The size of the W-boson

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Basic Idea

[1] All background on the topic can be found in: A. Maas, Brout-Englert-Higgs physics: From foundations to phenomenology (1712.04721 [hep-ph]; December 2017)

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- Elementary gauge fields are not gauge invariant $\rightarrow$ vanishing expectation values

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Basic Idea

- Elementary gauge fields are not gauge invariant $\rightarrow$ vanishing expectation values
- Different Approach: Consider the W-boson as a gauge invariant composite operator. [1]

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- For the mass spectrum of the standard model perturbation theory and the approach of gauge invariant composite operators yield the same results. [1]
- We want to get some more dynamical results where perturbation theory and the approach of gauge invariant composite operators differ, i.e. the radius, scattering processes...

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- No local $\mathrm{U}(1)$ hypercharge symmetry.

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Remarks on the custodial symmetry:

- Acts only on the Higgs.
- Can be made explicit by writing the Higgs as $\mathrm{SU}(2)$ matrix:

$$X(x) = \begin{pmatrix} \phi_1(x) & -\phi_2(x)^* \\ \phi_2(x) & \phi_1(x)^* \end{pmatrix}$$

Gauge invariance and composite operators

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Different Approach: Composite Operators

- No open gauge index $\rightarrow$ Non vanishing expectation value
- The W-boson is a triplet vector boson $\rightarrow$ We need a triplet vector gauge invariant composite operator
- The considered composite operator is

$$O^{\bar{a}}_{1_3^- \ \mu}(x) = tr\left[\tau^{\bar{a}} \frac{X(x)^\dagger}{\sqrt{\det\left(X(x)\right)}} D_\mu(x) \frac{X(x)}{\sqrt{\det\left(X(x)\right)}}\right] \quad . \tag{3}$$

The Radius

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$$-6\hbar \frac{\mathrm{d}R(q^2)}{\mathrm{d}q^2}\bigg|_{q^2=0} = \left\langle r^2 \right\rangle \tag{4}$$

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- The Lorentz structure of the three point function consists of 14 tensor objects.
- Gauge invariant perturbation theory[1] suggests to choose the tree-level tensor object.
- This yields for the form factor:

$$R(p,q,k) = \frac{\Gamma^{\bar{a}\bar{b}\bar{c}}_{\mu\nu\rho}(p,q,k)\,\left\langle O^{\bar{a}}_{\mu\,1_3^-}(p)\,O^{\bar{b}}_{\nu\,1_3^-}(q)\,O^{\bar{c}}_{\rho\,1_3^-}(k)\right\rangle}{\Gamma^{\bar{a}\bar{b}\bar{c}}_{\mu\nu\rho}(p,q,k)\,D^{\bar{a}\bar{d}}_{\mu\gamma}(p)\,D^{\bar{b}\bar{e}}_{\nu\eta}(q)\,D^{\bar{c}\bar{f}}_{\rho\zeta}(k)\,\Gamma^{\bar{d}\bar{e}\bar{f}}_{\gamma\eta\zeta}(p,q,k)} \quad . \tag{5}$$

Lattice technology

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The lattice setup is a 4 dimensional isotropic hypercubic lattice with lattice constant a and a lattice volume of $V = L^4$.

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The sets of parameters are:

A: Inverse lattice spacing: 435 GeV; Mass of the Higgs: $m_{0+} = 124$ GeV; coulping constant: $\alpha = 0.605$

B: Inverse lattice spacing: 335 GeV; Mass of the Higgs: $m_{0+} = 122$ GeV; coulping constant: $\alpha = 0.506$

C: Inverse lattice spacing: 255 GeV; Mass of the Higgs: $m_{0+} = 118$ GeV; coulping constant: $\alpha = 0.211$

D: Inverse lattice spacing: 151 GeV; Mass of the Higgs: $m_{0+} = 131$ GeV; coulping constant: $\alpha = 0.558$

The Higgs mass from the experiment is approximately 125 GeV.

Lattice results for the ratio

The following plots are the plots for the elementary and the composite vertex on a lattice with size $L = 20$

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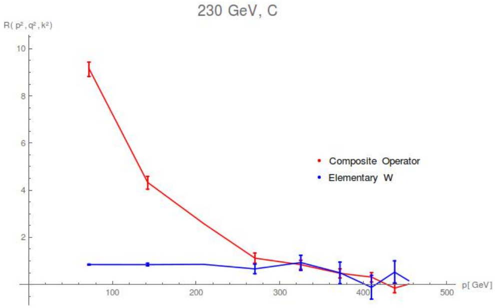

Determination of the Size

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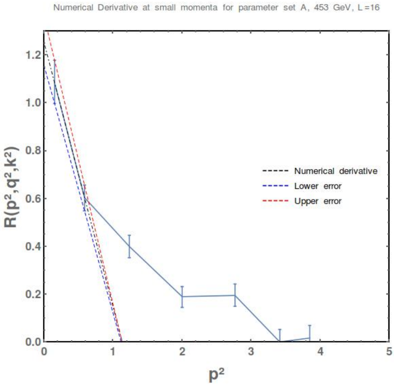
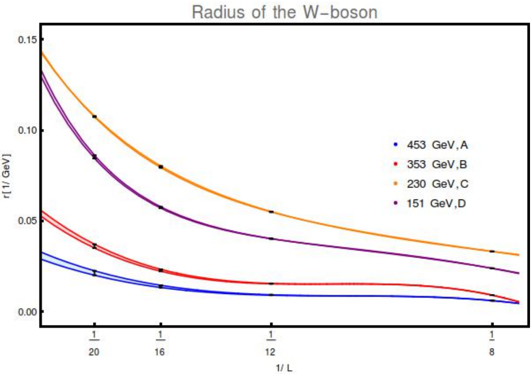
For the determination of the size the numerical derivative at small momenta was taken. This is a plot for the composite operator.



Values of the Radius of the composite operator in the continuum limit 1



Values of the radius of the composite operator in the continuum limit 2

Parameter set	Radius in [1/GeV]	Upper error in [1/GeV]	Lower error in [1/GeV]
A, 353 GeV	0.113	+0.008	-0.007
B, 253 GeV	0.222	+0.006	-0.006
C, 230 GeV	0.8	+0.2	-0.2
D, 151 GeV	1.22	+0.04	-0.11

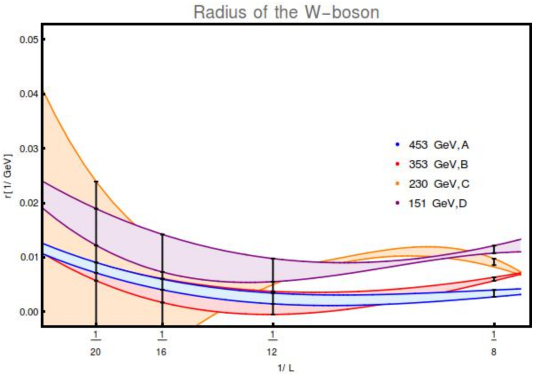
Table : Values of the radius of the composite operator in [1/GeV]

Values of the radius of the composite operator in the continuum limit 3

Parameter set	Radius in [fm]	Upper error in [fm]	Lower error in [fm]
A, 353 GeV	$2.2 * 10^{-2}$	$+1 * 10^{-3}$	$-1 * 10^{-3}$
B, 253 GeV	$4.3 * 10^{-2}$	$+1 * 10^{-3}$	$-1 * 10^{-3}$
C, 230 GeV	$1.6 * 10^{-1}$	$+4 * 10^{-2}$	$-4 * 10^{-2}$
D, 151 GeV	$2.41 * 10^{-1}$	$+8 * 10^{-3}$	$-2.2 * 10^{-2}$

Table : Values of the radius of the composite operator in [fm]

Values of the radius of the elementary fields in the continuum limit 1



Values of the radius of the elementary fields in the continuum limit 2

Parameter set	Radius in [1/GeV]	Upper error in [1/GeV]	Lower error in [1/GeV]
A, 353 GeV	0.036	+0.001	-0.001
B, 253 GeV	0.031	+0.001	-0.019
C, 230 GeV	0	+0.2	-0.008
D, 151 GeV	0.06	+0.01	-0.01

Table : Values of the radius of the elementary fields in [1/GeV]

Values of the Radius of the elementary fields in the continuum limit 3

Parameter set	Radius in [fm]	Upper error in [fm]	Lower error in [fm]
A, 353 GeV	$7.1 * 10^{-3}$	$+2 * 10^{-4}$	$-2 * 10^{-4}$
B, 253 GeV	$6.1 * 10^{-3}$	$+2 * 10^{-4}$	$-3 * 10^{-3}$
C, 230 GeV	0	$+4 * 10^{-2}$	$-2 * 10^{-2}$
D, 151 GeV	$1.3 * 10^{-2}$	$+3 * 10^{-3}$	$-3 * 10^{-3}$

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- The radius of the W-boson is much bigger than the radius of the electron($\approx 10^{-5}$fm).
- The radius of the elementary field is $\approx 10^{-1}$ smaller than the radius of the composite operator.

Outlook

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- One could calculate radii for different tensor structures, this approach is not restricted to tree-level.
- One could calculate the anomalous gauge coupling from these results.
- There is no experimental data on the radius of the gauge bosons of the weak interaction.

The End