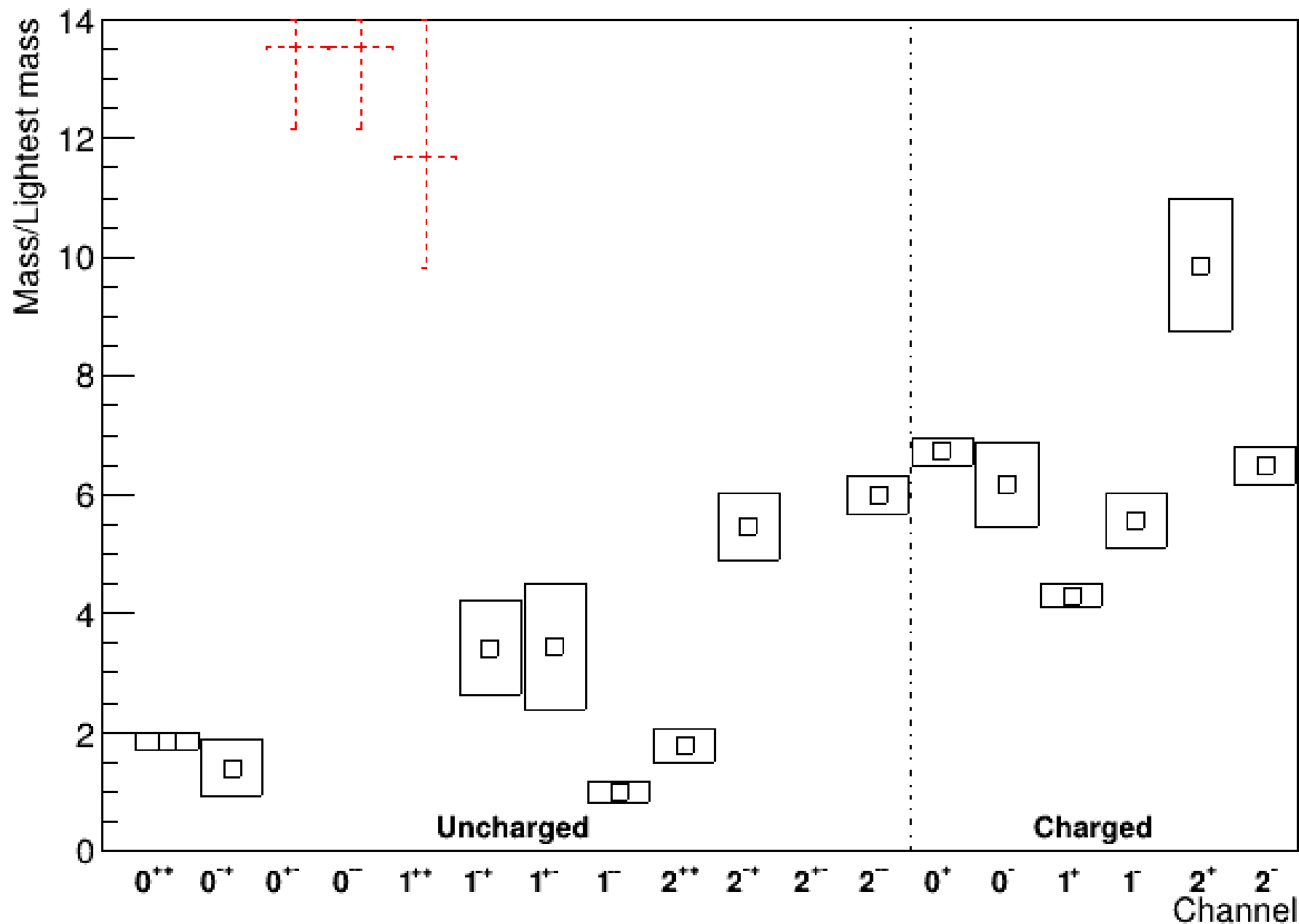


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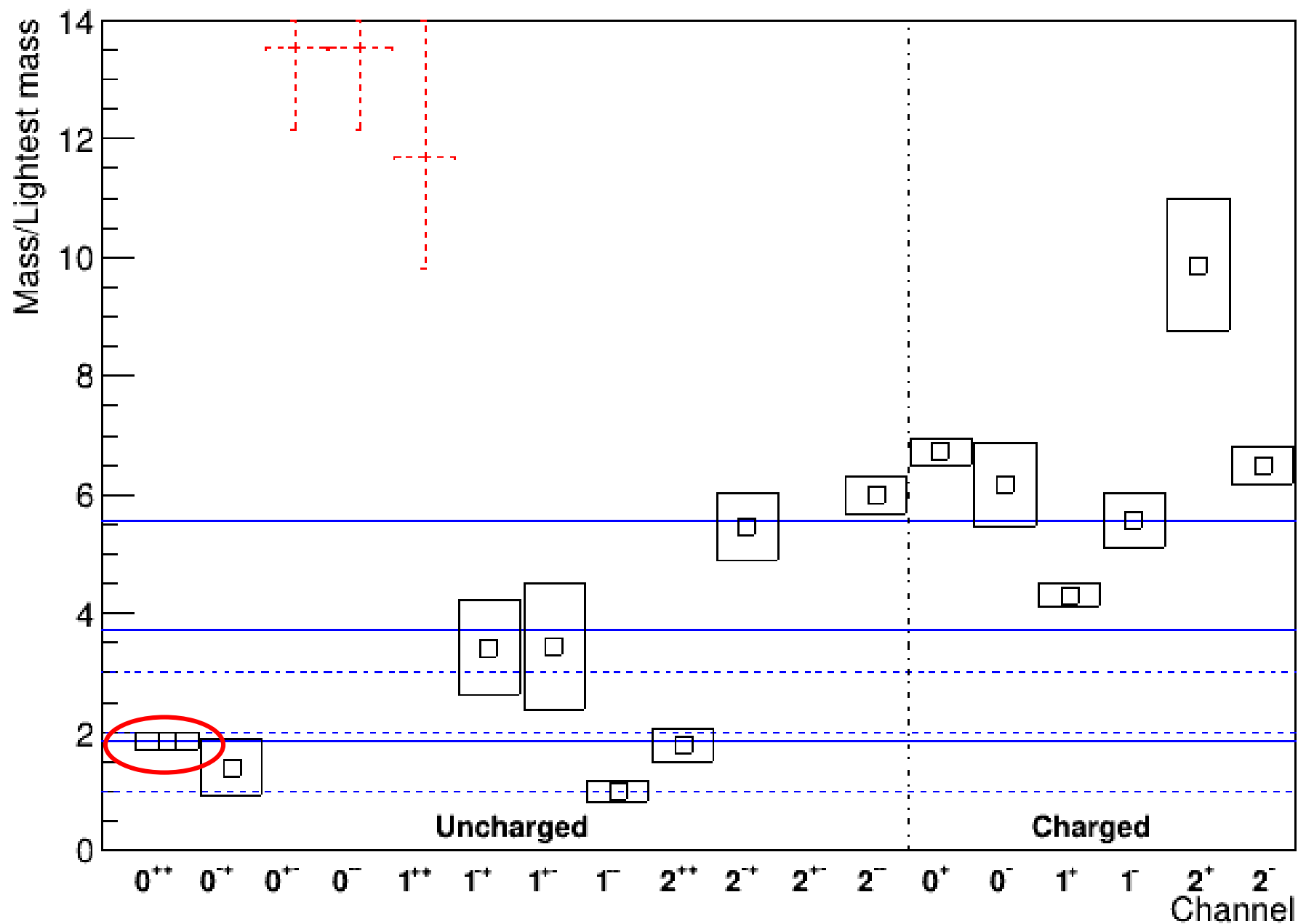


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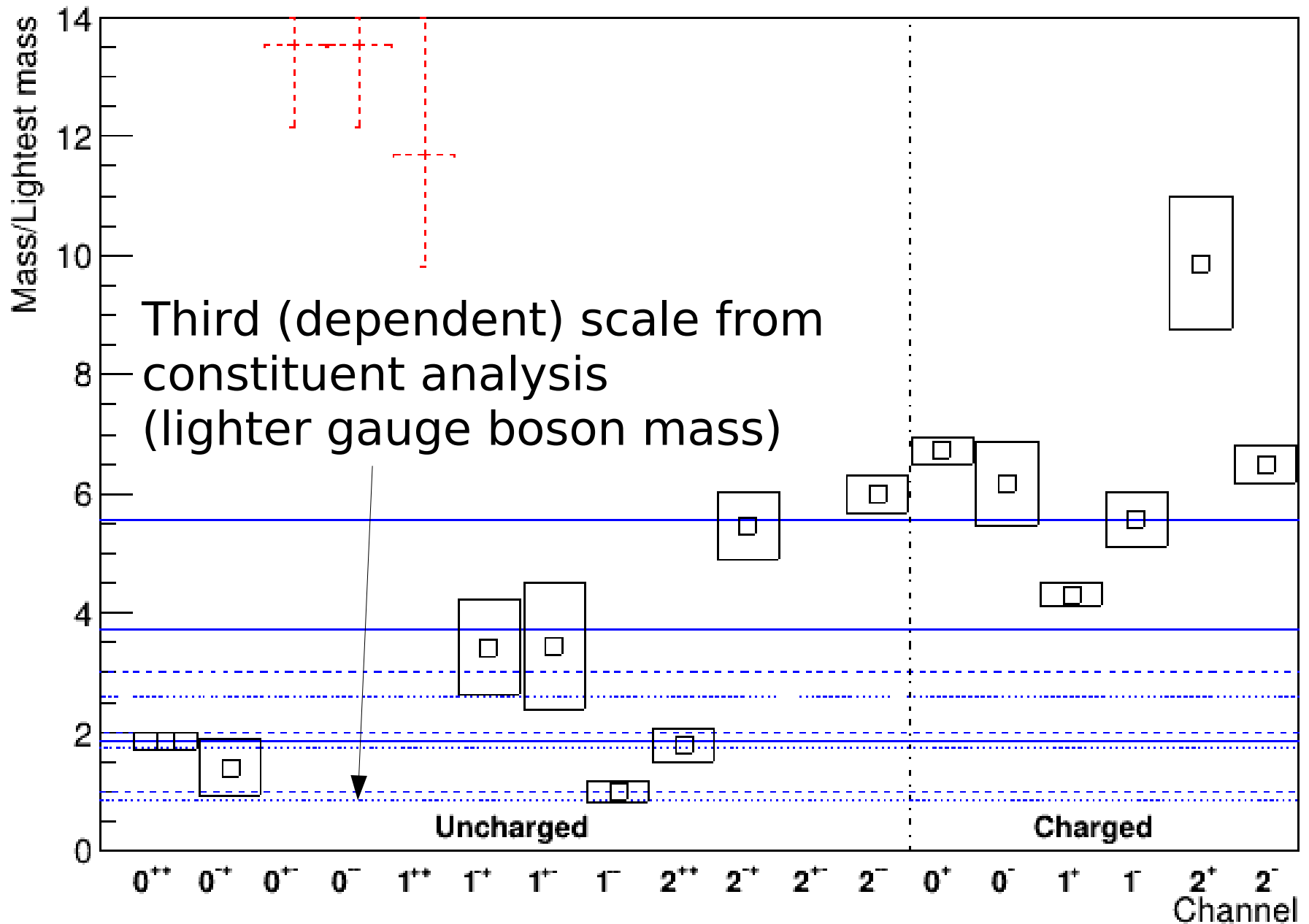


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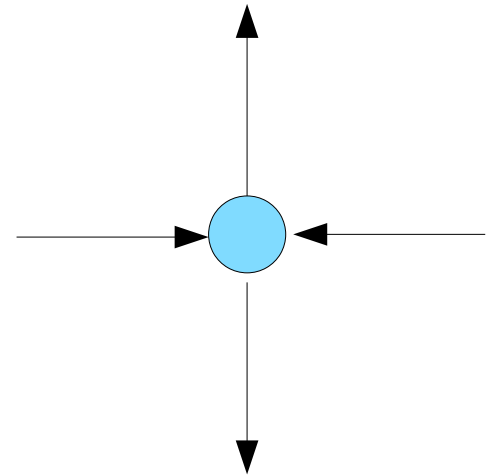
[Maas & Törek'18  
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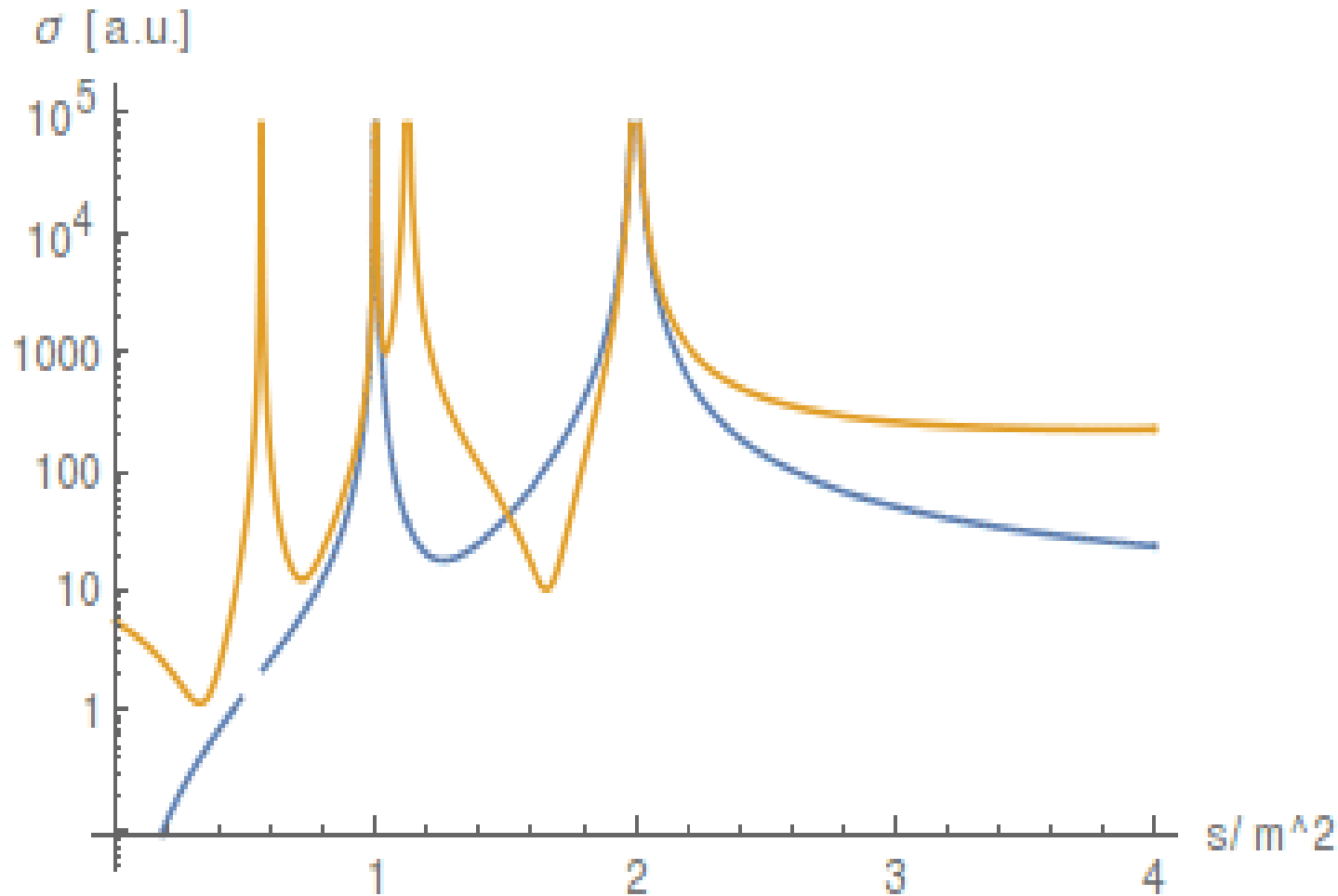
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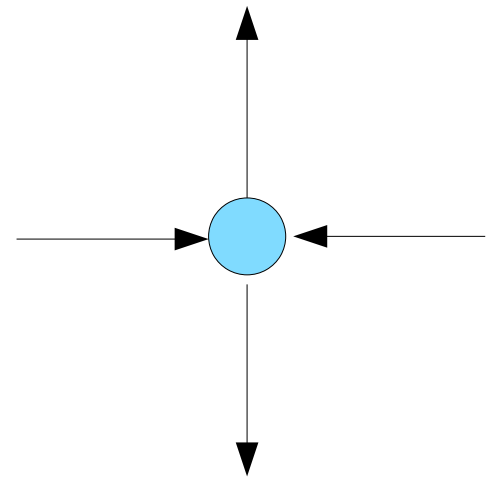
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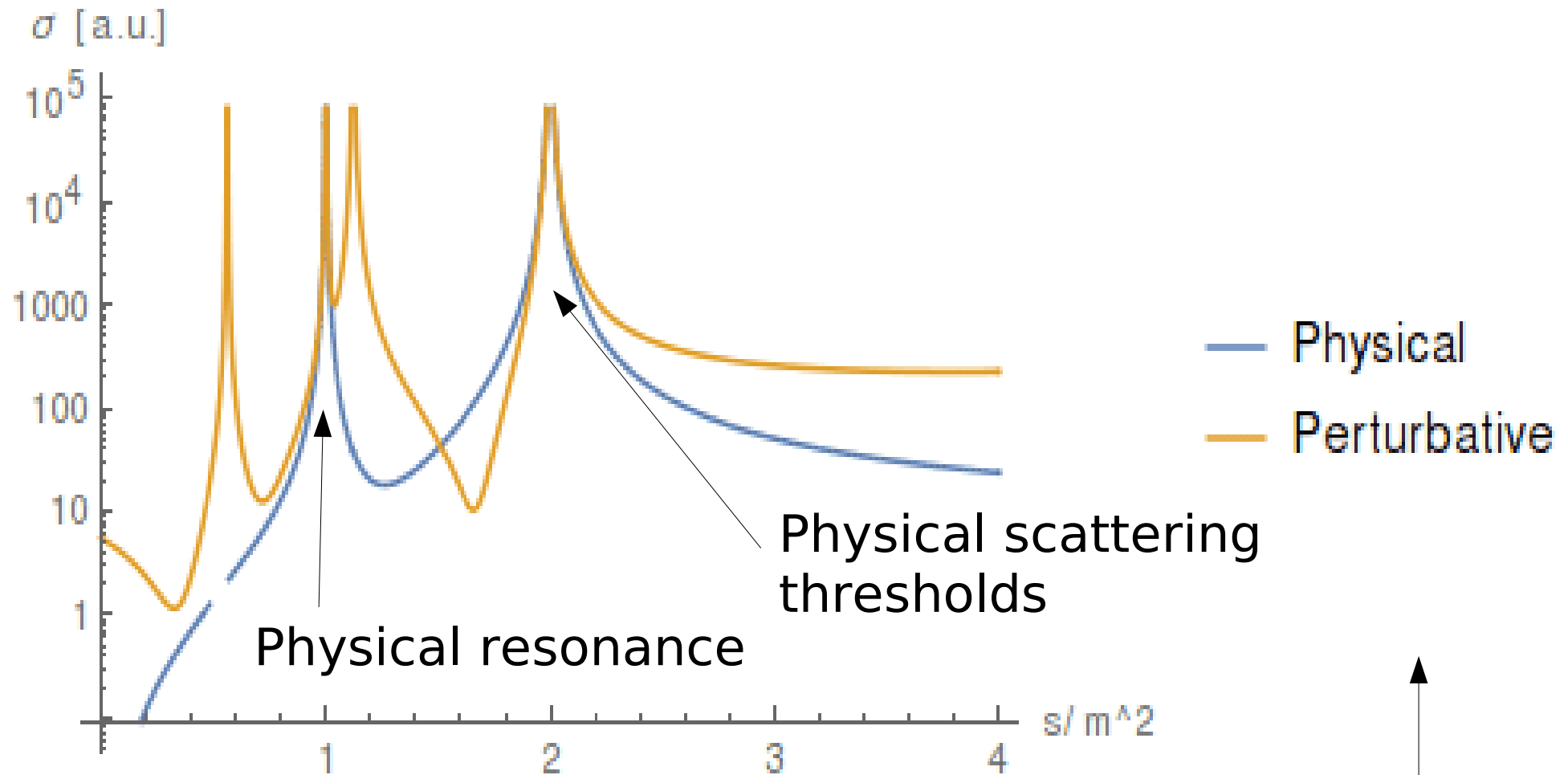
— Physical  
— Perturbative

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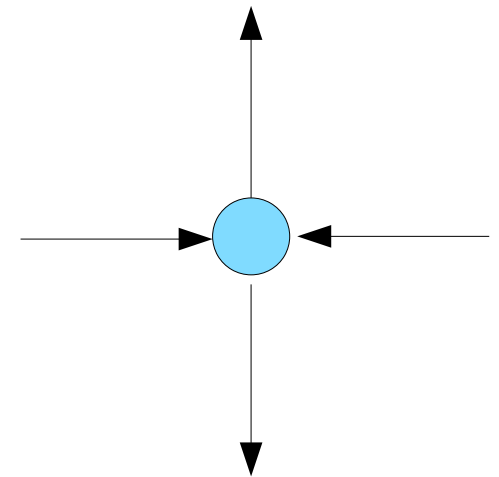


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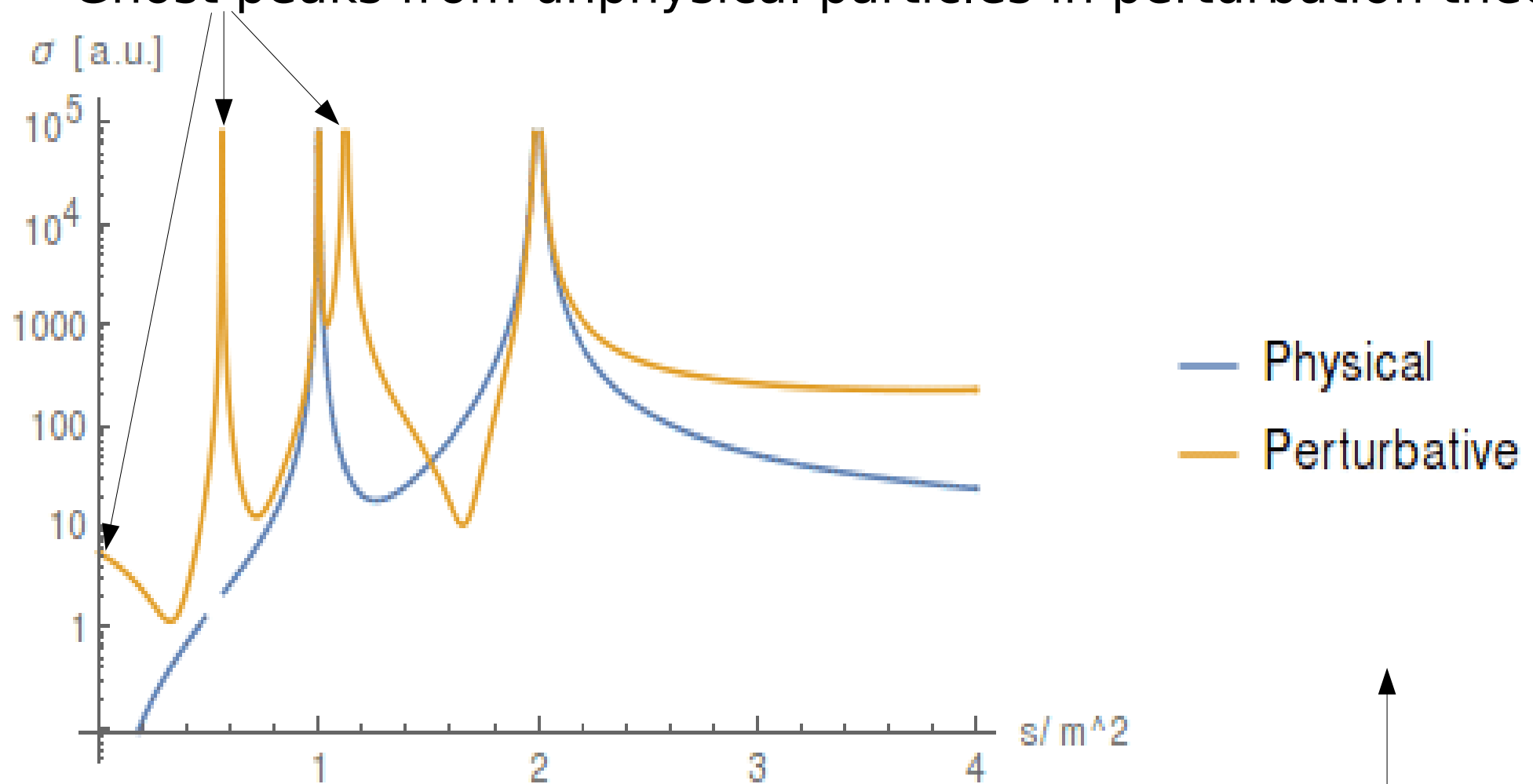
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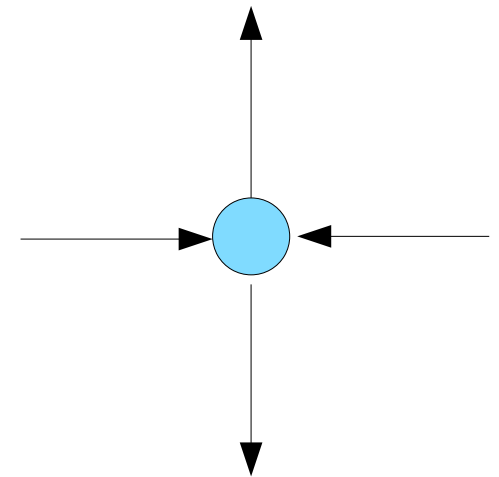
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Ghost peaks from unphysical particles in perturbation theory



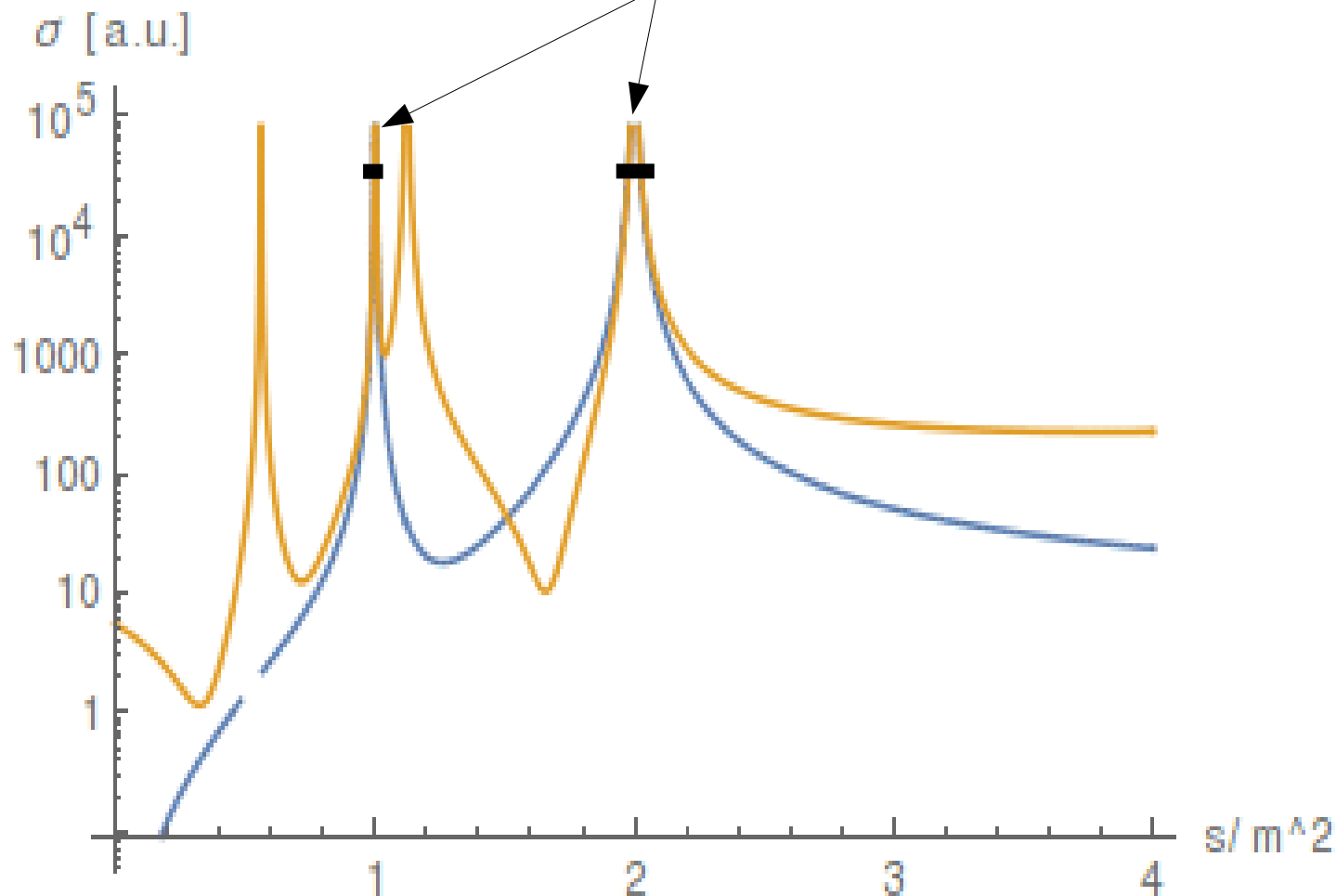
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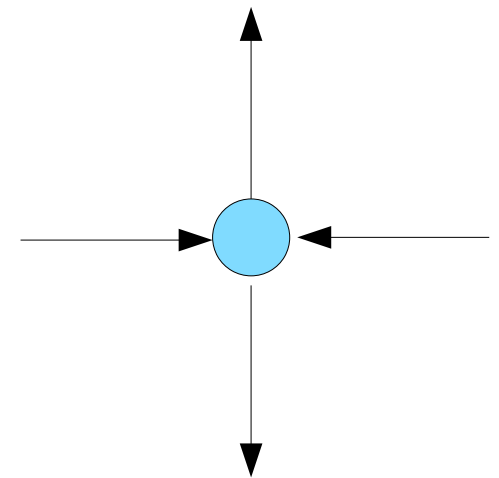
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Close to true structures identical!



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Maas, Markl, Müller'22]

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- FMS mechanism applicable
  - A 'BEH effect' for gravity
  - Technically much more involved
  - First predictions agree with dynamical triangulation results [Dai et al.'21]

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- Gives a new perspective on particle physics and quantum gravity

Philosophy of physics perspective: 2110.00616

Review: 1712.04721 Update: 2305.01960

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