Composite Massless Vector Bosons

And other surprises in GUTs

Axel Maas

4th of July 2022
Zeuthen/Online
Germany

UNI GRAZ
NAWI Graz
Natural Sciences
FWF
Der Wissenschaftsfonds.

What is this talk about?

- Why to care about GUTs?

Review: 1712.04721

What is this talk about?

- Why to care about GUTs?
- Gauge invariance and the Brout-Englert Higgs effect

Review: 1712.04721

What is this talk about?

- Why to care about GUTs?
- Gauge invariance and the Brout-Englert Higgs effect
- The standard model is special

Review: 1712.04721

What is this talk about?

- Why to care about GUTs?
- Gauge invariance and the Brout-Englert Higgs effect
- The standard model is special
- Implications for GUTs
    - Spectrum and compositness
    - Breaking patterns

Review: 1712.04721

What is this talk about?

- Why to care about GUTs?
- Gauge invariance and the Brout-Englert Higgs effect
- The standard model is special
- Implications for GUTs
  - Spectrum and compositness
  - Breaking patterns
- Where to go from here

Review: 1712.04721

Mysteries of the standard model

- Why is the hydrogen atom electrical neutral?
  - No coupling universality for Abelian charges
  - But necessary to cancel gauge anomalies

Mysteries of the standard model

- Why is the hydrogen atom electrical neutral?
  - No coupling universality for Abelian charges
  - But necessary to cancel gauge anomalies
- Why are gauge symmetries except the strong are linked to the Higgs?
  - Gauging of the SU(2)xU(1) subgroup of the O(4) Higgs global symmetry

Mysteries of the standard model

- Why is the hydrogen atom electrical neutral?
  - No coupling universality for Abelian charges
  - But necessary to cancel gauge anomalies
- Why are gauge symmetries except the strong are linked to the Higgs?
  - Gauging of the SU(2)xU(1) subgroup of the O(4) Higgs global symmetry
- Why do all gauge couplings become similarly strong at about the same energy of $\sim 10^{15}$ GeV?

Grand-unified theories

- Could all be emergent features?

Grand-unified theories

- Could all be emergent features?
- Scenario: Only one gauge interaction
  - Grand unification
  - Standard model low-energy effective theory

Grand-unified theories

- Could all be emergent features?
- Scenario: Only one gauge interaction
  - Grand unification
  - Standard model low-energy effective theory
- Standard scenario of double breaking
  - Two Brout-Englert-Higgs effect
  - One breaks at 10^{15} GeV
  - The other at the electroweak scale
  - Requires at least one more Higgs
    - Other particle content scenario-dependent

Consequences and status

- Requires leptoquarks
    - Surplus gauge bosons
    - Connects necessarily quarks and leptons
        - One gauge group only!
        - Color cannot be distinguished from electroweak weak charges

Consequences and status

- Requires leptoquarks
  - Surplus gauge bosons
  - Connects necessarily quarks and leptons
    - One gauge group only!
    - Color cannot be distinguished from electroweak charges
  - Violating baryon number and lepton number separately
    - → Lepton flavor universality violations?

Consequences and status

- Requires leptoquarks
    - Surplus gauge bosons
    - Connects necessarily quarks and leptons
        - One gauge group only!
        - Color cannot be distinguished from electroweak charges
    - Violating baryon number and lepton number separately
        - → Lepton flavor universality violations?
- Many (simple) scenarios ruled out

Consequences and status

- Requires leptoquarks
  - Surplus gauge bosons
  - Connects necessarily quarks and leptons
    - One gauge group only!
    - Color cannot be distinguished from electroweak charges
  - Violating baryon number and lepton number separately
    - → Lepton flavor universality violations?
- Many (simple) scenarios ruled out...
- ...but many more alive (w or w/o SUSY)

What’s the deal?

-

Gauge symmetry

A toy model

A toy model: Higgs sector of the SM

A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar

A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L = -\frac{1}{4} W_{\mu\nu}^{a} W_{a}^{\mu\nu}$$

$$W_{\mu\nu}^{a} = \partial_{\mu} W_{\nu}^{a} - \partial_{\nu} W_{\mu}^{a} + g f_{bc}^{a} W_{\mu}^{b} W_{\nu}^{c}$$

- Ws W_{μ}^{a} Ⓦ

- Coupling g and some numbers f^{abc}

A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Ws W_{μ}^{a} W
- Higgs h_i h
- Coupling g and some numbers f^{abc} and t_a^{ij}

A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W_{a}^{\mu\nu}+\left(D^{ij}_{\mu}h^{j}\right)^{+}D^{\mu}_{ik}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

$$D^{ij}_{\mu}=\delta^{ij}\partial_{\mu}-igW^{a}_{\mu}t^{ij}_{a}$$

- Ws W^{a}_{μ} W
- Higgs h_{i} h
- Couplings g, v, λ and some numbers f^{abc} and t_{a}^{ij}

A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Ws W_{μ}^{a} W
- Higgs h_i h
- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}
- Parameters selected for a BEH effect

A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Local SU(2) gauge symmetry

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-g f_{bc}^{a}W_{\mu}^{c})\phi^{b} \qquad h_{i}\rightarrow h_{i}+g\, t_{a}^{ij}\phi^{a}h_{j}$$

A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Local SU(2) gauge symmetry

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-g f_{bc}^{a}W_{\mu}^{c})\phi^{b} \qquad h_{i}\rightarrow h_{i}+g t_{a}^{ij}\phi^{a}h_{j}$$

- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a} \qquad h\rightarrow h\Omega$$

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain ‘spontaenous gauge symmetry breaking’: SU(2) → 1

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain ‘spontaenous gauge symmetry breaking’: SU(2) → 1
- Get masses and degeneracies at tree-level

Textbook approach

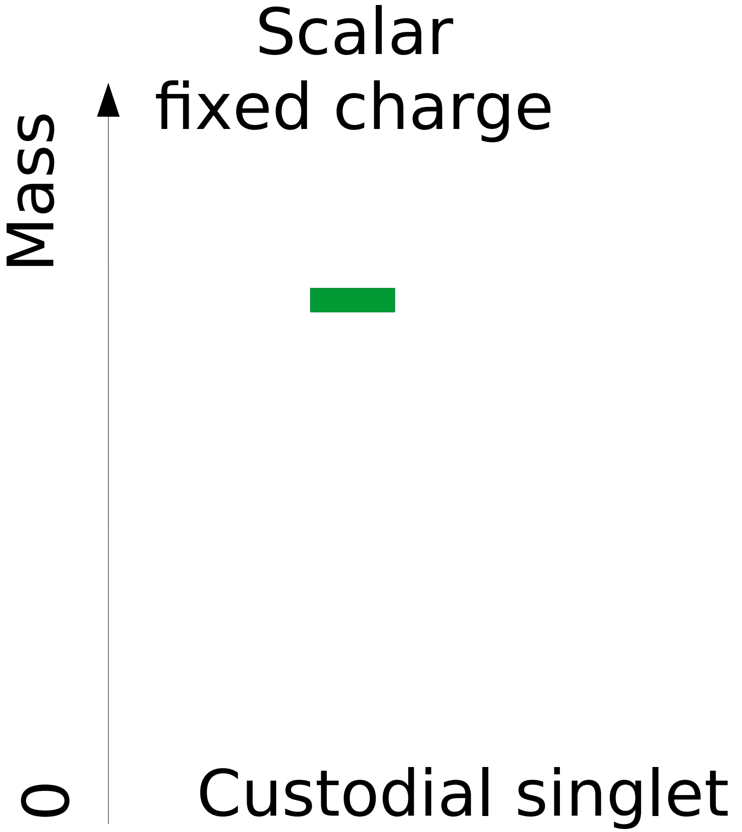
- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain ‘spontaenous gauge symmetry breaking’: SU(2) → 1
- Get masses and degeneracies at tree-level
- Perform perturbation theory

Physical spectrum

Perturbation theory

Physical spectrum

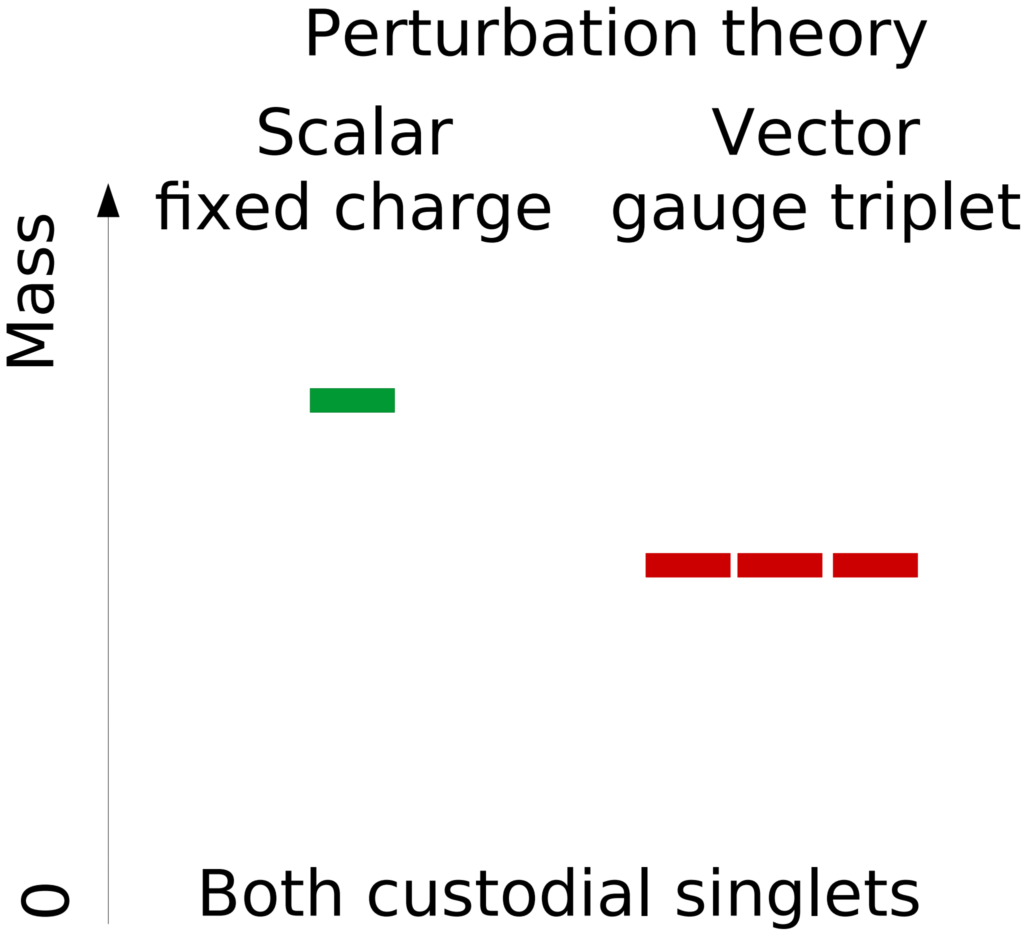
Perturbation theory



Scalar
fixed charge

Custodial singlet

Physical spectrum



Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Mass

0

Both custodial singlets

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
  - Change under a gauge transformation

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
  - Change under a gauge transformation
  - Gauge transformations are a human choice

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
  - Change under a gauge transformation
  - Gauge transformations are a human choice...
  - ...and gauge-symmetry breaking is not there

    [Elitzur'75, Osterwalder & Seiler'77, Fradkin & Shenker'78]

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
  - Change under a gauge transformation
  - Gauge transformations are a human choice...
  - ...and gauge-symmetry breaking is not there

    [Elitzur’75, Osterwalder & Seiler’77, Fradkin & Shenker’78]
    - Just a figure of speech
    - Actually just ordinary gauge-fixing

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
  - Change under a gauge transformation
  - Gauge transformations are a human choice...
  - ...and gauge-symmetry breaking is not there
    [Elitzur’75, Osterwalder & Seiler’77, Fradkin & Shenker’78]
    - Just a figure of speech
    - Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
    - Change under a gauge transformation
    - Gauge transformations are a human choice...
    - ...and gauge-symmetry breaking is not there

      [Elitzur'75, Osterwalder & Seiler'77, Fradkin & Shenker'78]
        - Just a figure of speech
        - Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities
    - And this includes non-perturbative aspects

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent
    - Change under a gauge transformation
    - Gauge transformations are a human choice...
    - ...and gauge-symmetry breaking is not there
      [Elitzur’75, Osterwalder & Seiler’77, Fradkin & Shenker’78]
        - Just a figure of speech
        - Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities
    - And this includes non-perturbative aspects...
    - ...even at weak coupling [Gribov’78,Singer’78,Fujikawa’82]

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
    - **Cannot** be the elementary particles
    - Non-Abelian nature is relevant

Physical states

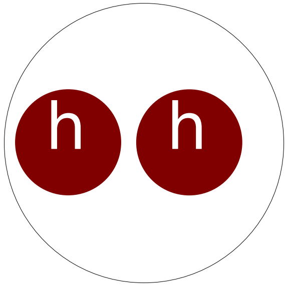
[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles

Physical states

[Fröhlich et al.'80, Banks et al.'79]

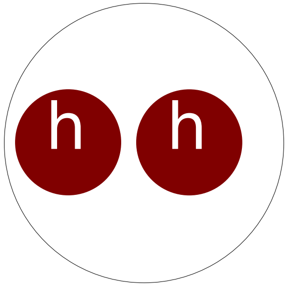
- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs

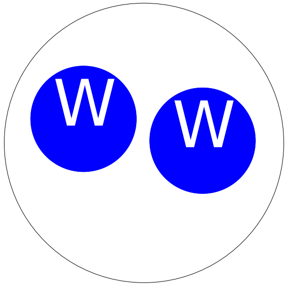


Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W

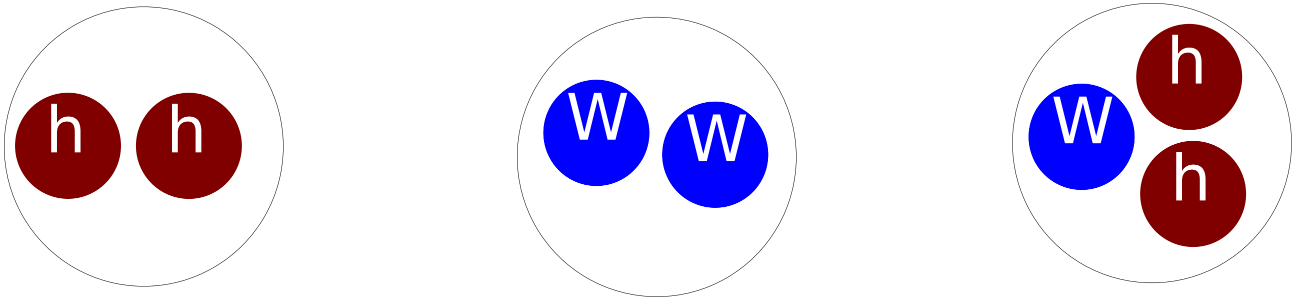




Physical states

[Fröhlich et al.'80,
Banks et al.'79]

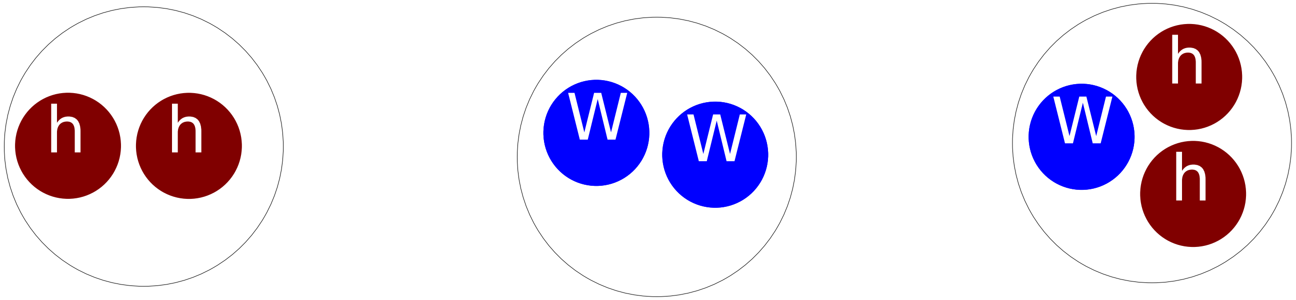
- Need physical, gauge-invariant particles
    - **Cannot** be the elementary particles
    - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
    - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.

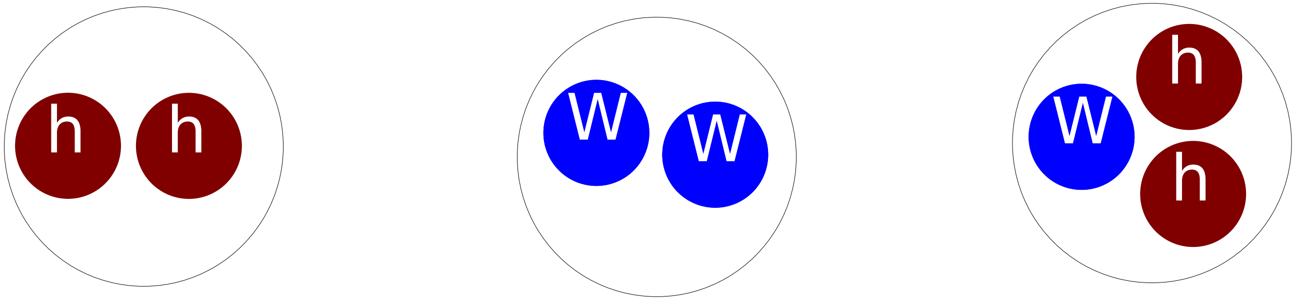


- Has nothing to do with weak coupling

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.

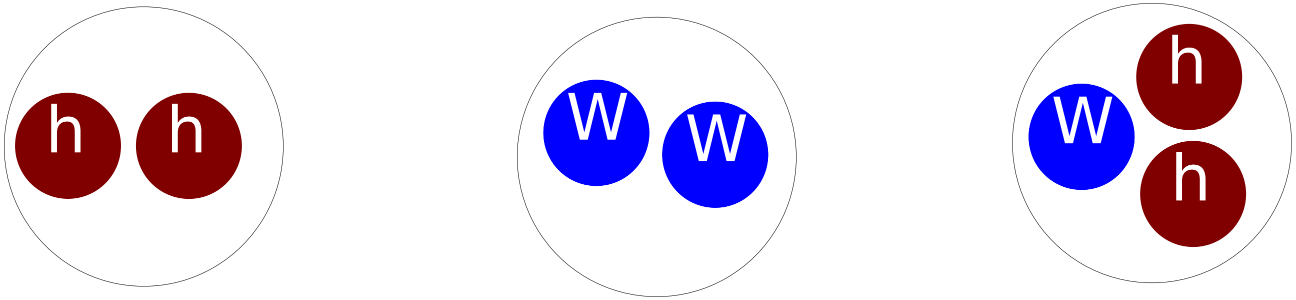


- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)
- Can this matter?

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
    - Different from perturbation theory
        - Operators limited to asymptotic, elementary, gauge-dependent states

How to make predictions

[Fröhlich et al.'80,'81,
Maas & Törek'16,'18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

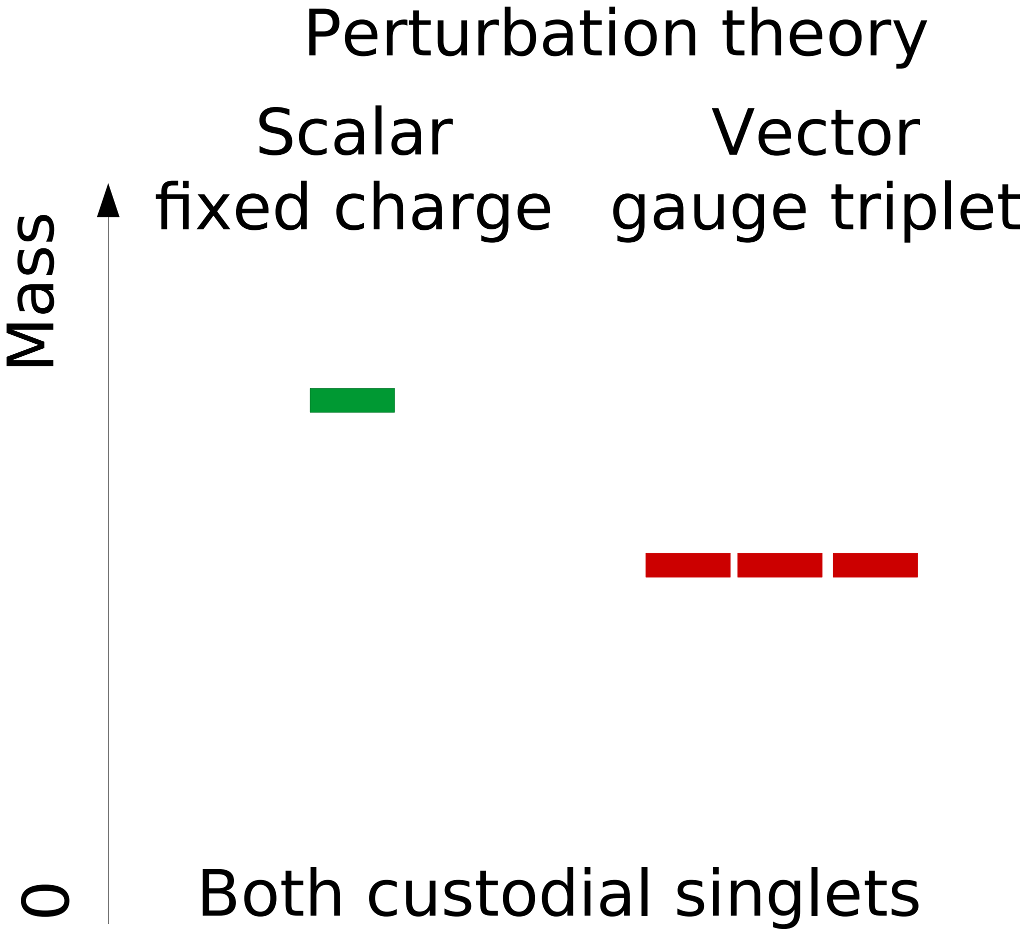
- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods!

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods! - Lattice

Physical spectrum



Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Both custodial singlets

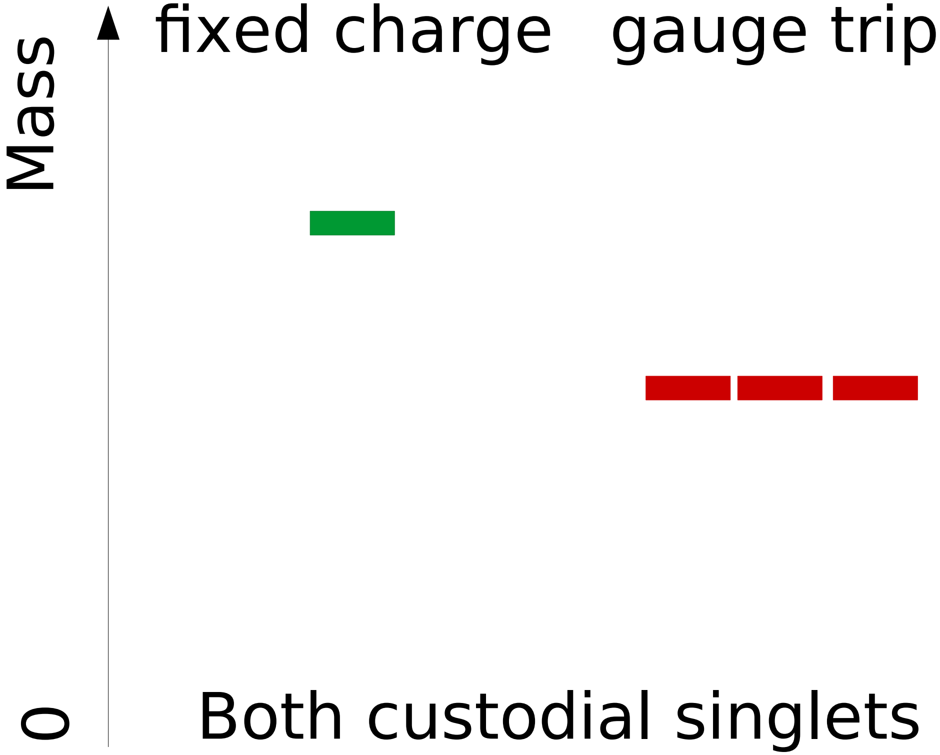
Experiment tells that somehow the left is correct

Physical spectrum

Perturbation theory

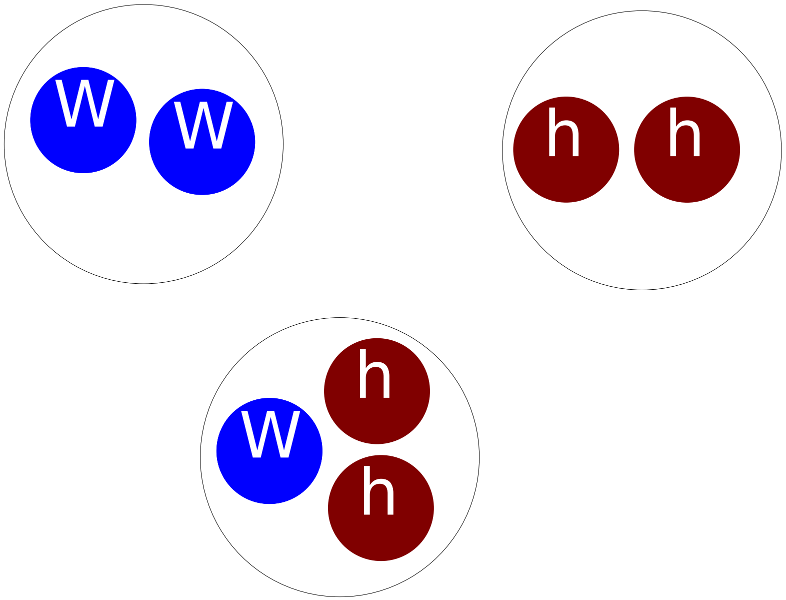
Scalar
fixed charge

Vector
gauge triplet



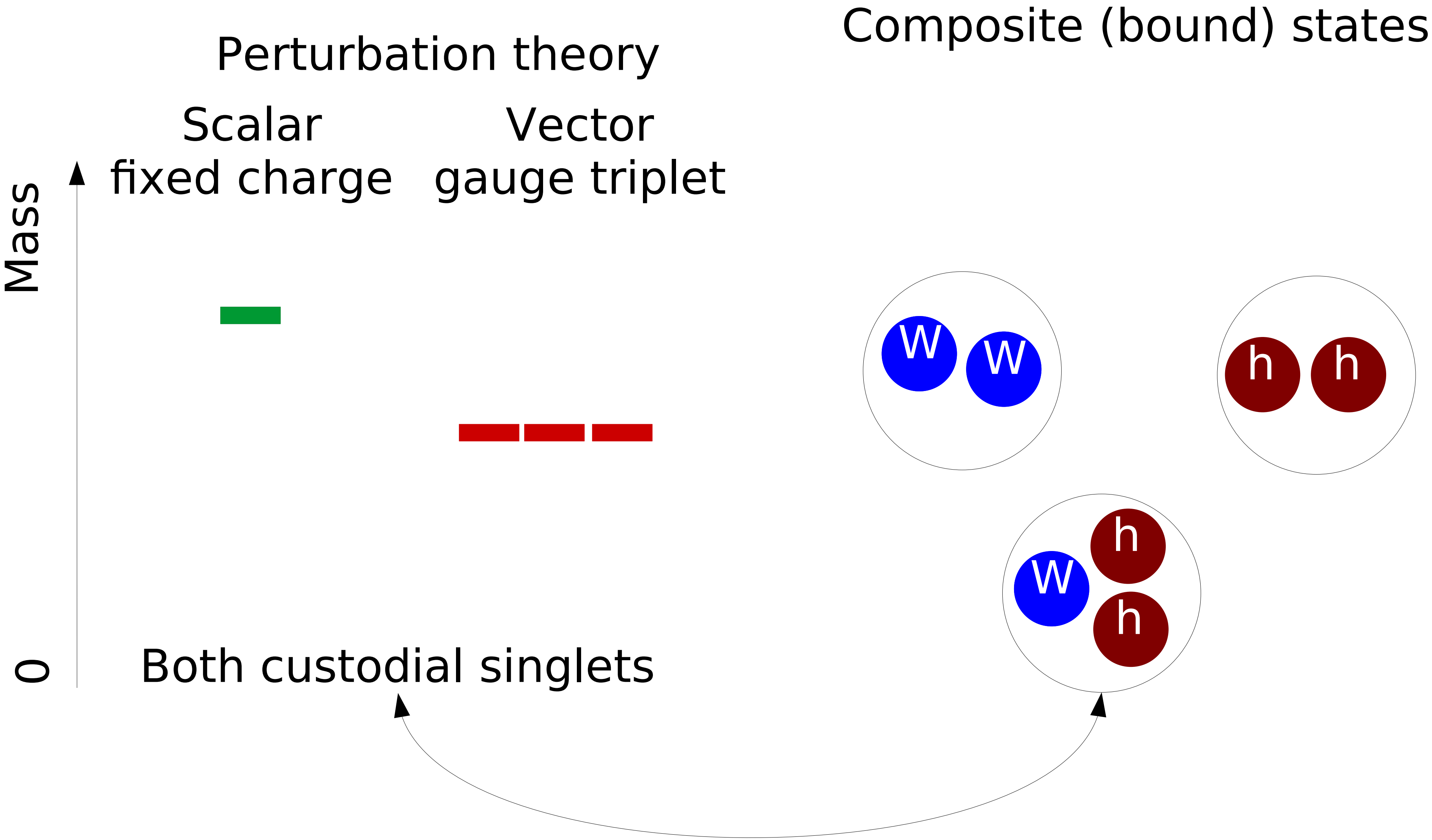
Both custodial singlets

Composite (bound) states



Experiment tells that somehow the left is correct
Theory say the right is correct

Physical spectrum



Perturbation theory

Composite (bound) states

Scalar
fixed charge

Vector
gauge triplet

Both custodial singlets

Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

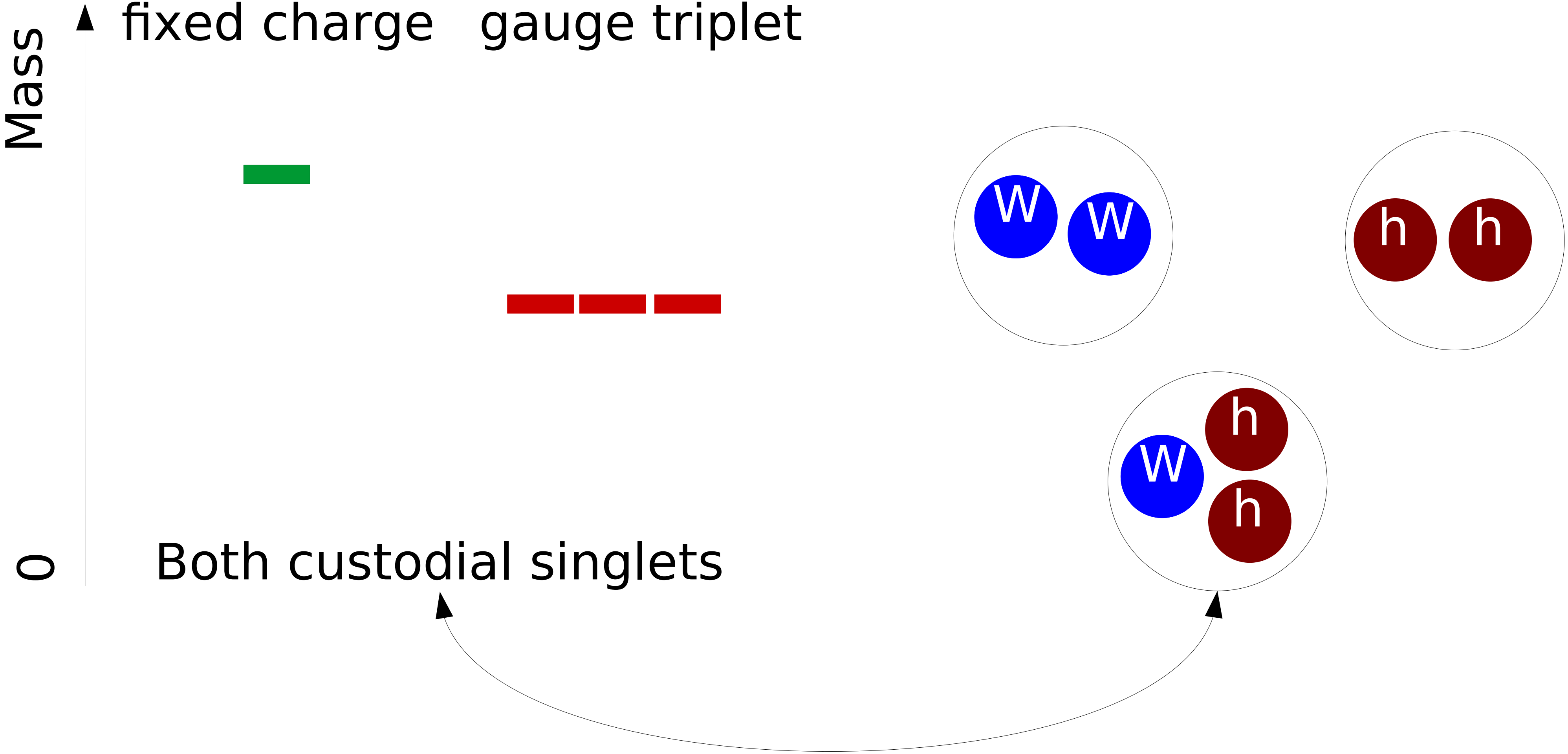
Physical spectrum

Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Composite (bound) states
Require
non-perturbative methods



Both custodial singlets

Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

Physical spectrum

[Maas'12, Maas & Mufti'14]

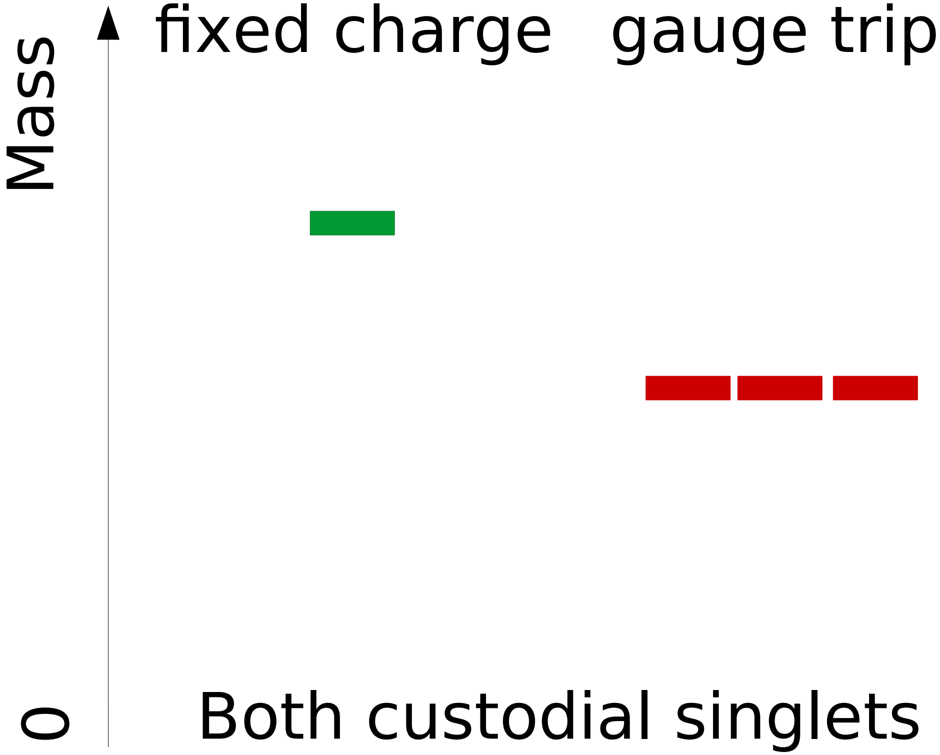
Perturbation theory

Gauge-invariant

Scalar
fixed charge

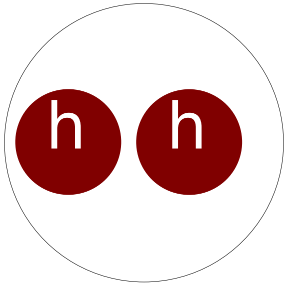
Vector
gauge triplet

Scalar
singlet



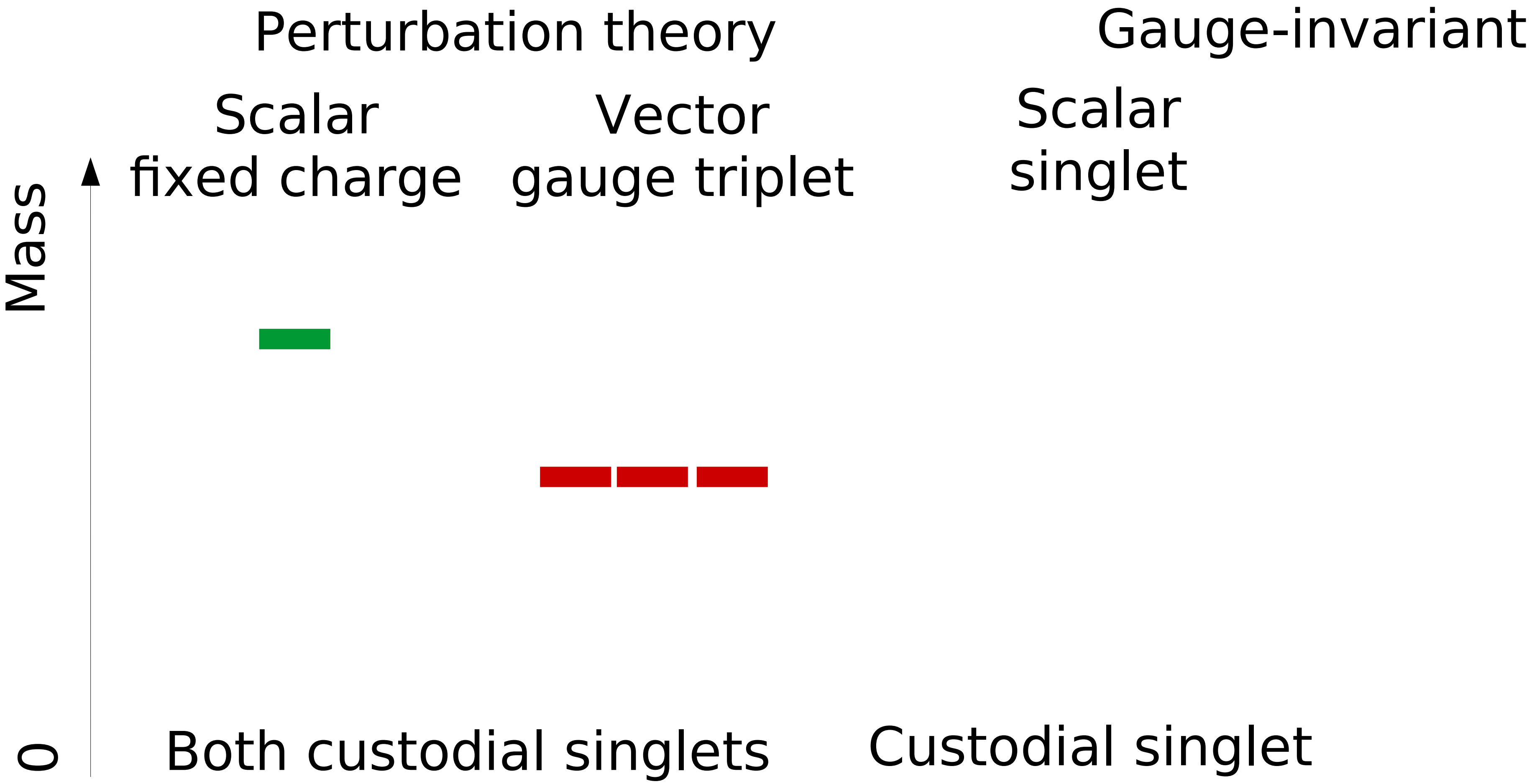
Both custodial singlets

$$h(x)^{+}\, h(x)$$



Physical spectrum

[Maas'12, Maas & Mufti'14]



Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

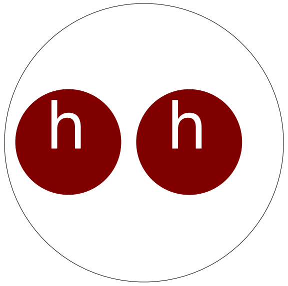
Gauge-invariant

Scalar
singlet

Both custodial singlets

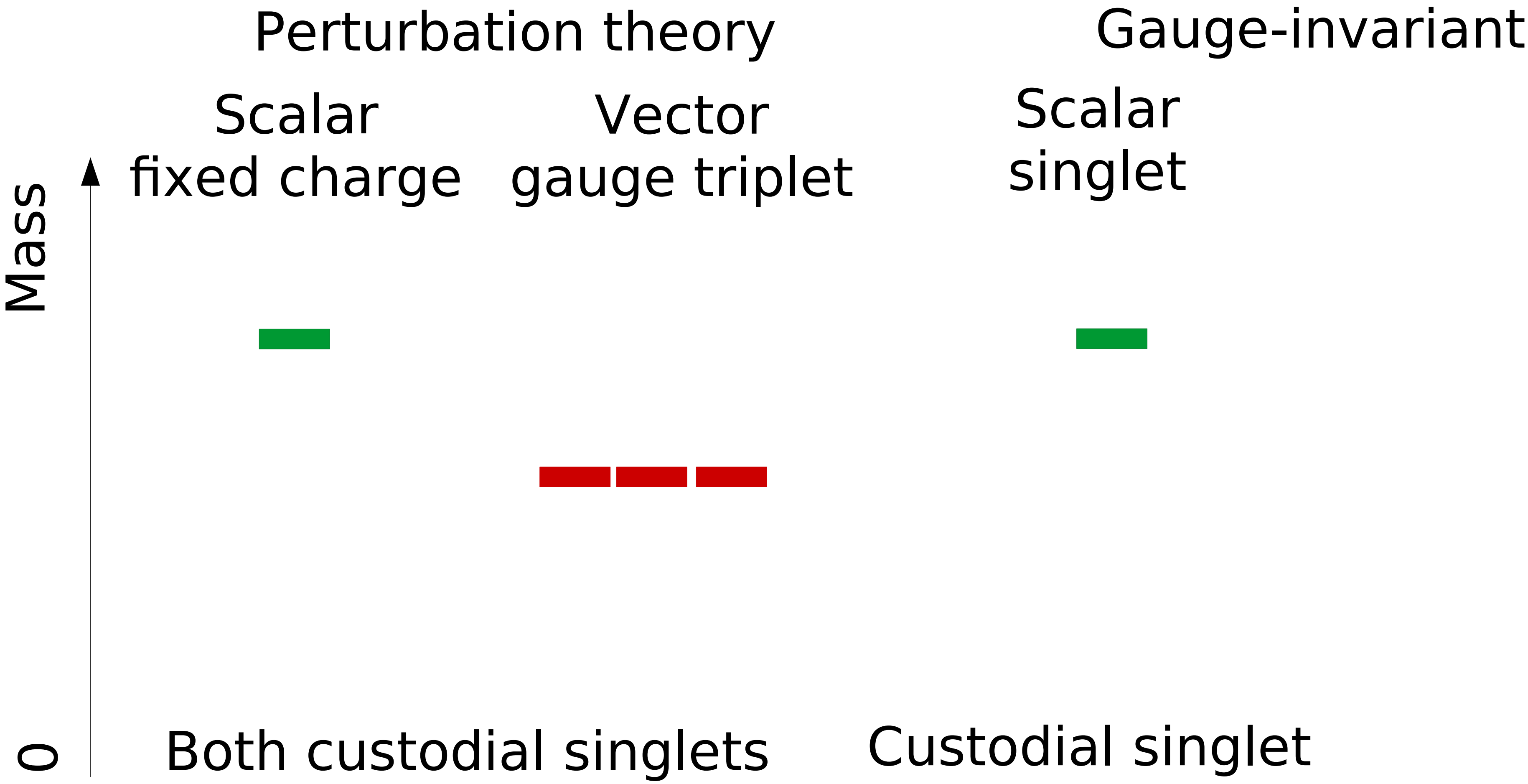
Custodial singlet

$$h(x)^{+}\,h(x)$$



Physical spectrum

[Maas'12, Maas & Mufti'14]



Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Both custodial singlets

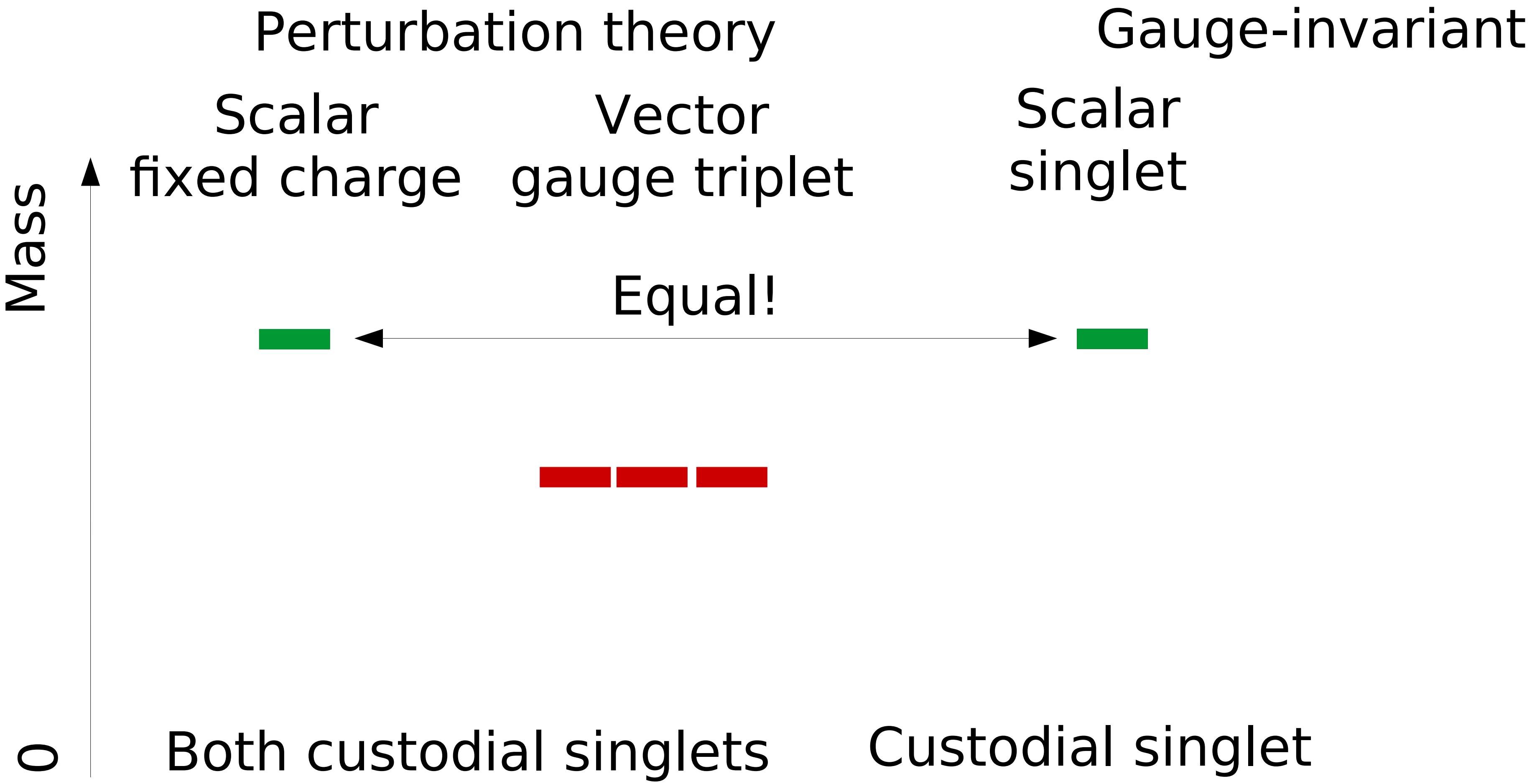
Gauge-invariant

Scalar
singlet

Custodial singlet

Physical spectrum

[Maas'12, Maas & Mufti'14]



Perturbation theory

Gauge-invariant

Scalar
fixed charge

Vector
gauge triplet

Scalar
singlet

Mass

Equal!

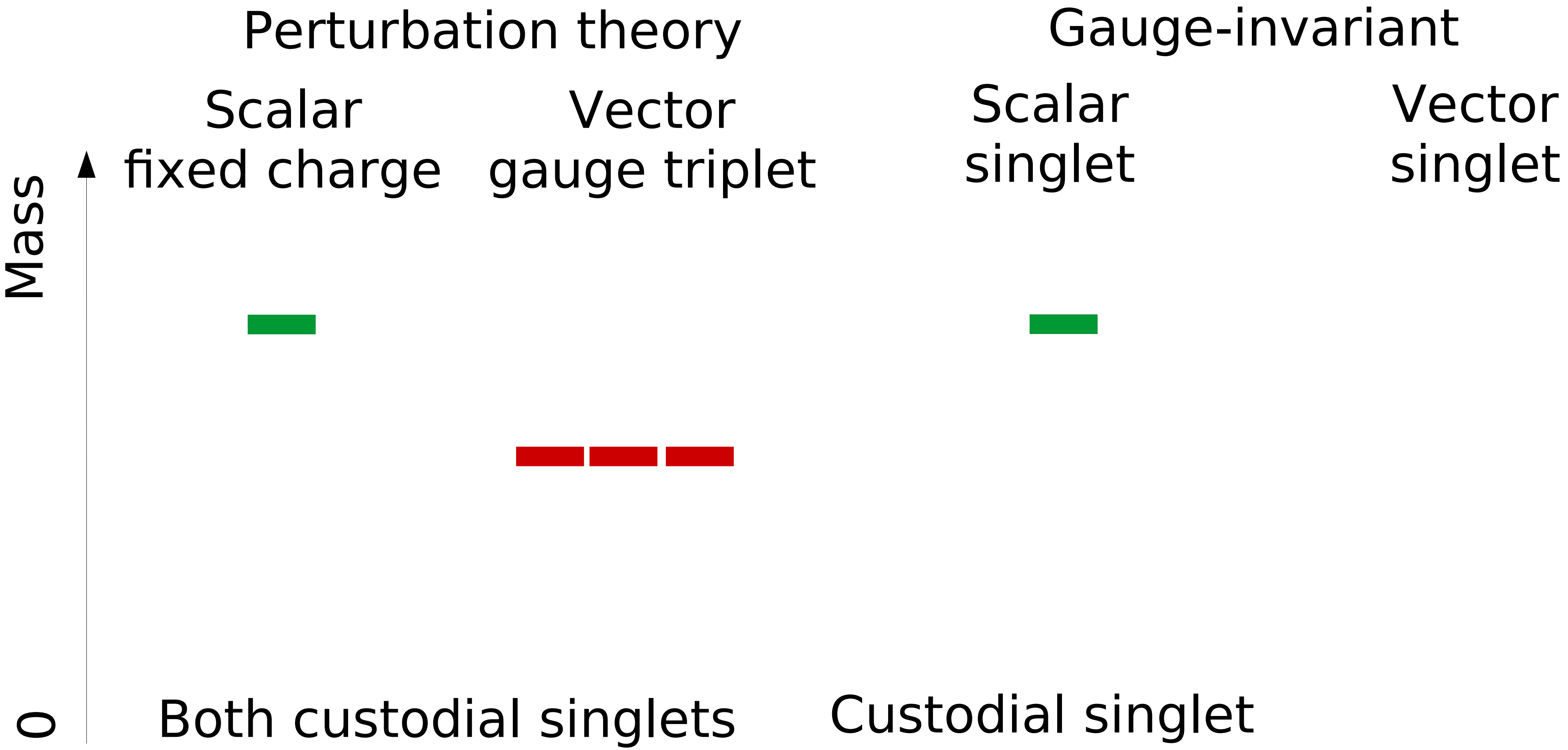
0

Both custodial singlets

Custodial singlet

Physical spectrum

[Maas'12, Maas & Mufti'14]


Perturbation theory

Gauge-invariant

Scalar
fixed charge

Vector
gauge triplet

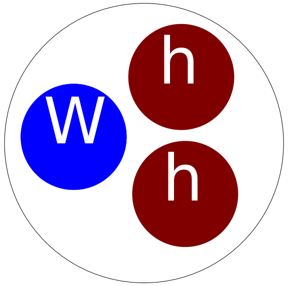
Scalar
singlet

Vector
singlet

Both custodial singlets

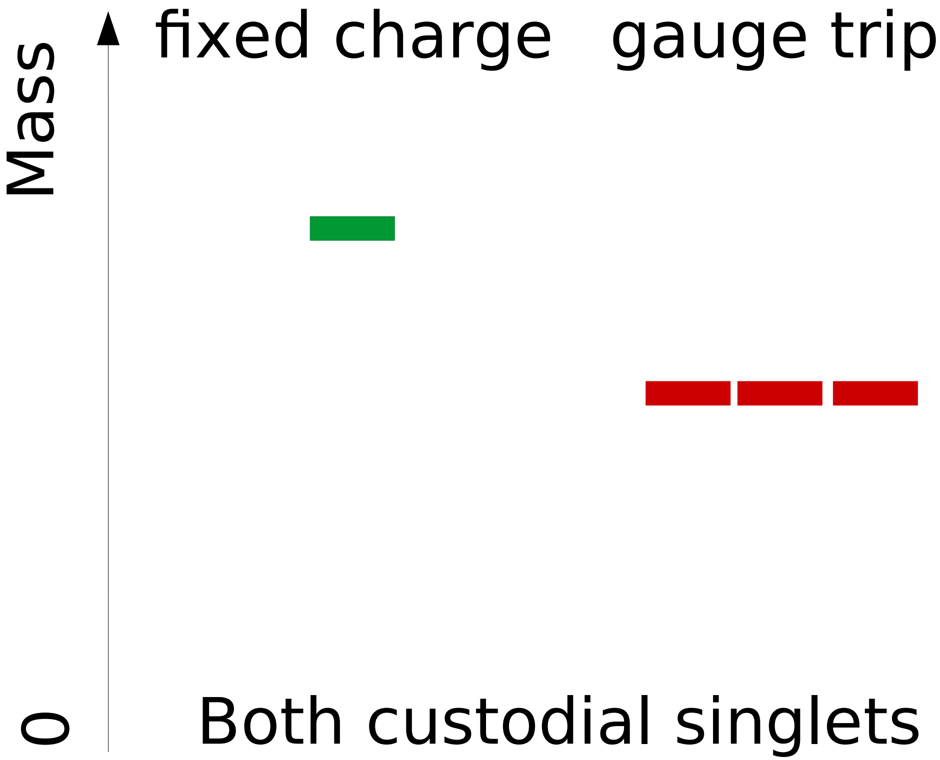
Custodial singlet

$$\mathrm{tr}\, t^a \frac{h^+}{\sqrt{h^+ h}} D_\mu \frac{h}{\sqrt{h^+ h}}$$



Physical spectrum

[Maas'12, Maas & Mufti'14]



Perturbation theory

Scalar fixed charge

Vector gauge triplet

Both custodial singlets

Gauge-invariant

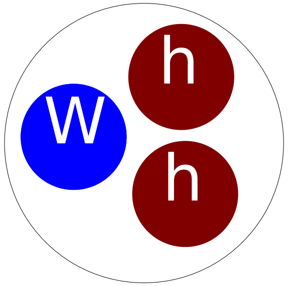
Scalar singlet

Vector singlet

Custodial singlet

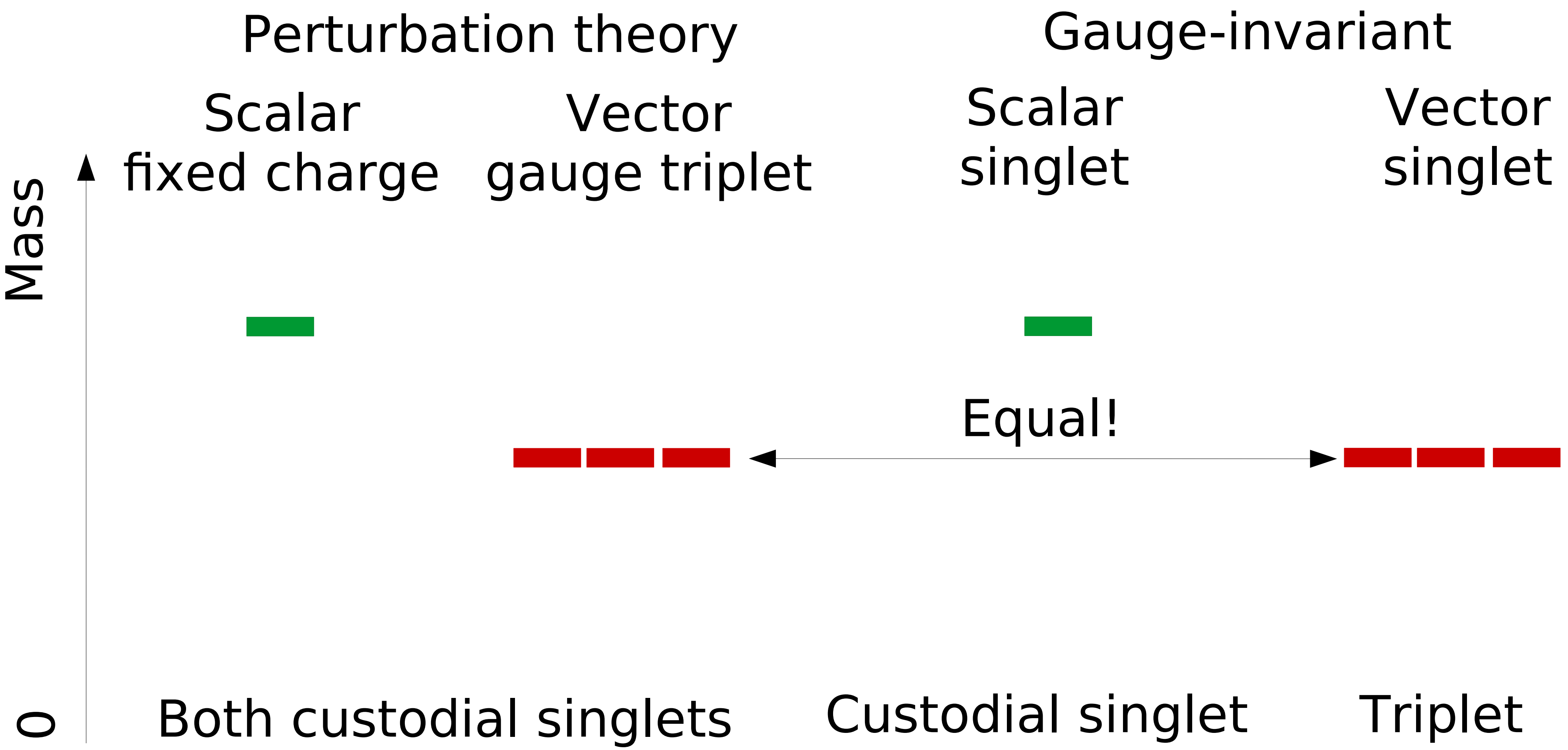
Triplet

$$tr\, t^{\textcircled{a}} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}$$



Physical spectrum

[Maas’12, Maas & Mufti’14]



Perturbation theory

Gauge-invariant

Scalar
fixed charge

Vector
gauge triplet

Scalar
singlet

Vector
singlet

Mass

0

Equal!

Both custodial singlets

Custodial singlet

Triplet

A microscopic mechanism
-
Why on-shell is important

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
    - Different from perturbation theory
        - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
    - Bound state structure – non-perturbative methods?

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas’12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y) \rangle$

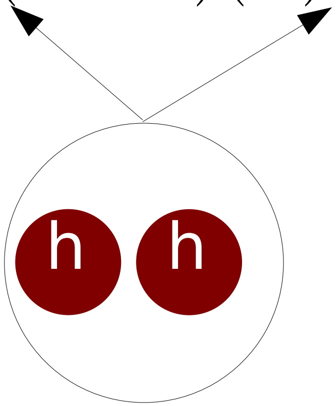
Higgs field

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y) \rangle$



Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2\langle \eta^+ (x)\eta(y)\rangle$$
$$+v\langle \eta^+ \eta^2+\eta^{+2} \eta\rangle+\langle \eta^{+2} \eta^2\rangle$$

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^{+} singlet: $\langle (h^{+} h)(x)(h^{+} h)(y)\rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^{2}\langle \eta^{+}(x)\eta(y)\rangle$$

$$+ v\langle \eta^{+}\eta^{2} + \eta^{+2}\eta\rangle + \langle \eta^{+2}\eta^{2}\rangle$$

3) Standard perturbation theory

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^{2}\langle \eta^{+}(x)\eta(y)\rangle$$

$$+\langle \eta^{+}(x)\eta(y)\rangle\langle \eta^{+}(x)\eta(y)\rangle + O(g,\lambda)$$

Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator

0$^+$ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2 \langle \eta^+ (x)\eta(y)\rangle$$
$$+ v\langle \eta^+ \eta^2 + \eta^{+2} \eta\rangle + \langle \eta^{+2} \eta^2\rangle$$

3) Standard perturbation theory

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2 \langle \eta^+ (x)\eta(y)\rangle$$
$$+ \langle \eta^+ (x)\eta(y)\rangle\langle \eta^+ (x)\eta(y)\rangle + O(g,\lambda)$$

4) Compare poles on both sides

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^{+} h)(x)(h^{+} h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^2\langle \eta^{+}(x)\eta(y)\rangle$$
$$+v\langle \eta^{+}\eta^2+\eta^{+2}\eta\rangle+\langle \eta^{+2}\eta^2\rangle$$

3) Standard perturbation theory

Bound state mass

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^2\langle \eta^{+}(x)\eta(y)\rangle$$
$$+\langle \eta^{+}(x)\eta(y)\rangle\langle \eta^{+}(x)\eta(y)\rangle+O(g,\lambda)$$

4) Compare poles on both sides

Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+h)(x)(h^+h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+h)(x)(h^+h)(y)\rangle = v^2\langle \eta^+(x)\eta(y)\rangle + v\langle \eta^+\eta^2+\eta^{+2}\eta\rangle + \langle \eta^{+2}\eta^2\rangle$$

3) Standard perturbation theory

Bound state mass

Higgs mass

$$\boxed{\langle (h^+h)(x)(h^+h)(y)\rangle} = v^2\boxed{\langle \eta^+(x)\eta(y)\rangle} + \langle \eta^+(x)\eta(y)\rangle\langle \eta^+(x)\eta(y)\rangle + O(g,\lambda)$$

4) Compare poles on both sides

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2\langle \eta^+(x)\eta(y)\rangle$$
$$+v\langle \eta^+ \eta^2 + \eta^{+2}\eta\rangle + \langle \eta^{+2}\eta^2\rangle$$

3) Standard perturbation theory

Standard Perturbation Theory

Bound state mass

$$\boxed{\langle (h^+ h)(x)(h^+ h)(y)\rangle} = v^2\boxed{\langle \eta^+(x)\eta(y)\rangle}$$
$$+\langle \eta^+(x)\eta(y)\rangle\langle \eta^+(x)\eta(y)\rangle + O(g,\lambda)$$

4) Compare poles on both sides

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\left\langle \left(\tau^i h^+ D_\mu h\right)\left(x\right)\left(\tau^j h^+ D_\mu h\right)\left(y\right)\right\rangle$

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle (\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y) \rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle (\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle = v^2 c_{ij}^{ab} \langle W_\mu^a(x) W^b(y)^\mu \rangle + \ldots$$

What about the vector?

[Fröhlich et al.'80,'81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle = v^2 c_{ij}^{ab}\langle W_\mu^a(x) W^b(y)^\mu\rangle + \ldots$$

Matrix from
group structure

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle = v^2 c_{ij}^{ab}\langle W_\mu^a(x) W^b(y)^\mu\rangle + \ldots$$

$$= v^2\langle W_\mu^i W_\mu^j\rangle + \ldots$$

Matrix from
group structure

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle = v^2 c_{ij}^{ab}\langle W_\mu^a(x) W^b(y)^\mu\rangle + \ldots$$
$$= v^2 \langle W_\mu^i W_\mu^j\rangle + \ldots$$

Matrix from
group structure

c projects custodial
states to gauge
states

What about the vector?

[Fröhlich et al.'80,'81 Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle(\tau^i h^+ D_\mu h)(x)(\tau^j h^+ D_\mu h)(y)\rangle = v^2 c_{ij}^{ab}\langle W_\mu^a(x) W^b(y)^\mu\rangle + \ldots$$

$$= v^2\langle W_\mu^i W_\mu^j\rangle + \ldots$$

Matrix from group structure

c projects custodial states to gauge states

Exactly one gauge boson for every physical state

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17
Maas & Sondenheimer’20]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2 \langle \eta^+ (x)\eta(y)\rangle$$
$$+ v\langle \eta^+ \eta^2 + \eta^{+2} \eta\rangle + \langle \eta^{+2} \eta^2\rangle$$

3) Standard perturbation theory

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2 \langle \eta^+ (x)\eta(y)\rangle$$
$$+ \langle \eta^+ (x)\eta(y)\rangle\langle \eta^+ (x)\eta(y)\rangle + O(g, \lambda)$$

4) Compare poles on both sides

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17
Maas & Sondenheimer’20]

1) Formulate gauge-invariant operator

0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2\langle \eta^+(x)\eta(y)\rangle$$

What about this?

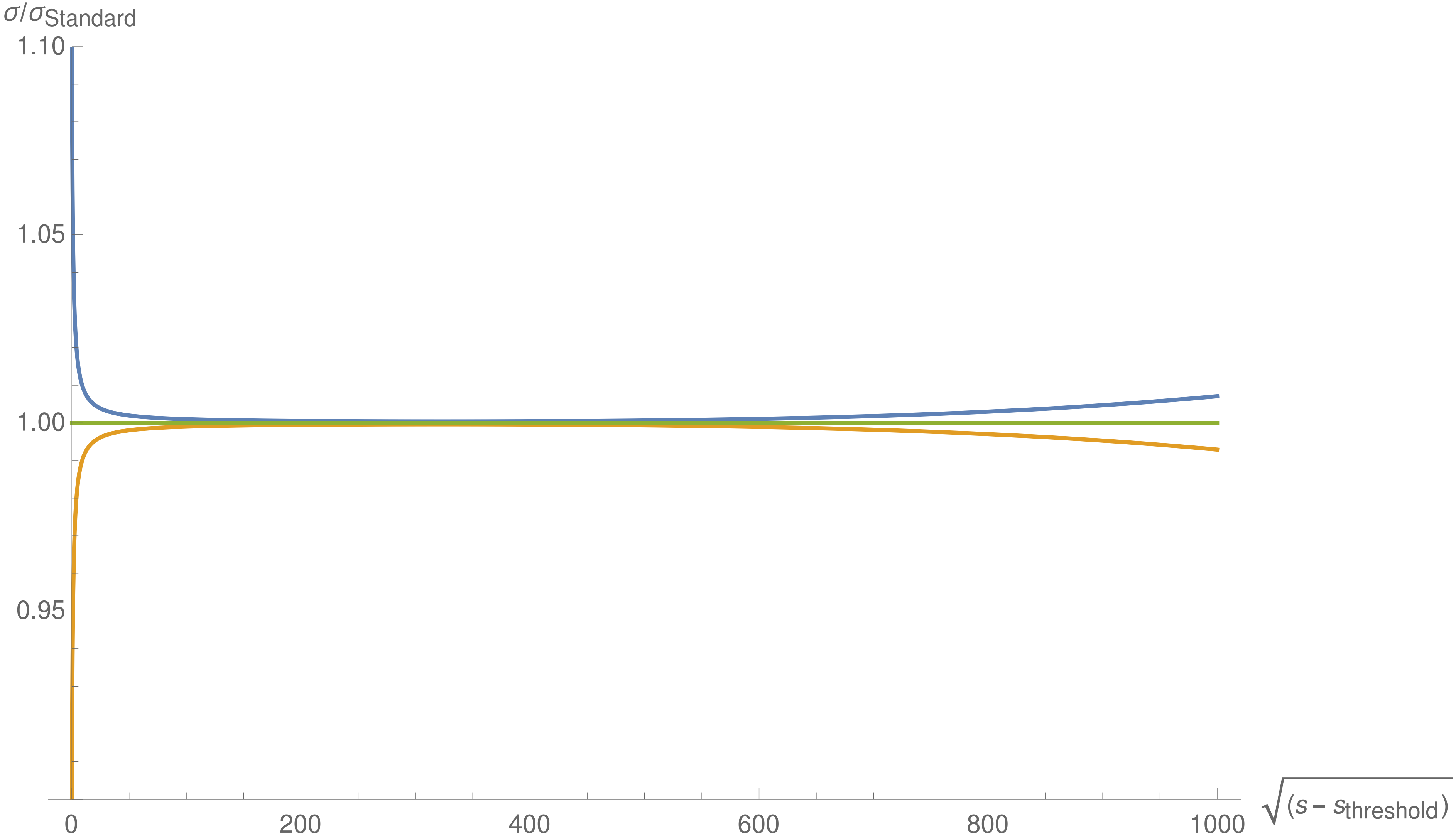
$$\boxed{+v\langle \eta^+ \eta^2+\eta^{+2}\eta\rangle+\langle \eta^{+2}\eta^2\rangle}$$

3) Standard perturbation theory

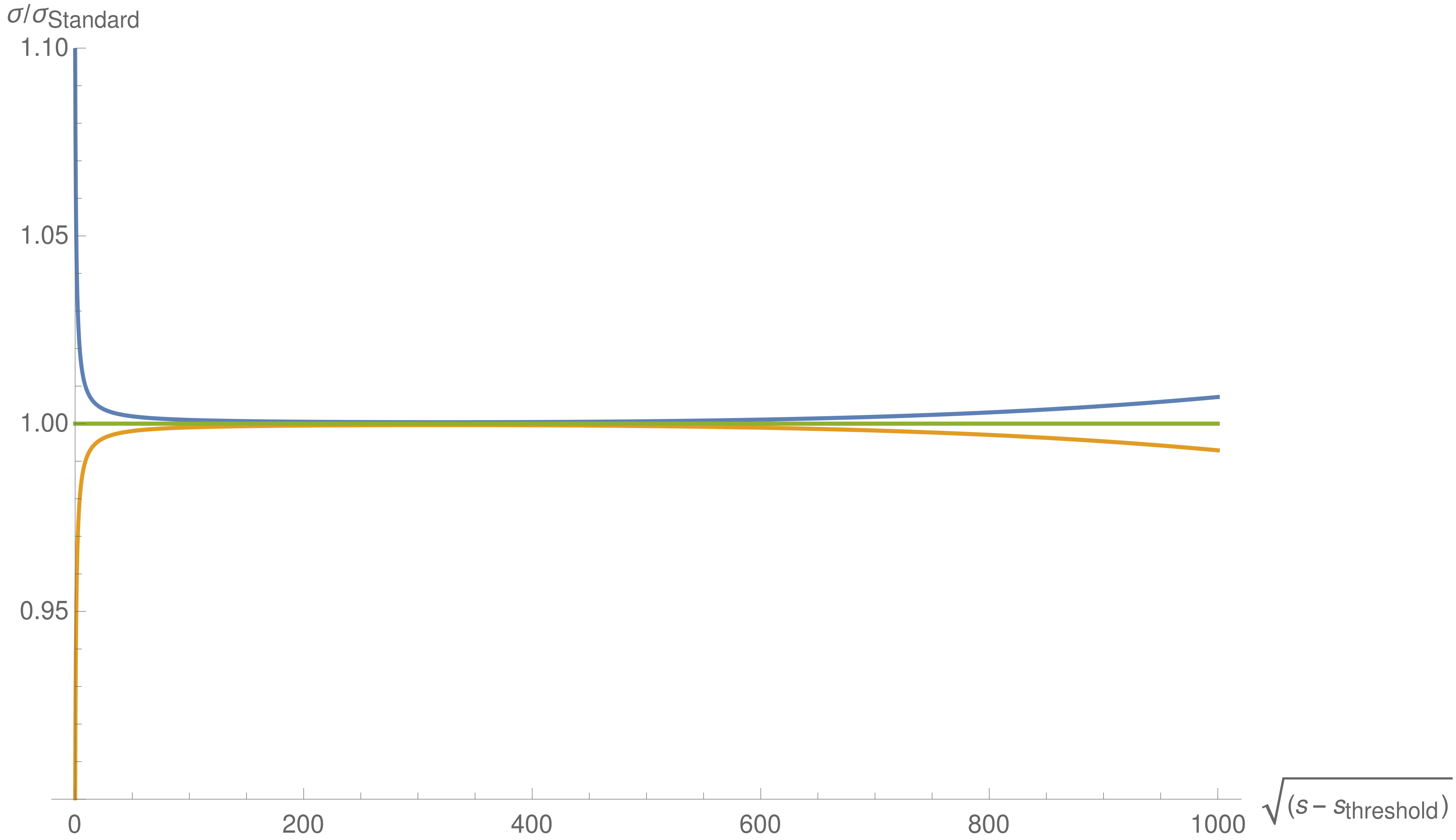
$$\langle (h^+ h)(x)(h^+ h)(y)\rangle = v^2\langle \eta^+(x)\eta(y)\rangle$$
$$+\langle \eta^+(x)\eta(y)\rangle\langle \eta^+(x)\eta(y)\rangle+O(g,\lambda)$$

4) Compare poles on both sides

Generic behavior: DIS-like

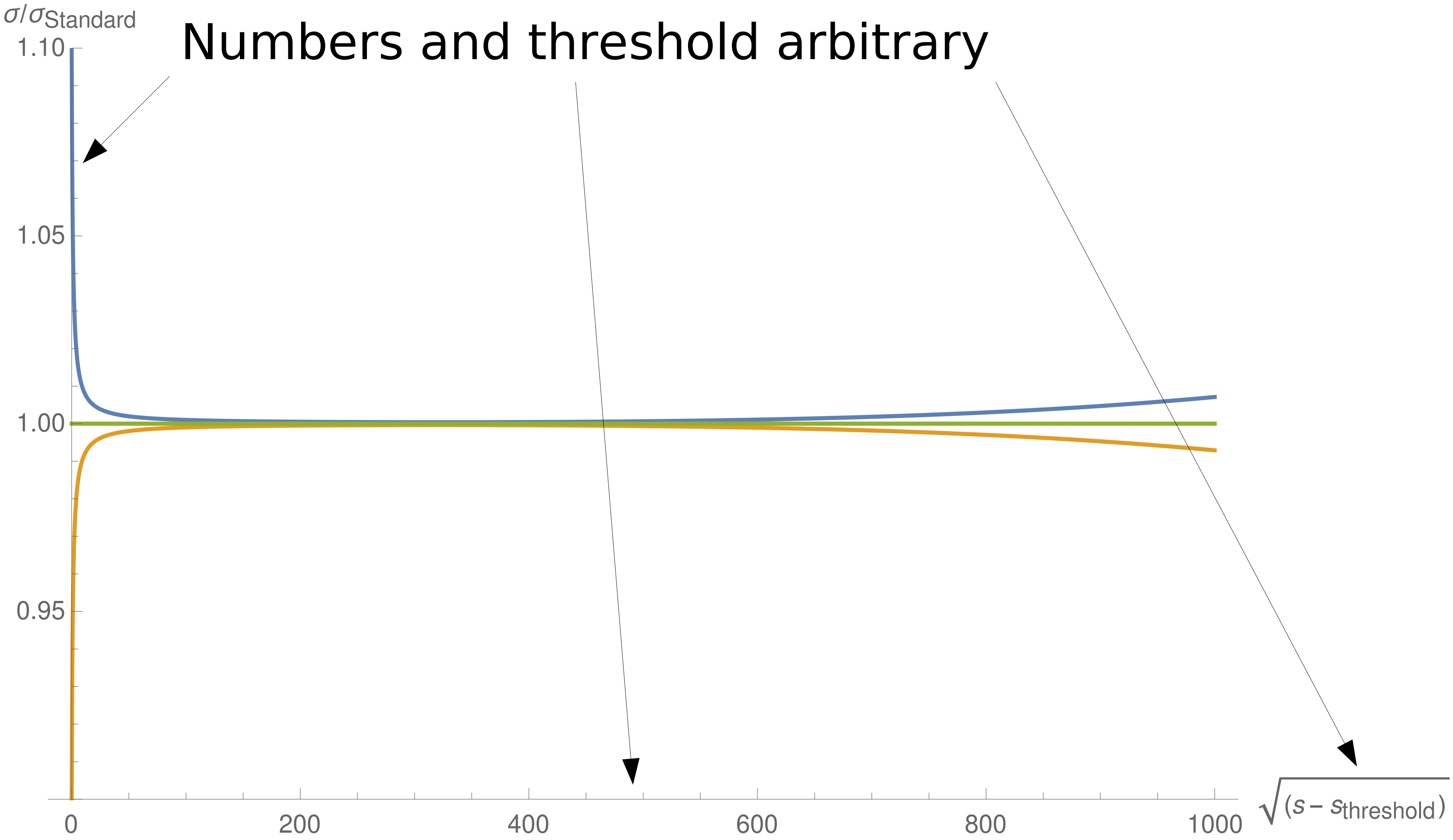


Generic behavior: DIS-like



Note: Works also with fermions [Afferrante et al.'20]

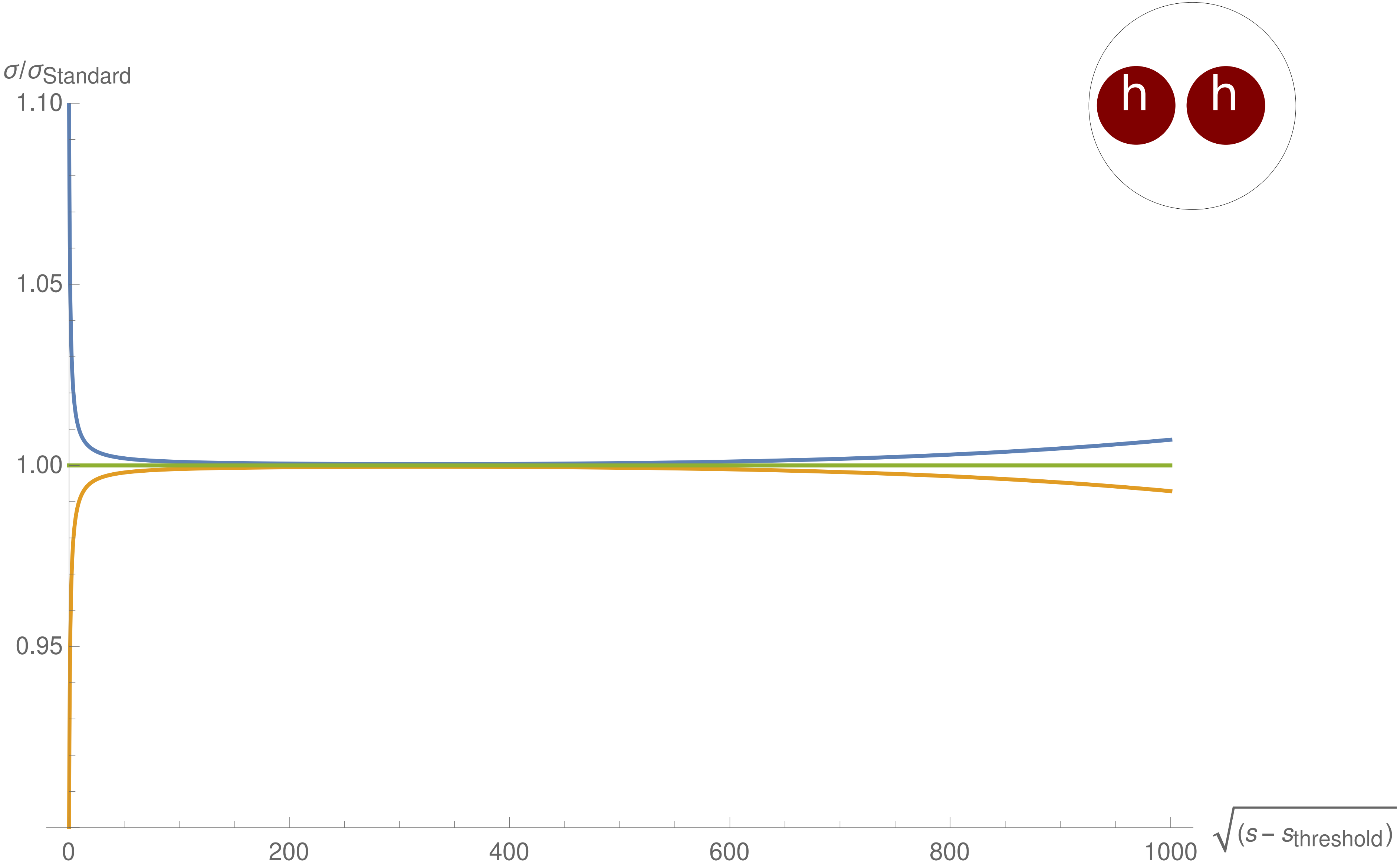
Generic behavior: DIS-like



Numbers and threshold arbitrary

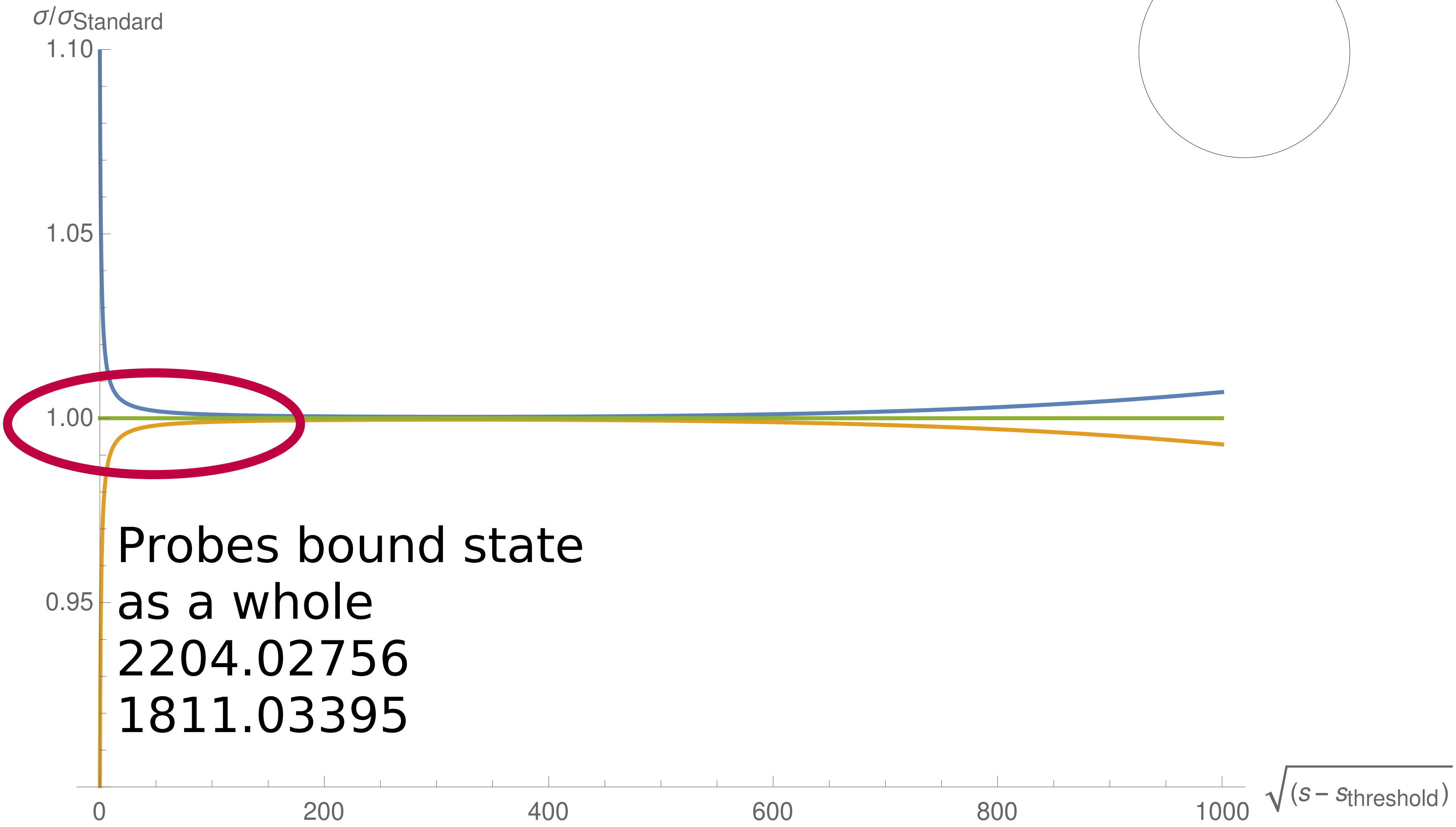
Note: Works also with fermions [Afferrante et al.'20]

Generic behavior: DIS-like



Note: Works also with fermions [Afferrante et al.'20]

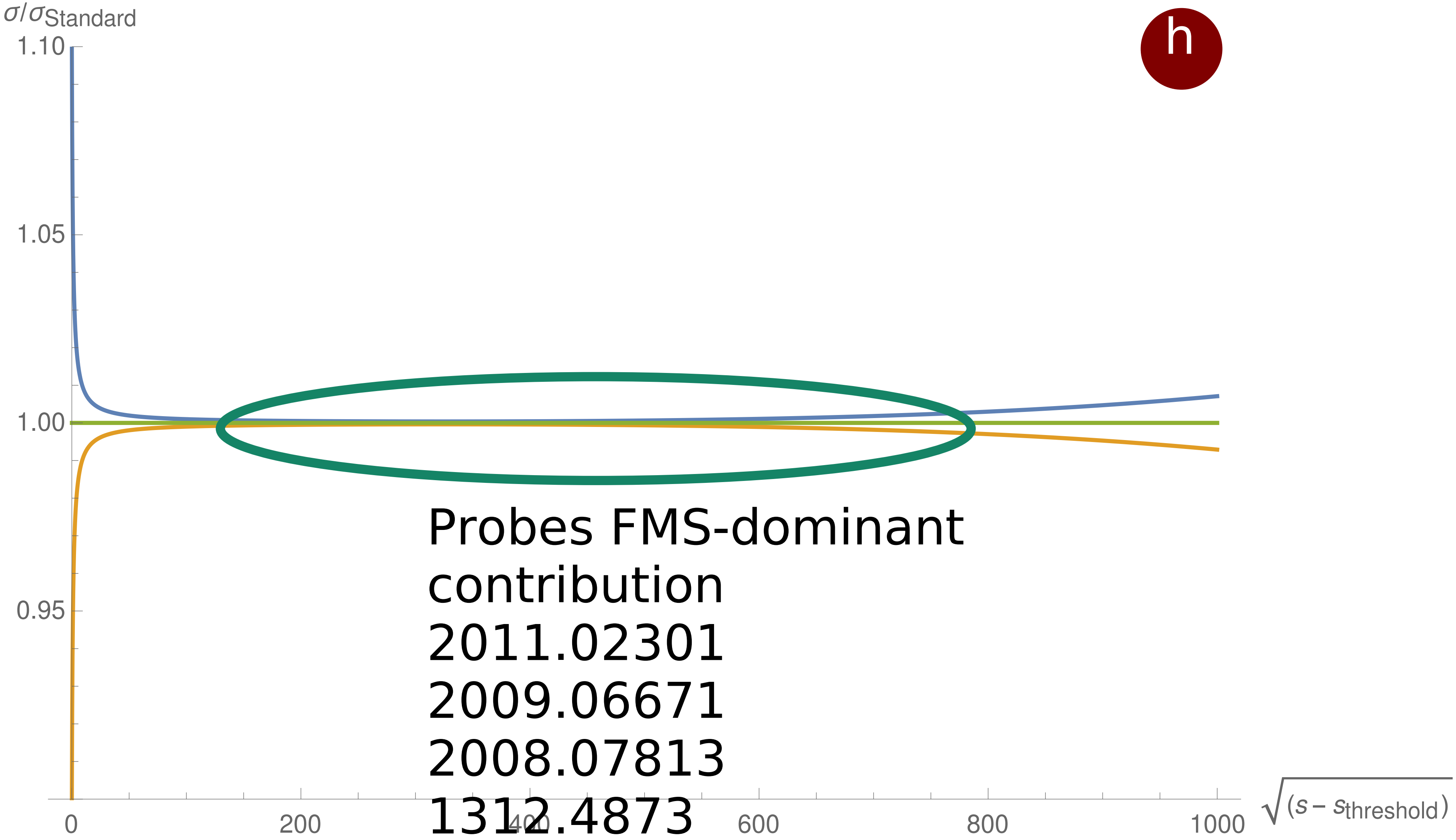
Generic behavior: DIS-like



Probes bound state
as a whole
2204.02756
1811.03395

Note: Works also with fermions [Afferrante et al.'20]

Generic behavior: DIS-like



Probes FMS-dominant contribution

2011.02301

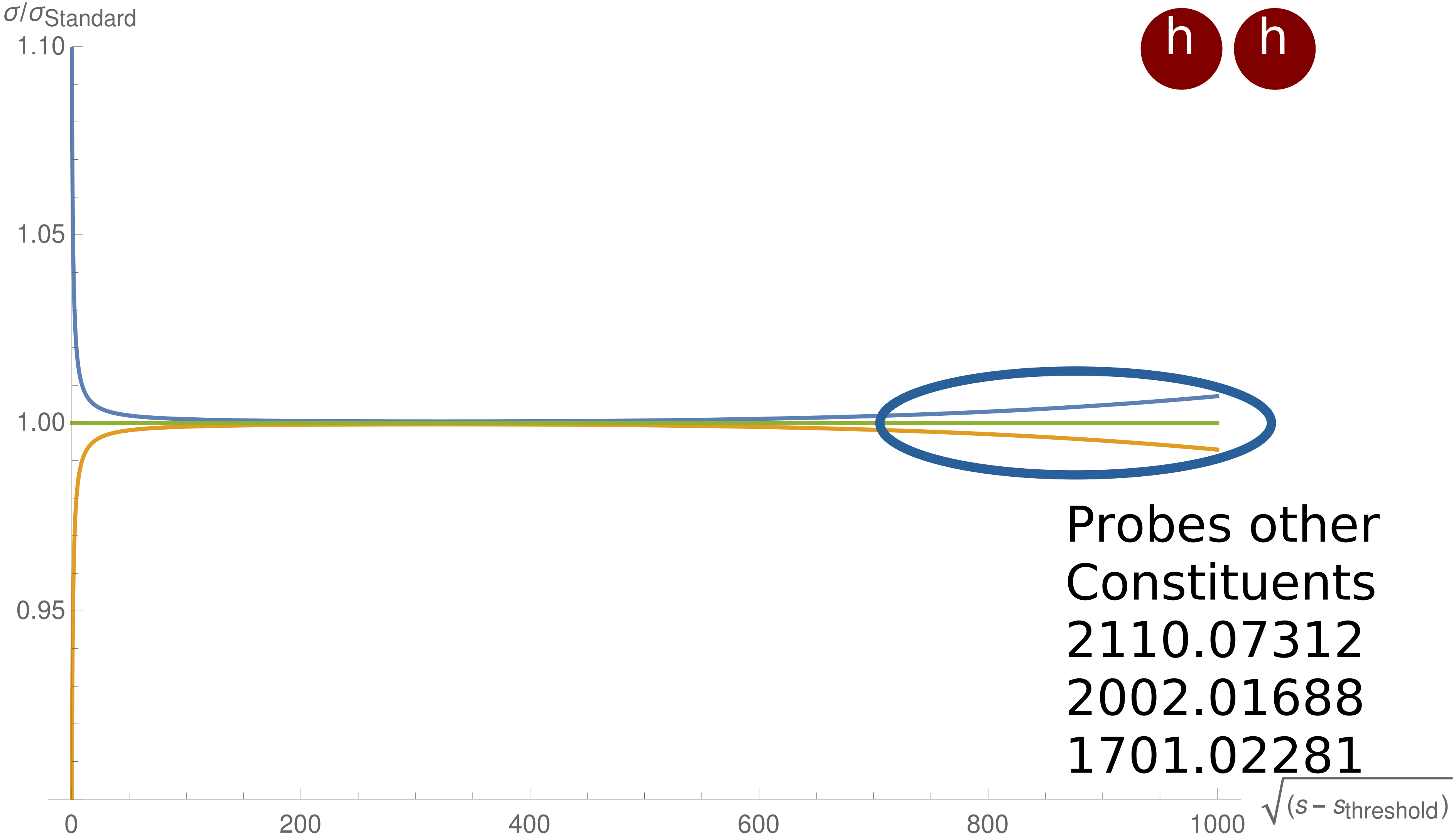
2009.06671

2008.07813

1312.4873

Note: Works also with fermions [Afferrante et al.'20]

Generic behavior: DIS-like



Probes other Constituents
2110.07312
2002.01688
1701.02281

Note: Works also with fermions [Afferrante et al.'20]

New physics
-
Qualitative changes

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special
    - Mapping of custodial symmetry to gauge symmetry
    - Fits perfectly degrees of freedom

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Török'17]

- Standard model is special
  - Mapping of custodial symmetry to gauge symmetry
  - Fits perfectly degrees of freedom
- Is this generally true?

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special
  - Mapping of custodial symmetry to gauge symmetry
  - Fits perfectly degrees of freedom
- Is this generally true?
  - No: Depends on gauge group, representations, and custodial groups

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special
  - Mapping of custodial symmetry to gauge symmetry
  - Fits perfectly degrees of freedom
- Is this generally true?
  - No: Depends on gauge group, representations, and custodial groups
  - Can work sometimes (2HDM) [Maas,Pedro'16]

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special
  - Mapping of custodial symmetry to gauge symmetry
  - Fits perfectly degrees of freedom
- Is this generally true?
  - No: Depends on gauge group, representations, and custodial groups
  - Can work sometimes (2HDM) [Maas,Pedro'16]
  - Generally qualitative differences

A toy model

- Consider an SU(3) with a single fundamental scalar

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W^{\mu\nu}_{a}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

- Ws W^{a}_{μ} W

- Coupling g and some numbers f^{abc}

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W^{\mu\nu}_{a}+\left(D^{ij}_{\mu}h^{j}\right)^{+}D^{\mu}_{ik}h_{k}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

$$D^{ij}_{\mu}=\delta^{ij}\partial_{\mu}-igW^{a}_{\mu}t^{ij}_{a}$$

- Ws W^{a}_{μ} W
- Higgs h_{i} h
- Coupling g and some numbers f^{abc} and t^{ij}_{a}

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu}+\left(D^{ij}_\mu h^j\right)^+ D^\mu_{ik}h_k+\lambda\left(h^a h_a^+ - v^2\right)^2$$

$$W^a_{\mu\nu}=\partial_\mu W^a_\nu-\partial_\nu W^a_\mu+gf^a_{bc}W^b_\mu W^c_\nu$$

$$D^{ij}_\mu=\delta^{ij}\partial_\mu-igW^a_\mu t^{ij}_a$$

- Ws W^a_μ W
- Higgs h_i h
- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Ws W_{μ}^{a} W
- Higgs h_i h
- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}
- Parameters selected for a BEH effect

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu}+\left(D^{ij}_\mu h^j\right)^+ D^\mu_{ik}h_k+\lambda\left(h^a h_a^+ -v^2\right)^2$$

$$W^a_{\mu\nu}=\partial_\mu W^a_\nu-\partial_\nu W^a_\mu+gf^a_{bc}W^b_\mu W^c_\nu$$

$$D^{ij}_\mu=\delta^{ij}\partial_\mu-igW^a_\mu t^{ij}_a$$

- Local SU(3) gauge symmetry

$$W^a_\mu\rightarrow W^a_\mu+(\delta^a_b\partial_\mu-g f^a_{bc}W^c_\mu)\phi^b \qquad h_i\rightarrow h_i+g\, t^{ij}_a\phi^a h_j$$

A toy model

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}+\lambda\left(h^{a}h_{a}^{+}-v^{2}\right)^{2}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Local SU(3) gauge symmetry

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-gf_{bc}^{a}W_{\mu}^{c})\phi^{b}$ $\qquad h_{i}\rightarrow h_{i}+g\,t_{a}^{ij}\phi^{a}h_{j}$

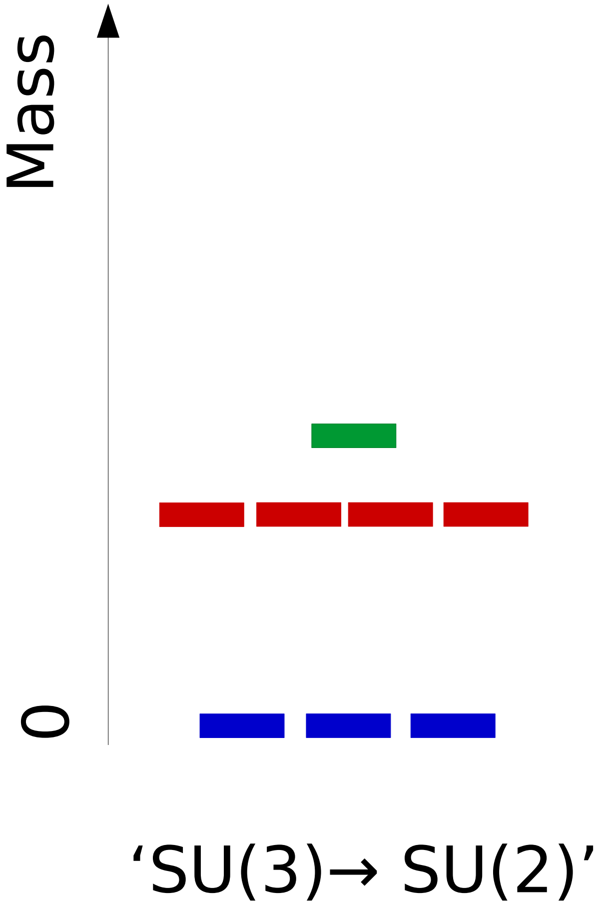
- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $\qquad h\rightarrow\exp(ia)h$

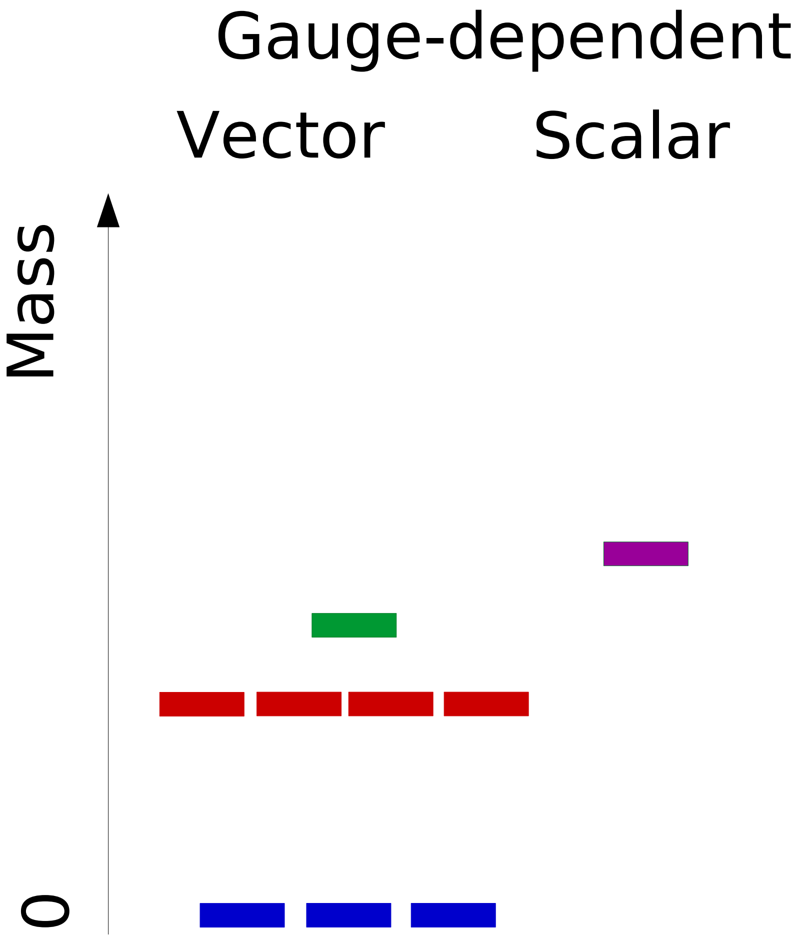
Spectrum

Gauge-dependent

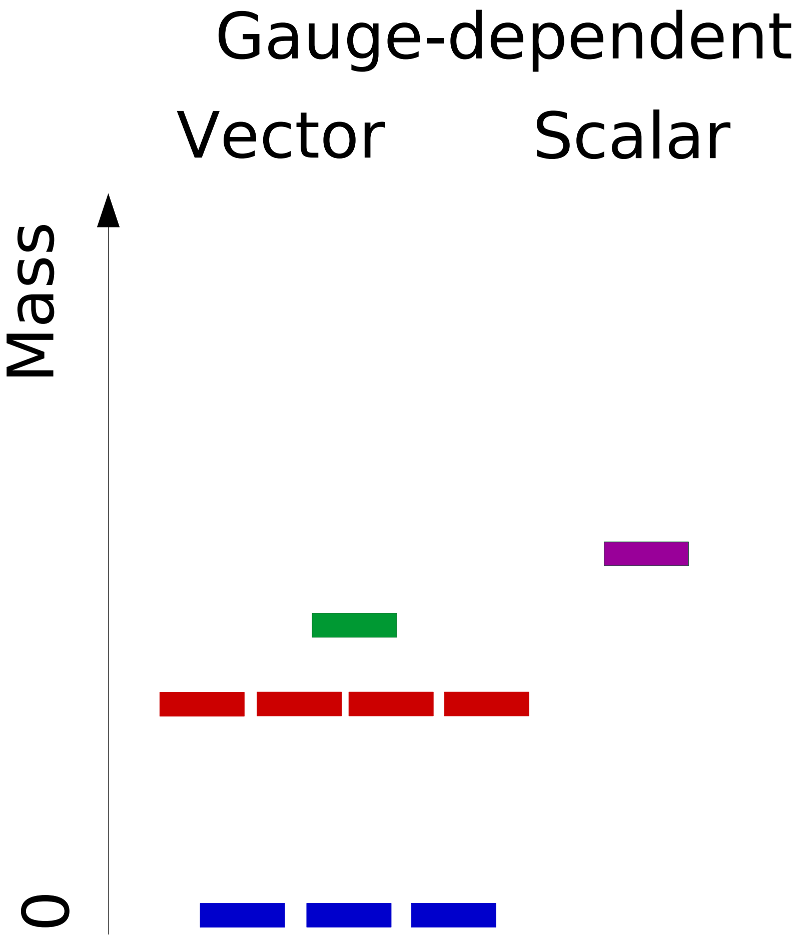
Vector



Spectrum



Spectrum



Confirmed in gauge-fixed lattice calculations [Maas et al.'16]

Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]

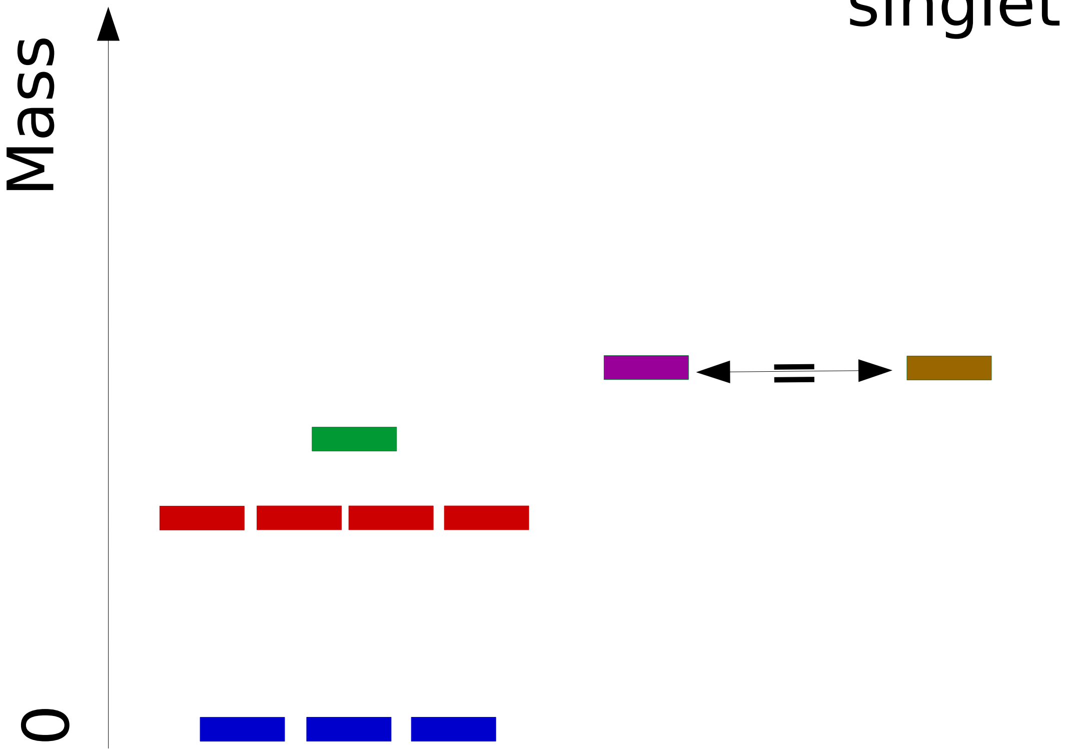
Gauge-dependent

Gauge-invariant

Vector

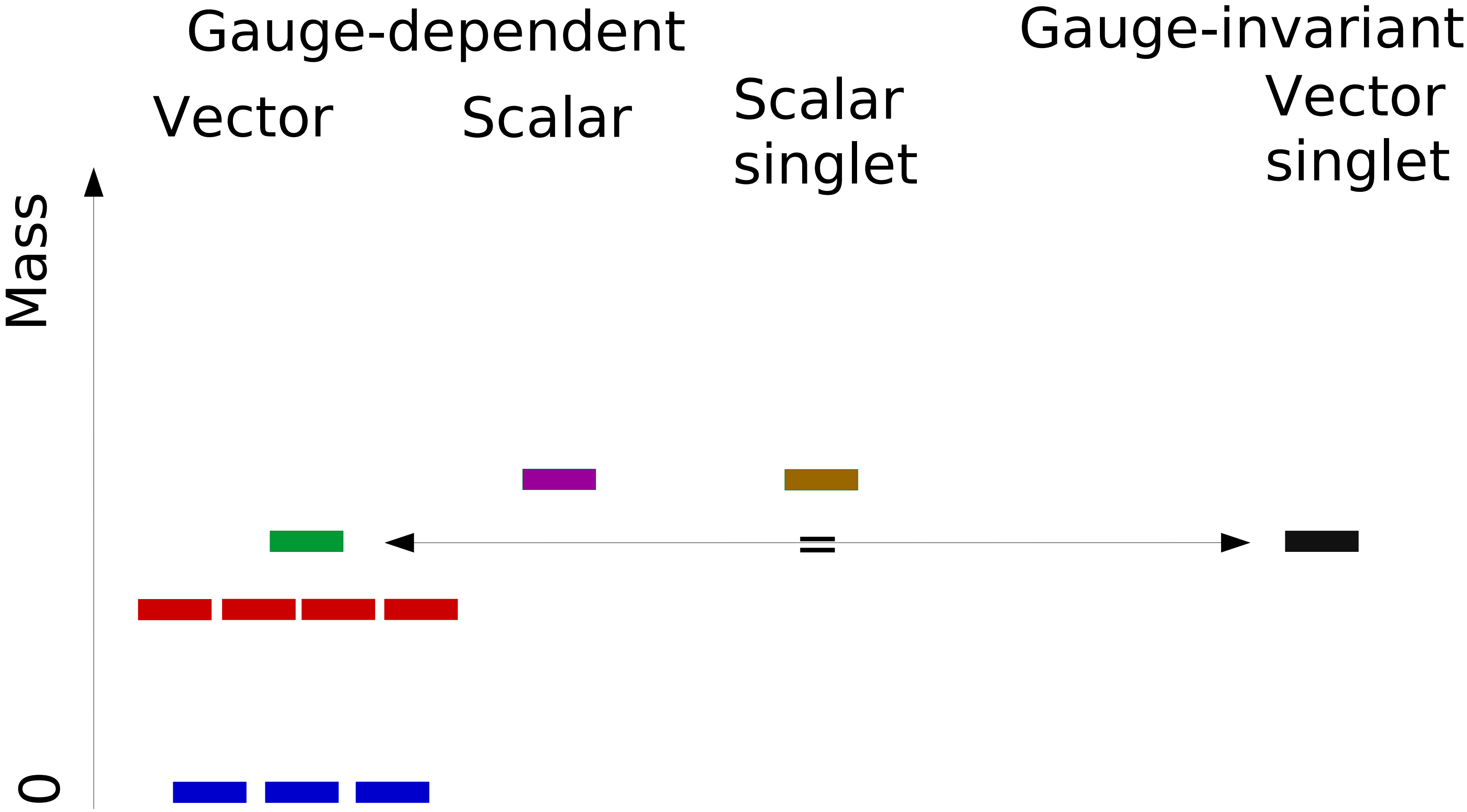
Scalar

Scalar singlet



Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]



Gauge-dependent

Gauge-invariant

Vector

Scalar

Scalar
singlet

Vector
singlet

What about the vector?

[Maas & Törek’16]

What about the vector?

[Maas & Törek'16]

1) Formulate gauge-invariant operator

1^- singlet

What about the vector?

[Maas & Törek'16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle$

What about the vector?

[Maas & Törek’16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

What about the vector?

[Maas & Török'16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle = v^2 c^{ab} \langle W_\mu^a(x) W^b(y)^\mu \rangle + \dots$$

What about the vector?

[Maas & Törek'16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle = v^2 c^{ab}\langle W_\mu^a(x) W^b(y)^\mu\rangle + \ldots$$

Matrix from
group structure

What about the vector?

[Maas & Törek’16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h = v + \eta$

$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle = v^2 c^{ab} \langle W^a_\mu(x) W^b(y)^\mu \rangle + \ldots$$

$$= v^2 \langle W^8_\mu W^8_\mu \rangle + \ldots$$

Matrix from
group structure

c^{ab} projects out
only one field

What about the vector?

[Maas & Török'16]

1) Formulate gauge-invariant operator

1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y)\rangle = v^2 c^{ab} \langle W_\mu^a(x) W^b(y)^\mu \rangle + \ldots$$
$$= v^2 \langle W_\mu^8 W_\mu^8 \rangle + \ldots$$

Matrix from group structure

c^{ab} projects out only one field

Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

A different kind of states

- Group theory forced same gauge multiplets and custodial multiples for SU(2)
  - Because Higgs is bifundamental
  - Remainder is bound state/resonance or not

A different kind of states

- Group theory forced same gauge multiplets and custodial multiples for SU(2)
  - Because Higgs is bifundamental
  - Remainder is bound state/resonance or not
- Now: Elementary states without analouge
  - No global symmetry to provide multiplet structure

A different kind of states

- Group theory forced same gauge multiplets and custodial multiples for SU(2)
  - Because Higgs is bifundamental
  - Remainder is bound state/resonance or not
- Now: Elementary states without analouge
  - No global symmetry to provide multiplet structure
- Now: States without elementary analouge
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue – U(1) acts as baryon number
  - Lightest must exist and be absolutely stable

Possible new states

- Quantum numbers are $J^{PC}{}_{\text{Custodial}}$

Possible new states

- Quantum numbers are $J^{PC}{}_{\text{Custodial}}$
    - Simpelst non-trivial state operator: $0^{++}{}_{1}$

$$\epsilon_{abc}\,\phi^{a} D_{\mu}\,\phi^{b} D_{\nu} D^{\nu} D^{\mu}\,\phi^{c}$$

Possible new states

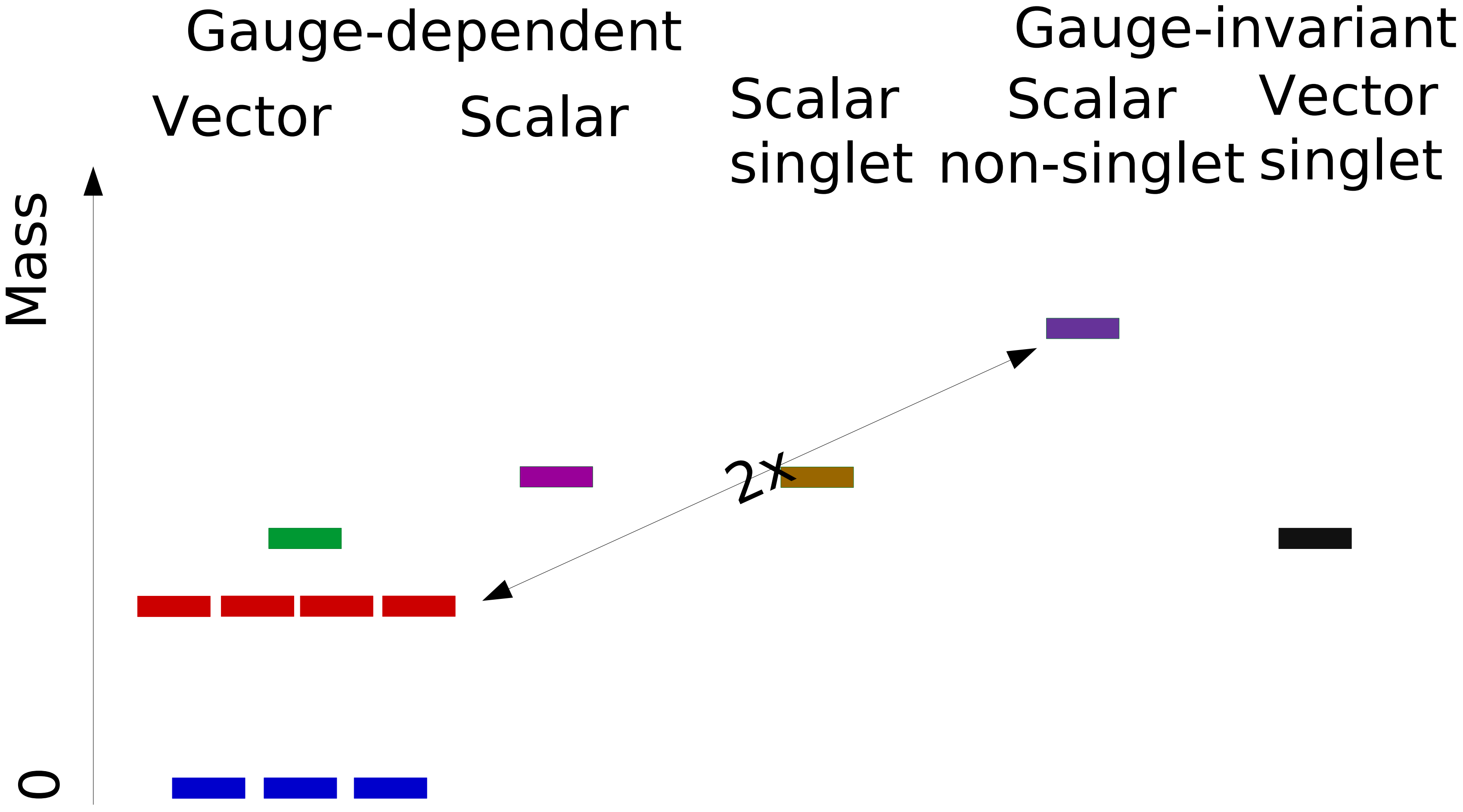
- Quantum numbers are $J^{PC}{}_{\text{Custodial}}$
  - Simpelst non-trivial state operator: $0^{++}{}_{1}$

$$\epsilon_{abc}\,\phi^{a}\,D_{\mu}\,\phi^{b}\,D_{\nu}\,D^{\nu}\,D^{\mu}\,\phi^{c}$$

- What is the lightest state?
  - Prediction with constituent model

Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]



Gauge-dependent

Vector
Scalar

Gauge-invariant

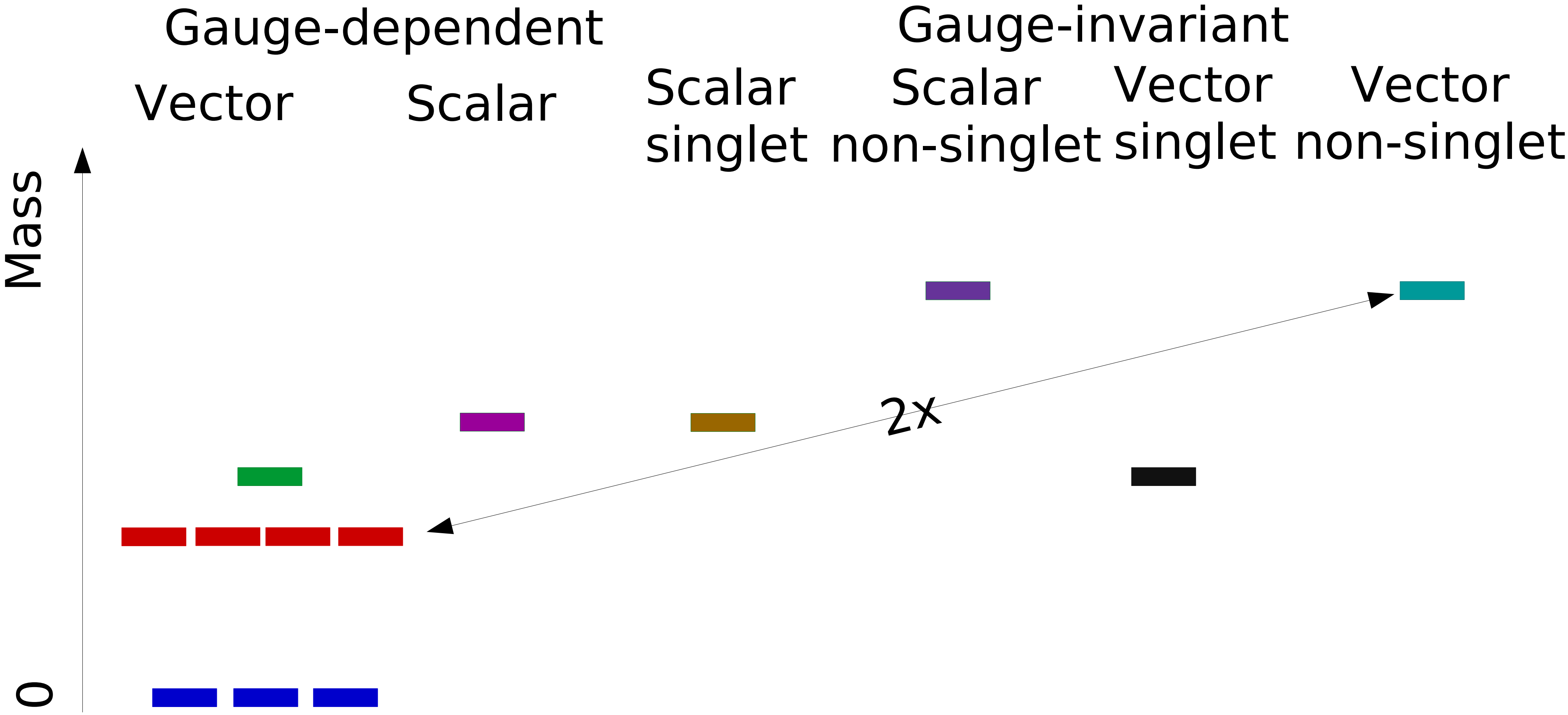
Scalar
singlet

Scalar
non-singlet

Vector
singlet

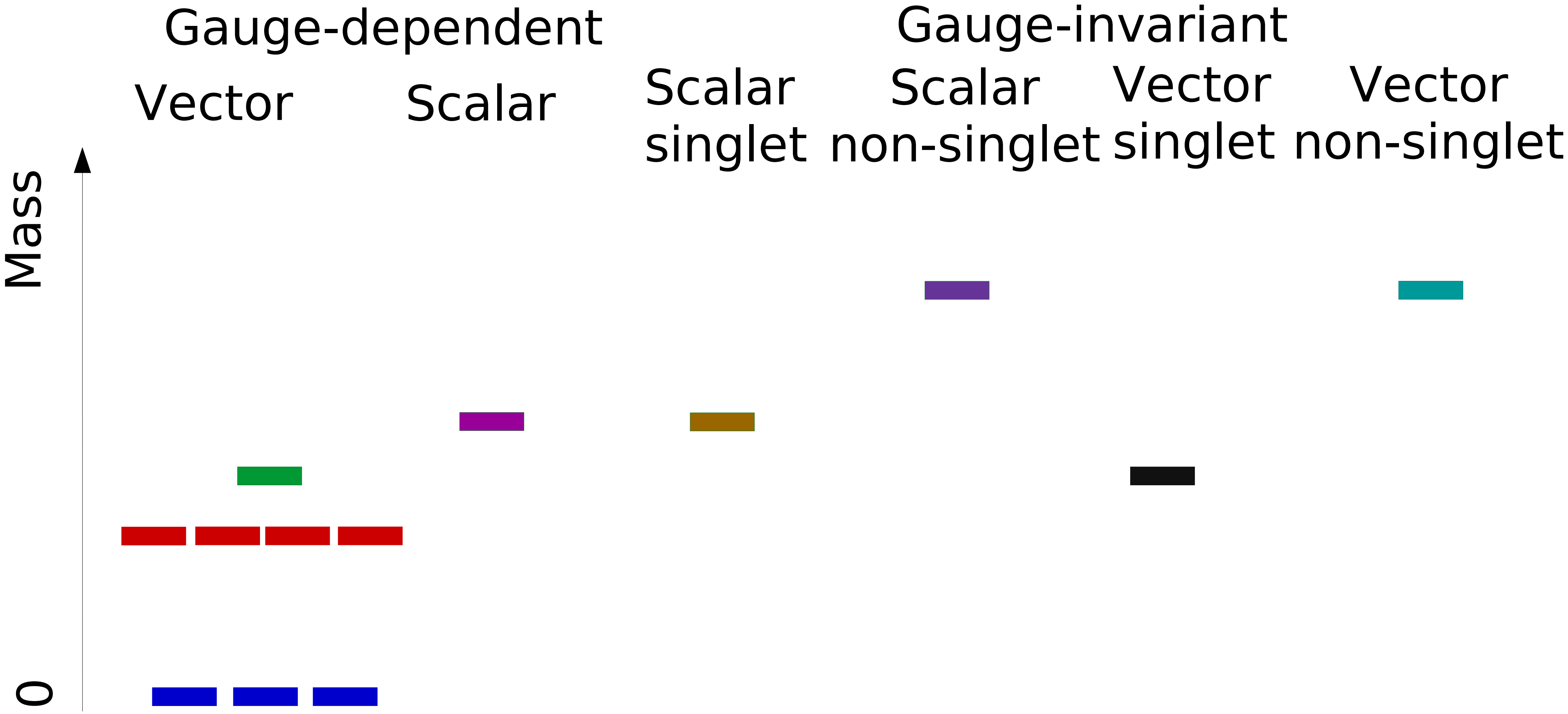
Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]



Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]



- Qualitatively different spectrum
- No mass gap!

Possible states

- Quantum numbers are $J^{PC}{}_{\text{Custodial}}$
    - Simpelst non-trivial state operator: $0^{++}{}_{1}$
    - $\epsilon_{abc}\phi^{a}D_{\mu}\phi^{b}D_{\nu}D^{\nu}D^{\mu}\phi^{c}$
- What is the lightest state?
    - Prediction with constituent model
    - Lattice calulations

Possible states

- Quantum numbers are $J^{PC}{}_{\mathrm{Custodial}}$
  - Simpelst non-trivial state operator: $0^{++}{}_1$
  - $\epsilon_{abc}\phi^a D_\mu \phi^b D_\nu D^\nu D^\mu \phi^c$
- What is the lightest state?
  - Prediction with constituent model
  - Lattice calulations
  - All channels: J<3
  - Aim: Ground state for each channel
    - Characterization through scattering states

Technical details

- Standard lattice discretization [Dobson et al.’21]
    - Pseudo-Heatbath
    - Flexible code for adjoint/fundamental Higgs

Technical details

- Standard lattice discretization [Dobson et al.'21]
  - Pseudo-Heatbath
  - Flexible code for adjoint/fundamental Higgs
- Variational analysis [Jenny et al.'22]
  - Extreme statistical fluctuations
    - APE smearing
    - Pre-/postseelection of operator basis
  - Fitting uses a predictor for plateaus
    - Substantial improvement compared to cosh

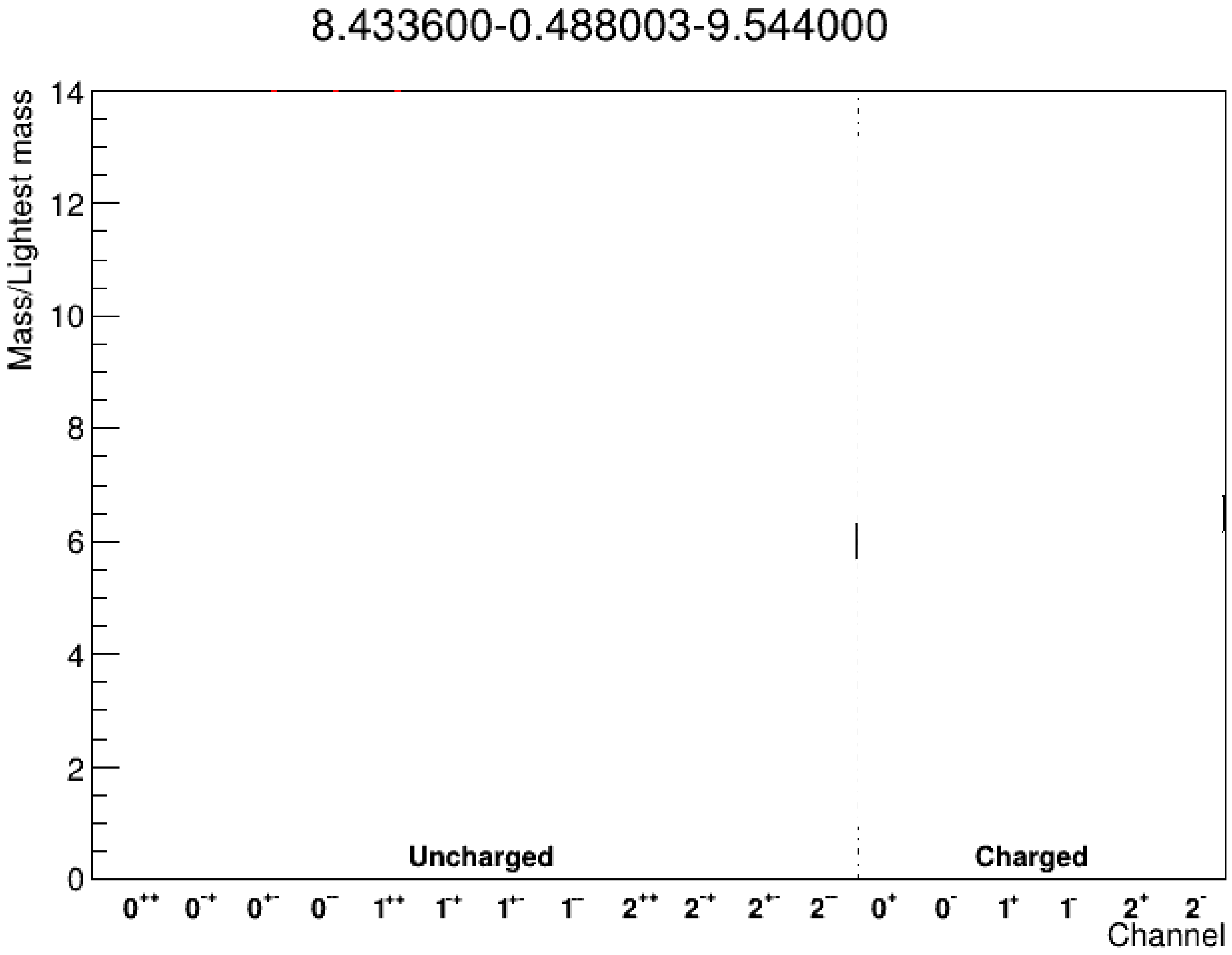
Technical details

- Standard lattice discretization [Dobson et al.’21]
  - Pseudo-Heatbath
  - Flexible code for adjoint/fundamental Higgs
- Variational analysis [Jenny et al.’22]
  - Extreme statistical fluctuations
    - APE smearing
    - Pre-/postseelection of operator basis
  - Fitting uses a predictor for plateaus
    - Substantial improvement compared to cosh
- Comparison to possible scattering states
  - Infinite volume analysis

Spectrum

PRELIMINARY

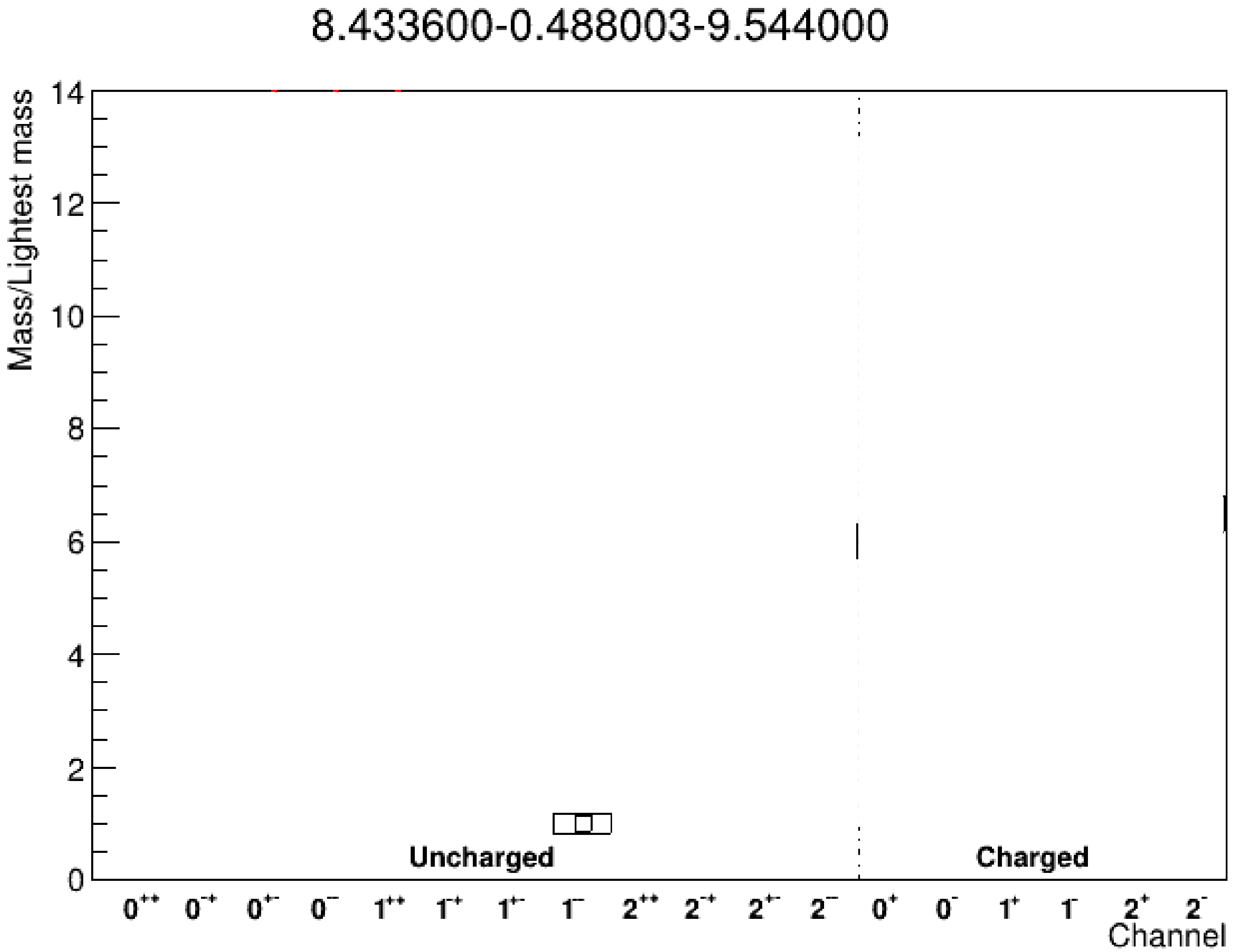
[Dobson et al.’21]



Spectrum

PRELIMINARY

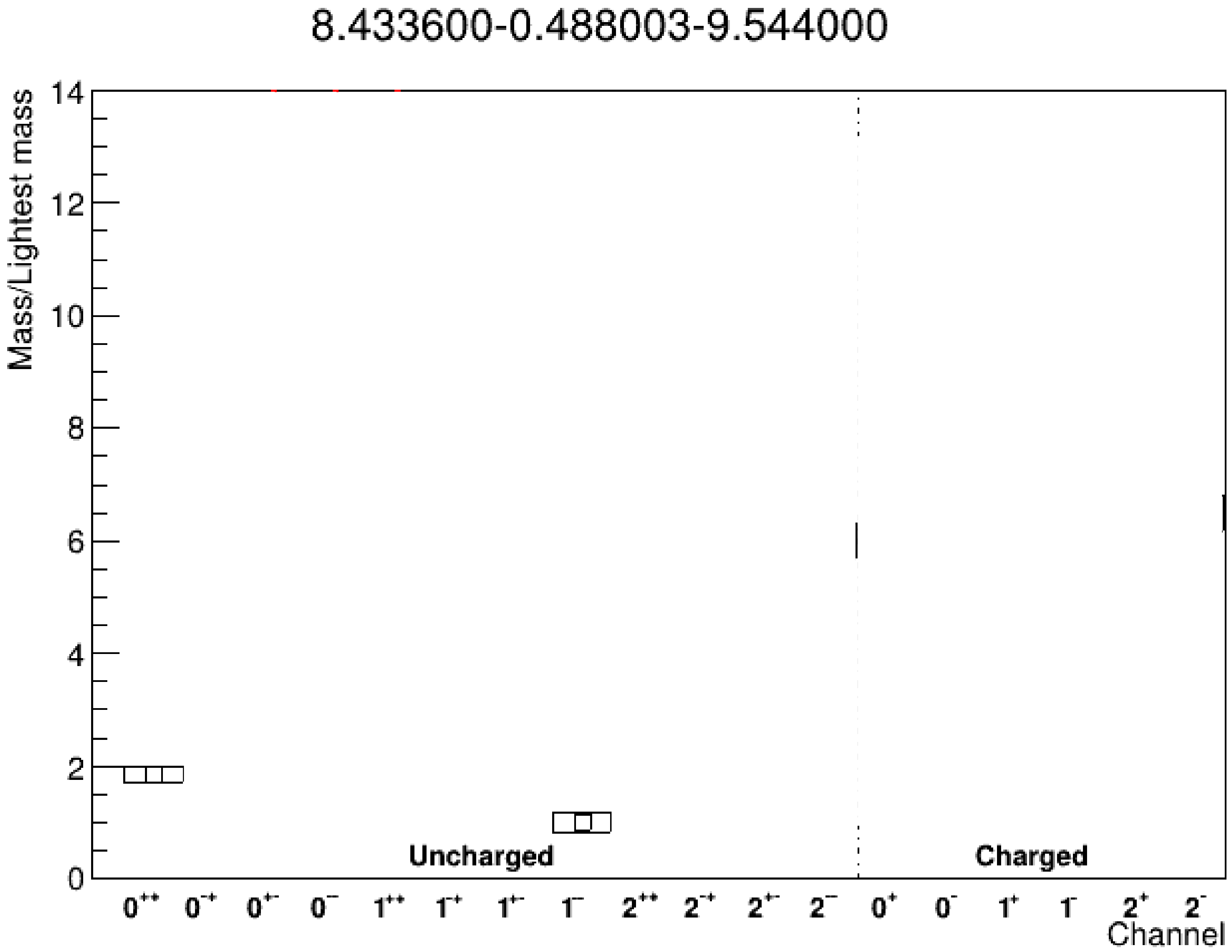
[Dobson et al.'21]



Spectrum

PRELIMINARY

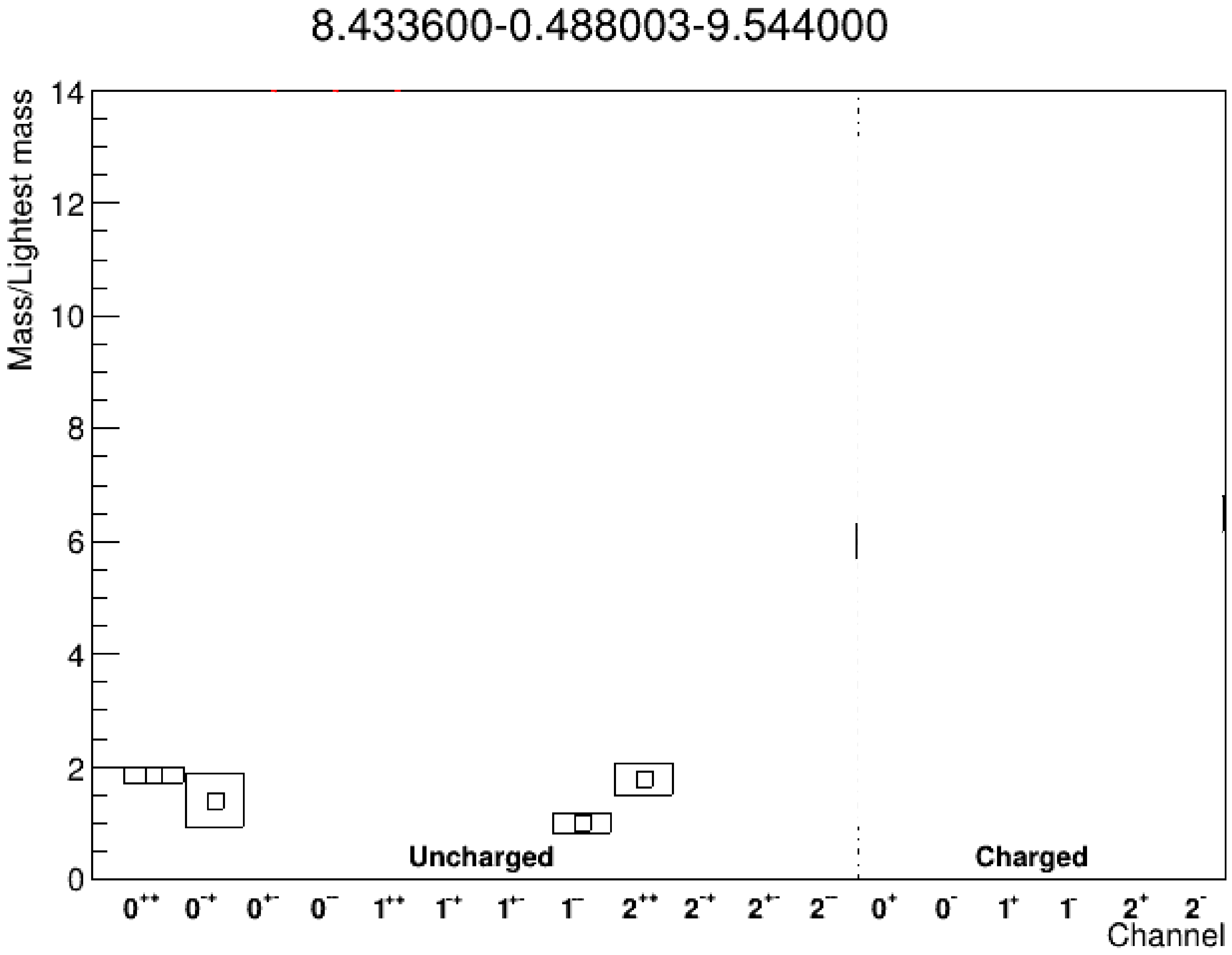
[Dobson et al.’21]



Spectrum

PRELIMINARY

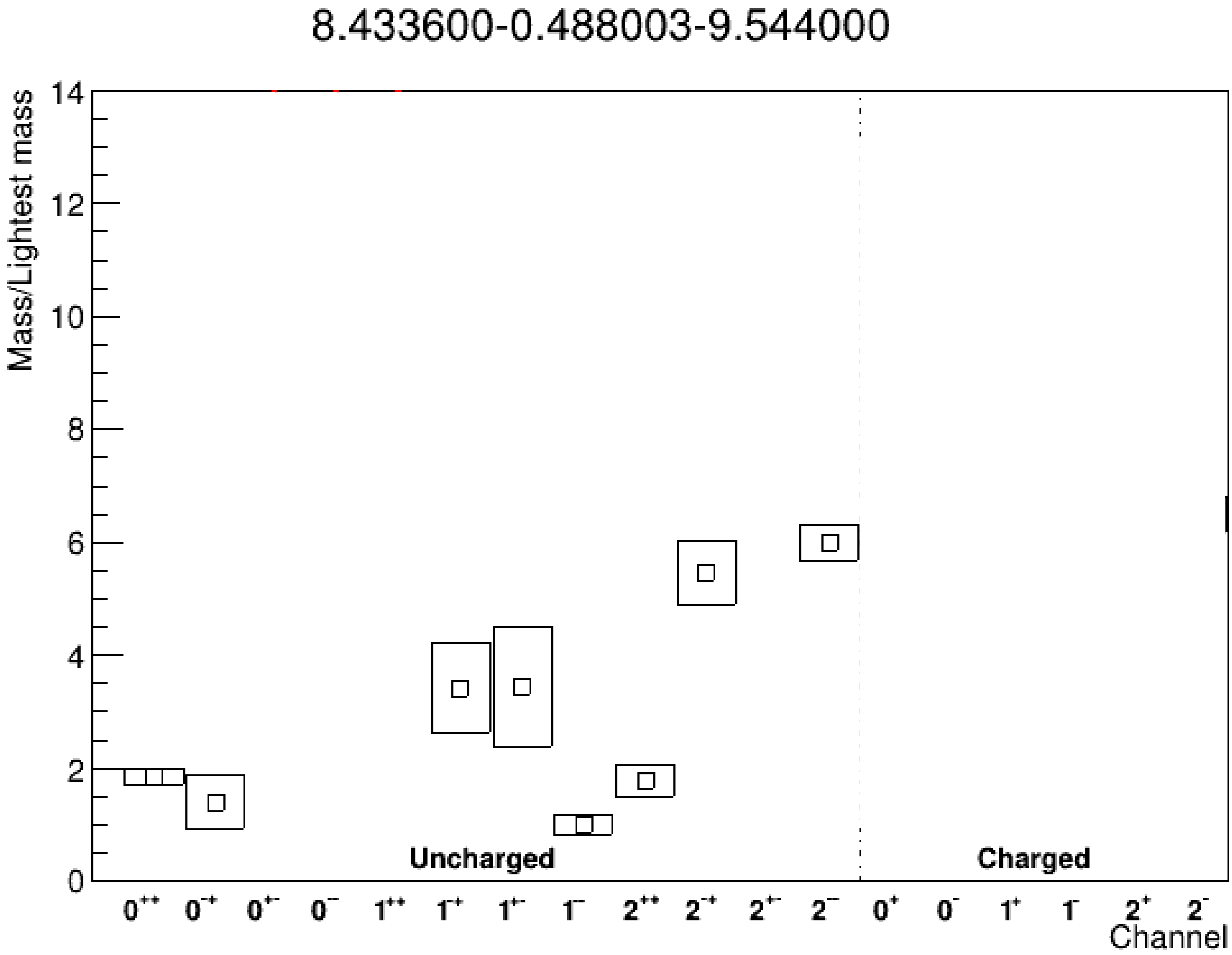
[Dobson et al.’21]



Spectrum

PRELIMINARY

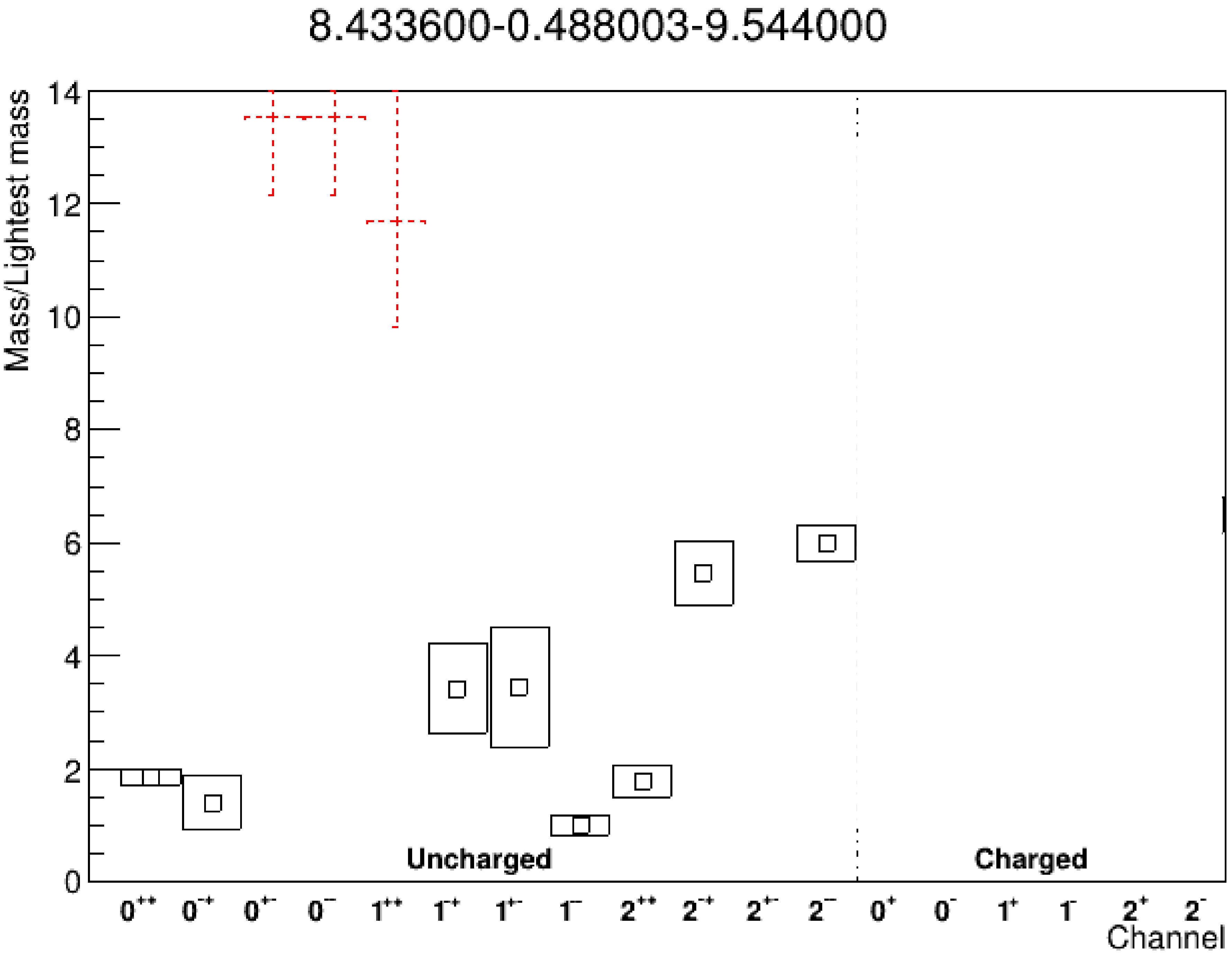
[Dobson et al.’21]



Spectrum

PRELIMINARY

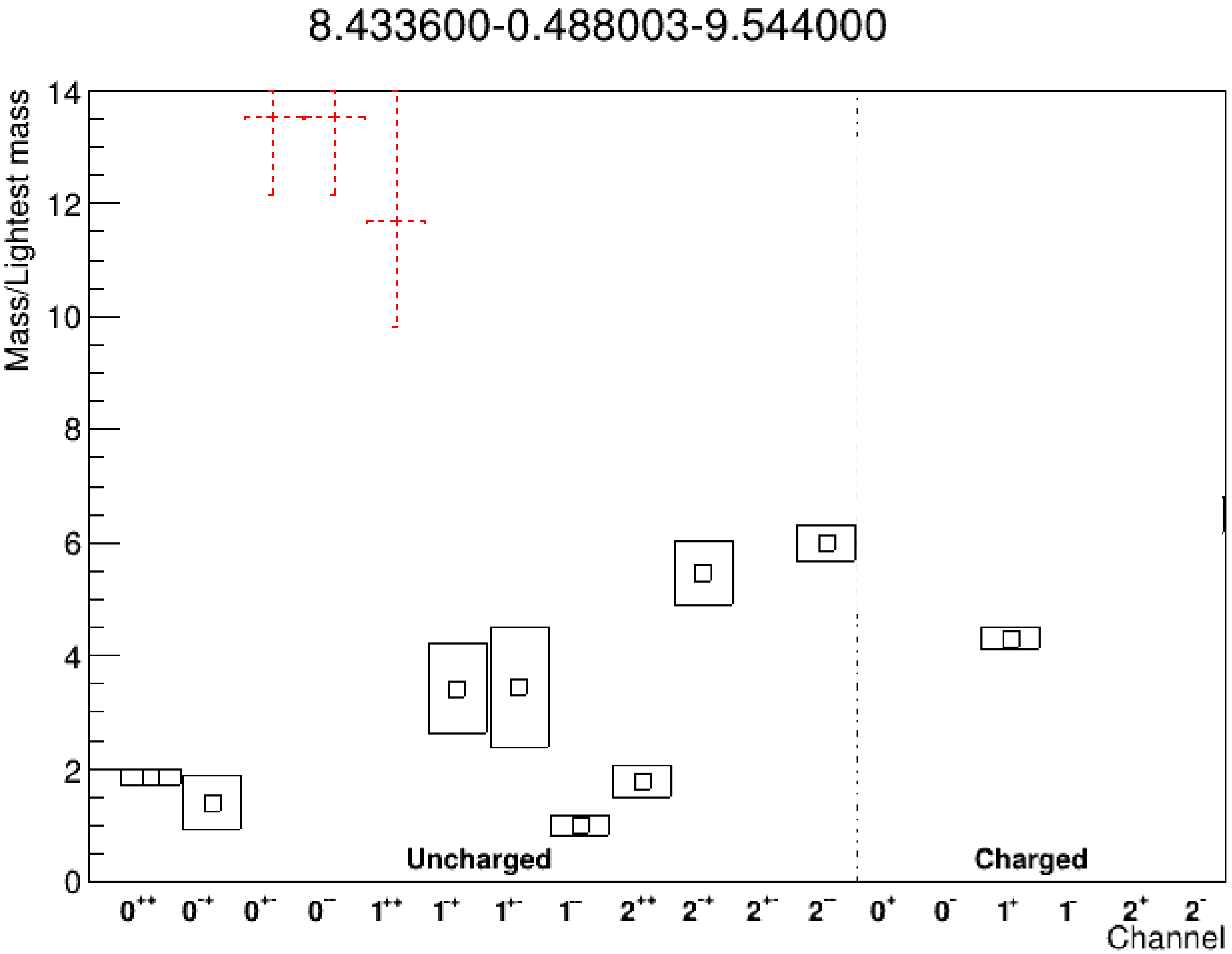
[Dobson et al.'21]



Spectrum

PRELIMINARY

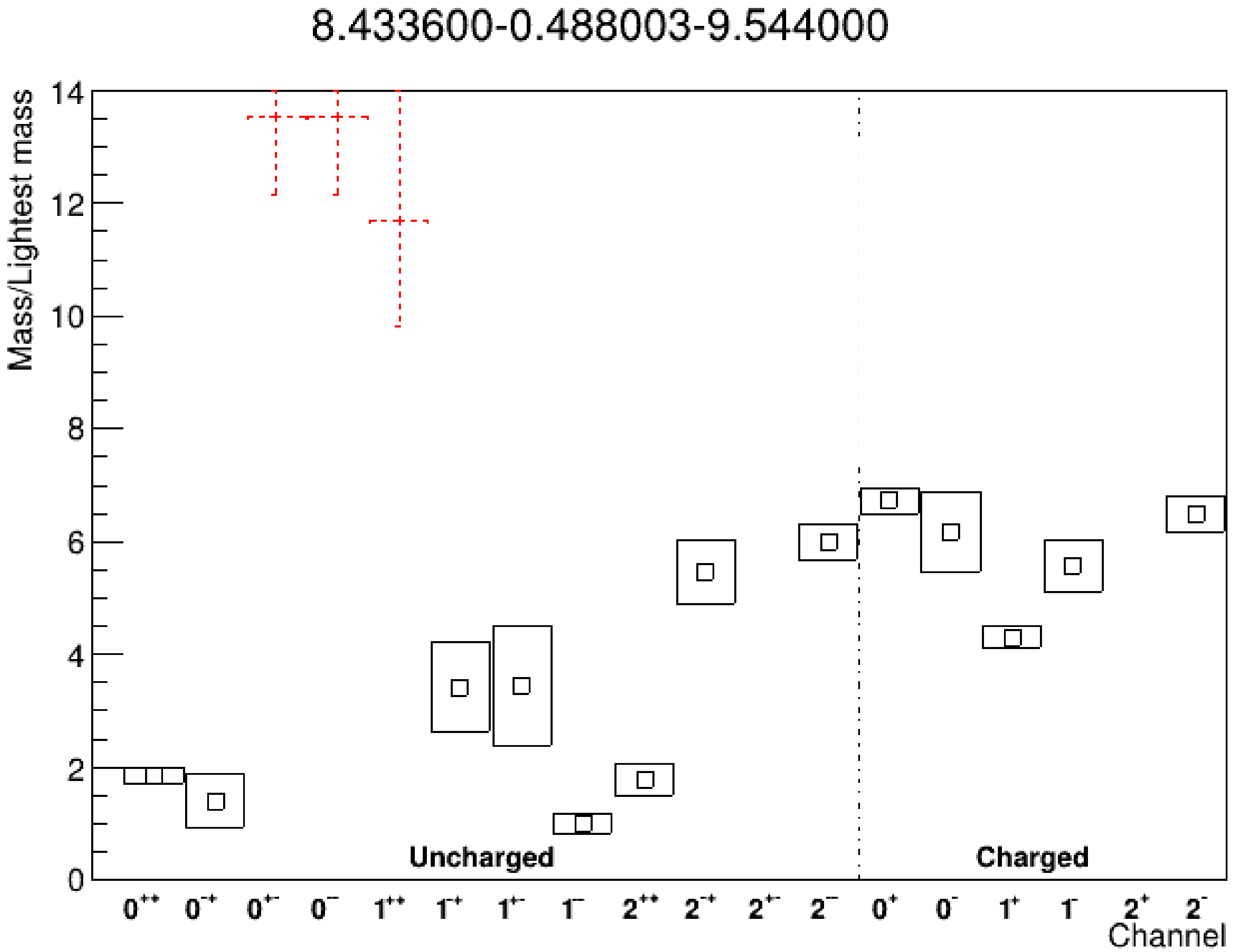
[Dobson et al.’21]



Spectrum

PRELIMINARY

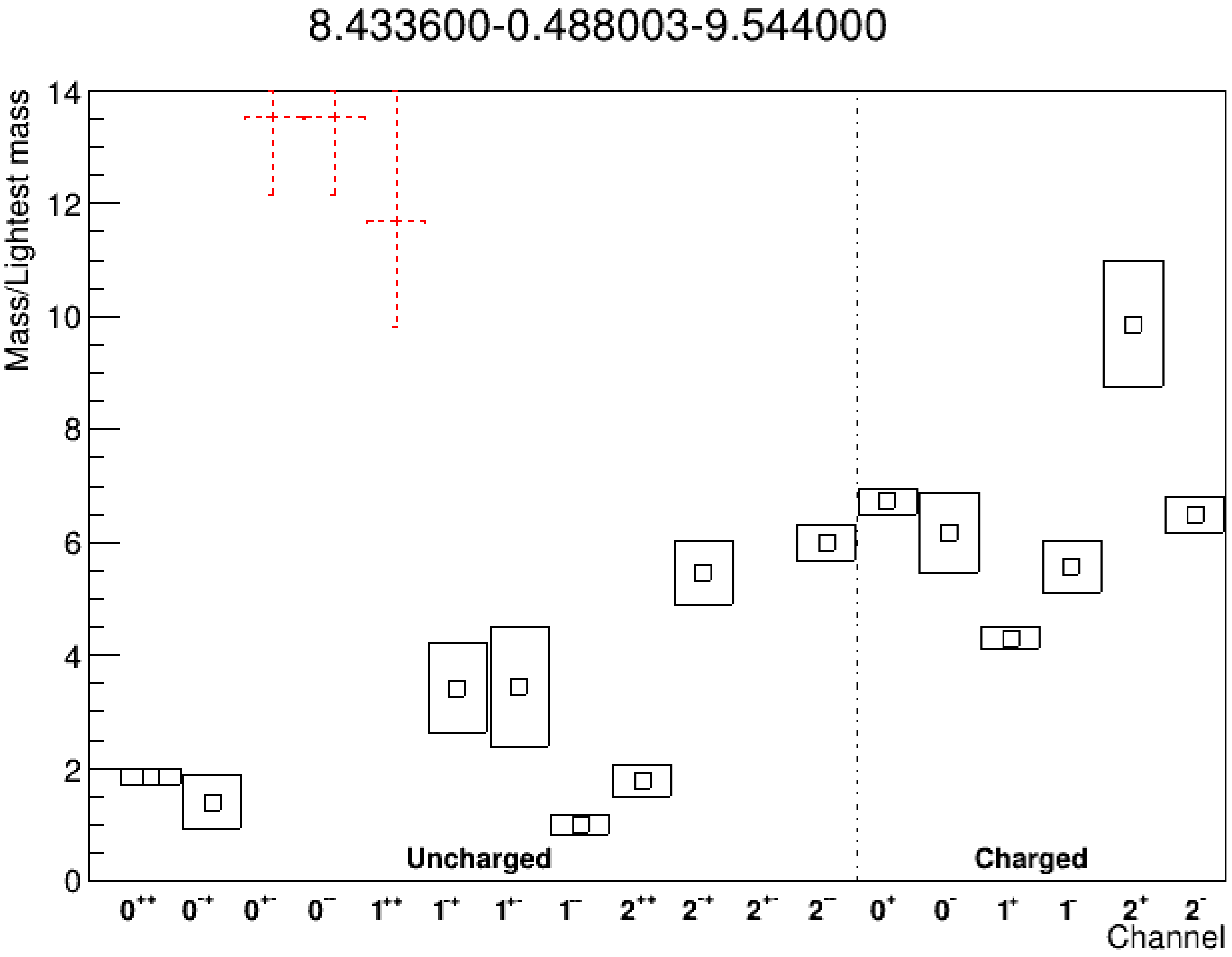
[Dobson et al.'21]



Spectrum

PRELIMINARY

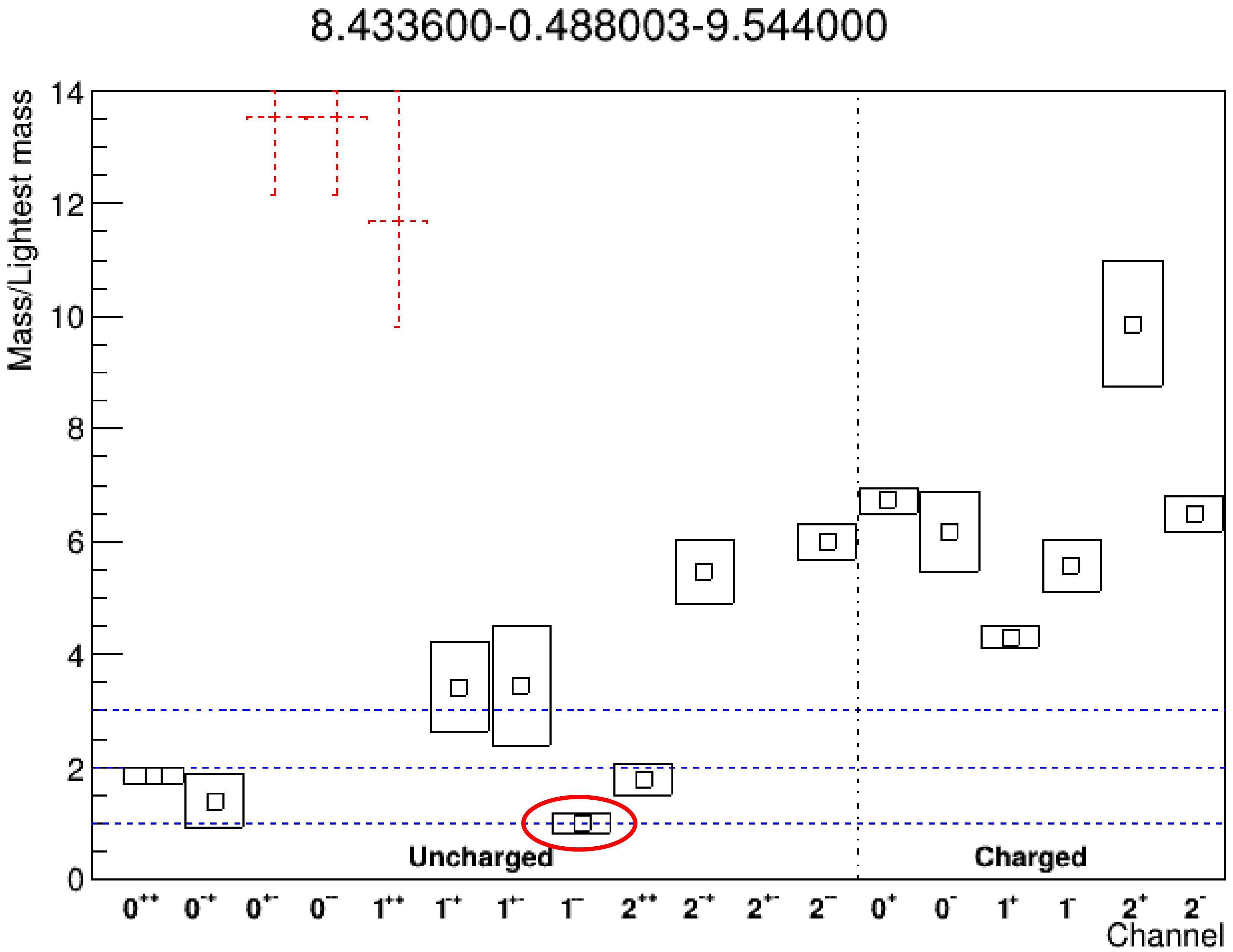
[Dobson et al.’21]



Spectrum

PRELIMINARY

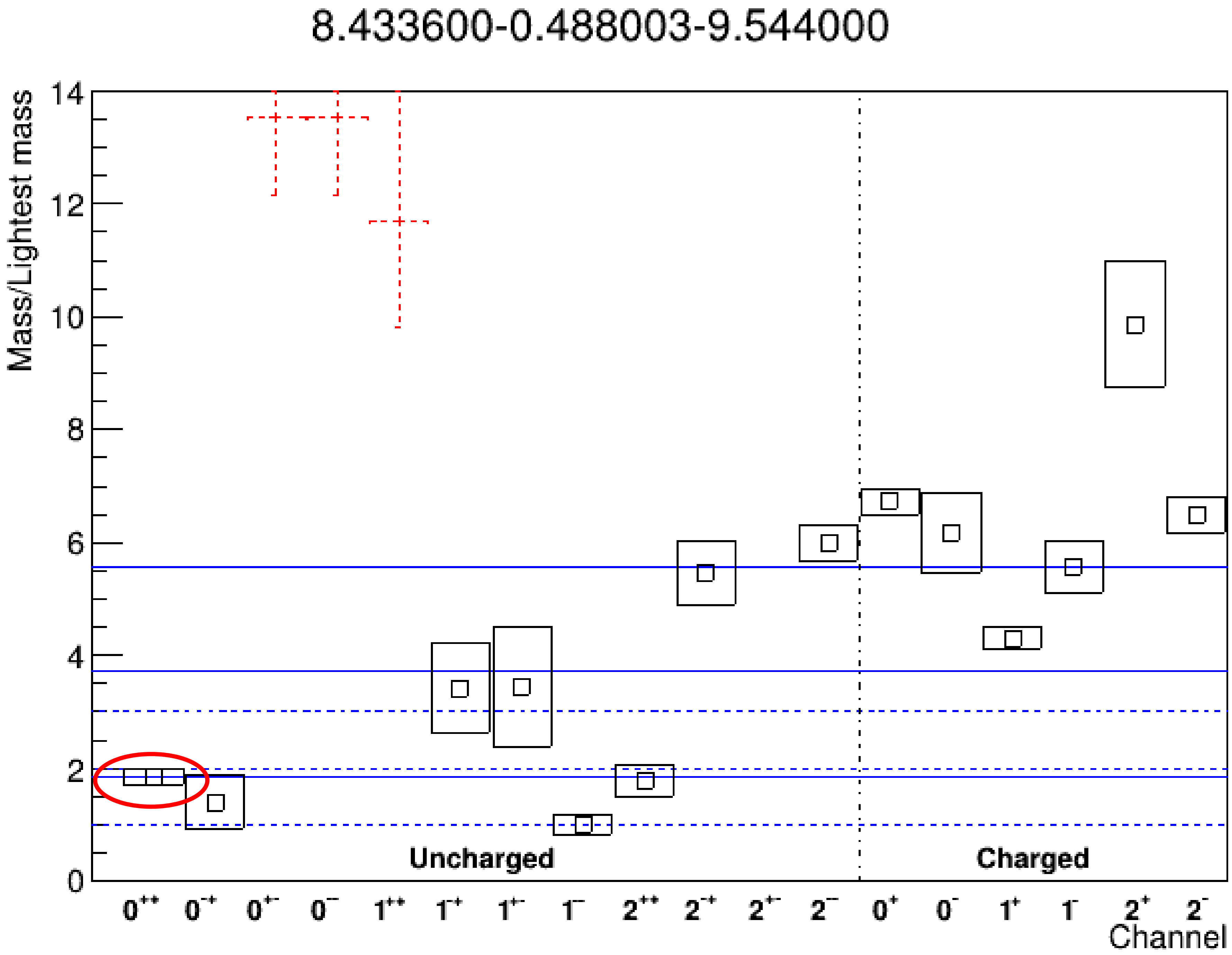
[Dobson et al.’21]



Spectrum

PRELIMINARY

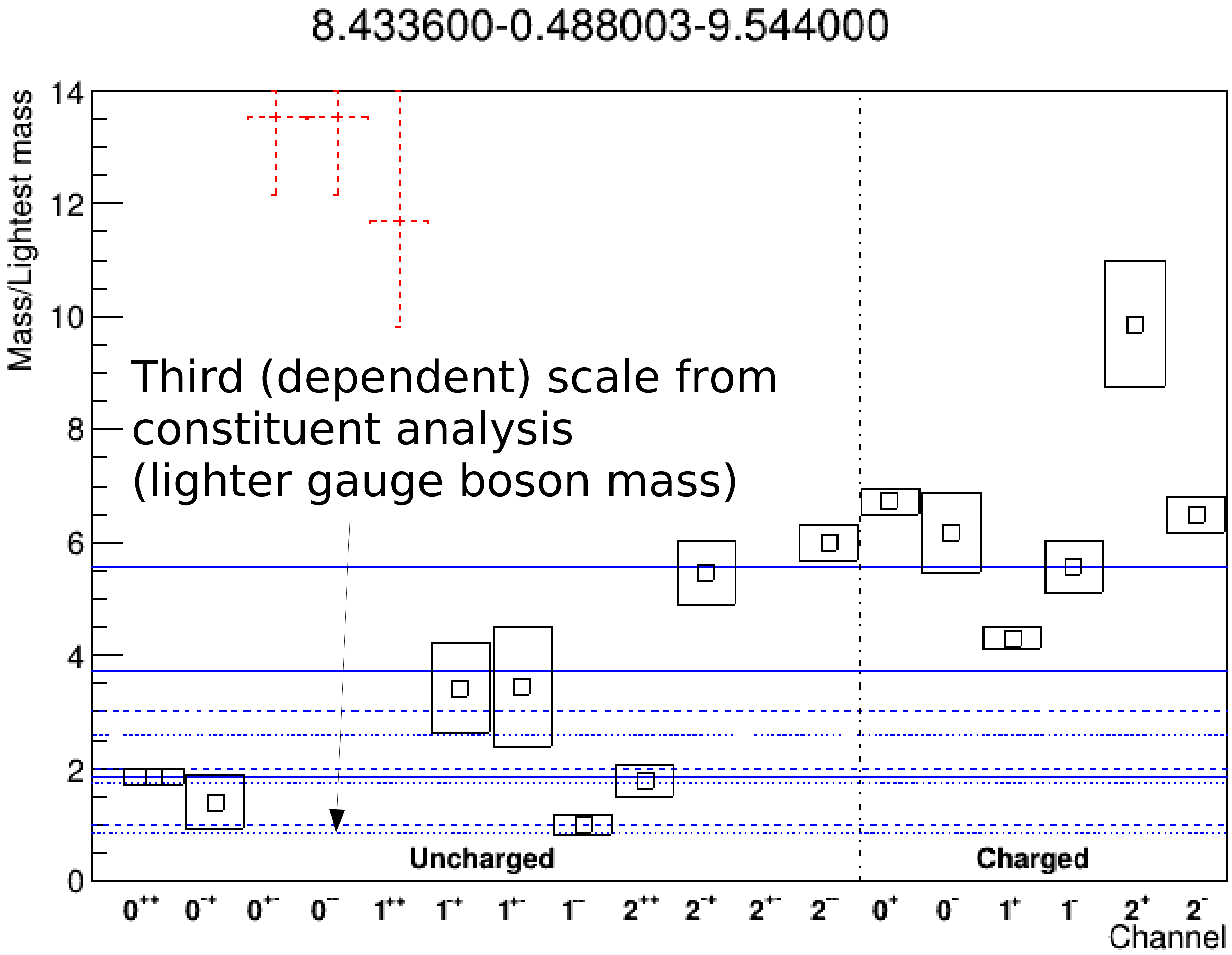
[Dobson et al.'21]



Spectrum

PRELIMINARY

[Dobson et al.’21]

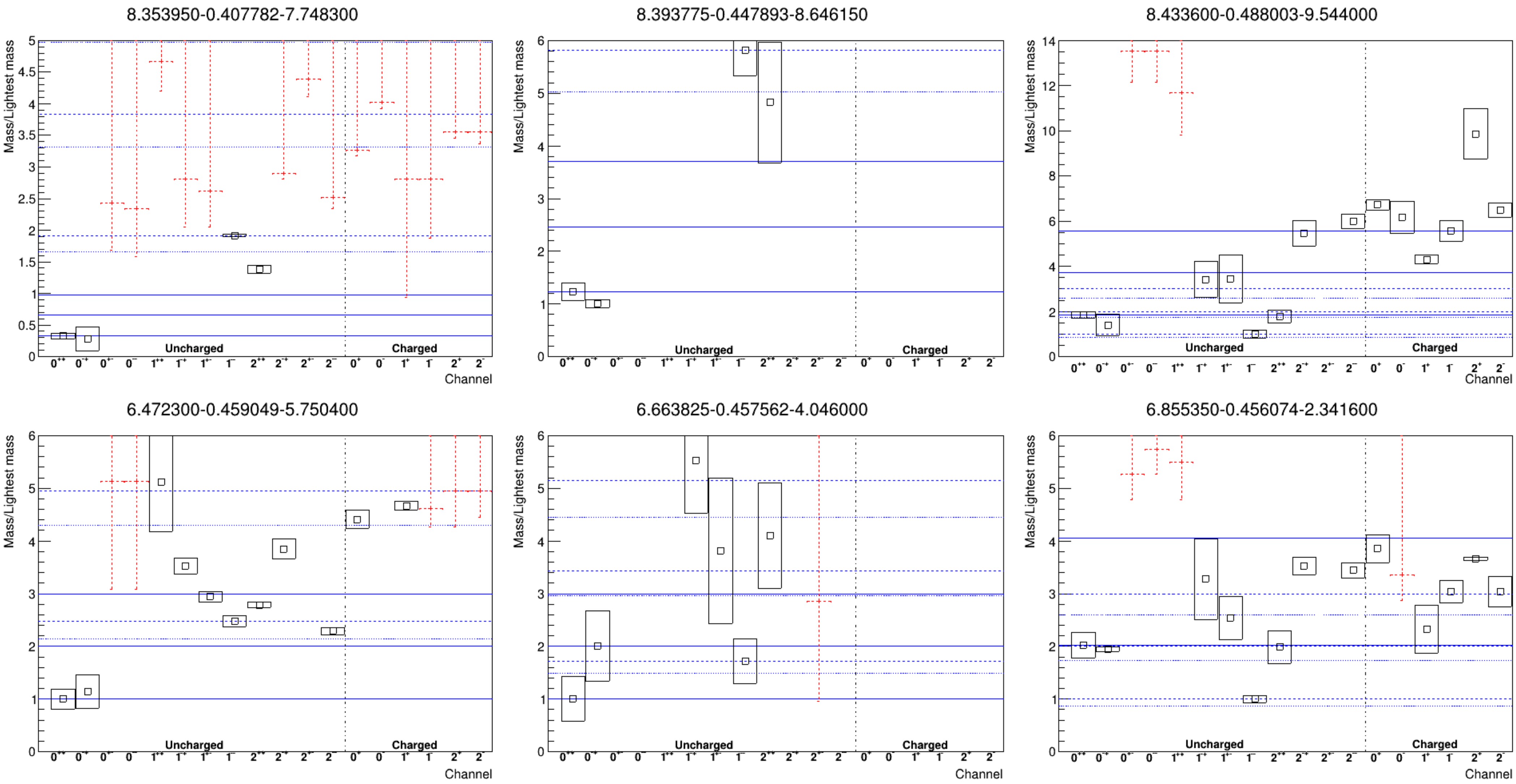


Third (dependent) scale from constituent analysis (lighter gauge boson mass)

Spectrum

PRELIMINARY

[Dobson et al.'21]

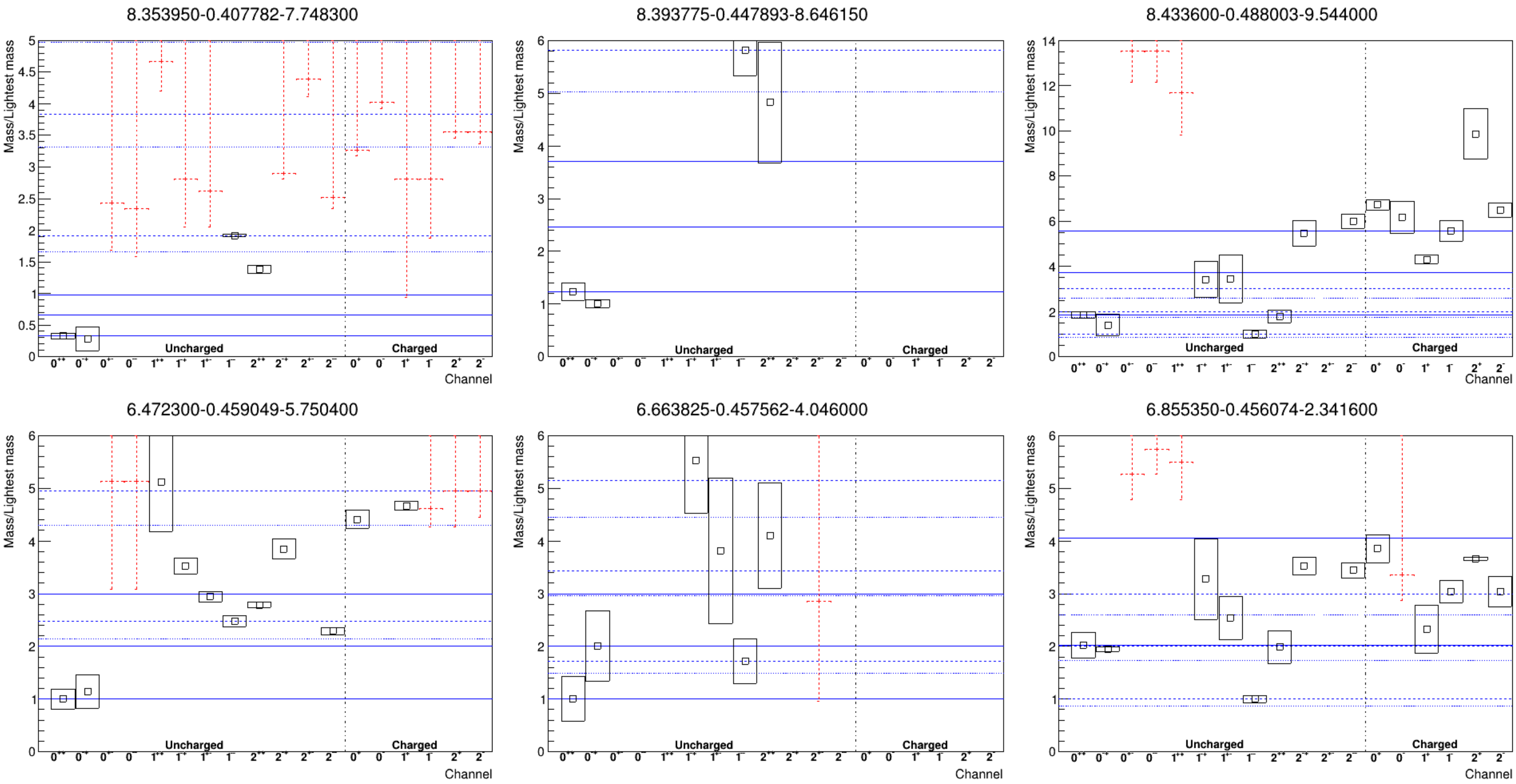


Spectrum

PRELIMINARY

[Dobson et al.’21]

QCD-like → BEH-like

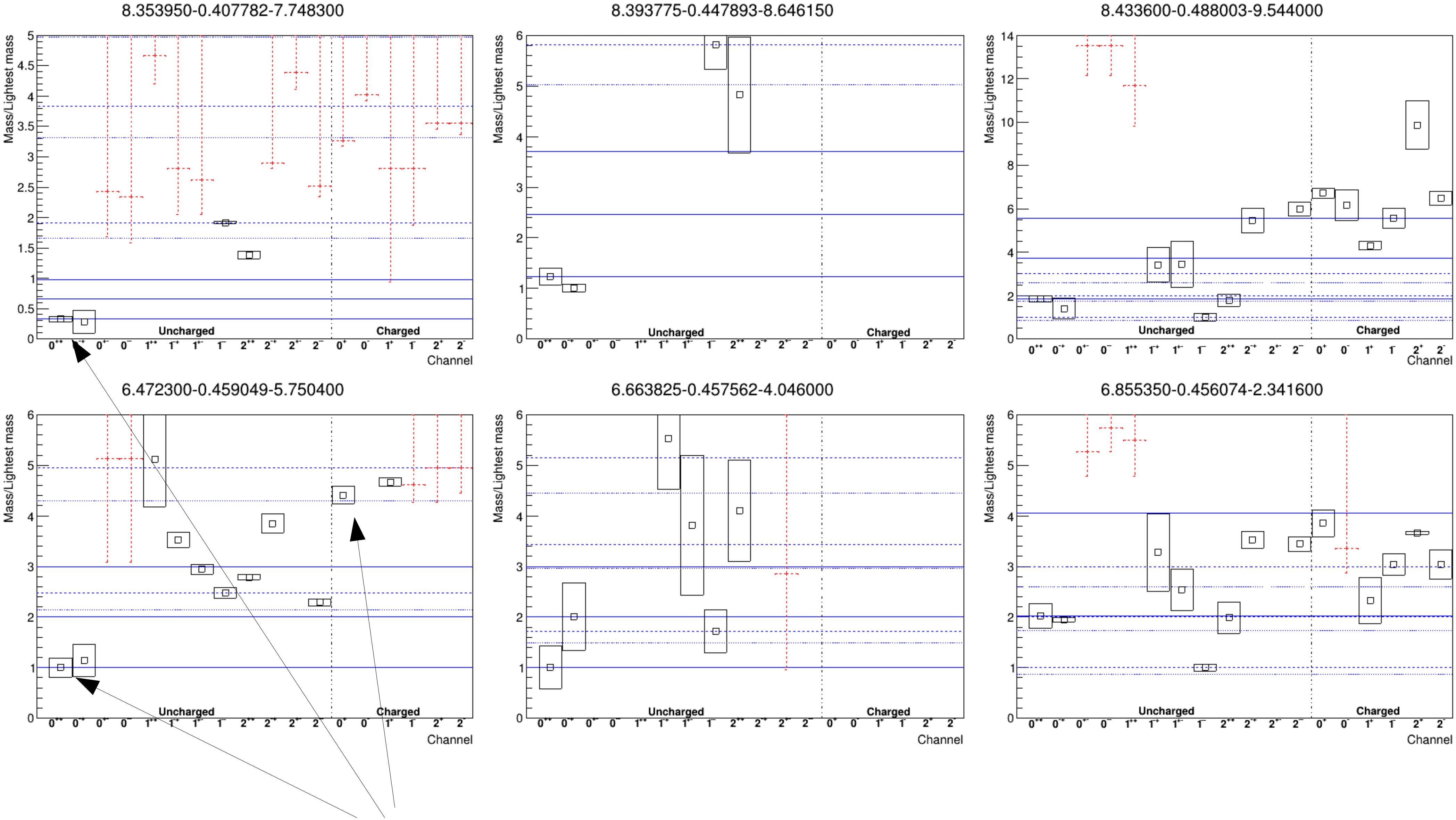


Spectrum

PRELIMINARY

[Dobson et al.’21]

QCD-like → BEH-like



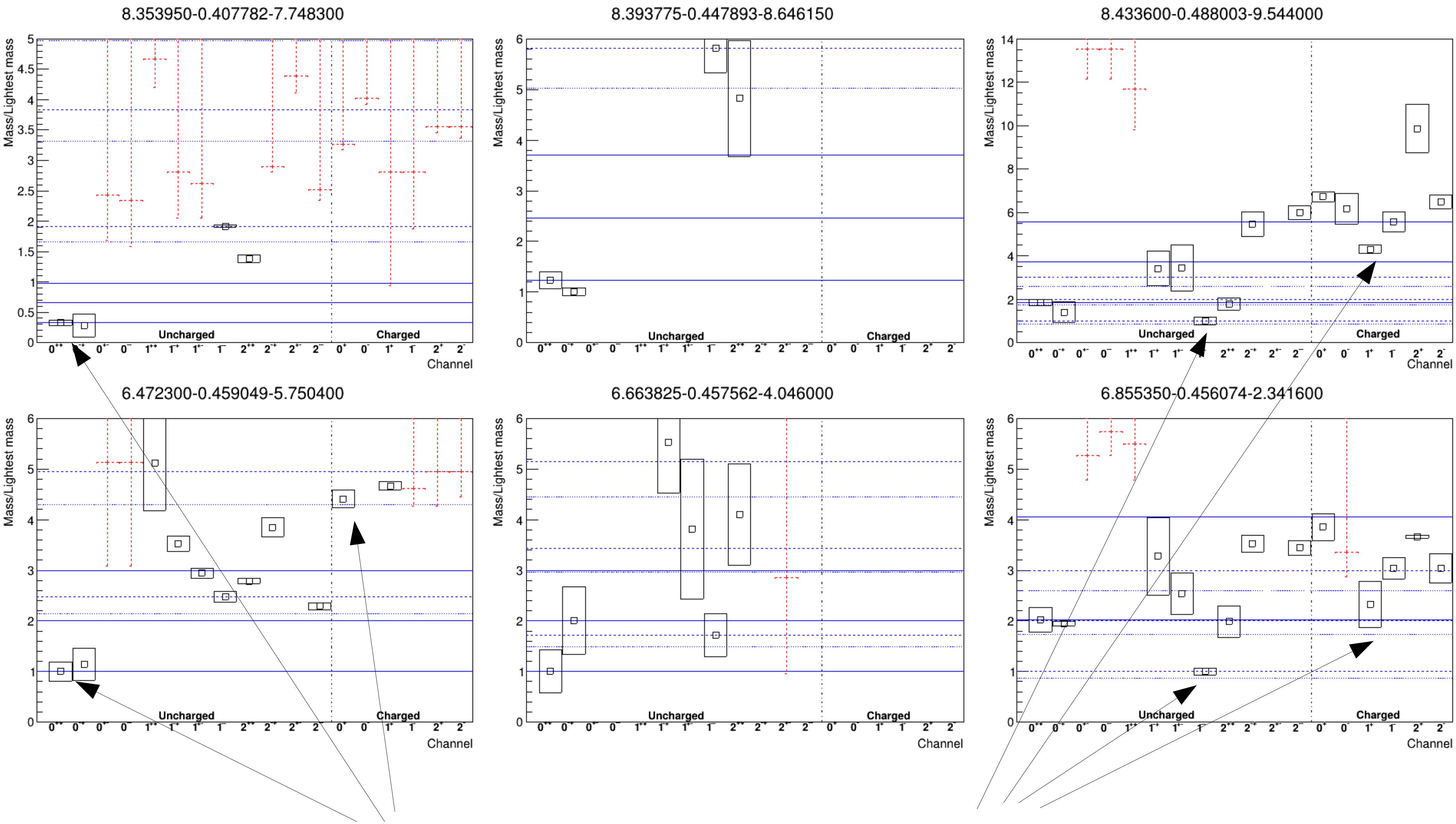
Change from scalars

Spectrum

PRELIMINARY

[Dobson et al.'21]

QCD-like → BEH-like



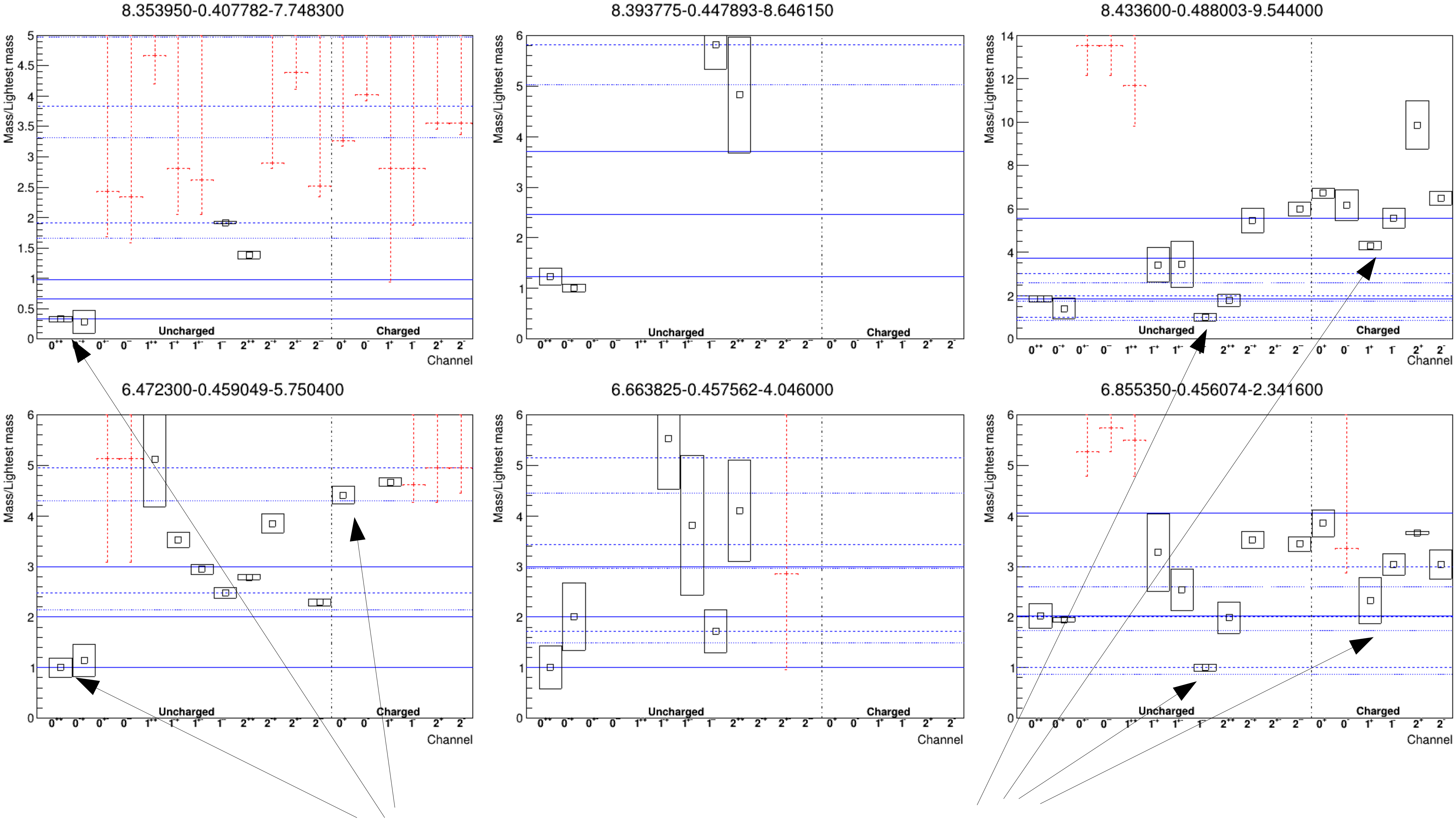
Change from scalars to vectors

Spectrum

PRELIMINARY

[Dobson et al.'21]

QCD-like → BEH-like



Change from scalars to vectors – like in SU(2)

What about the photon?

What about the photon?

- Spectrum gapped
  - OK for strong interactions (confinement)

What about the photon?

- Spectrum gapped
    - OK for strong interactions (confinement)
    - But there should be a photon

What about the photon?

[Maas et al.’17]

- Spectrum gapped
  - OK for strong interactions (confinement)
  - But there should be a photon
    - Photon also member of non-Abelian multiplet
    - Same arguments – need a gauge-invariant state

What about the photon?

[Maas et al.’17]

- Spectrum gapped
  - OK for strong interactions (confinement)
  - But there should be a photon
    - Photon also member of non-Abelian multiplet
    - Same arguments – need a gauge-invariant state
- Massless composite vector boson!
  - Made from massive objects

What about the photon?

[Maas et al.’17]

- Spectrum gapped
  - OK for strong interactions (confinement)
  - But there should be a photon
    - Photon also member of non-Abelian multiplet
    - Same arguments – need a gauge-invariant state
- Massless composite vector boson!
  - Made from massive objects
    - Without associated symmetry
    - Not a Goldstone of dilation symmetry breaking, like SM photon

A massless composite vector

- Appears impossible with a fundamental Higgs [Maas et al.’17, Sondenheimer’20]

A massless composite vector

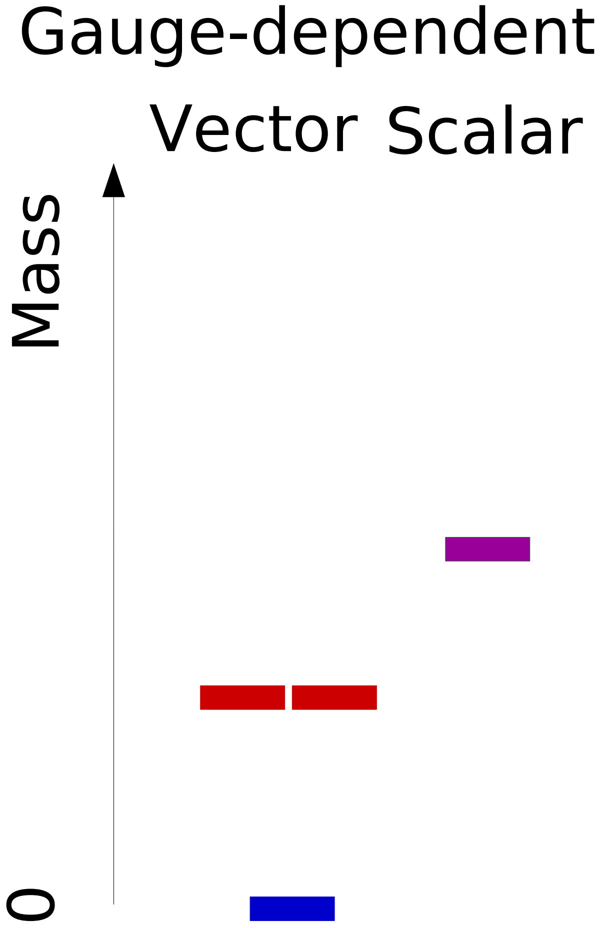
- Appears impossible with a fundamental Higgs [Maas et al.’17, Sondenheimer’20]
- Possible with an adjoint Higgs

Simpelst case

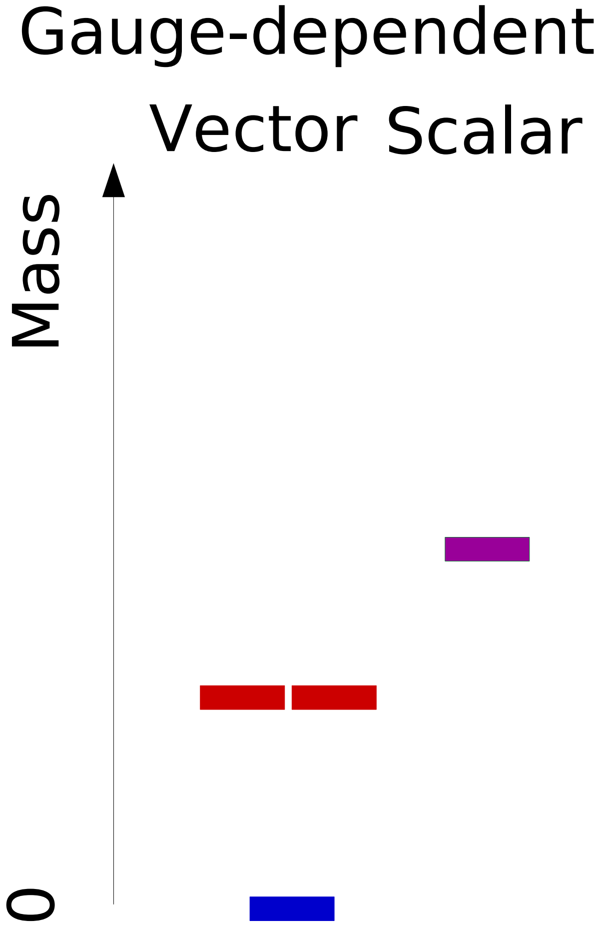
- SU(2) with an adjoint Higgs
    - Breaks by BEH to a U(1)

Simpelst case

- SU(2) with an adjoint Higgs
    - Breaks by BEH to a U(1)
    - Massless gauge boson
    - Two massive gauge boson
    - One massive scalar

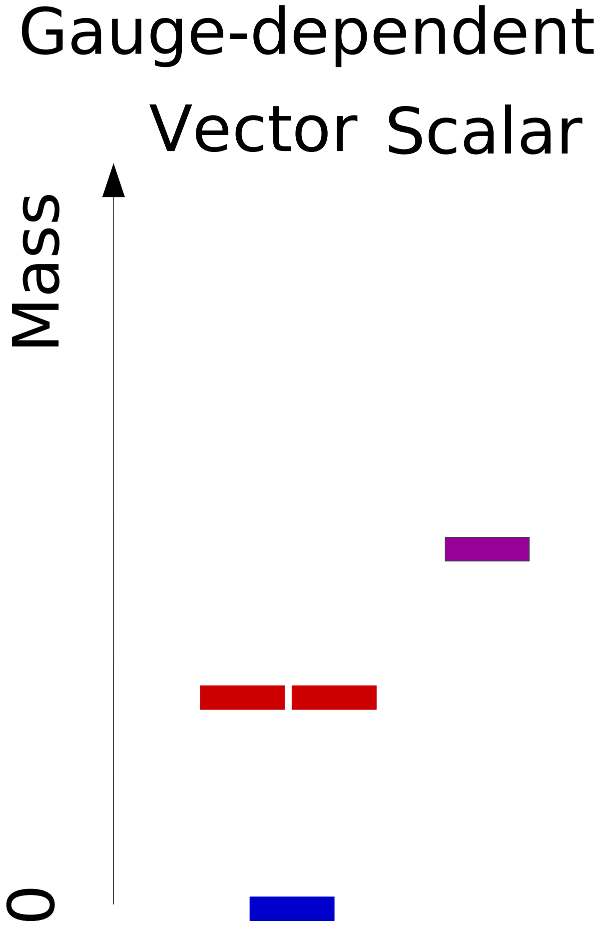


Simpelst case



- SU(2) with an adjoint Higgs
  - Breaks by BEH to a U(1)
  - Massless gauge boson
  - Two massive gauge boson
  - One massive scalar
- Should happen at arbitrary weak coupling
  - Check by calculating running coupling
    - MiniMOM scheme

Simpelst case



- SU(2) with an adjoint Higgs
  - Breaks by BEH to a U(1)
  - Massless gauge boson
  - Two massive gauge boson
  - One massive scalar
- Should happen at arbitrary weak coupling
  - Check by calculating running coupling
    - MiniMOM scheme
- Massless state: Autocorrelation bad!

A massless composite vector

- Appears impossible with a fundamental Higgs [Maas et al.'17, Sondenheimer'20]
- Possible with an adjoint Higgs
  - Simplest operator

$$\frac{p_\nu}{p^2} tr\left(\phi^a \tau^a F_{\mu\nu}\right)$$

A massless composite vector

- Appears impossible with a fundamental Higgs [Maas et al.’17, Sondenheimer’20]
- Possible with an adjoint Higgs
    - Simplest operator

$$\frac{p_\nu}{p^2} tr\left(\phi^a \tau^a F_{\mu\nu}\right)$$

    - Creates only a state in a moving frame
        - Indications existed early on [Lee et al.’85]
    - Seen also in 3d [Kajantie et al.’98]

A massless composite vector

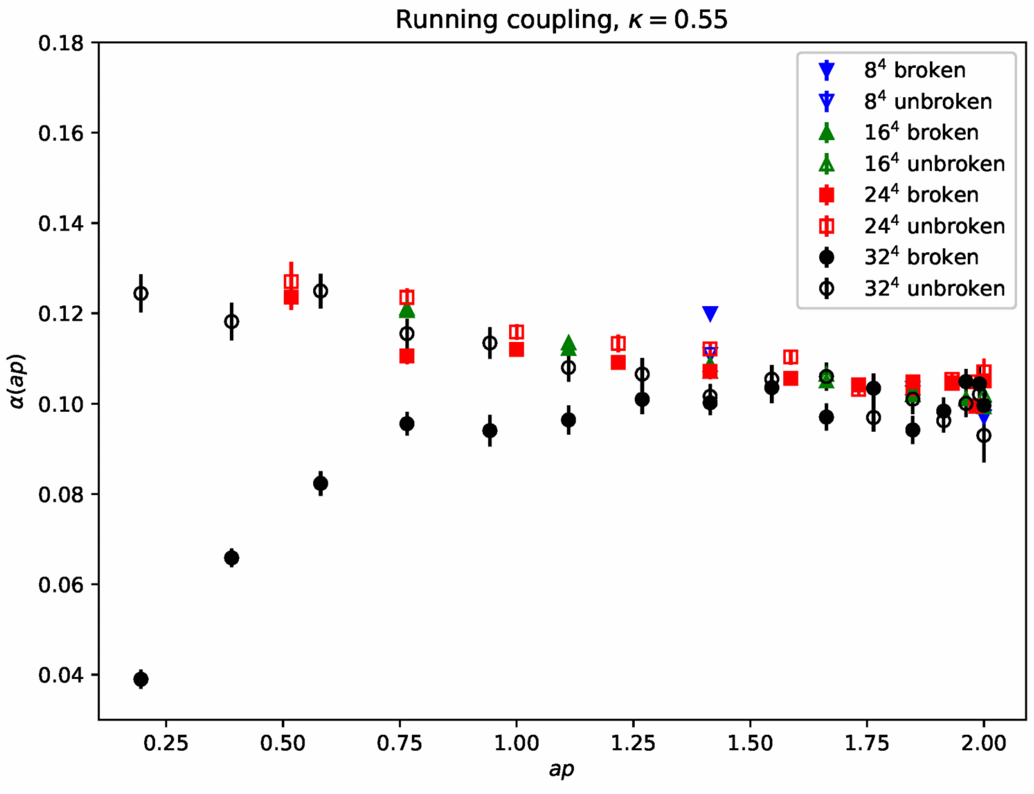
- Appears impossible with a fundamental Higgs [Maas et al.'17, Sondenheimer'20]
- Possible with an adjoint Higgs
  - Simplest operator

$$\frac{p_{\nu}}{p^2} tr\left(\phi^a \tau^a F_{\mu\nu}\right) = v\left(A_{\mu}^3\right)^T + O\left(v^0\right)$$

  - Creates only a state in a moving frame
    - Indications existed early on [Lee et al.'85]
  - Seen also in 3d [Kajantie et al.'98]
- FMS: Massless state [Maas et al.'17]

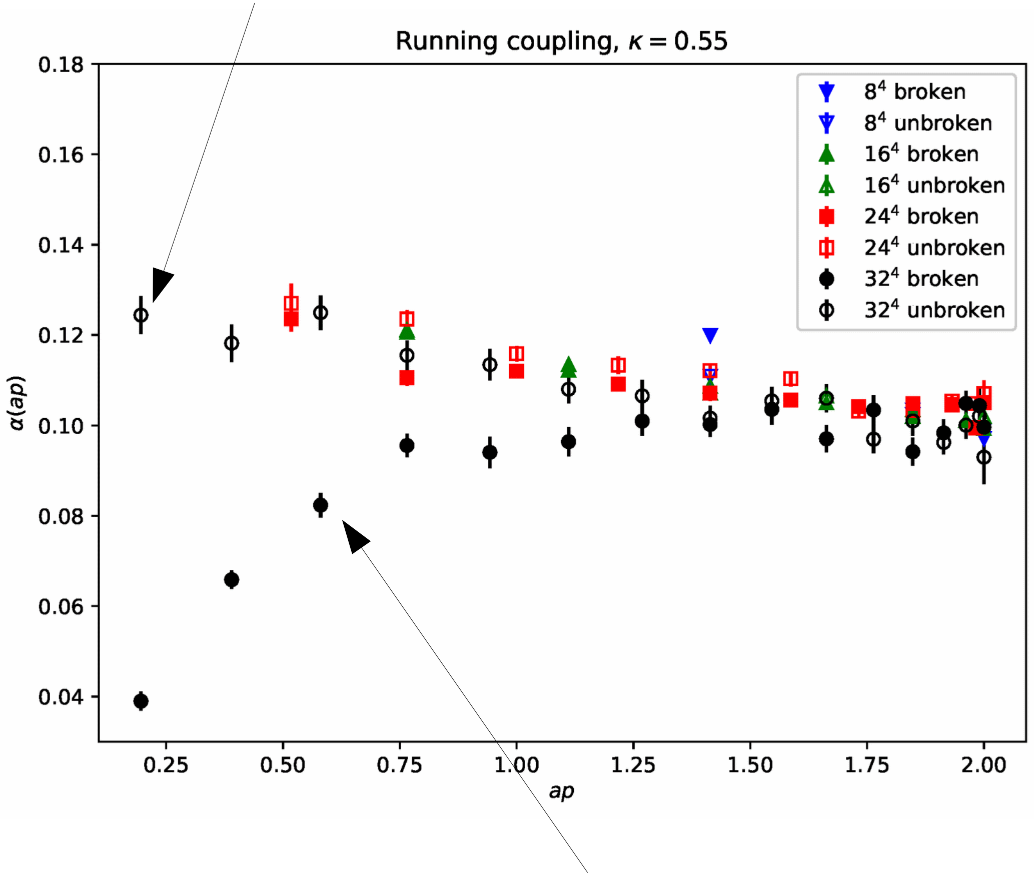
Result: Gauge-dependent

[Afferrante et al.'20]



Result: Gauge-dependent

[Afferrante et al.'20]



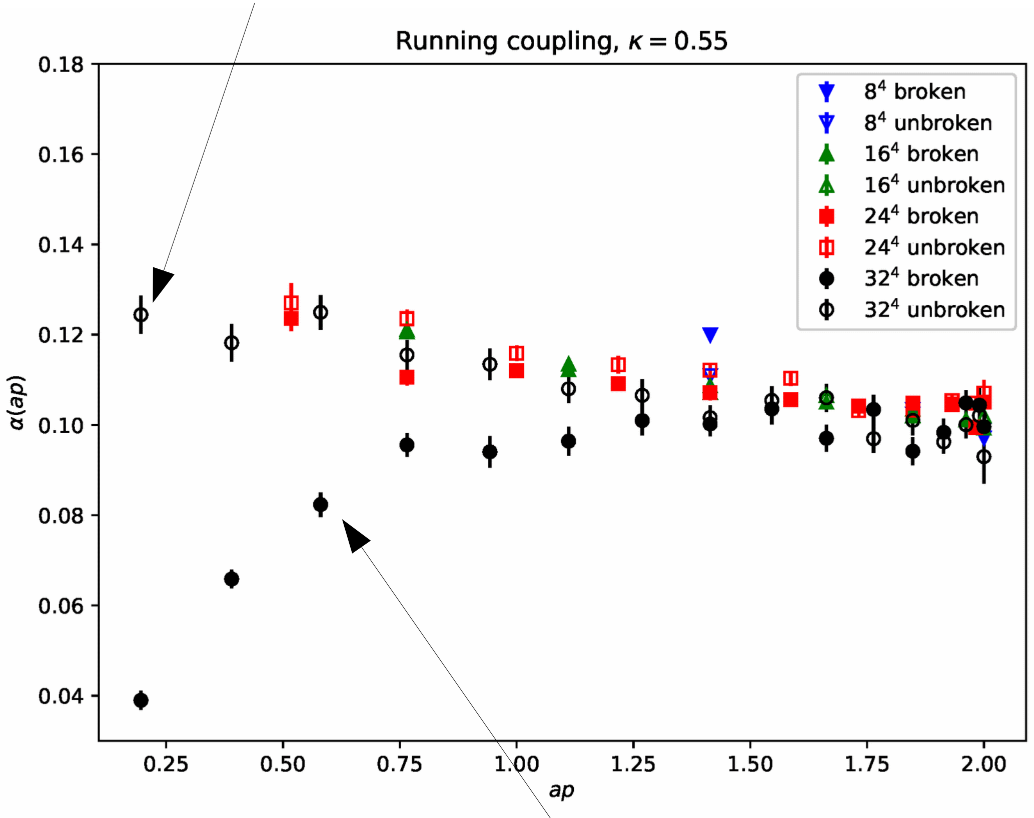
Gauge coupling to
massless gauge bosons
- still small

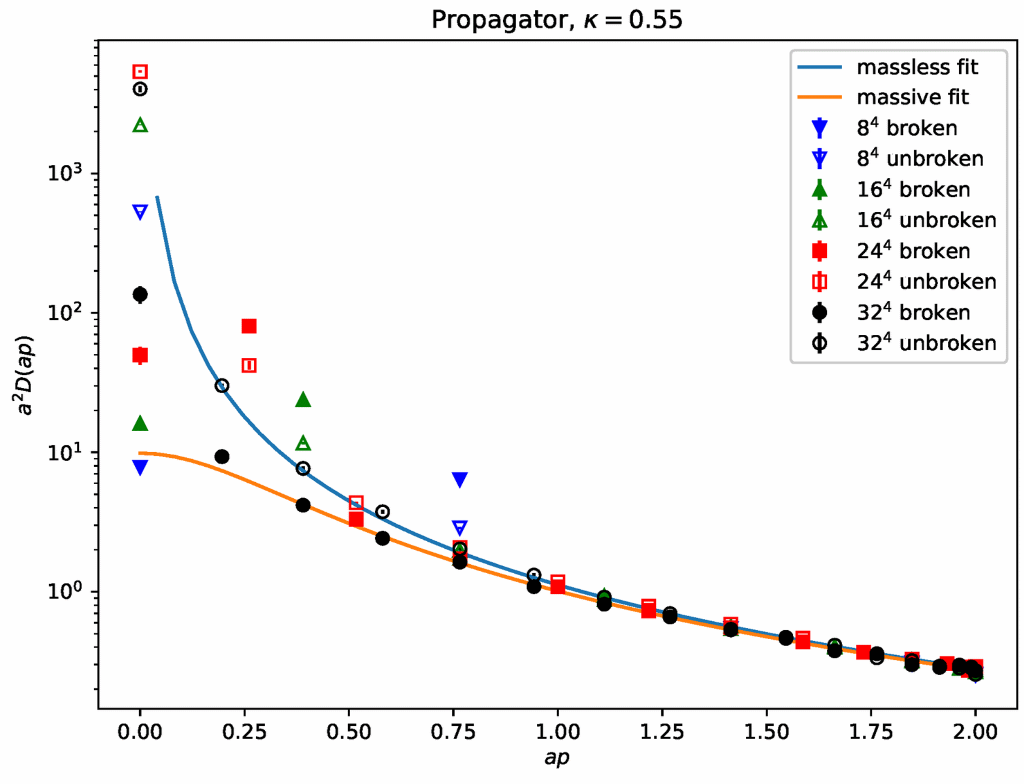
Gauge coupling between
massive gauge bosons

Result: Gauge-dependent

[Afferrante et al.’20]

Gauge coupling to
massless gauge bosons
- still small





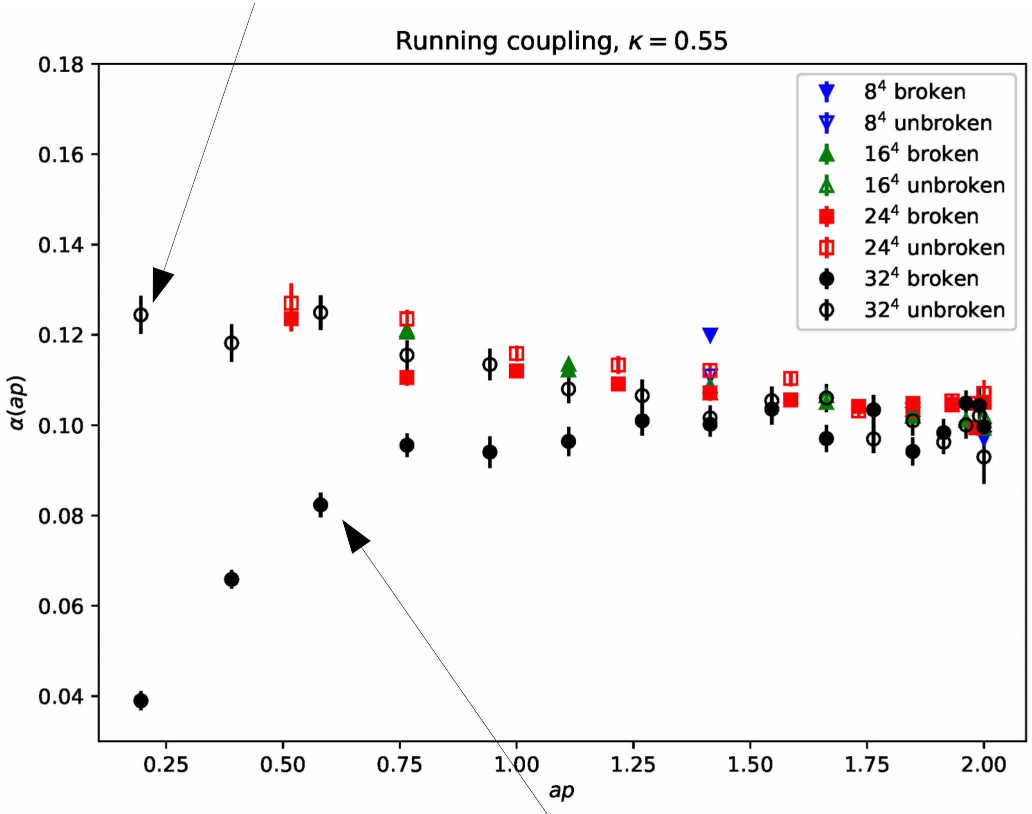
Gauge coupling between
massive gauge bosons

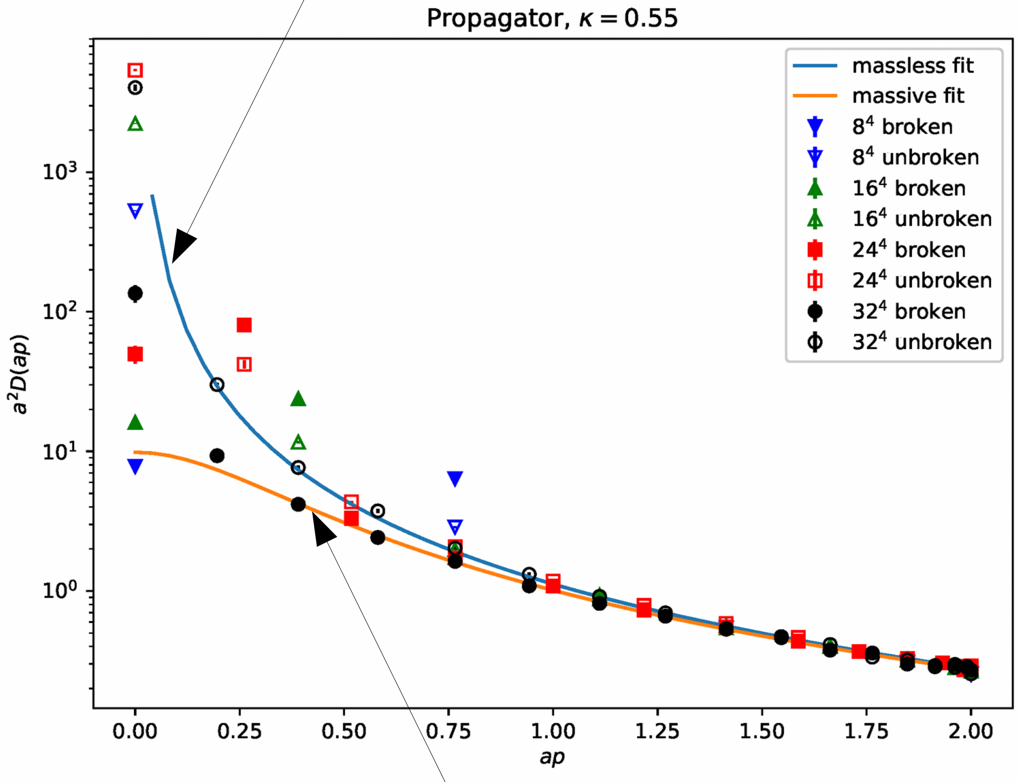
Result: Gauge-dependent

[Afferrante et al.'20]

Gauge coupling to
massless gauge bosons
- still small

Massless behavior



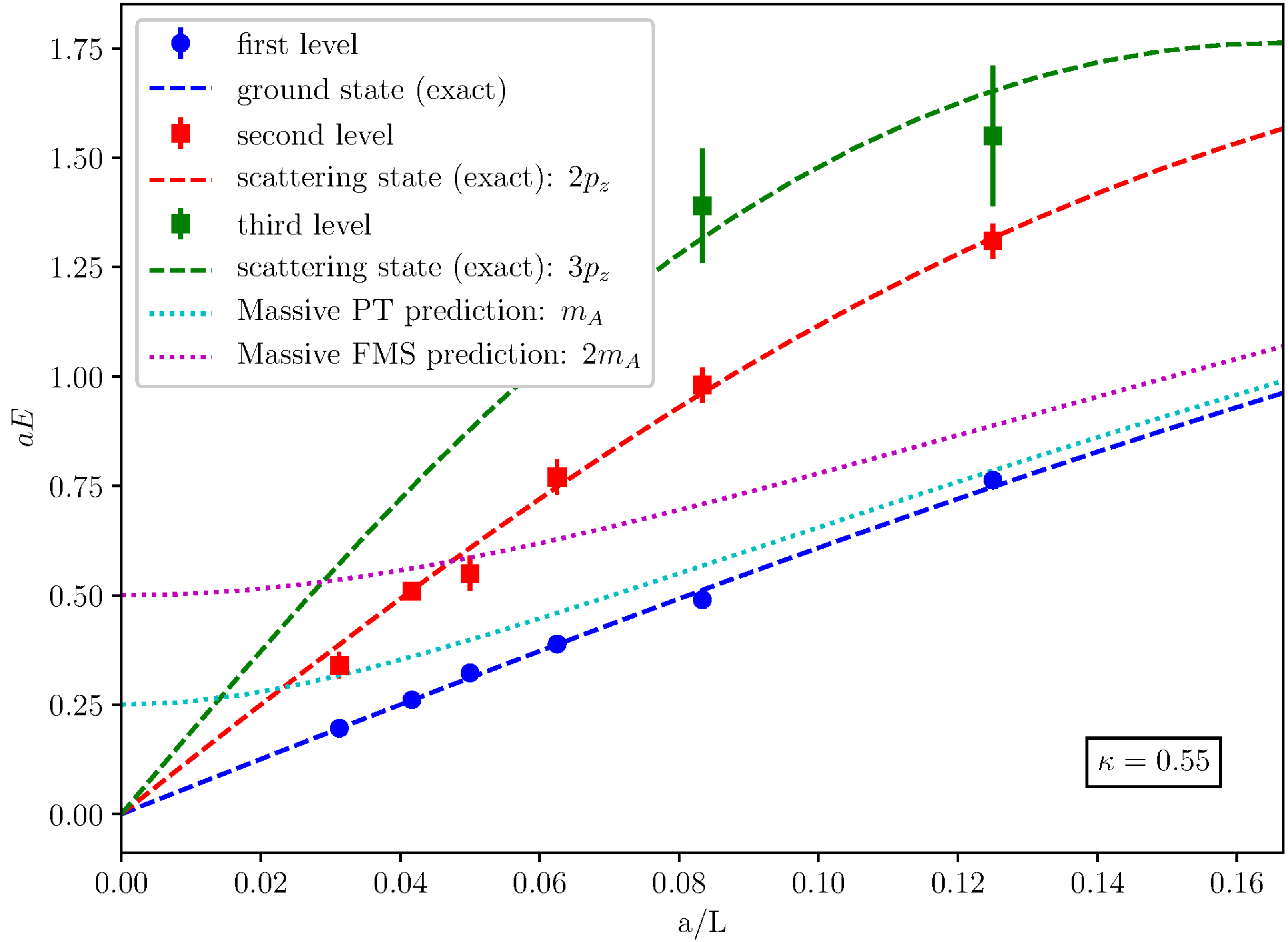


Gauge coupling between
massive gauge bosons

Massive behavior

Result

[Afferrante et al.'20]



No massive states seen yet – but no suitable methods available

What happens in a ‘real’ GUT?

What happens in a ‘real’ GUT?

- Multiple breaking patterns possible
  - Simplest is SU(5)

What happens in a ‘real’ GUT?

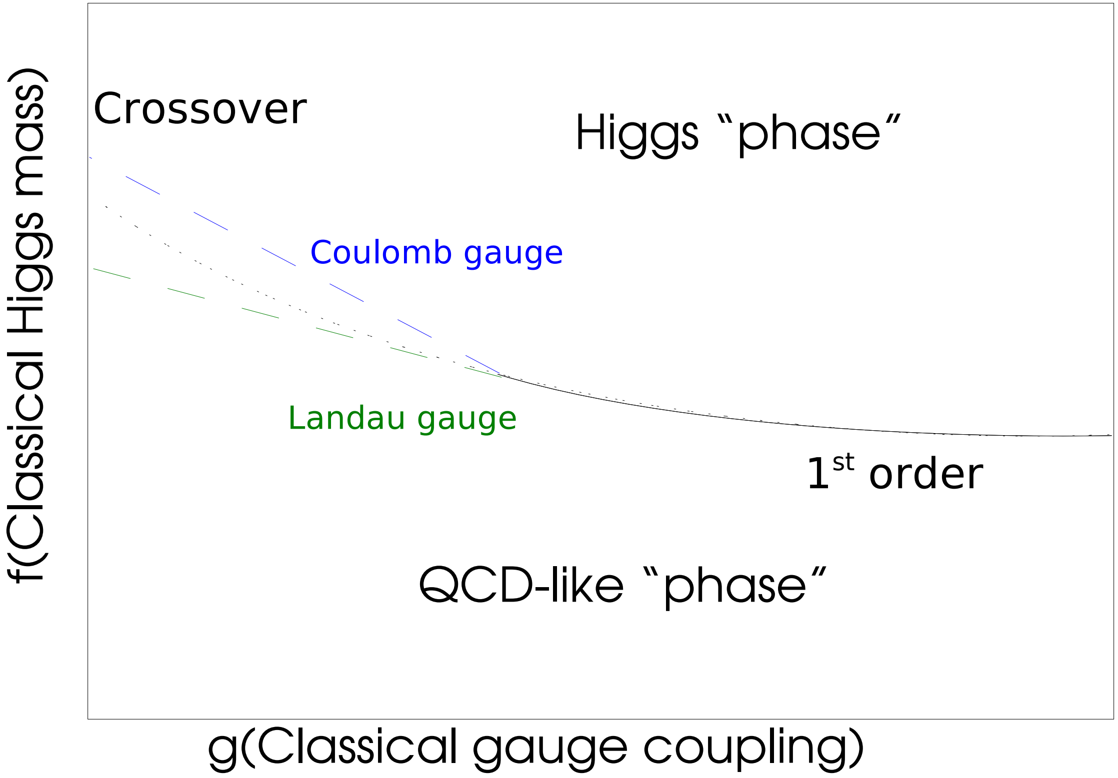
[Fradkin & Shenker'78
Seiler & Osterwalder’77
Caudy & Greensite'07]

- Multiple breaking patterns possible
  - Simplest is SU(5)
  - Potentially gauge-dependent!

What happens in a ‘real’ GUT?

[Fradkin & Shenker'78
Seiler & Osterwalder’77
Caudy & Greensite'07]

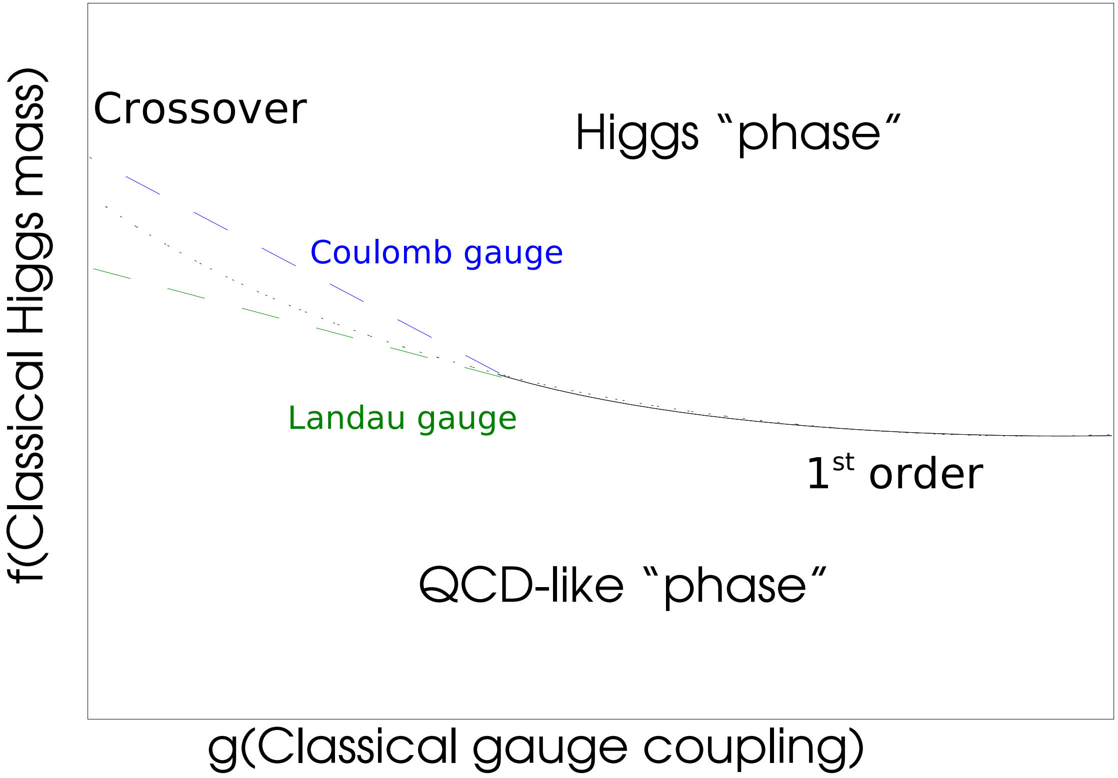
- Multiple breaking patterns possible
  - Simplest is SU(5)
  - Potentially gauge-dependent!
    - SU(2): no physical distinction even between QCD region and BEH region
      - BEH effect switches on as a function of gauge



What happens in a ‘real’ GUT?

[Fradkin & Shenker'78
Seiler & Osterwalder’77
Caudy & Greensite'07]

- Multiple breaking patterns possible
  - Simplest is SU(5)
  - Potentially gauge-dependent!
    - SU(2): no physical distinction even between QCD region and BEH region
      - BEH effect switches on as a function of gauge



- What is the physical realization?

Toy model

[Maas et al.’17]

- SU(3)+adjoint Higgs

Toy model

[Maas et al.’17]

- SU(3)+adjoint Higgs
  - Two Possible breaking patterns
    - SU(2)xU(1) and U(1)xU(1)

Toy model

[Maas et al.'17]

- SU(3)+adjoint Higgs
  - Two Possible breaking patterns
    - SU(2)xU(1) and U(1)xU(1)
  - Z_2 global symmetry

Toy model

[Maas et al.’17]

- SU(3)+adjoint Higgs
  - Two Possible breaking patterns
    - SU(2)xU(1) and U(1)xU(1)
  - Z_2 global symmetry
- Simplest theory with multiple breaking patterns

Toy model

[Maas et al.’17]

- SU(3)+adjoint Higgs
  - Two Possible breaking patterns
    - SU(2)xU(1) and U(1)xU(1)
  - Z_2 global symmetry
- Simplest theory with multiple breaking patterns
- How to even physically characterize patterns?

Toy model

[Maas et al.’17]

- SU(3)+adjoint Higgs
  - Two Possible breaking patterns
    - SU(2)xU(1) and U(1)xU(1)
  - Z_2 global symmetry
- Simplest theory with multiple breaking patterns
- How to even physically characterize patterns?
  - Not possible, gauge-dependent
  - Can any pattern be forced by gauge-fixing?

Different relevant quantities

- Global symmetry: Order parameter

$$\left\langle \left(\sum_x det\,\phi(x) \right)^2 \right\rangle$$

Different relevant quantities

- Global symmetry: Order parameter

$$\left\langle \left(\sum_x det\, \phi(x) \right)^2 \right\rangle$$

- Local symmetry: Characterize Higgs vev

$$\phi^g = v\left(\lambda^3 \cos\omega + \lambda^8 \sin\omega \right)$$

Different relevant quantities

- Global symmetry: Order parameter

$$\langle \left(\sum_x det\, \phi(x) \right)^2 \rangle$$

- Local symmetry: Characterize Higgs vev

$$\phi^g = v\left(\lambda^3 \cos\omega + \lambda^8 \sin\omega \right)$$

  - Angle and vev give gauge boson masses

Different relevant quantities

- Global symmetry: Order parameter

$$\left\langle \left(\sum_x det\, \phi(x) \right)^2 \right\rangle$$

- Local symmetry: Characterize Higgs vev

$$\phi^g = v\left(\lambda^3 \cos\omega + \lambda^8 \sin\omega\right)$$

  - Angle and vev give gauge boson masses
  - No gauge transformations connects Cartans
    - Cannot be chosen arbitrarily by gauge fixing
    - Only special angles give SU(2)xU(1)
    - All others give U(1)xU(1)

Different relevant quantities

- Global symmetry: Order parameter

$$\left\langle \left(\sum_x det\, \phi(x) \right)^2 \right\rangle$$

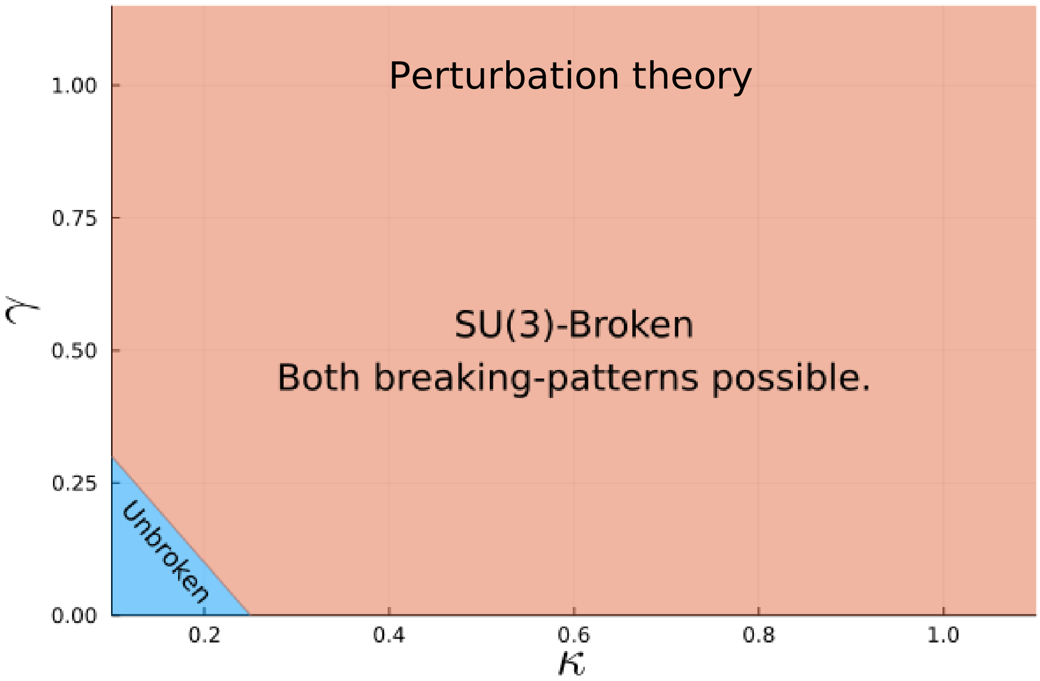
- Local symmetry: Characterize Higgs vev

$$\phi^g = v\left(\lambda^3 \cos\omega + \lambda^8 \sin\omega \right)$$

  - Angle and vev give gauge boson masses
  - No gauge transformations connects Cartans
    - Cannot be chosen arbitrarily by gauge fixing
    - Only special angles give SU(2)xU(1)
    - All others give U(1)xU(1)
- Possible vevs fixed by dynamics

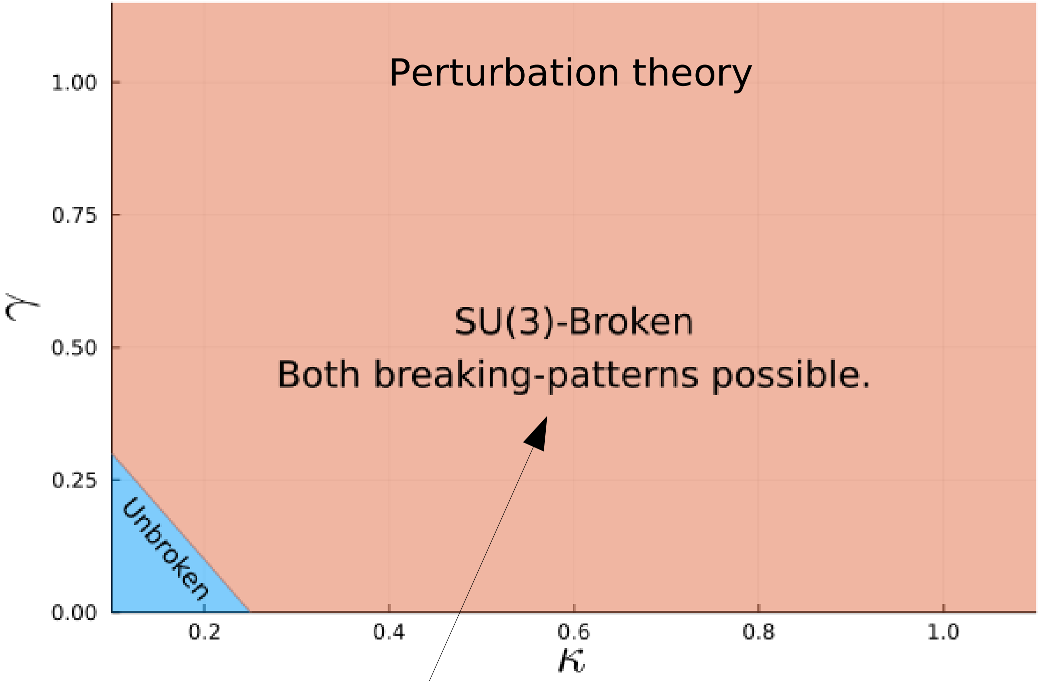
Expectations vs. reality

[Dobson et al.'21, in prep]



Expectations vs. reality

[Dobson et al.'21, in prep]

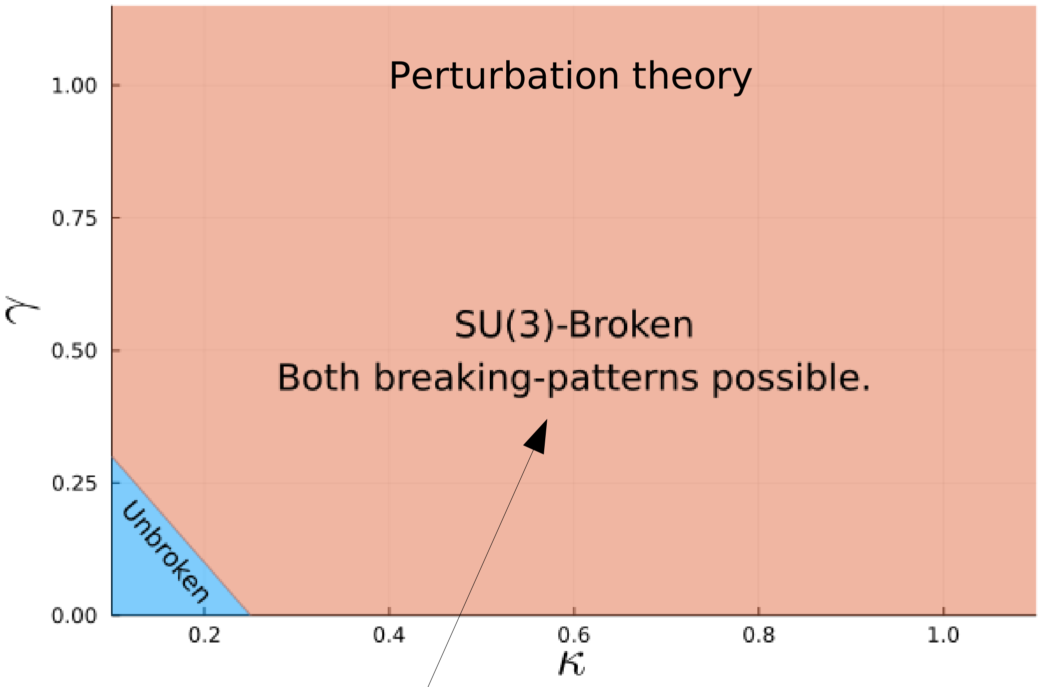


Fixed value of ω
by gauge condition

Expectations vs. reality

[Dobson et al.'21, in prep]

Modification at loop order

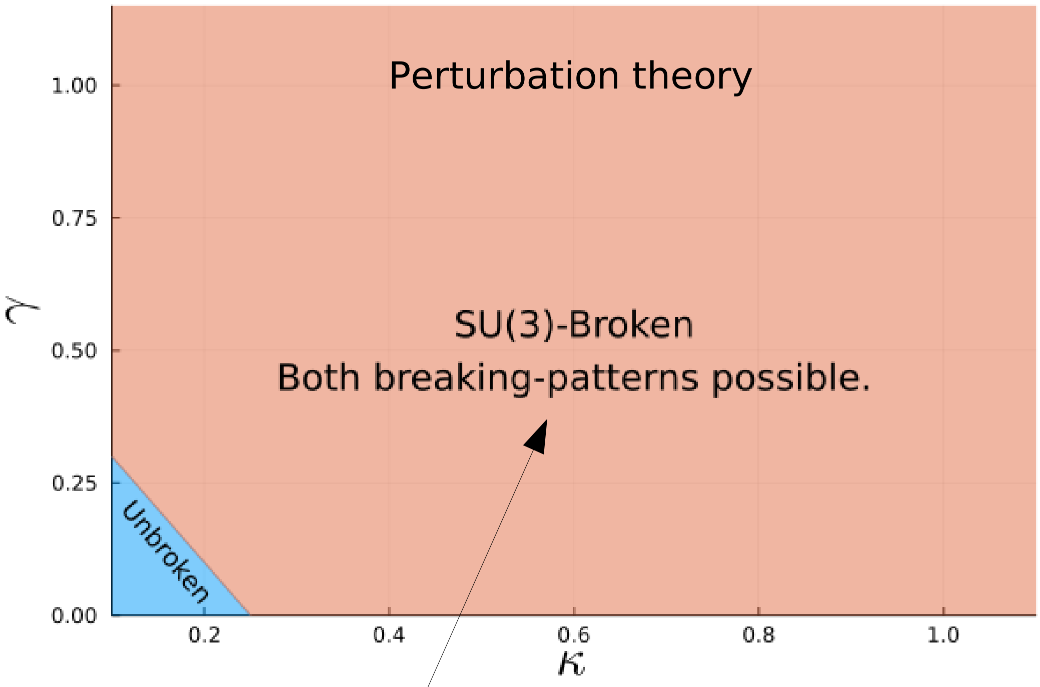


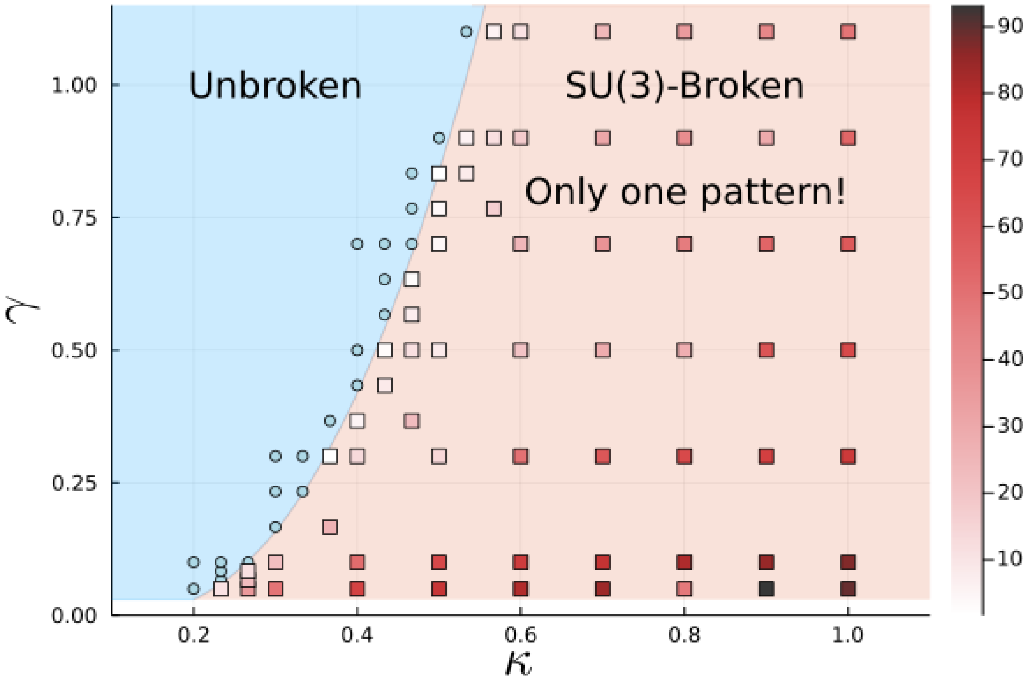
Fixed value of ω
by gauge condition

Expectations vs. reality

[Dobson et al.’21, in prep]

Modification at loop order



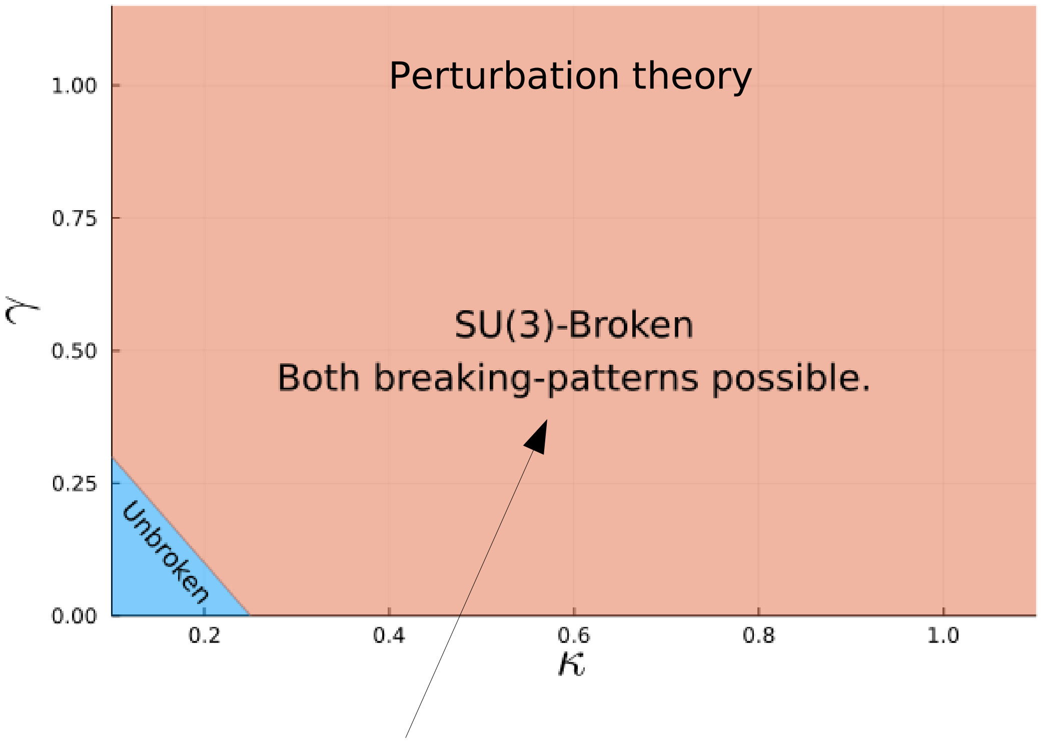


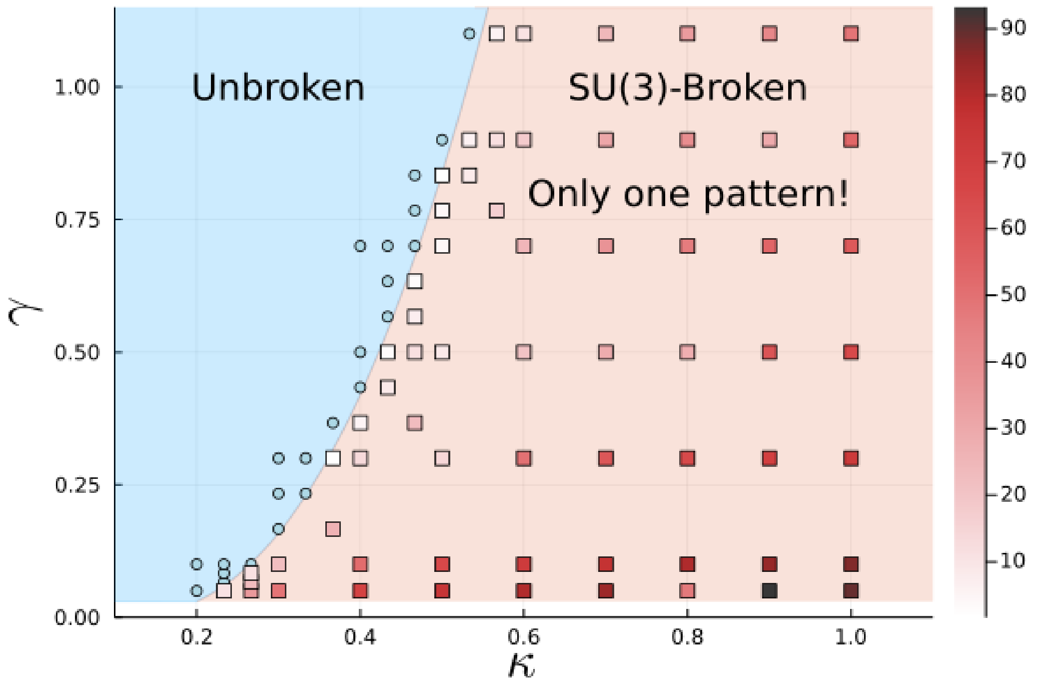
Fixed value of ω
by gauge condition

Expectations vs. reality

[Dobson et al.’21, in prep]

Modification at loop order





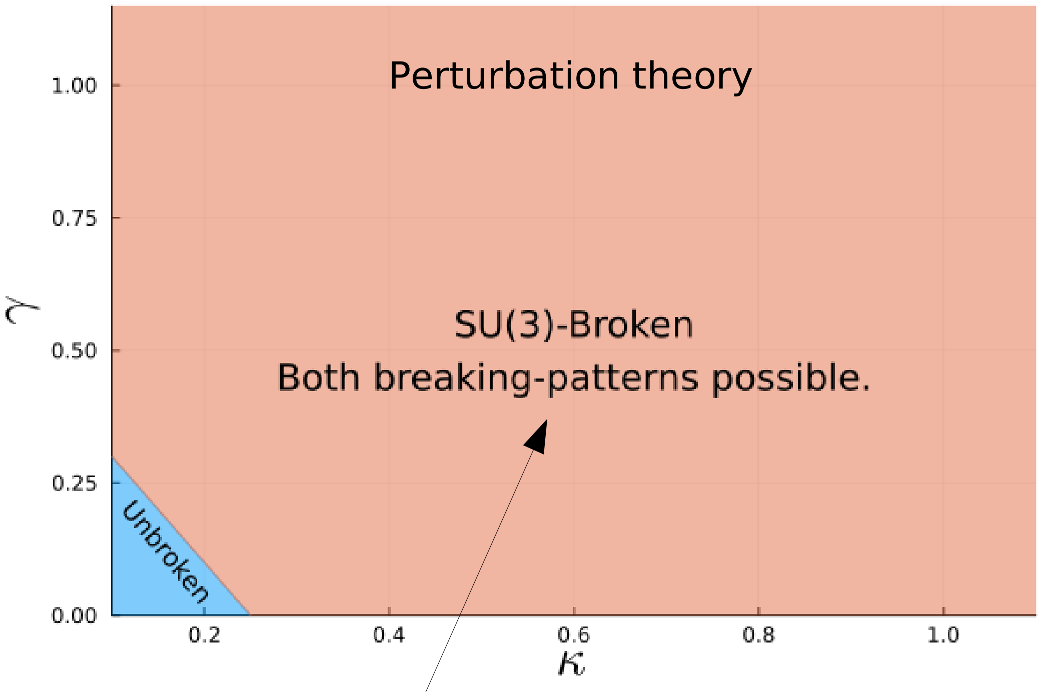
Fixed value of ω
by gauge condition

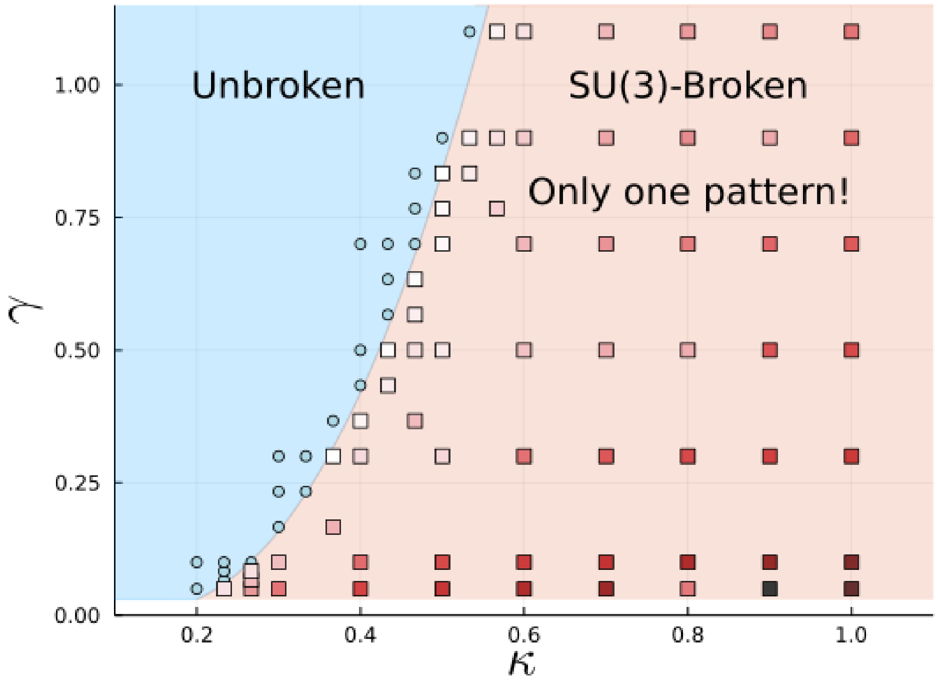
Depends on gauge –
in certain gauges there is
always a BEH effect!

Expectations vs. reality

[Dobson et al.’21, in prep]

Modification at loop order





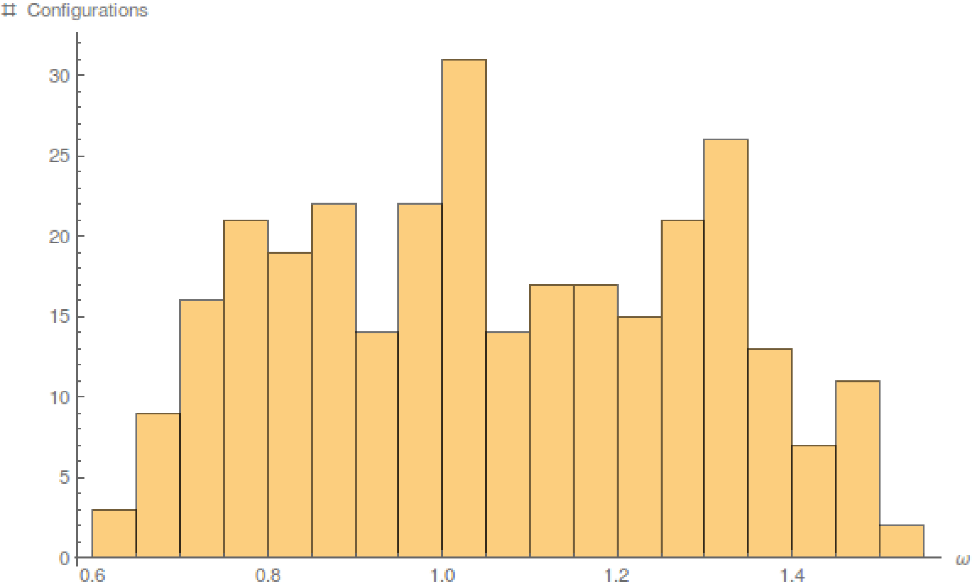
Fixed value of ω
by gauge condition

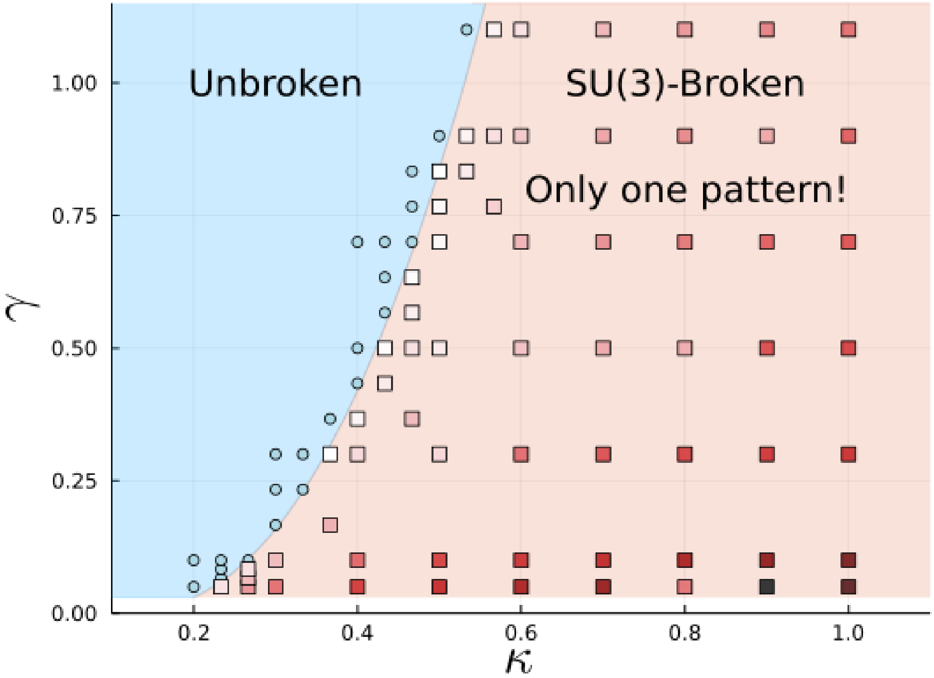
Depends on gauge –
in certain gauges there is
always a BEH effect!

“Gauge as perturbative
as possible”

Expectations vs. reality

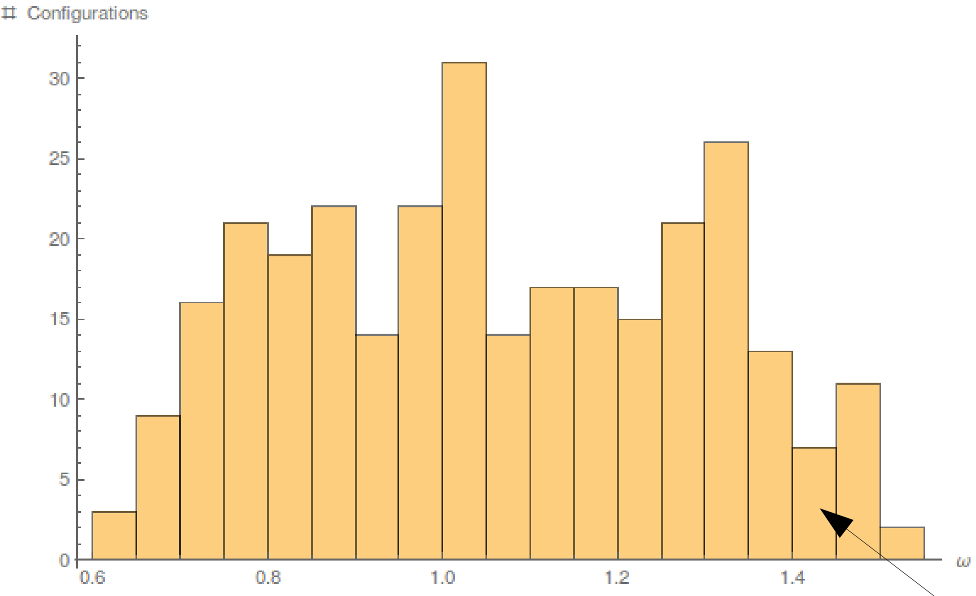
[Dobson et al.’21, in prep]

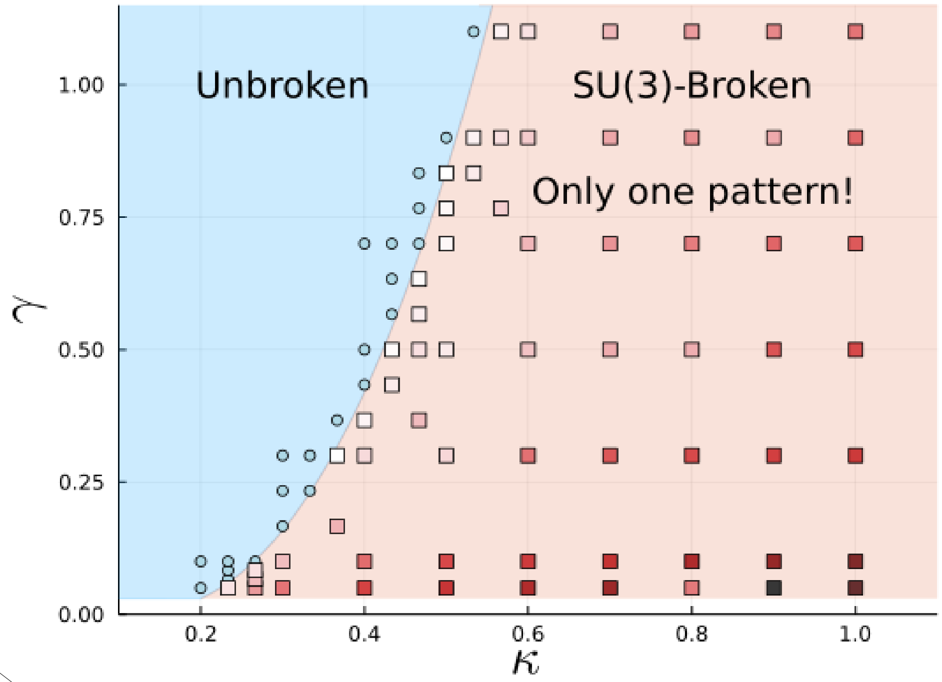




Expectations vs. reality

[Dobson et al.’21, in prep]

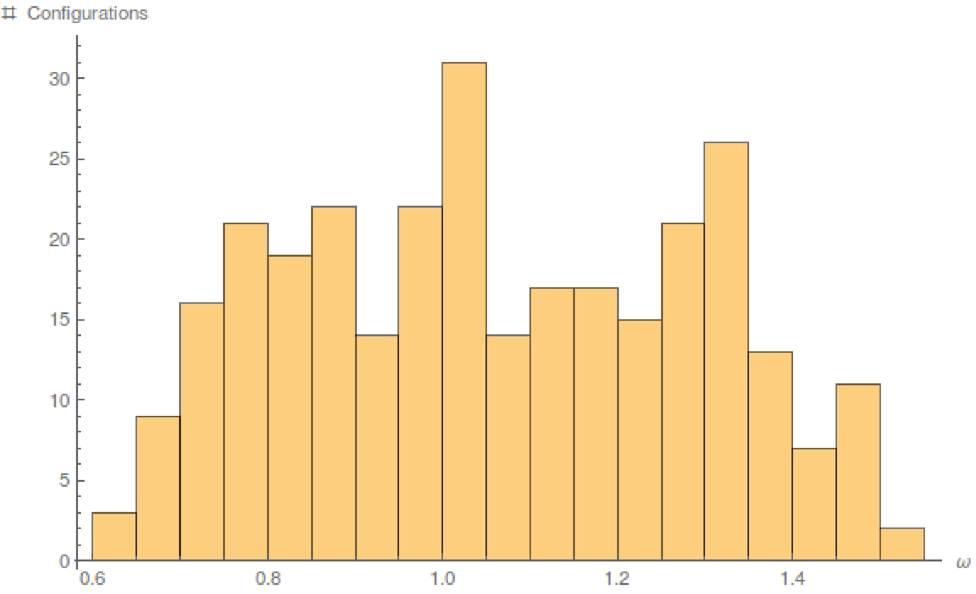


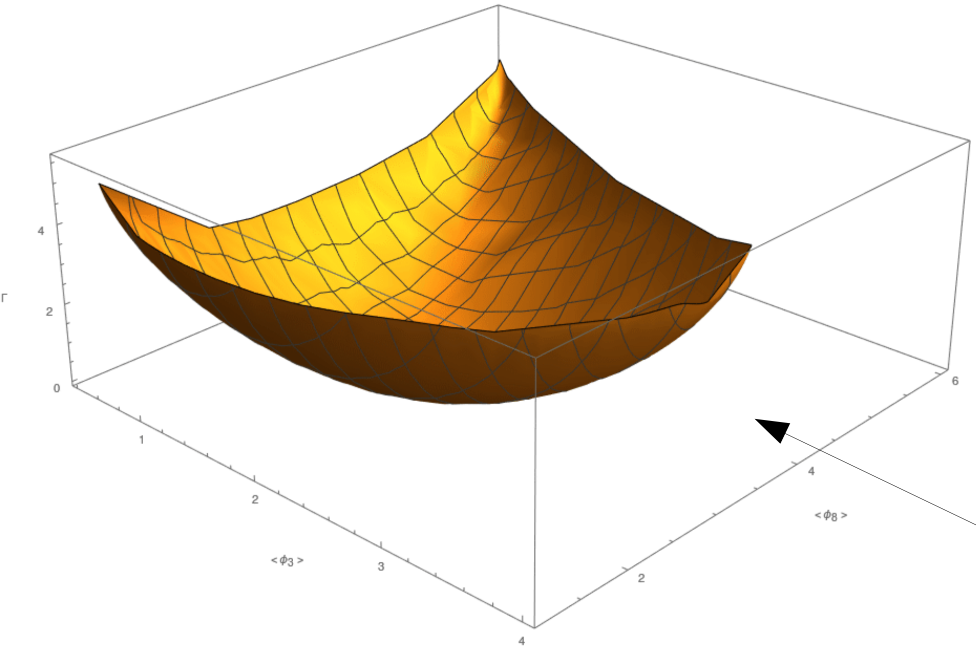


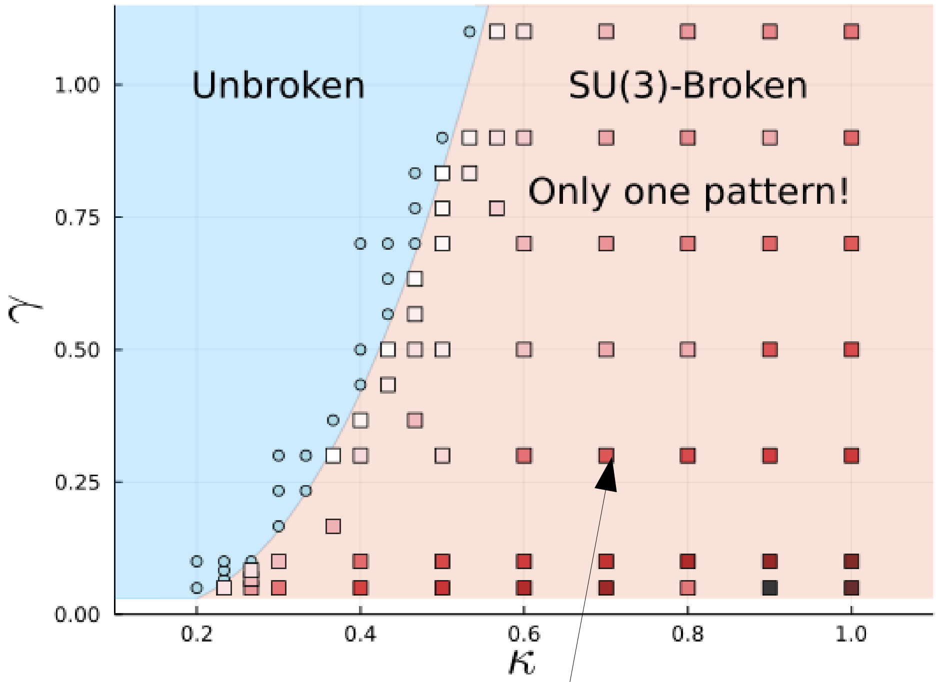
Every configuration has its “own” breaking direction, and cannot be forced to the perturbative one!

Expectations vs. reality

[Dobson et al.’21, in prep]







Unbroken

SU(3)-Broken

Only one pattern!

Averaging yields a quantum effective potential with unique minimum, giving overall pattern

Consequences

- Relation between parameters and breaking pattern may be different than in perturbation theory
  - Different spectrum
    - Both in perturbation theory and FMS

Consequences

- Relation between parameters and breaking pattern may be different than in perturbation theory
  - Different spectrum
    - Both in perturbation theory and FMS
- Spectrum predicted differently by FMS
  - As in the fundamental case

Consequences

- Relation between parameters and breaking pattern may be different than in perturbation theory
  - Different spectrum
    - Both in perturbation theory and FMS
- Spectrum predicted differently by FMS
  - As in the fundamental case
- Future: Needs to be checked
  - Difficult due to massless vector bosons

Bottom line for GUTs

Bottom line for GUTs

- Toy models representative for mechanisms

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
  - Different spectrum: Different phenomenology

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
    - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
  - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates: **All checked failed** [Maas et al.'17,Sondenheimer'19]
  - SU(5), SO(10), Pati-Salam,...

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
  - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates: **<u>All checked failed</u>** [Maas et al.'17,Sondenheimer'19]
  - SU(5), SO(10), Pati-Salam,...
    - None found so far that works

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
    - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates: **All checked failed** [Maas et al.'17,Sondenheimer'19]
    - SU(5), SO(10), Pati-Salam,...
        - None found so far that works
    - Depends on Higgs sector

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
  - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates: **All checked failed** [Maas et al.’17,Sondenheimer’19]
  - SU(5), SO(10), Pati-Salam,...
    - None found so far that works
  - Depends on Higgs sector
    - May still be possible

Bottom line for GUTs

- Toy models representative for mechanisms
- All results so far inconsistent with perturbation theory
- Consistent with FMS construction
  - Different spectrum: Different phenomenology
- Application of FMS to GUT candidates: **All checked failed** [Maas et al.'17,Sondenheimer'19]
  - SU(5), SO(10), Pati-Salam,...
    - None found so far that works
  - Depends on Higgs sector
    - May still be possible (hopefully?)

Summary: GUTs are different

- Gauge invariance is central

Summary: GUTs are different

- Gauge invariance is central
- Qualitative different results in GUTs
  - Invalidates most scenarios
  - SM special

Review: 1712.04721

Summary: GUTs are different

- Gauge invariance is central
- Qualitative different results in GUTs
    - Invalidates most scenarios
    - SM special
- Augmentation with FMS a possibility
    - Lattice tests successful

Summary: GUTs are different

- Gauge invariance is central
- Qualitative different results in GUTs
  - Invalidates most scenarios
  - SM special
- Augmentation with FMS a possibility
  - Lattice tests successful
- Nature of most particles different

@axelmaas

Review: 1712.04721

Summary: GUTs are different

- Gauge invariance is central
- Qualitative different results in GUTs
    - Invalidates most scenarios
    - SM special
- Augmentation with FMS a possibility
    - Lattice tests successful
- Nature of most particles different
    - Extension to fermions impacts experiments

[Maas et al.’21, Afferrante et al.’20, Fernbach et al.’20, Egger et al.’17]

Review: 1712.04721