Electroweak Pion decay in the Bethe-Salpeter approach

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Bound states in QCD and beyond
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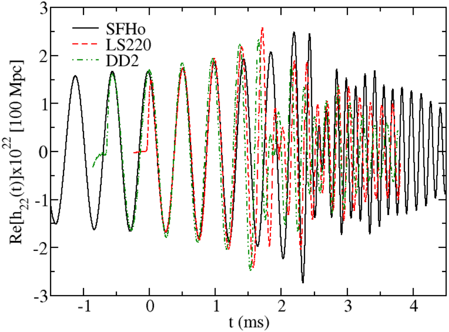
Der Wissenschaftsfonds.

Motivation

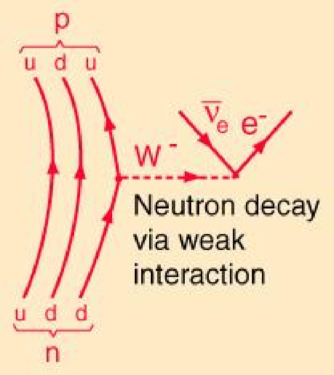
- System of binary neutron stars mergers
- Source of gravitational waves
- Very high neutrino flux
- Very dense matter $\Rightarrow$ opaque for neutrinos
- Back coupling of neutrinos influences gravitational waves
- Measurement shows the inner structure of neutron star mergers
- Electroweak interactions play an important role
- Consider QCD + electroweak interactions non-perturbatively



(Foucart et al. arXiv:1510.06398v2 [astro-ph],
Rosswog et al., Mon. Not. Roy. Astron. Soc. 342, 673 (2003),
Y. Sekiguchi et al., PRL 107, 051102 (2011),
J. A. Faber et al., Living Rev. Rel. 15, 8 (2012),
D. Neilsen et al., PRD 89, 104029 (2014),
C. Palenzuela et al., PRD 92, 044045 (2015),
O. L. Caballero arXiv:1603.02755 [nucl-th], . . .)

β-decay

- Full resolution of electroweak interactions is complicated
- β-decay captures the main features
- Look at the $\pi^{\pm}$-decay
- Electroweak interactions approximate by 4-Fermi-interaction
- Electroweak interactions violates parity
- Different energy scales + parity violation $\Rightarrow$ Lattice calculation unfeasible
- Functional methods (DSE, BSE, FRG, ...) prime candidate



http://hyperphysics.phy-astr.gsu.edu/hbase/particles/proton.html

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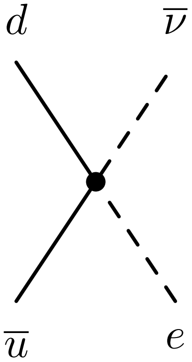


Ansatz

$$
\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{4-Fermi}} + \mathcal{L}_{e,\text{free}} + \mathcal{L}_{\nu,\text{free}} \\
=& \overline{\psi}_u \left[-\partial\!\!\!/ + m_u \right] \psi_u + \overline{\psi}_d \left[-\partial\!\!\!/ + m_d \right] \psi_d + \\
& g_s \overline{\psi}_u A\!\!\!/^i T^i \psi_u + g_s \overline{\psi}_d A\!\!\!/^i T^i \psi_d + \\
& \frac{1}{2} g_w \left\{ \left[\overline{\psi}^L_\nu \gamma^\mu \psi^L_e \right] \left[\overline{\psi}^L_u \gamma^\mu \psi^L_d \right] + \left[\overline{\psi}^L_e \gamma^\mu \psi^L_\nu \right] \left[\overline{\psi}^L_d \gamma^\mu \psi^L_u \right] \right\} + \\
& \overline{\psi}_e \left[-\partial\!\!\!/ + m_e \right] \psi_e + \overline{\psi}_\nu \left[-\partial\!\!\!/ + m_\nu \right] \psi_\nu + \mathcal{L}_{\text{Rest}} \\
\psi^L =& \frac{1}{2} (\mathbb{1} - \gamma^5) \psi
\end{aligned}
$$

- Numerical control via Masses m_e, m_ν and effective weak strength g_w

DSEs and BSEs

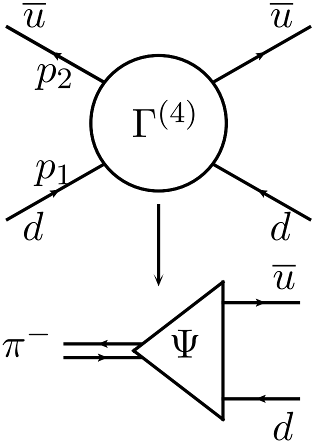
- BSEs: Bound state equations derived from DSEs and evaluated on the pole.
- Total momenta

$$P = p_1 - p_2$$

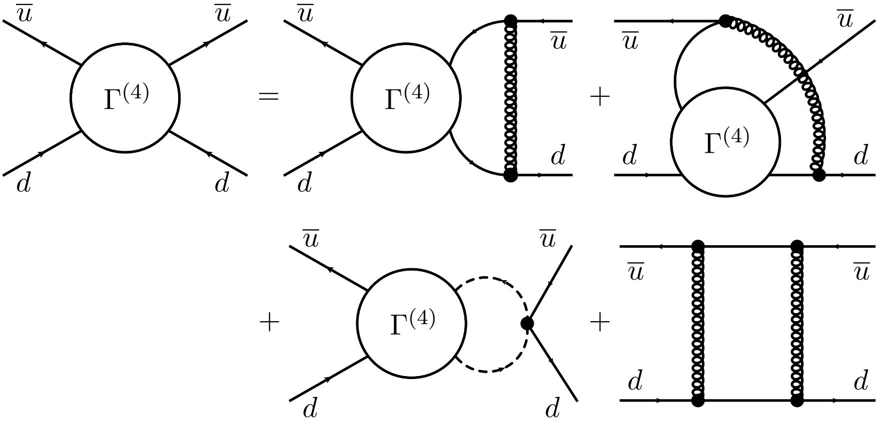
- At the pole M_{Pole}

$$\Gamma^{(4)} \propto \mathcal{N} \frac{\Psi\overline{\Psi}}{P^2 + M^2_{\text{Pole}}}$$

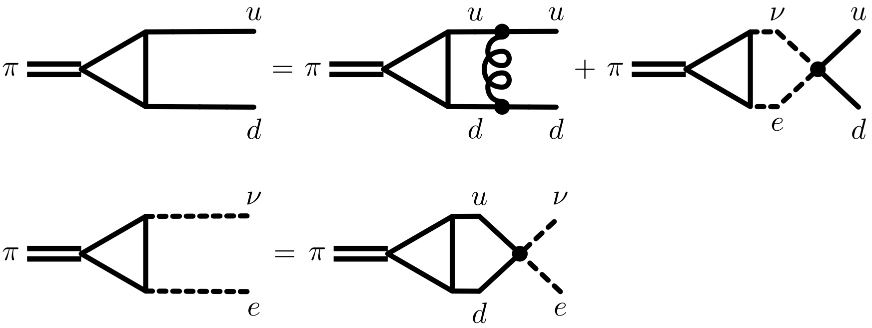
- Resonances: Pole in the 2nd Riemann sheet (Haag, Local Quantum Physics Fields, Particles, Algebras)



DSEs and BSEs



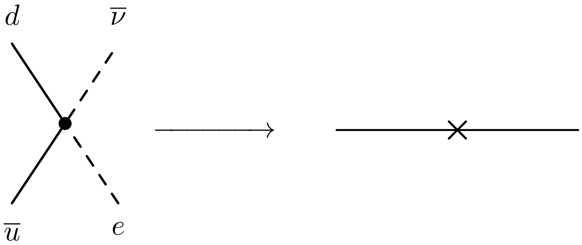
Coupled system of BSEs



- Solve the system self-consistent
- Needs the non-perturbative Quark propagator as input

Influence of broken C, P and flavor symmetry

- No results on non-perturbative backcoupling of C and P violation
- First: Investigate the effects of C and P violation on the simplest object: Quark propagator
- Analyse influence through explicit breaking term (A. Maas & W. Mian, EPJA (2017) **53**: 22 , arxiv:1611:08130)
- Couple with a reservoir of leptons



- Symmetry breaking $\Rightarrow$ More involved tensor structure
- Quark propagator from flavor A to flavor B

$$P_{AB}(p^2) = \tilde{A}_{AB}(p^2)\,\mathrm{i}\,\not{p} + \tilde{B}_{AB}(p^2)\mathbb{1} + \tilde{C}_{AB}(p^2)\,\mathrm{i}\,\not{p}\gamma^5 + \tilde{D}_{AB}(p^2)\gamma^5$$

Tree-level Propagator

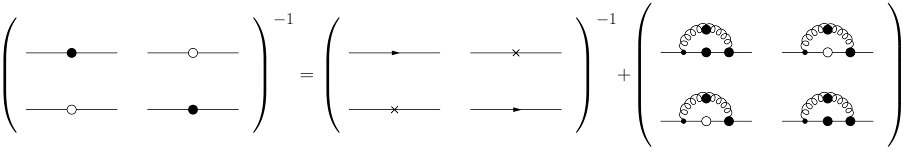
$$P_{0,uu}(p^2) = \frac{1}{N(p^2)} \left[(m_d^2 + (1 - 2g_{\mathrm{w}}^2)p^2)\, \mathrm{i}\, \not{p} + m_u(m_d^2 + p^2)\mathbb{1} + 2g_{\mathrm{w}}^2 p^2\, \mathrm{i}\, \not{p}\gamma^5 \right]$$

$$P_{0,ud}(p^2) = \frac{g_{\mathrm{w}}}{N(p^2)} \left[(m_u m_d - p^2)\, \mathrm{i}\, \not{p} - (m_u + m_d)p^2\mathbb{1} - (m_u m_d + p^2)\, \mathrm{i}\, \not{p}\gamma^5 \right] - \frac{g_{\mathrm{w}}(m_u - m_d)p^2}{N(p^2)}\gamma^5$$

$$N(p^2) = m_d^2 m_u^2 + (m_u^2 + m_d^2)p^2 + (1 - 4g_{\mathrm{w}}^2)p^4$$

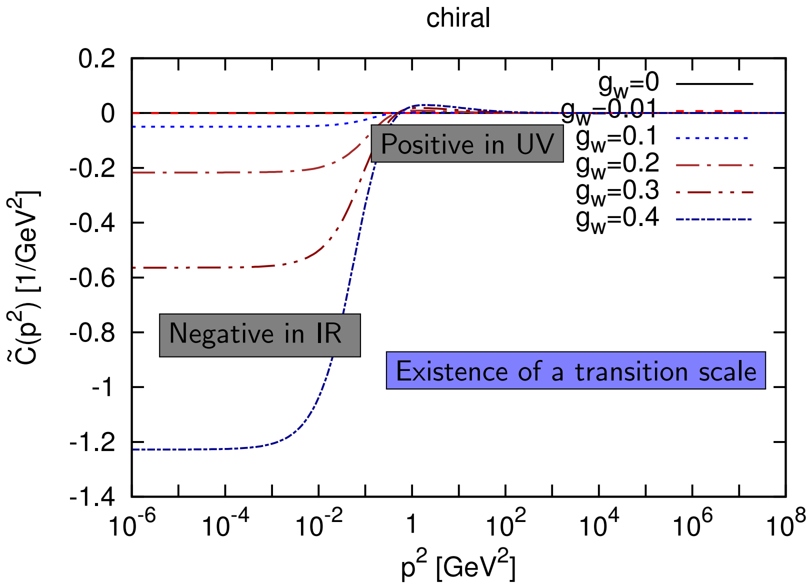
- Pseudo scalar channel of the mixed propagator (tree-level) is proportional to mass splitting
- $\Rightarrow$ All channels of full propagator have contributions from mass splitting

DSEs

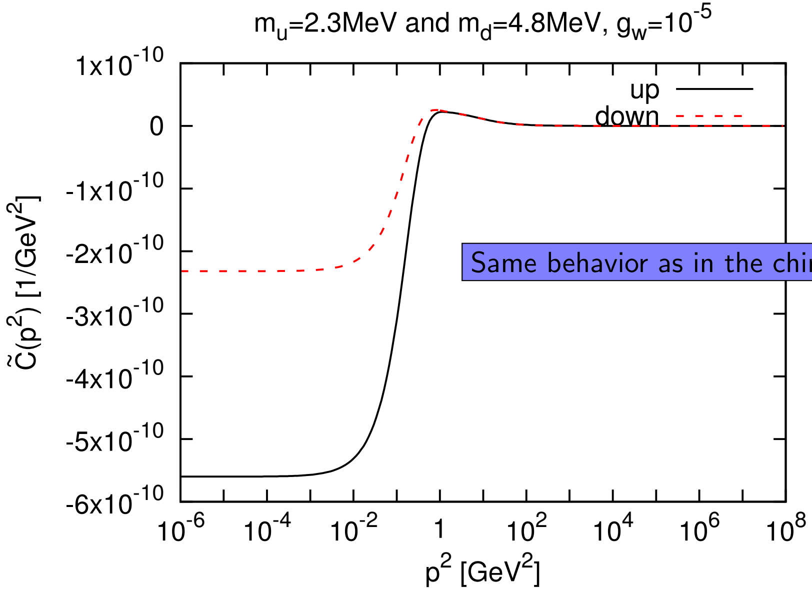


- Weak interaction: Non-vanishing off diagonal elements
- Rainbow-Ladder-Truncation
- Effective coupling: Maris-Tandy (P. Maris and P. C. Tandy, PRC 60, 055214 (1999))
- Important features like dynamical mass generation and chiral symmetry particularly well implemented (R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001), C. S. Fischer J. Phys. G32, R253 (2006), C. D. Roberts, J. Phys. Conf. Ser. 706, 022003 (2016))

Axial channel

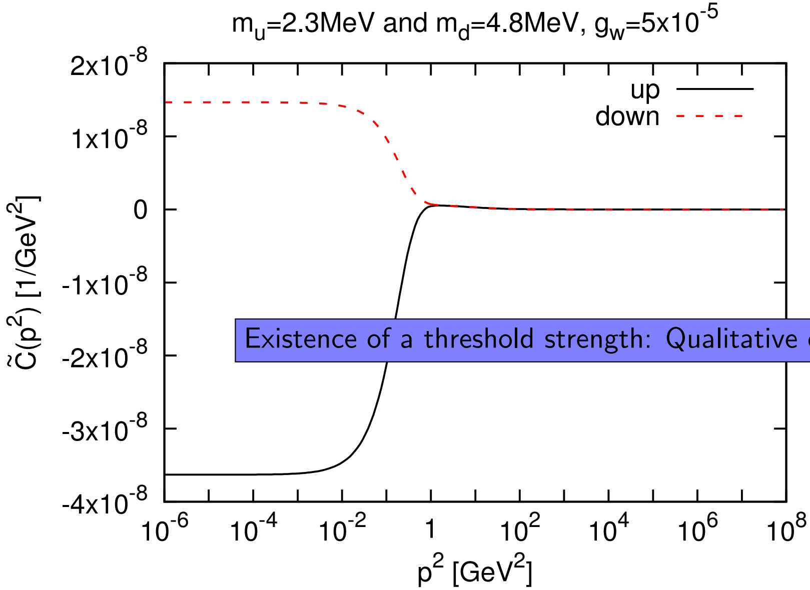


Axial channel



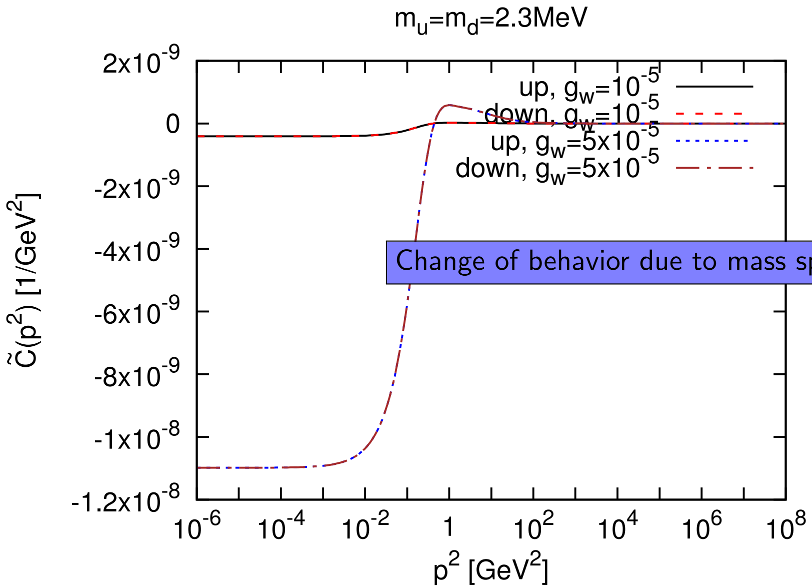
Same behavior as in the chiral limit

Axial channel



Existence of a threshold strength: Qualitative change

Effects of Mass Splitting on Axial channel



Change of behavior due to mass splitting

Right and Left Handed Projection

- Contributions from quark propagators to left $\tilde{L}$ and right handed $\tilde{R}$:

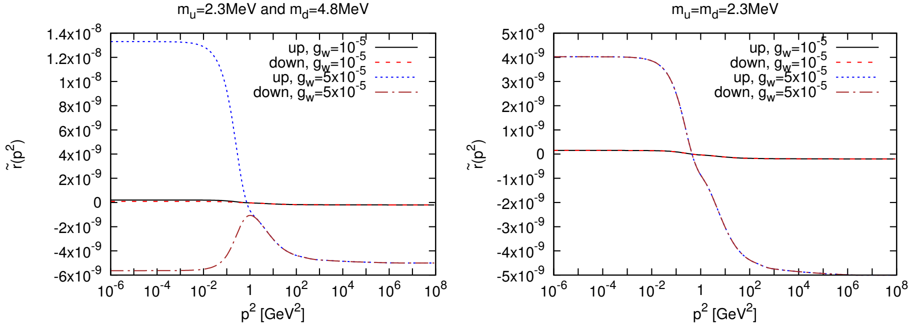
$$\begin{aligned}\tilde{L}_{AB} =& \tilde{A}_{AB} - \tilde{C}_{AB} \left(\propto \gamma^{\mu}(\mathbb{1} - \gamma^5)\right)\\ \tilde{R}_{AB} =& \tilde{A}_{AB} + \tilde{C}_{AB} \left(\propto \gamma^{\mu}(\mathbb{1} + \gamma^5)\right)\end{aligned}$$

- $\tilde{r}$: relative ratio for left handed to right handed contributions

$$\tilde{r}_{AB}(p^2) = \frac{\tilde{L}_{AB}(p^2) - \tilde{R}_{AB}(p^2)}{\tilde{L}_{AB}(p^2) + \tilde{R}_{AB}(p^2)} = -\frac{\tilde{C}_{AB}(p^2)}{\tilde{A}_{AB}(p^2)}$$

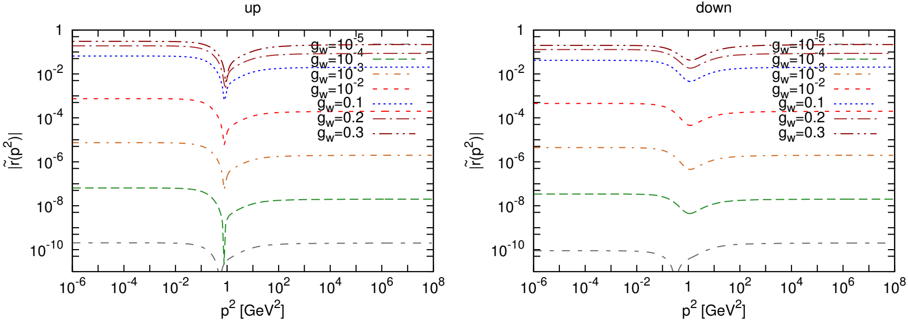
- More left handed or right handed contributions related to sign of $\tilde{C}$ for pure flavor quark propagators ($\tilde{A}$ always positive)

Parity Violation



- Below the threshold strength
  1. Dominantly right handed in the UV
  2. Dominantly left handed in the IR
- Above the threshold strength
  1. No qualitative change for up quark
  2. Change for down quark: Dominantly right handed in UV and IR
- Absolute value for the ratio is increased due to mass splitting

Parity Violation



- Absolute value is increased by two order of magnitude, when g_w is increased by one order of magnitude

Conclusion

- Consider QCD and electroweak interactions non-perturbative
- Goal:
  1. Dynamical decay process in functional approach
  2. β-decay in neutron stars
- First step: Dynamical weak Pion decay
- Influence of broken C, P and flavor symmetry at the level of the quark propagator can be studied through explicit breaking term
  1. Non-perturbative: Amplification of the backcoupling
  2. **Caution with perturbative extrapolation**
- Self-consistent backcoupling at the level of the Pion
- Results for the Pion on the way

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Thank you for your attention.